THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Networks of urban interaction

Growth and centrality in the complex geography of urban activity

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Cover:
Potential urban activity, simulated using the preferential centrality model (see section 8.5).

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ABSTRACT

How cities and regions grow and decline depend on technological, social and economic factors. Understanding the interplay of these forces is central in research efforts aiming to improve urban and transport planning. The purpose of this thesis is to explore how mathematical modelling and computer simulation can contribute to these efforts and a central aim is to achieve practically useful models with retained conceptual simplicity as well as correspondence to important empirical patterns.

The approach combines a spatially fine-grained representation of land, with processes of urban interaction based on the theory of complex networks. Urban activity at a location is modelled as the sum of all economic interactions stemming from that location. The potentials for interactions and activity are deduced mainly from spatial constraints, such as transport networks and land use regulations. Concepts that are studied include urban growth, accessibility and urban agglomeration.

For model validation, an extensive data set on Swedish land taxation values is used. These values are based on actual sales prices and rent levels and can thus be considered as reasonable proxies for urban economic activity. Comparisons are made between empirical data and model outcomes, both with regard to probability distributions and geographical distributions.

The empirical probability distribution of land values is found to be well approximated by a power law, strengthening the case for modelling the system as a complex network based on a process of multiplicative growth. By combining these principles with spatial interaction mediated by a transport network, the *preferential centrality* model is formulated. The activity predictions generated by this model reproduces empirical geographical patterns of land values.

The presented models provide explanatory links between the structure of transportation networks and the geographical distributions of urban economic activity. This makes them attractive as starting points for the further step of creating practically useful planning applications. For example, the models could be used to assess how specific transport infrastructure improvements influence urban expansion.

Keywords: urban growth, complex networks, centrality, transport infrastructure, urban activity, power law, Zipf’s law, accessibility, spatial interaction

Included publications

This thesis is based on the work contained in the following papers:

Paper I. C. Andersson, A. Hellervik, K. Lindgren, J. Tornberg, and A. Hagson. *Urban Economy as a scale-free network,* Physical Review E 68, 036124 (2003).

Contributions: CA and AH conceived of the presented idea and designed the study. JT obtained and pre-processed the data. A. Hagson and KL provided conceptual input. CA and AH jointly performed theoretical modelling, simulation, data analysis, interpretation of results and writing of the manuscript. KL supervised the project.

Paper II. C. Andersson, A. Hellervik, and K. Lindgren. *A Spatial Network Explanation of a Hierarchy of Urban Power Laws*, Physica A, 345: 227-244 (2005).

Contributions: CA and AH conceived of the presented idea and designed the study. CA and AH jointly performed theoretical modelling, simulation, data analysis, interpretation of results and writing of the manuscript. KL provided conceptual input and supervised the project.

Paper III. C. Andersson, K. Frenken, and A. Hellervik. *A Complex Network Approach to Urban Growth*. Environment and Planning A: Economy and Space. 38(10):1941-1964 (2006).

Contributions: CA and AH conceived of the presented idea and designed the study. KF provided theoretical input. CA and AH jointly performed theoretical modelling, simulation, data analysis and writing of the manuscript.

Paper IV. A. Hellervik, L. Nilsson, and C. Andersson. *Preferential centrality – A new measure unifying urban activity, attraction and accessibility*. Environment and Planning B: Urban Analytics and City Science 46.7: 1331-1346 (2019).

Contributions: AH and LN conceived of the presented idea. AH, LN and CA designed the study. AH performed theoretical modelling and simulation. AH and LN performed data analysis. LN performed the statistical analysis and created maps. CA supervised the project and contributed with conceptual input. All authors interpreted the results and wrote the manuscript.

Paper V. A. Hellervik, L. Nilsson, and C. Andersson. *Preferential centrality as a multi-regional model for spatial interaction and urban agglomeration.* Manuscript submitted and under review (2021).

Contributions: AH and LN conceived of the presented idea and designed the study. AH performed theoretical modelling, simulation and data analysis. AH wrote the manuscript in consultation with LN and CA. LN created maps. AH performed statistical analysis in consultation with LN. CA supervised the project. All authors interpreted the results, edited and approved the final version.

Acknowledgements

I have heard that writing a PhD thesis is not supposed to be a life´s work. However, 18 years have gone by since I programmed the first simulation of the urban growth model that is now an integral part of this thesis. Circumstances lured me to pursue more practical ambitions outside of academia, but there has always been a strong feeling of something important left unfinished.

I am deeply grateful to my supervisor Claes Andersson for allowing me to soothe this nagging mental itch. After all these years, you still believed in my ideas and cheered them on. Also, you are a great inspiration in the intricate craft of putting analytical thoughts on paper in an engaging way.

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Göteborg, September 2021

Alexander Hellervik

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Part I

1 Introduction

1.1 Background and aim

Cities and regions are shaped by a constant struggle to overcome the physical constraints hindering human and economic interaction. To this end, technological, social and economic structures are continuously developing into an increasingly intertwined urban system. Many of the components in this system can be described as being connected in networks. At the bottom, there are physical networks, such as rivers, roads, and rail-roads, and on top of them, we can find several layers of increasingly abstract networks of social and economic interactions.

The location, growth and decline of urban activities, such as workplaces, services and housing, are to a large extent dependent on access to other activities and how this access is mediated through transportation networks. At the same time, the transportation system itself is slowly transformed in response to the needs and locations of these urban activities. The slowly changing physical urban structures, such as buildings and transportation networks, set the stage for more rapid changes in activities and in their interactions.

The fact that physical structures might have a strong influence on many social and economic outcomes has spurred a growing interest in how policy and planning for land use and transportation can achieve broader societal aims. Considerations that are becoming increasingly important are, for example, limitation of climate and ecological impacts, energy consumption, quality of life, segregation and the balance between urban and rural development. The question has also been raised regarding the future role of large cities as centres for employment and activity. If trends regarding remote working become the new norm – what geographical transformations will that entail? What is clear is that behavioural changes, as well as changes in infrastructure and technology, can have profound impacts on the spatial organization of human and economic activities.

For the purpose of gaining a deeper understanding of all these processes and their linkages, it is indispensable to build models. Such models can be either plainly conceptual or also involve mathematical and algorithmic aspects. In the mathematical modelling of urban systems, approaches often fall within one of two categories, as discussed by Barthelemy (2016) and shown in Figure 1.

On one side of the modelling spectrum, we find conceptually and mathematically clear models aiming to capture general principles but without the intent of providing empirically verifiable predictions for specific cases. On the opposite side, we find complicated and data-hungry simulation models with detailed representations of many sub-processes. These latter models can be calibrated to achieve high empirical accuracy for a particular setting, but this comes with a large risk of overfitting.

Both approaches face substantial (but different) challenges when applied in actual urban and transport planning. In the former case, without empirical verification, it becomes difficult to trust any specific predictions. In the latter case, with too many parameters, there will always be a huge risk that conclusions are limited to the time and place represented by the calibration data. In a rapidly changing technological environment, this can quickly make a detailed model useless in practice.

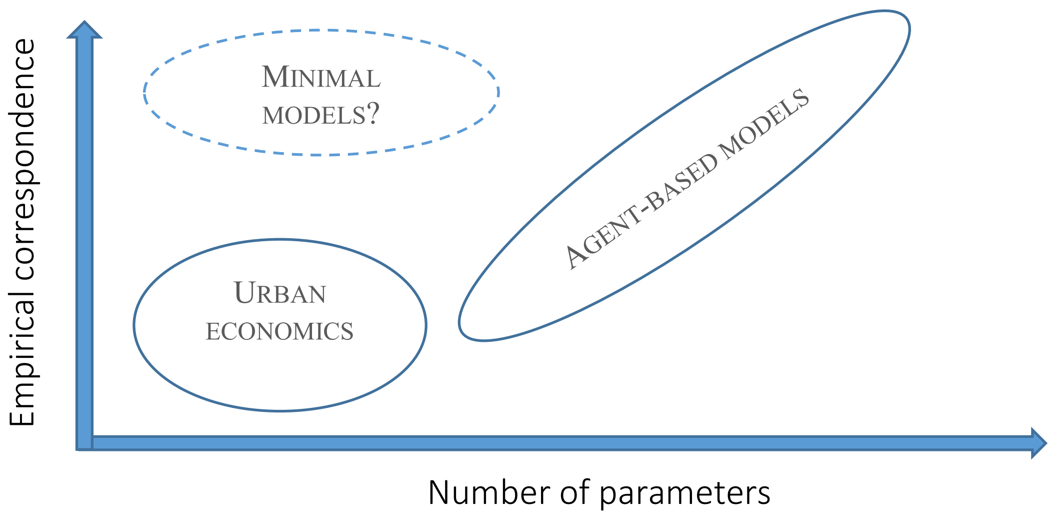

Figure 1. Adapted from Barthelemy (2016). The rationale for searching for new and minimal urban models.

This thesis is positioned in a line of urban modelling research (Wilson, 2000; Batty, 2012; Barthelemy, 2016) that tries to overcome this dichotomy by applying concepts and mathematical tools inspired by theories of complex systems, statistical physics and complex networks. The aim is to create minimal models with acceptable conceptual clarity, as few parameters as possible and satisfactory empirical validation.

To model urban systems in a way that is relevant in planning for the future, a solid grounding in basic principles is needed, and these principles need to be verified by observations. Models with a sole focus on phenomenology bear the risk of over-emphasizing aspects of the current state of the system that is being observed. A minimal model using less data might actually be more reliable in understanding processes that extend over time, not at least because we can anticipate that unknown non-marginal changes will occur in the future regarding transport technology and other system parameters.

1.2 Conceptual framework

Urban systems are in this thesis represented as outcomes of a co-evolutionary process involving land use and several layers of man-made networks, from physical infrastructure to abstract social and economic interaction networks. Urban activity at a location is assumed to consist of the aggregate interactions involving that location, which means that any non-interacting activity, by definition, is excluded from the modelling.

Figure 2 shows an overview of the underlying framework that forms the basis of the studied modelling approaches. Urban land use and its activities are modified by constraints from spatial planning and the spatial networks (such as streets and roads) together with the economic forces exerted by the interaction network.

The interaction network is shaped by the relative accessibility between different activities. Accessibility is determined in the interplay between different classes of activities that can take the role of both being attractions and origins for the interactions. Travel times (or generalised travel costs) make up the other necessary component in an accessibility calculation, and these are largely determined by the physical networks.

The physical network is shaped in part by transport planning and in part by the constraints of other urban land use and natural geography. In the physical network, we find flows of people, vehicles and goods. These flows are determined by the needs posed by the interaction network, but the actual routes taken are determined by the physical network structure. When flows grow large relative to capacity, negative feedback is initiated due to congestion, where travel times and accessibility, as well as the resulting interactions, might be dampened.

Since economic flow drives urban land use, positive feedback in the form of agglomerative forces can occur. This can be described as a mechanism where the urban interaction networks create conditions for their own further growth.

Between urban land use and the physical network, there is another, slower mutual dependency. On the one hand, the changes in urban land use drive the demand for new links and increased capacity in the physical network. On the other hand, the physical network can create necessary local conditions for the development of land. This means that the linkages between transport and land use are both direct and indirect, and these distinctions must be handled with care in any model or empirical study.

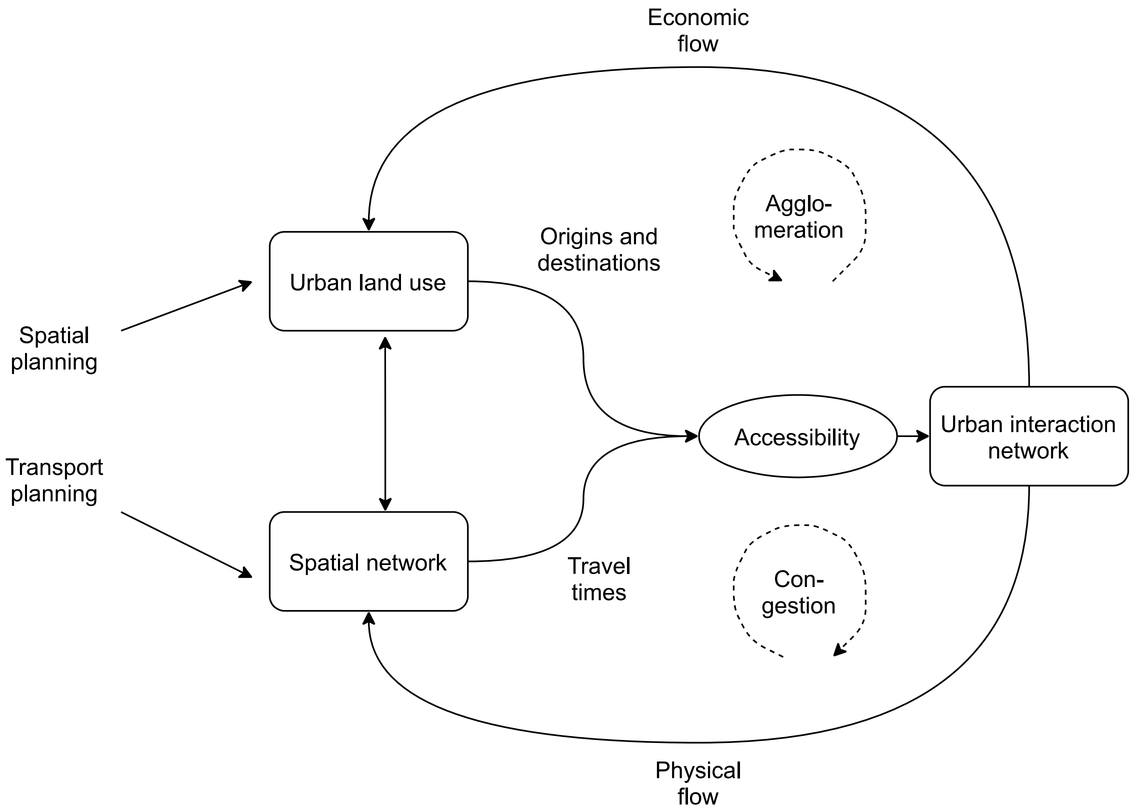

Figure 2. A conceptual framework for the formulation of urban models.

The picture painted in Figure 2 is, of course, only one of many possible ways to break apart and simplify the linkages between relevant subsystems. Another related conceptualisation is the land-use transport feedback cycle (Wegener, 1995). However, any conceptual framework can only serve as a starting point. The further steps of mathematical modelling and empirical tests are also crucial to determine the explanatory relevance of the framework.

1.3 Scientific context

There is a vast literature on using economic theory to describe and model cities and regions (e.g. Glaeser, 2008; Fujita et al., 1999). The research presented in this thesis is not based in that tradition. No microeconomic interpretation is presented, markets are not explicitly modelled, and there are no formal notions of supply and demand. With regard to the lack of connections to these economic concepts, the research efforts should instead be considered to be about the spatial relations between locations and the resulting forces on the geography of urban activity.

Although the research is not based on economic theory, *urban economic activity* is an important underlying concept, both with regard to empirical data and interpretation of model assumptions. The main empirical data source is urban land taxation values, which we consider to be a relevant proxy for urban activity.

It should also be noted that the empirical studies are not based on an econometric tradition. For instance, we have not attempted to derive an estimation of land value for particular locations. The presented methods could, however, be relevant with regard to land value assessment, but then many more empirical factors and methodological concerns should be taken into account.

Two of the included papers are published in physics journals. This does not mean that the presented research should be interpreted as a direct application of physics-based modelling to urban systems. The relation to physics should rather be considered as a shared approach with regard to modelling principles. Models should be based on a minimal number of simple principles, and model outcomes should be possible to interpret in relation to observations. Also, urban models should provide more than bare numerical predictions – they also need to contribute to relevant explanations in the context of geography and planning.

As discussed by Pumain (2020), regarding the increasing multi-disciplinary efforts to understand cities, there is yet no agreed-upon set of terminology, entities or core concepts. This, unfortunately, means that any reader of this thesis, regardless of their scientific field, will probably find some usage of a term or concept confusing or even erroneous.

1.4 About this thesis

The thesis is based on the work documented in the five included papers as summarised in Table 1. The first three papers were published during 2003-2006 and describes a complex network approach to urban growth. The work for the two later papers was performed during 2016-2021 and continue building upon the same conceptual framework but with a shift of focus toward urban centrality. In the meantime, from 2006 to 2016, the author has worked in practice, mainly with transport infrastructure planning, which has contributed to a shift in focus from abstraction towards models aimed at actual applications.

The long timeframe for the underlying research has had some impact on the consistency of terminology and notation between Papers I-III and Papers IV-V. One advantage of the duration has been increased clarity regarding what kinds of development of the original models were necessary to increase applicability in land use and transport planning. The introduction of an explicit representation of the transport network has been the most important of these extensions.

Paper	**Type of model**	**Empirical data**	**Empirical comparisons**	**Scale**
I	Urban Growth	Land taxation values	Cells	National
II	Urban Growth	Land taxation values, population data	Cells and clusters	National
III	Urban Growth	Land taxation values, population data	Cells and clusters	National
IV	Urban Centrality	Land taxation values, land use codes, road network	Property-based zones	Single city
V	Urban Centrality	Land taxation values, land use codes, road network	Property-based zones and administrative taxation areas	Regional and multi-regional

Table 1. Overview of included papers.

These are the main claims presented in the papers:

- Cities are not needed as entities when explaining the observed distributions of urban land value. Instead, a cellular (or zonal) representation can be considered more fundamental.
- Conceiving the urban geography as a system of interconnected networks provides a natural framework to model and explain the spatial distribution of economic activity.
- A spatial multiplicative growth model can explain several aspects of observed statistical distributions of urban activity.
- A novel centrality model, incorporating the underlying physical transportation network, as well as multiplicative attraction of activity, can predict spatial patterns of urban activity, both regionally and within cities.

The thesis consists of two parts. The first part contains a general framing of the included papers and the presented models within the contexts of complex systems, complex networks and urban modelling. The second part consists of the five included papers and supplementary material.

For complete descriptions of the discussed models, the reader is referred to paper II or III for the urban growth model and to paper IV for the centrality model.

2 Complex systems

Both the natural and social sciences have, during the last century, experienced enormous growth in activity and published results. With this growth has also come an accelerating branching into disciplines and sub-disciplines. These specialisations are naturally followed by a tendency to spend large efforts in understanding single components of systems.

The branching of knowledge-production is in part counterbalanced by efforts to synthesise different branches by reaching out to other disciplines. The science of complex systems is rooted in these efforts, with an aim to better understand how interaction according to simple rules can give rise to higher-order patterns.

Such emergent patterns are often not intuitively derivable from the behaviour of system components. One reason is that nonlinear interactions make aggregate patterns non-additive. This applies to many systems of both natural and social origin, with the exception of some designed systems, where elaborate care has been taken to isolate parts from each other to actively achieve linearity.

Complex systems science is far from a well-defined or well-delimited research area. Its perspectives are applicable to many branches of science, and its contributors come from very different scientific backgrounds. Some of these scientists are deeply rooted in a specific discipline and then focus on the complexity aspects arising in their particular problems. Others aim to transfer conclusions from one subject area to another, for example, from biology to economy.

There is also no agreed-upon definition of complexity, but a useful, intuitive notion is that truly complex systems lack compressed descriptions of their regularities (Gell-Mann, 1995). Cities and urban systems are great examples of this. Many small parts interact according to local rules, causing the whole system to behave in a way that is not particularly intended by any specific actor.

Complex systems theory provides a large toolbox with a diverse set of concepts and models. This opens up many interesting perspectives with regard to geography and urban systems. In this thesis, only a subset of these possibilities has been explored, namely the tools of multiplicative growth, simulation and network analysis.

An important caveat to consider when applying these tools to understand urban systems is that there seems to be little to find with regard to universal laws of complexity. This means that every model has to be tested in the actual context for its suitability for the intended purpose. Or, as stated by Wilson (2000), regarding natural science concepts applied to problems in other disciplines: "The application of these ideas then have to stand up within the fields in which they are applied."

3 Urban modelling

Cities and urban systems have been studied within many subject areas, including demography, economics, geography, architecture, transportation, sociology, ecology, history and anthropology. From a methods perspective, the list could be extended with the disciplines of philosophy, statistics, mathematics, physics, computer science and complex systems science. This plethora of perspectives makes it impossible to give a compact overview of all different approaches towards theory and modelling.

Within a more narrow scope of models that can be more directly related to the complexity perspective, early examples include the models of von Thunen (1826), Weber (1929) and Christaller (1933). These models are, however, mostly concerned with a static urban structure and are not well adapted to describe the spatial structure of modern growing cities. Von Thunen's ring model depends on the interaction with a pre-defined urban centre and has no notion of relations between other locations. Weber's model of location includes no concept of urban form at all, since all focus is on the decisions of a single firm. Christaller's model could be used to describe urban structure, but the dependence on a pre-defined hierarchy makes it difficult to include a realistic representation of spatial urban dynamics.

In paper III, a distinction is made between conceptual and topical models for urban growth. Conceptual models are mostly aimed at providing theoretical insights about general characteristics of urban systems, with less focus on numerical predictive power for actual, specific cities. Examples of such models are multiplicative growth models, models of correlated percolation, diffusion-limited aggregation, agent-based evolutionary models, and neoclassical economic models (Alonso, 1964; Alonso, 1972; Arthur, 1987; Axtell and Florida, 2001; Batty, 1991; Batty and Longley, 1994; Berry and W. L. Garrison, 1958; Dendrinos and Rosser, 1992; Fujita et al., 1999; Gabaix, 1999; Henderson, 1974; Lane, 2002; Makse et al., 1998; Manrubia et al., 1999; Marsili and Zhang, 1998; Reed, 2002; Rosser, 1998; Simon, 1955).

Topical models for urban growth are typically designed to answer questions about specific cities or regions. They often have a complicated structure with many submodels. A common framework is cellular automata (CA), with Tobler (1979) as a pioneer. CA has been further developed and explored by Couclelis (1985), White and Engelen (1997), Clarke et al. (1997), Itami (1988), Andersson et al. (2002b), White et al. (2015) and many others. Another line of topical urban modelling is characterised by the use of agent-based models, such as the Simpop model (Sanders et al., 1997).

In both conceptual and topical modelling, it is useful to make the distinction between extensive and intensive urban growth. Cities can grow both by expanding outwards (extensive growth) as well as by densification (intensive growth). Densification can also come in several forms; the first concerns new buildings on previously unbuilt land lots, the second takes place by replacing low buildings with higher ones, and the third is by an intensification of activity within existing buildings.

Intensification of activity could mean higher turnover, more customers and more densely packed workplaces. In an economic sense, local urban intensification could also occur due to changing services or products towards higher value per area. For a residential area, intensification could take the form of residents getting jobs with higher wages.

The urban growth models presented in papers I-III deal both with intensive growth and extensive growth, but the two processes are governed by separable parameters to acknowledge the significant qualitative differences. However, the processes are also modelled as interlinked since a primary extensive growth event can cause secondary intensive growth and vice versa.

The transportation system is an integral part of any city or region. Transportation serves the urban system by allowing the movement of people and goods. However, these opportunities for transport also change the opportunities for land use. Good transportation opportunities (i.e. good accessibility) can transform a location and change the land use (Hansen, 1959). New land might be developed, and already developed land can be intensified.

Over time, a growing urban system might produce increased flow, leading to congestion in the transportation system. New transportation links are built to alleviate congestion and sometimes also to form straighter connections. Negative aspects of transportation flows can also generate needs for detours and bypasses. All of these processes create a feedback loop back from land use to transportation (Wegener, 1995).

The feedback between land use and transportation is only implicitly included in the models presented in papers I-III since no transportation network is part of the explicit representation. The implicit linkage arises from the special growth potential of the urban perimeter. The perimeter is, compared to completely external locations, assumed to have better access to transportation infrastructure (Andersson et al., 2002a). Urban clusters with a large perimeter will thus experience a higher growth rate in the model.

4 Power laws and fat-tailed distributions

4.1 Power laws in natural and social systems

Power-law distributions, and more generally, fat-tailed distributions, are often observed in natural systems, as well as in social and artificial systems. A measured property is said to follow a power-law distribution if the probability frequency distribution $p(x)$ behaves as:

$$p(x) \sim x^{-\gamma},$$

for some exponent $\gamma > 0$. The power law property is usually observed only in the tail (i.e. for sufficiently large values of x), and the body of the distribution then has some other shape.

Power-law distributions belong to the wider class of fat-tailed (or heavy-tailed) distributions, which refers to distributions with a higher skewness than normal and exponential distributions. The high skewness implies an increased probability for observing large deviations from the mean.

Empirical observation of these distributions goes back to Pareto (1896) and Zipf (1949). The earliest examples regarded income distributions, city sizes and word frequencies. As more and more large datasets have become available, new systems with power-law signatures have been added to the list. For reviews, see Newman (2005), Clauset et al. (2009) and Farmer and Geanakoplos (2005). Examples include financial markets, firm sizes, scientific citations, web page hits, book sales, telephone calls, the intensity of wars, personal wealth and frequencies of family names.

Observing a fat-tailed distribution is valuable since it provides a starting point for modelling by pruning the huge tree of possible model specifications. If a property is found to be power-law distributed, certain underlying models become less plausible. This is because most generative models *do not* produce power laws, even if a power law in itself does not point towards any specific model. Any explanatory model of a system with power-law properties should be able to reproduce these power laws, especially if the model is claimed to be valid on multiple scales.

There has been a substantial discussion in the literature regarding if particular systems are best described by a power-law distribution or instead by some other fat-tailed distribution. A partial reason for this debate is probably the mistaken claim that all complex systems must have a power-law signature. The misconception could stem from the concept of self-organised criticality (Bak, 1996). Bak argued that many systems can self-organise into a critical state and noted that such critical systems do obey power-law scaling. However, criticality is not a necessary condition for power laws since plenty of alternative explanations are available (Sornette, 2002).

No claim is made in this thesis that power laws in themselves are signatures of a certain model. This also means that measuring and estimating if a data set exactly obeys a power law or not is somewhat beside the point. When studying the empirical properties of complex systems, any chosen specific functional form of the distribution will be approximate. If a power law is deemed to be a relevant description or not should then be determined within the context.

4.2 Urban power laws

A famous urban power law is Zipf's law (1949) for city sizes. In its simplest form, it states that (within a country) the second largest city has half the population of the largest city, and that the third city has a population one-third of the largest, and so on. This distribution is easily spotted in a log-log rank-size diagram.

With some generalization, Zipf's law can have a variable exponent, with the original law having this exponent set to one. Empirical studies show that the exponent differs between countries (e.g. Ioannides and Overman, 2002).

Most studies of urban size distributions have focused on cities as objects and population as the measured quantity. In this thesis, the main data source is instead land values, and it is shown in paper II that this data also follows fat-tailed distributions. In the same paper, it is also demonstrated that aggregate land values are strongly related to population. This means that similar models can be used to explain both land values and population numbers as long as we stay on the city scale.

The aggregate level of cities is, however, not the only relevant scale for studying urban probability distributions. Understanding processes internal to cities is also highly relevant. When zooming in to a cellular (or zonal) level, an additional power law is found with regard to land values. In papers I-III, this finding is discussed further as a basis for model formulation.

4.3 Models for generating power-law distributions

There are many possible explanations and suggested generative processes that can create power-law distributions (Mitzenmacher, 2003). Common to most explanations is that some assumption of non-additivity is needed. Additive processes tend to be governed by the law of large numbers, which results in normal (Gaussian) distributions.

The simplest generative mechanisms for fat-tailed distributions are different variations of multiplicative random processes, of which Simon's model (1955) is an early example. More generally, it can be shown that exponential growth combined with random survival times are sufficient to produce power-law distributions (Reed, 2002). The assumption of multiplicative (exponential) growth is thus a natural starting point for more elaborate modelling of urban systems (Gabaix, 1999).

Another mechanism that can give rise to power laws is optimization (Carlson and Doyle, 1999). Highly optimized tolerance (HOT) can explain power laws in certain designed systems, where trade-offs are made between robustness and cost of inputs and outputs. For urban systems, this might, however, not often be very plausible because these systems are seldom designed from the top-down. Instead, growth and design take place in a piece-wise manner, suggesting that evolutionary or incremental models are better suited as explanations.

4.4 Urban allometric scaling laws

The study of urban allometric scaling has been gaining increasing attention (Bettencourt et al., 2007; West, 2017). Similar types of scaling relations with regard to urban fractal geometry were also studied by Batty and Longley (1994). Allometric scaling concerns the power-law relation between different measured quantities of organisms and other systems. Such scaling laws for cities can also, in a general sense, be considered to be "urban power laws", but allometric scaling is not at all equivalent to power-law distributions, such as Zipf's law (Pumain, 2004).

In this thesis, allometric scaling is not the main focus. However, in Paper II, certain relations of this kind are demonstrated for urban clusters regarding area, perimeter, population and land price. It would be a worthwhile future effort to examine the theoretical connections between the presented models and allometric scaling theory.

According to Bettencourt (2013), certain network characteristics, such as space-filling, can explain many urban scaling laws. This opens up possibilities for interesting theoretical connections between the structure of physical networks and the resulting potential for urban activity.

5 Complex networks

5.1 Large networks

A network is a representation of a system as a set of nodes connected by a set of edges. Edges could be directed or undirected and can, in the general case, have different attributes, such as weights. In some networks, multiple edges can connect the same pair of nodes.

Graph theory is the mathematical domain where networks are studied. Traditionally, graph theory studied small networks, easy to visualise and analyse, such as the Seven Bridges of Königsberg studied by Euler (1741)[1].

However, many real networks have a large number of nodes. This makes many graph-theoretical computations overwhelming to perform by hand, for example, Dijkstra's algorithm (1959) for shortest paths. With the increasing power of computers, empirically based studies of much larger networks have thus become possible.

The possibility of computer simulation has also given rise to many models of graph formation (see below) as well as studies of dynamical processes taking place with the graph as a substrate (e.g. Holme and Saramäki, 2012). Epidemics, computer communication and city traffic, are examples of the latter.

In the theoretical treatment of large networks, it seldom makes sense to explicitly compute properties of specific graph configurations. This is an important reason why most modelling studies instead focus on statistical properties. This is in analogy to the futility of describing the location of all the gas molecules in a room using Newton's equations of motion. Instead, statistical physics is employed to derive aggregate properties, such as temperature and entropy. A commonly studied statistical property of large networks is the degree

[1] Euler did not actually use the terminology of graph theory, instead he referred to his theory as "geometry of position". (Hopkins and Wilson, 2004)

distribution, where the *degree* of a node refers to the number of connected edges.

A well-understood model of large random networks is the Erdös-Renyi model (1960). In this model, any two nodes have an equal, fixed probability of being connected to each other. All nodes and edges are independent. This results in a degree distribution that can be approximated by Poisson distribution.

Another way to model large networks is to assume high regularity, such as in lattice networks. A physical analogue for regular networks would be crystalline structures.

In between regular networks and Erdös-Renyi random networks, we find a class of large networks are that can be classified as *complex*. Such networks have more internal structure than purely random networks and more variations than lattice networks. In analogy to complex systems, the structure of complex networks can be explained by a small set of underlying mechanisms.[2] Many complex networks show fat-tailed or power-law signatures in their degree distributions (see references below).

A large body of literature now exists laying out both empirical and theoretical findings regarding complex networks. Examples include small-world networks (Watts and Strogatz, 1998), the internet (Pastor-Satorras et al., 2001), the World Wide Web (Albert et al., 1999), social networks (Holme, 2003; Jin et al., 2001), human sexual contacts (Liljeros et al., 2001), ecological networks (food webs) (Camacho et al., 2002), phone call networks (Aiello et al., 2000), citation networks (Redner, 1998), international trade (Serrano and Boguñá, 2003), power grids (Albert et al., 2004), flight networks (Barrat et al., 2004), brain activity networks (Bullmore and Sporns, 2009) and protein folding problems (Scala et al., 2001). See, for example, the reviews of Albert and Barabási (2002) and Newman (2003) for additional treatments of this subject.

5.2 Growing networks

In paper I, we have suggested that the power-law signature in land values could be due to the network characteristics of the underlying urban economy.

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[2] If no such mechanisms can be identified, these systems could be called *complicated networks* instead of *complex*, in accordance with the distinction between complex and complicated systems (Andersson et al., 2014).

However, a network structure in itself is not sufficient to explain the observed fat-tailed distribution. There must also be some plausible mechanism at work that gives rise to that degree distribution in the urban network.

As mentioned above, many complex networks have fat-tailed degree distributions, and there is not a single common generative mechanism for all of these types of networks. One very common characteristic, though, is that of network growth, i.e. the addition of new nodes and edges.

In a seminal paper, Barabási and Albert (1999) introduced a growth model for generating scale-free networks. The mechanism is simple, and the key component is that new edges attach preferentially to existing nodes according to the node degree. In the original model, there is a linear relationship between the node degree and the propensity for attracting new edges.

An intuitive explanation for preferential selection is to consider the edges instead of the nodes. If any edge is chosen with uniform probability, and then one of the two nodes connected by the edge is chosen randomly, the resulting probability for a node to be chosen will be proportional to the node degree.

Is this a plausible mechanism for the formation of an urban interaction network? If edges, as we claim in papers I-III, represent beneficial interactions between locations, then edges themselves could be natural triggers of new edge formation. Against this hypothesis can be raised the argument that the benefits represented by edges are not independent of each other. Edges connected to the same node could, taken together, be more or less beneficial than the simple sum of edges.

The argument can be summarised as: Many large networks have fat-tailed degree distributions, and several measurements on urban systems also have fat-tailed distributions. Also, there are many properties of urban systems that lend themselves very well to a network representation. Can we from this chain of reasoning deduce that it is the network properties of cities that give rise to urban power laws?

Not necessarily, since there are many other plausible explanations for how urban power laws could arise without explicitly referring to a network representation, for example, due to local growth processes constrained by certain economic forces (Gabaix, 1999).

Nevertheless, the relations between complex networks and urban power laws seem to be a fruitful starting point for the formulation of hypotheses and model building and also for empirical testing of the hypotheses and models.

5.3 Node deletion and rewiring

An important feature, shared by the Barabási-Albert and the underlying Simon-model (1955), is the constant addition of nodes. This assumption is probably unrealistic for many networks since it requires a rather quick growth rate of new nodes. A slow growth rate would imply a very long duration to reach a stationary state with regard to the degree distribution.

One possible way to have multiplicative growth in a system of stationary size is to remove entities at the same rate as they are created. Reed (2002) describes how random survival times combined with exponential growth can give rise to a power law, and it is natural to extend this model to networks. Hellervik and Rodgers (2007) have also shown that a model with preferential attachment and the creation and deletion of entities can generate a power-law distribution in a non-growing system.

A complementary process to the deletion of nodes is the rewiring of edges. Some networks have nodes that are static by their nature, for example, citation networks, where new publications are introduced into the network with a fixed set of links. After that, the publication sits idle, and the only growth occurring is the slow aggregation of new citations from publications added at a later stage.

Other networks constantly rewire, either by the deletion of edges matched by the addition of new edges or by the change of just one of the endpoints of the edge. Social networks, where edges are undirected and represent a mutual relationship between individuals, is an example of the former type. Demand and supply networks in the economy can often be of the latter kind.

5.4 Spatial networks

Many real networks are embedded in a spatial setting, which means that nodes have locations and that the edges have a spatial structure (Barthelemy, 2011).

The first urban network model introduced in paper I, is non-spatial, meaning that the distance between nodes is unrelated to the probability of forming a new connection. When modelling an urban system, this is a very strong assumption, which gives rise to the second model presented in the same paper.

The second model has similar properties as the first one, except for an additional spatial mechanism. The probability of forming a new connection is assumed to decrease with distance. The important conclusion is that the sought-for fat-tailed distributions are retained, regardless of many types of spatial constraints.

6 Urban networks

6.1 Physical networks

It is difficult to imagine an urban system without a network of physical connections. These physical networks can be of artificial or natural origin and are means of transporting people, goods, waste, energy or information.

In any modern city, we find a multitude of physical networks: roads, streets, railroads, water pipes, power grids and communication networks. These networks differ from non-physical networks in certain important ways. For example, since connections have costs dependent on the length, long edges are relatively rare. Physical networks are thus prime examples of spatial networks[3].

In the models presented in papers I-III, there are no explicit physical networks. Instead is the urban periphery is modelled as being in its own special category. This periphery category should be interpreted as unbuilt land with access to the urban physical networks, such as roads and utilities.

In papers IV and V, road networks are explicitly represented and are used as the main input data. It is the streets and roads that provide the possibility to calculate the travel times that are used to determine the interaction network and hence the resulting potentials for urban activity.

When representing a physical network, such as a street network, there are some choices to be made regarding nodes and edges. For travel time calculations, the most common choice puts street segments as edges and intersections as nodes. This is what has been used in papers IV-V.

However, another possible representation inverts the above-stated relations. The street segments (or longer contiguous entities) can be taken as nodes and intersections as edges – linking the streets. Such a representation lies at the heart of the strain of theory and model building, which is often called “Space syntax” (Hillier and Hanson, 1989). In this thesis, such a representation has not been used. However, when travel impedances are to be calculated in a walking network, such a “dual approach” (Batty, 2013) could be a valuable direction for future explorations.

3 All spatial networks are not physical, though. For example, social networks can have a strong spatial component, but their edges are not of a physical nature.

6.2 Transit networks

In uncongested settings, for car travel, biking and walking, the relationship between the physical network and the travel time is fairly straightforward. The calculation might involve speed limits and some other physical or regulatory restrictions, but on the whole, the travel time is basically taken as distance divided by average speed.

For public transit, however, the relationship is more complicated, mainly due to the dependence on a timetable and a stopping pattern. For example, arriving at a railroad track will not by itself transfer travellers to their desired destinations. Operational train services are also needed, and the final travel time will also depend on waiting time and the distance from the train stop to the destination.

These considerations make it more natural to represent public transport as its own network, on top of, but distinct from, the physical network. Nodes are stops, and edges represent lines of service between them. Weights are needed on the edges to account for frequency.

Public transit networks are thus not pure physical networks and also cannot be described fully as some derived flow of vehicles on the physical network. This is due to the planned nature of the time tabled service. Flight networks can also be included in this category. However, they are even less clearly tied to a physical network since the only physical infrastructure needed is the nodes (airports).

Ferry connections are usually also time-tabled, but with the special characteristic that they might carry road vehicles. When used in the latter sense, they can be considered as extensions of the physical road network but with a slow average speed that is heavily dependent on trip frequency. In the centrality model used in paper V, where a road network is considered on the national scale, ferry connections are treated in this manner.

Except for ferry connections for cars, public transit is not studied explicitly in this thesis. In papers I-III, all physical networks are abstractly represented, which means that the treatment of public transit can be viewed as being on par with car traffic. However, in papers IV-V, the lack of a public transit representation[4] must be viewed as a simplification that could affect results and

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[4] Bus service could however be considered to be implicitly represented in an average sense, since the included road network can be used also by buses.

conclusions. To include public transit is thus an important model improvement worth pursuing in future work.

6.3 Interaction networks

Interaction networks, such as social networks and business networks, are most commonly observed and described with individual agents (persons or firms) as nodes and their connections as edges. By adding location data to the nodes, a spatially embedded representation can be created. We can then describe the interactions between places, for example, between housing and workplaces, creating a commuting network.

The directionality of edges in an urban interaction network can be ambiguous. Both agents often benefit from the interaction – in this sense, the relationship is mutual and undirected. However, in a trade relation, the flow of money is directed, and the corresponding flow of services and goods is oppositely directed.

In most cases, one of the agents initiates the interaction by seeking to fulfil a demand, which will be delivered by the counterpart. In papers I-III, this is represented as "primary effects", where the initiator creates the first edge endpoint. In papers IV-V, the flow of money is explicit, with incoming and outgoing flows seeking a balance.

The following examples give an overview of how the presented interaction models can be interpreted.

Example 1: A firm needs a new employee, and interaction is established with a worker. Money flows from the workplace to the housing location of the worker. The worker provides labour by travelling to the work-place[5]. A reverse physical flow is also present, with the worker travelling back home.

Example 2: A household needs groceries, and interaction is established with a supermarket. Money flows from the household to the supermarket, and groceries flow in the opposite direction. Physically the interaction is facilitated by a shopping trip or home delivery.

Urban interaction networks are conceptually powerful, but they are also inherently hard to observe, making them problematic as objects for empirical scientific inquiry. We might obtain snapshots of interactions by using travel

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[5] For some remarks on remote work, see section 7.2

surveys or data from mobile devices, but these will necessarily be far from complete. In most cases, we are left with having to observe only the consequences of the interactions.

In Papers I-V, the edges in the interaction networks are never empirically studied. Instead, edges are implied by the model, and the values observed are proxies for the activity values at nodes.

6.4 Modelling of urban interaction networks

When designing a network model to represent urban interactions, the first step is to define what is represented by the nodes and the edges. As discussed above, a fundamental definition would use agents as nodes, with agents representing individuals in the system.

Using agents as nodes has the problem of agents not being spatially static, which would make the spatial representation very complicated. An ever-moving spatial network might be fascinating to study, but there is low hope of tractability.

Our choice has instead been to use locations as nodes, based on the fact that most agents perform their activities at one or a few recurring places. Many localised activities are also more persistent over time than particular agents. There are several different possible choices for locational entities, for example, buildings, organisational entities with a location (such as firms and households), legally defined land lots or geometrically created cells. The two latter choices have been employed in this thesis.

Edges represent interactions between zones (or cells). In the most general case, different types of interactions must be considered since both social and economic forces are at play in the urban system. However, in the spirit of constructing a minimal model, we have used the concept of a generalized economic interaction represented by single-valued edge weights.

Another aspect to consider is how the concepts of co-location and internal interactions (self-interactions) are handled. When a node is a single agent, this is rarely a problem, but when a node represents an aggregation such as a geographical zone, self-interaction can become significant since a large number of agents could reside and be bundled together within one node. However, numerical explorations of the presented models have shown that self-interaction does not significantly alter results[6].

In papers I-III, where the aim is to model a growth process, new nodes are introduced into the system at a constant rate. Since nodes represent land lots, the growing number of nodes corresponds to an urban system growing in physical extent. This could be interpreted either as the construction of new buildings or as existing rural buildings being transformed into urban use.

In papers IV-V, the aim is instead to model the urban activity pattern at a single snapshot in time[7]. The interaction network is assumed to develop on such a short time scale so that the physical network and urban extents can be approximated as static. This means that the set of nodes is constant in the model, and it is only the edges that are being updated. The nodes (that represent zones) are extracted from empirical data using a set of criteria. Hence, it is a cross-sectional picture of the network that is being studied.

Interaction edges are created either from new nodes (papers I-III) or by connecting already existing nodes (papers I-V). No difference is made between weights and multiple edges between the same pair of nodes. An additional edge between already connected nodes is equivalent to incrementing the weight of the existing edge.

In papers I-III, all edges remain forever once they are formed, which can be considered overly unrealistic. In papers IV-V, this assumption is relaxed, and edges are allowed to rewire until a stationary state is achieved.

How edges are attracted is a key modelling mechanism, determining both the resulting spatial structure and the statistical distributions. In all presented models, this attraction is modelled as a combination of background factors and preferential (multiplicative) factors. The background factors can be interpreted as influence caused by constant characteristics of the location itself, such as zone area and suitability for construction. These are assumed to be independent of the activity level. Conversely, the preferential factors are strongly dependent on activity, creating further attraction in a positive feedback loop.

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[6] For example, see supplementary material for Paper IV.

[7] An overarching aim is to achieve an empirically valid combined model, where the centrality model in papers IV-V could form the basis for a model of growing physical networks and growing urban extents. See also section 12.5.

6.5 A complex network model for urban growth

Papers II and III describe a complete urban growth model based on the framework of complex networks. Here follows a compact description of the model structure[8].

The geographical area is divided into equally sized cells, where every cell can be in either a *developed* or *undeveloped* state. All developed cells are assumed to contain some economic activity and are thus also nodes in the interaction network. The number of interactions (the node degree) for cell i is denoted by a_i. This can interchangeably be called the activity of the cell, and all *developed* cells have a_i>0. Cells adjacent to developed cells (according to queen contiguity) are called *perimeter* cells. All other cells are called *external.* The numbers of perimeter cells and external cells at iteration t are denoted $n_t{}^{(P)}$ and $n_t{}^{(E)}$.

There is also a set of constant spatial interaction strengths between every pair of cells, denoted by D_{ij}. These values are calculated based on the Euclidian distance d_{ij} between the cells using an interaction function $D_{ij} = f(d_{ij})$, for example, $f(d_{ij}) = (1 + d_{ij})^{-\beta}$.

The interaction network is updated in every iteration by adding edges according to the following rules:

1. Assign local weights R_i to every cell.

   $R_i = 1$ for developed cells (i.e. a_i>0).

   $R_i = b$ for perimeter cells, where b is a parameter.

   $R_i = b\epsilon n_t{}^{(P)} / n_t{}^{(E)}$ for external cells, where ϵ is a parameter.

2. Choose a cell i for primary growth (source of the new edge).
   a. Randomly choose between additive growth (with probability q) and preferential growth (with probability $1 - q$).
   b. For preferential growth, an already developed cell is chosen with probability in proportion to the degree a_i.
   c. For additive growth, choose among all cells with probability proportional to local weight R_i.

[8] The notation is here adapted for consistency with papers IV and V.

3. Choose a cell j for secondary growth (destination of the new edge)
    a. Again, choose randomly between additive growth (probability q) and preferential growth (probability $1 - q$).
    b. For preferential growth, an already developed cell is chosen with probability in proportion to $D_{ij}a_j$, i.e. in proportion to the activity dampened by the static spatial interaction with the primary cell.
    c. For additive growth, choose among all cells with probability in proportion to $D_{ij}R_j$.
4. Add an edge between the primary cell i and the secondary cell j. The node degree (activity) is thus increased by one for both cells.

The simulation can be started from an empty grid since additive external growth will make sure that some urban seeds are created. Figure 3 illustrates how the iterations can play out with different combinations of additive and preferential growth. During the growth process, more and more cells will be activated, and they will form urban clusters. Both node degrees and cluster sizes (both with regard to area and aggregate degree) will evolve into following fat-tailed distributions, well approximated by power laws, as shown in papers II and III.

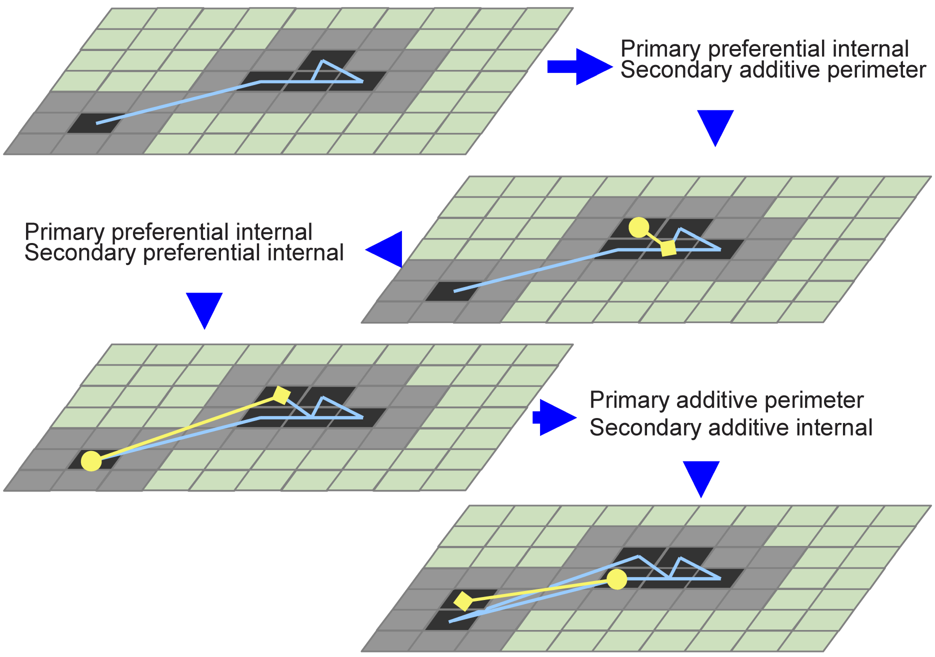


Figure 3. An example of the sequential process for updating the interaction network in the urban growth model. (This figure also appears in paper III.)

When comparing to other models of urban growth, two distinctions are important to make. The first is whether cities are modelled as composed entities or as emergent clusters. The second is whether the aim is to reproduce actual historical processes, or if the main interest is the statistical properties observed in current urban systems.

For example, the Simpop models (e.g. Bretagnolle and Pumain, 2010) use villages, towns, and cities as agents represented as points without spatial extent. These models have thus less spatial detail but more rules and parameters guiding the growth process, including a more detailed mechanism for the economy than what is implicit in our model. These model details give the Simpop models some capacity to describe actual historical transitions of settlements. The aim of the complex network model presented here is not that ambitious. Instead, the purpose is to show that these microscopic rules are sufficient for reproducing current statistical patterns on cellular and cluster levels.

Statistical patterns can also be observed in historical growth trajectories. Verbavatz and Barthelemy (2020) have shown that rather large fluctuations occur in interurban migration flows, casting into doubt the stability of empirically observed power-law distributions and their exponents. Our urban growth model does not in its current form capture such fluctuations, but they could be incorporated into the model formulation on the cellular level. One appealing possibility to generate significant volatility would be to let growth and decline events cascade over the interaction network.

6.6 Flow: Interaction materialised on a physical network

When an interaction network is to be realised within an urban system, it must use physical means to transport information, people and goods. This makes the interactions visible through the manifested flow in the underlying physical network. Observing the flow, however, will not give full information about the interactions since the same flow can result from different sets of origins and destinations.

There is also another sense in which interactions can be more intricate than physical flow. Two physically similar trips could carry very different amounts of economic and social value. For example, it would be difficult for an outside observer to discern the difference between an emergency drive to the hospital and a trip to the local convenience store.

Figure 4 shows how a modelled interaction network (from paper V) can be attributed to a physical network. The resulting flow has many visual similarities with measurements for traffic flow. However, an exact correspondence is not expected since the model does not include any concepts of actual vehicles or traffic.

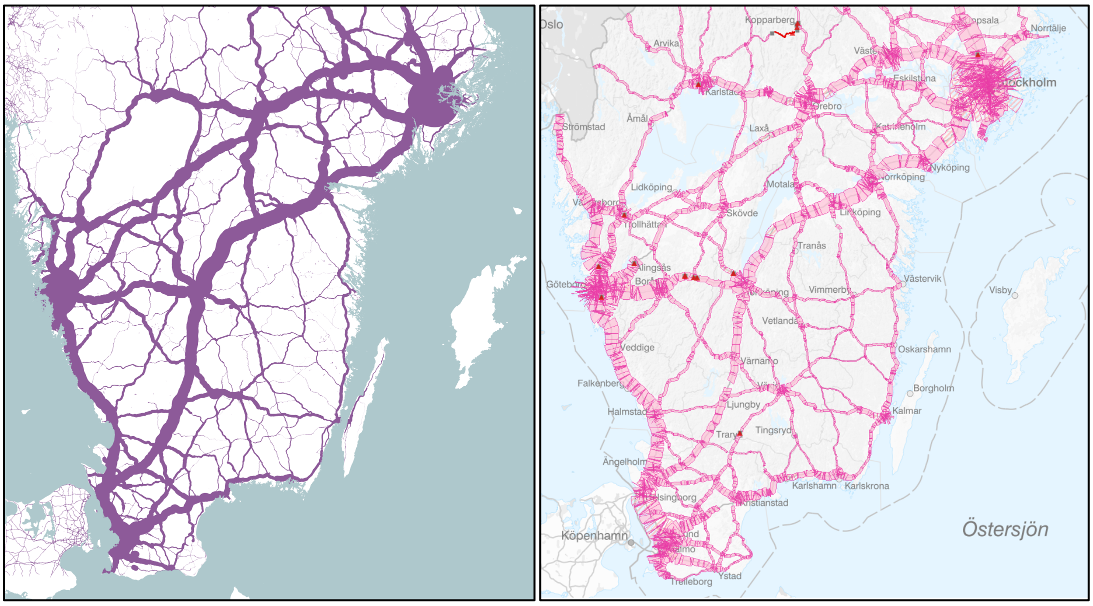


Figure 4. Left: Simulated flows on the major roads, for the model of southern Sweden, as implemented in Paper V. Right: Vehicular flow as measured by the Swedish Transport Administration

7 Urban activity

7.1 Definitions of urban activity

In this thesis, urban *activity* at a location is defined to be the summed value of interactions involving this location. In network terms, this corresponds to the weighted node degree. Activity is thus a generalised single value quantity representing both monetary economic value as well the intensity of non-monetised urban land use. Since the interaction-based definition makes activity additive, it can be used both at the level of land lots as well as on aggregates such as larger zones or cities.

In paper I, the definition is formulated as trade gains in “currency per unit area and unit time”. This goes back to the definition of interactions that must give some surplus to both endpoints of the edge.

In papers IV-V, activity is found by an assumed equilibrium between inflow and outflow of interaction. It could be argued that this definition is not equivalent to the definition in paper I since flow seems to measure turnover instead of surplus. The distinction has no importance for modelling results but could result in somewhat different interpretations.

The trade gain definition can also be interpreted as a willingness to pay for the interaction to occur. In many cases, but not all, this should stand in close relation to the monetary flow incurred by the interaction.

The idea behind a generalised concept of activity is that all types of urban land use should be measured by the same unit of account. Both services, workplaces and housing can then contribute to a unified quantity of activity. This makes land value a reasonable proxy for activity since the combined activities paying to reside at a location can be expected to have an aggregate activity level at least equal to the land rent.

If urban activity by definition is measured by interactions with other urban locations, there is room for an opposite definition for rural activity. Rural activity, such as agriculture and forestry, will then mainly interact with the land itself. Enjoyment of natural beauty could, in this dichotomy, also be classified as rural since the value and the choice of location for such activity stems from the characteristics of the landscape.

The rural and the urban networks are, however, in reality deeply integrated, in a way not truly visible in the models described in this thesis. In the models in papers I-III, the rural is represented as a constant (uniform) source of probability for spawning new clusters of activity. In paper IV, the local characteristics of zones can be used to represent a connection between land characteristics and urban activity. In paper IV, this is made explicit for coastal areas, which are given an extra local factor to represent the attraction of this particular type of location.

7.2 Observing activity

Using a generalised activity measure is valuable for creating simple models. Also, it is advantageous when dealing with empirical data since the measure is single-valued. In practice, however, it is not straightforward to observe urban economic activity. In this thesis, the main assumption is that economic activity affects land prices, opening up the possibility to compare model outcomes with empirical data.

Can we be sure that there is a linear and geographically homogenous relation between land price and urban activity? Probably not, since there will likely be many local, regional and national deviations from any linear relationship. For example, varying land-use regulations between cities could imply different amounts of land available for development. This means that the same amount of activity could cause different land prices in different cities.

Digital interaction might be another factor weakening the relationship between activity and willingness to pay to reside at a certain location. However, even when using digital communication, some fraction of the interaction will often be physical. For example, digitally interacting co-workers might also regularly meet at a physical location. This will certainly change the distribution of distances for interactions, in one sense making them less spatial[9], but the remaining spatial component might still be relevant for choosing a location.

For land price to remain a good indicator, there must be some competition. At least one other actor must stand in line to pay the price to pursue an equal amount of activity on the same piece of land. Physical land adaptations (such as buildings) must be assumed to be general enough so that competing agents can potentially achieve a comparable amount of total interaction at a location.

When considering remote work from this perspective, the digital economy might make employees more replaceable. This means that there will be ample opportunities for other agents to buy the house of a remote worker and start working remotely. The effect will be an expanding sphere of possible interaction, but this should still be observable through land prices. In other words, if remote work gives more people the opportunity to locate in more rural settings, then this should affect land prices for such locations.

Urban interactions and the resulting activity is not the sole factor affecting land price, even in a perfect market. Local environmental qualities, such as natural beauty, are obvious distorting factors. Potential negative interactions, such as criminality, might also cause deviations in the relationship between activity and price.

In paper V, an additional discussion can be found regarding the use of land price as a proxy for urban activity.

[9] Also, digital communications over distance is not without cost. Even computer systems might need to be in close physical contact to achieve competitiveness, due to time lags. One example is co-location for high-frequency trading (Aitken et al., 2017).

7.3 Modelling activity growth and agglomeration

In all the presented models, activity is assumed to grow in concert with the underlying interaction network. For the activity to grow at a location, new interactions must be created or attracted. Both the creation and attraction are assumed to be linearly related to the current activity, making the growth multiplicative, or in other words, *preferential*.

Using linear preferential attachment is obviously not the only possible modelling choice. For example, it could also be argued that low-activity locations should be more attractive than those that are already busy with intensive activity.

The main argument for using preferential attachment comes from empirical observations of urban fat-tailed distributions. Some mechanism of multiplicative growth seems to be the simplest assumption to generate these patterns. Large deviations from linear multiplicative growth would have caused deviations in the observed distributions.

Another argument comes from the economics of agglomeration. On average, there are more options when interacting with a location offering a higher activity level. This is in part due to economics of scale reducing the costs of delivering large amounts of similar goods and services from one location, for example, due to specialised buildings and infrastructure. Also, intensive urban activity could take the form of high diversity and specialisation, making a location attractive for interaction. More possible micro-mechanisms for agglomeration are discussed by Duranton and Puga (2004).

A non-economical but simpler explanation is discussed in section 5.2. If each edge represents an independent unit of activity, then attachment to a randomly chosen such unit will result in linear preferential growth.

The included papers do not hinge on any exact underlying mechanism for preferential growth since this is not necessary for achieving working models and empirical validation. However, a plausible interpretation on the micro-level would certainly improve the explanatory power. Also, a clear understanding would make it easier to estimate parameters in the model and to determine how well results generalize to other countries and contexts.

Most likely, the agglomeration processes captured by preferential attachment is a useful approximation for a collection of many underlying phenomena. This means that the parameters controlling preferentiality can be expected to vary, depending on region or country. Such variations are not further explored in this thesis but are highly relevant for further study.

In the economics literature, agglomeration is often described as a phenomenon taking place mainly on the city or regional scale, not for individual land lots. That is not in contradiction to the mechanisms of local preferential growth but could be viewed as a natural aggregation of the local growth mechanisms, possibly amplified by city-wide interactions.

Agglomeration of activity does not in itself imply increased productivity. It is also empirically challenging to assess the effects on productivity from increased agglomeration (Graham and Dender, 2011). However, productivity is not part of our presented models, and it is thus only the geographical agglomeration that we aim to model and explain.

It would also be less plausible that effects on productivity could be explained by minimal geographical interaction models since sector-specific economic factors might determine the productivity response. This implies that long-term trajectories of urban growth might deviate quite far from what any pure spatial model predicts. Changes in productivity could give feedback to the spatial process by attracting more activity and investments to a city or region. On the other hand, if productivity gains are only marginal, a minimal model could be expected to perform well, also in the long run.

In Paper III, the concept of spatial endogenous node fitness is introduced. This can be interpreted as an underlying growth potential exerted on every location without regard to the current activity. A spatial structure is exposed that is modifying the pure multiplicative growth, opening up for the hypothesis that the transportation network might have long-lasting effects on urban activity.

From the concept of spatial fitness, there is just a minor theoretical step towards the development of a centrality measure compatible with multiplicative growth. By letting all activity iteratively adjust until a stable state is achieved, a model outcome is obtained that is only dependent on the physical network, the interaction function and an agglomeration parameter. This forms the basis for the modelling and empirical investigations in papers IV-V.

8 Centrality and accessibility

8.1 Accessibility

The concept of accessibility can be used to describe the link between land use, activity and interactions. A metric of distance (or cost) is needed to calculate accessibility, together with some measure of the destinations to which the accessibility refers. A functional relationship between these factors and some method of summation are also necessary. Depending on these choices, many different accessibility measures can be obtained (Handy and Niemeier, 1997).

In network terms, accessibility can be described as the linkage between the physical network and the interaction network. The physical network and the agents' capabilities and preferences, together with the spatial distribution of opportunities, are what creates accessibility. Accessibility then forms the potential for interaction.

In papers IV-V, accessibility is in main focus, by the use of a detailed network travel time calculation. In papers I-III, accessibility is treated more simplistically, with Euclidian-distance based calculations.

In all included papers, it is mainly accessibility to urban activity that is being modelled. However, in paper V an additional simpler accessibility model is also studied for comparative purposes. This model measures accessibility to buildable land.

8.2 Network centrality

Just like accessibility, network centrality is not one single measure. Instead, network centrality is a broad category of approaches with the common theme of trying to infer the importance of nodes (or edges) within a network. This means that it is futile to compare all possible centrality measures to find one single "best" definition. Which concept of centrality to choose depends on the specific application and the adequate kind of node- and edge-importance.

In the literature about urban networks, many different centrality measures have been explored. It is not possible to review all of them in this thesis, but a few need to be mentioned.

Betweenness centrality

Betweenness centrality (Freeman, 1977) measures the number of paths through a node in a network, assuming that all paths (between all pairs of nodes) follow the shortest path. The actual paths and the resulting centrality will depend on which concept of distance is being used. Betweenness centrality corresponds closely to the concept of flow in a network.

Closeness centrality

The closeness centrality (Bavelas, 1950) of a node is proportional to the sum of the inverse of the distance to all other nodes. The nodes with the shortest average distance to all other nodes will thus get the highest centrality scores.

Degree centrality

Degree centrality is equal to the degree of a node, i.e. the number of connected edges, or more generally, the sum of weights of these edges. This means that only direct connections can affect the centrality value.

Eigenvector centrality

The eigenvector centrality (Bonacich, 1972) of a node is proportional to the sum of the eigenvector centrality of all the node´s neighbours. In this way, it is closely related to degree centrality but in a recursive fashion, giving higher importance to nodes that are connected to many other important nodes.

PageRank centrality

PageRank was developed by Brin and Page (1998) to find the centrality of web pages, using the network of hyperlinks as a basis. It is closely related to eigenvector centrality but also takes the directionality of edges into account.

8.3 Urban centrality

Urban centrality could refer to centrality in an urban network, but also more generally to the centrality of a location or a settlement. Such a centrality can be devised and analysed also without an explicit network, such as in Christaller's theory of central places (1933).

On a larger scale, there is also a possibility for modelling regional centrality. Can the location of a city, town or village within a regional area affect its social and economic performance? It would greatly simplify both theory and applied analysis if the same theory of centrality could be applied on several scales.

In paper V, the preferential centrality model (see also paper IV and section 8.5) is explored both within regions and on a multi-regional scale. From that analysis, it seems plausible that agglomerative mechanisms can give rise to regional centres and that the accessibility to these centres creates another level of centrality on a smaller scale.

Centrality in the physical network

The most studied urban networks, with regard to centrality, are the physical networks, especially the street network (e.g. Sevtsuk and Mekonnen, 2012; Porta et al., 2006b; Porta et al., 2006a; Hillier and Hanson, 1989; Agryzkov et al., 2019; Jiang, 2006).

One problematic aspect of considering only the physical network when calculating centrality is that the importance of certain physical connections might be over-emphasized. The exact topological configuration of nodes and edges is very influential on the result. Also, by most definitions of street centrality measures, the immediate physical neighbourhood becomes hugely important.

Using centrality measures with system-wide interactions, such as betweenness, is one way to remedy this problem. In such a calculation, all nodes can influence the result of all other nodes. However, this brings the risk of instead introducing too large long-range dependencies, raising the need for setting arbitrary geographical boundaries. This is a hint that some fundamental principle is missing in the model.

Another problem with using betweenness centrality is the focus on flow. Many essential aspects of urban activity are not directly related to physical flow, especially not to the flow of cars. A highway can be very considered central if it is flow (i.e. betweenness centrality) that is being measured, but nearby buildings might, in many cases, get only adverse effects.

Another remedy for the too localised measures is changing the representation of nodes to make them more extended. In space syntax, this is achieved by using streets (delimited in different ways) as nodes and intersections as edges. A similar effect can be achieved by using other distance metrics than Euclidian, for example, angular distance. Such a modelling choice will cause straight streets to become almost a single entity with regard to accessibility.

An obvious drawback of these modelling choices is that curves and intersections are not, in reality, the only impediments to interactions over long distances. Travel time is, for example, also a significant factor causing impedance in a transport system. More generally, the generalised cost could be considered. Such a generalised cost can include all the above-mentioned features and can also differ between transport modes.

8.4 Centrality in the interaction network

Following from the above discussion, we would like a centrality measure that extracts as much information as possible from the structure of the transport network while respecting:

- Generalized costs (including travel time and if needed also curvature and intersections), using some choice of paths and modes.
- Weights for nodes to acknowledge local differences between locations (such as different activity levels).
- Integration of information about the whole system, but with heavier weights placed on nearby locations.

To consider all of these factors, it seems like that we would end up with a somewhat complicated model, arguably quite far from a simple centrality measure. The risk of having to introduce arbitrary assumptions, parameters and data requirements seems overwhelming.

However, there is a modelling choice that can naturally solve many of these issues. The key idea is to shift focus from the physical network and instead study the interaction network. The interaction network can be explicitly measured, or it can be inferred from the physical network using some model. In papers IV and V, this line of modelling is pursued, with the choice of using an interaction network inferred from travel times calculated based on the physical network.

The interaction network tends to be very dense, with many nodes having connections to most others. Thus, the ability to that take into account weights on the interactions becomes crucial for attaining meaningful measures of centrality. Degree centrality and eigenvector centrality can easily work with weighted networks using the adjacency matrices.

The degree centrality a_j for node j in a weighted network becomes:

$$a_j = \sum_i M_{ij}\,, \tag{1}$$

where M_{ij} are the elements in the adjacency matrix.

If we assume that M_{ij} represents the edge weights in an interaction network, then the values can be inferred from the physical network, using some interaction model. A straightforward choice is to assign static weights R_j to each node (based on, for example, area of the zone represented by the node) and use these for both generating and attracting interactions, dampened over distance using a function $f(c_{ij})$ with generalised travel costs c_{ij} as input. The values c_{ij} are assumed to be static and can be calculated from the underlying transportation network.

The interaction weights can now be defined as $M_{ij} = R_j R_i f(c_{ij})$, and we get from Eq. (1) that

$$a_j = R_j \sum_i R_i f(c_{ij}). \tag{2}$$

From this, it is clear that degree centrality in the interaction network is proportional to accessibility in the physical network. What is being measured is then accessibility within the network, to the network itself, since no specific attractions are specified apart from the nodes.

If, instead of using static weights for generating interaction, we let the interactions in the weighted network be modulated by the centrality values themselves, we can arrive at the eigenvector centrality,

$$a_j = \frac{1}{\lambda} \sum_i a_i M_{ij}\,, \tag{3}$$

where λ is the largest eigenvalue, which in this case works only as a normalising factor without specific model interpretation.

Also, in the eigenvector case, the interactions can be inferred from the transportation network, together with some definition of zonal attraction W_j. Using the derivation of the flow S_{ij} from paper IV, which is summarised in Figure 5, we get

$$a_j = R_j \sum_i S_{ij} = R_j \sum_i \frac{a_i f(c_{ij})}{\sum_k R_k f(c_{ik})}, \tag{4}$$

where we have defined attractions to be equal to local weights, $W_j = R_j$.

By inspecting Eq. (3) and Eq. (4), we can identify

$$M_{ij} = \frac{R_j f(c_{ij})}{\sum_k R_k f(c_{ik})} \tag{5}$$

and $\frac{1}{\lambda} = 1$. The rationale for this centrality measure in an urban context is further discussed in paper IV. One interpretation is that eigenvector centrality measures the accessibility to centrality (in a recursive manner), or in other words: the accessibility to accessible locations.

For many networks, degree centrality and eigenvector centrality can give very similar results, except for specific highly connected nodes. In paper V, results are compared with empirical data, both for eigenvector centrality and degree centrality (called *static accessibility model* in the paper), showing that they are almost indistinguishable in this modelling context.

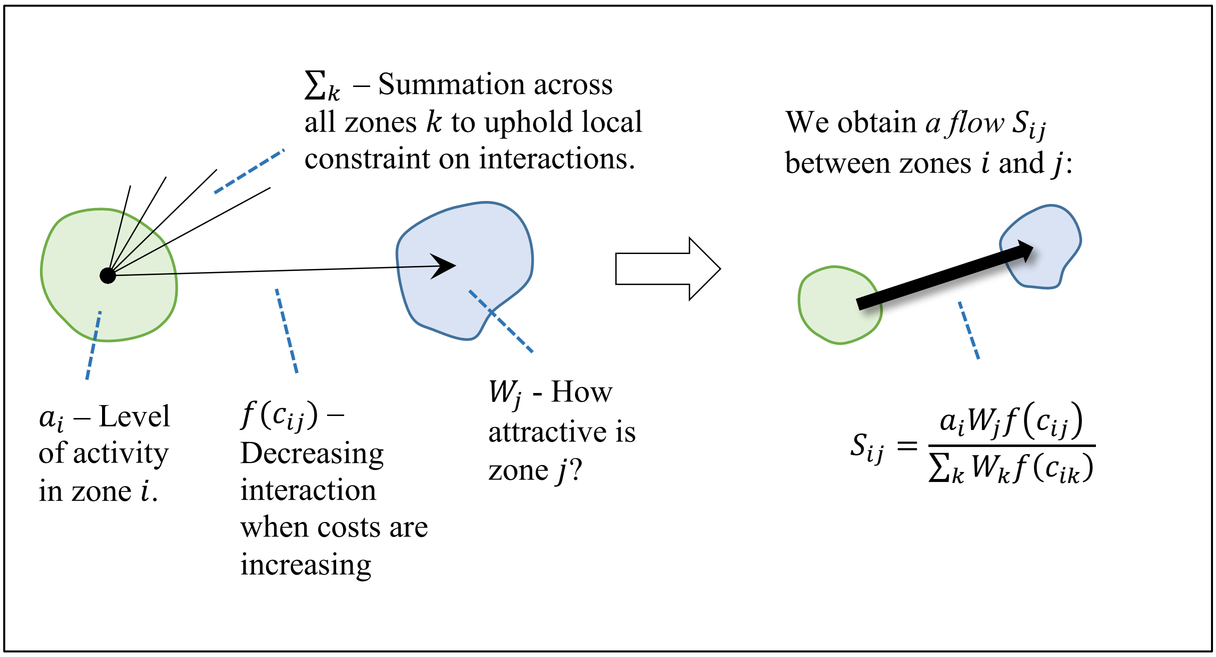


Figure 5. From activity and attraction to a flow of interaction. (Adapted from paper IV.)

8.5 Preferential centrality

In paper IV, the measure of preferential centrality is introduced. As described in Figure 6, it can be derived similarly to the eigenvector centrality by using the attraction $W_j = a_j + \alpha R_j$, where α is a parameter and R_j are the static weights.

In analogy to Eq. (4), we arrive at

$$a_j = W_j \sum_i \frac{a_i f(c_{ij})}{\sum_k W_k f(c_{ik})} = (a_j + \alpha R_j) \sum_i \frac{a_i f(c_{ij})}{\sum_k (a_k + \alpha R_k) f(c_{ik})}, \qquad (6)$$

which is a recursive definition of activity (centrality) since a_j appears on both sides. This means that the equation must be solved for all nodes simultaneously.

The centrality measure has been presented as a model specifically for urban systems, but with some generalisation, it can be applied to any weighted network. To simplify, we can assume that $R_j = 1$ for all nodes, and we can then write

$$a_j = (a_j + \alpha) \sum_i a_i M_{ij}, \qquad (7)$$

where $M_{ij} = \frac{D_{ij}}{\sum_k (a_k + \alpha) D_{ik}}$ and D_{ij} are static interaction strengths. However, the dynamic interactions M_{ij} are not static, but dependent on the centrality values of all other nodes.

In the simplest case D_{ij} could be taken as the elements of the pure adjacency matrix of a network, but it is important to note that the centrality measure hinges on two networks – the underlying static network determining D_{ij} and the dynamic interaction network that determines M_{ij}.

Preferential centrality can be derived from a few assumptions (see paper IV):

1. For every node, the sum of incoming interactions equals the sum of outgoing interactions
2. The centrality value for a node equals the sum of interactions.
3. Interactions are attracted to nodes based on a linear function of the centrality
4. Outgoing interactions are distributed according to connection strengths in the underlying static network

When applied to urban modelling, the assumptions can be interpreted as follows:

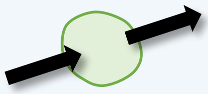
1. For every zone, the summed value of incoming interactions equals the summed value of outgoing interactions
2. The activity value for a zone equals the summed value of interactions.
3. Interactions are attracted to zones based on the linear function of activity and background attraction
4. Outgoing interactions are distributed according to an interaction function based on the generalized cost of connecting through the physical transportation network

This means that the physical network, together with values for background attraction, form the static basis for the resulting activity values. The interaction network is derived and thus only intermediary for the calculation. This is an essential feature of the centrality measure, making it possible to calculate centrality with only physical data as input.

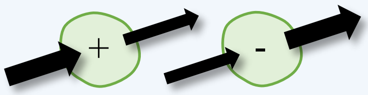
In spatial interaction modeling, activity a_i represents demand while attraction W_j represents supply. Interactions are thereby directed, going from demand to supply.

In the basic eigenvector centrality model, activity a_i represents *any* activity. Most types of activity generate both supply and demand on an aggregated level. Attraction W_j is refined into an intrinsic property R_j of the zone, reflecting suitability for development.

We posit that activity is in equilibrium when total interaction from a zone is in balance with total interaction to the zone. <u>Our task is then to find such a configuration of a_i to fulfil this for all zones.</u>



To achieve this, we iteratively adjust a_i across the zones. If interactions in and out are not in balance, the current estimate must be adjusted. We repeat until a convergence criterion has been reached.



In our preferential centrality model, we refine the definition of attraction to reflect a dynamic coupling with activity. Development suitability R_j now figures as one aspect of attraction W_j together with activity a_j. A parameter α is used to set the balance between these aspects.

Figure 6. From spatial interaction to activity modelling using preferential centrality. (This figure also appears in paper IV.)

Eigenvector and preferential centrality in a star network

To get a basic understanding of the workings of the presented centrality measures, some simplified network topologies can be studied. The first one is the star network (see Figure 7) with $N + 1$ nodes and an adjacency matrix with elements D_{ij}, with values $D_{ii} = 0$ and $D_{i1} = D_{1j} = 1$, and $D_{ij} = 0$ for all other relations.

Since there are no special node weights, we can use the simplified definition

$$a_j = \left(a_j + \alpha\right) \sum_i \frac{a_i D_{ij}}{\sum_k (a_k + \alpha) D_{ik}}. \tag{8}$$

For j>1, all nodes are equal, and we can, without loss of generality, set $a_j = a_2$. The solution is easily obtained:

$$a_1 = (a_1 + \alpha) \frac{N a_2}{(a_1 + \alpha)} = N a_2 \tag{9}$$

$$a_2 = (a_2 + \alpha) \frac{a_1}{N(a_2 + \alpha)} = \frac{a_1}{N} \tag{10}$$

Here it can be noted that the value of α does not affect the resulting centrality values, which means that in this particular case, preferential centrality will be equal to eigenvector centrality. If some interactions are introduced between peripheral zones, by setting $D_{22} > 0,$ we get

$$\begin{aligned} a_1 = (a_1 + \alpha) \left[\frac{a_1 D_{11}}{\sum_k (a_k + \alpha) D_{1k}} + N \frac{a_2 D_{21}}{\sum_k (a_k + \alpha) D_{2k}}\right] = \\ = \frac{N(a_1 + \alpha) a_2}{(a_1 + \alpha) + N(a_2 + \alpha) D_{22}} \end{aligned} \tag{11}$$

$$\begin{aligned} a_2 = (a_2 + \alpha) \left[\frac{a_1 D_{12}}{\sum_k (a_k + \alpha) D_{1k}} + \frac{N a_2 D_{22}}{\sum_k (a_k + \alpha) D_{2k}}\right] = \\ = \frac{a_1}{N} + \frac{N(a_2 + \alpha) a_2 D_{22}}{(a_1 + \alpha) + N(a_2 + \alpha) D_{22}}. \end{aligned} \tag{12}$$

The second equation can be rewritten

$$\frac{a_1}{N} = a_2 \left[\frac{(a_1 + \alpha)}{(a_1 + \alpha) + N D_{22} (a_2 + \alpha)}\right], \tag{13}$$

i.e. Eq. (12) yields the same result as Eq. (11).

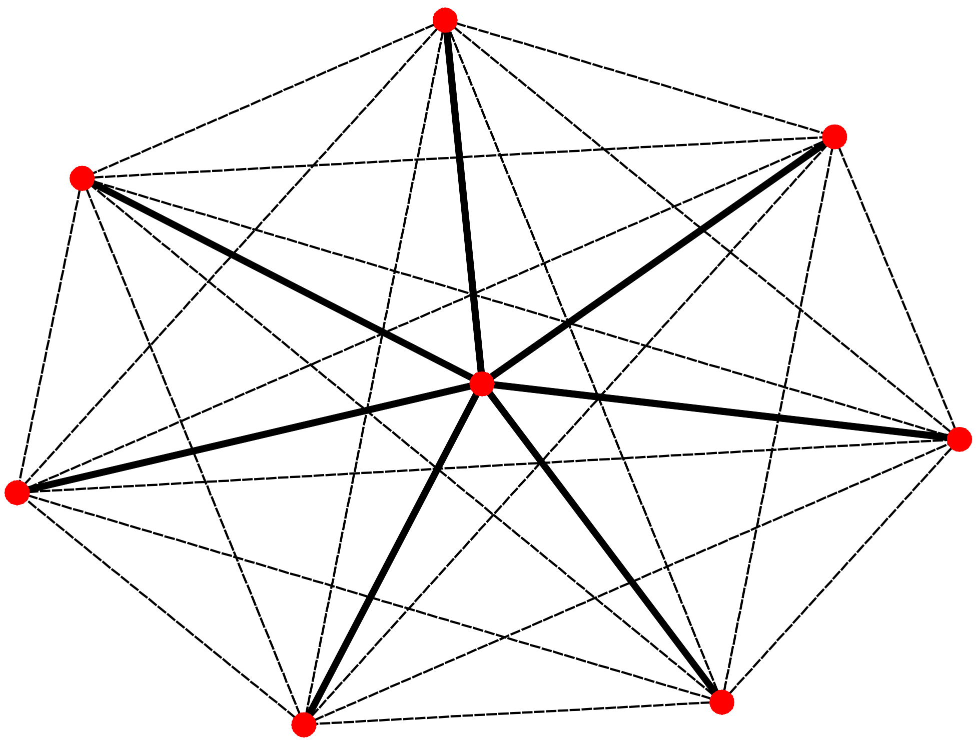


Figure 7. Star network with N=7. Bold lines have interaction D_{1j}=1. Dashed lines have interaction $D_{ij} = D_{22}$

Since the two equations are not independent, some additional assumption is needed to obtain a single solution. We can either choose to keep total activity constant or instead study relative activities. We choose now the latter option, since only relative activity values are of interest in this theoretical investigation. We assume that $a_1 = 1$, and from Eq. (12) we get

$$a_2 = \frac{\frac{(1+\alpha)}{N} + \alpha D_{22}}{(1+\alpha) - D_{22}}. \tag{14}$$

Now we can observe one necessary requirement for positive solutions,

$$(1+\alpha) > D_{22}, \tag{15}$$

which is fulfilled if D_{22}<1.

If $D_{22} = 0$, we retrieve the previous simpler solution, where α is without influence,

$$a_2 = \frac{1}{N}. \tag{16}$$

In the limit of large α, we get the eigenvector centrality solution,

$$a_2^{(eigenvector)} = \frac{1}{N} + D_{22}. \tag{17}$$

It can be noted that the eigenvector centrality, in this case, will be equal to relative degree centrality, $\frac{a_2^{(degree)}}{a_1^{(degree)}} = \frac{1+ND_{22}}{N}$.

We can also make some other observations:

- A large periphery (large N) reduces relative activity in the periphery, especially if the interaction is weak and agglomeration is strong (low α).
- Increased interaction (D_{22}) between periphery and periphery increases peripheral activity.
- Strong agglomeration (low α) reduces relative peripheral activity.

In the limit of large N, we get $a_2 = \frac{\alpha D_{22}}{(1+\alpha)-D_{22}}$, which makes it possible to create the curves shown in Figure 8, with the effect on a_2 of changing D_{22}, for different values of α. It is clear that the effect of an increasing preferentiality (i.e. decreasing α) is to strengthen the nonlinear response to changes in interaction within the periphery (D_{22}).

If agglomerative forces are strong, changing interaction can thus have effects that are difficult to predict. They could either be much smaller than expected or much more prominent. It would barely make sense to talk about a specific effect size in this context. Changes of interaction must thus be studied in a broader context, including network topology and assessments of the agglomerative strengths.

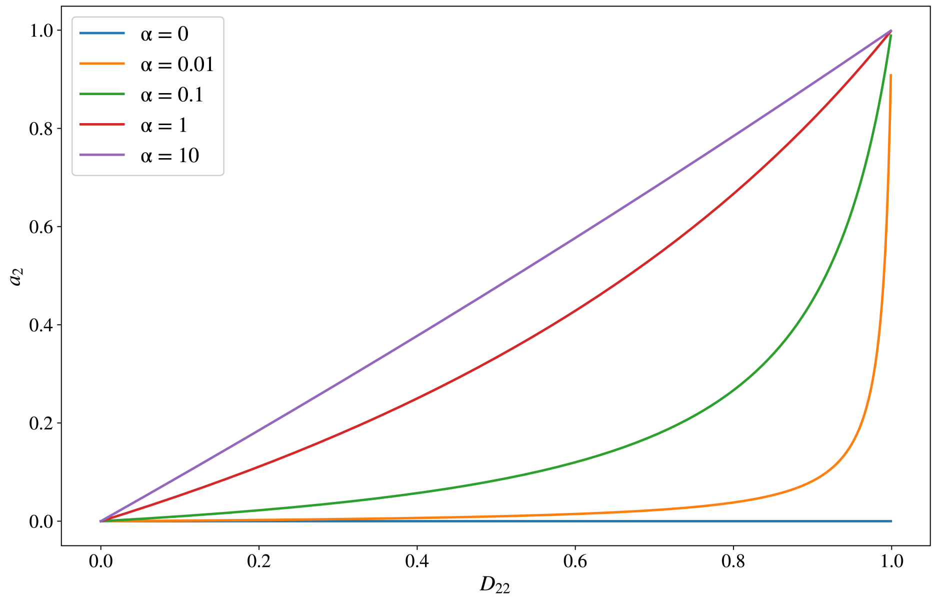


Figure 8. Peripheral activity in the star network (in the limit of large N), as a function of interaction (D_{22}) and preferentiality (α).

Preferential centrality in the one-dimensional city

When considering more complicated networks, numerical solutions provides the easiest way forward. A traditional urban toy model is the one-dimensional linear city. We can consider N locations distributed on a line and transport costs equal to the distance between locations.

Figure 9 shows the one-dimensional numerical solution of the preferential model for different values of the agglomeration parameter α. As agglomeration is increased (i.e. a lower value for α) the activity profile becomes increasingly sharp, despite that the interaction function is unchanged. This means that preferential centrality has the capacity to combine distinct concentrations of activity with long-range interactions. Both these features are important in an urban model aiming to achieve realistic results.

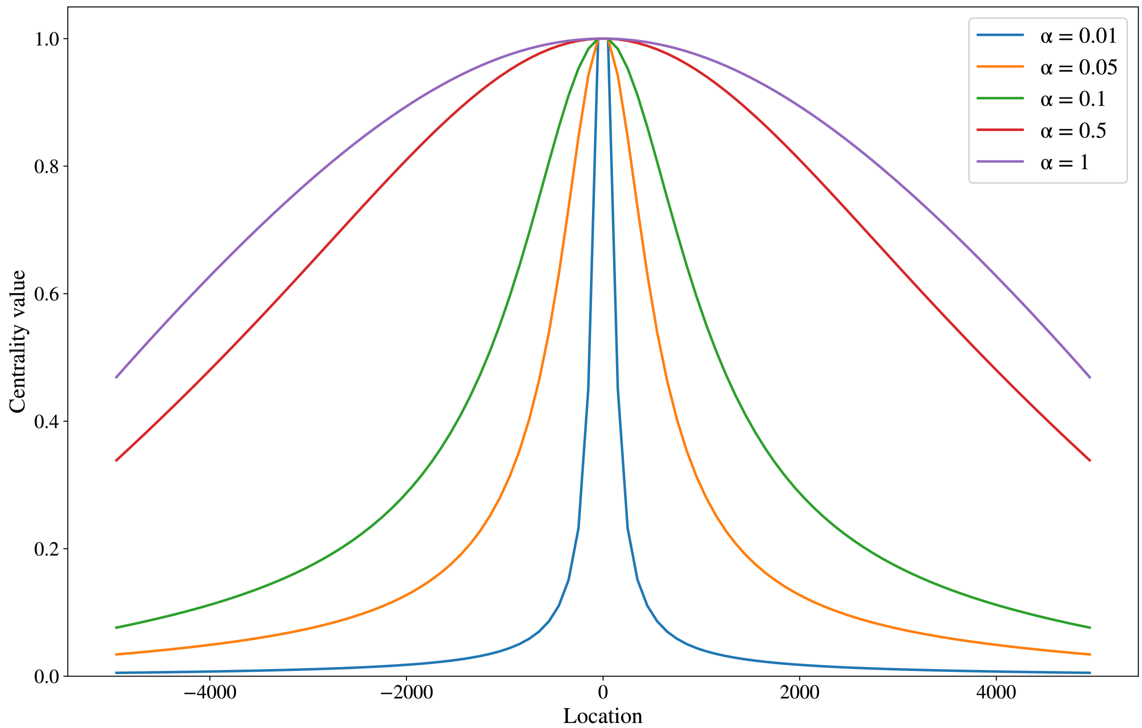


Figure 9. One-dimensional numeric results for the preferential centrality model, with varying values of α. The iterative solution is normalized with regard to the maximum value. The interaction function is $f(d_{ij}) = (d_{ij} + 5000)^{-2}$.

8.6 Implementation of the centrality model

To perform calculations for an actual urban system, the preferential centrality model requires a transport network. In the simplest case, no land-use data is needed since land can be represented by all cells within a certain distance of the network. For realistic results, however, a more detailed land use representation is needed, with some distinction between parts of the network where connections to land can plausibly be created. For example, it is not possible to access land directly from a motorway without some sort of junction.

In the model implementations used in papers IV and V (see also Figure 10), a zonal model based on property polygons is used. This makes it possible to have a more realistic spatial representation compared to cells, but the size difference between zones must be handled by adjusting the local weights.

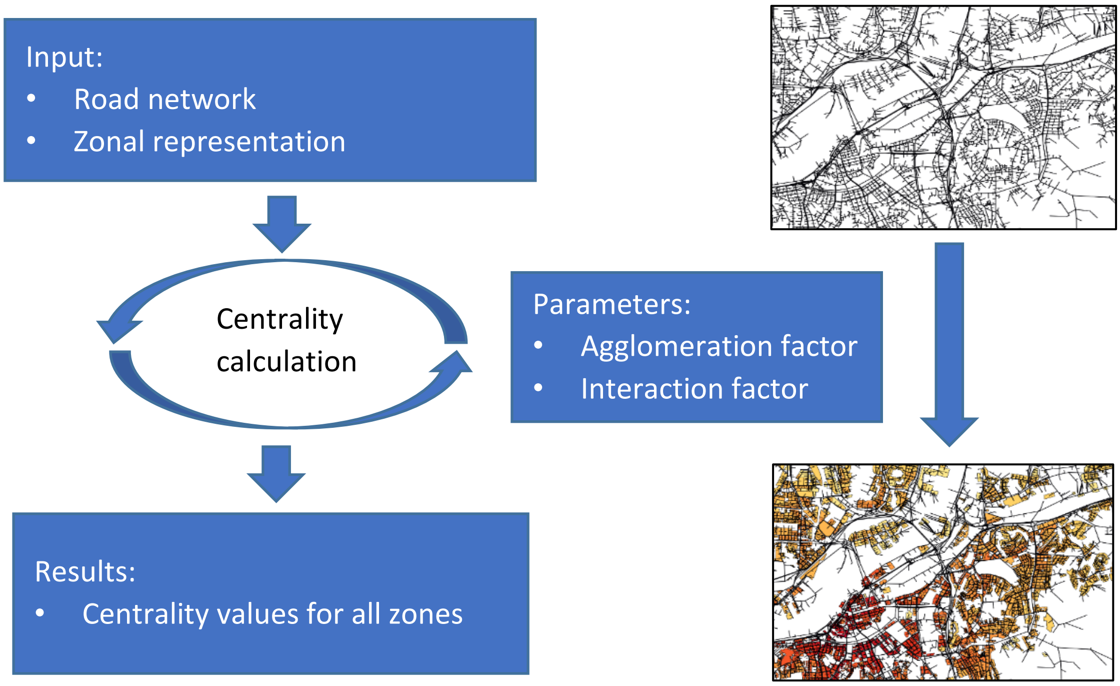


Figure 10. Implementation structure for the preferential centrality model.

The centrality calculation can be described as having two parts. The first part consists of calculating travel times from all zones to all others, and the second part of the algorithm uses these travel times as static inputs to adjust activity levels until an interactive equilibrium is reached (according to Eq. (6)).

The travel time calculation could, in principle, be made arbitrary detailed, with many nuances and parameters. It could also be calibrated using actual data on travel behaviour. However, no such calibration has been performed for the results obtained in this thesis.

In Paper V, the number of zones is too large for the all-to-all computation of travel times to be computationally feasible. This is solved by using an approximate travel time calculation, where representative nodes are chosen in a hierarchical fashion. This means that for shorter distances, an exact travel time is achieved, but for longer distances, travel times are instead calculated using the representative nodes.

One important feature of the model structure is that it only has two main parameters, α (agglomeration factor) and β (interaction factor). There are, however, also several intermediate parameters needed to calculate both the zonal weights (buffer sizes, road impediments, etc.) as well as the travel times (speed limits, start/end-penalty).

These intermediate parameters are of a more physical nature and can be considered less fundamental to the global model dynamics. Locally, choices for these parameter values can, however, have significant impacts on resulting activity levels for specific locations. In the supplementary material for paper IV, sensitivity analyses are presented where a selection of these parameters are varied.

9 Empirical results

9.1 The urban growth model

In Figure 11, empirical results are shown from the urban growth model developed in papers I-III and described in section 6.5. The same model simulation reproduces observed patterns both on the cellular level and concerning urban clusters. The clusters can be considered to correspond to cities but with an algorithmic delimitation instead of an administrative.

The clusters were identified by a simple method that has later been dubbed as CCA by Rozenfeld et al. (2008). Our analysis used land value as the delimiting factor for determining if a cell shall be considered urban or not. This is in contrast with many other similar studies where population data has been used.

For this urban growth model, there are only four essential parameters. The first is q which determines the ratio between additive and preferential growth. It can be determined from empirical data based on the relation between total activity and developed area.

The second and third parameters are b and ϵ, determining the rate of perimeter and external growth. The value of ϵ can be inferred from the ratio between the numbers of clusters and the total activity, and b is constrained by the values of ϵ and q together with the empirical ratio between the number of perimeter cells and the total activity.

The fourth parameter β controls the level of distance decay. This parameter cannot be estimated from cross-sectional activity data since it does not strongly affect the simulated activity levels. To find the empirically correct value for β for a particular application, additional interaction data would be needed, for example, data on the distribution of travel distances.

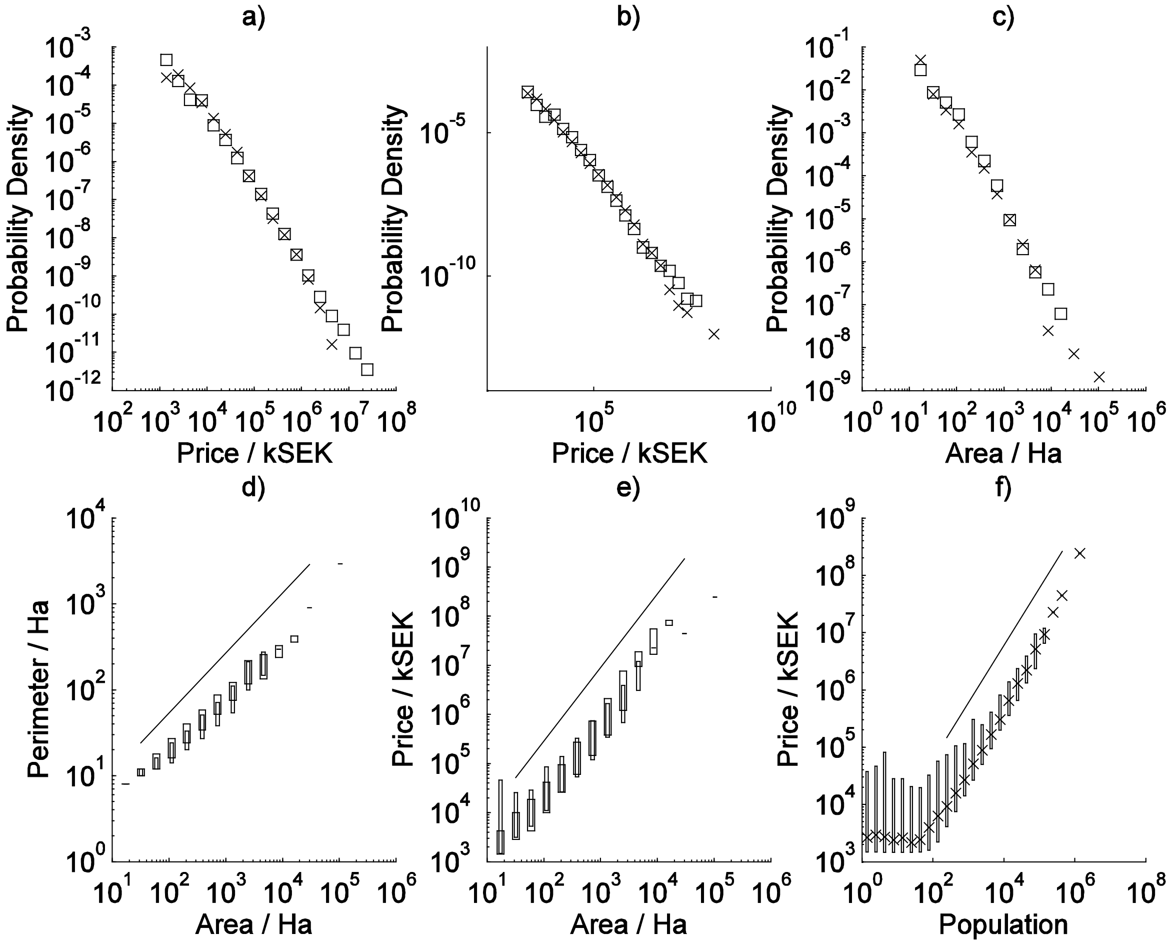


Figure 11. Empirical results for Sweden, compared with model outcomes for the urban growth model. (Figure reproduced from Paper III.)
a), b), and c): Squares denote simulated values and crosses denote empirical values.
a) Probability distribution for land value aggregated to cells with the size 400 x 400 m.
b) Probability distribution for land value aggregated to clusters.
c) Probability distribution for area aggregated to the same clusters as in b)
d) Empirical (broad boxes) and simulated (thin boxes) cluster areas are plotted against exponentially binned cluster perimeters.
e) Empirical (broad boxes) and simulated (thin boxes) land value aggregated to clusters are plotted against exponentially binned cluster perimeters.
f) Empirical cluster population plotted against exponentially binned empirical aggregated cluster land values. Crosses indicate the median values in the bins.

9.2 The urban centrality model

The preferential centrality model is compared to empirical land taxation values for a single city (paper IV), for a single region (paper V) and for a larger multi-regional area comprising the southern half of Sweden (paper V).

For the single city results, regressions are performed on the level of modelling zones (comparable in size to city blocks). An R^2-value of 0.58 is obtained that can be compared to 0.54 for a monocentric benchmark model. The monocentric model also has the drawback that a centre has to be manually specified. I.e. the preferential centrality model achieves better performance even when using a smaller amount of input data. Also, using spatial statistics, it is shown that residuals are less correlated for the preferential model compared to the monocentric model.

In paper V, where areas involving several cities are studied, it does not make sense to use a monocentric model as a benchmark. Instead, the outcomes of the preferential model are compared to those of a simple accessibility model. All models that are compared uses the same set of input data since the addition of other data sources would make comparisons difficult to interpret.

Regressions are performed on values aggregated to administrative tax assessment areas. The best single-regional model achieves an R^2-value of 0.65 that can be compared to the best accessibility model achieving 0.55. In the multi-regional case, the best result is 0.57 for the preferential model and 0.52 for the accessibility model. Also, studying the complementary cumulative distribution functions shows that only the preferential model achieves the broad distribution of values that is observed empirically.

Figure 12 shows results from the multi-regional model on the detailed level of model zones. Figure 13 shows empirical values and model values on the aggregated level used for regression.

To arrive at these results, two main parameters needed to be tuned, as demonstrated in paper V. The first parameter is α, controlling the rate of preferentiality (agglomeration), and the second is β, which describes how the amount of interaction decays with regard to travel time.

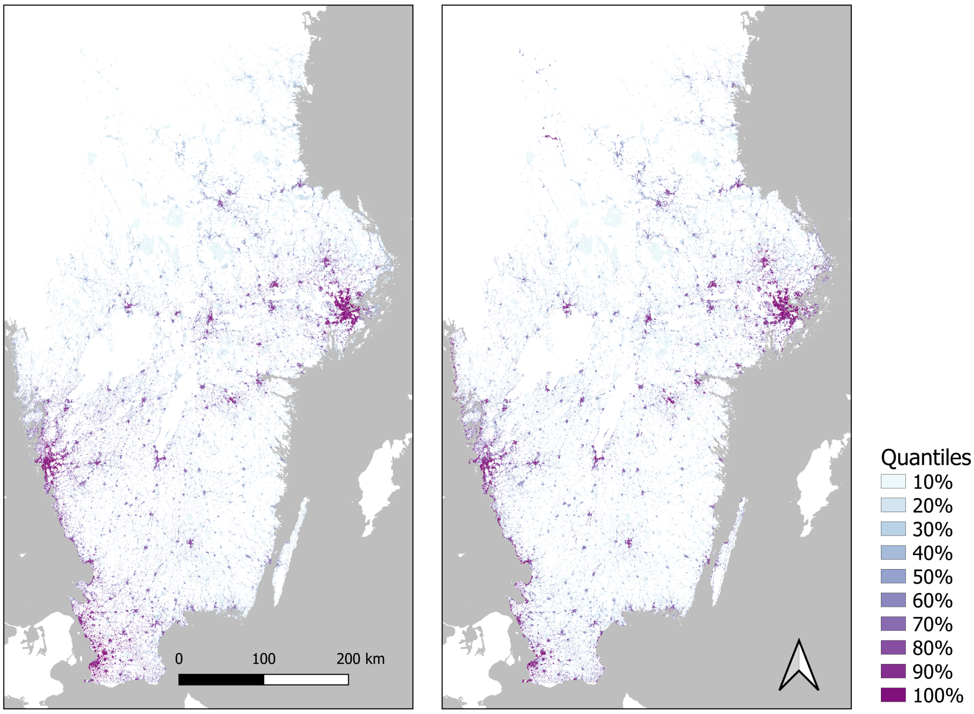


Figure 12. Comparing the multi-regional preferential centrality model (left) with empirical land taxation data (right). Values are shown on the zonal level relative to area.

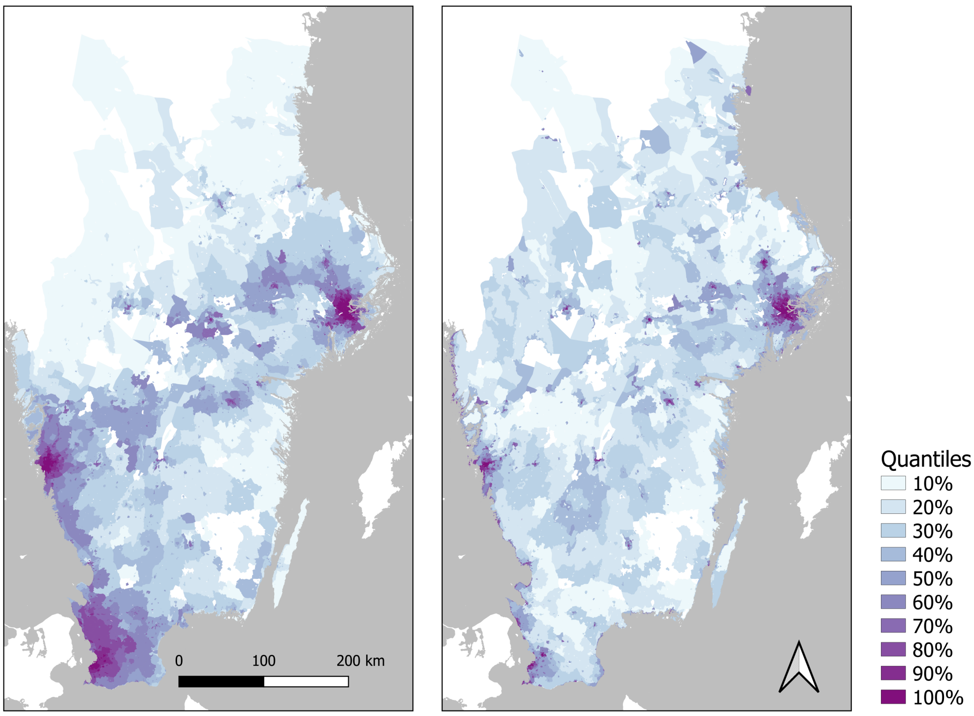


Figure 13. Comparing the multi-regional preferential centrality model (left) with empirical land taxation data (right). Values (model and empirical) are aggregated to larger administrative tax assessment areas. Shown values are relative to area.

10 Applications in planning

10.1 Motivation and possibilities

An overarching motivation for the construction of new and useful urban models is the possibility to answer relevant questions in planning. For this, we need models that are responsive to parameters and physical factors that are of interest in decision-making. We also need a certain level of trust in the models' predictions and an understanding of the limitations of the results.

In transport planning, there are two broad classes of questions suitable for this type of modelling. The first class encompasses the study of different scenarios, where models can be used to analyse the effects of system-wide variations, such as new technology, changing energy prices or new regulations. The other class of questions includes the planning of specific changes in the transportation system, such as new roads and railroads.

In urban planning, typical modelling questions arise with regard to both strategic and detailed planning. On the strategic level, a possible question to ask is at what pace it is suitable for a city to expand with regard to its urban footprint. On a more local level, specific development projects need to be evaluated with regard to consequences for their surroundings on several scales. Impacts of different zoning structures are also often of interest.

It is not very plausible that any single land-use and transport model should be able to reliably handle all these questions on all different scales. However, the preferential centrality modelling framework can potentially be adapted to address quite a wide range of questions. In actual planning, it is necessary to validate the different use-cases in their own contexts and in relation to existing operational models. The examples that follow should therefore only be considered as demonstrations of general model capability.

10.2 Scenario analysis

Many aspects of the calculated outcomes resulting from the preferential centrality model are controlled by the interaction and agglomeration parameters. They can be considered to vary slowly in time and space, and both are determined by combinations of technological, political, behavioural and economic factors.

Figure 14 shows the consequences of increasing agglomeration (lowering α) - activity becomes concentrated in a few dominant urban areas. Decreasing agglomeration, on the other hand, gives rise to a more broadly distributed pattern. If a large fraction of the agents in the system prefers to interact with the background instead of with other agents, this corresponds to low agglomeration. *Interaction with the background* can be interpreted as using natural resources (i.e. agriculture or forestry), enjoying natural amenities or just preferring a large living area.

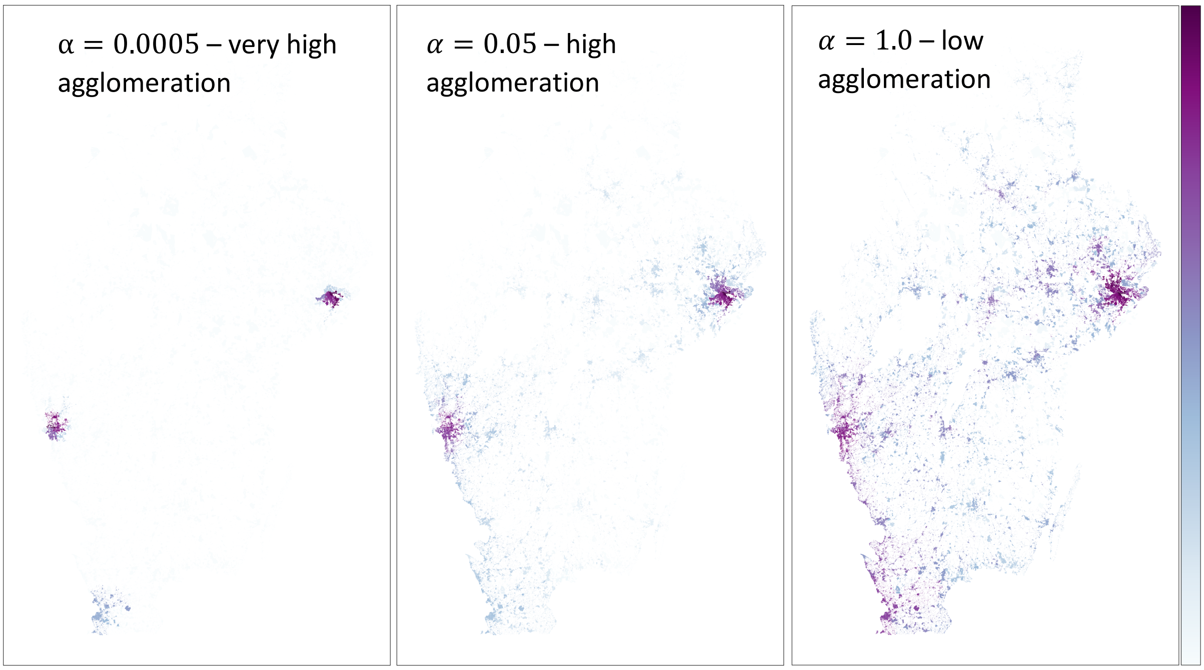


Figure 14. Results from the preferential model, for the southern half of Sweden, using different values for the agglomeration parameter α.

In Figure 15, the spatial interaction factor β is varied[10], while the agglomeration parameter is held constant at $\alpha = 0.1$. The effects can be likened to a change of scale. With many long-range interactions (low β), activity is spread widely, but there is only one dominant city. With intermediate β-values, a pattern with several large cities is revealed. With very high values of β, a scenario is achieved where there are almost no long-range interactions. The result is a more widespread spatial distribution of activity around a large number of local cores.

A change in β is most easy to interpret as a change in transportation technology or transportation costs. For a cost-change to actually affect β, there needs to be some non-linearity in the cost structure, i.e. different impacts per distance travelled, for long compared to short trips. A homogenous change in costs would not show up as a change in β.

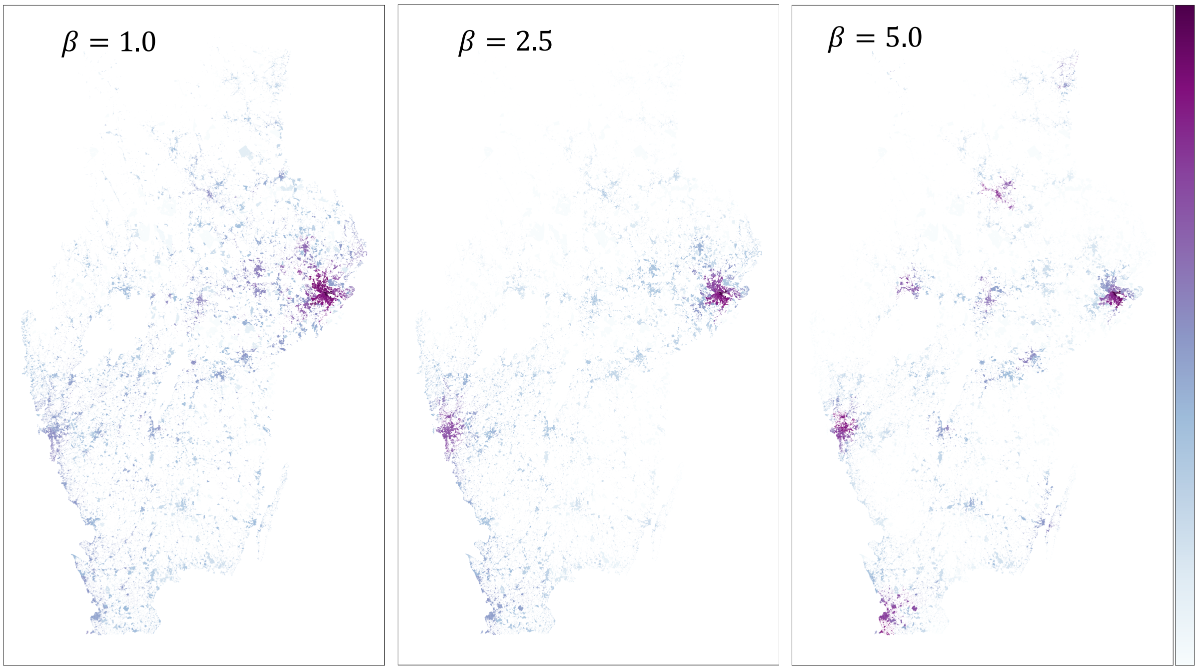


Figure 15. Results from the preferential model, for the southern half of Sweden, using different values for the interaction parameter β.

[10] The interaction parameter β is used in the interaction function $f(c_{ij}) = c_{ij}{}^{-\beta}$. See papers IV and V for more details.

10.3 Modifying the physical network

The preferential centrality model can be applied to analyse changes in the infrastructure network. Centrality is then calculated, with and without the network change, and relative differences in model outcomes can be studied.

An example is shown in Figure 16, where the network is changed by adding a motorway bypass around a major city. For comparative purposes, the total activity level is held fixed, regardless of network changes. This means that it is a redistribution of activity that is being modelled. Whether the modified spatial structure is more economically productive cannot be deduced from this model by itself.

For planning purposes, however, interesting questions can be posed regarding to what extent such modelled geographical redistributions are in line with local and regional goals for spatial development.

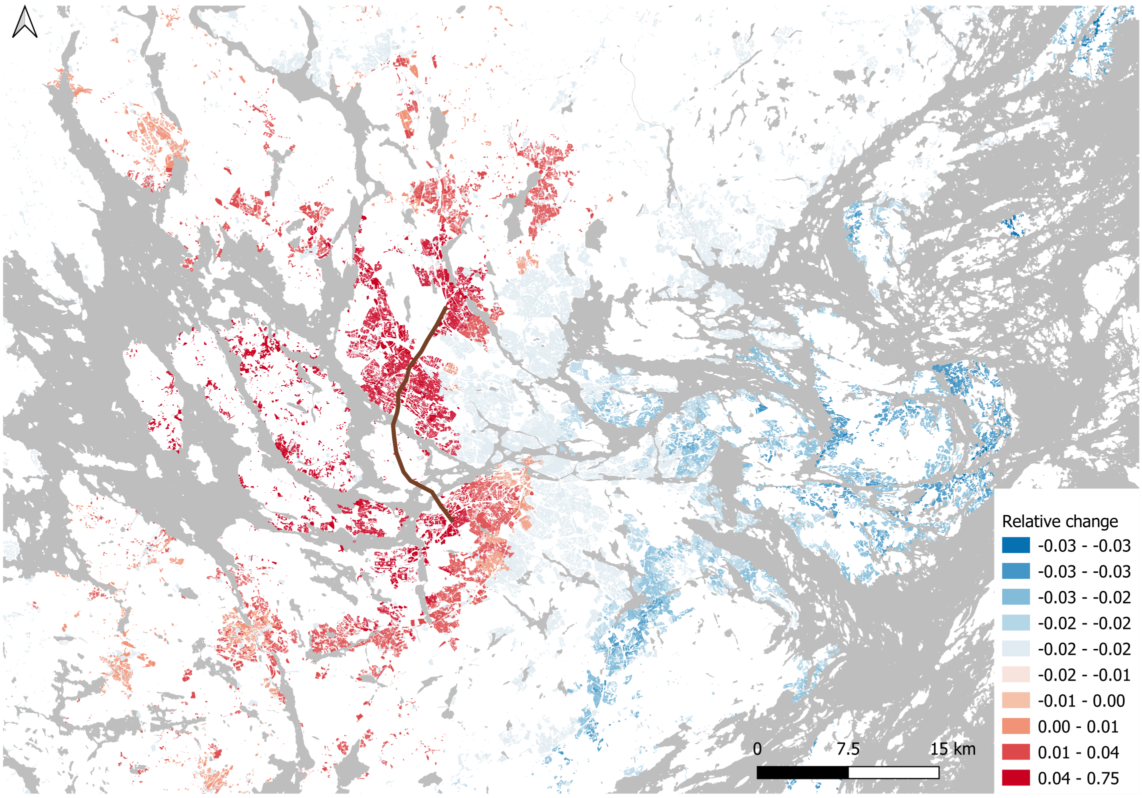


Figure 16. Simple case study of a bypass (thick brown line) around the Swedish capital, using the preferential centrality model. The colour scale is based on a quantile classification, with red indicating increased relative centrality.

10.4 Changes in flow and activity

In the preferential centrality model, interaction and activity are theoretically equivalent. This means that the concepts can be interchanged depending on the intended planning application. For example, the perspective of changing activity at different locations can be expected to be in focus for urban planners, and the perspective of changing flow in the physical network might be more relevant for transport planners.

The combination of the two perspectives into one modelling framework, where neither of the two is considered to be more fundamental, opens up for improved integration between land use and transport in actual planning practice.

In Figure 17, an example is shown where a network modification gives rise to changes in flow and new routes. Note that the changing flow of interactions cannot be interpreted as equal to changes in vehicular flow, as discussed in section 6.6. However, they can be expected to be closely related.

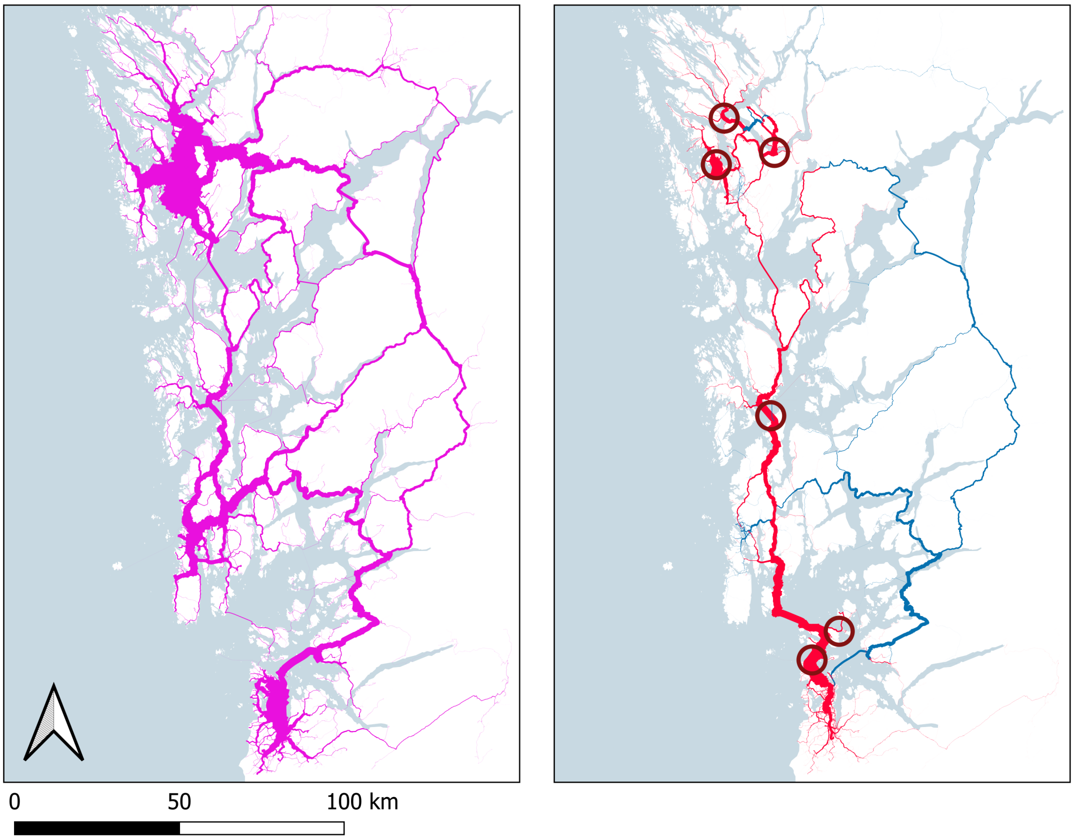


Figure 17. Changes in flow due to network modifications along the Norwegian coastline, according to the preferential centrality model. Left: Modelled flows before modification. Right: Differences in modelled flows due to the introduction of six fixed links (circles) replacing ferry connections. Red indicates increased flow, and blue indicates reduced flow.

11 Discussion

11.1 Urban activity

Since urban activity is notoriously difficult to measure fully, any interpretation of model results tends to be ambiguous. This concerns all the presented models, both for urban growth and urban centrality. If activity is predicted to increase at a location, this could take the shape of new or improved buildings; increased land values, population, and income levels; increased turnover; new workplaces; or any combination of these effects. Having a generalized measure of activity is thus a double-edged sword. It provides the basis for a clean and simple model structure but adds the mentioned complications in interpretation.

However, in a planning context, the ambiguity might also be a positive feature since a lot of details are kept outside of the model scope. The somewhat abstract model output could be used to guide decisions about what paths to take in the development of an urban area. Locations could be identified as having a high or low potential for generalized activity, and then the planning could adapt accordingly. Adaptation could occur either by accommodating the changes in activity by physical adjustments (such as new buildings) or by studying counteracting network alterations.

11.2 Multiplicative growth and agglomeration

In all the presented models, multiplicative growth (preferential attachment) is an essential mechanism. In the general case, there are some benefits of using the term *attachment* instead of *growth*. One such conceptual benefit is that an attachment process does not require absolute growth in the system. As long as local activity growth (attachment) is balanced by declining activity at other locations, fat-tailed distributions are retained.

To achieve a good fit between distributions of modelled activity and empirical data, it seems that the relation between activity size and activity attraction cannot deviate too much away from linearity. However, assuming such a *linear* preferential attachment is not obvious with regard to economic theory since it implies some sort of constant returns to scale. Many economic processes deviate from linearity, both into the sublinear domain with decreasing returns to scale; or towards the super-linear with increasing returns.

The analysed models are not aimed towards answering why linear preferential attachment might be so dominant. One hint could lie in the random selection of links, as discussed in section 5.2. However, for that explanation to make sense, edges in the interaction network must be reasonably independent of each other.

It should also be noted that the empirical studies reported in this thesis do not include any data on time dynamics. Theoretically, the models suggest some mechanisms of systems changing over time, both with regard to urban growth and changing centrality. Some of these assumptions should, in principle, be open to empirical investigations in future research.

11.3 Interaction and agglomeration

In the presented models, for urban growth as well as for centrality, the agglomeration parameters could in one limit be reduced to insignificance. In the urban growth model, this is called *uniform growth* (or additive growth), and in the case of centrality, the preferential model is reduced to the eigenvector model. In these model versions, spatial interaction is preserved, but there is no multiplicative attraction (i.e. no local agglomeration). This leads to model outcomes that deviate from empirically observed patterns.

This means that spatial interaction (or accessibility) in itself cannot be considered as a full explanation of urban growth and urban centrality. It is not enough that activities (such as jobs and housing) are located near each other. Some additional force of local agglomeration is also needed.

One suggestion of the local multiplicative agglomerative force is the movement of people (Hillier and Hanson, 1989). In our theoretical framework that would correspond to saying that it is the flow in the physical network that is the attractive driving force. It can, of course, not be disregarded that this is crucial for many urban economic activities such as shops and restaurants. However, the majority of the flow is not searching for random attractions along the way but is aiming for an intended destination. Movement-based models for agglomeration, such as betweenness centrality, are thus not plausible as full explanations. Mechanisms of destination-based attraction are also needed, as captured by the preferential centrality model.

In papers IV-V, dealing with interaction *without* agglomeration is represented by the accessibility and the eigenvector models that we use for comparison. The opposite case of agglomeration without interaction, is not explicitly considered but can be understood as a low β-value in combination with strong preferentiality.[11]

In paper V, it is observed that the regional model shows a higher empirical correspondence in comparison with the multi-regional model. This could possibly be because of the implicit assumption that interaction and agglomeration (represented by parameters α and β) behave uniformly across the system. If parameters then are tuned to fit observations in one region, adding other regions would tend to drag the performance down. There could also exist factors of regional productivity not directly linked to road network accessibility and agglomeration, for example, due to nationally allocated activities, such as public universities.

Also, international interactions (travel and trade) are not included in the current modelling setup, and these factors could cause regional variability. Railroads and information technology are other missing modelling components that are likely to affect results.

11.4 A minimal model?

In all types of modelling, there is always a tension between simplified and complicated models. Simplified models can be theoretically sound and analytically tractable but at the cost of empirical faithfulness. Complicated models might leave tractability aside to achieve empirical accuracy, at least in specific cases.

New theoretical approaches can, in principle, raise the quality in both ends of the modelling spectrum. More accurate mechanisms can be used, both to bring the simple models closer to empirical reality as well as to help the more complicated models to drop a few parameters without losing accuracy.

Compared to a simple accessibility model, preferential centrality seems to add empirical value. In relation to urban economics, however, some theoretical foundations in microeconomics are lost. Compared to full-blown LUTI-models, a preferential centrality model is missing many details, but the predictions on offer might be considered good enough. There is also the added value that a

[11] Using pure local agglomeration as a sole explanatory variable might not even make sense, since any real agglomeration takes place in a spatial setting, on several scales. If treated as a spatially uncorrelated factor, the agglomeration effect can then be both over- and underestimated.

centrality model avoids the LUTI-models' challenging requirements of input data and estimation of many different parameter values.

It is more difficult to assess if the added performance of preferential centrality compared to a simple accessibility model makes the extra effort worthwhile. The answer is probably dependent on the particular application. In situations where strong agglomerative forces are expected, there seem to be good reasons to use preferential centrality.

The modelling and empirical results suggest that multiplicative processes are plausible explanations for many urban statistical patterns. When studying preferential centrality, it can be found that increased preferentiality makes the model more sensitive to small changes in interaction. This could mean a more chaotic system with overall less predictability. However, this does not by itself suggests that a more linear model should be used. If the real system actually encompasses strong non-linear forces, models disregarding this would only give an illusory sense of predictability.

The lack of predictability should here be interpreted in the local sense, i.e. regarding what might happen to the activity level for a particular zone or for a particular city. Statistical properties of the systems, or the directions of change, could nevertheless be possible to model with more confidence.

11.5 Applications

An important aim in this search for new approaches in urban modelling is to achieve higher applicability in planning. With regard to planning, the centrality model presented in papers IV-V seems to have greater potential than the urban growth models presented in papers I-III.

The planning-relevant parameters in the urban growth model are the ratios of external growth, perimeter growth, and preferential growth, as well as the interaction parameter. Since no explicit road network is included in the model, all changes of the interaction function must be global, i.e. by political or technological changes. However, the outcome variable (distribution of land values and cluster sizes) are shown to not respond very strongly to the changes of interaction since all changes happen on the time scale of urban growth. This makes the model useful only when considering changes that take place over very long time scales.

The centrality model is better posed to study the short and intermediate time scales since the model responds immediately to changes in the physical network and to changes in the interaction function. Thus, both global changes, such as changing costs of driving, and local changes, such as new road infrastructure, can be studied.

The quick response of the model can be questioned with regard to realism. In reality, physical adaptations and changing activity take place on a longer time scale. The centrality model basically points toward a new equilibrium, providing only limited information about the system's path towards this new state.

These features have commonalities with many economic models, which build upon assumptions of partial or general equilibrium. Many of the same criticisms, and cautions, that has been raised towards such modelling probably also apply to spatial centrality modelling. In the end, it must be empirical validation that provides the trust needed for the practical application of any modelling approach.

11.6 Contribution of research

The contributions of the underlying publications can be summarised as follows.

Paper I

1. Empirical observation of a fat-tailed distribution in Swedish land values on a cellular scale.
2. Formulation of a conceptual link between complex networks and urban interaction.
3. A spatial complex network model is presented that can reproduce the empirically observed distribution.
4. Demonstration that spatial interactions do not significantly distort the power-laws generated by preferential attachment.

Paper II

1. Empirical observation of a fat-tailed distribution in Swedish land values for urban clusters.
2. Empirical observations for urban clusters regarding relations between land value, population, area and perimeter.
3. An improved spatial complex network model that can recreate the observed empirical distributions and relations.

Paper III

1. Interpretation of the urban growth model within a broader context in economic geography.
2. Adaptation to a spatial setting of the complex network concept of node fitness.

Paper IV

1. Derivation of eigenvector centrality in the interaction network, from a spatial interaction model based on the physical network.
2. Introduction of the preferential centrality measure by combining eigenvector centrality with preferential attachment.
3. Empirical spatial comparisons within a single city, showing high correspondence between preferential centrality and land taxation values.

Paper V

1. Extension of the analysis of preferential centrality from the urban to the regional and multi-regional scales.
2. Support for our hypothesis that the road network strongly influences the spatial distribution of economic activity both at the inter-city regional level, as well as on the intra-city local level.
3. Preferential centrality is shown to be a better explanation for empirical land value patterns in comparison to a simpler accessibility model.

In a wider context, the contribution can be summarised as a conceptual adaptation of the complex network framework to study urban interaction networks, specifically with regard to growth dynamics and fat-tailed distributions. The second part of the contribution regards the further model development to integrate the concepts of preferential attachment within a centrality framework.

The latter contribution makes it possible to study actual cities and regions based on transportation networks as input data, opening up for practical applications in transport and land use planning. The low data requirements for applying the preferential centrality model could make it well-suited for use in countries with limited availability of high-resolution socio-economic data.

11.7 Limitations

Regarding empirical tests, the research presented in this thesis has three main limitations. First, it is based solely on Swedish data, which can raise the question of how well results generalise to other parts of the world. Second, the primary source for validation data is property taxation. Other ways of measuring urban land use and activity could yield different results and potentially point towards other underlying mechanisms. It can also not be ruled out that some of the empirical observations are artefacts created by the tax assessment process. Third, all presented results are cross-sectional, which implies that some observed patterns might not be stable over time. Also, the underlying dynamical processes suggested by the modelling have not been empirically tested per se. It is only the resulting patterns produced by these processes that we have examined empirically.

Another limitation concerns the model representation of the transportation system in papers IV-V. It is the road network only that has been used as a basis for calculating spatial interaction, which implies an inadequate representation of interactions based on other physical networks. In reality, public transit, rail transport and flight networks can also have significant impacts on accessibility.

However, large parts of the represented road network are open to modes of transport that do not require a private car, such as travel by foot, cycle, taxi and bus. Since the network is the same, but the usage is different, these other modes can be considered to be implicitly included in the modelling. Nevertheless, an explicit and more careful treatment of these modes could potentially yield other model outcomes. The largest impact can be expected with regard to modes that use a separate infrastructure, such as rail.

Networks for utilities, such as water and electricity, can also be included in the category of physical networks, and these might also have large consequences for the potential of urban expansion. Peripheral development without access to existing utility networks might be costly compared to development in the perimeter of urban areas, where connections can be made easily. Peripheral development might also be discouraged by planning authorities for economic and environmental reasons (such as waste and water management).

Utility networks have not been explicitly considered in this thesis. Implicitly, they are represented in the urban growth models, by the different probabilities for development - externally and at the perimeter. In the centrality model, access to utilities could, in principle, be represented in the local zonal weights, but this has not yet been tested.

12 Further research

12.1 Improved cross-sectional analysis

Comparing empirical geographical distributions with model outcomes poses many problems. In a zone-by-zone comparison, such as in an OLS regression, there is a risk that local spatial peaks in model activity can be interpreted as large deviations if a corresponding empirical peak is present but with a spatial shift. The consequence might be that models without peak-generating processes (such as many models lacking multiplicative growth) can look like they are performing relatively well. This means that smooth (but unrealistic) model results tend to be disproportionally awarded.

The earth mover's distance (EMD, Kranstauber et al., 2017; Rubner et al., 2000) can provide a possible alternative metric that could be used to examine the *combined* fit of frequencies and locations in model output to empirical data. Using EMD could thus be an interesting option when examining how well different models reproduce empirical spatial patterns, where the underlying statistical distributions are fat-tailed.

12.2 Longitudinal studies

A natural next step to further validate the proposed models is to perform longitudinal studies. Historical changes in the physical networks can be studied in relation to changing activity, both empirically and within the models. We have access to Swedish land taxation data from 1988 and onwards, and in an already ongoing (2021-2024) research project, this data will be used to validate the preferential centrality model.

When analysing the effects of improved transport infrastructure, there are several methodological problems to consider. One such issue is due to that new infrastructure makes it easier to both import and export goods and services (Oosterhaven and Knaap, 2017). This means that effects on local economic activity cannot be expected to always be positive, nor always negative. Together with multiplicative forces causing low predictability in models, it is a daunting task to disentangle the causal linkages in an empirical study.

In the preferential centrality model framework, the issue can be exemplified as follows. A new road might influence residents in a small town to redirect their local shopping to a nearby larger city. This can be interpreted as increasing imports to the town and decreasing local economic activity. At the same time, the new road might make the town more attractive for commuting, causing increasing housing prices, i.e. increased activity. The net effect will depend on specific local factors, and the hypothesis implied by the preferential centrality model is that these effects can be inferred mainly from physical network characteristics.

12.3 Other transportation modes

Including public transit in the preferential centrality model is a necessary step for achieving applicability for metropolitan areas. A workable procedure to create multi-modal networks using timetable data has been presented by Gil (2015).

However, using current timetables is problematic when studying models for future cities. Transit networks cannot be considered as static as road networks, which blurs the separation of time scales between infrastructure changes and changing activity.

A possible remedy would be to develop models that can predict important aspects of transit networks. Transit characteristics could then be modelled as consequences of physical networks and urban activity. This would, however, add an additional layer of complexity to the modelling, and several iterations might be needed between modelled activity and modelled transit network.

12.4 Congestion and capacity

Capacity constraints are, in many cases, critical determinants for the performance of urban physical networks. Congested networks could be viewed as a diseconomy of agglomeration, balancing the agglomerative forces. To properly understand the dynamics of megacities, it is thus necessary to consider network capacity and congestion.

These mechanisms could be directly included in the preferential centrality modelling by using an external transport model (including effects of congestion) to create the travel times used in the interaction calculation.

However, more in line with the idea of a minimal model, would be to include a simple congestion mechanism within the centrality calculation. This would make it possible to capture the full feedback between activity, flow and congestion. It is an open question if an empirically relevant model of this kind is achievable.

12.5 Radiation centrality

The principles for distance decay that we have used are based on spatial interaction formulations stemming from the gravity model. Simini et al. (2012) have proposed an interesting alternative called the *radiation model,* which could in principle be incorporated into a centrality model, using a similar derivation as for preferential centrality.

The average flow S_{ij} of interaction between two zones can be calculated by the radiation model using

$$S_{ij} = a_i \frac{a_i W_j}{(a_i + s_{ij})(a_i + W_j + s_{ij})},$$

where a_i is the activity at zone i, and W_j is the attraction of zone j. The quantity s_{ij} could be defined[12] as the sum of the attraction values for all zones reachable from zone i, within the travel time between zones i and j.

By using the equilibrium assumption that the sum of incoming flow is equal to activity, we can find the radiation centrality by solving

$$a_j = W_j \sum_i \frac{a_i^2}{(a_i + s_{ij})(a_i + W_j + s_{ij})}.$$

What types of numerical solutions this can yield for different definitions of attraction has not yet been explored. One potential upside of this measure is that there is no parameter for distance decay, making the model in some sense simpler than preferential centrality based on the gravity model.

12.6 Growing physical networks and evolving urban systems

Several simple models for growing road networks have been proposed in the literature (e.g. Barthelemy and Flammini, 2008; 2009; Courtat et al., 2011; Rui and Ban, 2011). Connecting such a model to the preferential centrality model could provide further insights into the co-evolution between activity, land use and physical networks. By also including the above-mentioned congestion modelling, increased realism would be obtained in the growth process since

[12] The quantity s_{ij} was originally defined as the sum of population within a circle centred at zone i, with a radius equal to the distance between zones i and j. Here an adaptation is needed to account for the activity formulation and the structure of the road network.

congestion can be a significant factor determining decisions about network modifications.

Models for physical network growth, land use development and urban activity could be studied in a joint simulation framework. Empirical validation might be performed using historical changes in transportation technology, energy costs, and taxes since such factors would be exogenous to the joint model.

12.7 Designing urban systems

With a simplified joint simulation model as described above, a further step could be to use evolutionary methods for designing simulated urban systems according to some set of global performance metrics. The resulting development trajectories could then be studied to understand the possible consequences of different stylized planning principles for transport and land use. Hopefully, some guidance could be extracted regarding which principles that could be expected to result in robust and desired outcomes.

12.8 Spatial bubbles

The preferential centrality model has (if parameters are set to more extreme values) the interesting property of being able to form spatial bubbles, i.e. self-sustaining structures in otherwise homogenous spatial settings. This could be likened to how price bubbles might form in asset markets (Sornette et al., 1996).

Theoretical and empirical studies based on the concept of spatial bubbles could yield insights into sudden local changes in property markets, as well as processes such as gentrification.

Jointly with such exploratory modelling, a more thorough mathematical treatment of the preferential centrality model is needed, including studies of stability and multiplicity of solutions.

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Part II – Included Publications