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Mathematical modelling and methodology for cost optimization of variable renewable electricity integration

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Abstract

The global production of electricity contributes significantly to the release of carbon dioxide emissions. Therefore, a transformation of the electricity system is of vital importance in order to restrict global warming. This thesis concerns modelling and methodology of an electricity system which contains a large share of variable renewable electricity generation, such as wind and solar power.

The models developed in this thesis concern optimization of long-term investments in the electricity system. They aim at minimizing investment and production costs under electricity production constraints, using different spatial resolutions and technical detail, while meeting the electricity demand. Furthermore, they are able to capture some of the variation management strategies necessary for electricity systems that include a large share of variable renewable electricity. These models are very large in nature due to the high temporal resolution needed to capture the wind variations, and thus different decomposition methods are applied to reduce solution times. We develop two different decomposition methods: 1) Lagrangian relaxation combined with variable splitting solved using a subgradient algorithm, and 2) a heuristic decomposition approach using a consensus algorithm. In both cases, the decomposition is done with respect to the temporal resolution by dividing the year into 2-week periods. The decomposition methods are tested and evaluated for cases involving regions with different energy mixes and conditions for wind power. Numerical results show faster computation times compared to the non-decomposed models and capacity investment options similar to the optimal solutions given by the latter models.

Keywords: variable renewable electricity, variation management, electricity system modelling, long-term investment models, cost optimization, wind power integration, Lagrangian relaxation, variable splitting, consensus algorithm.

List of publications

This thesis is based on the work represented by the following papers:

I. **Granfeldt, C.**, Strömberg, A.-B., Göransson, L. (2021)
Managing the temporal resolution in electricity system investment models with a large share of wind power: An approach using Lagrangian relaxation and variable splitting.
Manuscript

II. **Göransson, L.**, Granfeldt, C., Strömberg, A.-B. (2021)
Management of Wind Power Variations in Electricity System Investment Models – A Parallel Computing Strategy.
SN Operations Research Forum, ***2**:25*, doi: 10.1007/s43069-021-00065-0.
url: `https://link.springer.com/article/10.1007/s43069-021-00065-0`

Author contributions

I. I worked on modeling, method development, implementation and writing the manuscript.

II. I worked on modeling and writing the manuscript.

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Caroline Granfeldt, Gothenburg 2021

Contents

1 Introduction

EU's roadmap 2050 establishes that the greenhouse gas emissions must decrease by some 85% until the year 2050 in order for global warming to be restricted to 2°C (COM(2011) 885, 2011). The electricity system contributes significantly to the emissions of carbon dioxide, both in the EU and globally (see for example Ritchie and Roser (2020) and underlying data sources). A transformation of the electricity system is therefore needed, and electricity investment models can be used as a tool to make informed decisions regarding future electricity generation, storage and transmission capacity. The mathematical optimization models describing the electricity system minimizes the investment and production costs of the system under electricity production constraints, while meeting the electricity demand.

The existing systems mostly consist of thermal power (IEA, 2020), and thus the traditional models are designed with this in mind. The characterization of such a system, dominated by dispatchable generation, include the ability to regulate the electricity production to meet instant demand. However, a cost-efficient reduction of greenhouse gas emissions from the electricity system is expected to imply a large-scale implementation of varying renewable electricity generation (VRE), such as wind and solar power. To be able to capture the variability in electricity generation from VRE, a realistic mathematical modeling of future electricity systems must include a fine discretization of time (IEA Wind TCP Task 25, 2018). Furthermore, one key strategy to reduce variability of wind power is geographical smoothing through trade. Thus, it is desired to consider a large geographical area in the modeling of the electricity system while accounting for the transmission bottlenecks within such an area.

There is, however, a conflict between a high temporal and spatial resolution and reasonable computing times for electricity system models. For real problem instances on the European scale, the challenge lies in finding this proper balance while introducing a large share of variable renewable electricity generation in the mathematical modelling.

1.1 Purpose and Aims

The purpose of the project is to formulate and analyze mathematical optimization models that capture strategies to manage the variability of variable renewable electricity generation. The research is focused on techniques and methodologies for decomposing and solving long-term investment models, where investments should be made into new electricity generation capacity. A key objective is to examine how the temporal and spatial resolution impacts the solution times using these decomposition methods, and—when possible—compare the solutions and solution times of the decomposed models to the non-decomposed model optimal solutions.

1.2 Limitations

Since the models are used to examine long-term capacity investments over large regions (i.e. entire countries), they are not suited for use to make decisions regarding how separate power plants should operate. Furthermore, electricity transmissions within each region are not considered in detail. Instead, the models use an aggregated continuous electricity generation and storage capacity for each region. The models consider electricity trade between regions, and the intention is that this is to be implemented in the project later on. However, at the moment, trade is accounted for in parts of the results.

We assume a perfect forecast of electricity demand and weather. Thus, there is no stochasticity in the models and they are purely deterministic. Demand and weather profiles in terms of, for example, wind speed and water inflow (from rain and melted ice) to hydropower turbines are based on data from previous years.

Solar power and different storage technologies (e.g. batteries) are currently considered in one of the three models presented in this thesis. Electricity used for heating, and power plants which deliver both electricity and heat (so-called CHP plants, combined heat and power) are not included in any of the models.

1.3 Outline

The remainder of this thesis is organized as follows. In Chapter 2, the electricity system and some of its modelling difficulties are described, along with a review of some previous modelling work. Chapter 3 presents and compares three different mathematical optimization models, including variables, constraints and objective functions. In Chapter 4, the scientific areas and methods used to solve these models are presented. Chapter 5 discusses how the models are implemented and solved using the methods introduced in the previous chapter. A summary of the appended papers are given in Chapter 6, and then finally Chapter 7 discusses conclusions and the main contributions of this thesis, as well as topics for future research.

2 The Electricity system and modelling

This chapter discusses the electricity system and some modelling difficulties that comes with it. It also gives an overview of some previous work done on electricity system modelling.

Electricity is an energy carrier which can be characterized using different properties, e.g. voltage, current, energy or power. In an electricity system, electric energy is produced in power plants and then transferred to electricity consumers connected to an electrical grid. Each power plant has an installed and available production capacity, typically measured in GW, which controls the amount of electric power that can be produced at any instant. (Specifically, one watt is defined as one joule per second and thus measures the rate of transfer.) The electrical energy produced in a power plant is often measured in GWh, and is thus the product of the power in gigawatts multiplied by the running time in hours.

The electricity system inside a region typically consists of different types of sources for electricity production. These can for example be coal power, nuclear power, hydropower, wind power, or natural gas turbines.

2.1 Electricity generation technologies

Thermal power plants are, as the name indicates, power plants where heat is converted to electricity. Water is heated to steam, which is then used to rotate turbines and generate electricity. Some different fuels used as heat sources are fossil fuels, nuclear energy, biofuels, and waste incineration. Some thermal power plants are combined to generate both electricity and heat (e.g. district heating) to consumers, but in this thesis only electricity is considered. The concept of *thermal cycling* refers to generating electricity at different demand levels. As the demand for electricity varies, some electricity generating units need to be turned on/off in response to these variations. However, every time a thermal power plant is turned off and on, the different components (e.g. boiler, steam lines, turbine) are exposed to stress caused by the large thermal and pressure variations, which then leads to maintenance costs.

In a *hydropower* plant, electricity is generated by leading water through turbines. The power extracted depends on the water volume and the height difference between the water's in- and outflow.

Wind power generates electricity by using the wind to provide mechanical power through turbines. Wind power is typically divided into onshore and offshore wind power. Investment and maintenance costs for offshore wind power are higher compared onshore wind power (Mone et al., 2017), but offshore wind is stronger and steadier compared to onshore wind.

Solar power is the technique to convert the energy from sunlight into electricity. This can be done by using photovoltaics (PV) or concentrated solar power (CSP). PVs use solar panels which contain photovoltaic cells that convert light into an electric current by the use of the photovoltaic effect. CSP is a technique which uses lenses or mirrors to concentrate sunlight into a small beam, and then use the resulting heat to generate electricity from steam turbines.

2.2 Electricity system modelling

The modelling of the electricity system can be done on different system levels, which vary with the types of questions that are asked. For example, the unit commitment problem studies how a set of electricity generators (e.g. power plants in a country, or turbines inside a power plant) should operate (i.e. dispatch electricity) in order to meet the demand at the lowest system cost (or highest revenue). Investment models look at cost-efficient investments in new capacity to meet future demand. These models typically consider a longer time horizon compared to the unit commitment problem, but instead lack in system details. This thesis addresses investment models, and therefore the focus here will be on those types of models.

In general, the investment models vary in three different dimensions: temporal scope, spatial scope and technological system detail. Figure 2.1 gives an illustration of the different model complexities, where a larger volume of

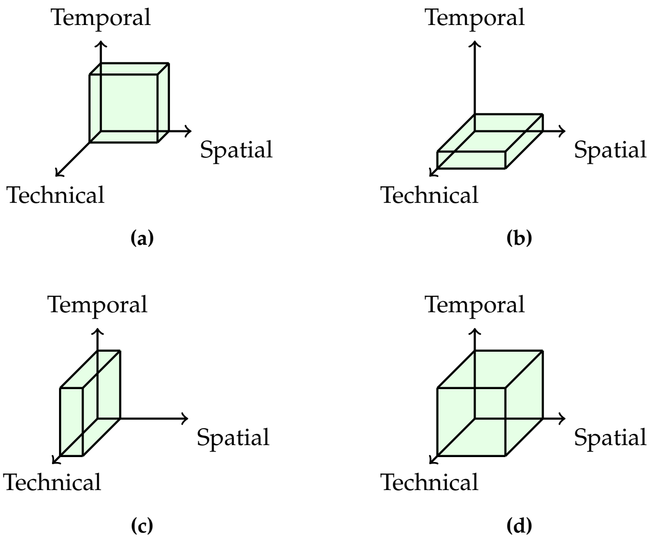


Figure 2.1: Illustrations of the problem complexity depending on the temporal resolution, spatial resolution, and technological system detail.
a) High temporal and spatial resolution, but low technological system detail
b) High spatial resolution and technological system detail, but low temporal resolution
c) High temporal resolution and technological system detail, but low spatial resolution
d) High temporal and spatial resolution, and high technological system detail

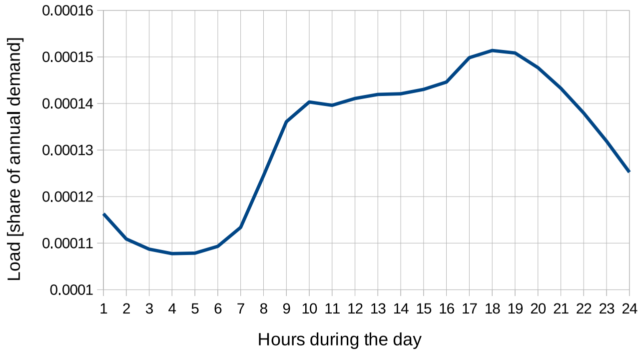
the cube indicates a more complex problem. The models tend to become very large if all dimensions have a high resolution/detail, and therefore electricity system investment models typically make sacrifices in resolution in at least one dimension.

The differences in temporal scope is related to the size of the time step, and the length of the time horizon. For models with a large share of VRE, a fine time resolution is needed in order to capture the variations, and thus the size of the time step needs to be in magnitude of a few hours. The time horizon typically ranges from a single year to a longer time span, for example all years up until 2050. The longer time resolution can be motivated by the lifetime of power plants, which is approximately 40 years for thermal power plants. The spatial scope can be everything from the unit commitment problem on a single power plant, to models which contain several countries. For the latter case, the spatial scope relates to the size of the interconnected transmission grids. Typically, for a high spatial resolution to be feasible, some simplifications in the technological system details need to be done. As mentioned earlier, electricity is produced by different electricity generation technologies, e.g. coal power, hydropower, wind power etc. The production capacity, measured in GW, determines an upper limit for how much electricity can be produced during a time instant. Instead of looking at separate power plants, assuming an aggregated capacity in each region will cause a loss of some system detail, but reduce the problem size significantly. Previous work by Göransson (2014) has, however, shown that the loss is marginal for the total system cost. The author also showed that the loss is marginal for the average full load hours for each electricity production type, including wind power. In terms of technological system details, besides using aggregated capacity, system details typically vary with the constraints included in the model. For example, using different types of storages such as batteries or hydrogen, which connect several time steps with each other, increases the model complexity. Also, hydropower connects over several time steps and is in that sense similar to storage constraints. Another complicating feature is caused by including thermal cycling in the modelling since this typically is modelled by integer variables (that are also connected over time). As will be seen in Section 3.1, however, this can be linearized to reduce complexity (Weber, 2005).

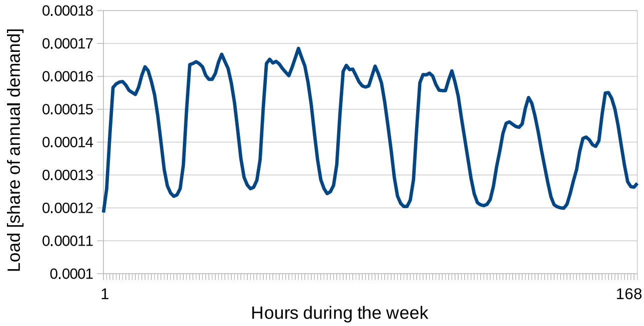
2.3 Variations in the electricity system

The variation of electricity demand is regular and related to our behavior as consumers. It can be divided into three main categories: seasonal variations, weekly variations, and diurnal variations. The seasonal variations has an annual cycle, where the demand varies throughout the year. In the northern European countries, the demand is higher during the darker and colder seasons due to electric heating, and lower in the warmer seasons when heating is not necessary. On the other hand, in southern Europe, the demand is higher during the summer months due to the need of air conditioning in buildings. Typically, less electricity is used during the nights compared to the days, and thus the demand also follows a diurnal cycle. However, the electricity use pattern on weekends compared to workdays also differs and therefore a weekly cycle exists. Figure 2.2 shows the electricity demand variations according to the described cycles.

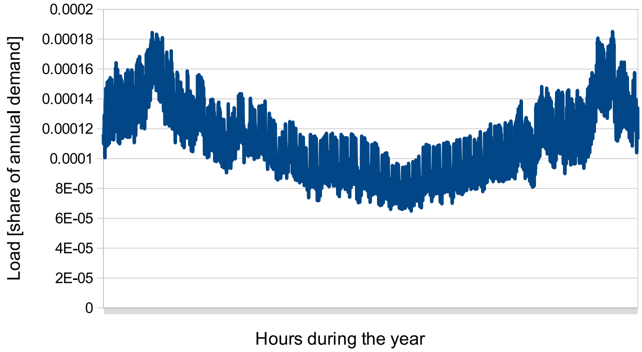
Traditionally, the electricity demand (usually referred to as load) is divided up in three different groups: base load, intermediate load, and peak load. Figure 2.3 demonstrates a load duration curve, where the load in a region has been sorted over all the hours of the year, from highest to lowest demand. The base load is consistent throughout the year, and typically the electricity production technologies that satisfy it run at full capacity at all hours. These production technology types generally have high investment costs, but low



(a) Diurnal variations of the electricity demand in region in Sweden.



(b) Weekly variations of the electricity demand in region in Sweden.



(c) Seasonal variations of the electricity demand in Sweden.

Figure 2.2: Load variations on different time scales.

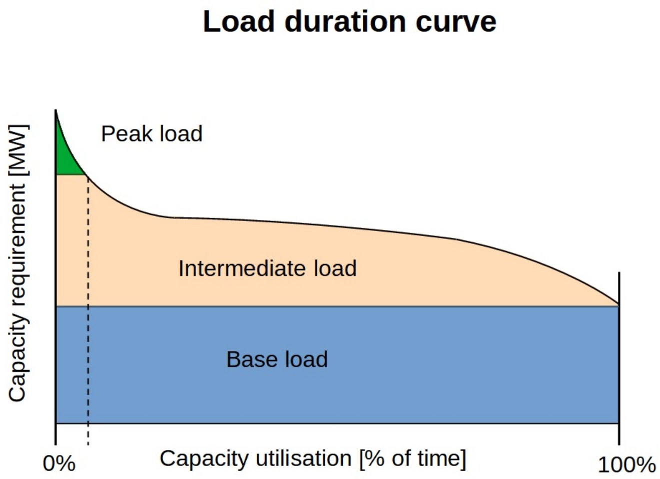


Figure 2.3: A load duration curve to illustrate how base, intermediate and peak load are used in the electricity system. The curve corresponds to the annual load, sorted according to the size of the load.

running costs. Examples include nuclear power and steam-engined power run by fossil fuels (e.g. coal). The peak load period has a significantly higher demand compared to the average load level and is fulfilled by for example gas turbines, that is, technologies that are expensive to operate but have comparatively low investment costs. Intermediate load corresponds to the period in between base load and peak load, and can for example be covered by combined cycle gas turbines, which combine several heat engines that all use the same source of heat, e.g. natural gas or biogas. The engines then work in tandem which allows them to extract heat energy from each other. These types of plants are better at following the load curve changes compared to base load production technologies.

The electricity system has historically been designed to meet the above mentioned load variations. However, variable renewable energy (VRE) sources such as wind and solar power are intermittent and non-dispatchable. This means that they are not always available as they depend on factors which can not be controlled. These factors include the weather and the location of wind turbines and solar panels, and thus different regions have different conditions. Nonetheless, while the wind variations are irregular, they can under some conditions still be fairly slow. If a large geographical scope (e.g. a group of wind farms, or all wind turbines in a country) is considered, there can be several days of high wind power production and then several days of low wind power production (Holttinen et al., 2009). Hence, a key strategy to manage variations from wind power includes a large geographical scope, and thus also trade between regions to smoothen the effect of wind variability.

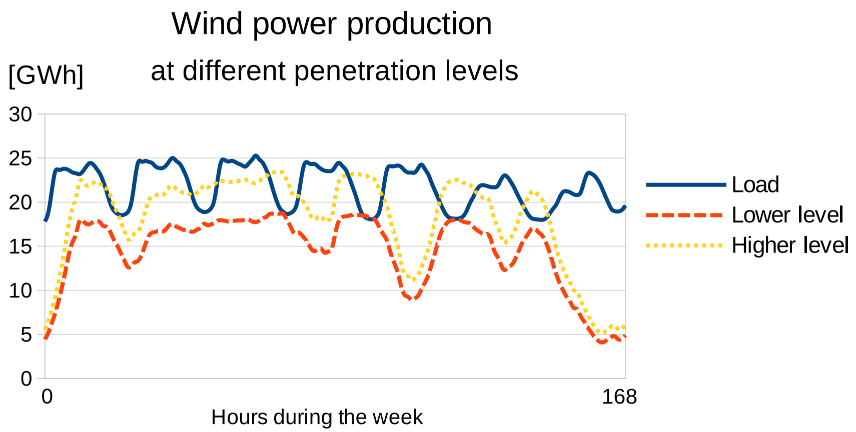


Figure 2.4: Example of wind power production at different penetration levels in a load curve during a week.

Figure 2.4 illustrates an example of the wind power production at different wind penetration levels during a week. For the lower level case, wind power will start to compete with base load production during times when the electricity demand is low. As discussed earlier, it is expensive to modify the output for base load production and thus wind power production will likely be *curtailed* during those hours. This means that the output from wind power production is deliberately decreased (even though more could have been produced) in order to balance the electricity supply and demand. If instead the case when the wind penetration level is higher is considered, there will be situations when the wind power production meets a larger share of the demand. In some situations, it could meet the entire demand for longer time periods. This implies that there are longer periods when other electricity production, such as base load, is not needed in the electricity system. Curtailing all the wind power production would in that case be too costly, and the alternative is then to turn off the base load production during these time periods. Furthermore, for the shorter time periods when demand can't be met entirely by wind power production only, it is not favorable to use base load production technologies. The reason for this is that since base load production is costly to invest in, and also expensive to start up, it is not cost-efficient to only use it during bursts of a few days a time. Instead, during these time periods intermediate and peak load production types will likely be used, since they better complement the wind power production pattern. However, to manage variations such as these in long and short term requires the solution of an optimization problem which should be captured by the electricity system investment models.

Solar power, just like the load, has regular variations that directly relates to the sunlight such that the production is highest in the middle of the day. The amplitude of the solar power production varies between days, but the general production pattern coincides with the demand curve; see Figure 2.5. This means that for low solar power penetration levels, it can replace peak load production. However, for higher penetration levels the solar power starts to compete with the base load production during the day. It would not be cost-efficient to only run base load production during the night, and therefore intermediate load production types would typically be used for these time periods. Moreover, the solar peak production is more narrow than the demand peak. This implies that when the sun is setting in the afternoon, the demand for electricity is still high and thus other electricity production need to cover that peak demand.

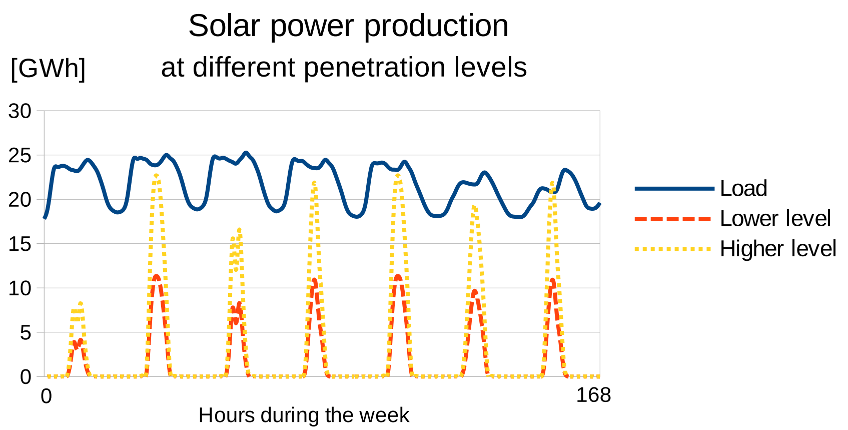


Figure 2.5: Example of solar power production at different penetration levels in a load curve during a week.

The operation of power plants in response to variations is a variation management strategy. It should be noted, however, that other variation management strategies for VRE integration exist. Typically, these can be found on the demand side, the production side, and by the use of storage units. In terms of storage, batteries or hydrogen storage can be used to store electricity when production is high, and discharged at a later point when production is lower. For example, storage units could be used to complement solar power to manage the peak demand in the afternoons. This flexibility also helps to avoid curtailing electricity since excess electricity from VRE production can be stored for later use. On the demand side, load shifting can be used to shift some of the load from hours with high demand to hours when there is less demand. This could for example be smart charging of electric vehicles, which means that the vehicles are charged (or discharged during peak load) when it is beneficial to the system in terms of load and electricity production. Peak load generation or reduced base load generation are examples of supply side variation management. Some of the mentioned variation management strategies have been evaluated by van Ackooij et al. (2021) using multi-objective optimization for costs and emissions on energy systems comprised of different energy mixes.

In conclusion, incorporating a high share of VRE sources, such as wind or solar power, into the energy mix requires additional variation management strategies compared to a system with a low share of VRE sources. For this to be successful, it is likely necessary to step away from the traditional way of designing the electricity system.

2.4 Previous work on electricity system modelling

To reduce computation time and computer memory requirements, previous work on electricity system investment models has typically focused on simplifying the time representation. Ringkjøb et al. (2018) reviewed 75 different modelling tools used for analysing energy and electricity systems with large shares of VRE. The authors identify that one of the remaining challenges is how to represent short-term variability in long-term studies. A methodological review of strategies to integrate short-term variations is given by Collins et al. (2017), who discuss methods to improve the time representation in long-term electricity system investment models that use traditional ways of time representation. Pfenninger et al. (2014) review several articles that discuss time representation for energy system models containing a substantial level of VRE.

Traditional time representation methods for electricity system investment models typically belong to a family of methods using *time slices*. Integral time slices can for example be a single time slice per year or a small set of seasonal and daily time slices to represent the differences in demand dependent on season, weekday, or time of day. Time slicing methods that are based on approximating the joint probability distribution of the load and VRE generation are for example developed by Wogrin et al. (2014) and Lehtveer et al. (2017). Another time slicing method is the *representative days* method, suggested by Nahmmacher et al. (2014), which identifies a number of 24-hour segments based on load and VRE patterns over a day. Time reduction methods based on these principles have been implemented and shown promising results for long-term investment models (see, for example, Mai et al. (2013), Gils (2016), Gerbaulet and Lorenz (2017), and Frew and Jacobson (2016)) and the methods have been compared and evaluated by Reichenberg et al. (2018).

However, the integral time slicing methods have traditionally not worked when considering a larger geographical scope which includes regional trade. The reason for this is that approximating the joint probability distribution is challenging since, unlike variations in load, variations in VRE generation do not follow a common pattern across a wide geographical scope. Thus, the integral time slicing methods can not properly account for wind and solar variations in models, in which a large geographical scope is considered. As discussed in Section 2.3, smoothing effects through trade is an important variation management strategy for VRE and thus, as is also concluded by Reichenberg et al. (2018), the integral time slicing method is not ideal for a multi-node electricity system model with large shares of VRE.

The representative days approach on the other hand can be employed in network models and therefore incorporate trade (see for example Frew et al. (2016)). This time representation can also handle short term storage, but not overnight storage since modelling of storage requires interconnected time steps. An alternative is to model over longer time periods, i.e. weeks, but this increases model complexity and thus computation time. Hence, a simplification in the spatial or technological system detail dimensions might be necessary to compensate for the increased complexity.

3 Mathematical modelling

The problem studied in this thesis, for which we have developed three mathematical models, consists of minimizing investment and operational costs while meeting the demand for electricity in a European electricity system. Europe has here been divided into several regions, chosen according to country borders and, if existent, infrastructural bottlenecks within the countries; see Figure 3.1.

We assume an aggregated capacity in each region, instead of looking at separate power plants. Furthermore, a time period stretching from 2020 to 2050 is studied where we consider both existing production capacity as well as new investments. The total time period is divided into different investment periods in order to account for the lifespan of different power plants, i.e. the production capacity lifespan.

Each region produces electricity (measured in GWh/h) to meet its electricity demand. For the second and third model, trade along the electricity grid is also possible between regions. Existing transmission lines are then considered, but it is also allowed to invest into new transmission capacity. Similar to that of electricity production capacity, the transmission capacity works as an upper limit for the electricity transmission.

Thermal cycling is included as previous work has shown that is has a substantial impact on the cost-optimal electricity system composition (Göransson et al., 2017). Furthermore, to keep the model linear, thermal cycling is accounted for using a relaxed unit commitment approach as described by Weber (2005). This method is explained in detail in Section 3.1, but to briefly summarize, variables are used to represent active (hot) production capacity in thermal power plants that is available for electricity generation in each time step. Moreover, there are some special constraints for renewable energy production, such as production and capacity limits from weather, competing land use, and population density. Hydropower has constraints for ramp rate, i.e. the rate at which the electricity output can increase or decrease, and balance constraints for electricity storage.

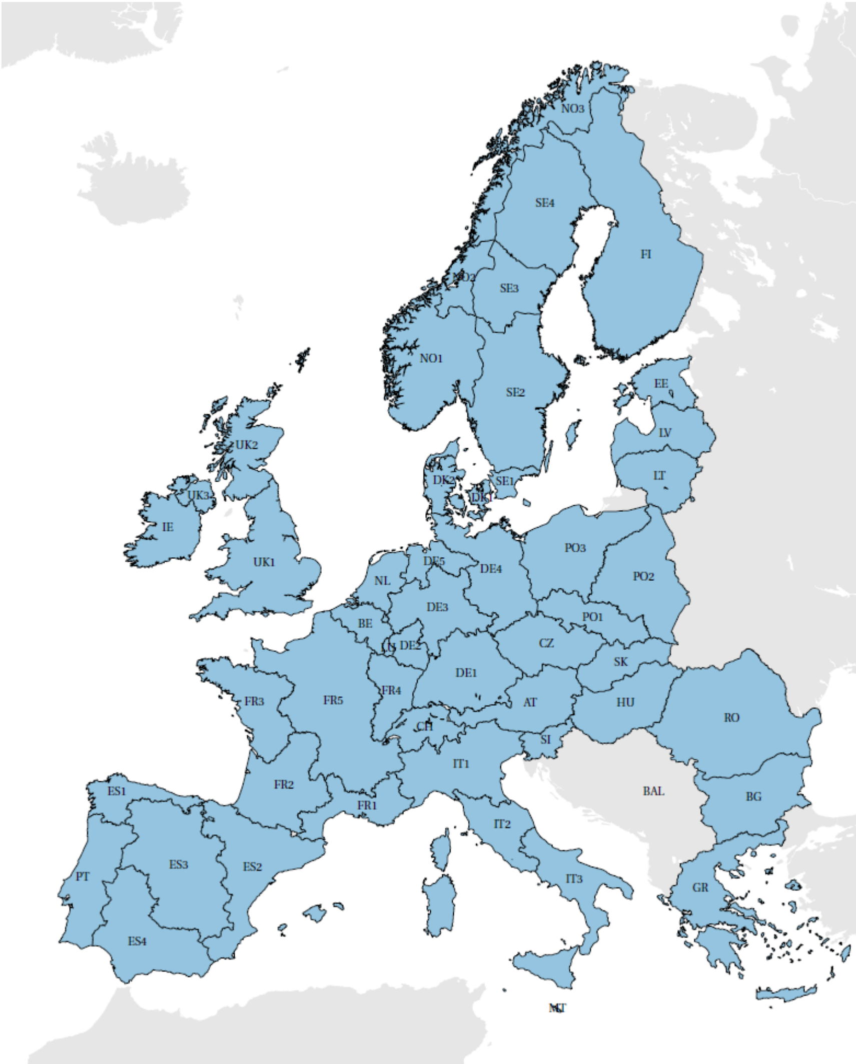


Figure 3.1: The different regions in Europe used in the model

Lastly, we should account for the total system carbon dioxide emissions. This can be represented by a hard constraint, or by using a tax (and thus penalize carbon-emitting technologies) in the objective. For reasons relating to the solutions methods, we have chosen to use the tax representation; see also Chapter 7 for further discussion regarding this.

3.1 A simple electricity system investment model

First, a basic and simplified model is considered. This linear model only considers one region and does not allow any import and export of electricity from other regions. The sets used in this model are listed in Table 3.1.

Table 3.1: The index sets used in the basic model

symbol		representation	member
$\mathcal{P}$	$:= \mathcal{P}_{\text{thermal}} \cup \mathcal{P}_{\text{ren}}$;	electricity generation technologies	p
$\mathcal{P}_{\text{thermal}}$	$\subset \mathcal{P}$;	thermal power technologies	p
$\mathcal{P}_{\text{ren}}$	$\subset \mathcal{P}$;	renewable technologies	p
$\mathcal{P}_{\text{wind}}$	$\subset \mathcal{P}_{\text{ren}}$;	wind technologies	p
$\mathcal{P}_{\text{hydro}}$	$\subset \mathcal{P}_{\text{ren}}$;	hydropower technologies	p
$\mathcal{S}$	$:= \{1, \ldots, S\} \subset \mathcal{I}$;	new capacity investment years	s
$\mathcal{T}$	$:= \{1, \ldots, T\}$;	time steps within a year	t
$\mathcal{T}_{\text{start}}(p)$	$\subset \mathcal{T} \cup \{0\}$;	hours in the start-up interval for technology $p \in \mathcal{P}$	t
$\mathcal{I}$	$:= \{S - I + 1, \ldots, S\}$;	investment periods, where $I = \lvert\mathcal{I}\rvert$	i
$\mathcal{I}^{\mathcal{P}}_{\text{active}}(p, s)$	$:= \mathcal{I} \cap \{s - U_p, \ldots, s\}$;	investment periods for each technology type $p \in \mathcal{P}$ with lifespan U_p that are active at year $s \in \mathcal{S}$	i

The set $\mathcal{P}$ represents all the electricity production technologies, e.g. hydropower, wind power, nuclear power, etc. This set contains thermal power technologies, $\mathcal{P}_{\text{thermal}}$, such as nuclear power and waste incineration plants. It also contains renewable electricity generation technologies, and in this model namely $\mathcal{P}_{\text{wind}}$ and $\mathcal{P}_{\text{hydro}}$. The set of renewables are then defined as $\mathcal{P}_{\text{ren}} := \mathcal{P}_{\text{wind}} \cup \mathcal{P}_{\text{hydro}}$. The modelling years are given by $\mathcal{S}$, and the set of time steps within a year is denoted $\mathcal{T}$. Since thermal cycling is considered in this model, $\mathcal{T}_{\text{start}}(p)$, is the set of hours in the start-up interval for technology $p \in \mathcal{P}$. This model uses the concept of investment periods, denoted $\mathcal{I}$, which are used to know at what year an investment in production capacity was made. This is relevant for two reasons: firstly, the model covers several years. Hence, if an investment in technology type p is made in year s, it should not be possible to use that invested capacity prior to the year s. Secondly, since each production technology has a lifetime (dependent on the specific technology type $p \in \mathcal{P}$), then in order to know for how long that invested capacity can be used it is crucial to know when the investment was made. Therefore, the set $\mathcal{I}^{\mathcal{P}}_{\text{active}}(p, s)$ contains the investment periods for each technology type $p \in \mathcal{P}$ (with its own lifespan U_p) that are active at year $s \in \mathcal{S}$. Note that $\mathcal{S} \subset \mathcal{I}$ holds.

A detailed list of the set elements is presented in Chapter 5, and a full nomenclature list is given in Appendix A.1. Note that some sets, variables, and parameters in Appendix A.1 are non-existent in this model. Additionally, the indices for regions are not present in this model.

The mathematical constraints and objective for the problem are described below. All the decision variables are subject to non-negativity constraints.

Meeting the demand

We begin by introducing the decision variables

$$\begin{aligned} x_{pist} = \;& \text{generated electricity of technology type } p \in \mathcal{P} \text{ in year } s \in \mathcal{S} \\ & \text{for investment period } i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s, p) \text{ and time step } t \in \mathcal{T}, \end{aligned}$$

that are measured in GWh/h. Let d_{st} denote the demand in year $s \in \mathcal{S}$ at time step $t \in \mathcal{T}$. The constraints

$$\sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} x_{pist} \geq d_{st}, \qquad s \in \mathcal{S},\, t \in \mathcal{T}, \tag{3.1.1}$$

imply that the produced electricity meets the demand during all modelled time steps and years.

Generation limits

Variables are needed to represent the capacity installed in the system. Thus, define the decision variables

$$y_{pi} = \text{installed capacity of technology type } p \in \mathcal{P} \text{ in investment period } i \in \mathcal{I},$$

measured in GW. Let b^{gen}_{pi} be the existing production capacity of production type $p \in \mathcal{P}$ for investment period $i \in \mathcal{I} \setminus \mathcal{S}$. The investment variables y_{pi} are fixed such that no new investments can be made prior to "now", according to

$$y_{pi} = b^{\text{gen}}_{pi}, \qquad p \in \mathcal{P},\, i \in \mathcal{I} \setminus \mathcal{S}. \tag{3.1.2}$$

Thermal cycling

The idea of thermal cycling is that for thermal power plants, the capacity that has been taken out of operation has a minimum down-time that corresponds to the time it takes to start-up the capacity before it can generate electricity once again. Furthermore, and more importantly, the start-up cost for a unit is typically high and thermal cycling constraints can be used to capture this property. To model this, let us begin by defining two new types of decision variables:

$$\begin{aligned} z_{pist} &= \text{available hot capacity of technology type } p \in \mathcal{P}_{\text{thermal}} \text{ in year } s \in \mathcal{S} \\ &\quad\ \text{for investment period } i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p) \text{ and time step } t \in \mathcal{T}; \\ z^{+}_{pist} &= \text{started hot capacity of technology type } p \in \mathcal{P}_{\text{thermal}} \text{ in year } s \in \mathcal{S} \\ &\quad\ \text{for investment period } i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p) \text{ from time step } t-1 \in \mathcal{T} \\ &\quad\ \text{to time step } t \in \mathcal{T}, \end{aligned}$$

measured in GWh/h. The capacity that is currently up and running in a thermal power plant is referred to as *hot capacity* or *available capacity*; see Figure 3.2a. The electricity generation should never exceed the available hot capacity. Likewise, it is required to generate a minimum level of electricity depending on the available hot capacity in order for it to stay hot. Let ϕ_p denote the share corresponding to the minimum load level. This yields the constraints

$$\phi_p z_{pist} \leq x_{pist} \leq z_{pist}, \qquad i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p),\ p \in \mathcal{P}_{\text{thermal}},\ s \in \mathcal{S},\ t \in \mathcal{T}. \quad (3.1.3)$$

Furthermore, to connect the started hot capacity to the available hot capacity, for $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$, $p \in \mathcal{P}_{\text{thermal}}$, and $s \in \mathcal{S}$ the following constraints are used:

$$z^{+}_{pist} \geq \begin{cases} z_{pist} - z_{p,i,s,t-1}, & t \in \mathcal{T} \setminus \{1\}, \\ z_{pist} - z_{pisT}, & t = 1. \end{cases} \quad (3.1.4)$$

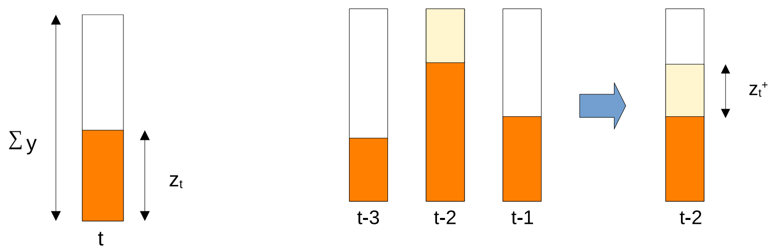
The difference in available hot capacity between the time steps t and $t-1$ will then correspond to the started hot capacity. It is costly to start new capacity and hence the variable z^{+}_{pist} is penalized in the objective function. Thus, z^{+}_{pist} will be zero if $z_{pist} \leq z_{p,i,s,t-1}$.

Moreover, $t_p \in \mathcal{T}_{\text{start}}(p)$, $p \in \mathcal{P}_{\text{thermal}}$, hours back in time, the started hot capacity is limited by the available hot capacity z_{pist}. This yields, for $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$,

$p \in \mathcal{P}_{\text{thermal}}$, $s \in \mathcal{S}$, and $t \in \mathcal{T}$, the constraints

$$\sum_{j \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} y_{pj} - z^{+}_{pist} \geq \begin{cases} z_{p,i,s,t-t_p}, & t_p \in \mathcal{T}_{\text{start}}(p) \setminus \{t, \dots, T\}, \\ z_{p,i,s,T+t-t_p}, & t_p \in \mathcal{T}_{\text{start}}(p) \setminus \{0, \dots, t-1\}, \end{cases} \tag{3.1.5}$$

which linearize the start-up constraints that are more intuitively modeled as integers. The idea is that capacity that has been taken out of operation has a minimum down-time before it can be started again. As a simple example, consider Figure 3.2b. Here, the minimum down-time is three; thus we look at the hot capacity three time steps back. The amount of capacity that is available for start-up is capacity that has not been used in any of these time steps. The hot capacity is the largest for step $t-2$; therefore it will limit z^{+}_{t} the most.



(a) Hot capacity is limited by the total amount of installed capacity in time step t.

(b) The minimum down-time for hot capacity limits the possible start-up capacity.

Figure 3.2: Illustrations for thermal cycling.

Renewables

Policies in some countries require that the total renewable electricity generation (wind and hydropower) should stay above some minimum level. Let f_s denote this level for the year $s \in \mathcal{S}$. Then, this requirement is modelled by the constraints

$$\sum_{p \in \mathcal{P}_{\text{ren}}} \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \sum_{t \in \mathcal{T}} x_{pist} \geq f_s, \qquad s \in \mathcal{S}. \tag{3.1.6}$$

There is an upper limit on production of wind power due to weather and climate. Let θ_{pt} be a profile, given as share of total installed capacity, for technology $p \in \mathcal{P}_{\text{wind}}$ at time step $t \in \mathcal{T}$. Then, this limit is modelled as

$$x_{pist} \leq \theta_{pt} \sum_{j \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} y_{pj}, \qquad i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p),\ p \in \mathcal{P}_{\text{wind}},\ s \in \mathcal{S},\ t \in \mathcal{T}. \tag{3.1.7}$$

Not all areas are suitable for installation of wind farms since wind speed and terrain varies across the regions. Thus, for reasons regarding land exploitation, there is an upper limit on the possible investments in wind capacity; let W_p be that upper limit for wind type $p \in \mathcal{P}_{\text{wind}}$. Then, this limit is modelled as

$$\sum_{i \in \mathcal{I}} y_{pi} \leq W_p, \qquad p \in \mathcal{P}_{\text{wind}}. \tag{3.1.8}$$

Define the decision variables

$$w_{st} = \text{hydropower storage in year } s \in \mathcal{S} \text{ at time step } t \in \mathcal{T},$$

given in GWh. Let g_t denote the inflow into the reservoirs at time step $t \in \mathcal{T}$. The inflow is assumed to be the same over the years. For $p \in \mathcal{P}_{\text{hydro}}$ and $s \in \mathcal{S}$, the hydropower balance constraint is then modelled as

$$w_{st} + g_t - \tau \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} x_{pist} \geq \begin{cases} w_{s,t+1}, & t \in \mathcal{T} \setminus \{T\}, \\ w_{s1}, & t = T, \end{cases} \tag{3.1.9}$$

where τ [h] denotes the length of the time step. In this model presentation, $\tau = 1$ for simplicity but still necessary to include for the dimension analysis.

The hydropower storage has an upper limit denoted H, modelled as

$$w_{st} \leq H, \qquad s \in \mathcal{S},\ t \in \mathcal{T}. \tag{3.1.10}$$

The production level for hydropower can't change too quickly. Thus, ramping rate constraints are required. Let δ^{inc} and δ^{dec} denote shares corresponding to the maximum change level. The constraints (3.1.11) and (3.1.12) imply an upper limit on the rate of increase and decrease, respectively, of the storage level. For $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$, $p \in \mathcal{P}_{\text{hydro}}$, and $s \in \mathcal{S}$, thus

$$(1 + \delta^{\text{inc}}) x_{pist} \geq \begin{cases} x_{p,i,s,t+1}, & t \in \mathcal{T} \setminus \{T\}, \\ x_{pis1}, & t = T; \end{cases} \tag{3.1.11}$$

$$(1 + \delta^{\text{dec}}) x_{pist} \leq \begin{cases} x_{p,i,s,t+1}, & t \in \mathcal{T} \setminus \{T\}, \\ x_{pis1}, & t = T. \end{cases} \tag{3.1.12}$$

Lastly, no new investments in hydropower capacity are allowed. Thus,

$$y_{ps} = 0, \qquad p \in \mathcal{P}_{\text{hydro}},\ s \in \mathcal{S}. \tag{3.1.13}$$

Emissions

The emissions arise from running the power plants (fuel etc.), but also from start-ups of plants since fuel is needed for this. Furthermore, there are extra emissions when not running on full capacity, due to reduced efficiency. Let e_{pi} denote the emissions released, measured in CO_2/MWh, by technology type $p \in \mathcal{P}$ in investment period $i \in \mathcal{I}$. Let e^+_{pi} and $\tilde{e}_{pi}$ denote the emissions released from start-ups and from running on part-load, respectively, for technology type $p \in \mathcal{P}_{\text{thermal}}$ in investment period $i \in \mathcal{I}$. Let the auxiliary variable e^{tot}_{st} denote the total emissions for year $s \in \mathcal{S}$ and time step $t \in \mathcal{T}$, which is then expressed as

$$e^{\text{tot}}_{st} := \sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} e_{pi} x_{pist} + \sum_{p \in \mathcal{P}_{\text{thermal}}} \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \big(e^+_{pi} z^+_{pist} + \tilde{e}_{pi}(z_{pist} - x_{pist}) \big). \tag{3.1.14}$$

Objective

The objective is to minimize the total system costs. The sum (3.1.15a) considers the investment costs in electricity production technologies. Here, c^{invtech}_{ps} is the investment cost (with annuity costs included) for technology type $p \in \mathcal{P}$ in year $s \in \mathcal{S}$, and c^{omf}_{p} is the fixed operation and maintenance costs for technology type $p \in \mathcal{P}$. The sum (3.1.15b) considers the costs of electricity production, where c^{run}_{pi} are the run costs for technology type $p \in \mathcal{P}$ where the investment was made in period $i \in \mathcal{I}$. The sum (3.1.15c) describes the additional costs for thermal power technology types $p \in \mathcal{P}_{\text{thermal}}$. Specifically, c^+_{ps} is the start-up cost for hot capacity in year $s \in \mathcal{S}$, and $\tilde{c}_{ps}$ is the additional cost for running on part-load capacity in year $s \in \mathcal{S}$. Lastly, the sum (3.1.15d) describes the costs for carbon dioxide emissions. The objective function is defined as

$$\sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} (c^{\text{invtech}}_{ps} + c^{\text{omf}}_{p}) y_{ps} \tag{3.1.15a}$$

$$+ \sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} c^{\text{run}}_{pi} x_{pist} \tag{3.1.15b}$$

$$+ \sum_{p \in \mathcal{P}_{\text{thermal}}} \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \Big(c^{+}_{ps} z^{+}_{pist} + \tilde{c}_{ps} (z_{pist} - x_{pist}) \Big) \tag{3.1.15c}$$

$$+ \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} c^{\text{CO}_2}_{s} e^{\text{tot}}_{st}. \tag{3.1.15d}$$

Note that for the investment costs in (3.1.15a), we only consider the investment periods that coincide with the years $s \in \mathcal{S}$. Thus, any costs for prior investments are not counted. In (3.1.15b) and (3.1.15c), the total costs cover all active investment periods.

We assume that the total costs for investments and operation of a power plant is evenly distributed across all of its hours of operation, and the formula for present value of an annuity is used to calculate the annualized investment cost.

3.2 Full-scale electricity system investment model

For the full-scale electricity system investment model, the basic model is used as a foundation and built upon to include several regions, compared to the single region in the basic model. With this comes the option to import and export electricity, and thus the added model constraints describe a transmission and load balance for the electricity grid in the form of network balance constraints. Hence, the problem is represented using a linear network model with additional side constraints.

The sets used in this model are listed in Table 3.2. It contains some additional sets compared to the basic model. The set $\mathcal{L}$ represents all the countries that the different regions in the model belong to. The different regions are defined by the set $\mathcal{R}$, and the set $\mathcal{R}(l)$ contains all regions that belong to country $l \in \mathcal{L}$. Transmission lines between regions are given by the set $\mathcal{A} \subset \mathcal{R} \times \mathcal{R}$. Thus, from region $q \in \mathcal{R}$ to region $r \in \mathcal{R}$, there exists a transmission line if $(q, r) \in \mathcal{A}$. The model uses different types of cables for transmission, and they are given by the set $\mathcal{K}$. Similar to the set of active investment periods for the electricity generation technologies, a set of active periods for the transmission technologies is needed. This set is denoted $\mathcal{I}^{\mathcal{K}}_{\text{active}}(s)$, and it contains the active investment periods at year $s \in \mathcal{S}$. It is assumed that the different types of transmission technologies have a life span that outlives the model years, and therefore the set is not dependent on the transmission types $k \in \mathcal{K}$.

References to the data are discussed in Chapter 5, and a full nomenclature list is given in Appendix A.1. The mathematical constraints and objective for the problem are described below. All the decision variables are restricted to be non-negative.

Table 3.2: The index sets used in the full-scale model

symbol		representation	member
$\mathcal{L}$		countries	l
$\mathcal{R}$		regions	r
$\mathcal{R}(l)$	$\subset \mathcal{R}$;	regions within a country $l \in \mathcal{L}$	r
$\mathcal{A}$	$\subset \mathcal{R} \times \mathcal{R}$;	transmission lines between regions	q, r
$\mathcal{K}$		technologies for transmission	k
$\mathcal{P}$	$:= \mathcal{P}_{\text{thermal}} \cup \mathcal{P}_{\text{ren}}$;	electricity generation technologies	p
$\mathcal{P}_{\text{thermal}}$	$\subset \mathcal{P}$;	thermal power technologies	p
$\mathcal{P}_{\text{ren}}$	$\subset \mathcal{P}$;	renewable technologies	p
$\mathcal{P}_{\text{wind}}$	$\subset \mathcal{P}_{\text{ren}}$;	wind technologies	p
$\mathcal{P}_{\text{hydro}}$	$\subset \mathcal{P}_{\text{ren}}$;	hydropower technologies	p
$\mathcal{S}$	$:= \{1, \ldots, S\} \subset \mathcal{I}$;	new capacity investment years	s
$\mathcal{T}$	$:= \{1, \ldots, T\}$;	time steps within a year	t
$\mathcal{T}_{\text{start}}(p)$	$\subset \mathcal{T} \cup \{0\}$;	hours in the start-up interval for technology $p \in \mathcal{P}$	t
$\mathcal{I}$	$:= \{S - I + 1, \ldots, S\}$;	investment periods, where $I = \lvert\mathcal{I}\rvert$	i
$\mathcal{I}^{\mathcal{P}}_{\text{active}}(p, s)$	$:= \mathcal{I} \cap \{s - U_p, \ldots, s\}$;	investment periods for each technology type $p \in \mathcal{P}$ with lifespan U_p that are active at year $s \in \mathcal{S}$	i
$\mathcal{I}^{\mathcal{K}}_{\text{active}}(s)$	$:= \{S - I + 1, \ldots, s\}$;	investment periods that are active at year $s \in \mathcal{S}$	i

Transmission and load balance

The first constraint describes a transmission balance for the grid. Introduce the two variables, both measured in GWh/h,

x_{prist} = generated electricity of technology type $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ at year $s \in \mathcal{S}$ for investment period $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s, p)$ and time step $t \in \mathcal{T}$;

v_{kqrst} = electricity traded with transmission type $k \in \mathcal{K}$ from region $q \in \mathcal{R}$ to region $r \in \mathcal{R}$, such that $(q, r) \in \mathcal{A}$, at year $s \in \mathcal{S}$ and time step $t \in \mathcal{T}$.

Let d_{rst} denote the electricity demand in region $r \in \mathcal{R}$ at year $s \in \mathcal{S}$ and time step $t \in \mathcal{T}$. Then for each $r \in \mathcal{R}$, $s \in \mathcal{S}$ and $t \in \mathcal{T}$, the demand fulfillment is formulated as

$$\sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} x_{prist} + \sum_{k \in \mathcal{K}} \sum_{q:(q,r) \in \mathcal{A}} v_{kqrst} - \sum_{k \in \mathcal{K}} \sum_{j:(r,j) \in \mathcal{A}} v_{krjst} \geq d_{rst}. \quad (3.2.1)$$

Thus, all electricity generation and import to a certain region during a certain year and time step, minus the export has to meet the demand for electricity.

The transmission is limited by the transmission capacity, measured in GW. It is possible to invest into new capacity, but there is some capacity previously installed. Let b^{tra}_{kqr} be a parameter which denotes the installed transmission capacity on transmission line $(q, r) \in \mathcal{A}$ of transmission type $k \in \mathcal{K}$. Define

u_{kqri} = total investments in transmission capacity for transmission type $k \in \mathcal{K}$ from region q to region r, $(q, r) \in \mathcal{A}$, at investment period $i \in \mathcal{I}$.

These variables are then limited by the existing capacity such that

$$u_{kqri} = b^{\text{tra}}_{kqr}, \qquad k \in \mathcal{K},\ (q,r) \in \mathcal{A},\ i \in \mathcal{I} \setminus \mathcal{S}. \quad (3.2.2)$$

Furthermore, the transmission capacity for arcs $(q, r) \in \mathcal{A}$ and $(r, q) \in \mathcal{A}$ should be the same, modelled as

$$u_{kqri} = u_{krqi}, \qquad k \in \mathcal{K},\ (q,r) \in \mathcal{A},\ i \in \mathcal{I}. \quad (3.2.3)$$

This assumes that transmission is always possible in both directions, but for each solution trade will only occur in one direction due to it otherwise being inefficient. The directed arcs are defined such that if $(q, r) \in \mathcal{A}$ then $(r, q) \in \mathcal{A}$.

The transmission should not exceed the total transmission capacity—both new and old—modelled as

$$v_{kqrst} \leq \sum_{i \in \mathcal{I}^{\mathcal{K}}_{\text{active}}(s)} u_{kqri}, \qquad k \in \mathcal{K},\ (q,r) \in \mathcal{A},\ s \in \mathcal{S},\ t \in \mathcal{T}. \quad (3.2.4)$$

Generation limits

Define the decision variables

y_{pri} = installed capacity of technology type $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ at investment period $i \in \mathcal{I}$.

Let b^{gen}_{pri} be existing production capacity, with $p \in \mathcal{P}$, $r \in \mathcal{R}$ and $i \in \mathcal{I}$. Similar to the basic model, fix the investment variable y_{pri} to existing capacity for previous years, as

$$y_{pri} = b^{\text{gen}}_{pri}, \qquad p \in \mathcal{P},\ r \in \mathcal{R},\ i \in \mathcal{I} \setminus \mathcal{S}. \tag{3.2.5}$$

Thermal cycling

As in the basic model, we work with continuous variables to represent hot capacity and start-up constraints. Introduce the decision variables

z_{prist} = available hot capacity of technology type $p \in \mathcal{P}_{\text{thermal}}$ in region $r \in \mathcal{R}$ at year $s \in \mathcal{S}$ for investment period $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$ and time step $t \in \mathcal{T}$,

z^{+}_{prist} = started hot capacity of technology type $p \in \mathcal{P}_{\text{thermal}}$ in region $r \in \mathcal{R}$ at year $s \in \mathcal{S}$ for investment period $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$ from time step $t-1 \in \mathcal{T}$ to time step $t \in \mathcal{T}$,

both measured in GWh/h. The electricity should never exceed the available hot capacity, and there is a minimum level of necessary electricity generation depending on hot capacity. Let ϕ_p denote some share corresponding to the minimum load level. This yields the constraints

$$\phi_p z_{prist} \le x_{prist} \le z_{prist}, \quad i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p),\ p \in \mathcal{P}_{\text{thermal}},\ r \in \mathcal{R},\ s \in \mathcal{S},\ t \in \mathcal{T}. \tag{3.2.6}$$

Furthermore, for $p \in \mathcal{P}_{\text{thermal}}$, $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$ and $s \in \mathcal{S}$ the started hot capacity is connected to the available hot capacity by the constraints

$$z^{+}_{prist} \ge \begin{cases} z_{prist} - z_{p,r,i,s,t-1}, & t \in \mathcal{T} \setminus \{1\}, \\ z_{prist} - z_{prisT}, & t = 1. \end{cases} \tag{3.2.7}$$

Like in the basic model, z^{+}_{prist} is penalized in the objective function and will thus be zero if $z_{pist} \le z_{p,i,s,t-1}$.

For $p \in \mathcal{P}_{\text{thermal}}$, $r \in \mathcal{R}$, $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$, $s \in \mathcal{S}$, $t \in \mathcal{T}$, the minimum down-time constraints are modeled as

$$\sum_{j \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} y_{prj} - z^{+}_{prist} \ge \begin{cases} z_{p,r,i,s,t-t_p}, & t_p \in \mathcal{T}_{\text{start}}(p) \setminus \{t,\dots,T\}, \\ z_{p,r,i,s,T+t-t_p}, & t_p \in \mathcal{T}_{\text{start}}(p) \setminus \{0,\dots,t-1\}. \end{cases} \tag{3.2.8}$$

Renewables

As stated in the basic model, policies in some countries require that the total renewable electricity generation (wind and hydropower) stays above some minimum level. Assume this level is denoted f_{ls}, so that each country $l \in \mathcal{L}$ has its own level for year $s \in \mathcal{S}$. Then, this constraint is expressed as

$$\sum_{p \in \mathcal{P}_{\text{ren}}} \sum_{r \in \mathcal{R}(l)} \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \sum_{t \in \mathcal{T}} x_{prist} \geq f_{ls}, \qquad l \in \mathcal{L},\ s \in \mathcal{S}. \tag{3.2.9}$$

There is an upper limit on wind power production due to weather and climate. Let θ_{prt} be a profile, given as share of total installed capacity, for technology $p \in \mathcal{P}_{\text{wind}}$ in region $r \in \mathcal{R}$ at time step $t \in \mathcal{T}$. Then, the limit is modelled as

$$x_{prist} \leq \theta_{prt} \sum_{j \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} y_{prj}, \qquad i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p),\ p \in \mathcal{P}_{\text{wind}},\ r \in \mathcal{R},\ s \in \mathcal{S},\ t \in \mathcal{T}. \tag{3.2.10}$$

The upper limit on possible investments in wind capacity for wind type $p \in \mathcal{P}_{\text{wind}}$ in region $r \in \mathcal{R}$ is denoted W_{pr}. The limit is then modelled by the constraints

$$\sum_{i \in \mathcal{I}} y_{pri} \leq W_{pr}, \qquad p \in \mathcal{P}_{\text{wind}},\ r \in \mathcal{R}. \tag{3.2.11}$$

Introduce the decision variable

$$w_{rst} = \text{hydropower storage in region } r \in \mathcal{R} \text{ at year } s \in \mathcal{S} \text{ at time step } t \in \mathcal{T},$$

measured in GWh. Furthermore, let g_{rt} be the inflow into the reservoirs in region $r \in \mathcal{R}$ at time step $t \in \mathcal{T}$, assumed to be the same over the years. The hydropower balance constraint is then, for $p \in \mathcal{P}_{\text{hydro}}$, $r \in \mathcal{R}$ and $s \in \mathcal{S}$, modelled as

$$w_{rst} + g_{rt} - \tau \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} x_{prist} \geq \begin{cases} w_{r,s,t+1}, & t \in \mathcal{T} \setminus \{T\}, \\ w_{rs1}, & t = T. \end{cases} \tag{3.2.12}$$

As in the basic model, $\tau = 1$ [h] represents the time step length.

The upper limit for the hydropower storage, H_r for each $r \in \mathcal{R}$, is modelled as

$$w_{rst} \leq H_r, \qquad r \in \mathcal{R},\ s \in \mathcal{S},\ t \in \mathcal{T}. \tag{3.2.13}$$

Let δ_r^{inc} and δ_r^{dec} denote the maximum ramping rate increase and decrease, respectively. The ramping rate constraints are then given by

$$(1+\delta_r^{\text{inc}})x_{prist} \geq \begin{cases} x_{p,r,i,s,t+1}, & t \in \mathcal{T} \setminus \{T\}, \\ x_{pris1}, & t = T. \end{cases} \tag{3.2.14}$$

$$(1+\delta_r^{\text{dec}})x_{prist} \leq \begin{cases} x_{p,r,i,s,t+1}, & t \in \mathcal{T} \setminus \{T\}, \\ x_{pris1}, & t = T. \end{cases} \tag{3.2.15}$$

Finally, it is not allowed to invest in new hydropower capacity. Therefore,

$$y_{prs} = 0, \qquad p \in \mathcal{P}_{\text{hydro}},\ r \in \mathcal{R},\ s \in \mathcal{S}. \tag{3.2.16}$$

Emissions

Similar to the basic model, let e_{pri} denote the emissions released, measured in CO_2/(GWh/h), by technology type $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ for capacity made in investment period $i \in \mathcal{I}$. Let e_{pri}^{+} and $\tilde{e}_{pri}$ denote the emissions released from start-ups and from running on part-load, respectively, for technology type $p \in \mathcal{P}_{\text{thermal}}$ in region $r \in \mathcal{R}$ and investment period $i \in \mathcal{I}$. Lastly, let e_{st}^{tot} denote the auxiliary variable that represents the total released emissions for year $s \in \mathcal{S}$ and time step $t \in \mathcal{T}$. The definition constraint for the total system emissions released, for every $s \in \mathcal{S}$ and $t \in \mathcal{T}$, is

$$\begin{aligned} e_{st}^{\text{tot}} := \sum_{r \in \mathcal{R}} \Bigg(& \sum_{p \in \mathcal{P}} \sum_{i \in \mathcal{I}_{\text{active}}^{\mathcal{P}}(s,p)} e_{pri} x_{prist} \\ & + \sum_{p \in \mathcal{P}_{\text{thermal}}} \sum_{i \in \mathcal{I}_{\text{active}}^{\mathcal{P}}(s,p)} \big(e_{pri}^{+} z_{prist}^{+} + \tilde{e}_{pri}(z_{prist} - x_{prist}) \big) \Bigg). \end{aligned} \tag{3.2.17}$$

Objective

Similar to the basic model, the objective is to minimize the total system costs. The sum (3.2.18a) considers the investment costs in electricity production technologies, with c_{ps}^{invtech} representing the investment cost (with annuity costs included) for technology type $p \in \mathcal{P}$ in year $s \in \mathcal{S}$ and c_p^{omf} is the fixed operation and maintenance costs for technology type $p \in \mathcal{P}$. The sum (3.2.18b) considers the costs of electricity production, with c_{pri}^{run} denoting the run costs for technology type $p \in \mathcal{P}$ in region $r \in \mathcal{R}$, where the investment is done in investment period $i \in \mathcal{I}$. The sum (3.2.18c) describes the additional costs for thermal power technology types $p \in \mathcal{P}_{\text{thermal}}$ using c^{+}_{prs} as the start-up cost for hot capacity in region $r \in \mathcal{R}$ and year $s \in \mathcal{S}$, and $\tilde{c}_{prs}$ as the additional cost for running on part-load capacity, and the sum (3.2.18d) corresponds to the total system emissions costs. The sum (3.2.18e) corresponds to the investment costs in new transmission capacity, with c^{invtra}_{kqr} being the investment cost in transmission capacity of type $k \in \mathcal{K}$ between regions $(q, r) \in \mathcal{A}$. This cost parameter is halved compared to the real cost in order to compensate for the network representation of using directed arcs, while in reality electricity is sent in either direction on the same transmission line. Finally, the sum (3.2.18f) covers the transmission cost of sending electricity using transmission technology $k \in \mathcal{K}$ on transmission line $(q, r) \in \mathcal{A}$. The cost parameter is denoted c^{tra}_{kqr}. The objective function to minimize is then given by

$$\sum_{s\in\mathcal{S}} \Bigg(\sum_{p\in\mathcal{P}} \sum_{r\in\mathcal{R}} \left(c^{\text{invtech}}_{ps} + c^{\text{omf}}_{p}\right) y_{prs} \tag{3.2.18a}$$

$$+ \sum_{p\in\mathcal{P}} \sum_{r\in\mathcal{R}} \sum_{i\in\mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \sum_{t\in\mathcal{T}} c^{\text{run}}_{pri} x_{prist} \tag{3.2.18b}$$

$$+ \sum_{p\in\mathcal{P}_{\text{thermal}}} \sum_{r\in\mathcal{R}} \sum_{i\in\mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \sum_{t\in\mathcal{T}} \left(c^{+}_{prs} z^{+}_{prist} + \tilde{c}_{prs}\left(z_{prist} - x_{prist}\right)\right) \tag{3.2.18c}$$

$$+ \sum_{t\in\mathcal{T}} c^{\text{CO}_2}_{s} e^{\text{tot}}_{st} \tag{3.2.18d}$$

$$+ \sum_{k\in\mathcal{K}} \sum_{(q,r)\in\mathcal{A}} c^{\text{invtra}}_{kqr} u_{kqrs} \tag{3.2.18e}$$

$$+ \sum_{k\in\mathcal{K}} \sum_{(q,r)\in\mathcal{A}} \sum_{t\in\mathcal{T}} c^{\text{tra}}_{kqr} v_{kqrst} \Bigg). \tag{3.2.18f}$$

3.3 Hours-to-Decades model

The major difference between the full-scale model presented in Section 3.2 and the Hours-to-Decades model is that the latter model 1) includes solar power, hydrogen storage, and batteries, 2) disregards explicit yearly connections and instead links the years implicitly by adding operational costs to the objective, and 3) is decomposed into 2-week segments, within each of which the chronology is retained, providing 26 separate submodels. The purpose of this decomposition is to take advantage of the possibility to solve the submodels in parallel, which thus would shorten the computing times. The Hours-to-Decades model is solved by the use of a heuristic algorithm, i.e. a *consensus loop* which enables information to be exchanged between the submodels. Each separate subproblem represents the problem of meeting the demand for electricity while minimizing investment and operational costs in its 2-week segment. In the consensus loop, information from the solutions is gathered in capacity–cost curves in which the capacity invested in all 2-week segments have the lowest cost, while additional capacity invested in a subset of the segments is more expensive. The smaller the subset of segments, the more expensive the capacity. The solution process is iterated until there is consensus, i.e. until the capacity–cost curves are unchanged between iterations. The methodology for solving this model will be further discussed in Chapter 4, while the remainder of this section will focus on the electricity system investment model.

Table 3.3 provides a list of the sets used in this model. The notation here differs from the previous two models as it was originally primarily aimed at an energy system modelling audience.

The set $\mathcal{I}$ is the set of all regions. The set $\mathcal{P}$ contains electricity generation technologies $\mathcal{P}^{\text{gen}} := \mathcal{P}^{\text{wind}} \cup \mathcal{P}^{\text{therm}} \cup \mathcal{P}^{\text{solar}}$, i.e. wind power, thermal power, and solar power. It also contains $\mathcal{P}^{\text{bat}}$, $\mathcal{P}^{\text{electrolysis}}$, and $\mathcal{P}^{\text{hydrogen}}$, which are the sets of battery technology, electrolyzer technology, and hydrogen storage technology, respectively. The set $\mathcal{Q}$ is the set of all transmission technologies, which are the same as in the full-scale model. The set $\mathcal{S} := \{1, \ldots, S\}$ contains all 2-week segments, where $S = 26$. The set of time steps is denoted $\mathcal{T}_s := \{(s-1)T + 1, \ldots, sT\}$ for $s \in \mathcal{S}$. The model uses thermal cycling constraints similar to the previous models, and therefore $\mathcal{K}_p$ denotes the set of hours in the start-up interval for technology $p \in \mathcal{P}_{\text{thermal}}$. The set of cost classes $\mathcal{R}$ are calculated by the cost-capacity curves in the consensus loop, and are used to determine the cost of an investment.

Table 3.3: The index sets used in the Hours-to-Decades model

symbol	representation	unit
$\mathcal{I}$	set of all regions	i, j
$\mathcal{P}$	$:= \mathcal{P}^{\text{bat}} \cup \mathcal{P}^{\text{electrolysis}} \cup \mathcal{P}^{\text{hydrogen}} \cup \mathcal{P}^{\text{gen}}$; set of all technology aggregates	p
$\mathcal{P}^{\text{bat}}$	set of all battery technologies	p
$\mathcal{P}^{\text{electrolysis}}$	set of all electrolyzer technologies	p
$\mathcal{P}^{\text{hydrogen}}$	set of all hydrogen storage technologies	p
$\mathcal{P}^{\text{gen}}$	$:= \mathcal{P}^{\text{wind}} \cup \mathcal{P}^{\text{therm}} \cup \mathcal{P}^{\text{solar}}$; set of all electricity generation technologies	p

$\mathcal{P}^{\text{wind}}$	set of all wind technologies	p
$\mathcal{P}^{\text{therm}}$	set of all thermal technologies	p
$\mathcal{P}^{\text{solar}}$	set of all solar technologies	p
$\mathcal{Q}$	set of technologies for transmission	q
$\mathcal{S}$	$:= \{1, \ldots, S\}$; set of all 2-week segments (typically, $S = 26$)	s
$\mathcal{T}_s$	$:= \{(s-1)T+1, \ldots, sT\}$; set of all time steps in the 2-week segment $s \in \mathcal{S}$	t
$\mathcal{K}_p$	$:= \{0, \ldots\}$; set of hours in the start-up interval for technology $p \in \mathcal{P}_{\text{thermal}}$	k
$\mathcal{R}$	set of cost classes, i.e. the steps in the cost–supply curve	r

A list of the set elements and a full nomenclature list is given in Chapter 5 and Appendix A.2, respectively. The constraints and objective function for the model are given below.

Network balance and generation limits

Define the decision variables

w_{ipr} = installed electricity generation and storage capacity in technology $p \in \mathcal{P}$ in region $i \in \mathcal{I}$ and cost class $r \in \mathcal{R}$,

h_{ijqr} = installed transmission capacity between regions $i \in \mathcal{I}$ and $j \in \mathcal{I} \setminus \{i\}$, using transmission technology $q \in \mathcal{Q}$ in cost class $r \in \mathcal{R}$.

The consensus loop to be described in Chapter 4 computes cost class potentials, denoted M^{e}_{ipr} and M^{h}_{ijqr}, which acts as upper limit for the respective investment variables. This yields the constraints

$$w_{ipr} \leq M^{\text{e}}_{ipr}, \qquad i \in \mathcal{I},\ p \in \mathcal{P},\ r \in \mathcal{R}, \tag{3.3.1}$$

$$h_{ijqr} \leq M^{\text{h}}_{ijqr}, \qquad i \in \mathcal{I} \setminus \{j\},\ j \in \mathcal{I},\ q \in \mathcal{Q},\ r \in \mathcal{R}. \tag{3.3.2}$$

Introduce the decision variables

$$
\begin{aligned}
g_{ipt} &= \text{electricity generation in region } i \in \mathcal{I} \text{ of production type } p \in \mathcal{P}^{\text{gen}} \\
&\quad\ \text{during time step } t \in \mathcal{T}_s,\ s \in \mathcal{S}, \\
g_{ipt} &= \text{energy storage in region } i \in \mathcal{I} \text{ in battery type } p \in \mathcal{P}^{\text{bat}} \\
&\quad\ \text{during time step } t \in \mathcal{T}_s,\ s \in \mathcal{S}, \\
g_{ipt} &= \text{hydrogen storage in region } i \in \mathcal{I} \text{ in battery type } p \in \mathcal{P}^{\text{hydrogen}} \\
&\quad\ \text{during time step } t \in \mathcal{T}_s,\ s \in \mathcal{S}, \\
e_{ijt} &= \text{exported electricity from region } i \in \mathcal{I} \text{ to region } j \in \mathcal{I} \setminus \{i\} \\
&\quad\ \text{in time step } t \in \mathcal{T}_s,\ s \in \mathcal{S}, \\
b_{ipt}^{\text{charge}} &= \text{charging of battery technology } p \in \mathcal{P}^{\text{bat}} \text{ in region } i \in \mathcal{I} \text{ and time} \\
&\quad\ \text{step } t \in \mathcal{T}_s,\ s \in \mathcal{S}, \\
b_{ipt}^{\text{discharge}} &= \text{discharging of battery technology } p \in \mathcal{P}^{\text{bat}} \text{ in region } i \in \mathcal{I} \text{ and} \\
&\quad\ \text{time step } t \in \mathcal{T}_s,\ s \in \mathcal{S}.
\end{aligned}
$$

Let d_{it}^{hydrogen} denote the electricity consumption of the electrolyzer. The demand for electricity, D_{it}, must be met in all regions at all times. This is expressed, for $i \in \mathcal{I}$ and $t \in \mathcal{T}_s,\ s \in \mathcal{S}$, by the constraints

$$\sum_{p \in \mathcal{P}^{\text{gen}}} g_{ipt} \geq D_{it} + d_{it}^{\text{hydrogen}} + \sum_{j \in \mathcal{I} \setminus \{i\}} e_{ijt} + \sum_{p \in \mathcal{P}^{\text{bat}}} \left(b_{ipt}^{\text{charge}} - b_{ipt}^{\text{discharge}} \right). \tag{3.3.3}$$

The import and export of electricity are required to be balanced, and the export may not exceed the installed transmission capacity, as expressed by the relations

$$-e_{ijt} = e_{jit} \leq \sum_{m \in \mathcal{I} \setminus \{i\}} \sum_{q \in \mathcal{Q}} \sum_{r \in \mathcal{R}} h_{imqr}, \quad i \in \mathcal{I} \setminus \{j\},\ j \in \mathcal{I},\ t \in \mathcal{T}_s,\ s \in \mathcal{S}, \tag{3.3.4}$$

$$e_{ijt}^{\text{pos}} = |e_{ijt}| = \max \{ e_{ijt}, e_{jit} \}, \qquad i \in \mathcal{I} \setminus \{j\},\ j \in \mathcal{I},\ t \in \mathcal{T}_s,\ s \in \mathcal{S}. \tag{3.3.5}$$

Let $\theta_{ipt} \in [0, 1]$ be a profile, which is weather-dependent for wind and solar power but equals 1 for all $p \in \mathcal{P}^{\text{therm}}$. The level of electricity generation may not exceed the installed capacity, which is weighted by the weather profile:

$$g_{ipt} \leq \sum_{r \in \mathcal{R}} w_{ipr} \theta_{ipt}, \qquad i \in \mathcal{I},\ p \in \mathcal{P} \setminus \mathcal{P}^{\text{electrolysis}},\ t \in \mathcal{T}_s,\ s \in \mathcal{S}. \tag{3.3.6}$$

Battery storage

Flow batteries and lithium ion batteries are amongst the investment options in the model. An energy balance constraint is needed to manage the storage of each battery type. The battery storage level during the last time step of each 2-week segment $s \in \mathcal{S}$ constrains the battery storage level in the first time step of the same 2-week segment. Here, g_{ipt} is the storage level of the battery, $\eta_p b_{ipt}^{\text{charge}}$ is the charging of the battery where η_p is the efficiency of battery type $p \in \mathcal{P}^{\text{bat}}$, and $b_{ipt}^{\text{discharge}}$ is the discharging of the battery. For each $i \in \mathcal{I},\ p \in \mathcal{P}^{\text{bat}}$ and $s \in \mathcal{S}$ the constraints are expressed as

$$g_{ipt} + \eta_p b_{ipt}^{\text{charge}} - b_{ipt}^{\text{discharge}} \geq \begin{cases} g_{i,p,t+1}, & t \in \mathcal{T}_s \setminus \{sT\}, \\ g_{i,p,t-(T-1)}, & t = sT. \end{cases} \tag{3.3.7}$$

Each battery type has an installed storage capacity, and the charging and discharging of batteries may not exceed this limit:

$$b_{ipt}^{\text{charge}} \leq \sum_{r \in \mathcal{R}} w_{ipr}, \qquad i \in \mathcal{I},\ p \in \mathcal{P}^{\text{bat}},\ t \in \mathcal{T}_s,\ s \in \mathcal{S}, \tag{3.3.8}$$

$$b_{ipt}^{\text{discharge}} \leq \sum_{r \in \mathcal{R}} w_{ipr}, \qquad i \in \mathcal{I},\ p \in \mathcal{P}^{\text{bat}},\ t \in \mathcal{T}_s,\ s \in \mathcal{S}. \tag{3.3.9}$$

Hydrogen storage

Hydrogen storage uses an electrolysis process, which is based on using electricity to split water into hydrogen and oxygen. This process takes place inside an electrolyzer. The investments in hydrogen storage are stimulated by introducing a demand for electricity in hydrogen production for industry. Let D_i^{hydrogen} be the industry demand for hydrogen which is evenly distributed over the year, in region $i \in \mathcal{I}$. The hydrogen production in the electrolyzer is given by $\eta_p d_{it}^{\text{hydrogen}}$ for $p \in \mathcal{P}^{\text{hydrogen}}$, $i \in \mathcal{I}$ and $t \in \mathcal{T}_s,\ s \in \mathcal{S}$, where η_p denotes the efficiency of charging the hydrogen storage. Furthermore, the storage level and the charging and discharging of the hydrogen storage during the last time step of the 2-week segment are used to constrain the hydrogen storage level in the first time step of the same 2-week segment. Thus, for $i \in \mathcal{I},\ p \in \mathcal{P}^{\text{hydrogen}}$ and $s \in \mathcal{S}$, this is modelled as

$$g_{ipt} + \eta_p d_{it}^{\text{hydrogen}} - D_i^{\text{hydrogen}} \geq \begin{cases} g_{i,p,t+1}, & t \in \mathcal{T}_s \setminus \{sT\}, \\ g_{i,p,t-(T-1)}, & t = sT. \end{cases} \tag{3.3.10}$$

The electricity consumption of the electrolyzer, d_{it}^{hydrogen}, may not exceed the installed electrolyzer capacity:

$$d_{it}^{\text{hydrogen}} \leq \sum_{r \in \mathcal{R}} w_{ipr}, \qquad i \in \mathcal{I},\ p \in \mathcal{P}^{\text{electrolysis}},\ t \in \mathcal{T}_s,\ s \in \mathcal{S}. \tag{3.3.11}$$

Wind and solar power

The wind power capacity in technology $p \in \mathcal{P}^{\text{wind}}$ and region $i \in \mathcal{I}$ is limited by the regional resources A_{ip}, which implies an upper bound on wind power investments:

$$\sum_{r \in \mathcal{R}} w_{ipr} \leq A_{ip}, \qquad i \in \mathcal{I},\ p \in \mathcal{P}^{\text{wind}}; \tag{3.3.12}$$

For solar power, there is a total resource constraint for each modeled region $i \in \mathcal{I}$:

$$\sum_{r \in \mathcal{R}} \sum_{p \in \mathcal{P}^{\text{solar}}} w_{ipr} \leq \sum_{p \in \mathcal{P}^{\text{solar}}} A_{ip}, \qquad i \in \mathcal{I}. \tag{3.3.13}$$

Thermal cycling

As in the previous models, thermal cycling is here accounted for by applying the relaxed unit commitment approach suggested by Weber (2005). Let

g_{ipt}^{active} = capacity that is active and available for generation in each time step $t \in \mathcal{T}_s,\ s \in \mathcal{S}$, in region $i \in \mathcal{I}$ and within each technology aggregate $p \in \mathcal{P}^{\text{therm}}$.

g_{ipt}^{on} = capacity started in each time step $t \in \mathcal{T}_s,\ s \in \mathcal{S}$, in region $i \in \mathcal{I}$ and within each technology aggregate $p \in \mathcal{P}^{\text{therm}}$;

The electricity generation should stay below the active capacity in each time step. Moreover, the minimum load share of the active capacity for technology $p \in \mathcal{P}^{\text{therm}}$ is given by ϕ_p, and the electricity generation is not allowed to be below this level. Hence, the following inequalities are included in the model:

$$\xi_p^{\min} g_{ipt}^{\text{active}} \leq g_{ipt} \leq g_{ipt}^{\text{active}}, \qquad i \in \mathcal{I},\ p \in \mathcal{P}^{\text{therm}},\ t \in \mathcal{T}_s,\ s \in \mathcal{S}. \tag{3.3.14}$$

As in the previous models, the active capacity is limited in each time step by the sum of the started capacity and the active capacity in the previous time step. However, for the first time step of each 2-week segment, except the first segment, the active capacity in the previous time step is represented by the active capacity in the last time step of the previous segment, as given by the previous iteration of the consensus loop (see Chapter 4.4), i.e. by $G^{\text{active}}_{i,p,t-1}$. Moreover, for the first time step of the first segment (i.e. for $t = 1$), the active capacity in the last time step of the last segment is used, as given by the previous iteration of the consensus loop, i.e. by $G^{\text{active}}_{i,p,ST}$. For $i \in \mathcal{I}$ and $p \in \mathcal{P}^{\text{therm}}$, these relations are modelled by the inequalities

$$g^{\text{active}}_{ipt} \leq g^{\text{on}}_{ipt} + \begin{cases} g^{\text{active}}_{i,p,t-1}, & t \in \mathcal{T}_s \setminus \{(s-1)T+1\}, \quad s \in \mathcal{S}, \\ G^{\text{active}}_{i,p,t-1}, & t = (s-1)T+1, \qquad\qquad s \in \mathcal{S} \setminus \{1\}, \\ G^{\text{active}}_{i,p,ST}, & t = 1. \end{cases} \tag{3.3.15}$$

Define the variable

c^{cycl}_{ipt} = resulting thermal cycling costs in region $i \in \mathcal{I}$ for technology type $p \in \mathcal{P}^{\text{therm}}$ in time step $t \in \mathcal{T}_s,\ s \in \mathcal{S}$.

As in the previous models, the start-up cost is proportional to the started capacity g^{on}_{ipt}, while the part-load cost is proportional to the difference between the active generation capacity and the generation level. In order to avoid boundary effects on the last time step of the 2-week segment, we include a value for the active capacity which is proportional to the solution given in the previous iteration. This scaling is based on the start-up cost $C^{\text{on}}_{i,p,t+1}G^{\text{on}}_{i,p,t+1}$ and active capacity $G^{\text{active}}_{i,p,t+1}$ paid in the first hour of the following 2-week segment. For each $i \in \mathcal{I}$ and $p \in \mathcal{P}^{\text{therm}}$, these constraints are expressed as

$$c^{\text{cycl}}_{ipt} \geq C^{\text{on}}_{ipt} g^{\text{on}}_{ipt} + C^{\text{part}}_{ipt} \left(g^{\text{active}}_{ipt} - g_{ipt} \right) - G_{ipt}, \qquad t \in \mathcal{T}_s,\ s \in \mathcal{S}, \tag{3.3.16}$$

where

$$G_{ipt} := \frac{g^{\text{active}}_{ipt}}{2} \cdot \begin{cases} 0, & t \in \mathcal{T}_s \setminus \{sT\}, \quad s \in \mathcal{S}, \\ \frac{C^{\text{on}}_{i,p,t+1} G^{\text{on}}_{i,p,t+1}}{G^{\text{active}}_{i,p,t+1}}, & t = sT, \qquad\qquad s \in \mathcal{S} \setminus \{S\}, \\ \frac{C^{\text{on}}_{ip1} G^{\text{on}}_{ip1}}{G^{\text{active}}_{ip1}}, & t = ST. \end{cases} \tag{3.3.17}$$

Hence, if thermal capacity is active in the end of one 2-week segment and also in the beginning of the subsequent 2-week segment, the start-up cost for that capacity is shared equally between the segments.

As explained previously, thermal generation is subject to a start-up time, i.e. it takes some time for a thermal power plant to heat up before it can deliver electricity. Thus, in the model, once capacity is deactivated, it cannot become active again during the interval $\mathcal{K}_p$, which encompasses the time-steps k in the start-up interval. For $i \in \mathcal{I}$ and $p \in \mathcal{P}^{\text{therm}}$, this is expressed as

$$g_{ipt}^{\text{on}} \leq \sum_{r \in \mathcal{R}} w_{ipr} - g_{i,p,t-k}^{\text{active}}, \qquad t \in \mathcal{T}_s,\ s \in \mathcal{S},\ k \in \mathcal{K}_p \setminus \{t, \dots, sT\}. \tag{3.3.18}$$

Objective

For each 2-week segment $s \in \mathcal{S}$, the objective function to be minimized is expressed as

$$\begin{aligned}
c_s^{\text{tot}} := &\sum_{i \in \mathcal{I}} \sum_{p \in \mathcal{P}} \sum_{r \in \mathcal{R}} C_p^{\text{inv}} \lambda_{iprs}^{\text{e}} w_{ipr} && \text{(3.3.19a)}\\
&+ \sum_{i \in \mathcal{I}} \sum_{p \in \mathcal{P}} \sum_{t \in \mathcal{T}_s} \left(C_{pt}^{\text{run}} g_{ipt} + c_{ipt}^{\text{cycl}} \right) && \text{(3.3.19b)}\\
&+ \sum_{q \in \mathcal{Q}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I} \setminus \{i\}} \sum_{r \in \mathcal{R}} C_q^{\text{h-inv}} \lambda_{ijqrs}^{\text{h}} h_{ijqr} && \text{(3.3.19c)}\\
&+ \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{I} \setminus \{i\}} \sum_{t \in \mathcal{T}_s} C_t^{\text{exp}} e_{jit}^{\text{pos}}, && \text{(3.3.19d)}
\end{aligned}$$

where (3.3.19a) represents the costs for investments in the different technologies in the different regions, (3.3.19b) the running costs of the different technologies in the different regions at all time steps within the 2-week segment, (3.3.19c) the costs for investments in technologies for transmission of electricity between the regions, and (3.3.19d) the costs of transmitting electricity between the regions in each time step within the 2-week segment.

Here, for the cost class $r \in \mathcal{R}$ and segment $s \in \mathcal{S}$, the parameters $\lambda_{iprs}^{\text{e}}$ and $\lambda_{ijqrs}^{\text{h}}$ represent shares of the investment costs for electricity generating technologies $p \in \mathcal{P}$ in region $i \in \mathcal{I}$ and transmission technologies $q \in \mathcal{Q}$ between regions $i \in \mathcal{I}$ and $j \in \mathcal{I} \setminus \{i\}$, respectively.

3.4 Comparison of the different models

The three models presented in Sections 3.1, 3.2, and 3.3, respectively, are very alike in terms of constraints. There are however some differences, of which some effect the mathematical structure of the models.

The basic model (Section 3.1) does not consider several regions and therefore does not include electricity trade. As discussed earlier, trade is a powerful tool to smoothen the variations from wind power. It is arguably also more realistic since electricity trade is used within Europe today. However, since the spatial resolution increases, the problem size does as well and therefore the solution times grows rapidly. If trade is included (as in the full-scale model; Section 3.2), the model expands and includes a network structure. Furthermore, variations from seasons differ between the models. For the Hours-to-Decades model (Section 3.3), seasonal variability is represented by accounting for all 2-week segments in a year. However, dimensioning of seasonal storage (i.e. storages shifting electricity from summer to winter or vice versa) is not possible with this approach. If storage technologies (besides hydro power) were included in the basic model and the full-scale model, seasonal storage dimensioning would not be an issue.

On the other hand, there are some differences which are not important to the model structure. The Hours-to-Decades model includes solar power technologies, as well as storage technologies. Expanding the other models to include solar power technologies do not change their respective model structure since solar power does not yield any additional constraints. The storage technologies come with the benefits of demand shifting, and is thus important for variation management. However, the Hours-to-Decades model uses run-of-river hydro power technologies, which implies that the electricity generated by these technologies must be used instantaneously and can't be saved for later use. The basic and full-scale models, on the other hand, allow the inflow water to be stored in reservoirs. Thus, the constraints for hydro power in these latter models possess the same structure as the storage constraints in the Hours-to-Decades model.

Moreover, the models serve different purposes and can be used to answer different questions. The Hours-to-Decades model presents an initial idea of how to decompose the problem as it gives very promising results and converges in only a few iterations of the consensus algorithm. The full-scale model acts a starting point for the relaxation as it contains many of the features necessary to analyze for a variable renewable electricity integration, but is demanding to solve for high resolution data. The basic model can be used to explain or evaluate the results from the Hours-to-Decades model and the full-scale model. Furthermore, it can be used to provide feasible solutions for the full-scale model. If the latter model is decomposed and then solved by a subgradient algorithm (see Chapter 4), a step length rule such as the Polyak step length rule (see Section 4.3) is necessary. This step rule requires an upper bound on the objective function value, which is provided by any feasible solution to the full-scale model.

4 Mathematical methods and their theory

The models introduced in Chapter 3 are by nature very large and therefore practically impossible to solve in reasonable solution times. To counteract this, different mathematical methods need to be used. For example, the Hours-to-Decades model in Paper II uses a heuristic where the time dimension is discretized into 2-week segments, allowing 26 problems to—in theory—be solved in parallel. Information between these segments is exchanged by the use of a consensus loop; see Section 4.4. The backside of this method is, however, that since it is a heuristic, optimality can not be guaranteed. Other more mathematically robust methods exploit the model structure by using Lagrangian duality and Lagrangian relaxation, which first came to light by the seminal work of Held and Karp (1971). Some decomposition methods relating to electrical energy applications are also covered in Sagastizábal (2012). Sections 4.1–4.3 discusses the decomposition methods used in Paper I.

4.1 Lagrangian duality and Lagrangian relaxation

Many large optimization problems are structured such that they consist of several smaller separate problems connected by some overlapping, typically complicating, constraints. Each separate problem is, however, often more easily solvable in comparison to the full problem. Lagrangian dual methods, such as Lagrangian relaxation, can take advantage of this problem structure; see Guignard (2003).

The idea of Lagrangian relaxation is to relax the connecting constraints such that the remaining problem is separable into several subproblems. Consider the linear optimization problem to

$$\begin{array}{lrcl} \text{minimize} & z := & \boldsymbol{c}^\top \boldsymbol{x}, & \\ \text{subject to} & & \boldsymbol{g}(\boldsymbol{x}) & \leq \boldsymbol{0}^m, \\ & & \boldsymbol{x} & \in \boldsymbol{X}, \end{array} \tag{4.1.1}$$

where $\boldsymbol{c} \in \mathbb{R}^n$, $\boldsymbol{x} \in \mathbb{R}^n$, $\boldsymbol{g} : \mathbb{R}^n \to \mathbb{R}^m$, $\boldsymbol{X} \subset \mathbb{R}^n$, $m, n \in \mathbb{Z}_+$. We also assume that $\{\boldsymbol{x} \in \boldsymbol{X} \,|\, \boldsymbol{g}(\boldsymbol{x})) \leq \boldsymbol{0}^m\} \neq \emptyset$ such that there exists a feasible solution. Here, $\boldsymbol{g}(\boldsymbol{x}) \leq \boldsymbol{0}^m$ are connecting constraints while the remaining set $\boldsymbol{x} \in \boldsymbol{X}$ is separable; see Figure 4.1. Define the Lagrangian function $L : \mathbb{R}^{m+n} \to \mathbb{R}$ such

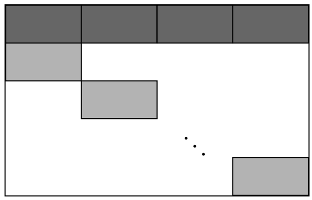


Figure 4.1: A block diagonal matrix besides the rows representing the connecting constraints. Here, the dark area corresponds to the connecting constraints $\boldsymbol{g}(\boldsymbol{x})$, while the lighter area is the separable set $\boldsymbol{x} \in \boldsymbol{X}$.

that $L(\boldsymbol{x}, \boldsymbol{\pi}) := \boldsymbol{c}^\top \boldsymbol{x} + \boldsymbol{\pi}^\top \boldsymbol{g}(\boldsymbol{x})$, using the Lagrangian multipliers $\boldsymbol{\pi} \in \mathbb{R}^m$. The Lagrangian dual problem is then defined as

$$\max_{\boldsymbol{\pi} \geq \boldsymbol{0}} \quad h(\boldsymbol{\pi}), \tag{4.1.2}$$

where

$$h(\boldsymbol{\pi}) \quad := \quad \min_{\boldsymbol{x} \in \boldsymbol{X}} L(\boldsymbol{x}, \boldsymbol{\pi}) \; = \min_{\boldsymbol{x} \in \boldsymbol{X}} \left(\boldsymbol{c}^\top \boldsymbol{x} + \boldsymbol{\pi}^\top \boldsymbol{g}(\boldsymbol{x}) \right) \tag{4.1.3}$$

and $h : \mathbb{R}^m \to \mathbb{R}$ is denoted the Lagrangian dual function. Here, for some $\boldsymbol{\pi} \geq \boldsymbol{0}^m$, the problem of minimizing the Lagrangian function L over its first argument $\boldsymbol{x} \in X$ is referred as the subproblem, and since we assumed a block diagonal structure of the set $\boldsymbol{x} \in \boldsymbol{X}$, this problem is separable.

Let z^* denote the optimal objective function value in problem (4.1.1). By weak duality, $h(\boldsymbol{\pi}) \leq z^*$ then holds that for any $\boldsymbol{\pi} \geq \boldsymbol{0}$; that is, any feasible solution to the dual problem provides a lower bound on the optimal objective value of the original problem. Furthermore, any feasible solution $\bar{\boldsymbol{x}}$ provides an upper bound for z^* since the inequality $z^* \leq z(\bar{\boldsymbol{x}})$ holds. Moreover, strong duality implies that the equality $z^* = h^*$ holds.

4.2 Variable splitting

For the three models presented in Chapter 3, relaxing over the time dimension is tricky since some variables are time-independent. To illustrate this, we look at the Example 1.

Example 1. Divide the year into time periods (for example, segments of two weeks as in the Hours-to-Decades model) over the year such that the set $\mathcal{T}_n$ contains the time steps in a period $n \in \mathcal{N} := \{1, ..., N\}$. Define the variables $x_t \in X_t \subseteq \mathbb{R}_+$ and $y \geq 0$, representing the electricity generation in time step $t \in \mathcal{T}_n$, $n \in \mathcal{N}$, and the invested capacity, respectively. Then, consider the problem to

$$\begin{array}{llll} \underset{y, x_t}{\text{minimize}} & c^{\text{inv}} y + \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}_n} c_t^{\text{run}} x_t, & & \\ \text{subject to} & x_t \leq y, & t \in \mathcal{T}_n, \ n \in \mathcal{N}, & \\ & x_t \in X_t, & t \in \mathcal{T}_n, \ n \in \mathcal{N}, & \\ & y \geq 0, & & \end{array} \tag{4.2.1}$$

where c^{inv} and c_t^{run}, $t \in \mathcal{T}_n$, $n \in \mathcal{N}$, are investment costs and run costs, respectively. This problem is a simplified version of the models introduced in Chapter 3. Here, the variable y is complicating the problem since it is time independent. The constraints $x_t \leq y$ can be relaxed, but they represent an important property of the electricity system (that you can not produce more electricity than the installed capacity). Thus they are very important for the model structure and should not be relaxed. This is where the concept of *variable splitting* comes in, introduced by Jörnsten and Näsberg (1986). The main idea is to split a variable into several variables, and then add the constraints that they should all equal the original variable. Thus, let us introduce splitting variables for the investment variable in problem (4.2.1) by letting $y_n^{\text{split}} := y$, $n \in \mathcal{N}$. This gives the equivalent problem formulation

$$\begin{array}{lll} \underset{y, y_n^{\text{split}}, x_t}{\text{minimize}} & \frac{c^{\text{inv}}}{N} \sum_{n \in \mathcal{N}} y_n^{\text{split}} + \sum_{n \in \mathcal{N}} \sum_{t \in \mathcal{T}_n} c_t^{\text{run}} x_t, & \\ \text{subject to} & y_n^{\text{split}} = y, & n \in \mathcal{N}, \\ & x_t \leq y_n^{\text{split}}, & t \in \mathcal{T}_n, \ n \in \mathcal{N}, \\ & x_t \in X_t, & t \in \mathcal{T}_n, \ n \in \mathcal{N}, \\ & y \geq 0. & \end{array}$$

Now, let π_n denote the Lagrangian dual variables corresponding to the relaxation of the constraints $y_n^{\text{split}} = y$, $n \in \mathcal{N}$. The Lagrangian dual function is then defined by the minimization of the subproblem, according to

$$h(\boldsymbol{\pi}) := \left(\begin{array}{lll} \min\limits_{y, y_n^{\text{split}}, x_t} & \left[\sum\limits_{n\in\mathcal{N}} \frac{c^{\text{inv}}}{N} y_n^{\text{split}} + \sum\limits_{n\in\mathcal{N}} \sum\limits_{t\in\mathcal{T}_n} c_t^{\text{run}} x_t + \sum\limits_{n\in\mathcal{N}} \pi_n (y - y_n^{\text{split}}) \right] & \\ \text{s.t.} & x_t \le y_n^{\text{split}}, \quad t \in \mathcal{T}_n,\ n \in \mathcal{N}, & \\ & x_t \in X_t, \qquad t \in \mathcal{T}_n,\ n \in \mathcal{N}, & \\ & y \ge 0 & \end{array} \right) \tag{4.2.2a}$$

$$= \min_{y\ge 0} \sum_{n\in\mathcal{N}} \pi_n y \tag{4.2.2b}$$

$$+ \sum_{n\in\mathcal{N}} \left(\begin{array}{ll} \min\limits_{y_n^{\text{split}}, x_t} & \left[\left(\frac{c^{\text{inv}}}{N} - \pi_n \right) y_n^{\text{split}} + \sum\limits_{t\in\mathcal{T}_n} c_t^{\text{run}} x_t \right] \\ \text{s.t.} & x_t \le y_n^{\text{split}}, \quad t \in \mathcal{T}_n, \\ & x_t \in X_t, \qquad t \in \mathcal{T}_n \end{array} \right). \tag{4.2.2c}$$

The corresponding dual problem is

$$\begin{array}{ll} \text{maximize} & h(\boldsymbol{\pi}), \\ \text{subject to} & \pi_n \in \mathbb{R}, \quad n \in \mathcal{N}, \end{array} \tag{4.2.3}$$

and we can see that it has some additional properties:

$$\max_{\boldsymbol{\pi}\in\mathbb{R}^N} \left\{ \min_{y\ge 0} \sum_{n\in\mathcal{N}} \pi_n y \right\} \implies \begin{cases} y = 0, & \text{when } \sum\limits_{n\in\mathcal{N}} \pi_n \ge 0, \\ y \to \infty, & \text{when } \sum\limits_{n\in\mathcal{N}} \pi_n < 0. \end{cases} \tag{4.2.4}$$

Thus, for $\sum_{n\in\mathcal{N}} \pi_n < 0$, the subproblem (4.2.2b) is unbounded. Therefore, this implicit constraint on the dual variables can be expressed explicitly in the dual problem. Furthermore, since the inequality $\sum_{n\in\mathcal{N}} \pi_n \ge 0$ implies that $y = 0$ in the solution to (4.2.2b), we can remove the variable y in the subproblem when the dual problem is subject to this constraint. Thus, with $y = 0$ the subproblem (4.2.2a) is separable over $n \in \mathcal{N}$, which means that the solution process can be parallelized.

The analogous reasoning for the subproblem in (4.2.2c) yields that if the inequality $\pi_n > \frac{c^{\text{inv}}}{N}$ holds, the nth subproblem will be unbounded, $n \in \mathcal{N}$. Hence we include the explicit constraints $\pi_n \le \frac{c^{\text{inv}}}{N}$, $n \in \mathcal{N}$, in the dual problem. The

Lagrangian dual problem can thus be stated as to

$$\begin{aligned} \text{maximize} \quad & \hat{h}(\boldsymbol{\pi}), \\ \text{subject to} \quad & \sum_{n \in \mathcal{N}} \pi_n \geq 0, \\ & \pi_n \leq \frac{c^{\text{inv}}}{N}, \qquad n \in \mathcal{N}, \end{aligned} \tag{4.2.5}$$

where

$$\hat{h}(\boldsymbol{\pi}) = \sum_{n \in \mathcal{N}} \left(\begin{array}{ll} \min\limits_{y_n^{\text{split}}, x_t} & \left[\left(\frac{c^{\text{inv}}}{N} - \pi_n \right) y_n^{\text{split}} + \sum\limits_{t \in \mathcal{T}_n} c_t^{\text{run}} x_t \right] \\ \text{s.t.} & y_n^{\text{split}} \geq x_t \in X_t, \quad t \in \mathcal{T}_n \end{array} \right). \tag{4.2.6}$$

□

As demonstrated by the above example, the starting point for the variable splitting approach is to reformulate the problem such that copies of some primal variables are introduced. Constraints are added to ensure consistency between the original variables and the copies, and then these consistency constraints are Lagrangian relaxed. The method was developed simultaneously by different researchers, and is therefore also referred to as *Lagrangian decomposition* (Guignard and Kim, 1987) or *variable layering* (Glover and Klingman, 1988).

4.3 Subgradient algorithm

The subgradient algorithm was developed by N. Z. Shor in 1962; see Shor (1991) for a full review of the early history of nonsmooth optimization. This method has often been applied to solve optimization problems, especially together with Lagrangian duality. Larsson et al. (1996) formulated the conditional subgradient method, which combines subgradient methods with subgradient projection methods. However, we begin by a definition.

Definition 1. A vector $\boldsymbol{\gamma} \in \mathbb{R}^n$ is a *subgradient* of the concave function h at $\bar{\boldsymbol{\pi}} \in \mathbb{R}^n$ if the inequality

$$h(\boldsymbol{\pi}) \leq h(\bar{\boldsymbol{\pi}}) + \boldsymbol{\gamma}^\top (\boldsymbol{\pi} - \bar{\boldsymbol{\pi}}) \tag{4.3.1}$$

holds for all $\boldsymbol{\pi} \in \mathbb{R}^n$. The set of subgradients of h at $\bar{\boldsymbol{\pi}}$ is called the *subdifferential*, denoted $\partial h(\bar{\boldsymbol{\pi}})$. □

Geometrically, a subgradient is a vector defining a supporting hyperplane to the epigraph of the function h containing the point $\bar{\boldsymbol{\pi}}$. The subgradient algorithm is provided in Algorithm 1. Here, we assume that $\boldsymbol{\pi} \in \Pi$, such that Π is the feasible set for the multipliers $\boldsymbol{\pi}$.

Algorithm 1 Subgradient algorithm

1: Initiate $\boldsymbol{\pi}^0 \in \Pi$ and $h^0_{\text{best}} = h(\boldsymbol{\pi}^0)$. Let $k := 0$.
2: Find a subgradient to h at the point $\boldsymbol{\pi}^k$
$\implies$ solve the subproblem $\min_{\boldsymbol{x} \in \boldsymbol{X}} L(\boldsymbol{x}, \boldsymbol{\pi}^k)$
which gives a solution $\boldsymbol{x}(\boldsymbol{\pi}^k)$.
A subgradient to h is then given by $\boldsymbol{\gamma}^k := \boldsymbol{g}(\boldsymbol{x}(\boldsymbol{\pi}^k))$
3: For some $\alpha_k > 0$, the new point $\boldsymbol{\pi}^{k+1} := \text{Proj}_{\Pi}\{\boldsymbol{\pi}^k - \alpha_k \boldsymbol{\gamma}^k\}$
4: Update $h^{k+1}_{\text{best}} := \max\{h^k_{\text{best}}, h(\boldsymbol{\pi}^{k+1})\}$
5: Termination criteria fulfilled $\implies$ stop.
Otherwise, let $k := k + 1$ and go to 2

Step lengths α_k

The step lengths α_k are chosen according to some rule which guarantees convergence. According to Andréasson et al. (2020)[p.181], the first rule is the divergent series step length rule. It requires that

$$\alpha_k > 0,\ k = 0, 1, \dots; \quad \lim_{k \to \infty} \alpha_k = 0; \quad \sum_{k=0}^{\infty} \alpha_k = +\infty. \tag{4.3.2}$$

The second rule adds to (4.3.2) the square-summable restriction

$$\sum_{k=0}^{\infty} \alpha_k^2 < +\infty. \tag{4.3.3}$$

The conditions in (4.3.2) allow for convergence to any point from any starting point, since the total step is infinite, but convergence is therefore also quite slow; the additional condition in (4.3.3) means that fast sequences are selected. The third rule, and the one used in this work, is the Polyak step length rule (Polyak, 1969) which has seen much use in practice. It is defined as

$$\alpha_k = \frac{\theta_k \left(h^* - h(\boldsymbol{\pi}^k)\right)}{||\boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}(\boldsymbol{\pi}^k)||^2} \tag{4.3.4}$$

under the conditions

$$0 < \epsilon_1 \leq \theta_k \leq 2 - \epsilon_2 < 2, \quad k = 0, 1, 2, ... \tag{4.3.5}$$

Here, θ_k acts as a scaling parameter for the step length, and the parameters ϵ_1 and ϵ_2 define positive limits for the scaling parameter.

Polyak showed that using (4.3.4) together with (4.3.5) guarantees theoretical convergence to an optimal solution to the dual problem (4.1.2). However, the dual optimal value h^* is typically not known. If so, an upper bound $\bar{h} \geq h^*$ can be used instead to achieve finite convergence to an ϵ-optimal solution, where we define ϵ-optimal as $h(\boldsymbol{\pi}^k) \geq 2h^* - \bar{h} - \epsilon$ for any $\epsilon > 0$; see (Polyak, 1969, Theorem 4). For the full-scale model, an upper bound can be found by solving the basic model for each examined region. This gives a feasible solution to the full-scale model, but likely not optimal since no trade between regions is included. The upper bound is given as the sum of the total system costs for all the included regions.

The value of the scaling parameter θ_k can be chosen in different ways. One method that in practice has been shown to give fast convergence to an optimal solution is presented by Caprara et al. (1999). The authors presents the use of an adaptive method, where the value of the parameter is updated every p number of subgradient iterations:

$$\theta_{k+1} = \begin{cases} \frac{1}{2}\theta_k, & \text{if } \bar{h} - \underline{h} > 0.1 \cdot |\underline{h}|, \\ \frac{3}{2}\theta_k, & \text{if } \bar{h} - \underline{h} < 0.01 \cdot |\underline{h}|, \\ \theta_k, & \text{otherwise}, \end{cases} \qquad k = 1, 2, ..., \tag{4.3.6}$$

where

$$\bar{h} = \max_{r=k-p+1,\dots,k} h(\boldsymbol{\pi}^r) \quad \text{and} \quad \underline{h} = \min_{r=k-p+1,\dots,k} h(\boldsymbol{\pi}^r). \tag{4.3.7}$$

For every $p = 3$ subgradient iterations, the best and worst lower bounds found during the last p iterations are compared. If the difference is more than 10%, this means that too large steps are taken by the algorithm and thus the scaling parameter is halved. On the other hand, if the difference is less than 1%, this means the step length can be increase and thus the scaling parameter is multiplied by $\frac{3}{2}$. The values of p, 0.1, and 0.01 should be modified depending on the results from the algorithm. Note that this method does not require $0 < \theta_k < 2$.

Generating primal solutions by the use of ergodic sequences

In general, subgradient optimization methods often identify near-optimal dual solutions, but do not directly provide solutions to the primal problem. The conditional subgradient method constructs a sequence $\{\boldsymbol{x}(\boldsymbol{\pi}^k)\}$ of solutions to the Lagrangian subproblem, but these solutions are typically not feasible in the original primal problem since they will not satisfy the relaxed constraints. Thus, the sequence $\{\boldsymbol{x}(\boldsymbol{\pi}^k)\}$ does not converge to the optimal primal solution. To remedy this, *ergodic sequences of subproblem solutions* can be generated.

As first presented by Larsson et al. (1999), ergodic sequences creates approximations of primal solutions by averaging the solutions from the subproblems. The authors showed that the ergodic sequences in the limit produce optimal solutions to the original problem. An enchanced version in terms of convergence speed was introduced by Gustavsson et al. (2015). This version exploits more information from later subproblem solutions than from earlier ones. The ergodic sequence $\{\tilde{\boldsymbol{x}}_k\}$ is defined as

$$\tilde{\boldsymbol{x}}_k := \sum_{s=0}^{k-1} \mu_s^k \boldsymbol{x}(\boldsymbol{\pi}^s); \quad \sum_{s=0}^{k-1} \mu_s^k = 1; \quad \mu_s^k \geq 0, \quad s = 0, ..., k-1, \tag{4.3.8}$$

where the convexity weights μ_s^k are chosen according to the s^n-rule:

$$\mu_s^k := \frac{(s+1)^n}{\sum_{r=0}^{k-1}(r+1)^n}, \quad s = 0, ..., k-1, \quad k = 1, 2, ..., \quad n \geq 0. \tag{4.3.9}$$

For $n > 0$, the s^n-rule results in an ergodic sequence in which the later iterates are assigned higher weights than the earlier ones. For increasing values of n, the weights are shifted towards later iterates. See Definition 1 in Gustavsson et al. (2015) for further details.

It remains however to combine the ergodic sequences with the variable splitting approach. The idea used in this research is to calculate each subproblem's average investments over the subgradient iterations. This converges in the limit for each subproblem. However, this provides N (possibly) different solutions for the investments. Furthermore, all these solutions are most likely not feasible in the original full-scale model since the different subproblems have different profiles for weather and demand. Thus, a heuristic to combine these solutions is currently applied (see Paper I).

4.4 Consensus algorithm

The consensus algorithm is used to connect the models separated by the 2-week segments. When the algorithm begins, the starting point is that all 2-week segments share the investment cost equally, i.e. the costs are weighted by $1/26$. The 26 different electricity system models are then solved, and information on investments in different types of generation, transmission, and storage capacity in each 2-week segment is collected to form one capacity–cost curve for each technology and region. The idea is that the investments form the basis for the investment cost in the subsequent solve. The cost of the capacity that is invested in all 2-week segments is weighted by $1/26$, however, if, e.g. only k problems have made the investment, the capacity is weighted by $1/k$'th of the investment cost for all 2-week segments in the next iteration. Figure 4.2 illustrates an example of a capacity–cost curve.The remainder of this section presents the capacity–cost curve construction for generation technologies; the construction for transmission technologies are done analogously. Figure 4.3 presents a

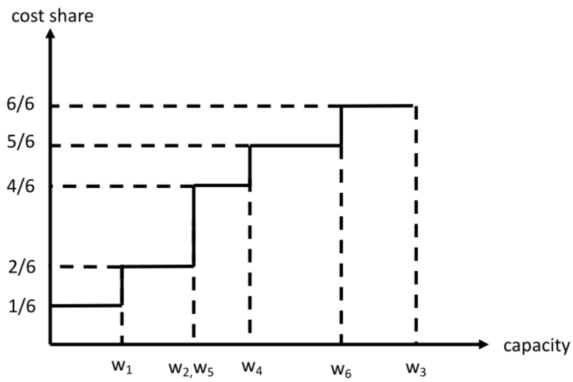


Figure 4.2: An example of a capacity–cost curve. Let $w_1 = 20$, $w_2 = 40$, $w_3 = 100$, $w_4 = 55$, $w_5 = 40$, and $w_6 = 80$. Then, $L = \{w_1, w_2, w_3, w_4, w_5, w_6\}$ and sorted as $L = \{w_1, w_2, w_5, w_4, w_6, w_3\}$

schematic illustration of the modelling methodology. The capacity–cost curves are composed by 26 steps, where the length of the first step corresponds to the capacity investment level that is common to all 26 subproblems. The length of the second step represents, in addition to the first step, the capacity investment which is shared by all the 2-week segments except one, and so on. In order to determine the lengths of the steps, the number R_{ips} of 2-week segments that have lower or equal levels of installed capacity of technology $p \in \mathcal{P}$ in region

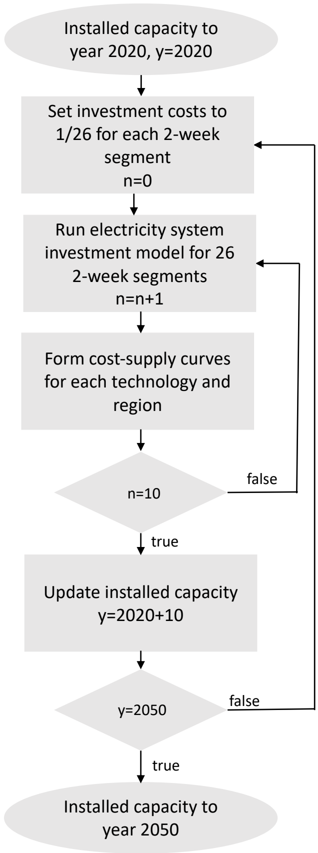


Figure 4.3: A schematic illustration of the modelling methodology.

$i \in \mathcal{I}$ than the 2-week segment $s \in \mathcal{S}$ is calculated as[1]

$$R_{ips} = 1 + S - \sum_{u \in \mathcal{S}} [w_{ipu} \leq w_{ips}], \qquad i \in \mathcal{I},\ p \in \mathcal{P},\ s \in \mathcal{S}, \tag{4.4.1}$$

where $\mathcal{S}$ is the set of 2-week segments. It follows that the length of the first step

[1]The *Iverson bracket* (Iverson, 1962) returns 1 if the expression within the brackets is true; otherwise it returns 0.

in the capacity–cost curve M^{e}_{i,p,r_1} is given by

$$M^{\text{e}}_{i,p,r_1} = \frac{\displaystyle\sum_{s\in\mathcal{S}}[R_{ips} = 1]w_{ips}}{\sum_{s\in\mathcal{S}}[R_{ips} = 1]}, \qquad i \in \mathcal{I},\ p \in \mathcal{P}, \tag{4.4.2}$$

where r_1 is the first element in the set of cost classes $\mathcal{R}$. For $i \in \mathcal{I}$ and $p \in \mathcal{P}$, the lengths of the subsequent steps in the capacity–cost curve are calculated sequentially as

$$M^{\text{e}}_{i,p,r_m} = \frac{\sum_{s\in\mathcal{S}}[R_{ips} = m]w_{ips}}{\sum_{s\in\mathcal{S}}[R_{ips} = m]} - \sum_{n=1}^{m-1} M^{\text{e}}_{i,p,r_n}, \qquad m \in \{2,\ldots,|\mathcal{R}|\}. \tag{4.4.3}$$

The length of the last step in the capacity–cost curve is set to be very large, i.e. three times the maximum annual load in the respective region. The height of each step in the capacity–cost curve, i.e. the weight of the investment, is given by the number of 2-week segments sharing the investment, as

$$\lambda^{\text{e}}_{i,p,s,r_m} = \frac{1}{S-(m-1)}, \qquad m \in \{1,\ldots,|\mathcal{R}|\}. \tag{4.4.4}$$

This cost is slightly modified in two ways: 1) the cost share is lower in the first iterations in order to enable the capacity with a high investment costs to stabilize before extinction, and 2) the cost share is lower for those 2-week segments that have not invested in the capacity that other 2-week segments have. This "rebate" is then reduced with the iteration number. Hence, for $i \in \mathcal{I}$, $p \in \mathcal{P}$, and $s \in \mathcal{S}$, it holds that

$$\lambda^{\text{e}}_{i,p,s,r_m} = \frac{\alpha_{nips}}{S-\beta_n(m-1)}, \qquad m \in \{1,\ldots,|\mathcal{R}|\},\ n \in \{1,\ldots,10\}, \tag{4.4.5}$$

where the choices for the parameters α_{nips} and β_n in each iteration n are listed in Table 4.1. The parameter α_{nips} can take on a high ($\alpha^{\text{high}}_{nips}$) or low ($\alpha^{\text{low}}_{nips}$) value depending on whether or not investments have been made for the corresponding region, technology, and 2-week segment (i, p, s).

The construction of the capacity–cost curve is summarized in Algorithm 2.

Table 4.1: Parameter values used in the consensus loop

iteration number (n)	$\alpha^{\text{low}}_{nips}$	β_n	$\alpha^{\text{high}}_{nips}$
1	0.5	0.5	0.1
2	0.6	0.6	0.1
3	0.7	0.7	0.2
4	0.8	0.8	0.2
5	0.8	0.9	0.3
6	0.8	1.0	0.4
7	0.8	1.0	0.5
≥ 8	0.8	1.0	0.6

Algorithm 2 Creating the capacity–cost curve

1: Create a list L of the capacities such that $L := (w_{ip1}, w_{ip2}, \ldots, w_{ipS})$.
2: Sort the list L in ascending capacity size order. Each unique element represents a step in the capacity–cost curve.
3: The height of each step in the capacity–cost curve, i.e. $\lambda^{\text{e}}_{iprs}$, is determined by the number of 2-week segments sharing the investment. For each element, the number of 2-week segments sharing the investment corresponds to $\mathcal{S}$ reduced by the order of the element in the list L.
4: **if** $\exists$ duplicates in list L **then**
    remove duplicates from the list L
5: The length of the steps corresponds to capacity, such that each new step occurs at the values present in the reduced list $L*$. The potential of each cost class, M^{e}_{i,p,r_m}, is given by the capacity in the capacity–cost curve reduced by the capacity of the prior step.

Yearly linkages

In traditional electricity system investment models, the represented years are linked by the investment variables. However, the Hours-to-Decades model disregards any possible influence that future years might have on investments. This is based on the hypothesis that investments are made only to meet exactly the demand for electricity in the cost-optimal system, largely ignoring future needs in terms of capacity.

The cost of CO_2 emissions, investment costs (due to learning), efficiencies and discount rate can change between years and may influence the investment decisions. For scenarios with gradually increasing costs for generation capacity or operation over the years, this increase is likely to impact investments and needs to be transferred to prior years. Electricity generation technologies that rely on fossil fuels are for example typically subject to a gradual increase in operational costs over the decades considered, which reduces the cost-competitiveness of these technologies in the long-term perspective. Under the assumption that the total cost for investments and operation of a power plant is evenly distributed across all of its hours of operation, some of the operational costs from later years need to be shifted to earlier years. The net present value of these future operational cost (with interest rate δ) is added to the objective function. Thus, for $p \in \mathcal{P}$ and $t \in \mathcal{T}_s$, $s \in \mathcal{S}$, we define the additional operational costs, C^{add}_{pty}, as

$$C^{\text{add}}_{pty} := \frac{1}{Z_p} \sum_{n=y}^{y+Z_p} \frac{1}{(1+\delta)^{(n-y)}} (C^{\text{run},n}_{pt} - C^{\text{run},y}_{pt}), \qquad y \in \mathcal{Y}, \tag{4.4.6}$$

where $y \in \mathcal{Y}$ is the year considered, i.e. the year in which investments are made, and Z_p is the lifetime of technology $p \in \mathcal{P}$. The costs (4.4.6) are added to the running cost C^{run}_{pt}, in the objective function (3.2.18) for the respective years.

5 Model implementation

In this chapter, we apply the mathematical theory from Chapter 4 to the full-scale model. But first, we begin by discussing some of the data used in the different models.

5.1 Data used in this thesis

The investment costs and fixed operation and maintenance costs are based on the World Energy Outlook (IEA, 2016). The costs for wind power are, however, based on Energistyrelsen (2016). For the Hours-to-Decades implementation, the wind power costs are based on the costs presented by Mone et al. (2017). The models use annualised investment costs where a 5% interest rate is assumed.

Technology learning for thermal generation is included as gradual improvement in the efficiencies of these technologies, which is reflected in a reduced variable cost for later years in the model. The cost of carbon dioxide emissions varies also between years as it is assumed to become more expensive in the forthcoming years. Moreover, the variable costs do not include costs from thermal cycling generation. Instead, the start-up costs and part-load costs are included explicitly as part of the thermal cycling constraints in the model. These costs, including the minimum load level, are based on the report by Jordan and Venkataraman (2012). The cycling properties of nuclear power are based on the paper by Persson et al. (2012), who describe a start-up time of 20h and a minimum load level of 70%. Biogas is assumed to be produced through the gasification of solid biomass, with 70% conversion efficiency. The cost of the gasifier equipment is included in the form of 20 €/MWh added to the fuel cost, rather than being incorporated into the investment cost of the biogas technologies, since biogas is storable, which means that the gasifier equipment may attain a much higher number of full-load hours compared to the power plant consuming the biogas. The total cost of the gasification equipment is taken from Thunman et al. (2019), and 8,000 full-load hours are assumed.

Wind power sites are ordered in classes. Offshore sites are represented by one class while onshore sites are organized into several classes corresponding to different wind conditions, where each class is represented as one generation technology. These classes are defined differently for the different models and based on different data. For the basic and full-scale model, wind power generation profiles are based on a code set presented by Mattsson et al. (2021), which combines data with high spatial resolution and high temporal resolution. The wind resource in each region is divided into 10 different wind classes with different annual capacity factors and generation patterns dependent on local differences in topography. For the Hours-to-Decades model, the wind power generation profiles are calculated for wind turbines with low specific power (200 W/m^2), with the power curve and losses proposed by Johansson et al. (2017). The wind speed input data comprise a combination of the MERRA and ECMWF ERA-Interim data for year 2012, whereby the profiles from the former are re-scaled with the average wind speeds from the latter (see ECMWF (2010), Lucchesi (2012), and Olauson and Bergkvist (2015)). The high resolution of the wind profiles from the ERA-Interim data was processed into wind power generation profiles and put together into twelve wind classes for each region. The wind farm density is set to 3.2 $MW/(km)^2$ and is assumed to be limited to 10% of the available land area, accounting for protected areas, lakes, water streams, roads, and cities (Nilsson and Unger, 2014).

Solar PV is modelled as mono-crystalline silicon cells installed with optimal tilt with one generation profile for each region. Solar radiation data from MERRA is used to calculate the generation with the model presented by Norwood et al. (2014), including thermal efficiency losses. The cost and technical data for variation management technologies are based on Energistyrelsen (2012). The hydrogen storage is assumed to be of the large-scale, steel lined cavern type.

5.2 Lagrangian relaxation

Similar to the Hours-to-Decades model, we wish to separate the annual time steps of the full-scale model into M-week periods (where $M = 2$ in the Hours-to-Decades model) and solve them in parallel. Thus, some new sets and parameters are necessary:

- $\mathcal{N} = \{1, \ldots, N\}$ is the set of time periods within a year, with $M := \lfloor \frac{52}{N} \rfloor$,

- $\mathcal{T}_n = \{(n-1)\Omega + 1, \dots, n\Omega\}$ is the set of time steps within each time period $n \in \mathcal{N}$, where $\Omega = \frac{24 \times 7 \times M}{\tau}$ is the length of a time period and τ is the time step length.

Furthermore, let $y^{\text{split}}_{prin} := y_{pri}$ and $u^{\text{split}}_{kqrin} := u_{kqri}$ for all $n \in \mathcal{N}$, such that we introduce splitting variables (see Section 4.2) for the investment variables.

Now, consider again the objective function (3.2.18) representing the total system cost. Using the above notation, it can be equivalently expressed as

$$C^{\text{tot}} := \sum_{n \in \mathcal{N}} C^{\text{split}}_n \tag{5.2.1}$$

where the system cost in each time period $n \in \mathcal{N}$ is defined as

$$C^{\text{split}}_n := \sum_{p \in \mathcal{P}} \sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} \left(\frac{c^{\text{invtech}}_{ps} + c^{\text{omf}}_{p}}{N}\right) y^{\text{split}}_{prsn} \tag{5.2.2a}$$

$$+ \sum_{p \in \mathcal{P}} \sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \sum_{t \in \mathcal{T}_n} c^{\text{run}}_{pri} x_{prist} \tag{5.2.2b}$$

$$+ \sum_{p \in \mathcal{P}_{\text{thermal}}} \sum_{r \in \mathcal{R}} \sum_{s \in \mathcal{S}} \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \sum_{t \in \mathcal{T}_n} \left(c^{+}_{prs} z^{+}_{prist} + \tilde{c}_{prs}(z_{prist} - x_{prist})\right) \tag{5.2.2c}$$

$$+ \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}_n} c^{CO_2}_{s} e^{\text{tot}}_{st} \tag{5.2.2d}$$

$$+ \sum_{k \in \mathcal{K}} \sum_{(q,r) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \frac{c^{\text{invtra}}_{kqrs}}{N} u^{\text{split}}_{kqrsn} \tag{5.2.2e}$$

$$+ \sum_{k \in \mathcal{K}} \sum_{(q,r) \in \mathcal{A}} \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}_n} c^{\text{tra}}_{kqr} v_{kqrst}. \tag{5.2.2f}$$

The investment variables in the model constraints should also be replaced by the splitting variables, and we add to the model the constraints

$$y^{\text{split}}_{prsn} = y_{prs}, \qquad p \in \mathcal{P},\ r \in \mathcal{R},\ s \in \mathcal{S},\ n \in \mathcal{N}; \tag{5.2.3}$$

$$u^{\text{split}}_{kqrsn} = u_{kqrs}, \qquad k \in \mathcal{K},\ (q,r) \in \mathcal{A},\ s \in \mathcal{S},\ n \in \mathcal{N}. \tag{5.2.4}$$

These constraints are then Lagrangian relaxed using Lagrangian multipliers π^{one}_{prsn} and π^{two}_{kqrsn}, respectively.

As for the other model variables, we need to handle the constraints that consider multiple time steps. More specifically, it is "the seams" that make the problem non-separable over time periods. Consider again the constraints (3.2.7). Using the notation above, for all $p \in \mathcal{P}_{\text{thermal}}$, $r \in \mathcal{R}$, $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$, and $s \in \mathcal{S}$, they can be equivalently expressed as

$$z^{+}_{prist} \geq \begin{cases} z_{prist} - z_{p,r,i,s,t-1}, & t \in \mathcal{T}_n \setminus \{(n-1)\Omega + 1\},\ n \in \mathcal{N}, \\ z_{prist} - z_{p,r,i,s,t-1}, & t = (n-1)\Omega + 1,\ n \in \mathcal{N} \setminus \{1\}, \\ z_{prist} - z_{p,r,i,s,T}, & t = 1. \end{cases} \tag{5.2.5}$$

Here, it is the second and third sets of constraints—which correspond to the seams—that need to be Lagrangian relaxed in order to make the model separable. For this, we denote the Lagrangian multipliers $\pi^{\text{three}}_{prist}$.

Moreover, for all $p \in \mathcal{P}_{\text{thermal}}$, $r \in \mathcal{R}$, $s \in \mathcal{S}$, and $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$, the constraints (3.2.8) can be written as

$$\sum_{j \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} y^{\text{split}}_{prjn} - z^{+}_{prist} \geq$$
$$\begin{cases} z_{p,r,i,s,t-t_p}, & t_p \in \{m \in \mathcal{T}_{\text{start}}(p) : t - m \in \mathcal{T}_n\},\ t \in \mathcal{T}_n,\ n \in \mathcal{N}, \\ z_{p,r,i,s,t-t_p}, & t_p \in \{m \in \mathcal{T}_{\text{start}}(p) : t - m \in \mathcal{T}_{n-1}\},\ t \in \mathcal{T}_n,\ n \in \mathcal{N} \setminus \{1\}, \\ z_{p,r,i,s,N\Omega+t-t_p}, & t_p \in \{m \in \mathcal{T}_{\text{start}}(p) : t - m \leq 0\},\ t \in \mathcal{T}_n,\ n = 1. \end{cases} \tag{5.2.6}$$

Note that here the splitting variables are used for investments in electricity generation; they have thus replaced the original investment variables. The first constraint corresponds to time steps for z_{prist} within the M-week period, while the second constraint represents the time steps in the previous M-week period. The third constraint regards the special case of the first and last M-week periods during the year. Here, we Lagrangian relax the second and third constraint using Lagrangian multipliers $\pi^{\text{four}}_{prist}$.

Furthermore, for all $p \in \mathcal{P}_{\text{hydro}}$, $r \in \mathcal{R}$, and $s \in \mathcal{S}$, the constraints (3.2.12) are replaced by

$$w_{rst} + g_{rt} - \tau \sum_{i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} x_{prist} \geq \begin{cases} w_{r,s,t+1}, & t \in \mathcal{T}_n \setminus \{n\Omega\},\ n \in \mathcal{N}, \\ w_{r,s,t+1}, & t = n\Omega,\ n \in \mathcal{N} \setminus \{N\}, \\ w_{r,s,1}, & t = T. \end{cases} \tag{5.2.7}$$

Once again, the second and third constraints should be relaxed, using Lagrangian multipliers π^{five}_{prst}.

For all $p \in \mathcal{P}_{\text{hydro}}$, $r \in \mathcal{R}$, $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$, and $s \in \mathcal{S}$, the constraints (3.2.14) and (3.2.15) can be written as

$$(1+\delta^{\text{inc}}_r)x_{prist} \geq \begin{cases} x_{p,r,i,s,t+1}, & t \in \mathcal{T}_n \setminus \{n\Omega\},\ n \in \mathcal{N}, \\ x_{p,r,i,s,t+1}, & t = n\Omega,\ n \in \mathcal{N} \setminus \{N\}, \\ x_{p,r,i,s,1}, & t = T; \end{cases} \tag{5.2.8}$$

$$(1+\delta^{\text{dec}}_r)x_{prist} \leq \begin{cases} x_{p,r,i,s,t+1}, & t \in \mathcal{T}_n \setminus \{n\Omega\},\ n \in \mathcal{N}, \\ x_{p,r,i,s,t+1}, & t = n\Omega,\ n \in \mathcal{N} \setminus \{N\}, \\ x_{p,r,i,s,1}, & t = T. \end{cases} \tag{5.2.9}$$

The second and third constraints in each of the expressions (5.2.8) and (5.2.9) should be relaxed, and we denote the corresponding Lagrangian multipliers π^{six}_{prist} and $\pi^{\text{seven}}_{prist}$.

Moreover, the constraints (3.2.9) should be relaxed using Lagrangian multipliers π^{eight}_{ls}.

We also add to the subproblems the constraints (5.2.10) and (5.2.11), which limit the hot capacity and the start-up capacity, respectively. Note that these constraints are redundant in the original problem, but are here used to strengthen the dual formulation by making the subproblems tighter. These constraints are formulated as

$$z_{prist} \leq \sum_{j \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} y_{prj}, \quad i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p),\ p \in \mathcal{P}_{\text{thermal}},\ r \in \mathcal{R},\ t \in \mathcal{T}; \tag{5.2.10}$$

$$z^{+}_{prist} \leq \sum_{j \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} y_{prj}, \quad i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p),\ p \in \mathcal{P}_{\text{thermal}},\ r \in \mathcal{R},\ t \in \mathcal{T}. \tag{5.2.11}$$

The new objective will consist of C^{tot} from (5.2.1) and the added penalty terms, which are derived from the Lagrangian relaxation of the constraints described above. The penalty term H^{relax} is defined as

$$H^{\text{relax}} := \sum_{n \in \mathcal{N}} H^{\text{one}}_n + \sum_{n \in \mathcal{N} \setminus \{1\}} H^{\text{two}}_n + \sum_{n \in \mathcal{N} \setminus \{N\}} H^{\text{three}}_n + H^{\text{four}}, \tag{5.2.12}$$

where H_n^{one} is defined for each $n \in \mathcal{N}$ as

$$H_n^{\text{one}} := \sum_{p\in\mathcal{P}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \pi_{prsn}^{\text{one}} \left(y_{prs} - y_{prsn}^{\text{split}} \right) \tag{5.2.13a}$$

$$+ \sum_{k\in\mathcal{K}} \sum_{(q,r)\in\mathcal{A}} \sum_{s\in\mathcal{S}} \pi_{kqrsn}^{\text{two}} \left(u_{kqrs} - u_{kqrsn}^{\text{split}} \right) \tag{5.2.13b}$$

$$+ \sum_{l\in\mathcal{L}} \sum_{s\in\mathcal{S}} \pi_{ls}^{\text{eight}} \left(f_{ls} - \sum_{t\in\mathcal{T}_n} \sum_{p\in\mathcal{P}_{\text{ren}}} \sum_{r\in\mathcal{R}(l)} \sum_{i\in\mathcal{I}_{\text{active}}^{\mathcal{P}}(s,p)} x_{prist} \right). \tag{5.2.13c}$$

For $n \in \mathcal{N} \setminus \{1\}$, H_n^{two} is defined as:

$$H_n^{\text{two}} := \sum_{t=(n-1)\Omega+1} \sum_{p\in\mathcal{P}_{\text{thermal}}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{I}_{\text{active}}^{\mathcal{P}}(s,p)} \pi_{prist}^{\text{three}} \left(z_{prist} - z_{p,r,i,s,t-1} - z_{prist}^{+} \right) \tag{5.2.14a}$$

$$+ \sum_{t\in\mathcal{T}_n} \sum_{p\in\mathcal{P}_{\text{thermal}}} \sum_{t_p\in\{m\in\mathcal{T}_{\text{start}}(p):t-m\in\mathcal{T}_{n-1}\}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{I}_{\text{active}}^{\mathcal{P}}(s,p)} \pi_{prist}^{\text{four}} \left(z_{p,r,i,s,t-t_p} - \sum_{j\in\mathcal{I}_{\text{active}}^{\mathcal{P}}(s,p)} y_{prjn}^{\text{split}} + z_{prist}^{+} \right). \tag{5.2.14b}$$

For $n \in \mathcal{N} \setminus \{N\}$, we define H_n^{three} as

$$H_n^{\text{three}} := \sum_{t=n\Omega} \sum_{p\in\mathcal{P}_{\text{hydro}}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \pi_{prst}^{\text{five}} \left(w_{r,s,t+1} - w_{rst} - g_{rt} + \tau \sum_{i\in\mathcal{I}_{\text{active}}^{\mathcal{P}}(s,p)} x_{prist} \right) \tag{5.2.15a}$$

$$+ \sum_{t=n\Omega} \sum_{p\in\mathcal{P}_{\text{hydro}}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{I}_{\text{active}}^{\mathcal{P}}(s,p)} \pi_{prist}^{\text{six}} \left(x_{p,r,i,s,t+1} - (1+\delta_r^{\text{inc}}) x_{prist} \right) \tag{5.2.15b}$$

$$+ \sum_{t=n\Omega} \sum_{p\in\mathcal{P}_{\text{hydro}}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{I}_{\text{active}}^{\mathcal{P}}(s,p)} \pi_{prist}^{\text{seven}} \left((1+\delta_r^{\text{dec}}) x_{prist} - x_{p,r,i,s,t+1} \right). \tag{5.2.15c}$$

Additional relaxed constraints for the specific values of n that relate to the first and last period in $\mathcal{N}$ are given by H^{four}:

$$H^{\text{four}} := \sum_{p\in\mathcal{P}_{\text{thermal}}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \pi^{\text{three}}_{pris1}\left(z_{pris1} + z_{prisT} - z^{+}_{pris1}\right) \tag{5.2.16a}$$

$$+ \sum_{t\in\mathcal{T}_1} \sum_{p\in\mathcal{P}_{\text{thermal}}} \sum_{t_p\in\{m\in\mathcal{T}_{\text{start}}(p):t\leq m\}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \pi^{\text{four}}_{prist}\left(z_{p,r,i,s,N\Omega+t-t_p} - \sum_{j\in\mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} y^{\text{split}}_{prj1} + z^{+}_{prist}\right) \tag{5.2.16b}$$

$$+ \sum_{p\in\mathcal{P}_{\text{hydro}}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \pi^{\text{five}}_{prsT}\left(w_{rs1} - w_{rsT} - g_{rT} + \tau \sum_{i\in\mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} x_{prisT}\right) \tag{5.2.16c}$$

$$+ \sum_{p\in\mathcal{P}_{\text{hydro}}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \pi^{\text{six}}_{prisT}\left(x_{pris1} - (1+\delta^{\text{inc}}_r)x_{prisT}\right) \tag{5.2.16d}$$

$$+ \sum_{p\in\mathcal{P}_{\text{hydro}}} \sum_{r\in\mathcal{R}} \sum_{s\in\mathcal{S}} \sum_{i\in\mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)} \pi^{\text{seven}}_{prisT}\left((1+\delta^{\text{dec}}_r)x_{prisT} - x_{pris1}\right). \tag{5.2.16e}$$

The Lagrangian problem, which is separable into $n \in \mathcal{N}$ subproblems, is then defined as

$$h(\boldsymbol{\pi}) = \left\{ \begin{array}{ll} \text{minimum} & \sum_{n\in\mathcal{N}} C^{\text{split}}_n + H^{\text{relax}}, \\ \text{subject to} & (5.2.2), (5.2.13), (5.2.14), (5.2.15), (5.2.16), \\ & (3.2.1), (3.2.2), (3.2.3), (3.2.4), (3.2.5), (3.2.6), \\ & (3.2.10), (3.2.11), (3.2.13), (3.2.16), (3.2.17), \\ & (5.2.5), (5.2.6), (5.2.7), (5.2.8), (5.2.9) \end{array} \right\}. \tag{5.2.17}$$

The dual problem is defined as

$$\begin{array}{lll} \max & h(\boldsymbol{\pi}) & \\ \text{s.t.} & \sum_{n\in\mathcal{N}} \pi^{\text{one}}_{prsn} \geq 0, & p\in\mathcal{P},\ r\in\mathcal{R},\ s\in\mathcal{S}, \\ & \pi^{\text{one}}_{prsn} \leq \frac{1}{N}\left(c^{\text{invtech}}_{ps} + c^{\text{omf}}_{p}\right), & p\in\mathcal{P},\ r\in\mathcal{R},\ s\in\mathcal{S},\ n\in\mathcal{N}, \\ & \sum_{n\in\mathcal{N}} \pi^{\text{two}}_{kqrsn} \geq 0, & k\in\mathcal{K},\ (q,r)\in\mathcal{A},\ s\in\mathcal{S}, \end{array}$$

$$\pi^{\text{two}}_{kqrsn} \leq \frac{1}{N} c^{\text{invtra}}_{kqr}, \qquad k \in \mathcal{K},\ (q,r) \in \mathcal{A},\ s \in \mathcal{S},\ n \in \mathcal{N},$$
$$\boldsymbol{\pi}^{\text{three}},\ \boldsymbol{\pi}^{\text{four}},\ \boldsymbol{\pi}^{\text{five}} \geq \mathbf{0},$$
$$\boldsymbol{\pi}^{\text{six}},\ \boldsymbol{\pi}^{\text{seven}},\ \boldsymbol{\pi}^{\text{eight}} \geq \mathbf{0}. \tag{5.2.18}$$

5.3 Solving the Lagrangian dual problem

To solve the Lagrangian dual problem, the subgradient algorithm from Section 4.3 is applied. As a termination criterion we use the duality gap such that the algorithm terminates when the following criterion is satisfied:

$$\frac{\text{UBD} - \text{LBD}}{\text{UBD}} < \epsilon^{\text{gap}}. \tag{5.3.1}$$

Here, UBD is an upper bound on the objective value of the primal problem and LBD is a lower bound on the optimal objective value of the primal problem. The left–hand–side is defined as the relative duality gap, and we say that it should be smaller than the duality gap max ϵ^{gap}, which is defined as a small positive number. Since we are working with an LP, strong duality implies that the duality gap is zero in the optimal solution. However, we let $\epsilon^{\text{gap}} := 10^{-4}$.

Lower bounds on the optimal objective value of the primal problem are given by the Lagrangian function evaluated for the Lagrangian multipliers, and thus a result of the subgradient algorithm. The upper bound is given by a feasible solution to the primal problem. For this, we can use the simple model to solve the electricity system investment problem for each region $r \in \mathcal{R}$. This solution is naturally feasible in the full-scale model as well, since both models contain the same type of constraints albeit the option for electricity trade.

Updating the multipliers

If the dual problem has constraints on the dual variables, then the update from the subgradient algorithm may lead to values on the multipliers that are infeasible. If so, the multiplier needs to be projected on the feasible region. Let

$$\bar{\boldsymbol{\pi}} := \boldsymbol{\pi}^k - \alpha_k \boldsymbol{\gamma}^k$$

as in the subgradient algorithm update. For the multiplier vectors $\boldsymbol{\pi}^{\text{three}}$, $\boldsymbol{\pi}^{\text{four}}$, $\boldsymbol{\pi}^{\text{five}}$, $\boldsymbol{\pi}^{\text{six}}$, $\boldsymbol{\pi}^{\text{seven}}$, and $\boldsymbol{\pi}^{\text{eight}}$, the projection according to step 3 in Algorithm 1 can be made separately for each element of each vector. Thus, the projection is given by

$$\pi_j^{k+1} := \begin{cases} \bar{\pi}_j, & \bar{\pi}_j \geq 0, \\ 0, & \bar{\pi}_j < 0, \end{cases}$$

where j refers to element j in the respective multiplier vector.

For multiplier π^{one}_{prsn}, then for each $p \in \mathcal{P},\ r \in \mathcal{R},\ s \in \mathcal{S}$ with $c := \dfrac{c^{\text{invtech}}_{ps} + c^{\text{omf}}_{p}}{N}$, the projection corresponds to solving the problem to

$$\begin{aligned} \text{minimize} \quad & \tfrac{1}{2}||\boldsymbol{\pi} - \bar{\boldsymbol{\pi}}||^2 = \tfrac{1}{2}\sum_{n\in\mathcal{N}}(\pi_n - \bar{\pi}_n)^2, \\ \text{subject to} \quad & \sum_{n\in\mathcal{N}} \pi_n \geq 0, \\ & \pi_n \leq c, \quad n \in \mathcal{N}, \end{aligned} \tag{5.3.2}$$

with the KKT conditions (where λ and ψ_n are multipliers of the constraints)

$$\begin{aligned} \begin{pmatrix} \pi_1 \\ \vdots \\ \pi_N \end{pmatrix} - \begin{pmatrix} \bar{\pi}_1 \\ \vdots \\ \bar{\pi}_N \end{pmatrix} + \sum_{n\in\mathcal{N}} \psi_n \boldsymbol{e}_n - \lambda \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} &= 0, \\ \psi_n(\pi_n - c) &= 0, \qquad n \in \mathcal{N}, \\ \lambda(-\sum_{n\in\mathcal{N}} \pi_n) &= 0. \end{aligned} \tag{5.3.3}$$

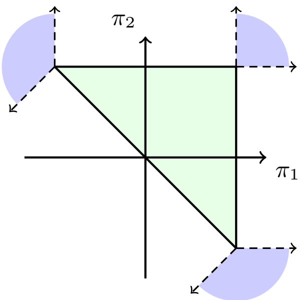


Figure 5.1: The projection problem for $N = 2$. Here, the green area is the feasible region. For points inside the blue cones, the projection should be onto the corresponding extreme points. For any other points outside the green area, the projection is defined by the corresponding shortest Euclidean distance to the triangle. The blue cones are spanned by the gradients of the active constraints in the extreme points.

The projection problem, illustrated in Figure 5.1, has the property that it is a convex quadratic program. Therefore, the Lagrangian dual of this problem is also a convex quadratic program. To see this, Lagrangian relax the first constraint. Then, using the Lagrange multiplier λ, the Lagrangian function is

$$L(\boldsymbol{\pi}, \lambda) := \frac{1}{2} \sum_{n \in \mathcal{N}} (\boldsymbol{\pi} - \bar{\boldsymbol{\pi}})^2 - \lambda \sum_{n \in \mathcal{N}} \pi_n = \sum_{n \in \mathcal{N}} \left(\frac{1}{2} (\pi_n - \bar{\pi}_n)^2 - \lambda \pi_n \right).$$

Let $L_n(\pi_n, \lambda) := \frac{1}{2}(\pi_n - \bar{\pi}_n)^2 - \lambda \pi_n$, such that $L(\boldsymbol{\pi}, \lambda) = \sum_{n \in \mathcal{N}} L_n(\pi_n, \lambda)$ holds. It follows that the function L is separable with respect to $n \in \mathcal{N}$, where L_n is a quadratic, convex, and differentiable function with respect to its first argument.

Now, define

$$h_n(\lambda) := \begin{array}{ll} \text{minimum} & L_n(\pi_n, \lambda) \\ \text{subject to} & \pi_n \leq c \end{array} = \begin{array}{ll} \text{minimum} & \frac{1}{2}(\pi_n - \bar{\pi}_n)^2 - \lambda \pi_n \\ \text{subject to} & \pi_n \leq c \end{array}$$

so that the Lagrangian dual function is given by

$$h(\lambda) = \sum_{n \in \mathcal{N}} h_n(\lambda) = \sum_{n \in \mathcal{N}} \left(\begin{array}{ll} \text{minimum} & \frac{1}{2}(\pi_n - \bar{\pi}_n)^2 - \lambda \pi_n \\ \text{subject to} & \pi_n \leq c \end{array} \right).$$

For a constant value of λ, the minimum for L_n over $\pi_n \in [-\infty, c]$ is attained when either $\frac{\partial L_n(\pi_n, \lambda)}{\partial \pi_n} = 0$ or $\pi_n = c$. Since

$$\frac{\partial L_n(\pi_n, \lambda)}{\partial \pi_n} = \pi_n - \bar{\pi}_n - \lambda = 0 \iff \pi_n = \bar{\pi}_n + \lambda,$$

it follows that $L_n(\cdot, \lambda)$ attains its minimum for

$$\pi_n = \begin{cases} \bar{\pi}_n + \lambda, & \bar{\pi}_n \leq c - \lambda, \\ c, & \bar{\pi}_n > c - \lambda. \end{cases} \tag{5.3.4}$$

Hence, the function h can be expressed according to the following:

$$\begin{aligned} h(\lambda) &= \sum_{n \in \mathcal{N} : \bar{\pi}_n \leq c - \lambda} \left(\frac{1}{2}\lambda^2 - \lambda(\bar{\pi}_n + \lambda) \right) + \sum_{n \in \mathcal{N} : \bar{\pi}_n > c - \lambda} \left(\frac{1}{2}(c - \bar{\pi}_n)^2 - \lambda c \right) \\ &= \sum_{n \in \mathcal{N} : \bar{\pi}_n \leq c - \lambda} \left(-\frac{1}{2}\lambda^2 - \lambda \bar{\pi}_n \right) + \sum_{n \in \mathcal{N} : \bar{\pi}_n > c - \lambda} \left(\frac{1}{2}(c - \bar{\pi}_n)^2 - \lambda c \right). \end{aligned}$$

The function h is clearly a quadratic and concave function of λ. The Lagrangian dual problem is given by

$$\begin{array}{ll} \text{maximum} & h(\lambda), \\ \text{subject to} & \lambda \geq 0. \end{array}$$

The dual function is differentiable, and thus the derivative can be used to find the maximum. Hence, the dual problem is maximized when $\frac{\partial h(\lambda)}{\partial \lambda} = 0$ or $\lambda = 0$:

$$\left.\frac{\partial h(\lambda)}{\partial \lambda}\right|_{\lambda} = -\sum_{n \in \mathcal{N}: \bar{\pi}_n + \lambda \le c} (\lambda + \bar{\pi}_n) - \sum_{n \in \mathcal{N}: \bar{\pi}_n + \lambda > c} c$$
$$\implies \frac{\partial h(\lambda)}{\partial \lambda} = -\sum_{n \in \mathcal{N}} \min\{\bar{\pi}_n + \lambda\, ; c\} = 0. \tag{5.3.5}$$

The partial derivative $\frac{\partial h(\lambda)}{\partial \lambda}$ is decreasing (though not strictly) for increasing

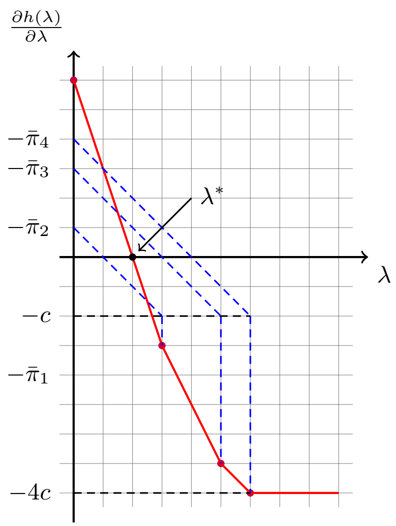


Figure 5.2: The partial derivative as a function of λ. The optimal value λ^* is where the partial derivative is equal to zero.

values of λ. Hence, it will be zero for a specific value of λ. As an example, consider Figure 5.2, where we use the property

$$-\min\{\bar{\pi}_n + \lambda\, ; c\} = \max\{-(\bar{\pi}_n + \lambda)\, ; -c\}.$$

The gradient of the function h changes in the break points corresponding to $\lambda = c - \bar{\pi}_n$, $n \in \mathcal{N}$. Thus, between two break points, the point at which $\frac{\partial h(\lambda)}{\partial \lambda} = 0$ holds will be passed, and this corresponds to the optimal value λ^*. Using the optimal value λ^*, the optimal values π_n^*, $n \in \mathcal{N}$, are then given by (5.3.4). The method discussed here to find the projection point is described in Algorithm 3.

If there does not exist a point $\lambda \ge 0$, for which $\frac{\partial h(\lambda)}{\partial \lambda} = 0$ holds, then this implies that $\frac{\partial h(\lambda)}{\partial \lambda} = -\sum_{n \in \mathcal{N}} \min\{\bar{\pi}_n + \lambda\, ; c\} \neq 0 \implies \bar{\pi}_n > 0$ for all $n \in \mathcal{N}$. Thus, if

$\bar{\pi}_n > 0$ for all $n \in \mathcal{N}$, then $\lambda^* = 0$ and the function $h(\lambda)$ has no zero derivative for $\lambda \geq 0$.

Algorithm 3 Algorithm to solve the projection problem

1: Calculate all break points $\lambda_n := c - \bar{\pi}_n$, $n \in \mathcal{N}$
2: Calculate the partial derivative for all break points, i.e. $\frac{\partial h(\lambda_n)}{\partial \lambda}$, $n \in \mathcal{N}$
3: Find λ_i and λ_j such that

$$\frac{\partial h(\lambda_i)}{\partial \lambda} = \min_{n: \frac{\partial h(\lambda_n)}{\partial \lambda} \geq 0} \left\{ \frac{\partial h(\lambda_n)}{\partial \lambda} \right\} \quad \text{and} \quad \frac{\partial h(\lambda_j)}{\partial \lambda} = \max_{n: \frac{\partial h(\lambda_n)}{\partial \lambda} \leq 0} \left\{ \frac{\partial h(\lambda_n)}{\partial \lambda} \right\}$$

4: If $\lambda_i = \lambda_j$, then $\lambda^* := \lambda_i = \lambda_j$. Otherwise, λ^* is given by the linear interpolation

$$\lambda^* := \frac{(0 - \frac{\partial h(\lambda_i)}{\partial \lambda})(\lambda_j - \lambda_i)}{\frac{\partial h(\lambda_j)}{\partial \lambda} - \frac{\partial h(\lambda_i)}{\partial \lambda}} + \lambda_i$$

5: Using λ^*, calculate π_n^*, $n \in \mathcal{N}$, according to (5.3.4)

It should be noted that the Lagrangian update is analogous for the dual variables π_{kqrsn}^{two} with $c := \frac{c_{ps}^{\text{invtech}} + c_p^{\text{omf}}}{N}$, where $k \in \mathcal{K}$, $(q, r) \in \mathcal{A}$, $s \in \mathcal{S}$ and $n \in \mathcal{N}$.

6 A summary of the appended papers

Paper I: Managing the temporal resolution in electricity system investment models with a large share of wind power: An approach using Lagrangian relaxation and variable splitting

In this paper, we formulate a long-term electricity system investment model that accounts for some variation management strategies to capture the variations from intermittent electricity production technologies.

The model is decomposed, using Lagrangian relaxation in combination with variable splitting. The decomposition results in 26 subproblems—each representing a time period of two weeks—which can be solved in parallel. The Lagrangian dual problem is solved by using a subgradient algorithm, which leads to 26 different subproblem solutions each iteration of the algorithm. Ergodic sequences are used to create a single solution in terms of production technology investments for each of the 26 subproblems, and these 26 investment solutions are then combined through a heuristic algorithm.

The decomposed model is implemented and solved using `Julia` (Bezanson et al., 2017) and `JuMP` (Dunning et al., 2017) together with the `Gurobi` (Gurobi Optimization, LLC, 2021) solver. Three different cases that includes a single region without the possibility for trade are examined: 1) Hungary (HU), a region with a large share of nuclear power, coal power and gas power plants, 2) Ireland (IE), a region with good wind power conditions, and 3) southern Sweden (SE2), a region with hydropower and a large share of nuclear power. The solutions found by the decomposed model provide capacity investments similar to the optimal solution for investments provided by the non-decomposed model. The heuristic used to combine the subproblem solutions can be further developed as it tends to overestimate the wind power capacity which increases the total system costs. The most important extension to this work is to implement data to include the possibility to trade with neighbouring regions, and also to include solar power and battery technology options

This paper is in the form of a manuscript (Granfeldt et al., 2021) that is to be submitted to a journal. The initial ideas were presented at the Swedish Operations Research Conference, Linköping (2017), and some later ideas on the Swedish Operations Research Conference, Nyköping (2019) and virtually at EUROPT 2021 Workshop on Advances in Continuous Optimization, Toulouse (2021).

Paper II: Management of wind power variations in electricity system investment models: A parallel computing strategy

In this paper, we develop a mathematical model and a heuristic method which account for variation management strategies in a long-term electricity system investment model. The Hours-to-Decades model discretizes the time dimension into 2-week segments and solves the resulting 26 separate problems in parallel. Information between the segments is then exchanged in a consensus loop, and the main idea is that the investments from the different solutions form the basis for the investment costs in the subsequent solve. This process is then iterated until consensus for the investments made by all 26 problems is reached.

The model is implemented in `GAMS` and then solved using `CPLEX` on a system with 32 cores and 256 GB RAM. Different cases are considered, with some variation management options including storage and trade. The different regions considered are: (1) Ireland, which is a region with good conditions for wind power, (2) Ireland and UK (regions UK1, UK2, UK3, and IE), for the case when trade for Ireland is considered, (3) central Spain, a region with good conditions for solar power, and (4) Iberian Peninsula (regions ES1, ES2, ES3, ES, and PT) for central Spain trade. The solutions found by the Hours-to-Decades model possess an increased total system cost of approximately 1 % compared to the same electricity investment model but with connected time. The resulting energy mix shows that the Hours-to-Decades model responds to variation management similar to the connected-time model. When it comes to the computation times, the Hours-to-Decades model are able to solve the problem faster than a time-connected model. This is even more prominent when several regions with trade are included. As an example, when two regions are considered the Hours-to-Decades model takes around half an hour to solve while the run time for time-connected model is several days. A drawback of the method is, however, that it cannot dimension seasonal storages, which is to store energy during summer or winter, and discharge it during the other respective season. The results of the cases studied indicates, however, that seasonal storage capacity can be dimensioned post-process.

Our heuristic method targets the combination of wind variation management and trade in electricity system models. Nevertheless, if wind power or trade are not of relevance for the investigated regions, representative days or integral time slicing are likely more efficient modelling methodologies.

This paper was initially presented by Lisa Göransson at the International Energy Workshop, Paris (2019), and later published in *SN Operations Research Forum*, ***2**:25* (see Göransson et al. (2021)).

7 Conclusions and ideas for future research

We have developed three electricity system models that can be used as a tool to analyze long-term investments in an electricity system that contains a large share of variable renewable electricity generation. We have further developed two decomposition methods to decrease computation times for all the models presented in this thesis. The first method, which is the one used in the Hours-to-Decades model, can be seen as a heuristic approach to variable splitting. This approach gave us the initial idea to develop the second method which uses Lagrangian relaxation combined with variable splitting.

Moreover, using a decomposition over 2-week periods, the most important part of the original model structure is kept and therefore the two methods have reached reasonable solutions within a few iterations. In each subproblem, we basically solve the entire electricity system model but for a smaller time interval. Hence, for both the full-scale model and the Hours-to-Decades model, the subproblems have a structure which is very similar to their respective non-decomposed model. For each of the two models, this furthermore leads to subproblem solutions which are very similar to each other. Therefore, the optimal solution provided by each subproblem is decent in terms of a solution to the non-decomposed problem. As a comparison, consider a decomposition over groups of variable types instead. For example, electricity generation and investments could be one group, and hot capacity and start-up capacity another group. Here, we assume that we relax constraints that connect different groups which then makes the problem separable into a few subproblems. The most important model structures would immediately be lost, and most likely very many iterations of the subgradient algorithm would be required before the method converges.

The decomposition methods developed in this thesis also make the models parallelizable with the potential to reduce computation times. Moreover, the subproblems of a model take approximately the same amount of time to solve since they have the same model structure and several parameters with the same value (e.g. costs). There is therefore not much idle time in the subgradient algorithm. For example, if a single subproblem had taken much longer time to solve compared to the other subproblems, then each iteration of the subgradient algorithm would include some "waiting time" before the subgradient update could be done. This is however not the case if all subproblems finish solving their model simultaneously.

Another reflection is that if we would model the emission limits as a hard constraint, it would need to be relaxed for the optimization problem to become separable. It would hence be penalized in the objective using different dual variables for each subproblem, the values of the dual variables being updated in the subgradient iterations. Thus, providing from the start a reasonable cost for the emissions in the form of taxes in the original problem will 1) make the Lagrangian problem separable with respect to emissions, since these are not constrained, 2) lower the complexity of the model, since fewer dual variables are required, and 3) provide better subproblem solutions, since emissions are penalized with the same costs in all of the subproblems.

7.1 Future perspective

There exists several ideas for how our research can be extended. A first step is to further develop the transmission and trade aspect of the full-scale model. The theory for it is presented in this thesis but it has yet not been implemented and tested. Another aspect is to extend the full-scale model to include solar power and different storage technologies such as batteries and hydrogen storage. Beside the aforementioned technologies, it is possible to include electric vehicles in the models as a variation management strategy.

However, the suggested extensions of the models will increase the computational effort required for their solution, which then calls for further development of the problem decomposition and of algorithms for solving the resulting problems. The convergence to optimal solutions in the limit for ergodic sequences used together with the variable splitting approach needs to be examined. Furthermore, the heuristic used to combine the subproblem solutions can be further developed.

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A Nomenclature

A.1 Full-scale model

Table A.1: The index sets used in the full-scale model

symbol		representation	member
$\mathcal{L}$		countries	l
$\mathcal{R}$		regions	r
$\mathcal{R}(l)$		regions within a country $l \in \mathcal{L}$	r
$\mathcal{A}$	$\subset \mathcal{R} \times \mathcal{R}$;	transmission lines between regions	q, r
$\mathcal{P}$		electricity generation technologies	p
$\mathcal{P}_{\text{ren}}$	$\subset \mathcal{P}$;	renewable technologies	p
$\mathcal{P}_{\text{wind}}$	$\subset \mathcal{P}_{\text{ren}}$;	wind technologies	p
$\mathcal{P}_{\text{hydro}}$	$\subset \mathcal{P}_{\text{ren}}$;	hydropower technologies	p
$\mathcal{K}$		technologies for transmission	k
$\mathcal{S}$	$:= \{1, \ldots, S\} \subset \mathcal{I}$;	new capacity investment years	s
$\mathcal{T}$	$:= \{1, \ldots, T\}$;	time steps within a year	t
$\mathcal{T}_{\text{start}}(p)$	$\subset \mathcal{T} \cup \{0\}$;	hours in the start-up interval for technology $p \in \mathcal{P}$	t
$\mathcal{I}$	$:= \{S - I + 1, \ldots, S\}$;	investment periods, $I = \lvert\mathcal{I}\rvert$	i
$\mathcal{I}^{\mathcal{P}}_{\text{active}}(p, s)$	$:= \mathcal{I} \cap \{s - U_p, \ldots, s\}$;	investment periods for each technology type $p \in \mathcal{P}$ with lifespan U_p that are active at year $s \in \mathcal{S}$	i
$\mathcal{I}^{\mathcal{K}}_{\text{active}}(s)$	$:= \{S - I + 1, \ldots, s\}$;	investment periods for each transmission type that are active at year $s \in \mathcal{S}$	i

Table A.2: The variables used in the full-scale model

symbol	restriction	explanation	unit
y_{pri}	≥ 0	Investments in capacity (both new and old) for technology $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ during investment period $i \in \mathcal{I}$	GW
u_{kqri}	≥ 0	Investments in new transmission capacity for transmission type $k \in \mathcal{K}$ between regions q and r, $(q,r) \in \mathcal{A}$, at investment period $i \in \mathcal{I}$	GW
x_{prist}	≥ 0	Generated electricity of technology type $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ using technology from investment period $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$ in year $s \in \mathcal{S}$ and time step $t \in \mathcal{T}$	GWh/h
v_{kqrst}	≥ 0	Electricity traded with transmission type $k \in \mathcal{K}$ from region q to region r, $(q,r) \in \mathcal{A}$, in year $s \in \mathcal{S}$ at time $t \in \mathcal{T}$	GWh/h
w_{rst}	≥ 0	Stored hydropower in region $r \in \mathcal{R}$, year $s \in \mathcal{S}$ at time step $t \in \mathcal{T}$.	GWh
z_{prist}	≥ 0	Available hot capacity of technology type $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ using technology from investment period $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$, in year $s \in \mathcal{S}$ at time $t \in \mathcal{T}$	GWh/h
z^{+}_{prist}	≥ 0	Increase in hot capacity from time step $t-1 \in \mathcal{T}$ to $t \in \mathcal{T}$ for technology type $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ using technology from investment period $i \in \mathcal{I}^{\mathcal{P}}_{\text{active}}(s,p)$ in year $s \in \mathcal{S}$	GWh/h
e^{tot}_{st}	≥ 0	Auxiliary definition variable for the total system emissions at year $s \in \mathcal{S}$ in time step $t \in \mathcal{T}$	tonnes CO_2

Table A.3: The parameters used in the full-scale model

symbol	representation	unit
b^{gen}_{pri}	Existing electricity generation capacity of technology $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ in investment period $i \in \mathcal{I} \setminus \mathcal{S}$	GW
b^{tra}_{kqr}	Existing transmission capacity of technology $k \in \mathcal{K}$ on transmission line $(q, r) \in \mathcal{A}$	GW
c^{invtech}_{ps}	Investment cost for technology type $p \in \mathcal{P}$ during year $s \in \mathcal{S}$. Includes an annuity factor	k€/GW
c^{omf}_{p}	Fixed operation and maintenance costs for technology type $p \in \mathcal{P}$	k€/GW
c^{run}_{pri}	Run cost for technology type $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ using technology from investment period $i \in \mathcal{I}$	k€/(GWh/h)
c^{invtra}_{kqrs}	Investment cost of new transmission capacity of transmission technology $k \in \mathcal{K}$ on transmission line $(q, r) \in \mathcal{A}$ during year $s \in \mathcal{S}$. This cost is divided by two to compensate for double arcs since the network is expressed only with non-negative arcs. Includes an annuity factor	k€/GW
c^{tra}_{kqr}	Transmission cost of transmission technology $k \in \mathcal{K}$ on transmission line $(q, r) \in \mathcal{A}$	k€/(GWh/h)
c^{+}_{prs}	Upstart cost for technology type $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ in year $s \in \mathcal{S}$	k€/(GWh/h)
$\tilde{c}_{prs}$	Part load cost for technology type $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ using technology at year $s \in \mathcal{S}$	k€/(GWh/h)
$c^{\text{CO}_2}_{s}$	The costs for emissions at year $s \in \mathcal{S}$	k€/tonnes CO_2
d_{rst}	Demand in region $r \in \mathcal{R}$ at year $s \in \mathcal{S}$ and time step $t \in \mathcal{T}$	GWh/h
e_{pri}	Emissions per produced GWh of technology $p \in \mathcal{P}$ in region $r \in \mathcal{R}$ using technology from investment period $i \in \mathcal{I}$	tonnes CO_2/(GWh/h)
$\tilde{e}_{pri}$	Extra emissions when running on part-load for technology type $p \in \mathcal{P}_{\text{thermal}}$ in region $r \in \mathcal{R}$ using technology from investment period $i \in \mathcal{I}$	tonnes CO_2/(GWh/h)
e^{+}_{pri}	Upstart emissions for technology type $p \in \mathcal{P}_{\text{thermal}}$ in region $r \in \mathcal{R}$ using technology from investment period $i \in \mathcal{I}$	tonnes CO_2/(GWh/h)

θ_{prt}	Weather profile for renewable technologies $p \in \mathcal{P}_{\text{ren}}$ in region $r \in \mathcal{R}$ at time step $t \in \mathcal{T}$	share
g_{rt}	Inflow into hydro power from rain, ground etc. in region $r \in \mathcal{R}$ during time step $t \in \mathcal{T}$	GWh
δ_r^{inc}	The maximum ramping rate for water level increase in hydropower in region $r \in \mathcal{R}$	share
δ_r^{dec}	The maximum ramping rate for water level decrease in hydropower in region $r \in \mathcal{R}$	share
W_{pr}	Maximum capacity of wind, i.e. land availability, for wind technology $p \in \mathcal{P}_{\text{wind}}$ in region $r \in \mathcal{R}$	GW
ϕ_p	Minimum load level for technology $p \in \mathcal{P}_{\text{thermal}}$	share
f_{ls}	Minimum RES load level in country $l \in \mathcal{L}$ during year $s \in \mathcal{S}$	GWh/h
U_p	The lifespan of technology type $p \in \mathcal{P}$	years
H_r	Upper limit for hydropower storage in region $r \in \mathcal{R}$	GWh
T	The number of total time steps in the model	
S	The total number of years where it is possible to make new investments in capacity	
I	The total number of investment periods in the model	

A.2 Hours-to-Decades model

Table A.4: The index sets used in the Hours-to-Decades model

symbol	representation	member
$\mathcal{I}$	set of all regions	i, j
$\mathcal{P}$	$:= \mathcal{P}^{\text{bat}} \cup \mathcal{P}^{\text{electrolysis}} \cup \mathcal{P}^{\text{hydrogen}} \cup \mathcal{P}^{\text{gen}}$; set of all technology aggregates	p
$\mathcal{P}^{\text{bat}}$	set of all battery technologies	p
$\mathcal{P}^{\text{electrolysis}}$	set of all electrolyzer technologies	p
$\mathcal{P}^{\text{hydrogen}}$	set of all hydrogen storage technologies	p
$\mathcal{P}^{\text{gen}}$	$:= \mathcal{P}^{\text{wind}} \cup \mathcal{P}^{\text{therm}} \cup \mathcal{P}^{\text{solar}}$; set of all electricity generation technologies	p
$\mathcal{P}^{\text{wind}}$	set of all wind technologies	p
$\mathcal{P}^{\text{therm}}$	set of all thermal technologies	p
$\mathcal{P}^{\text{solar}}$	set of all solar technologies	p
$\mathcal{Q}$	set of technologies for transmission	q
$\mathcal{S}$	$:= \{1, \ldots, S\}$; set of all 2-week segments (typically, $S = 26$)	s
$\mathcal{T}_s$	$:= \{(s-1)T + 1, \ldots, sT\}$; set of all time steps in the 2-week segment $s \in \mathcal{S}$	t
$\mathcal{K}_p$	$:= \{0, \ldots\}$; set of hours in the start-up interval for technology $p \in \mathcal{P}_{\text{thermal}}$	k
$\mathcal{R}$	set of cost classes, i.e., the steps in the cost–supply curve	r
$\mathcal{Y}$	set of years	y

Table A.5: The variables used in the Hours-to-Decades model

symbol	restriction	explanation	unit
w_{ipr}	≥ 0	investment in region $i \in \mathcal{I}$ in generation technology $p \in \mathcal{P}^{\text{gen}}$ in cost class $r \in \mathcal{R}$	GW
w_{ipr}	≥ 0	investment in storage capacity in region $i \in \mathcal{I}$, technology $p \in \mathcal{P}^{\text{bat}} \cup \mathcal{P}^{\text{hydrogen}}$ in cost class $r \in \mathcal{R}$	GWh
h_{ijqr}	≥ 0	investment in transmission capacity between regions $i, j \in \mathcal{I}$ using transmission technology $q \in \mathcal{Q}$ in cost class $r \in \mathcal{R}$	GW
g_{ipt}	≥ 0	electricity generation in region $i \in \mathcal{I}$, technology $p \in \mathcal{P}^{\text{gen}}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	GWh/h
g_{ipt}	≥ 0	battery storage in region $i \in \mathcal{I}$, technology $p \in \mathcal{P}^{\text{bat}}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	GWh
g_{ipt}	≥ 0	hydrogen storage in region $i \in \mathcal{I}$, technology $p \in \mathcal{P}^{\text{hydrogen}}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	GWh
e_{ijt}		electricity export from region $i \in \mathcal{I}$ to region $j \in \mathcal{I}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$ ($e_{ijt} < 0$ represents import to i from j)	GWh/h
e_{ijt}^{pos}	≥ 0	absolute value of electricity export from region $i \in \mathcal{I}$ to region $j \in \mathcal{I}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	GWh/h
c_{ipt}^{cycl}	≥ 0	resulting thermal cycling costs in region $i \in \mathcal{I}$ for technology $p \in \mathcal{P}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	k€/h
b_{ipt}^{charge}	≥ 0	battery charging in region $i \in \mathcal{I}$, technology $p \in \mathcal{P}^{\text{bat}}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	GWh/h
$b_{ipt}^{\text{discharge}}$	≥ 0	battery discharging in region $i \in \mathcal{I}$, technology $p \in \mathcal{P}^{\text{bat}}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	GWh/h
g_{ipt}^{active}	≥ 0	activated thermal capacity in region $i \in \mathcal{I}$, technology $p \in \mathcal{P}^{\text{therm}}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	GW
g_{ipt}^{on}	≥ 0	started thermal capacity in region $i \in \mathcal{I}$, technology $p \in \mathcal{P}^{\text{therm}}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	GW
d_{it}^{hydrogen}	≥ 0	electricity consumption in the electrolyzer in region $i \in \mathcal{I}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	GWh/h

Table A.6: The parameters used in the Hours-to-Decades model

symbol	representation	unit
S	number of 2-week segments	1
T	number of time steps in each 2-week segment	1
C_p^{inv}	investment cost of technology $p \in \mathcal{P}^{\text{gen}}$	k€/GW
C_p^{inv}	investment cost of storage capacity for technology $p \in \mathcal{P}^{\text{bat}} \cup \mathcal{P}^{\text{hydrogen}}$	k€/GWh
$C_{q,i,j}^{\text{h-inv}}$	investment cost of transmission technology $q \in \mathcal{Q}$ between regions $i, j \in \mathcal{I}$	k€/GW
$\lambda_{ipsr}^{\text{e}}$	share of the investment cost for technology $p \in \mathcal{P}$ in region $i \in \mathcal{I}$ taken by cost class $r \in \mathcal{R}$ and segment $s \in \mathcal{S}$	1
α_{nips}, β_n	parameters used to compute $\lambda_{ipsr}^{\text{e}}$ in iteration n of the consensus loop	1
$\lambda_{ijqrs}^{\text{h}}$	share of the investment cost for transmission technology $q \in \mathcal{Q}$ between regions $i, j \in \mathcal{I}$ taken by cost class $r \in \mathcal{R}$ and segment $s \in \mathcal{S}$	1
C_{pt}^{run}	running cost of technology $p \in \mathcal{P}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	k€/GWh
C_t^{exp}	cost of transmitting electricity at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	k€/GWh
M_{ipr}^{e}	cost class potential for generation technology $p \in \mathcal{P}$ in region $i \in \mathcal{I}$ and cost class $r \in \mathcal{R}$	GW
M_{ijqr}^{h}	cost class potential for transmission technology $q \in \mathcal{Q}$ between regions $i, j \in \mathcal{I}$ in cost class $r \in \mathcal{R}$	GW
D_{it}	demand for electricity in region $i \in \mathcal{I}$ at time $t \in \cup_{s \in \mathcal{S}} \mathcal{T}_s$	GWh/h
D_i^{hydrogen}	electricity demand for hydrogen in region $i \in \mathcal{I}$	GWh/h
η_p	efficiency of technology $p \in \mathcal{P}$	1
A_{ip}	regional resources based on land available in region $i \in \mathcal{I}$ for technology $p \in \mathcal{P}$	GW
ξ_p^{min}	minimum share of load for $p \in \mathcal{P}^{\text{therm}}$	1
C_{ipt}^{on}	start-up cost in region $i \in \mathcal{I}$ for technology $p \in \mathcal{P}^{\text{therm}}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	k€/(GW·h)
C_{ipt}^{part}	part-load cost in region $i \in \mathcal{I}$ for technology $p \in \mathcal{P}^{\text{therm}}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	k€/GWh
$G_{i,p,t}^{\text{active}}$	activated thermal capacity from previous iteration in region $i \in \mathcal{I}$, technology $p \in \mathcal{P}^{\text{therm}}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	GW

$G^{\text{on}}_{i,p,t}$	started thermal capacity from previous iteration in region $i \in \mathcal{I}$, technology $p \in \mathcal{P}^{\text{therm}}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	GW
θ_{ipt}	weather profile for region $i \in \mathcal{I}$ of technology $p \in \mathcal{P}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$	1
C^{add}_{pty}	additional, future, running cost for technology $p \in \mathcal{P}$ at time step $t \in \mathcal{T}_s$, $s \in \mathcal{S}$ and year $y \in \mathcal{Y}$	k€/GWh
Z_p	technical life-time of technology $p \in \mathcal{P}$	years