THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

OPTIMIZATION CAPABILITIES FOR CRUSHING PLANTS

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"*As you will find in multivariable calculus, there is often a number of solutions for any given problem*." - John Forbes Nash Jr. (1928 - 2015)

ABSTRACT

Responsible production and minimal consumption of resources are becoming competitive factors in the industry. The aggregates and minerals processing industries consist of multiple heavy mechanized industrial processes handling large volumes of materials and are energy-intensive. One such process is a crushing plant operation consisting of rock size reduction (comminution) and particle size separation (classification) processes. The objective of the crushing plant operation for the aggregates industry is to supply specific size fractions of rock material for infrastructure development, while the objective in minerals processing is to maximize material ore throughput below a target size fraction for the subsequent process. The operation of a crushing plant is complex and suffers variabilities during the process operation, resulting in a drive for optimization functionality development. Process knowledge and understanding are needed to make proactive decisions to enable operations to maintain and elevate performance levels.

To examine the complex relationships and interdependencies of the physical processes of crushing plants, a simulation platform can be used at the design stage. Process simulation for crushing plants can be classified as either steady-state simulation or dynamic simulation. The steady-state simulation models are based on instantaneous mass balancing while the dynamic simulation models can capture the process change over time due to non-ideal operating conditions. Both simulation types can replicate the process performance at different fidelities for industrial applications but are limited in application for everyday operation. Most companies operating crushing plants are equipped with digital data-collection systems capturing continuous production data such as mass flow and power draw. The use of the production data for the daily decision-making process is still not utilized to its full potential. There are opportunities to integrate optimization functions with the simulation platform and digital data platforms to create decision-making functionality for everyday operation in a crushing plant. This thesis presents a multi-layered modular framework for the development of the optimization capabilities in a crushing plant aimed at achieving process optimization and process improvements. The optimization capabilities for crushing plants comprise a system solution with the two-fold application of 1) Utilizing the simulation platform for identification and exploration of operational settings based on the stakeholder's need to generate knowledge about the process operation, 2) Assuring the reliability of the equipment model and production data to create validated process simulations that can be utilized for process optimization and performance improvements.

During the iterative development work, multiple optimization methods such as multi-objective optimization (MOO) and multi-disciplinary optimization (MDO) are applied for process optimization. An adaptation of the ISO 22400 standard for the aggregates production process is performed and applied in dynamic simulations of crushing plants. A detailed optimization method for calibration and validation of process simulation and production data, especially for mass flow data, is presented. Standard optimization problem formulations for each of the applications are demonstrated, which is essential for the replicability of the application. The proposed framework poses a challenge in the future development of a large-scale integrated digital solution for realizing the potential of production data, simulation, and optimization. In conclusion, optimization capabilities are essential for the modernization of the decision-making process in crushing plant operations.

Keywords: Modelling, Dynamic Simulations, Crushing, Screening, Process Optimization, Process Improvement, Digital Twin, Multi-Disciplinary Optimization (MDO), Multi-Objective Optimization (MOO), Key Performance Indicators (KPIs), Calibration, Production Data

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Kanishk Bhadani

Gothenburg, April 2022

PUBLICATIONS

Appended Publications: The thesis contains the following research papers which contribute to the core findings of this work.

Paper A: Bhadani, K., Asbjörnsson, G., Hulthén, E., Bengtsson, M., & Evertsson, M. (2017). State of the Art in Application of Optimization Theory in Minerals Processing. *European Symposium on Comminution and Classification*, Izmir, Turkey, September 11-14, 2017.

Paper B: Bhadani, K., Asbjörnsson, G., Hulthén, E., & Evertsson, M. (2018). *Application of multi-disciplinary optimization architectures in mineral processing simulations*. Minerals Engineering, 128, 27–35.

Paper C: Bhadani, K., Asbjörnsson, G., Hulthén, E., Bengtsson, M., & Evertsson, M. (2019). Comparative Study of Optimization Schemes in Mineral Processing Simulations. *IMPC 2018 - 29th International Mineral Processing Congress*, September 17-21, 2018, 464–473.

Paper D: Bhadani, K., Asbjörnsson, G., Hulthén, E., & Evertsson, M. (2020). *Development and implementation of key performance indicators for aggregate production using dynamic simulation*. Minerals Engineering, 145.

Paper E: Bhadani, K., Asbjörnsson, G., Schnitzer, B. M., Quist, J., Hansson, C., Hulthén, E., & Evertsson, M. (2021). *Applied Calibration and Validation Method of Dynamic Process Simulation for Crushing Plants*. Minerals, 11(9).

Paper F: Bhadani, K., Asbjörnsson, G., Hulthén, E., Hofling, K., & Evertsson, M. (2021). *Application of Optimization Method for Calibration and Maintenance of Power-Based Belt Scale.* Minerals, 11(4).

Additional Publications: The following research papers do not contribute to the finding of this thesis but were instrumental in improving the author's understanding of the research field.

Paper G: Bhadani, K., Asbjörnsson, G., Bepswa, P., Mainza, A., Andrew, E., Philipo, J., Zulu, N., Anyimadu, A., Hulthén, E., & Evertsson, M. (2021). Experimental and simulation-driven improvements for coarse comminution circuit using *Plantsmith* process simulator - A case study of Geita Gold Mine, Tanzania. *Conference in Minerals Engineering* (*Konferens i mineralteknik*), Luleå, Sweden, February 8-9, 2021.

Paper H: Bhadani, K., Asbjörnsson, G., Bepswa, P., Mainza, A., Andrew, E., Philipo, J., Zulu, N., Anyimadu, A., Hulthén, E., & Evertsson, M. (2021). Simulation-driven development for coarse comminution process - A case study of Geita gold mine, Tanzania using *Plantsmith* process simulator. *Proceedings of the Design Society*, 2021, 1, 2681-2690.

WORK DISTRIBUTION

Paper A: Bhadani, Asbjörnsson and Hulthén initiated and conceptualized the idea. Bhadani carried out the literature study and wrote the paper with Asbjörnsson, Hulthén, Bengtsson and Evertsson as active reviewers.

Paper B: Bhadani initiated the idea and carried out the development & implementation of the optimization methods. Asbjörnsson provided models for process simulation. Bhadani wrote the paper with Asbjörnsson, Hulthén and Evertsson as active reviewers.

Paper C: Bhadani, Asbjörnsson, Hulthén and Evertsson initiated the idea. Bhadani carried out the development & implementation of the optimization methods. Asbjörnsson provided models for process simulation. Bengtsson provided constructive input for optimization methods. Bhadani wrote the paper with Asbjörnsson, Hulthén and Evertsson as active reviewers.

Paper D: Bhadani, Asbjörnsson, Hulthén and Evertsson initiated the idea. Bhadani carried out the development & implementation of the key performance indicators. Asbjörnsson provided models for process simulation and Hulthén supported with real-time process data. Bhadani wrote the paper with Asbjörnsson, Hulthén and Evertsson as active reviewers.

Paper E: Bhadani, Asbjörnsson, Hulthén and Evertsson initiated the idea. Bhadani conducted experiment and simulation work with support from Asbjörnsson. Bhadani developed and implemented the optimization and validation methods and Asbjörnsson provided constructive feedback. Hanson facilitated the experimental work at the industrial site and supported the laboratory analysis of experimental samples. Schnitzer provided constructive input for mathematical formulations. Bhadani wrote the paper with Asbjörnsson, Quist, Hulthén and Evertsson as active reviewers.

Paper F: Bhadani, Asbjörnsson, Hulthén and Evertsson initiated the idea. Bhadani conducted experimental and simulation work with support from Asbjörnsson. Bhadani developed and applied the optimization method and the deviation tracking method together with Asbjörnsson expertise. Hofling and Hulthén facilitated the experimental work and process data collection at the industrial site. Bhadani wrote the paper with Asbjörnsson, Hulthén and Evertsson as active reviewers.

TABLE OF CONTENTS

Appendix

ABBREVIATIONS

ACO	Ant colony optimization
CSS	Closed-side setting
CV	Conveyor
CEF	Conveyor error factor
CER	Conveyor error ratio
FSM	Finite state machine
FIFO	First-in-first-out
GA	Genetic algorithm
IDF	Individual-discipline feasible
ISO	International Organization for Standardization
KKT	Karush-Kuhn-Tucker condition
KPI	Key performance indicators
MILP	Mixed-integer linear programming
MDO	Multi-disciplinary optimization
MDF	Multi-discipline feasible
MOO	Multi-objective optimization
NLP	Non-linear programming
OFAT	One-factor-at-a-time
OEE	Overall equipment effectiveness
PSO	Particle swarm optimization
PGSL	Probabilistic global search Lausanne
PSD	Product size distribution
PID	Proportional-integral-derivative controller
RMSE	Root mean square error
SA	Screen aperture
SQP	Sequential quadratic programming
SOO	Single-objective optimization
SPV	Sub-process value

NOTATIONS

F_{80}	80% passing size on the product size distribution of feed
P_{80}	80% passing size on the product size distribution of product
η	Conveyor efficiency
A_{pk}	Correlation matrix
f'	First derivative of objective function
$(\ )_i$	Functions or variables applied only to a sub-process i
$(\ )^0$	Functions or variables at their initial values
$(\ )^*$	Functions or variables at their optimal values
$(\ )_0$	Functions or variables shared between more than one sub-process
∇f	Gradient
H	Hessian
$(\bar{\ })$	Independent copies of variables distributed to other sub-process
L	Lagrange
λ, μ	Lagrange multipliers
m	Mass
$\dot{m}$	Mass flow
N	Number of sub-processes
f	Objective function
$\Re^n$	Real number
f''	Second derivative of objective function
$\mathbf{c}^c$	Vector of consistency constraints
$\mathbf{y}$	Vector of coupling variables or output from sub-process analysis
$\mathbf{c}$	Vector of design constraints
$\mathbf{p}$	Vector of design parameters or system parameters
$\mathbf{x}$	Vector of design variables or system variables
$\mathbf{h}$	Vector of equality constraints
$\mathbf{g}$	Vector of inequality constraints
$\boldsymbol{\gamma}$	Vector of material characterization properties
$\mathbf{f}$	Vector of objective functions
xsize	Vector of standard sieve size

1 INTRODUCTION

This chapter aims to:

> *Introduce a generic overview of a crushing plant.*
>
> *Provide an outline of the need for optimization capabilities in crushing plant operations.*
>
> *Introduce the area of research and the scope for the development of optimization capabilities.*
>
> *Formulate the research questions.*

Infrastructure development is one of the cornerstones of human success in modern times and facilitates economic growth and mobility. Modern infrastructure developments such as roads, railways, housing, and commercial buildings are directly dependent on the supply of crushed rock materials (aggregates). Aggregates are used in the construction industry as a base material and are classified into products such as sand, gravel, clay, and crushed stone [1]. The industrial processes used in aggregates production after mining activities are comminution (rock size reduction) and classification processes (particle size separation) [2], which are often referred to in combination as a crushing plant. Crushing plants are also used in the minerals processing industry for ore size reduction in the coarse particle size range. The common types of equipment present in a crushing plant are rock size reduction units (e.g., jaw crushers, gyratory crushers, cone crushers, etc.), particle separation units (e.g., vibratory screens, cyclones, etc.), material transport (e.g., conveyors, trucks), and material storage (e.g., bins, stockpiles, etc.).

1.1 DRIVERS FOR OPTIMIZATION CAPABILITIES IN CRUSHING PLANTS

According to a recent report from the European Aggregates Association (UEPG), the average consumption of aggregates is around 6 tonnes per capita per year within the European Union and 10 tonnes per capita per year in Sweden [3]. According to the reports from the Swedish Geological Survey (SGU), the production of mineral ores in Sweden accounted for 87.9 million tonnes in 2020 [4] and aggerates production for 100.2 million tonnes in 2019 [5], both of which are continuously increasing from previous years' production. Similar trends are also shown in worldwide mining data by the British Geological Survey, wherein the production volumes of major minerals (e.g., bauxite, iron ore, copper ore, etc.) have been increasing in comparison with data from previous years [6]. Another trend observed within the aggregates and minerals processing industries in Sweden is that the number of mine sites is decreasing while the volume of production is increasing [4, 5]. This can be attributed to many factors, such as improved equipment technology, stricter environmental laws, the economy of operation, availability of resources, changing ore conditions, and so on. In Sweden, the estimated carbon footprint during the processing of aggregates in a crushing plant alone (excluding mining, transportation, etc.) accounts for 3.5–5.4 kg CO_2 eq./ton of crushed rock material depending on the energy source – electricity-driven or diesel-driven [7]. Irrespective of the choice of energy, there is potential to further reduce the carbon footprint with improvements in the operation of crushing plants.

Given the large volume of material processed in crushing plants (in both the aggregates and minerals processing industries) in a limited number of sites, there is a need and a possibility for developing methods that can result in more responsible production, such as improved resource- and energy-efficiency and reduced waste material. The increased mechanization has resulted in the automation of process operations, which has in turn resulted in increased throughput, process control and energy control. On the other side, the implication of increased mechanization has brought less flexibility in operation. In particular, the process operation performance has increased sensitivity to stoppages, which in turn is expensive for the company operating the plants. The operations of crushing plants are complex and require a broad understanding by the personnel involved, which is developed by training and with experience. This creates a need to develop methods and tools which can facilitate and support the operation of the crushing plant to maximize utilization within the given resources. The main driver for this research is the industry that wants to be competitive (profitable) and adoptive to customer demands and changing conditions. These drivers are also in line with UN sustainability goals (especially Goal 9: Build resilient infrastructure, promote inclusive and sustainable industrialization and foster innovation) for innovation in the aggregates and minerals processing industries to increase resource-use efficiency in comparison with today's performance [8].

Crushing plant operations are often classified as a continuous production process and this process is complex, with interdependencies among various equipment and sub-processes. To capture these complex relationships, a process simulation, which is one of the cost-effective tools accepted by the industry, is often used. There are two types of process simulations used for crushing plants in operation: steady-state simulation and dynamic simulation [9, 10]. Steady-state simulation represents an instantaneous mass balancing of the crushing plant circuit, while dynamic simulation can imitate time-dependent phenomena in the crushing plant circuit such as discrete and gradual changes due to material delays, start-up sequence, wear, control, etc. [10]. The dynamic simulation platform comes closer to representing the actual operations of a crushing plant than steady-state simulations. However, it is time-consuming, complex to set up and requires expertise to interpret results. Most of the commercial simulation tools available to the industry are steady-state, for example, *JKSimMet* (JKTech) [11], *Bruno*™ (Metso:Outotec) [12], *Integrated Extraction Simulator – IES* (Orica) [13], *Plant Designer* (Sandvik) [14], *Plantsmith* (Roctim) [15], etc. The commercial software has been successful in helping designers to plan and create virtual crushing circuits but is limited in providing functionality such as optimization of production in daily plant operations. Inclusion of such optimization functionality can bring value to customers, wherein daily production operations can be steered in flexible ways to meet customer demands, especially in aggregates production. Decision support from a simulation tool can be useful for daily production operations in a crushing plant on the condition that the underlying models and process performance are calibrated and validated regularly. This implies that there is a need to collect appropriate data to maintain trust within the simulation systems.

Recent technological developments and inclinations towards digitalization provide new opportunities for industry (coined as Industry 4.0) [16]. The question that arises here is how the companies operating crushing plants are taking advantage of the digitalization transitions and what the transition means at the functional level of the core operations. Services such as cloud-based production data collection help large organizations operating multiple crushing plant sites to connect and access their sites on a central platform. This can enable an organization to compare process performances, schedule product delivery, identify best practices, etc. However, the detailed needs of data collection and how it can be used for process improvements and process optimization in the context of crushing plant operations need to be investigated. Typically, continuous production data collected in aggregates production are mass flow, power consumption and different operational parameters from various equipment in a process. Limited work has been performed to integrate the existing simulation platforms for crushing plants and digital data collection tools present in today's modern companies. This poses a new challenge to the crushing plant owners to determine how future plants should be built to operate and how to transition the existing plant to digital platforms. The development of meaningful optimization functionality for a simulation platform in the context of crushing plant operations is one of the proposed solutions to the changing needs of the industry. This can enable operators and plant managers of a crushing plant to perform knowledge-based flexible and profitable operations. Further, the development can result in the democratization of the expertise knowledge of crushing plant simulations to operators and plant managers for everyday operation.

1.2 Context of Optimization Functionality for a Crushing Plant

Figure 1 presents an overview of a generic three-stage crushing plant. The purpose of the crushing plant is to reduce the rock material size from up to 1000 mm to below 30 mm depending on the need for the aggregates products or the requirements from the subsequent steps in minerals processing. The need for multiple stages in a crushing plant is based on equipment technological capabilities, for example, a cone crusher capability is based on the top size in feed, maximum allowed reduction ratio, power rating, chamber type, etc. [17]. After the mining operation (blasting, drilling, etc.), the rock material is transported (trucks, front loaders, conveyors, etc.) to the primary crushing stage, in which the material is fed through a grizzly screen. The purpose of the grizzly screen is to bypass the lower size fraction material and feed the oversized material to the primary crusher (jaw crusher, gyratory crusher, etc.). The crushed rock from the primary crushing process is stored in a stockpile to create a buffer zone in operation and flexibility of maintenance of equipment. The material is further reduced in size in secondary and tertiary crushing processes, typically by using cone crushers. Material is fed to the crusher using a vibratory feeder, surge bin or conveyor, depending on the installation. Vibratory screens are used to separate the desired products and the oversize material is recirculated into the circuit. The material transport within the circuit is carried out using belt conveyors. The crushing plant circuit has certain inbuilt flexibility, such as bypassing material to re-direct flow and recirculating material for re-crushing (closed circuit or open circuit).

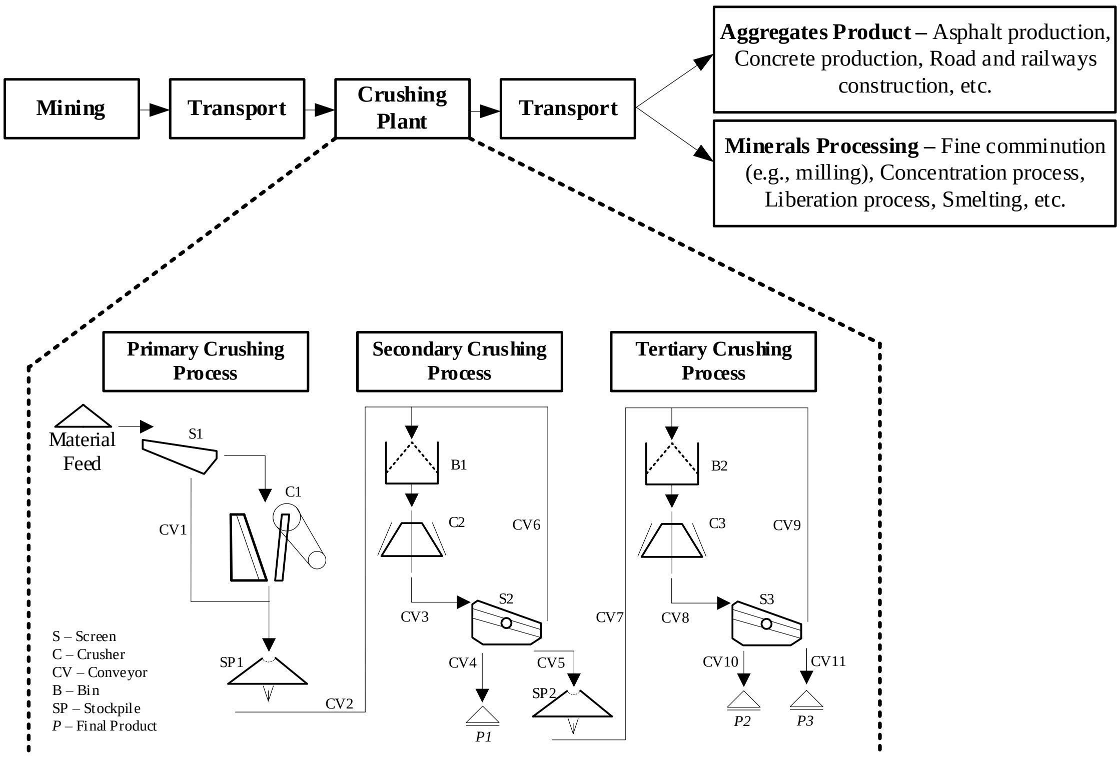


Figure 1. An illustration of a crushing plant circuit in relation to the value chain of aggregates and minerals processing.

In aggregates production, the aim is often to produce multiple products based on the size specification (e.g., 0/16 mm, 11/16 mm, 5/8 mm, etc.). There are several criteria for defining product quality for aggregates (e.g., grading, flakiness index, etc.) depending on the target use and certification requirements [18]. The need for different products in aggregates production is often dependent on the customer requirements and market demands, which in turn leads to the prerequisite of maintaining the desired stock levels in order to be competitive. It has been noted that many aggregates sites have stocks of over-produced products which are non-sellable due to limited demand. Although the present technology in aggregates production is limited to eliminating by-products, there are opportunities to reduce the ratio of non-desired products to desired products. For minerals processing, the target of the crushing plant is usually to produce materials in one or two size ranges for the subsequent processing steps (e.g., 0/12 mm for ball-mill feed). The product quality criteria are limited in minerals processing, while the focus of process operations is usually on maximizing circuit throughput of material below a target fraction while minimizing downtime and energy consumption.

Managing a crushing plant for daily operations in aggregates production poses challenges as the plant is a complex system with interdependencies between different equipment. Mathematical optimization is a powerful tool that can be used to generate knowledge about a system in relation to the defined conditions. Papalambros and Wilde [19] defined the optimization of a system as a *decision-making process* of choosing the *best alternative* which meets the *original need* within the *available means*. In relation to the crushing plant in operation as a system, the primary value-adding functions of the system are size reduction and separation of rock material followed by performance aspects such as throughput, energy consumption and quality. The *need* is dependent on the objective of the plant operations (performance), which can differ based on the stakeholder's perspective. For example, an operator can aim to minimize the downtime of process operations, a plant manager can aim to maximize process throughput, and a salesperson can aim to minimize the lead time to product delivery. Multiple combinations of operational settings are possible for various equipment (e.g., crusher settings, screen settings etc) in a crushing plant, which refer to the *alternatives* in the operation. The circuit design and equipment type present in a crushing plant are the *available means* in operation, which limits the number of possibilities of operation. For a crushing plant in operation, *optimization capability* can be defined as the process of choosing the *best* operational settings for a *given circuit configuration* to meet the *performance need* of the stakeholder.

The capability of process simulation to replicate crushing plant performance and implementation in full-scale plant design and operation has increased over the past couple of decades [9, 10, 20, 21], although the application to daily process operations is limited due to limited functionality. Knowledge of process simulation is also limited to certain experts in the companies and a wider set of users is needed to create a tangible impact on the processed volume in the crushing plant. Adding optimization capability can enhance the usability of the simulation platform and help support plant managers and operators in taking proactive decisions for changes in the requirements for plant operation. The use of a simulation platform is a more cost-efficient method than the experimental trial-and-error approach. However, multiple aspects, such as continuous simulation configuration, calibration, and validation, are needed to create reliable models for optimization. Concrete measurements of improvements achieved by optimization capabilities in physical plants are also required, which poses a need to evaluate the correct set of data. Moreover, the users (plant managers and operators) in crushing plants have expressed the desire to simplify the use of simulation platforms for optimization through the use of a simple push-button, which poses a need for investigation of methods to make this possible. To address the abovementioned challenges, method developments for implementing optimization capability, performance improvements, simulation and data reliability are presented in this thesis.

1.3 RESEARCH OUTLINE

The research project aims to develop and apply optimization capabilities to crushing plant process simulations. The focus is on the crushing plant operation (see Figure 1), which is characterized as a continuous dynamic production process. The simulation platform used to replicate crushing plant performance is based on the dynamic process simulation in MATLAB/Simulink developed by Asbjörnsson [10]. The research has been developed with a primary focus on the aggregates production industry and with a limited scope for the minerals processing industry. The primary crushing plant equipment and their corresponding mathematical models included in the research are cone crushers, vibratory screens, belt conveyors, material feeders and bins. The objective of this research is to investigate different optimization methods for applications of process improvements and process optimization and to generate needed knowledge about the crushing plant system. Further on in the thesis, the use of optimization methods for process model calibration and validation, and data reliability are investigated. The purpose of the research is to understand and increase the process performance of a crushing plant in operation. The research is limited to the evaluation of crushing plant performance related to equipment settings and excludes human-interaction studies. Further on, the research builds on the previous work (e.g., dynamic simulation approach, equipment models, data acquisition system, real-time optimization etc.) carried out by Asbjörnsson [10], Hulthén [22] and Evertsson [17]. The consideration of certain physical and chemical properties of the rock material, such as ore grade, is excluded from this work. An investigation of the influence of control systems on the dynamic performance of crushing plants is not included.

1.4 RESEARCH QUESTIONS

The scope of the thesis can be described by the following research questions, with a brief description of investigations carried out under each research question:

RQ1: What are the optimization system requirements for developing optimization capabilities in crushing plant operations?

- *In this research question, the aim was to investigate aspects such as the purpose of optimization application, problem formulation, possible optimization methods and algorithms, optimization results and implementation, and practical implications. It is assumed from the beginning that a computer-based simulation platform and optimization can be used as a foundation for the development.*

RQ2: How can the process performance objectives be formulated for carrying out process optimization and process improvements in crushing plant operations?

- *Based on the preliminary findings of research question 1, the aim was to perform an explorative study to understand and develop process objectives suitable for improving crushing plant performance.*

RQ3: What are the critical requirements on the process simulation platform, equipment models, experimental and process data to be used in the optimization system?

- *The iterative learning from the optimization application led to the need to investigate methods that could be used to create a reliable simulation platform for crushing plant optimization studies. Different data associated with crushing plants are also studied.*

The research questions are addressed throughout the thesis and answers to them are presented in Chapter 5 – Discussion and Conclusion. During this PhD project, several papers have been written by the author of this thesis which contribute to the research findings. Table 1 presents an illustration of the appended papers' contributions in relation to the research questions.

Table 1. The contribution of the appended papers to the research questions. A larger sphere represents a strong relation to the research question while a smaller sphere represents a minor contribution.

Appended Papers / **Research Questions**	**Paper A:** State of the Art in Application of Optimization Theory in Minerals Processing	**Paper B:** Application of Multi-Disciplinary Optimization Architectures in Mineral Processing Simulations	**Paper C:** Comparative Study of Optimization Schemes in Mineral Processing Simulations	**Paper D:** Development and Implementation of Key Performance Indicators for Aggregate Production Using Dynamic Simulation	**Paper E:** Applied Calibration and Validation Method of Dynamic Process Simulation for Crushing Plants	**Paper F:** Application of Optimization Method for Calibration and Maintenance of Power-Based Belt Scale
RQ1	●	●	●	•	•	•
RQ2	•	•	•	●	•	•
RQ3	•	•	•	•	●	●

2 SCIENTIFIC APPROACH

This chapter aims to:

> *Introduce and describe the research methodology used in this thesis.*
>
> *Introduce research evaluation aspects.*
>
> *Introduce fundamentals of optimization methods.*
>
> *Describe optimization algorithms and methods used in this thesis.*

The research was performed in the Chalmers Rock Processing Systems (CRPS) research group, which is a part of the Machine Element Group, Division of Product Development at the Department of Industrial and Materials Science at the Chalmers University of Technology. The research group has been active in the field of modelling comminution and classification equipment and processes for over three decades. The research output presented in this thesis has been performed in close collaboration with major aggregates producers in Sweden and utilizes findings from previous research conducted at CRPS [10, 17, 21-23].

2.1 RESEARCH METHODOLOGY

The research methodology applied is characterized as a combination of two approaches: a problem-based approach and a system theory approach. The adoption of a problem-based approach towards understanding cone crushers was demonstrated by Evertsson [17]. The problem-based approach is described as a systematic search for new knowledge by focusing on the problem and iterative method development by understanding fundamental principles of operation and characteristics of the problem [17]. The general system theory approach is based on the fundamental principle that a complex system can be presented as a combination of the various sub-systems and their interactions for a given defined boundary [24, 25]. Asbjörnsson [10] combined the problem-based approach with the system theory approach for the development of a dynamic simulation system for crushing plants. Svedensten [21] and Hulthén [22] proposed that early implementation of the results to the industry is a critical success factor as it adds a validation process and checks the applicability for industrial use.

The modified research methodology used in this thesis is shown in Figure 2 (based on Asbjörnsson [10] and Evertsson [17]). The modification is aimed at addressing the multidisciplinary nature of research together with the problem-based approach and industrial case studies. The development work relies on both simulation studies and industrial implementation studies to increase the maturity of the research output.

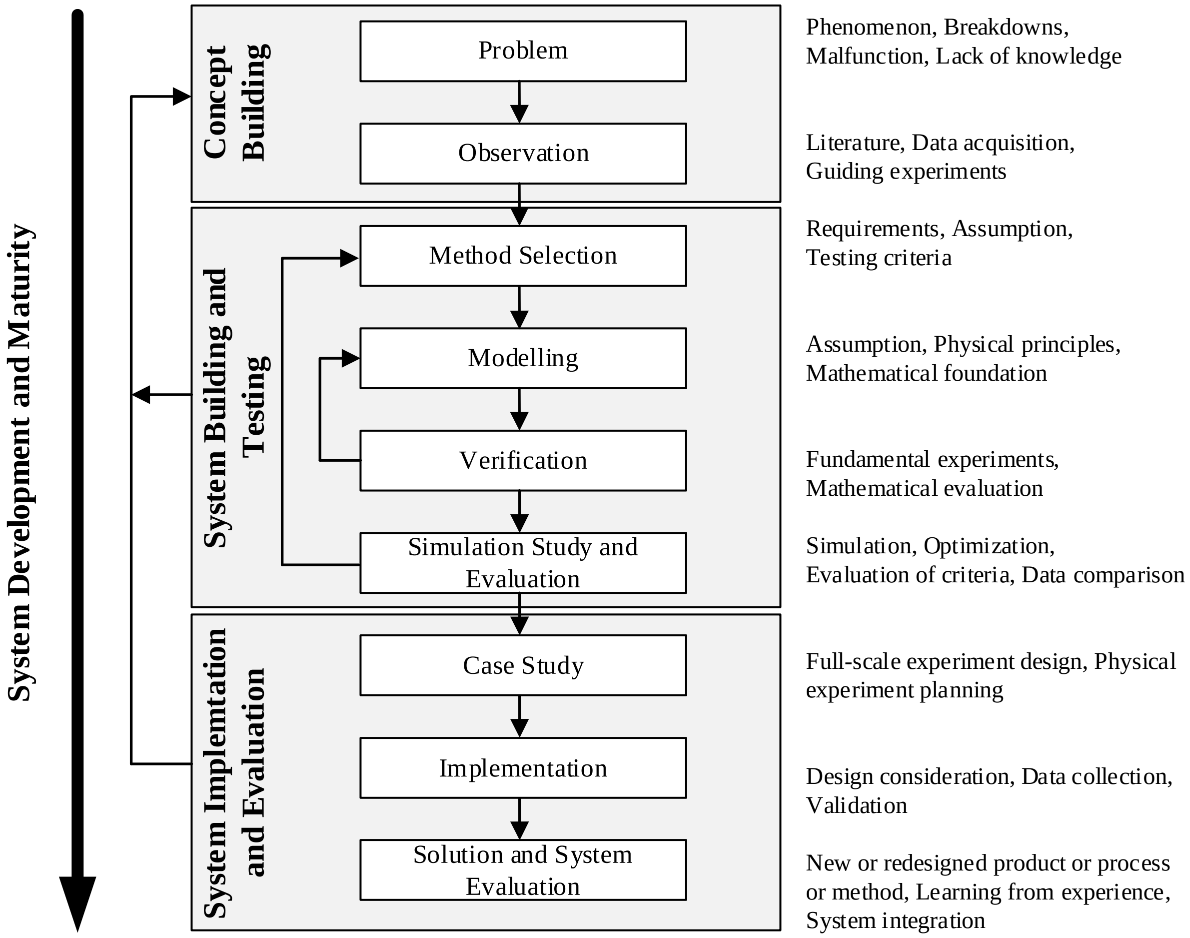


Figure 2. Applied research methodology based on problem-based approach and system theory [10, 17].

The research was initiated by the identification of possible knowledge gaps and the industrial relevance of optimization applications for crushing plants. This resulted in the formulation of a set of problems that were individually studied using literature study, explorative method searches, and knowledge exchange with experts. At this stage, multiple problems with clearly defined distinct scopes were conceptualized.

Each of the defined problems was undertaken for the system building and testing phase with an iterative mindset. The process began with the identification of suitable methods to solve the problem with a clear presentation of the underlying requirements and assumptions. This was followed by iterative modelling and verification processes. The modelling of the problem was based on the mathematical foundation while verification was based on a check of the correctness of the implementation. This phase mainly relied on the simulation studies and the results were evaluated based on the suitable testing criteria. The iterative nature of the process led to gaining new insights about the system in consideration and feedback to the concept building phase to revise the defined problems.

As the system development progressed with new insights and gained certain confidence levels, full-scale industrial implementation cases were formulated. This step required physical experimental design and planning followed by execution and data collection activities. The methods applied were evaluated with validation against the industrial data, which led to an increased maturity in comparison with the simulation studies. As the process was further iterated, system integration was performed to build the solution for the initial defined problems.

2.2 RESEARCH EVALUATION

The research activities performed in this thesis involve the use of both computer simulation and physical experimentation. This requires the research quality to be evaluated using central concepts: verification, validation, and reliability. Sargent [26] defined model verification as the process of ensuring that computer code captures the solution model with the correct implementation process, while model validation is a process of evaluating the accuracy of the model to its intended use. Bryman and Bell [27] defined research validity as the integrity of the research conclusion and reliability as the repeatability of the results. The specific claims made for a study are characterized as internal validity, while the generic claims made are characterized as external validity [27]. According to Pedersen et al. [28], the research validity is categorized as structural validity and performance validity for both theoretical and empirical research. Structural validity represents a qualitative process to evaluate if the system is built with sufficient information to demonstrate the application of the results, while performance validity represents a quantitative process to determine the accuracy of the results [28]. The multidisciplinary nature of this research poses a challenge to include both qualitative and quantitative evaluation of the research work [29]. For individual experimental studies, reliability is defined as the extent to which experimental outputs are consistent on repeated trials and thus provide confidence in the measuring procedure [30]. In comparative simulation studies, it is recommended to use the same underlying assumptions for a particular simulation model under study to increase the reliability of the results. These different aspects of the verification, validation, and reliability of the individual research activities will be discussed in Chapter 5 – Discussion & Conclusions.

2.3 OPTIMIZATION FUNDAMENTALS

The mathematical optimization toolbox for engineering design problems consists of several concepts which enable the decision-making process [19]. A general set of concepts associated with the description of an optimization application are as follows:

- System concept and system function
- Mathematical models and mathematical relationships
- System variables and system parameters
- Optimization problem definitions
- Optimization algorithms
- Optimization methods

The following section briefly describes each concept based on the interpretation of multiple pieces of literature and is primarily based on Papalambros and Wilde [19] and Arora [31]. This is relevant for understanding the optimization application performed in the thesis.

2.3.1 System Concept and System Function

A system can be defined as a collection of units representing the mathematical behaviour of a physical phenomenon that is intended to perform a specific set of system functions by taking a set of inputs and producing a set of outputs [19]. System function can be defined as the value-adding activity of the physical phenomenon. Figure 3 represents a general representation of a system concept, and it is implicit that the system is triggered by the input values and the output is governed by the dynamic behaviour of the mathematical functions. Each instance of the output of the system function $\mathbf{y}(\mathbf{x},\mathbf{p})$ is distinguished for a set of inputs $(\mathbf{x},\mathbf{p})$ and is called a *state of the system*. The system can produce outputs that can be discrete or continuous depending on mathematical functions.

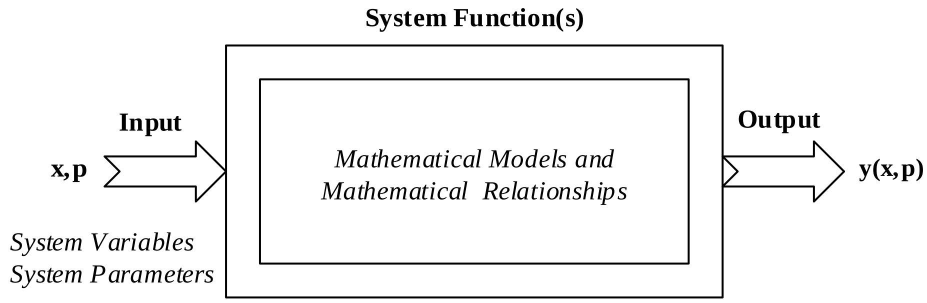


Figure 3. System concept for optimization application.

2.3.2 Mathematical Model and Mathematical Relationship

A mathematical model is an approximate representation of complex physical reality under a set of assumptions [19]. Mathematical models exist in multiple degrees of fidelity depending on the intended use of the model and the level of quality can vary [32]. The types of mathematical models can be characterized as empirical models (developed based on experimental data), mechanistic models (developed based on the physics of the problem), phenomenological models (developed based on experimental data and physical phenomena) and so on. In essence, a mathematical model is a mathematical function describing physical system functions. Mathematical relationships are a set of equations that combine the input values of the system with the mathematical models to create a set of the outputs of interest. Mathematical models are developed based on the study of the system while the mathematical relationships are created based on the intended use of the mathematical models.

2.3.3 System Variables and System Parameters

System variables (**x**) represent the vector of input to the system function, which can be altered in the mathematical model to create multiple system states. The system parameters (**p**) are the vector of input to the system that are set to specific values for a particular mathematical model. The state of the system is dependent on the combination of system variables and system parameters. The limits on the system variables and system parameters are based on the knowledge of the development of the mathematical model in relation to the application.

2.3.4 Optimization Problem Definition

The standard optimization problem formulation in negative null form is defined in Equation 2.1, where the objective is to minimize a function $f(\mathbf{x},\mathbf{p})$ for a given vector of inequality constraint $\mathbf{g}(\mathbf{x},\mathbf{p})$ and equality constraint $\mathbf{h}(\mathbf{x},\mathbf{p})$ [19]. The system variables vector ($\mathbf{x}$) is bounded by set constraints (χ) which belong to the real number ($\Re^n$) for n dimensions. Mathematical relationships are used to define various functions in the optimization problem definition.

$$
\begin{aligned}
&\min_{\mathbf{x}} f(\mathbf{x},\mathbf{p}) \\
&\text{subject to} \\
&\mathbf{h}(\mathbf{x},\mathbf{p}) = 0 \\
&\mathbf{g}(\mathbf{x},\mathbf{p}) \le 0 \\
&\mathbf{x} \in \chi \subseteq \Re^n
\end{aligned}
\tag{2.1}
$$

The following describes fundamental definitions related to the optimization problem:

- *Objective Function*: An objective function *f*(**x**,**p**) is a representation of the goal of the optimization problem. The objective function is either described as minimization form or maximization form. In practical problems, the objective function is defined based on the stakeholder's needs.

- *Constraint Function*s: A constraint provides a set of requirements that the optimization solution set needs to fulfil. The constraints are of two types: equality constraints **h**(**x**,**p**) and inequality constraints **g**(**x**,**p**). The problem formulation shown in Equation 2.1 can also be termed a constrained optimization problem. Excluding the constraints and variable limits can turn the problem into an unconstrained optimization problem.

- *Optimizer* ($\mathbf{x}^*$): A converged optimization solution consists of a set of optimal solution points for the variables and is called the *optimizer*.

- *Optimum* ($f^*(\mathbf{x},\mathbf{p})$): The value of the objective function at the optimal solution points is called the *optimum*.

The optimization solution needs to be analysed to find the feasibility and boundedness of the solution point. The term feasibility means the solution set is meeting the requirements (constraints) defined in the optimization problem, while the term boundedness means the optimizer set is within the defined limits of upper and lower bounds of the defined variable for a well-posed problem. The solution set also needs to be reviewed to determine whether it is a local optimization or a global optimization result.

2.3.5 Optimization Algorithm

The optimization algorithm presents a numerical or logical scheme for solving an optimization problem. It represents a set of processes carried out to find the optimal solution for the optimization problem. These can be broadly classified as a gradient-based algorithm (e.g., interior-point method, sequential quadratic programming, etc.) or a non-gradient-based algorithm (e.g., genetic algorithm, particle swarm, pattern search, etc.) [19, 31]. There are other multitude variants of the optimization algorithms which can be referenced under other categories of optimization algorithms [31, 33, 34].

2.3.5.1 Gradient-Based Algorithm

Gradient-based algorithms are based on the fundamental mathematical application of derivatives or partial derivatives of a function *f*(**x**) at a system variable's (**x**) point. Table 2 presents an overview of the necessary and sufficient conditions for optimality for single and multivariable optimization problem functions [19, 31]. In most practical applications, the optimization formulation is defined as a multivariable problem, and it is essential to understand the behaviour of the objective function (linear or non-linear).

In a practical implementation of mathematical models using simulation software, the functions are differentiated using a numerical approach considering non-linear behaviour. For example, forward difference, backward difference, and central difference methods can be applied to calculate the derivative of a function at a point. *Taylor series* expansion can be applied to estimate a value of a function at an incremental point ($x+\Delta x$) with a known value of function *f*(*x*) at a point *x*, as shown in Equation 2.2. [31]

$$f(x+\Delta x) = f(x) + \Delta x f'(x) + \frac{\Delta x^2}{2!} f''(x) + \frac{\Delta x^3}{3!} f'''(x) + \dots \tag{2.2}$$

Table 2. Overview of the optimality condition for the application of gradient-based algorithms.

	Single Variable	**Multi Variables**
First-Order Necessary Condition	For $f(x) \rightarrow$ Differentiable Function If Gradient: $f'(x) = \frac{df}{dx}(x^*) = 0$ then $x^* \rightarrow$ Local minimum, Local maximum or Saddle point	For $f(\mathbf{x}) \rightarrow$ Differentiable Function If Gradient: $\nabla f(\mathbf{x}^*) \triangleq \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \cdots, \frac{\partial f}{\partial x_n}\right) = 0$ then $\mathbf{x}^* \rightarrow$ Local minima, Local maxima or Saddle points where n = vector size of $\mathbf{x}$
Second-Order Sufficient Condition	For $f(x) \rightarrow$ Differentiable Function If $f'(x) = \frac{df}{dx}(x^*) = 0$ and $f''(x) = \frac{d^2 f}{dx^2}(x^*) > 0$ then $x^* \rightarrow$ Local minimum	For $f(\mathbf{x}) \rightarrow$ Differentiable Function If Gradient: $\nabla f(\mathbf{x}^*) \triangleq \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \cdots, \frac{\partial f}{\partial x_n}\right) = 0$ and Hessian: $\mathbf{H}(\mathbf{x}) \triangleq \begin{pmatrix} \frac{\partial^2 f}{\partial x_1^2} & \cdots & \frac{\partial^2 f}{\partial x_1 x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial^2 f}{\partial x_n x_1} & \cdots & \frac{\partial^2 f}{\partial x_n^2} \end{pmatrix}$ $\mathbf{H}(\mathbf{x})$ is positive definite then $\mathbf{x}^* \rightarrow$ Local minima where n = vector size of $\mathbf{x}$

The forward difference and backward difference formulas can further be derived from Equation 2.2, as shown in Equations 2.3 and 2.4, respectively, where $\mathrm{O}(\Delta x)$ represents that the formula is first-order accurate [31].

$$f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x} + \mathrm{O}(\Delta x) \tag{2.3}$$

$$f'(x) = \frac{f(x) - f(x-\Delta x)}{\Delta x} + \mathrm{O}(\Delta x) \tag{2.4}$$

Combining Equations 2.3 and 2.4 results in the central difference formula, as shown in Equation 2.5 [31].

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{\Delta x} + \mathrm{O}(\Delta x^2) \tag{2.5}$$

Similarly, the second-order derivative can be estimated using Equation 2.6 [31].

$$f''(x) = \frac{f(x+\Delta x) - 2f(x) + f(x-\Delta x)}{\Delta x^2} \tag{2.6}$$

The equivalent of the *Taylor series* (Equation 2.2) can be applied for multivariable ($\mathbf{x}$) problems of size n to perform linear and quadratic approximation, as shown in Equation 2.7 [31].

$$f(\mathbf{x}+\Delta\mathbf{x}) = f(\mathbf{x}) + \nabla f(\mathbf{x})^T \Delta\mathbf{x} + \frac{1}{2}\Delta\mathbf{x}^T \mathbf{H}(\mathbf{x})\Delta\mathbf{x} + \cdots \tag{2.7}$$

Newton and Quasi-Newton Methods

The Newton method is the simplest gradient-based iterative algorithm based on the linear approximation of the function using first-order *Taylor series* expansion. It can be used to solve unconstrained optimization problems with convex functions where the solution is iteratively updated based on the initial point of variables ($\mathbf{x}_0$).[19]

Algorithm : Newton Method

Assumption: Function is differentiable and Hessian can be defined

Input – Objective function $f(\mathbf{x},\mathbf{p})$, inital variable value $(\mathbf{x}_0)$

Output – Optimum $f^*(\mathbf{x},\mathbf{p})$ and optimizer $(\mathbf{x}^*)$

Initiate

0 : Assume inital feasible point k at $(\mathbf{x}_k = \mathbf{x}_0)$

Repeat

1 : Define and calculate Gradient $\nabla f(\mathbf{x}_k)^T$ and Hessian $\mathbf{H}(\mathbf{x}_k)$

2 : Calculate new variable value: $\mathbf{x}_{k+1} = \mathbf{x}_k - [\mathbf{H}(\mathbf{x}_k)]^{-1}\nabla f(\mathbf{x}_k)$

Until $\rightarrow$ Convergence critieria reached

For many practical problems, the calculation of the second derivative to determine the Hessian is limited. The quasi-Newton method provides an updated version of the Newton method wherein the calculation of the Hessian is iteratively estimated and updated. The estimation of the Hessian can be performed using the *DFP* formula (Davidon-Fletcher-Powell) or the *BFGS* formula (Broyden-Fletcher-Goldfarb-Shanno). [19]

Algorithm : Quasi-Newton Method

Assumption: Function is differentiable and Hessian can be estimated

Input – Objective function $f(\mathbf{x},\mathbf{p})$, inital variable value $(\mathbf{x}_0)$, and inital Hessian value $\mathbf{H}_0(\mathbf{x})$

Output – Optimum $f^*(\mathbf{x},\mathbf{p})$ and optimizer $(\mathbf{x}^*)$

Initiate

0 : Assume inital feasible point k at $(\mathbf{x}_k = \mathbf{x}_0)$ and Hessian $\mathbf{H}_k(\mathbf{x}_k) = \mathbf{H}_0(\mathbf{x}_k)$

Repeat

1 : Define and calculate Gradient $\nabla f(\mathbf{x}_k)^T$

2 : Calculate new variable value: $\mathbf{x}_{k+1} = \mathbf{x}_k - [\mathbf{H}(\mathbf{x}_k)]^{-1}\nabla f(\mathbf{x}_k)$

3 : Compute new $\hat{\mathbf{H}}(\mathbf{x}_\mathbf{k})$ and update $\mathbf{H}_{k+1} = \mathbf{H}_k + \hat{\mathbf{H}}(\mathbf{x}_\mathbf{k})$

Until $\rightarrow$ Convergence critieria reached

Sequential Quadratic Programming

Sequential quadratic programming (SQP) is suitable for solving constrained optimization problems and is a popular non-linear programming (NLP) algorithm. To understand SQP, two important concepts need to be explained: Lagrange multiplier and Karush-Kuhn-Tucker (KKT) conditions. For a constrained optimization problem as shown in Equation 2.1, a Lagrange function can be defined, as in Equation 2.8, where λ and μ are Lagrange multipliers associated with equality and inequality constraints. The KKT condition for finding the local minimizer for the stated problem in Equation 2.8 is shown in Table 3. The solved value of Lagrange multipliers indicates if the constraint is active or inactive in each problem [19].

$$\min_{\mathbf{x},\boldsymbol{\lambda},\boldsymbol{\mu}} L(\mathbf{x},\boldsymbol{\lambda},\boldsymbol{\mu},\mathbf{p}) = f(\mathbf{x},\mathbf{p}) + \boldsymbol{\lambda}^T \mathbf{h}(\mathbf{x},\mathbf{p}) + \boldsymbol{\mu}^T \mathbf{g}(\mathbf{x},\mathbf{p}) \tag{2.8}$$

Table 3. Overview of the optimality condition for the application of gradient-based algorithms for constrained optimization problem formulation [19].

	Karush-Kuhn-Tucker (KKT) Conditions
First-Order Necessary Condition	For $\min_{\mathbf{x},\boldsymbol{\lambda},\boldsymbol{\mu}} L(\mathbf{x},\boldsymbol{\lambda},\boldsymbol{\mu},\mathbf{p}) = f(\mathbf{x},\mathbf{p}) + \boldsymbol{\lambda}^T \mathbf{h}(\mathbf{x},\mathbf{p}) + \boldsymbol{\mu}^T \mathbf{g}(\mathbf{x},\mathbf{p})$ If $\mathbf{h}(\mathbf{x}^*,\mathbf{p}) = 0, \mathbf{g}(\mathbf{x}^*,\mathbf{p}) \le 0,$ $\nabla f(\mathbf{x}^*,\mathbf{p}) + \boldsymbol{\lambda}^T \nabla \mathbf{h}(\mathbf{x}^*,\mathbf{p}) + \boldsymbol{\mu}^T \nabla \mathbf{g}(\mathbf{x}^*,\mathbf{p}) = 0^T$, where $\boldsymbol{\lambda} \ne 0, \boldsymbol{\mu} \ge 0, \boldsymbol{\mu}^T \mathbf{g} = 0$ then $\mathbf{x}^* \to KKT$ point
Second-Order Sufficient Condition	For $\min_{\mathbf{x},\boldsymbol{\lambda},\boldsymbol{\mu}} L(\mathbf{x},\boldsymbol{\lambda},\boldsymbol{\mu},\mathbf{p}) = f(\mathbf{x},\mathbf{p}) + \boldsymbol{\lambda}^T \mathbf{h}(\mathbf{x},\mathbf{p}) + \boldsymbol{\mu}^T \mathbf{g}(\mathbf{x},\mathbf{p})$ $\mathbf{x}^* \to KKT$ point If $\mathbf{H}(L(\mathbf{x}^*,\boldsymbol{\lambda},\boldsymbol{\mu},\mathbf{p}))$ is positve definite $\partial \mathrm{x}^T \mathbf{H}(L(\mathbf{x}^*,\boldsymbol{\lambda},\boldsymbol{\mu},\mathbf{p}))\partial \mathrm{x} > 0 \ \forall \ \partial \mathrm{x} \ne 0$ then $\mathbf{x}^* \to$ Local minima

Algorithm : Sequential Quadratic Programming

Input – Objective function $f(\mathbf{x},\mathbf{p})$, equality constraint $\mathbf{h}(\mathbf{x},\mathbf{p})$ and inequality constraint $\mathbf{g}(\mathbf{x},\mathbf{p})$, inital variable value $(\mathbf{x}_0)$, and inital Langrange multiplier $(\boldsymbol{\lambda}_0,\boldsymbol{\mu}_0)$

Output – Optimum $f^*(\mathbf{x},\mathbf{p})$ and optimizer $(\mathbf{x}^*)$

Initiate

0 : Assume inital feasible point at $(\mathbf{x}_k = \mathbf{x}_0)$ and Langrange multiplier $(\boldsymbol{\lambda}_k = \boldsymbol{\lambda}_0, \boldsymbol{\mu}_k = \boldsymbol{\mu}_0)$

Repeat

1: *S*olve quadratic sub-problem to determine search direction $(\mathbf{s}_k)$, $\boldsymbol{\lambda}_{k+1}$, $\boldsymbol{\mu}_{k+1}$

$$\min_{\mathbf{s}_k} q(\mathbf{s}_k) = f(\mathbf{x}_k,\mathbf{p}) + \nabla f(\mathbf{x}_k,\mathbf{p})^T \mathbf{s}_k + \frac{1}{2}\mathbf{s}_k^T \nabla_{xx}^2 L(\mathbf{x}_k,\boldsymbol{\lambda}_k,\boldsymbol{\mu}_k,\mathbf{p})\mathbf{s}_k,$$

where $L(\mathbf{x}_k,\boldsymbol{\lambda}_k,\boldsymbol{\mu}_k,\mathbf{p}) = f(\mathbf{x}_k,\mathbf{p}) + \boldsymbol{\lambda}^T \mathbf{h}(\mathbf{x}_k,\mathbf{p}) + \boldsymbol{\mu}^T \mathbf{g}(\mathbf{x}_k,\mathbf{p})$

Subject to

$\mathbf{g}(\mathbf{x}_k,\mathbf{p}) + \nabla \mathbf{g}(\mathbf{x}_k,\mathbf{p})^T \mathbf{s}_k \le 0$

$\mathbf{h}(\mathbf{x}_k,\mathbf{p}) + \nabla \mathbf{h}(\mathbf{x}_k,\mathbf{p})^T \mathbf{s}_k = 0$

2 : Update new $\mathbf{x}_{k+1} = \mathbf{x}_k + \mathbf{s}_k$

Until $\to$ Convergence critieria reached

Interior-Point Method

The interior-point method is suitable for constrained optimization problems with a large number of constraints and the algorithm moves inside the feasible region to reach the optimal solution [31]. The problem is initiated by adding a Barrier function to the formulation. A Barrier function is typically a logarithmic function added to the inequality constraint to penalize the objective function [19]. The estimates of the Lagrange multipliers in the formulation show the activity of the constraints.

Algorithm : Interior-Point Method using Barrier Function

Input – Objective function $f(\mathbf{x},\mathbf{p})$, inequality constraint $\mathbf{g}(\mathbf{x},\mathbf{p})$, inital variable value $(\mathbf{x}_0)$

Output – Optimum $f^*(\mathbf{x},\mathbf{p})$ and optimizer $(\mathbf{x}^*)$

Initiate

0 : Penalizing objective function : $T(\mathbf{x},r) = f(\mathbf{x}) - r\sum_{j=1}^{m}\ln(-g(\mathbf{x}))$

Assume inital feasible point at $(\mathbf{x}_k = \mathbf{x}_0)$

Repeat

1 : Find interior point $\mathbf{x}_k$ and

select monotonically decreasing sequence: $\{r_k\} \to 0$ when $k \to \infty$, Set $k = 0$

2: Solve $\min T(\mathbf{x},r_k) = f(\mathbf{x}) - r\sum_{j=1}^{m}\ln(-g(\mathbf{x}))$ using unconstrained method with $\mathbf{x}_k$ as starting point

3 : Update new $\mathbf{x}_{k+1} = \mathbf{x}^*(r_k)$

Until $\to$ Convergence critieria reached

2.3.5.2 Non-Gradient-Based Algorithm

Non-gradient-based algorithms are a class of algorithms that overcomes the limitations of gradient-based algorithms, such as the need for continuous and differentiable function and handling of discrete variables. Examples of non-gradient-based algorithms include genetic algorithms (GAs), particle swarm optimization (PSO), ant colony optimization (ACO), etc. These algorithms are also termed heuristic optimization methods as there are no assumptions made on the mathematical form of the function, they require expensive computation, and they have no convergence proofs. [19, 31]

Genetic Algorithm

Genetic algorithms (GAs) were developed based on inspiration from the natural evolution process and are suitable for an objective function that is non-linear and stochastic in behaviour [35]. A classic GA is based on a series of steps as shown in Figure 4 [19]. The algorithm is initiated with an initial set of design variable points for the given design space and is encoded as a chromosome, which is defined as the initial population. Fitness values are calculated which represent the objective function values at the given population. Based on the evaluation of the fitness values, a certain set of design variable points are selected as a parent. The high-performing part of the population represented by fitness values are given a higher chance to reproduce and the best-performing design points are directly transferred to the next-generation population as elite individuals. Crossover is defined as a possibility to define a new set of design variables through a combination of design points selected in the parent set, and mutation is defined as the alternation that generates slightly new design points. The crossover and mutation steps create a new population of design points for the next-generation population. The process is repeated until the termination criteria is reached [19].

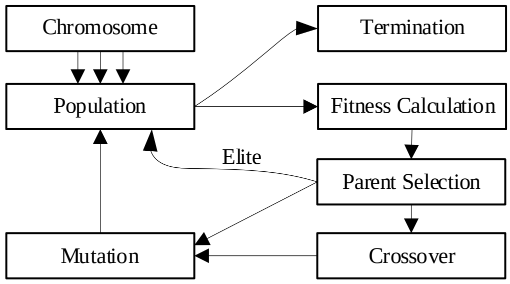


Figure 4. Generic flow description of genetic algorithm [19].

2.3.6 Optimization Methods

An optimization method is a process of implementing multiple optimization problems present in a complex system. To solve the optimization problems, two optimization methods have been applied in this thesis, namely, multi-disciplinary optimization (MDO) architecture and multi-objective optimization (MOO). The optimization methods follow a set of notations for defining the optimization problem in the form of MDO architecture and MOO, which is shown in Table 4 [36].

Table 4. Notation for defining an optimization problem.

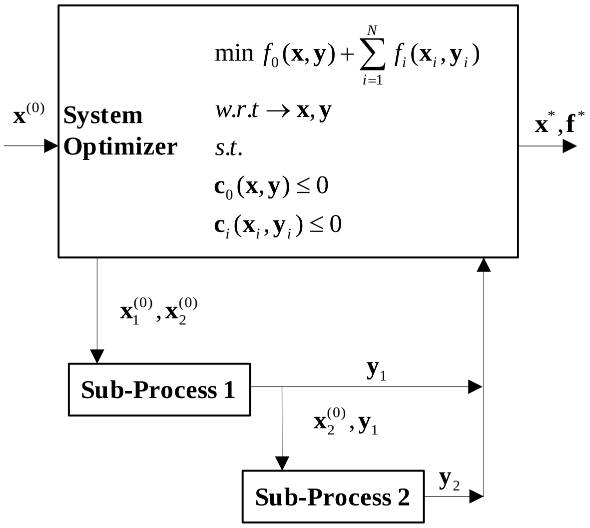
Symbol	Definition
$\mathbf{x}$	Vector of design variables
$\mathbf{y}$	Vector of coupling variables or output from sub-process analysis
f	Objective function
$\mathbf{c}$	Vector of design constraints
$\mathbf{c}^c$	Vector of consistency constraints
N	Number of sub-processes
$(\ )_0$	Functions or variables shared between more than one sub-process
$(\ )_i$	Functions or variables applied only to a sub-process i
$(\bar{\ })$	Independent copies of variables distributed to other sub-process
$(\ )^0$	Functions or variables at their initial values
$(\ )^*$	Functions or variables at their optimal values

2.3.6.1 Multi-Disciplinary Optimization (MDO) Architecture

The MDO architecture is a representation of organizing, coordinating and solving a set of optimization problems defined for a cross-disciplinary problem [36]. Two simple MDO architectures have been used in this study: multi-discipline feasible (MDF) and individual-discipline feasible (IDF). The optimization problem formulation and the algorithms of the two architectures are shown in Figures 5 and 6, which are based on the work of Martins and Lambe [36].

Multi-Discipline Feasible (MDF)

The MDF architecture presented in Figure 5 is monolithic in nature as it contains a single level of the optimization problem. The optimization problem is solved by sequentially evaluating each sub-process involved in the system. The objective function consists of two sets of functions, i.e., function (f_o), which is shared between the sub-processes, and function (f_i), which represents individual sub-process i. Similarly, the problem definition contains two sets of constraints, i.e., constraint ($\mathbf{c}_o$), which is shared between the sub-processes, and constraint ($\mathbf{c}_i$), which represents individual sub-process i.



Algorithm:

Input- Design variables $\mathbf{x}$

Output- Coupling variables $\mathbf{y}^*$, optimized variables $\mathbf{x}^*$, objective functions $\mathbf{f}^*$

0: Initiate MDF loop

Repeat

1: Evaluate sub-process 1 and update $\mathbf{y}_1$

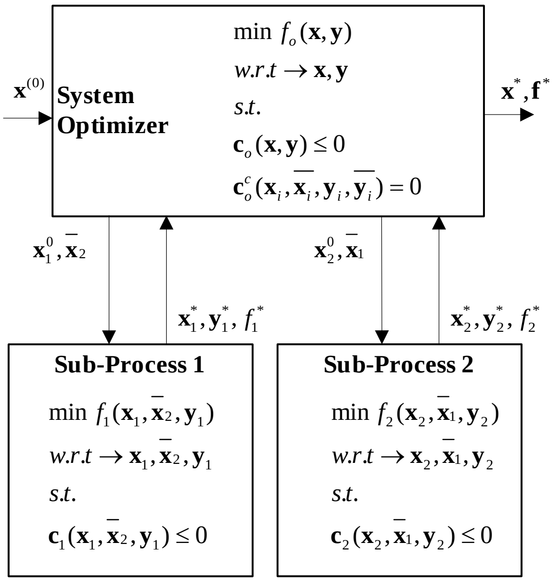
2: Evaluate sub-process 2 and update $\mathbf{y}_2$

Until →MDF converged

Figure 5. Optimization problem formulation and algorithm for the monolithic MDF architecture.

<u>Individual-Discipline Feasible (IDF)</u>

The IDF architecture, presented in Figure 6, is a distributed architecture that contains two levels of optimization problems. The system optimization problem is iteratively solved by concurrently solving the individual sub-process optimization problem in parallel. The system optimization problem contains an objective function (f_o) and constraint ($\mathbf{c}_o$), which are shared between sub-processes, and an additional consistency constraint ($\mathbf{c}^c$) is introduced to maintain the consistencies of the design variables. Each sub-process optimization problem consists of the objective function (f_i) and constraint ($\mathbf{c}_i$) belonging to sub-process i. The individual sub-process optimization receives independent copies of the design variables ($\bar{\mathbf{x}}$) belonging to the other sub-process through the system optimizer. The sub-process optimizer delivers a local optimal value for the design variable ($\mathbf{x}_i^*$) and function value (f_i^*) to the system optimizer.



Algorithm:

Input- Design variable $\mathbf{x}$

Output- Optimized variable $\mathbf{x}^*$, objective function $\mathbf{f}^*$

0: Initiate system optimizer iteration

Repeat

1: Compute sub-process objective and constraints

For each sub-process i,

1.0 Initiate sub-process optimization

Repeat

1.1 Evaluate sub-process i

1.2 Compute sub-process i objective and constraints

1.3 Compute new design point for sub-process (i+1)

Until 1.3 → Optimization i has converged

End for

2. Compute new system design points

Until 2 → System optimization has converged

Figure 6. Optimization problem formulation and algorithm for the distributed IDF architecture.

2.3.6.2 Multi-Objective Optimization (MOO)

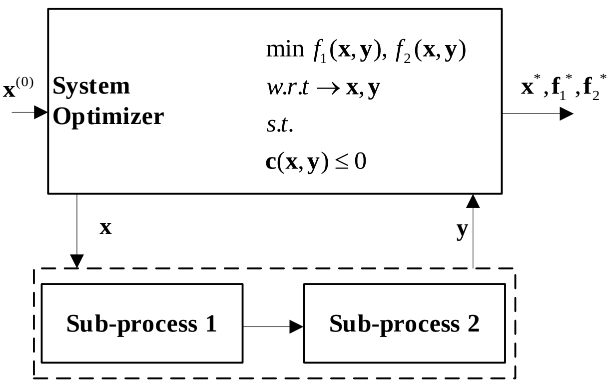
The MOO method represents a synchronized optimization of multiple objective functions involved in each problem. The central concept for using MOO is to generate trade-off curves (Pareto optimality) between various objective functions. The MOO problem can be solved using various approaches, such as the weighted-sum approach or constraint-based approach, and use of a heuristic algorithm, for example, a genetic algorithm [37]. A generic example for *k* objective function optimization problem formulation for a weighted-sum approach as compared with a constrained-based approach is given in Table 5. In the weighted-sum approach, the weight factor is changed to generate the Pareto front, while in the constraint-based approach, the problem is converted to single-objective optimization and other objective functions are parametrically varied with a target value of ε [31, 37].

Table 5. Comparison of weighted-sum approach and constrained-based approach for Pareto optimality.

Weighted-Sum Approach	**Constrained-Based Approach**
$\min_{\mathbf{x}} \sum_{k=1}^{K} w_k f_k(\mathbf{x},\mathbf{p})$ subject to $\mathbf{h}(\mathbf{x},\mathbf{p}) = 0,\ \mathbf{g}(\mathbf{x},\mathbf{p}) \le 0,$ $\sum_{k=1}^{K} w_k = 1$ $\mathbf{x} \in \chi \subseteq \Re^n.$	$\min_{\mathbf{x}} f_1(\mathbf{x},\mathbf{p})$ subject to $\mathbf{h}(\mathbf{x},\mathbf{p}) = 0,\ \mathbf{g}(\mathbf{x},\mathbf{p}) \le 0,$ $f_k(\mathbf{x},\mathbf{p}) - \varepsilon_k \le 0$ $\mathbf{x} \in \chi \subseteq \Re^n.$

<u>*MOO Using Genetic Algorithm*</u>

A general form for defining a MOO problem using a GA is shown in Figure 7 [35, 38]. The system optimizer parses the design variables (**x**) to the process simulation. The simulation returns the output variable (**y**) to the system optimizer, and this process is repeated until the convergence criteria are achieved. The optimization problem contains multiple objective functions (f_1, f_2), also referred to as fitness functions, and a set of constraints (**c**). The choice of objective functions and problem formulations is critical in generating the relevant results using this approach.



Algorithm:
Input- Design variable **x**
Output- Pareto front for multiple-objective functions ($\mathbf{f}_1^*$, $\mathbf{f}_2^*$) and optimized variable set ($\mathbf{x}^*$)
0: Initiate population
Repeat
    Repeat
    0.1: Crossover
    0.2: Mutation
    0.3: Fitness computation
    Until → Population complete
1: Selection of parental population
Until → Termination condition

Figure 7. Optimization problem formulation and algorithm for the MOO problem using genetic algorithm.

3 LITERATURE REVIEW

This chapter aims to:

> *Introduce process and equipment modelling for crushing plants.*
>
> *Describe recent research in process simulation, production data and optimization for crushing plants.*
>
> *Identify needs and gaps within the application of optimization methods for crushing plants.*

The essential function of a crushing plant in aggregates production is the breakage of rock (coarse comminution process) and separation of its fragments (classification process) based on size and shape [39]. A crushing plant in minerals processing has the same functionality and is integrated with fine comminution and classification processes such as milling, high-pressure grinding roller, hydrocyclone separation, and so on [2, 39]. Extensive research has been conducted over the past 50 years, particularly in the field of numerical simulation, to investigate and find ways to develop and operate crushing plants in both the minerals processing and aggregates processing industries [9, 39]. These simulation methods and models were developed to address the industrial needs to understand, diagnose, and capture performance aspects of equipment and processes [9, 39], and more importantly, the needs of users such as engineers and developers for their utilization [40]. The journey of fundamental understanding comminution (theories of comminution) [41-43], classification [44, 45], process modelling and experimental studies [9, 10, 39] has had a tangible impact on the industry, with multiple commercial software applications in use and considerable progress in the research community [46]. The drive towards increasing the industrial utility of different developed models and methods still exists, with requirements from the industry for controlling energy and product yield. The simulation methods can simulate scenarios for achieving the reduction of energy usage and control of the process yield, but a gap remains between simulations and actual utilization of the simulation tools for daily operation to solve industrial needs. This further adds new challenges in utilizing and integrating different technological advancements to address the industrial needs.

3.1 PROCESS SIMULATION OF CRUSHING PLANTS

Numerous studies have research has been conducted over the past decades on numerical simulation of crushing plants [9, 40, 45, 47]. The process simulation for crushing plants is classified as either steady-state or dynamic simulation. The steady-state process simulation is based on the instantaneous mass balance of the process in all nodes, see Equation 3.1, where $\dot{m}$ is mass flow rate, and *i* and *j* are the number of material streams at input and output of a node [9]. In a circuit application, the mass balancing is applied to multiple nodes and iterated until a convergence criterion is achieved and the output contains one set of mass flow rates at different nodes of the circuit.

$$\sum_i \dot{m}_{i,in} = \sum_j \dot{m}_{j,out} \tag{3.1}$$

E.g. One input and two outputs in a splitter: $\dot{m}_{1,in} = \dot{m}_{1,out} + \dot{m}_{2,out}$

The application of steady-state process simulations for plant design, optimization and comparison of different circuit configurations has been shown [48-50]. King [51] presented an extensive and systematic demonstration of the use of a steady-state simulation tool to improve the production of fine materials in a comminution circuit of a uranium plant. Asbjörnsson [10] demonstrated that the steady-state process simulations for crushing plants are limited to predicting different operational scenarios such as changes in the process over time and non-ideal operating conditions.

The initial work for the dynamic process modelling for crushing plants was carried out by Whiten [52], who introduced the idea of transition from the steady-state to the dynamic-state model to include the effect of material delays during physical processing. Liu and Spencer [53] showed the application of the PID (proportional-integral-derivative) controller in the grinding circuit, and Sbárbaro and del Villar [54] demonstrated the application of a model-based control system to further propel the development of dynamic simulation in crushing plants. Asbjörnsson [10] presented the capability of the dynamic simulation to capture discrete and gradual changes happening in the crushing plant due to delays, start-ups, discrete events, wear, etc. In the dynamic process simulation developed by Asbjörnsson [10], each equipment model includes the derivative for mass m and properties γ of the material to time as given in Equations 3.2 and 3.3, respectively, where n represents different material characterization properties, e.g., product size distribution, material strength, density, shape, etc. The blending of the material is represented by a perfect mix model.

$$\frac{dm(t)}{dt} = (\dot{m}_{i,in}(t) - \dot{m}_{j,out}(t)) \tag{3.2}$$

$$\frac{d\boldsymbol{\gamma}(t)}{dt} = \frac{\dot{m}_{i,in}(t)}{m(t)}(\boldsymbol{\gamma}_{i,in}(t) - \boldsymbol{\gamma}(t)), \text{ where } \boldsymbol{\gamma}(t) = \begin{bmatrix} \gamma_1(t) \\ \vdots \\ \gamma_n(t) \end{bmatrix} \tag{3.3}$$

The dynamic process simulation has been applied to study the different implementations of the control system, such as model predictive control [55], robustness study [56], production improvement [20] and operator industrial training [57]. Due to the advantage of traceability of material in the circuit, dynamic simulation is useful for industrial applications such as debottlenecking in crushing circuits [10], equipment sizing [40], and disturbance analysis [10, 40, 56].

3.2 EQUIPMENT MODELLING IN CRUSHING PLANTS

Equipment modelling in a crushing plant is usually aimed at describing, explaining, and mathematically presenting relationships between elements of the rock breakage process (cone crusher, jaw crusher, etc.) and between elements of the rock separation process (vibratory screen, hydrocyclone, etc). Napier-Munn and Lynch [40] presented a classification of the modelling for crushers and screens based on three fundamental bases:

- Mechanistic models based on the physics of the equipment.
- Phenomenological models based on an intellectual construct describing the phenomenon.
- Empirical models based on mathematical convenience.

The population balance model, introduced by Epstein [58], is a commonly used phenomenological model to represent particle size reduction in comminution equipment [9], for example, cone crushers [45], high-pressure grinding rolls [59], grinding mills [60], etc. It is characterized as a probability-based model and is dependent on a large empirical dataset generated by testing different materials [9, 10]. Evertsson [17] presented a mechanistic model for cone crushers relying on the geometry of the crusher chamber and the sequence of operations of the crushing process within the equipment which can offer higher predictability of actual conditions [61]. The screening process, using equipment such as mechanical vibratory screens, can be modelled using efficiency curves based on a probabilistic function [44], an exponential sum expression [45], or a polynomial function [62]. Other models for vibratory screens include an analytical model [63, 64] which provides higher fidelity in the simulations. Equipment such as bins and stockpiles have been modelled using a perfect-mix principle, a first-in-first-out (FIFO) principle and a mechanistic-model principle [65]. Conveyors in the process usually act as a material delay unit and are modelled as a state-space model [66].

3.3 Operation, Control and Optimization of Crushing Plants

The operation of an industrial-scale crushing plant is complex and performance depends on multiple aspects ranging from raw material and individual equipment to the management of the operation. Asbjörnsson [10] presented a system-wide overview on the complexity of the development of process simulation for crushing plants and showcased several factors that can influence plant performance (see Figure 8). The process operation includes changes and variations due to both controllable factors such as the setting of machine parameters for single equipment and uncontrollable factors such as wear and segregation during operation [10, 67, 68].

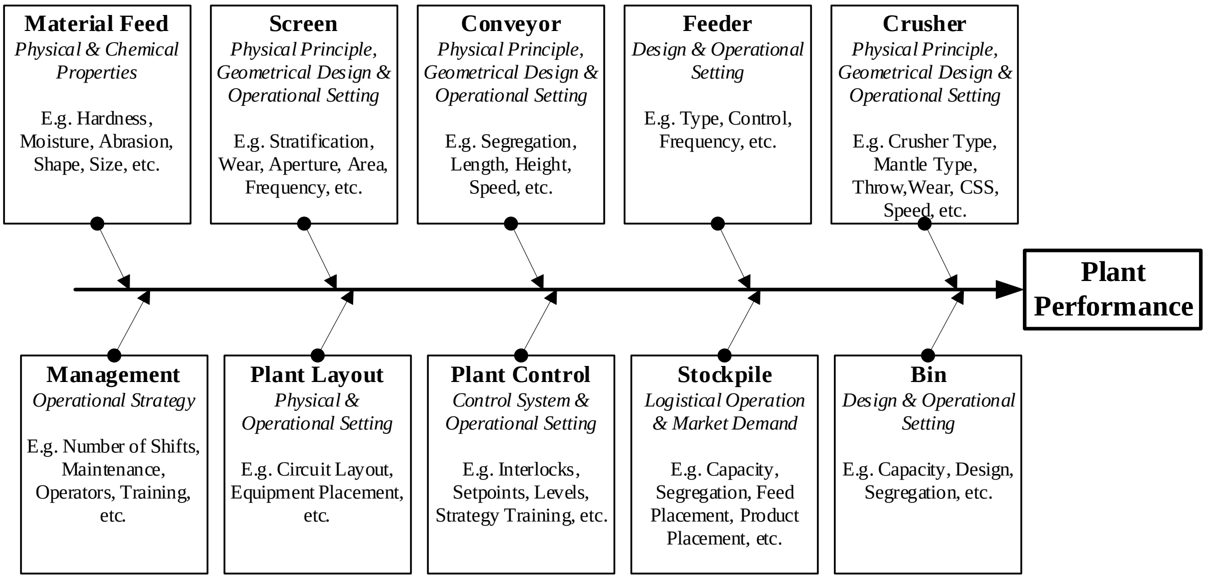


Figure 8. Factors influencing plant performance for a crushing plant [10].

The physical operation of a crushing plant is built of multiple layers in the control system application together with the manual operation to manage regular operations and the changes due to variability in material feed, wear in crushers and screens, etc. [10, 22, 69]. Equipment involved in a crushing plant layout needs to be coordinated and controlled during the operation to produce products. To manage differences, various types of control systems are in place to steadily operate the crushing plant. These include simple controls such as electrical switches (on/off), and interlocks to advance process control such as model predictive control [10, 22]. For individual equipment, the control systems are installed to maintain and regulate operation based on the equipment purpose and safety requirements, while for a process, the control systems can be installed to stabilize or regulate the process operation [10, 22, 70, 71]. The supervisory controller is usually applied to achieve process optimization based on the target for real-time plant performance [72], for example, model predictive control [55] and real-time optimization using a finite state machine (FSM) algorithm [73]. These advanced applications of the control systems are still limited in daily industrial practices and operation requires multiple manual interventions [46].

Given existing plant operations with limited utilization of control systems, improvements in crushing plants are still carried out through an iterative process and manual decisions by operators and plant managers. The manual decisions by personnel can potentially be supported by an optimization tool. Research within the area of minerals processing circuits has identified the use of numerous different objective functions for optimization, which was driven by the need for technical and economic measurements of the processes [74-76]. Such performance indicators are used to demonstrate the optimization applications by the range of research applications, for example, production and operation costs [77-79], profit and quality [80], material throughput rate [61, 73, 81], technical parameters [82], yield and energy [83], net present value [84], yield and cost [85], grade engineering [86, 87], crusher geometrical design [88, 89], etc. The scope of the optimization objective function has been varied depending on the application and there is no clear consensus on how these process objectives are classified and how are they useful for operational change decisions for operators or plant managers.

The methods used for optimization problem solutions are largely based on stochastic and heuristic optimization algorithms such as genetic algorithms and genetic evolutionary algorithms [79, 82-84, 87, 89, 90], while limited gradient-based algorithms are applied in minerals processing circuits. [75, 76]. The common reasoning for the choice of such algorithms is that the optimization problem is described as complex and non-linear. Other optimization algorithms used are mixed-integer linear programming (MILP) [91] and probabilistic global search Lausanne (PGSL) [78]. The factors contributing to the choice of method and algorithm are based on ease of application, computation cost and model behaviour (linear or non-linear). The selection of the method and algorithm can be questioned if the choice is due to a lack of understanding of the optimization problem or the objective functions used are complex or there is a vast amount of information to handle. A common denominator from the above-stated optimization research papers is lack of repeatability, as most optimization problems posed are not presented in a standardized form (e.g., negative null form) as generally represented in mathematical optimization. Additional perspectives from the operation management research are seen in studies of specific indicators that have been applied to minerals processing circuit improvements such as overall equipment effectiveness, availability, performance, and effectiveness [92-94]. The ISO 22400 standard states a set of key performance indicators for managing manufacturing operations [95-98], although it has not been utilized to its full potential for continuous production processes such as in crushing plants.

3.4 MODERN TRENDS IN INDUSTRY

Real-time measurement of production data is a common practice in many production industries, driving the need for digital solutions to capture, filter, structure, and store data, followed by performing analytics and knowledge generation [16, 99]. According to Kusiak [16], a smart manufacturing system is defined by autonomy, evolution, simulation and optimization of production, which is reflected by the degree of the physical process being captured in cyberspace. Grieves and Vickers [100] defined a digital twin as a virtual system that can assist engineers in the design, testing, manufacturing, and use of a product for the discrete production industry. Modern manufacturing industries are developing solutions towards integrated decision-making capabilities where the application of digital twins can be found for product and production development of discrete production [101-103], yet the application of these findings to the minerals processing industry (characterized by continuous production) remains limited in application [104]. Recent trends in minerals processing industry development are automation and control [71, 105, 106], machine learning [107-109], and big data management [99], and these are propelling a drive towards new technological developments. An important aspect that is gaining traction is sustainability indicators [110], such as requirements for environmental product declarations in the aggregates industry [111, 112].

Complex and large optimization problems exist in the development of other mechanical products in, for example, the aerospace and automotive industries, especially concerning the interaction of two or more disciplines such as structures and aerodynamics [19, 36, 113, 114]. To address and manage these complex interactions and their optimization, multi-disciplinary optimization (MDO) architectures have been used to attain global optimization and exploit the power of parallel computing [115]. Martins and Lambe [36] reviewed multiple MDO architectures applied in various engineering applications and showed that MDO methods are suitable to handle large complex optimization problems in engineering systems. The MDO architecture is a representation of how various sub-disciplines involved in an engineering system optimization problem are organized and how their strategies are set up to achieve optimal values for design variables of the engineering system [19, 36]. Similarly, complex dependencies exist between various processing units of crushing plants which require new ways of understanding and computing optimization. There are opportunities to use MDO for minerals processing simulations and integration of multiple systems to create useful decision-making tools.

4 RESULTS

This chapter aims to:

> *Present a system-wide overview of optimization capabilities.*

> *Describe the requirements in the development of the optimization capabilities for crushing plants.*

> *Demonstrate the application of performance indicators.*

> *Demonstrate the application of optimization methods for process optimization, model calibration and data reliability.*

The iterative development in this thesis resulted in the exploration and application of multiple optimization approaches which are suitable for crushing plant optimization. A multi-layered modular framework for the development of the optimization capabilities in a crushing plant together with the individual studies performed at various abstraction levels is presented in this chapter [Papers A-F].

4.1 MODULAR FRAMEWORK FOR OPTIMIZATION CAPABILITIES DEVELOPMENT

A multi-layered modular framework for performing optimization functions for industrial use is presented in Figure 9. The framework consists of two systems – a physical system and a simulation system – both with four sub-levels interacting parallelly. The interaction between the physical system and simulation system (digital twin) describes different abstraction levels and use in the equipment and process performance mapping.

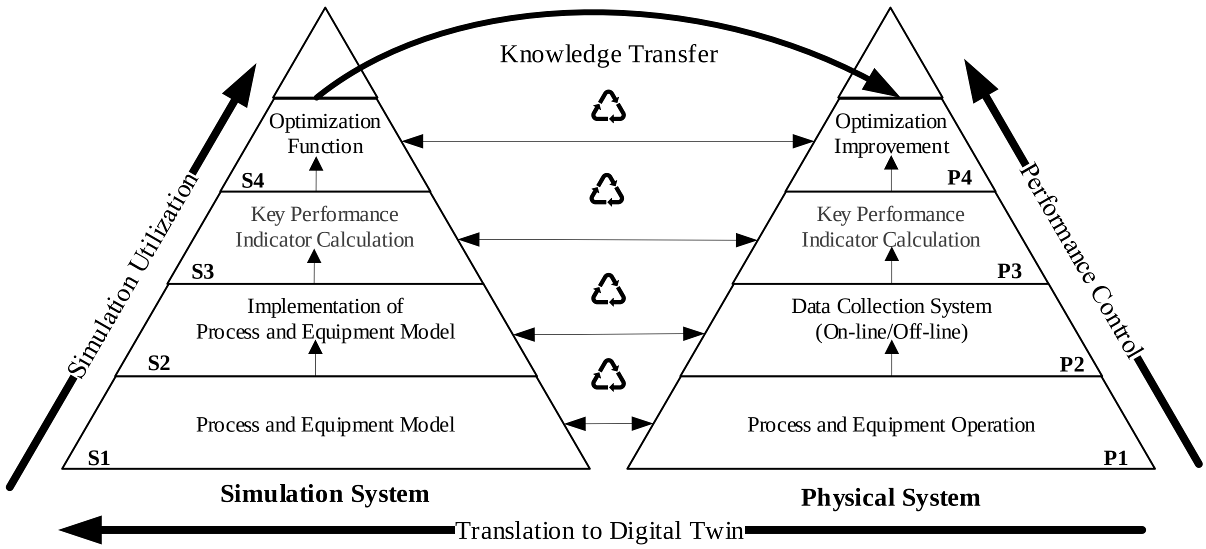


Figure 9. A modular framework for the implementation of optimization capabilities for crushing plants.

The optimization function is hierarchically built over the digital twin of the crushing plant pertaining to the process operation. The desired optimization of the physical system is achieved by translating, modelling, and simulating the physical process, which is then followed by the optimization routine. The results from the simulation system are transferred to the physical system for implementation. The continuous integration of the data into simulation for performance improvement and optimization is a necessary condition for creating a powerful decision-making tool.

4.1.1 Physical System

The physical system in this framework represents the physical process operation of a crushing plant. The base level (P1: Process and Equipment Operation) represents multiple interconnected primary pieces of equipment (crusher, screens, conveyors, bins, etc.) and supporting auxiliary equipment (wheel loader, pumps, etc) in an operation. It is assumed that the process performance is mainly affected by the control of the primary set of equipment, although there are effects from the auxiliary equipment. Individual equipment operation is based on their physical and mechanical principles. Process operation is dynamic and includes the effects of the interconnected equipment, control systems, and material flow.

The second level (P2: Data Collection System) consists of multiple data-acquisition methods for capturing both offline and online production data. The online data represent continuous data captured by various sensors present in the process operation, such as mass flow, power draw, pressure, speed, temperature and so on, belonging to either individual equipment or the process. The data also include measurement or estimation of machinery settings (soft sensor) of individual equipment, for example, cone crusher main shaft position (closed-side setting estimate), material level in the hopper, etc. The offline data represents the data that is determined discretely in the laboratory by collecting samples of material from the process operation such as material strength, product size distribution, flakiness index, etc. The offline data also includes equipment settings that are measured or noted based on the physical operation of the process, for example, crusher’s eccentric throw, screen aperture, various geometric dimensions of the equipment, etc.

The third level (P3: Key Performance Indicator (KPI) Calculation) includes the calculation of process production performance using the data collected during plant operation. This indicator presents how well the operation is carried out within individual equipment or the process. The KPIs can be analysed continuously as trends (seconds, minutes, hours) or discrete time steps (days, weeks, months, years) depending on the application need. The KPIs are useful for implementing incremental improvements in production by performing root-cause analysis or for benchmarking similar crushing processes for continuous improvements and capturing the best practices of operation.

The fourth level (P4: Optimization and Improvement) in a physical system context means control of the production performance towards the desired needs. The control of production with respect to aggregates production entails controlling the production rate, production volume, energy impact, product quality, etc. of different aggregates products based on the market or industrial needs. The iterative learning from the previous step and practices can bring about improvements in production performance. A simulation system can assist in making decisions aimed at distinctly steering production performance towards the desired target. The improvements and changes are indicated by the change in KPIs, which are calculated based on the data collected from the physical plant. The utilization of the four steps in the physical system can lead to performance control of the crushing plant.

4.1.2 Simulation System

The simulation system represents a parallel digital replica of the physical crushing plant process. The purpose of the simulation system is to understand, predict, explore, and modify the physical system. The base level (S1: Process and Equipment Model) consists of mathematical modelling of physical equipment and processes. The individual equipment model is fundamentally developed based on mechanistic, phenomenological, probabilistic, or data-driven principles. The fidelity of the equipment model is dependent on multiple criteria such as the underlying assumptions, degree of data supplied, experimental procedures, validity and so on. The equipment models are combined to produce a process simulation for a crushing plant. The process simulation is broadly classified into two categories: steady-state simulation and dynamic simulation. The advantage of the dynamic simulation is that it provides a closer replication of the actual physical operation than the steady-state simulation models, although the optimization methods can be run on both types of process simulation models. However, dynamic simulation requires more computational power and knowledge than steady-state simulation for its configuration and operation.

The second level (S2: Implementation of Process and Equipment Model) represents the procedures involved in configuration, calibration, validation and verification of the process and equipment model. The configuration is defined as setting up the simulation model based on the physical crushing plant layout and settings. The calibration refers to comparing and modifying the selected process and equipment model output to controlled experimental data (online or offline). This is achieved by performing an experimental survey and a model error-minimization process using a suitable optimization method. The validation step refers to proving that the simulation output is in line with the physical output. The numerical value of the errors during the calibration and validation steps represents the accuracy of the simulation. The verification step is a process of re-checking the practical implementation of the procedure and numerical model used for the simulation. The different data required in this step is retrieved from the parallel physical system (P2).

The third level (S3: Key Performance Indicator (KPI) Calculation) is similarly based on the physical system KPI calculation. The simulation system KPIs can be used for process improvement by iteratively studying the plant under consideration. The advantage of the simulated KPIs is that a cost-efficient, trial-and-error new setting combination can be performed. It also provides opportunities for simulating KPIs for which there is limited sensor technology present for physical operation given that the output of interest is predicted by the underlying process and equipment model.

For the fourth level (S4: Optimization Function), KPIs are used as objective functions and constraint functions depending on the optimization problem definition. The optimization function consists of a set of different optimization methods which is used to either generate operational settings or explore the trade-off between KPIs of interest for given requirements. The results from the optimization function are translated back to the physical system to optimize the production operation for the given need of the stakeholder. In essence, the higher level of utilization of the simulation system can lead to increased performance control of the physical system.

4.2 DEVELOPMENT OF OPTIMIZATION FUNCTIONALITY FOR CRUSHING PLANTS

The optimization function is aimed at exploring non-intuitive solutions for a defined problem towards designing, operating, and controlling a crushing plant. Explorative studies were carried out to understand and implement optimization methods for crushing plant operational application [Papers A, B and C]. Figure 10 presents a summarized process of optimization function implementation for the crushing plant, which was developed from a top-down approach to the problem.

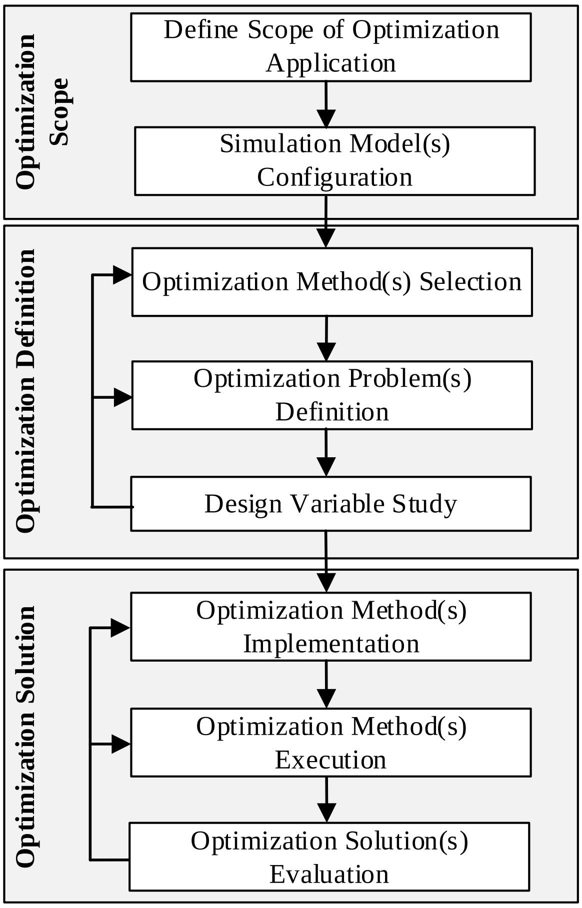


Figure 10. Overview of optimization function implementation in the simulation system.

4.2.1 Optimization Scope

The first task in starting the optimization procedure is to define the scope of the optimization application. Based on the literature review in Paper A, a general classification scheme is established to define the scope of the optimization (see Table 6) consisting of two dimensions: *State of Application Area Units* and *State of Development Stage*.

Table 6. Classification scheme to define the scope of optimization application [Paper A].

	State of Development Stage →		
State of Application Area Unit ↓	**Design**	**Operations**	**Control**
Equipment		Papers E, F	
Sub-Process		Papers B, E, F	
Main Process		Paper C	

The state of application area units characterizes the abstraction level based on the hierarchical position of physical entities in the crushing plant. This is categorized as *Equipment*, *Sub-Process* and *Main Process*. The equipment represents an individual physical unit (e.g., crusher, screen, etc.) in the processing operation, while the sub-process (e.g., primary, secondary, tertiary crushing, etc.) represents a collection of equipment performing a specific functionality for the main process of the physical crushing plant.

The state of development stage represents the purpose of the optimization and is divided into three categories: *Design Stage*, *Operation Stage,* and *Control Stage*. The *Design Stage* covers the optimization application towards developing and designing a completely new process or equipment. This can also include the re-configuration of an existing design concept for a process or equipment. The validity of the optimization results is dependent on the fidelity of mathematical models used for the process and equipment optimization, while the possibility for the validation of the optimization results is limited. The *Operation Stage* deals with the optimization application towards understanding (trade-offs between KPIs) and finding operational settings of an existing process or piece of equipment based on the user requirements. The validity of the optimization results is dependent on the type of mathematical models together with the calibration process applied. The optimization results can be implemented into real-time operations and the results can be validated by collecting and comparing them with operational data. The *Control Stage* includes the optimization application towards regulatory control and supervisory control of the process and equipment under real-time operation. The application is related to stabilizing or regulating an existing process or equipment towards maintaining its nominal performance. The usefulness of the optimization application can be observed from the real-time plant performance control. For Papers B and C, the application of the optimization function was carried out to find suitable operating parameters and trade-offs for a fixed crushing plant layout and are categorized under *Operation Stage*.

The process simulation model can be configured based on the scope and then used as an underlying mathematical model for running an optimization function. For both Papers B and C, a dynamic simulation model for crushing plants developed by Asbjörnsson [10] was used. In Paper B, the scope of the optimization application was to investigate the operating parameters for an existing layout of an aggregates production plant (see Figure 11 for circuit layout). The purpose was to find a balance between the goal of the individual sub-processes and the overall process goal. In Paper C, the purpose of the optimization application was to study the trade-off and balancing point between the two sub-processes of a conceptual operation of a crushing plant for minerals processing (see Figure 12 for circuit layout).

4.2.2 Optimization Definition

Based on the scope and the purpose of the optimization application, a multitude of optimization approaches can be applied. In Paper B, an explorative study was performed to investigate the application of two multi-disciplinary optimizations (MDO) architectures, namely, multi-discipline feasible (MDF) and individual-discipline feasible (IDF). In Paper C, an additional implementation of the multi-objective optimization (MOO) using a genetic algorithm (GA) was carried out to demonstrate the Pareto front for the multiple objectives present in the crushing plant.

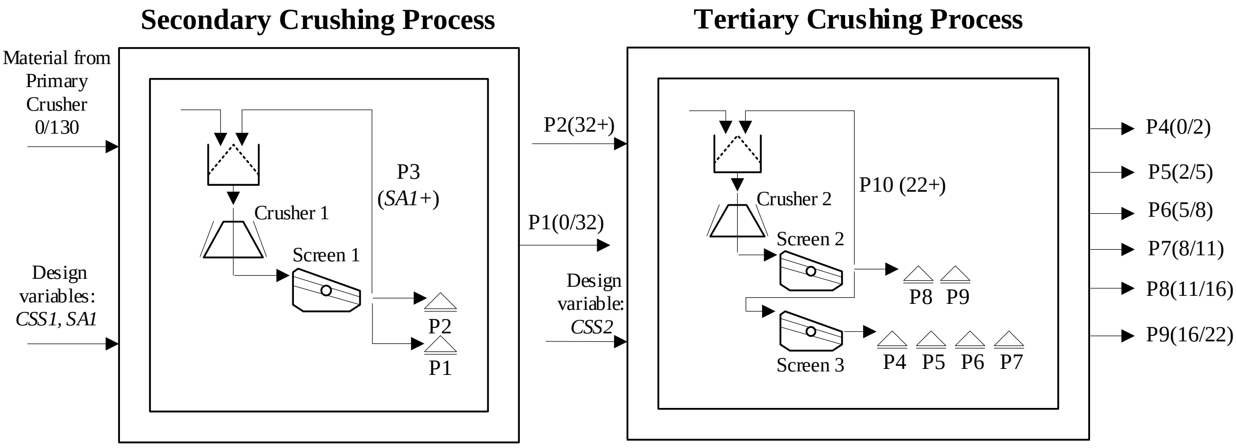


Figure 11. Crushing plant for aggregates production consisting of two sub-processes [Paper B].

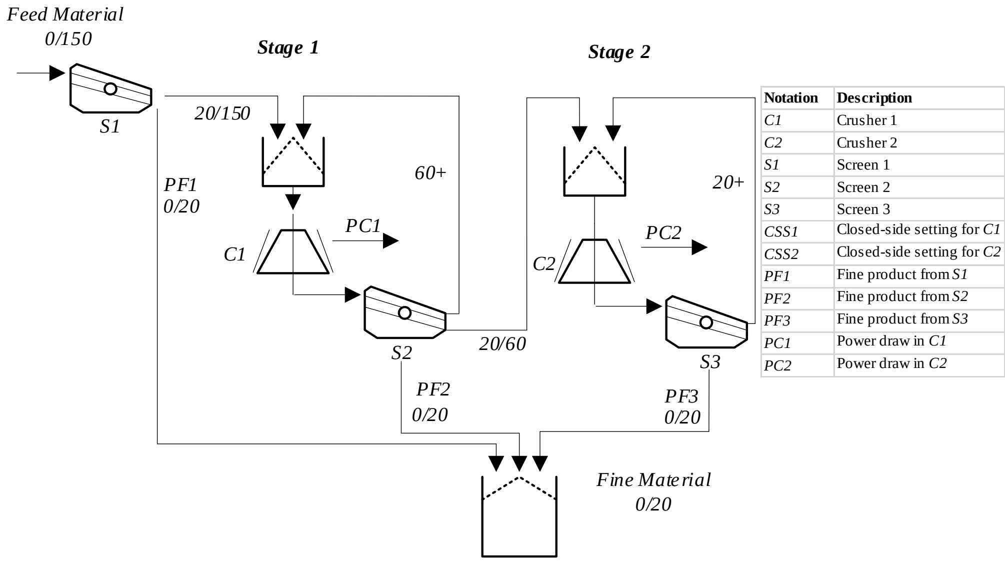


Figure 12. Crushing plant layout for a two-stage coarse comminution plant [Paper C].

The different optimization methods applied were based on exploring the functionality of each towards the creation of decision-making results. Typically, multi-domain optimization problems exist in a crushing plant with a set of conflicting objectives. The interactions and dependencies between sub-processes (e.g., primary crushing process, secondary crushing process, etc.) are a function of the characteristics of material streams in the process, which classifies the multi-domain optimization problem as loosely coupled and hierarchical with typical plant layout in most existing operations. For example, a change in the operational setting of a tertiary cone crusher can have a limited impact on the previous secondary crusher setting, unless there are recirculating material streams, and can have a higher impact on the subsequent screen performance. A change in *CSS* (closed-side setting) in the primary crushing process influences the characteristics of the material produced in this process, which in turn influences the performance of the subsequent secondary crushing process. In conclusion, the sub-processes are loosely coupled with input design variables of the other sub-processes (meaning that the effect of change of the design variable is limited to the specific piece of the equipment in that sub-process) but are strongly coupled with the material output of one sub-process to another [Papers B and C].

Given the understanding of the operational design variable in a crushing plant, standard optimization problem definition in negative null form is used to determine the goal (objective functions) and the requirements (constraint functions). The objective functions which have been applied for optimization are throughput rate [Papers B and C], sub-process value (SPV) [Paper B], and power consumption [Paper C]. The SPV function is a simple function representing the technical-economic performance based on approximate cost, selling price, and throughput rate of the aggregates product [Paper B].

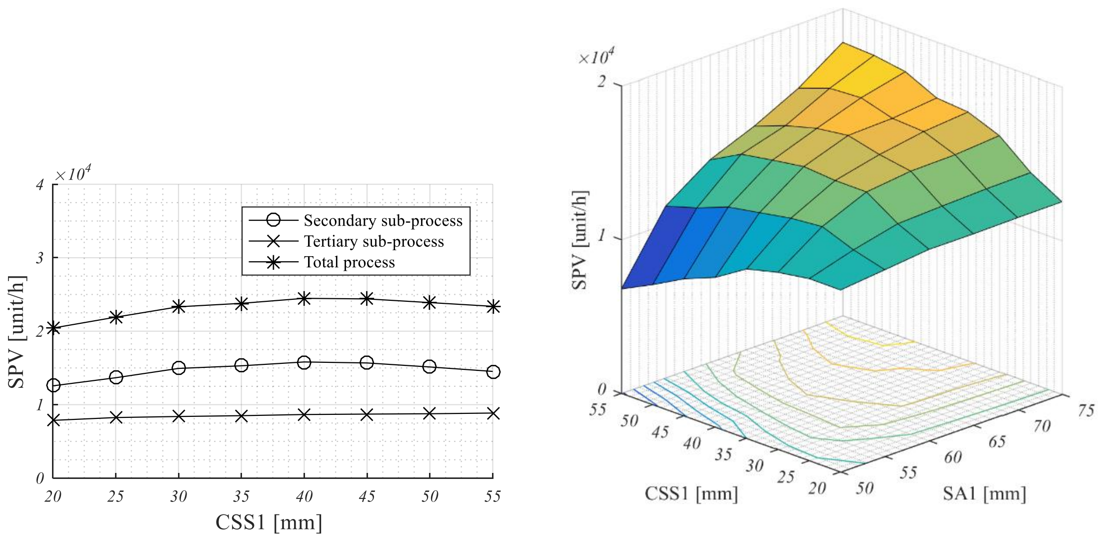
Table 7 presents examples of optimization problem formulations applied for a two-stage crushing plant in the negative-null form [Paper B]. In MDF formulation, the objective function is the weighted sum function to maximize the total throughput of the crushing plant and sum of the SPVs of the individual sub-process, where w is the weight factor, P_j is throughput for product j, V_j is the value added in the sub-process for the product j, and i represents the two sub-processes. The problem is constrained by maintaining the reduction ratio (F_{80}/P_{80}) of the crushers in the two sub-processes to a value of 4. The optimization problem highlights the design variables (operational settings of the crushers and screen), which can change. The design variables are *CSS1* and *CSS2*, which represent the closed-side setting of the crushers in the secondary and tertiary crushing processes, respectively, and *SA1* represents the topdeck screen aperture setting of the secondary crushing process. The optimization formulation also presents the upper (x_{ub}) and lower (x_{lb}) limits for each design variable. The IDF formulation is a bi-level optimization problem formulation, where the objective function of the system optimizer is to maximize the total throughput of the crushing plant while the objective function of the individual sub-process is to maximize the SPV of each crushing stage. The system optimizer maintains the consistency between the duplicate variables by introducing consistency constraints. The consistency constraint is introduced in the IDF system optimizer to minimize the norm between the design variables and the duplicate copy of the design variables. The details of the plant layout, functions and other descriptions can be found in Paper B.

The optimization problem formulation is dependent on the choice and purpose of the optimization method. The optimization problem is decoupled and is represented by a simple objective function in the case of the distributed IDF approach [Papers B and C]. The optimization problem formulation for monolithic MDF is rather comprehensive in comparison with the IDF problem formulation. The optimization problem formulation in a standard format is essential for replication of the results. The IDF formulation is particularly modular in design, which means that it is comparatively easier to add more sub-processes within a system optimizer than it is in the MDF formulation. Also, unlike the MDF formulation, the IDF formulation maintains the discreteness in the objective function and is independent of the effect of weight.

Table 7. Comparison of applied MDF and IDF optimization problem definitions for a two-stage crushing plant [Paper B].

<table>
<tr><td colspan="3">MDF - Single level optimization problem</td></tr>
<tr><td colspan="3">System Optimization
$\max \sum_{j=1, j\neq 2,3,10}^{10} wP_j(x,y) + \sum_{i=1}^{2}\sum_{j=1,4}^{3,10} P_{ij}V_{ij}(x_i, y_i)$
$w.r.t \rightarrow x, y$
$x = \left\{(CSS1, SA1)_1, (CSS2)_2\right\}$
$y = \left\{(P1, P2, P3)_1, (P4, ..., P10)_2\right\}$
$s.t.$
$(F_{80} / P_{80})_1 - 4 \leq 0$
$(F_{80} / P_{80})_2 - 4 \leq 0$
$x_{lb} = \left\{20, 50, 10\right\}, x_{ub} = \left\{50, 65, 30\right\}$</td></tr>
<tr><td colspan="3">Distributed IDF - Bi-level optimization problem</td></tr>
<tr><td>System Optimization
$\max \sum_{j=1, j\neq 2,3,10}^{10} P_j(x, \bar{x}, y)$
$w.r.t \rightarrow x, \bar{x}, y$
$x = \left\{(CSS1, SA1)_1, (CSS2)_2\right\}$
$\bar{x} = \left\{(\overline{CSS}1, \overline{SA}1)_1, (\overline{CSS}2)_2\right\}$
$y = \left\{(P1, P2, P3)_1, (P4, ..., P10)_2\right\}$
$s.t.: \left\| x - \bar{x} \right\| = 0$
$x_{lb} = \left\{20, 50, 10\right\}, x_{ub} = \left\{55, 65, 30\right\}$
$\bar{x}_{lb} = \left\{20, 50, 10\right\}, \bar{x}_{ub} = \left\{55, 65, 30\right\}$</td>
<td>Secondary Crushing Process Optimization
$\max \sum_{j=1}^{3} P_j V_j(x_1, \bar{x}_2, y_1)$
$w.r.t \rightarrow x_1, \bar{x}_2, y_1$
$x_1 = \left\{(CSS1, SA1)_1\right\}, \bar{x}_2 = \left\{(\overline{CSS2})_2\right\}$
$y_1 = \left\{(P1, P2, P3)_1\right\}$
$s.t.: (F_{80} / P_{80})_2 - 4 \leq 0$
$x_{lb} = \left\{20, 50\right\}, x_{ub} = \left\{55, 65\right\}$</td>
<td>Tertiary Crushing Process Optimization
$\max \sum_{j=4}^{10} P_j V_j(x_2, \bar{x}_1, y_2)$
$w.r.t \rightarrow x_2, \bar{x}_1, y_2$
$x_2 = \left\{(CSS2)_2\right\}, \bar{x}_1 = \left\{(\overline{CSS1}, \overline{SA1})_1\right\}$
$y_2 = \left\{(P4, ..., P10)_2\right\}$
$s.t.: (F_{80} / P_{80})_2 - 4 \leq 0$
$x_{lb} = \left\{10\right\}, x_{ub} = \left\{30\right\}$</td></tr>
</table>

Before solving the optimization problem, it can be good to know the dependencies of the design variable to the defined objective functions for interpretation of the optimization results. Design variable study is a parametric study of the individual design variables (or combination of variables) towards the objective function to gain insights into the defined problem and potential improvements. Figure 13 shows an example of the design variable studies for the SPV function applied in the crushing plant in Paper B. As seen in Figure 13 (a), the OFAT (One-factor-at-a-time) study reveals the monotonic increasing and decreasing behaviour of SPV in secondary sub-processes based on *CSS1*. Figure 13 (b) shows a surface model of the influence of two factors (*CSS1* and *SA1*) on SPV, indicating the magnitude and behaviour of the function. It can be noted that the 2D SPV graph for the secondary sub-process presented in Figure 13 (a) is an extract of the 3D graph shown in Figure 13 (b) at the *SA* value of 65 mm. It can also be noted in Figure 13 (a) that the SPV value is maximum around the *CSS1* value of 40 mm. Design variable study is useful for understanding the mathematical response of the objective functions with respect to the important design variables.



(a) (b)

Figure 13. (a) SPV for varying CSS1 (For SA1 = 65 mm, CSS2 = 15 mm), (b) Influence of two variables (CSS1 and SA1) on the SPV for secondary sub-process (For CSS2 = 15 mm) [Paper B].

4.2.3 Optimization Solution

The implementation of the optimization method requires the creation of a coordinating algorithm to pose the optimization problem formulated. The implementation includes linking the optimization problem with the configured simulation model. Figure 14 shows a pictorial overview comparison of the MDF and IDF architecture communication, illustrating how MDF iteration is sequentially performed with the simulation model while the bi-level distributed IDF is parallelly solved [Paper B]. SQP was used to solve the optimization problem in Paper B. Figure 15 presents a comparison of the IDF architecture and MOO using GA communication, wherein the MOO problem considers the simulation model as black box, meaning that the algorithm brute-forces the model based on the applied settings in the GA to produce trade-off curves [Paper C].

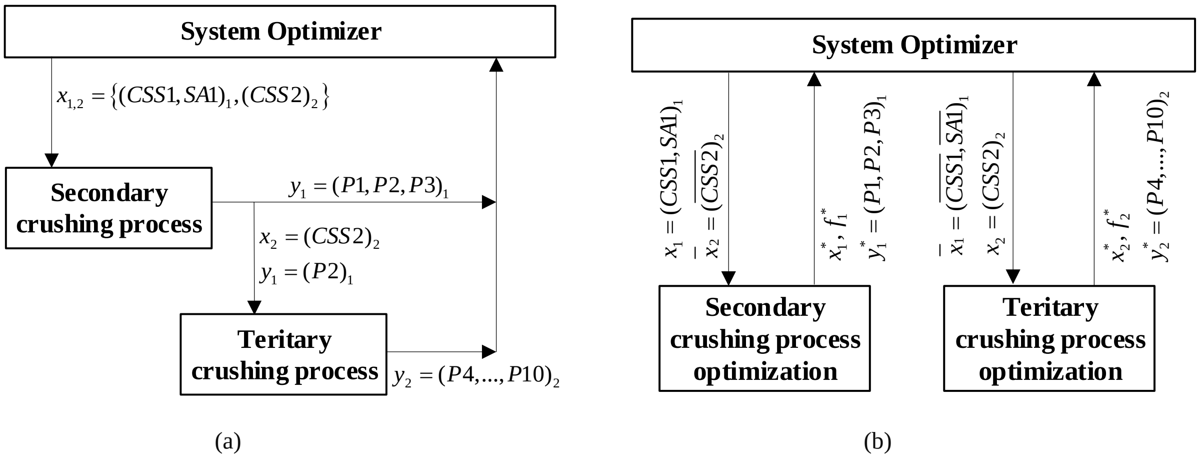


Figure 14. Comparison of (a) MDF and (b) IDF problem communication model for a two-stage crushing plant [Paper B].

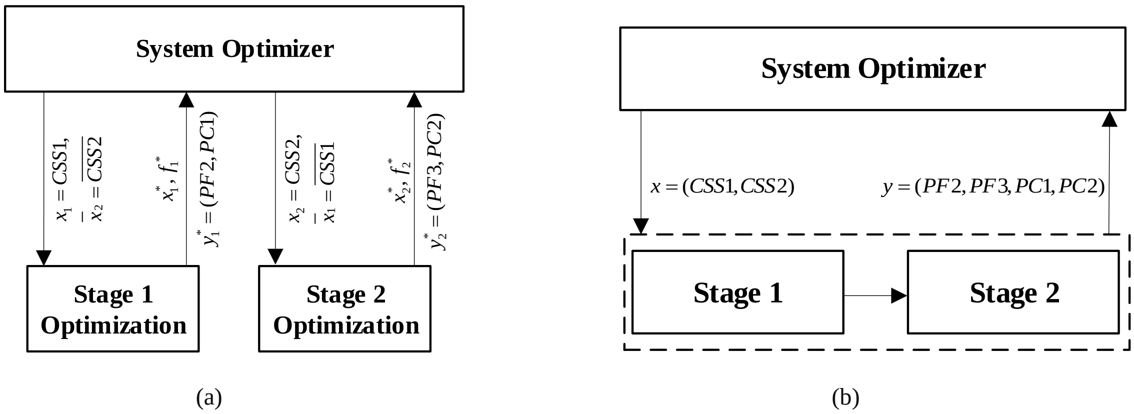


Figure 15. Comparison of (a) IDF and (b) MOO (using GA) problem communication model for a two-stage crushing plant [Paper C].

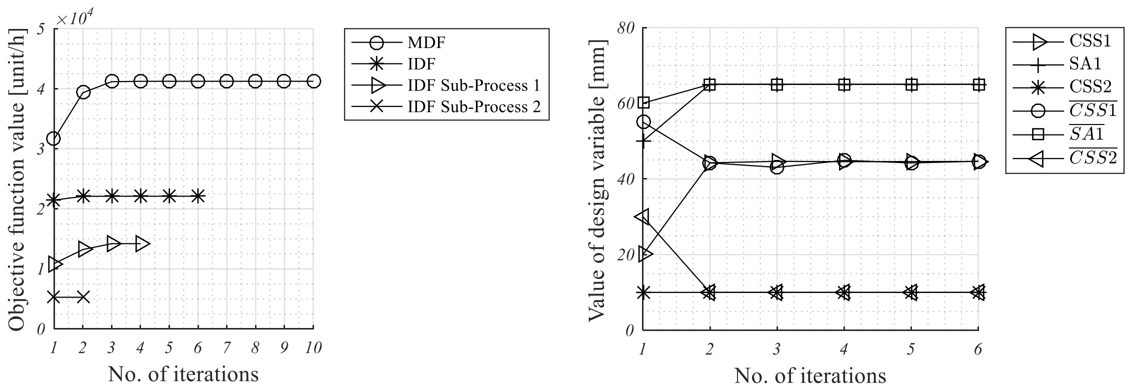
The optimization method execution requires multiple micro-level decision entities for running an optimization method and algorithm. Several settings need to be selected, such as:

- Choice of solver algorithm, e.g., sequential quadratic programming (SQP), GA
- Tolerance criteria in the functional evaluation (objective and constraint)
- Convergence criteria, maximum number of iterations, etc. for the optimization algorithm
- Strategies of weights in the objective functions
- Initial start points for design variable(s)

Iterative learning is required to develop a recommendation of these settings. The choice governs the computational time, result quality and practical implications. For example, the MDO algorithms were found to be sensitive to the initial starting point of the algorithm, which is in line with the literature study of the gradient-based algorithm [Paper B]. It is recommended to test the MDO algorithm at different start points to address the lack of robustness in the application. In Paper C, the results from a genetic algorithm are dependent on the initial definition of population size, number of generations, etc.

The solution obtained from the optimization method needs to be evaluated for two categories: convergence analysis and physical relevance. The convergence analysis deals with the understanding of the iterations required for the optimization algorithm, constraint activity and the behaviour of solution point(s) (optima and optimizers). The number of iterations reflects the computation time, while the satisfaction of the constraint function reflects the feasibility of the solution point(s). The solution point(s) reflect the boundedness of the solution based on the upper and lower limits of variables defined in the optimization problem formulation. The solution points(s) also need to be checked for local optimization and global optimization, which can be reflected together with the understanding developed from the design variable study. The physical relevance of the optimization results is to check whether the solution is feasible to the practical operation. This can be discussed with the experienced personnel in the management and operation of such processes of the plant. The practical implementation of optimization results can be carried out after this stage.

An example of the convergence analysis for the distributed IDF optimization problem is shown in Figure 16 [Paper B]. The design variables and their duplicate copies reached convergence in six iterations. It can be observed that the two variables, *CSS2* and *SA1*, are hitting the boundary value limits defined in the problem, while the *CSS1* is converging to an interior optimum value (*CSS1**= 44 mm). This is in line with the observation made in Figure 13 (a). For the posed problem, the solution is recommending operating the process with a maximum value of *SA1*, meaning opening the screen aperture at the secondary process to allow for an increase in the material flow to the tertiary process. The minimum value *CSS2* means operating the tertiary crusher at the lowest *CSS2* setting possible while operating the secondary crusher at a specific value of *CSS1**. Changing the posed problem's objective function and design variable limits can result in different recommended settings.



(a) (b)

Figure 16. Example of a convergence study for distributed IDF formulation a) number of iterations required for the convergence of the algorithm where constraints are satisfied, b) Design variable behaviour for the respective iterations [Paper B].

An example of the Pareto-front result for the MOO problem using GA for two objectives in a two-stage crushing plant is shown in Figure 17 [Paper C]. The purpose of the optimization application was to explore the solution space to find a trade-off between the maximization of the production of the fine materials and the minimization of the power drawn by crushers. The design variables for the problems were *CSS1* and *CSS2* for crushers in stages 1 and 2, respectively. The choice of the solution is based on the reasoning of the solution space. As seen in Figure 17 (a), there is a trade-off between the production of the fine materials and power, which resulted in three categories of solution space in Figure 17 (b).

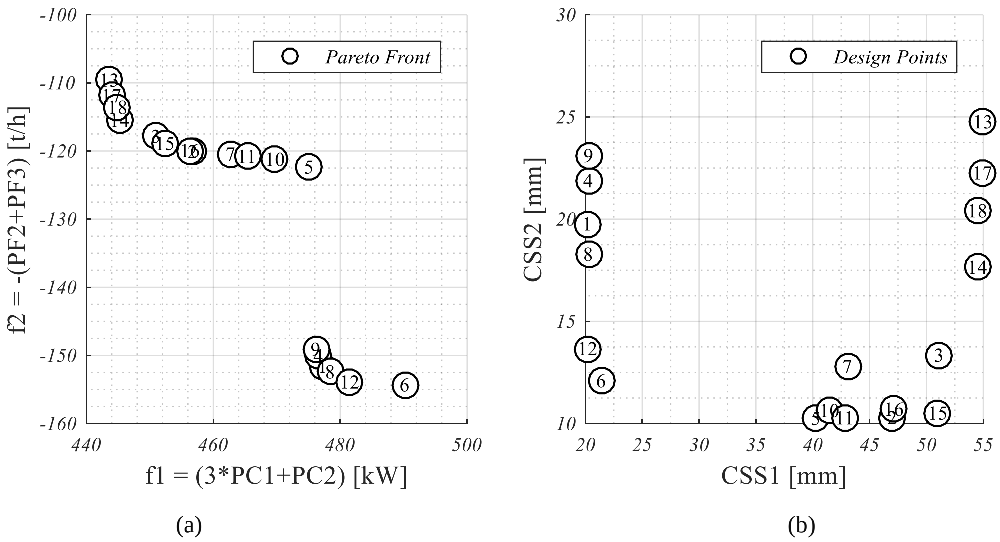


Figure 17. Example of a) Pareto front and b) design points of the MOO problems using GA [Paper C].

- Category 1 is solution numbers 13, 17, 18, and 14, which consume a lower range of power and produce a lower range of fine materials. The corresponding solution set shows that the *CSS1* is operating at the upper boundary, meaning that the first crusher is completely open and performing the least amount of work, while the *CSS2* is also at a fairly open setting, resulting in low consumption of power.
- Category 2 is solution numbers 9, 4, 1, 8, 12, and 6, which produce a large amount of fine materials at the expense of higher power consumption. The solution set recommends operating the *CSS1* at the lower boundary, meaning closing crusher 1 to the lowest settings, while the *CSS2* is varying within the limits defined in the problem definition. These solutions will have an implication on the maintenance of the first crusher.
- Category 3 is solution numbers 5, 10, 11, 7, 16, 2, 15, and 3, which produce a relatively higher fraction of fine materials than the category 1 solutions, while the values in the power consumption vary in a larger range. These are interior solutions to the defined values of *CSS* limits in the optimization problem, especially for *CSS1*. The solutions are tending to operate the second crusher (*CSS2*) at the lower boundary of the *CSS* range. These solutions will have an implication on the maintenance of the second crusher.

The user can select the most competitive solution depending on the judgement considering the physical plant, maintenance implications and production targets. Changing the defined problem definition by changing, e.g., objective functions, weights, constraint functions and variable limits would result in a different set of Pareto solutions.

4.3 PERFORMANCE INDICATORS FOR CRUSHING PLANTS

Performance indicators consist of a set of meaningful measurable entities of the crushing plant which can be used to carry out process improvements and process optimization. The performance improvement is iterative and can be carried out based on the process diagnostics with the support of the performance indicators. The process optimization can be carried out by using the performance indicator as an objective function in the optimization function. A three-stage crushing plant for aggregates production was used to demonstrate the implementation of the KPIs, as shown in Figure 18.

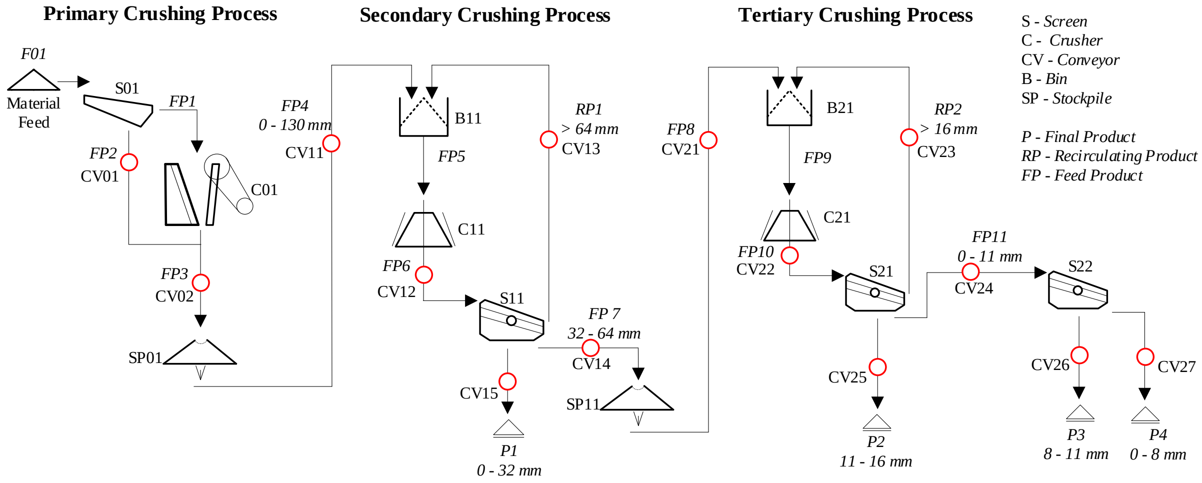


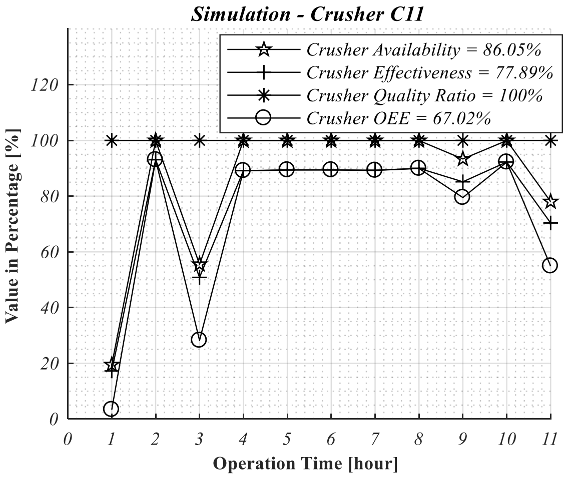
Figure 18. A three-stage crushing plant for aggregates production [Paper D].

In Paper D, a set of key performance indicators (KPIs) has been developed based on the ISO 22400 Standards [95-98], as shown in Table 8. The KPIs were calculated based on the dynamic process simulation output and in parallel real-time measured production data of the process. The real-time data for the physical crushing plant was retrieved from the data collection system, consisting of a cloud-based solution for plant monitoring. The real-time data contained measurements of mass flow rate from conveyors, various process and equipment set points, and power consumption.

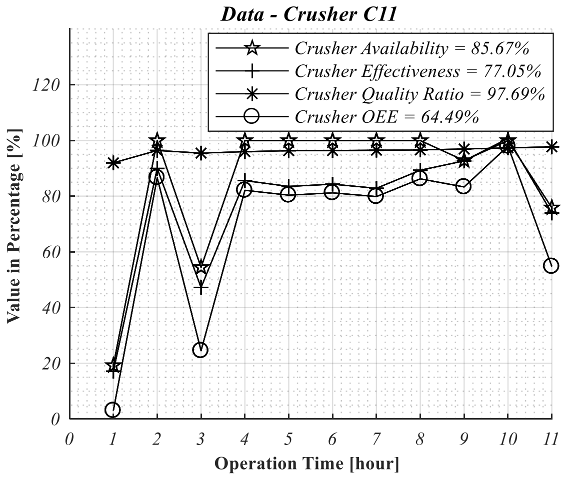
Table 8. List of KPIs developed for the measurement of performance in aggregates production [Paper D].

Measurement Basis	**KPI**	**Unit**
Planned and Real-Time	Equipment Utilization	%
	Equipment Availability	%
	Process Availability	%
Logistical Quantities and Quality	Throughput Rate	tph
	Equipment Effectiveness	%
	Process Effectiveness	%
	Yield of Product	%
	Quality Ratio	%
Power Consumption	Specific Power	kW/tph
	Direct Power Effectiveness	%
Overall Performance	Overall Equipment Effectiveness Index	%
	Overall Process Effectiveness Index	%

An example of the KPI calculation for crusher *C11* in the aggregates production plant is shown in Figure 19. It can be noticed that the KPIs have the capability to demonstrate the start-up and shutdown sequences of the crusher operation and highlight performance losses happening during the operations. The KPI value drop due to unscheduled stops occurring in the crushing operation can also be seen in crusher *C11* at around the 3rd hour of the operation. The dynamic process simulation predicts the average KPIs over a shift to be close to the real-time data-based KPIs that can be seen for the crusher, although there are differences at the hourly calculations. This can be attributed to the assumptions and degree of information brought into the simulation. The KPIs can be applied to different time intervals (e.g., 30 min to 30 days) depending on the requirement [Paper D].



(a)



(b)

Figure 19. Crusher C11 performance based on process (a) simulation and (b) real-time data [Paper D].

From an industrial implementation perspective, performance improvement is possible from a multitude of solution types. For example, reducing the downtime of a crusher can lead to meeting the target production value, meaning the more up-time, the more production. But it does not necessarily contribute to the desired product type in production, for which the operational setting of the crushing needs to be modified. Modifying operation settings again can have consequences towards increasing or decreasing the downtime of the equipment and subsequent equipment performance change (screening load change). Other solutions, such as modifying the crusher feed, monitoring power consumption, or a physical investigation of equipment operation, can lead to insights on the improvement opportunities. It can be concluded that looking into the KPIs of the crushing plant can provide the first glimpse of the ample improvement opportunities, and simulation tools can be handy to test and try new solutions.

To address the output difference between physical operation and simulation output, a novel approach to represent error propagation within the implementation of the KPIs is presented (see Figure 20). The choice of the type of equipment and process models directs the degree of fidelity of the simulation results. Each underlying equipment model is developed based on practical assumptions and the accuracy of the model is also dependent on the experimental calibration procedure applied. This induces an error in the estimation of reality and is denoted by $\pm\varphi_1$. The process simulation is configured with a combination of multiple equipment models, resulting in an error denoted by $\pm\varphi_2$, and the process simulation also receives input from physical processes, adding an error of $\pm\varphi_3$. In addition, the data captured during the physical operation also contains errors due to measurements, denoted by $\pm\delta_1$, arising from sensor accuracy and maintenance status. Based on the underlying process simulation error ($\pm\varphi_4$) and data measurement errors ($\pm\delta_1$), the KPIs calculation can be compared by calculating errors ($\pm\omega_R$) between the physical and simulation systems. It is of interest to estimate the accuracy in each stage of development, as the reliability of the simulation-based decisions is dependent on it. The errors associated with the equipment model and process simulation are calculated in Paper E, while errors associated with data measurements are presented in Paper F.

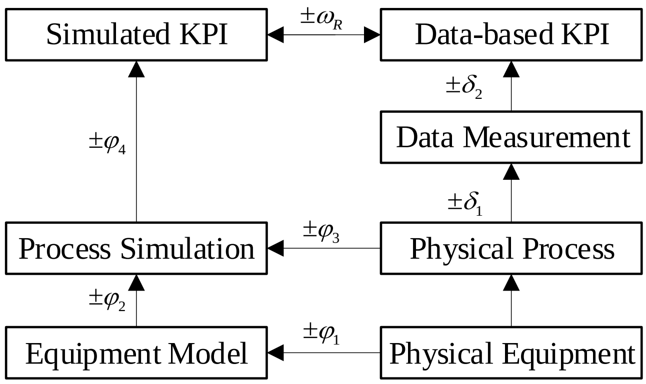


Figure 20. Error propagation in the calculation of KPI from the simulated process and real-time data [Paper D].

In conclusion, the KPIs are useful for operators and plant managers of a crushing plant to identify performance improvement opportunities and can be viewed as a support tool for the decision-making process. Transformation of the KPIs using ISO 22400 is beneficial in benchmarking performance standards between multiple crushing plant processes and equipment operation.

4.4 Simulation Calibration and Validation for Crushing Plants

As mentioned before, one of the underlying requirements for achieving process improvement and optimization using simulation is to maintain a low degree of error between the output of simulation and physical operation data. The choice of equipment and process model, together with the calibration and validation process, influences the outcome. An extensive study is performed to demonstrate the application of the optimization method for calibration of the equipment model in Paper E.

Paper E presents an experimental survey and production data-based methodology together with the use of optimization methods to calibrate and validate crushing plant simulation. Figure 21 represents a schematic view of the pillars for the model calibration for equipment and process simulation based on the source of the data. The first source of data is laboratory data, which includes standard material characterization tests of, for example, material density, compressive stress, moisture content, breakage, etc. The second source is experimental survey data, which includes controlled experiments performed on individual equipment or processes to collect belt-cut material samples. The results from this provide a snapshot performance of the process and equipment at different operational settings. The third source of data is production data, which refers to controlled data of crushing plant operation, such as mass flow, power, process set points, and control signals. These data sets are characterized as continuous time-dependent data and influenced by more than one equipment behaviour. The three data forms create a different abstraction of the information captured from crushing plants.

To create utilization of simulation platforms for daily operation performance control (refer to Figure 9), there is a need to re-think the approach by which different existing models are built and used. Most mechanistic and phenomenological models of equipment are developed based on laboratory and experimental data together with the understanding of the machinery. Transforming the existing models to use and adapt to different data sources requires computationally efficient optimization methods in order to fit the model to the data. Another approach for modelling is to develop a completely new data-driven approach based on production data such as machine learning. In both cases, there is also a need to design new ways of performing the experimental survey to collect controlled data from the crushing plant.

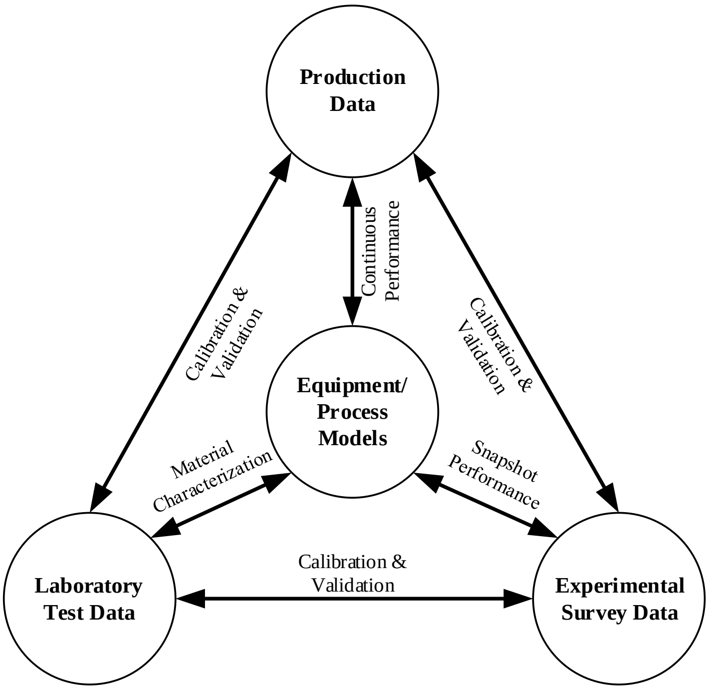


Figure 21. Generalized overview of the model and process calibration based on the sources of data [Paper E].

Figure 22 presents the layout of the tertiary crushing stage used in Paper E. A novel approach to experimental design for collecting the experimental and production data was also described in this paper; see Figure 23. The essence of the design of the experiment was to capture continuous production data (mass flow and power) together with snapshot performance of equipment (crusher and screen) through belt-cut sampling. The crusher settings were altered in different tests. The snapshot data were used to calibrate the individual equipment model used in the dynamic process simulation, while the production data were used to validate the process simulation; see Figure 24.

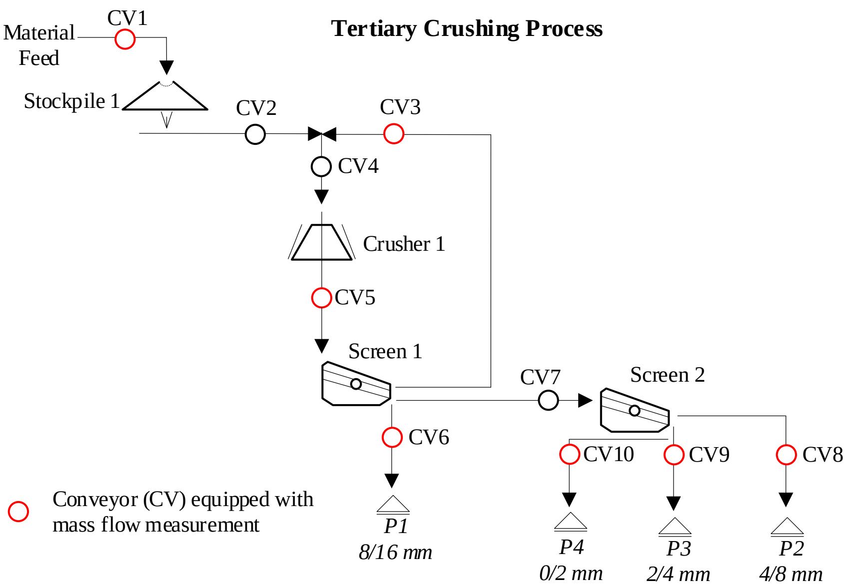


Figure 22. A tertiary crushing stage for an aggregates production plant [Paper E].

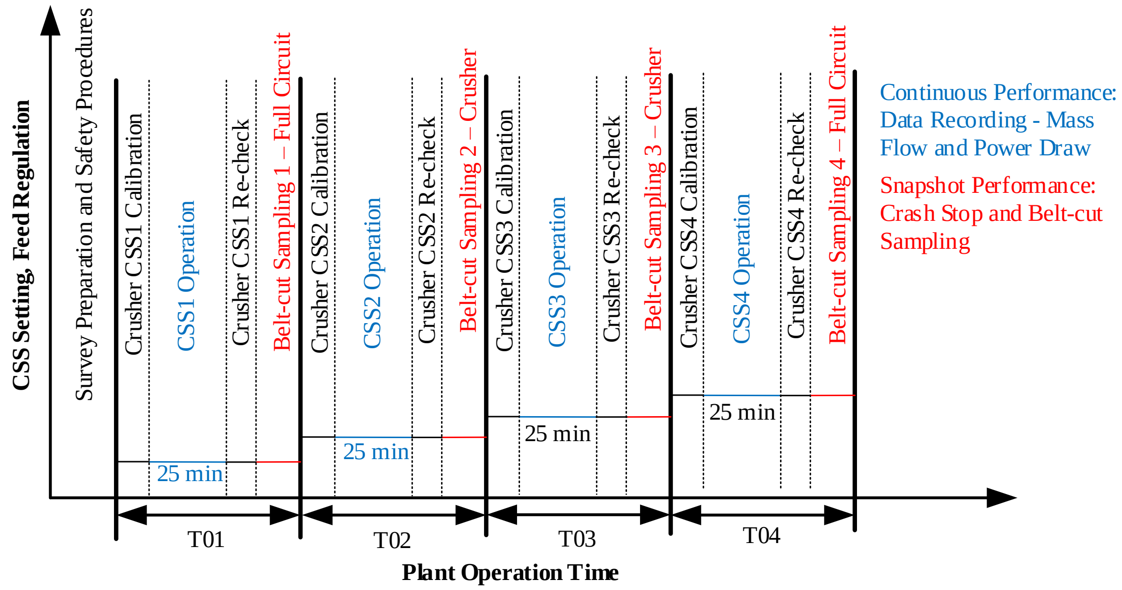


Figure 23. Experimental sequence applied for mapping the circuit performance of the crushing plant [Paper E].

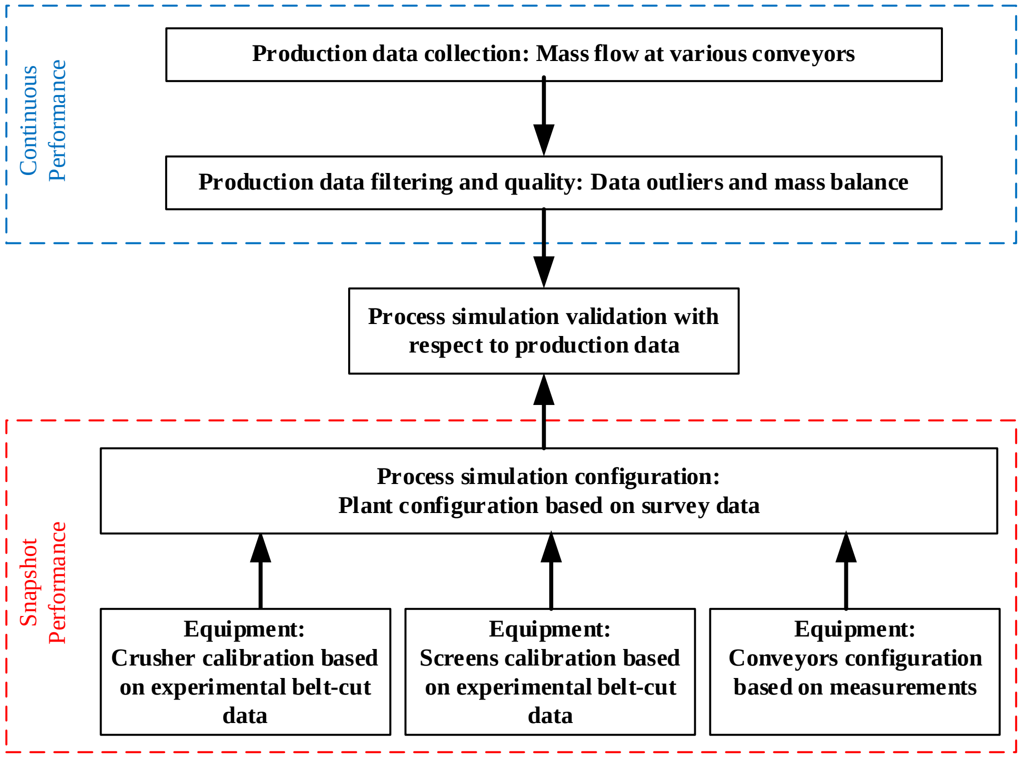


Figure 24. Steps for calibration and validation of dynamic process simulation of the crushing plant [Paper E].

A unique and simple optimization approach using an unconstrained gradient-based approach (quasi-Newton method) was applied to calibrate the fast-mechanistic crushing model; see Table 9. The optimization problem definition is a weighted sum approach with two sequential optimizations: Capacity Optimization and Product Size Distribution Optimization with 10 variables (k). In Capacity Optimization, the objective function is to minimize the sum of the relative errors between the crusher-measured capacity (Cap_{Di}) and the simulated capacity (Cap_{Si}) for the n number of tested settings of CSS. In the PSD Optimization for the crusher, the objective function is to minimize the weighted (w_j) sum of errors for the data ($PSDf_{Dji}$) and the simulation ($PSDf_{Sji}$) for the values of the n number of tested settings.

The PSD Optimization problem is posed in the frequency domain of the size distribution for the *m* number of given sieve sizes (**xsize**), and *mat* represents the nominal material characterization values for a given material type.

Table 9. Crusher optimization problem formulation for the fast-mechanistic model [Paper E].

Crusher Optimization Problem Formulation	
Capacity Optimization $\min \sum_{i=1}^{n} \left\| \frac{(Cap_{Di} - Cap_{Si})}{Cap_{Di}} \right\|$ $w.r.t \rightarrow x_k, [k = 10],$ *where* $[\mathrm{x}_k]_0 = [1]$ $n = 4$ $Optimizer \;=\; x_k^*$	**Product Size Distribution (PSD) Optimization** $\min \sum_{i=1}^{n} \sum_{j=1}^{m} w_j \left\| (PSDf_{Dji} - PSDf_{Sji}) \right\|$ $w.r.t \rightarrow x_k, [k = 1, 2, ..., 9],$ *where* $[\mathrm{x}_k]_0 = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 1\ 1]$ $n = 4,\ m = 25$ $Optimizer = \mathrm{x}_k^* \bullet mat$
Weighted Function $z_j = \log_2(xsize_j) + \left\|\log_2(\min(\mathbf{xsize})\right\|$ $w_j = \begin{cases} z_j / (\max(\mathbf{z})) \forall (j = 1, 2, ..., 24) \\ 1, (j = 25) \end{cases}$ *where* $\mathbf{xsize} =$ $[360; 250; 125; 90; 63; 45; 31.5; 22.4; 16; 11.2; 8; 5.6; 4; 2.8; 2; 1.4; 1; 0.7; 0.5; 0.35; 0.25; 0.177; 0.125; 0.088; 0.063]$	

The purpose of the weighted function (w_j) is to steer and compensate for the distribution of the number of data points available at different sieve size ranges (**xsize**) (see Figure 25). The function weighs more on the coarse end of the particle size range than on the fine end of the particle size range, which is useful to avoid over- and underfitting of the model with respect to the data. The details of the model calibration can be found in Paper E.

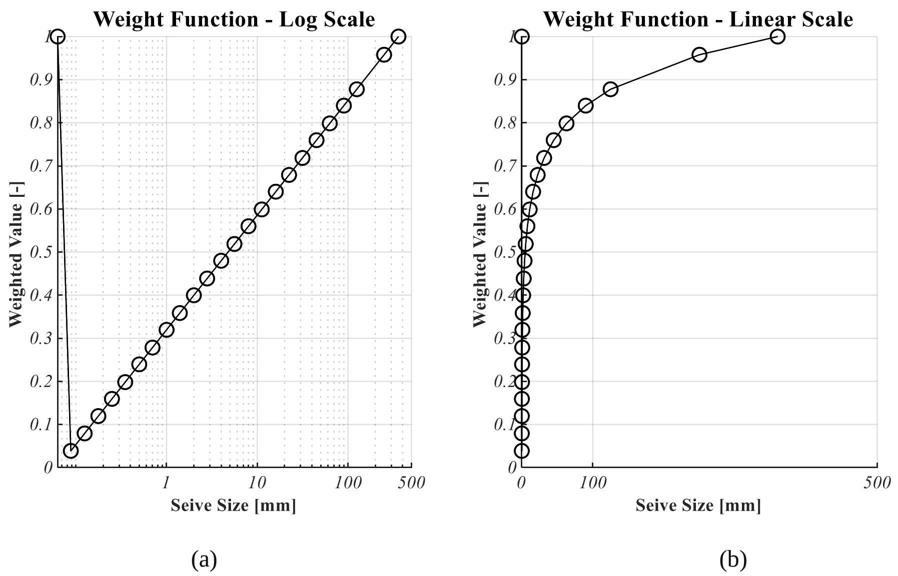


Figure 25. Graphical representation of the weighted function used in the optimization problem formulation [Paper E].

Figure 26 represents crusher calibration results for different tested *CSSs* in the frequency domain. It was crucial to work with the frequency domain for the optimization problem rather than the cumulative domain, as the former avoids accumulated error in different sieve size data. In essence, the problem was decoupled for every sieve size fraction and test condition. Similarly, a modified Whiten [39] partition curve for the screen model was applied and calibrated with the tested screen samples using unconstrained gradient-based optimization (quasi-Newton method). The calibration process resulted in a single set of model variable values for all tested conditions. The detailed results are provided in Paper E.

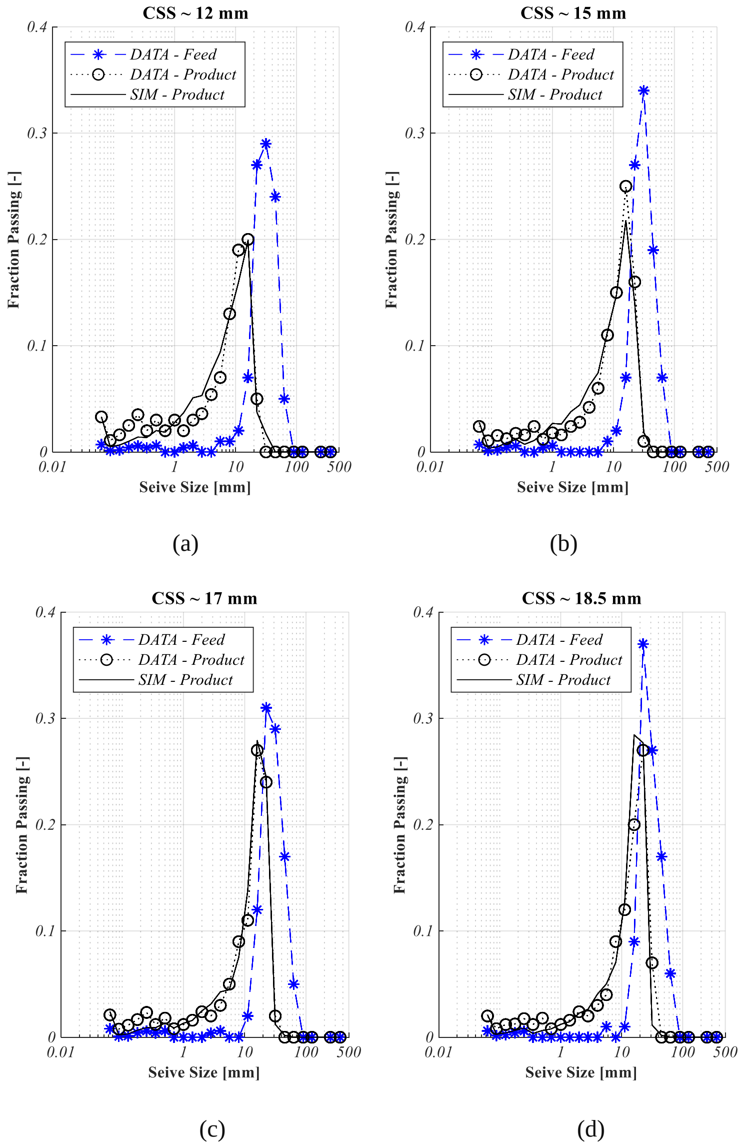


Figure 26. Crusher calibration results to the belt-cut experimental data in the frequency domain of the product size distribution [Paper E].

The configured and calibrated dynamic process simulation results were validated against the production data; see Figure 27. Table 10 presents the root mean square error (RMSE) values for each product stream and test conditions. The RMSE values are low for most cases, except product *P8/16* mm and *P16+* mm. The origin of the error can be either associated with the crusher model or screen model or the production data itself. Evidence of dynamic interaction effect in the performance of screens and crushers was found, especially at the coarser end of the product range where the capacity and PSD of the crusher product dynamically affects the performance of the screen [Paper E].

Overall, for the process performance prediction, it is satisfactory to use such models for process optimization and process planning for aggregates production. Closeness to the operational prediction is key to generating reliable results and increasing the degree of accuracy at this stage, which in turn increases confidence in the results of process improvement and optimization. Additionally, a new form of model and calibration process is needed to achieve seamless integration of the physical and simulation systems of the crushing plant.

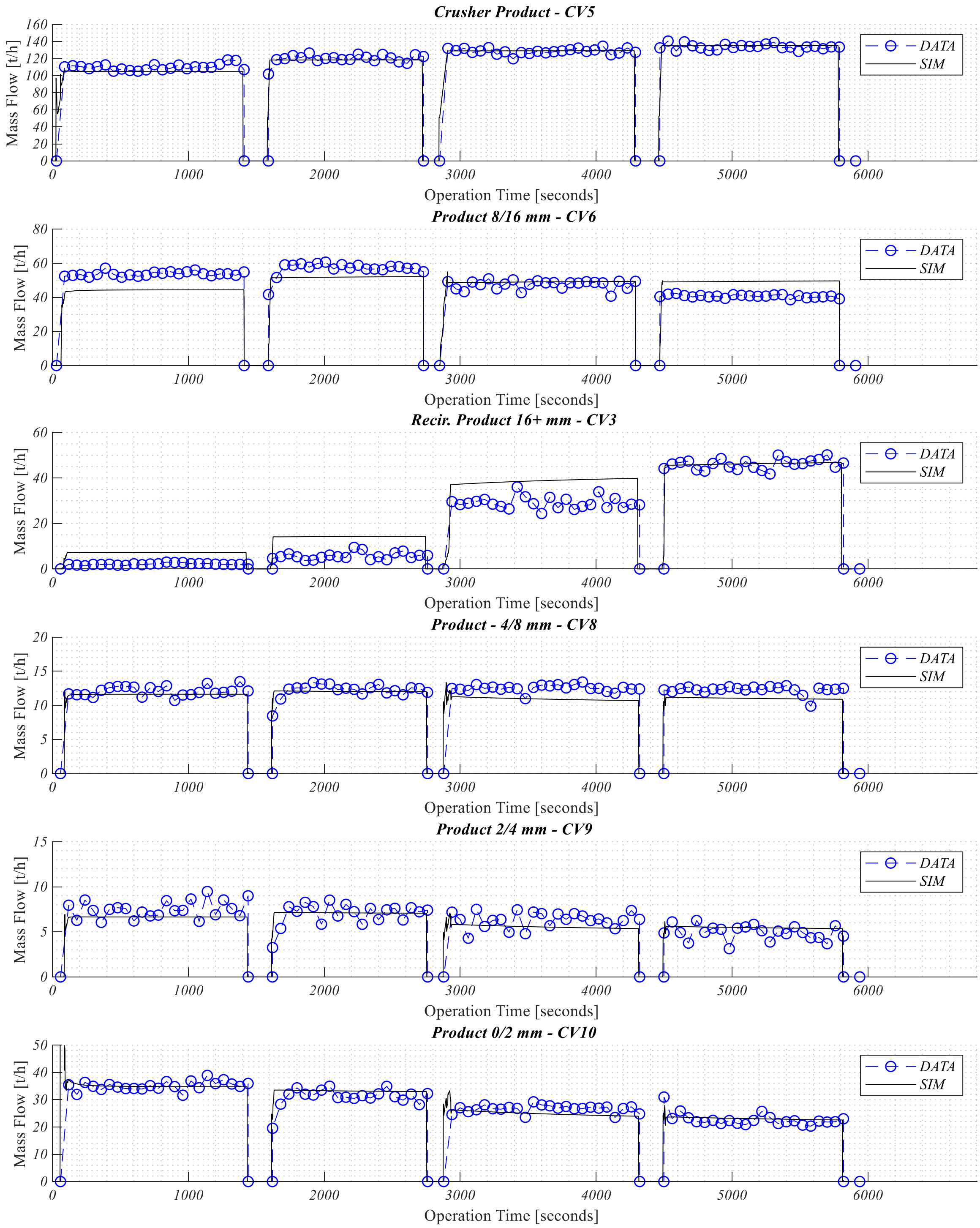


Figure 27. Dynamic simulation process results compared with production data for the four test conditions. [Paper E].

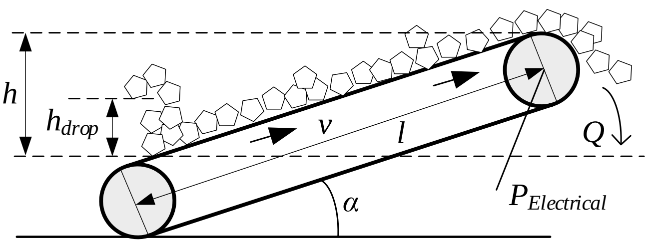
Table 10. RMSE calculation between process simulation and production data [Paper E].

Product Stream	**T01**	**T02**	**T03**	**T04**
Crusher Product	6.42	3.90	3.22	3.40
P8/16 mm	14.38	6.05	4.20	8.75
P16+ mm	5.10	8.40	11.38	7.72
P4/8 mm	2.56	0.70	1.77	1.48
P2/4 mm	2.06	0.78	1.28	1.01
P0/2 mm	1.90	2.42	2.22	1.50

4.5 CRUSHING PLANT MASS FLOW DATA

Crushing plant data consist of offline and online data, as described in Section 4.1.1: Physical System. Among many different data types in the data collection system, mass flow measurements are critical data that are utilized in the calculation of product volume, production rate, energy impact, etc. The improvement and optimization of the crushing plant performance is directly dependent on the accountability of the mass flow. Accurate estimation of the mass flow from the physical process is required for sales, legal compliance, and process simulation validation. There are multiple mass flow measurement systems present in the industrial application based on different principles, for example, load cell-based, laser profilometer, ultrasonic sensor, and power-based belt scale. These systems vary largely on aspects such as cost, accuracy, and maintenance.

In Paper F, a cost-effective method was shown for the calibration of a power-based belt scale together with the monitoring and detection of deviations in the estimation for the mass flow. The principle behind the mass flow estimation is that the power draw from a conveyor belt is dependent on the load on the conveyor, conveyor speed, geometrical design, and overall efficiency of the conveyor [116]. Figure 28 presents the working principle of the power-based belt scale, where $\dot{m}$ is the mass flow rate, P_{Load} is the power required to lift material, C_{Geom} is a geometrical constant and η is the total efficiency of a conveyor.



$$\dot{m} = \frac{(P_{Electrical} - P_{Idle}) \cdot \eta}{gh + v^2 + v\sqrt{2gh_{drop}}\sin\alpha} = \frac{P_{Load} \cdot \eta}{C_{Geom}}$$

Figure 28. The working principle of a conveyor lifting material [38] [Paper F].

The crushing plant layout used in Paper F is presented in Figure 29, which is a tertiary stage of a three-stage aggregates production plant, and each conveyor is equipped with a power-based belt scale. An error minimization optimization problem was applied to calibrate accessible conveyors (access to physical measurement) and the mass balancing property of the circuit is used to calibrate the non-accessible conveyors (limited or no access to physical measurements); see Table 11.

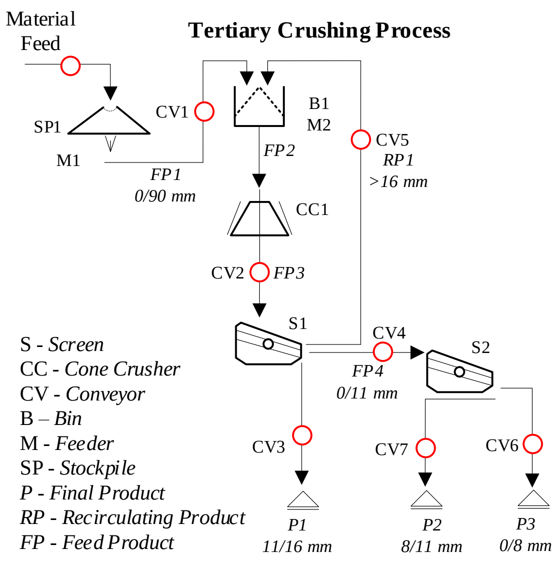


Figure 29. Crushing plant layout for the implementation of conveyor mass flow calculation [Paper F].

Table 11. Optimization problem formulation and deviation tracking for power-based belt scale [Paper F].

Accessible Conveyor Optimization Problem Formulation	**Non-accessible Conveyor Optimization Problem Formulation**
$\min \sum e_i = \sum \frac{Q_{iM} - Q_{iC}}{Q_{iM}}$ $w.r.t. \rightarrow \eta$ $s.t.$ $0.75 \le \eta \le 0.99$ $where$ $Q_{iC} = \int_{t_1}^{t_2} \frac{P_{iLoad}(t) \cdot \eta}{C_{iGeom}} dt$ $Q_{iM} = \int_{t_1}^{t_2} \dot{m}_i(t) dt$	$\min \sum \varepsilon_k$ $w.r.t. \rightarrow \eta_u$ $s.t.$ $0.75 \le \eta_u \le 0.99$ $where$ $\varepsilon_k = \left\Vert \sum_{i=1}^{n} Q_{i,k} - \sum_{j=1}^{m} Q_{j,k} \right\Vert$ $Q_i = \int_{t_1}^{t_2} \dot{m}_i dt = \frac{\eta_i}{C_{iGeom}} \int_{t_1}^{t_2} P_{iLoad}(t) dt$ $Q_j = \int_{t_1}^{t_2} \dot{m}_j dt = \frac{\eta_j}{C_{jGeom}} \int_{t_1}^{t_2} P_{jLoad}(t) dt$
Conveyor Error Factor (CEF)	***Conveyor Error Ratio (CER)***
$CEF_p = \frac{\sum_{k=1}^{k_{max}} (A_{pk} * \varepsilon_k)}{\sum_{k=1}^{k_{max}} A_{pk}}$	$CER_p = \frac{CEF_p}{\sum_{k=1}^{k_{max}} \varepsilon_k}$

The generalized error minimization optimization problem for the efficiency calibration of an individual conveyor is given, where i is the number of test samples, e_i is the relative error function, Q_{iC} is calculated accumulated mass, Q_{iM} is measured accumulated mass for a tested time interval, and t_1 and t_2 represent the start and end times for the calibration for one test. For a given time of operation, at any node in a crushing circuit, the accumulated incoming material mass is equal to the accumulated outgoing mass under negligible material loss conditions. The error function is given by ε_k, where k is a node in the circuit, n and m are the numbers of mass measuring units before and after the selected node in the plant circuit, respectively, and η_u is the non-accessible conveyor efficiency.

The solution and choice of the algorithm in calibrating non-accessible conveyors is dependent on circuit layout and resulting mass balancing equations. If the number of error equations is less than or equal to more than the number of non-accessible conveyors, an under-constrained, full constrained, or over-constrained optimization problem is formulated, respectively. The optimization problems can be solved using gradient-based constrained optimization algorithms such as the interior-point algorithm.

The power-based belt scale is a cost-effective solution for mass flow estimation but, as any other measurement system, it requires re-calibration. The need can arise due to either gradual changes over time, such as component wear, or more instant changes like a rock getting stuck in a roller. The reliability of the mass flow estimation is tracked using the principle of mass balancing of the system. A correlation matrix A_{pk} is developed between the conveyor mass flow and the error function based on the layout of the circuit. *Conveyor Error Factor* (*CEF*) and *Conveyor Error Ratio* (*CER*) are calculated using the A_{pk} matrix; see Table 10. *CEF* indicates the total mean error associated with each conveyor (*p*) with respect to the entire system and *CER* indicates the proportion of the error contributed by each conveyor (*p*) to the entire system. The detailed description can be found in Paper F.

The value of the *CER* is ranked, and the conveyor with the higher value is investigated first. The value of *CEF* indicates the magnitude of the deviation. The impact of this magnitude is dependent on the rated capacity of the conveyor. Based on the values of *CER* and *CEF* together, decisions are made:

- If the values are within allowed statistical limits, retain the efficiency value of the conveyors. If the values are deviating towards a certain direction, create an alert for operators to inspect the conveyor for any change in physical operation.
- If the values are above the allowed limit, initiate re-compensation of the deviating conveyor with a new value of efficiency. This is carried out by modifying non-accessible conveyor optimization problem formulation depending on the identified deviating conveyor. In this case, the efficiency value of the deviating conveyor is set as an unknown variable.

As an example to test the calculation of the correlation matrix, values of *CER*, and *CEF*, two hypothetical test cases were carried out for the crushing plant in Paper F (see Table 12). A 10% change in efficiency value for conveyors *CV4* and *CV7* was performed in Test 1 and Test 2, respectively. The method detected *CV4* to be deviating and the magnitude of *CEF* was also significantly increased. For Test 2, changing *CV7* resulted in an alert for both *CV6* and *CV7* because of the possible relations which are created using the correlation matrix. This is limited on account of plant layout and conveyor connections.

Table 12. Testing of CEF and CER values for hypothetical test cases [Paper F].

	Correlation Matrix						**Day 1**		**Test 1**		**Test 2**	
$\mathbf{A_{pk}}$	ε_1	ε_2	ε_3	ε_4	ε_5	ε_6	*CEF*	*CER*	*CEF*	*CER*	*CEF*	*CER*
CV_1	0	0	1	1	1	0	2.87	0.12	42.28	0.11	11.81	0.12
CV_2	1	0	1	0	0	1	6.41	0.26	42.28	0.11	11.81	0.12
CV_3	1	0	0	1	1	1	4.30	0.18	60.76	0.16	15.05	0.16
CV_4	1	1	0	1	0	0	2.87	0.12	**118.06**	**0.32**	12.33	0.13
CV_5	1	0	1	0	0	1	6.41	0.26	42.28	0.11	11.81	0.12
CV_6	0	1	0	0	1	1	4.02	0.17	43.96	0.12	**27.81**	**0.29**
CV_7	0	1	0	0	1	1	4.02	0.17	43.96	0.12	**27.81**	**0.29**
B_1	0	0	1	1	1	0	2.87	0.12	42.28	0.11	11.81	0.12
Day1: ε_k	6.1	1.7	5.3	0.7	2.5	7.8	$\sum \varepsilon_k = 24.17$	Calibration Data				
Test 1:ε_k	113.6	121.5	5.3	119	2.5	7.8	$\sum \varepsilon_k = 369.86$	Change in *CV4* efficiency by + 10%				
Test 2:ε_k	6.1	30.1	5.3	0.7	29.3	24	$\sum \varepsilon_k = 95.64$	Change in *CV7* efficiency by + 10%				

It was also observed through Tests 1 and 2 that the magnitude of CEF is dependent on the size and capacity of the conveyor as *CV4* is of higher capacity than *CV7*, which needs to be taken into account in the decision for the conveyor re-calibration. Periodic calibration of the conveyors is required to maintain reliability within the data collection system. This is a step towards developing a robust mass measurement solution where the system can detect changes. The proposed methodology can lead to design rules for the implementation of an automatic calibrating mass flow system using a power-based belt scale technique.

5 DISCUSSION & CONCLUSIONS

This chapter aims to:

> *Present and discuss the most important conclusions drawn in this thesis.*
> *Answer the research questions stated in Chapter 1.*
> *Discuss the validity of the research.*
> *Discuss industrial relevance and future work.*

This thesis aimed to investigate and develop optimization capabilities for crushing plants. Multiple studies were performed at various abstraction levels to develop and demonstrate the functionality needed to realize optimization capabilities for crushing plants [Papers A-F]. During the iterative development work, the application of optimization methods multi-objective optimization (MOO) and multi-disciplinary optimization (MDO) were applied at the process level using dynamic process simulation of a crushing plant [Papers B and C]. An implementation of the ISO 22400 standard for aggregates production in dynamic process simulation was demonstrated and compared with the production data from an industrial crushing plant operation [Paper D]. A detailed optimization method for calibration and production data-based validation of the dynamic process simulation including cone crusher and screen models was presented [Paper E]. To assure the quality of the production data, an application of an optimization method for calibration and maintenance of power-based belt scales together with mass flow deviation tracking was shown [Paper F]. A multilayer holistic perspective for building optimization capabilities for crushing plants was demonstrated.

5.1 ANSWERS TO RESEARCH QUESTIONS

The following answers are given to the research question stated in this thesis together with the reclarification of the set of investigations performed under each research question.

RQ1 - What are optimization system requirements for developing optimization capabilities in crushing plant operations?

The following aspects are needed to perform a generic optimization application for a crushing plant in operation:

- *Define the purpose of optimization system capability*
- *Frame optimization problem formulation for the crushing plant*
- *Compare various possible optimization methods and algorithm applications*
- *Explore the underlying requirements to produce a suitable optimization result*
- *Evaluate different optimization methods for practical applications*

A multi-layered modular framework for the implementation of optimization capability for crushing plants is presented (see Figure 9). To achieve improvements and optimization in the physical system for stakeholders' requirements, the use of a parallel simulation system is proposed. The simulation system consists of four layers – equipment and process modelling, model implementation, key performance indicators and an optimization function with consistent connection with the physical system. These represent components of the framework in an interconnected system solution that are required to build optimization capabilities for crushing plants in operation.

A classification scheme to define the purpose (design, operations, and control) and scope (equipment, sub-process, and main process) of the optimization application is presented in Paper A. The classification scheme helps in defining the model requirements as a system perspective, wherein the interaction elements of the underlying simulation model such as input, output, value-adding functions, variables, and parameters are clearly presented. The classification scheme is useful for communicating the purpose of the optimization application to the research and industry communities.

The application of multiple optimization approaches – multi-objective optimization (MOO) and multi-disciplinary optimization (MDO) – are demonstrated in Paper B and Paper C for crushing plant optimization. In Paper B, two MDO architecture frameworks – multi-discipline feasible (MDF) and individual discipline feasible (IDF) – were applied in the context of a crushing plant consisting of two sub-processes. In Paper C, a comparative study between the IDF and MOO using a genetic algorithm was carried out. The MDO methods used in the studies were aimed at finding a balance point between the various objectives of the crushing plant, while the MOO using the GA method was aimed at exploring the spectrum of the solution space using Pareto front. The MDO methods help in decoupling multiple optimization problems which exist in the complex relationship of crushing plant operations. The purpose of the optimization application directs the choice of methods and computation time, and it is recommended to use more than one method to generate a comparison of the results and check the triangulation of the solution points.

In Papers B and C, a standard representation of the optimization problem formulation in negative null form was presented for a variety of optimization cases. This is required for the problem replication as the global optimization results are dependent on the problem definition of the optimization, which can be subject to change based on the stakeholder's needs. To perform iterative calculation in the selected optimization method, it is necessary to model the communication between the optimization algorithm and the crushing plant model. A modular approach needs to be maintained to clearly define the interaction of the objective function, design variables, parameters, and constraints with the model in the crushing plant simulation. It is necessary to maintain the validity of the process simulation and check the convergence graphs for the optimization results. The results obtained from the given crushing plant optimization problem need to be intuitively evaluated concerning the equipment and process knowledge for practical application, which is a subjective process. This is especially required when the solution points generated are at the boundary optima.

RQ2 - How can the process performance objectives be formulated for carrying out process optimization and process improvements in crushing plant operations?

The following steps are performed to present process performance objectives for a crushing plant in operation:

- *Define different performance indicators of the crushing plant*
- *Formulate the calculations of performance indicators*
- *Demonstrate the application of performance indicators for the crushing plant*

Process optimization and process improvements are complementary approaches to increase crushing plant performance with respect to defined objectives or goals from stakeholders' points of view. Process improvements can be defined as increasing the process performance through multiple means, such as iterative learning and experience. Process optimization, on the other hand, can be defined as a mathematical process of employing a numerical approach to define the scope of the problem, solve the problem, and generate alternative solutions towards the desired goal of the stakeholder. To achieve both process optimization and process improvements, mathematical representations of the objectives with respect to the crushing plant technical performance are required.

A process objective can be defined as a mathematical function that indicates the process performance of interest for a crushing plant operation. In Paper D, a list of key performance indicators (KPIs) has been developed and implemented based on the ISO 22400 standard for aggregates crushing plants in dynamic process simulation. The KPIs can represent equipment performance or process performance depending on the scope of the application. Results of the implementation in a dynamic process simulation bring the KPIs study closer to the real-time production operation. For the process improvement, a new conceptual operation strategy can be tested and verified in a simulation system before implementation in the physical operation of the plant. For process optimization purposes, the KPIs can be used as an objective function in the optimization problem formulation, for example, maximizing product yield, maximizing product throughput, etc. In Papers, B and C, the application of process performance indicators such as technical function (e.g., throughput rate, power consumption) and techno-economic function (e.g., sub-process value) for optimization objective functions have been demonstrated. These functions represent the different goals for the optimization problem formulation, and implementation on the simulation platform allows for a cost-effective method of exploration. It is also possible to customize KPIs for optimization depending on the organizational needs.

RQ3 - What are the critical requirements on the process simulation platform, equipment models, experimental and process data to be used in the optimization system?

The following steps were needed to identify the requirements for using a simulation platform to carry out optimization application:

- *Identify the process simulation calibration and validation requirements*
- *Identify the equipment model calibration and validation requirements*
- *Investigate the opportunities and limitations of experimental and production data collection processes*
- *Investigate methods for evaluating production data reliability*

Optimization routines are implemented with an interaction to the dynamic process simulation for the crushing plant. The reliability of the optimization results is a function of the configured, calibrated, validated, and verified process simulation. The reliability of the process simulation, in turn, is a function of the equipment model, and data used for the calibration and validation of the model. Papers E and F present the application of optimization methods for process calibration and power-based belt scale calibration, respectively.

To assure the reliability of the dynamic process simulation, a controlled method for equipment model (crusher and screen) calibration using a computationally effective optimization method for multi-point test conditions was presented in Paper E. A novel approach to defining optimization problem formulation with a weighted function for a cone crusher model is shown. The weighted function enables compensation of the distribution of the data points available at different sieve size ranges to generate a good model fit. Both the objective function for crusher and screen model fitting optimization problem formulation use frequency distribution data for the sieve size range rather than the use of cumulative distribution. A method for controlled experimental design for collecting multiple forms of data (belt-cut samples and production data) was demonstrated. The validation of the process simulation was performed by comparison of the root mean square error (RMSE) values between the results of the dynamic process simulation and the collected production data. During the investigation process, the complex relationships and interdependencies between crusher performance and screen performance were also highlighted. The implication of the changing screen performance with respect to the crusher products leads to material quality (grade) change for the aggregates.

Any process improvements and process optimization performed at a crushing plant need to be evaluated with the use of production data. One type of production data critical for the crushing plant operation is the mass flow data. In Paper F, an application of an optimization method for calibration of accessible and non-accessible power-based belt scale units for an industrial crushing plant is demonstrated. The calibration process is a two-fold process. Firstly, physical measurements were performed on the accessible conveyors to calibrate the unknown factors for the power-based belt scale by minimizing the error between the physical measurements and the recorded power data. The second process is the utilization of the mass balancing property of the crushing plant operation to estimate the unknown factors for the non-accessible conveyors. Further on, a novel approach to tracking the deviation in mass flow during operation using a correlation matrix is presented. Data sanity check is an important step in utilization for model calibration, simulation process validation, quantification of process improvements and optimization.

In essence, the use of optimization problem formulations in a standard form such as the negative-null form for both model calibration and sensor calibration is required for the replication of the results. Gradient-based optimization algorithms are found to be viable for solving such problems. Learning from the applied optimization methods showed that the knowledge about the system under study is generated iteratively. The formulation of optimization problems as constrained or unconstrained depends on the understanding of the physical system and knowledge about the model and its limitations.

5.2 Research Validity

The research presented in this thesis is a combination of explorative and confirmative studies employing both simulation-based studies and experimental-based studies. The research output presented includes multiple interdisciplinary fields and a subjective approach is applied to evaluate the research validity.

Multiple optimization methods have been explored at different abstraction levels within the development of optimization capabilities for crushing plants. The developed and applied optimization methods are based on a theoretical understanding of the optimization concepts [19, 31, 35, 37] and previous research work within aggregates and minerals processing simulation [Papers A-F]. The optimization studies were evaluated using certain testing criteria such as convergence analysis and comparison of the results by different methods [Papers B, C]. The development of KPIs was based on the ISO 22400 standard and the results of the simulation studies were compared to the physical system [Paper D]. The calibration and validation studies for the crushing plant simulation shown in Paper E were experimentally based together with a reliance on a theoretical framework of models and optimization methods. These arguments support the high internal validity [27] of the results obtained from the individual studies conducted for this thesis. The optimization methods applied [Papers B, C, E, F] and developed KPI models [Paper D] have external validity [27] as they can be transferred to other simulation studies within other comminution and classification processes, with necessary modifications.

The use of previous theoretical knowledge also supports the structural validity of the results [28], although the specific results and values obtained in different studies were from crushing plant sites and case-study specific. The results in Papers E and F were quantitively evaluated and compared with experimental data, leading to high performance validity of the methods demonstrated. The comparison of different optimization methods in each simulation study was carried out at equivalent parameter settings of the underlying process simulation, thus allowing fair reliability of the comparative results obtained in Papers B and C [30]. Systems thinking together with the problem-based approach helped in compartmentalizing each research problem in the thesis. This also assisted in the modularization of the solution. Limited system integration of different developed methods is performed at this stage and will be the next natural step in the development.

5.3 Industrial and Optimization Relevance

Optimization capabilities for crushing plants is a system solution with the two-fold application of:

- Utilizing the simulation platform for identification and exploration of operational settings based on the stakeholder's need to generate knowledge about the process operation [Papers A, B, C and D].
- Assuring the reliability of equipment models, process models and production data to create validated process simulations that can be utilized for process optimization and performance improvements [Papers E and F].

The simulation platform can be utilized to explore the potential and limits of the physical operation with an aim towards process optimization and process improvements. The underlying simulation model and data need to be validated, which requires experimental procedures, suitable optimization methods and automation for increased utilization. This further entails development in a digitally integrated solution wherein the simulation platform can continuously interact with production data, and optimization methods can reside in the simulation system to assist a user in plant operation.

Optimization is a misused term that can be found in many pieces of literature, reports, communication, etc. and is often misinterpreted as referring to improvements, which represent the betterment of a solution as compared to another solution. Process improvement is an iterative process that can be carried out based on experience and understanding of the process, while process optimization is a mathematical process. Process optimization can be carried out by utilizing the optimization methods together with the underlying simulation and performance measurement system. The data collection system together with the real-time performance calculations can help in the distinction of benefits of the process improvement and process optimization.

For operations purposes, the user, such as the operator or plant manager, can set the goal of the optimization using either the historical performance or new requirements to find recommended operational settings. It could also be used as an explorative tool to get recommended settings for new requirements and to produce trade-offs between conflicting objective functions. For design purposes, the optimization can be used to explore the performance boundaries of a new process layout.

Application of optimization needs to be transparent with clear boundary conditions. To clearly define objectives, constraints, and design variables, it is necessary to present the optimization problem formulation in a standard form (e.g., negative null form). This also assists in making practical choices in the industrial application as the solution can be debated, technically analysed, and evaluated for a suitable use case.

Simulation and equipment model calibration and validation are currently performed by only a limited number of experts in research and companies. The development of suitable optimization methods within a simulation system can enable the democratization of such processes, making them simple and easy to perform for a wider set of users.

The reality of a crushing plant operation is of course much more complex than described in the simulation environment. Dynamic simulation is shown to be suitable for use in both KPI calculation and optimization application as it captures a closer approximation of the real situation than is obtained with steady-state simulation. The KPI improvements and optimization are performed using bulk performance measurements for a user-specified time duration, although the reality is always dynamic. It is also computationally not viable to optimize every time step in process operation, while it is essential to assess the findings in dynamic simulation. Changes in the operational settings of the process can influence the stability of the process. This can occur due to equipment capacity limits, the configuration of conveyors, plant layout, etc., which can be evaluated in a dynamic simulation. The output of the operation can also change over time due to the maintenance status, wear, material change, etc. of the crushing plant, for which dynamic simulation is suitable for evaluation. Currently, the use of KPIs is very popular in the industry to make decisions and there is a need for customization for individual company demands. The calculation of the KPIs relies on the production data and a decision made using KPIs is as good as the range and quality of the data collected.

5.4 Future Work

The thesis presented multiple methods in a toolbox that can be implemented at different abstraction levels for developing optimization capabilities in crushing plants. The methods developed are based on individual studies performed with different industrial partners. The following presents a list of research and developments that can be undertaken to bring further insights to the framework of optimization capability for crushing plants:

5.4.1 Method Development

- Production data characterization techniques: This relates to determining and evaluating the different requirements on the capture of production data of a crushing plant. This can include developing guidelines and methods for handling data and evaluating data robustness for different use cases. This in turn can enable the translation of continuous production data for different uses: evaluating optimization results, key performance indicator mapping, environmental impact calculations, model calibration, model validation, etc.
- Equipment Modelling: The modification of existing equipment models used in process simulation to comply with the production data instead of laboratory data can be performed. This can eliminate the need for expensive experimental work performed in the industry. New model types such as machine learning can also be explored for such applications.
- Simulation process calibration using continuous production data can enable maintenance of the reliability of the results. This can be developed by performing a controlled experimental procedure for crushing plants to collect production data together with applying suitable optimization methods. Dynamic simulation of crushing plants is appropriate for such applications. The development can further enable a cost-effective and seamless method of simulation use for the industry.
- Optimization objective functions can be developed to include aspects such as demand, sales, cost, revenue, and environment for aggregates production. This is further needed to create a range of objective function libraries that can be used by different stakeholders involved in the crushing plant. The use of other gradient-based optimization methods can be explored.

5.4.2 Method Implementation

- Integration of individual components of the optimization capabilities for crushing plants in an IT solution can be performed. This is a development process that requires close industrial collaboration and can lead to the development of new product features.

- Creating an industrial case study for end-to-end implementation of optimization results in an industrial use context is required for testing the framework presented in this thesis.

The methods applied in the thesis were developed by focusing on the aggregates processing industry. There are possibilities to extend the methods for the mineral processing industry, which needs a detailed investigation. The optimization functions developed at this stage are directed towards aggregates production, and the transferability of these to minerals processing in mining will be studied in future work.

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