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Nanomechanics driven by the superconducting proximity effect

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Abstract

We consider a nanoelectromechanical weak link composed of a carbon nanotube suspended above a trench in a normal metal electrode and positioned in a gap between two superconducting leads. The nanotube is treated as a movable single-level quantum dot (QD) in which the position-dependent superconducting order parameter is induced as a result of Cooper pair tunneling. We show that in such a system, self-sustained bending vibrations can emerge if a bias voltage is applied between normal and superconducting electrodes. The occurrence of this effect crucially depends on the direction of the bias voltage and the relative position of the QD level. We also demonstrate that the nanotube vibrations strongly affect the dc current through the system, a characteristic that can be used for the direct experimental observation of the predicted phenomenon.

1. Introduction

Nanoelectromechanical systems (NEMS) provide a promising platform for investigations into the quantum mechanical interplay between mechanical and electronic subsystems [1, 2]. One of the most important phenomena providing the foundation of NEMS functionality is the generation of self-sustained mechanical oscillations by a dc flow [3–8]. This effect is itself an interesting problem from a fundamental point of view, opening new possibilities for mass and force sensing [9, 10], while its underlying physical processes show potential applications for mechanical cooling [11]. Self-sustained mechanical oscillations were first observed in a carbon nanotube (CNT)-based transistor [12], with further studies later verifying their transport signatures [13–15]. Recently, the experimental observation of self-driven oscillations of a CNT-based quantum dot (QD) in the Coulomb blockade regime has been reported [16].

Superconducting elements incorporated into NEMS extend the horizon of this phenomenon, namely through the effects of superconducting phase coherence; see, for example, the following reviews [17, 18]. Interplay between electromechanical effects and phase coherence gives new and unusual properties to a number of normal metal/superconducting hybrid junctions [19–21]. In particular, it has recently been shown that in a normal metal–suspended CNT–superconductor transistor, Andreev reflection [22, 23] may give rise to a cooling of the mechanical subsystem [24] or generate a single-atom lasing effect [25] if certain conditions are fulfilled.

The mechanical functionality of NEMS is to a large extent determined by the physical principles underlying the interaction between the electronic and mechanical subsystems. In all studies mentioned above, this interaction was due to the *localization* of the charge [4–6] or spin [7, 8] carried by electrons in the movable part of the system. In the present paper, we consider a fundamentally new type of electromechanical coupling based on the *quantum delocalization* of Cooper pairs. We demonstrate that such

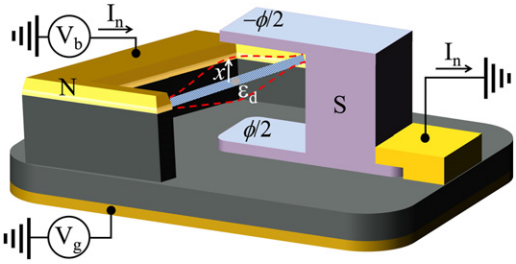

Figure 1. Schematic illustration of the nanoelectromechanical device under consideration. A CNT is suspended in a gap between two edges of a normal electrode (N) and tunnel-coupled to it. The electronic energy levels of the CNT are tuned such that only one energy level with energy ε_d, which is well separated from the other levels, is considered. Bending of the CNT in the x direction between two superconducting leads (S) affects the values of the tunneling barriers between them. Bias voltage V_b is applied to the normal electrode.

coupling can promote a self-saturated mechanical instability resulting in the generation of *self-sustained* mechanical oscillations. It is also shown that these oscillations significantly increase the average current through the system, making it possible to carry out direct experimental detection.

2. Model and dynamics

A sketch of the NEMS investigated in this paper is presented in figure 1. A single-walled CNT is suspended above a trench in a bulk normal metal electrode biased by a constant voltage V_b. Two superconducting leads with the superconducting phase difference ϕ are positioned near the middle of the nanotube in such a way that the bending of the nanotube moves it closer to one electrode and further away from the other. We consider a case when the nanotube is short ($L \lesssim 1\ \mu$m), thus the distances between the quantized electronic levels inside the nanotube are much greater than the other energy parameters, allowing one to treat it as a single-level QD. The bending dynamics of the CNT are reduced to the dynamics of the fundamental flexural mode. We suppose that the amplitude of this mode, x, is larger than the amplitude of zero-point oscillations, and will consider it as a classical mechanical oscillator with mass m and frequency ω. The dynamics is described by Newton's equation,

$$\ddot{x} + \omega^2 x = -\frac{1}{m}\,\mathrm{Tr}\left\{\hat{\rho}\frac{\partial H(x)}{\partial x}\right\}, \tag{1}$$

where

$$H = H_d + H_l + H_t \tag{2}$$

is the Hamiltonian of the electronic subsystem. The first term H_d represents the single-level QD,

$$H_d = \sum_\sigma \varepsilon_d d_\sigma^\dagger d_\sigma, \tag{3}$$

where $d_\sigma^\dagger(d_\sigma)$ is the creation (annihilation) operator of an electron with spin projection $\sigma = \uparrow, \downarrow$ on the dot. The Hamiltonian $H_l = H_l^n + H_l^s$ describes the normal and superconducting leads, respectively, with

$$H_l^n = \sum_{k\sigma} (\varepsilon_k - eV_b) a_{k\sigma}^\dagger a_{k\sigma}, \tag{4}$$

$$H_l^s = \sum_{kj\sigma} \left(\varepsilon_k c_{kj\sigma}^\dagger c_{kj\sigma} - \Delta_s (e^{\imath\phi_j} c_{kj\uparrow}^\dagger c_{-kj\downarrow}^\dagger + \mathrm{h.c.})\right). \tag{5}$$

Here, $a_{k\sigma}^\dagger(a_{k\sigma})$, and $c_{kj\sigma}^\dagger(c_{k\sigma})$ are the creation (annihilation) operators of an electron with quantum number k and spin projection σ in the normal and superconducting $j = 1, 2$ leads, respectively, and $\Delta_s e^{\imath\phi_j}$ is the superconducting order parameter (in the j electrode). Note that the energies $\varepsilon_d, \varepsilon_k$ are counted from the Fermi energy of the superconductors. In what follows, we set $\phi_1 = -\phi_2 = \phi/2$.

The Hamiltonian $H_t = H_t^n + H_t^s$ describes the tunneling of electrons between the dot and the leads, where

$$H_t^n = \sum_{k\sigma} t_0^n (a_{k\sigma}^\dagger d_\sigma + \mathrm{h.c.}), \tag{6}$$

$$H_{\rm t}^{\rm s} = \sum_{kj\sigma} t_j^{\rm s}(x)(c_{kj\sigma}^{\dagger} d_\sigma + \text{h.c.}). \tag{7}$$

The position-dependent superconducting tunneling amplitude $t_{1(2)}^{\rm s}(x) = t_0^{\rm s}\, {\rm e}^{(-1)^j(x+a)/2\lambda}$, where 2λ is the characteristic tunneling length and a is a parameter for asymmetry. For a typical CNT-based nanomechanical resonator, $2\lambda \sim 0.5$ nm [26]. We concentrate our attention on the symmetric case $a = 0$ and leave the asymmetric one for discussion elsewhere.

The time evolution of the electronic density matrix $\hat{\rho}$ is described by the Liouville–von Neumann equation ($\hbar = 1$),

$$\imath \partial_t \hat{\rho} = [H, \hat{\rho}], \tag{8}$$

which together with equation (1) forms a closed system of equations that describe the nanoelectromechanics of our system. In this paper, we restrict ourselves to the case $\Delta_{\rm s} \gg |eV_{\rm b}| \gg \Delta_{\rm d} \sim \Gamma_{\rm n}$, where $\Delta_{\rm d} = (2\pi)\nu_{\rm s}|t_0^{\rm s}|^2$ and $\Gamma_{\rm n} = (2\pi)\nu_{\rm n}|t_0^{\rm n}|^2$, with $\nu_{\rm s(n)}$ the density of states in the superconducting (normal) electrode.

To describe the electronic dynamics of the QD, we use the reduced density matrix approximation in which the full density matrix of the system is factorized to the tensor product of the equilibrium density matrices of the normal and superconducting leads and the density matrix of the dot as $\hat{\rho} = \hat{\rho}_{\rm n} \otimes \hat{\rho}_{\rm s} \otimes \hat{\rho}_{\rm d}$. Using the standard procedure, one can trace out the degrees of freedom of the leads and obtain the following equation for the reduced density matrix $\hat{\rho}_{\rm d}$ [6] (in the deep subgap regime $\Delta_{\rm s} \to \infty$),

$$\partial_t \hat{\rho}_{\rm d} = -\imath \left[H_{\rm d}^{\rm eff}, \hat{\rho}_{\rm d}\right] + \mathcal{L}_{\rm n}\{\hat{\rho}_{\rm d}\}, \tag{9}$$

where

$$H_{\rm d}^{\rm eff} = H_{\rm d} + \Delta_{\rm d}(x,\phi) d_\downarrow d_\uparrow + \Delta_{\rm d}^*(x,\phi) d_\uparrow^\dagger d_\downarrow^\dagger, \tag{10}$$

$$\Delta_{\rm d}(x,\phi) = \Delta'(x,\phi) + {\rm i}\Delta''(x,\phi) = \Delta_{\rm d}\, \cosh(x/\lambda + {\rm i}\phi/2).$$

Above, $\Delta_{\rm d}(x,\phi)$ is the off-diagonal order parameter induced by the superconducting proximity effect [24, 27], and $\Delta'^{,''}(x,\phi)$ are real functions. The Lindbladian term in equation (9) reflects the incoherent electron exchange between the normal lead and QD. The latter in the high bias voltage regime, $|eV_{\rm b}| \gg \varepsilon_0, k_{\rm B}T$, takes the form

$$\mathcal{L}_{\rm n}\{\hat{\rho}_{\rm d}\} = \Gamma_{\rm n} \sum_\sigma \begin{cases} 2d_\sigma^\dagger \hat{\rho}_d d_\sigma - \left\{d_\sigma d_\sigma^\dagger, \hat{\rho}_d\right\}, & \kappa = +1; \\ 2d_\sigma \hat{\rho}_d d_\sigma^\dagger - \left\{d_\sigma^\dagger d_\sigma, \hat{\rho}_d\right\}, & \kappa = -1; \end{cases} \tag{11}$$

where $\kappa = {\rm sgn}(eV_{\rm b})$.

Figure 2 represents the electronic dynamics on the dot for $\kappa = \pm 1$. From figure 2, one can see that not all electron processes are allowed due to the parameter scales in this work. In the subgap regime, single-electron transitions between the dot and the superconducting leads are prohibited, and thus only an exchange of Cooper pairs occurs. Moreover, because of the high bias voltage, single-electron tunneling between the dot and the normal leads is enabled exclusively in one direction (from the lead to the dot, see figure 2(a), or vice versa, figure 2(b)), establishing that our model is electron–hole symmetric. As a consequence, the QD density matrix $\hat{\rho}_{\rm d}$ acts in the Hilbert space $\mathcal{H}_4$, which may be presented as a direct sum of two $\mathcal{H}_2$ spaces via $\mathcal{H}_4 = \mathcal{H}_{\rm e} \oplus \mathcal{H}_{\rm CP}$ spanned over state vectors $|\uparrow\rangle = d_\uparrow^\dagger|0\rangle$, $|\downarrow\rangle = d_\downarrow^\dagger|0\rangle$, and $|0\rangle$, $|2\rangle = d_\uparrow^\dagger d_\downarrow^\dagger|0\rangle$ (with $d_{\uparrow,\downarrow}|0\rangle = 0$).

The superselection rule, which forbids the superposition of states with integer and half-integer spins, allows us to present the density matrix $\hat{\rho}_{\rm d}$ as a direct sum of two density matrices $\hat{\rho}_{\rm d} = \hat{\rho}_{\rm e} \oplus \hat{\rho}_{\rm CP}$ acting in the $\mathcal{H}_2$ Hilbert space spanned over state vectors $|\uparrow\rangle, |\downarrow\rangle$ and $|0\rangle, |2\rangle$, respectively. Moreover, taking into account spin-rotation symmetry, one can conclude that $\hat{\rho}_{\rm e}$ should be proportional to the unit matrix, $\hat{\rho}_{\rm e} = \rho_{\rm e}\hat{I}$, while $\hat{\rho}_{\rm CP}$ can be written in the form $\hat{\rho}_{\rm CP} = \frac{1}{2}R_0\hat{I} + \frac{1}{2}\sum_i R_i\sigma_i$, where σ_i, $(i = 1, 2, 3)$ are the Pauli matrices.

Then by introducing the dimensionless time $\omega t \to t$ and displacement $x/\lambda \to x$, and taking into account the normalization condition Tr $\hat{\rho}_{\rm d} = 1$, we get the following closed system of equations for $x(t)$ and $R_i(t)$,

$$\ddot{x} + x = -\xi \left[\sinh(x)\cos\left(\frac{\phi}{2}\right) R_1 - \cosh(x)\sin\left(\frac{\phi}{2}\right) R_2\right], \tag{12}$$

$$\alpha \dot{\vec{R}} = \hat{L}\vec{R} - \kappa\vec{e}_3, \tag{13}$$

where $\vec{R} = (R_1, R_2, R_3)^{\rm T}$, $\vec{e_3} = (0,0,1)^{\rm T}$, $\xi = \Delta_{\rm d}/(m\lambda^2\omega^2)$ is the nanoelectromechanical coupling parameter, and $\alpha = \omega/(2\Gamma_{\rm n})$ is the adiabaticity parameter. For a typical CNT-based NEMS, one can

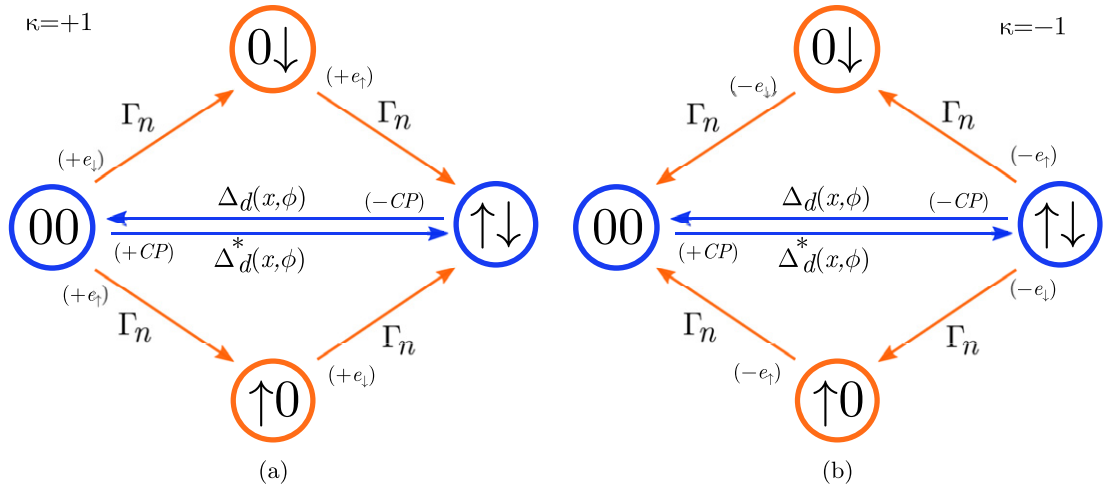

Figure 2. Diagrams representing the transitions between electronic states in the QD. The single-electron states change due to transitions from the empty to the single-occupied QD and then from the single-occupied to the double-occupied one (indicated by orange arrows). In the high bias voltage regime, the tunneling of electrons (a) or holes (b) with spin ↓ or ↑ is allowed only from the normal lead to the dot and forbidden in the opposite direction. Transitions between the empty and double-occupied QD are due to coupling with the superconducting leads (indicated by blue arrows).

estimate $\xi \sim 10^{-3} \ll 1$ [26, 28]. The matrix $\hat{L}$ is defined as follows,

$$\hat{L}(x) = \begin{pmatrix} -1 & \tilde{\varepsilon}_{\rm d} & -\tilde{\Delta}''(x,\phi) \\ -\tilde{\varepsilon}_{\rm d} & -1 & -\tilde{\Delta}'(x,\phi) \\ \tilde{\Delta}''(x,\phi) & \tilde{\Delta}'(x,\phi) & -1 \end{pmatrix}, \tag{14}$$

where $\tilde{\varepsilon}_{\rm d} \equiv \varepsilon_{\rm d}/\Gamma_{\rm n}, \tilde{\Delta}_{\rm d} \equiv \Delta_{\rm d}/\Gamma_{\rm n}$.

The system of equations (12) and (13) has an obvious static solution $x_{\rm st} = 0 + \mathcal{O}(\xi)$, $\vec{R}_{\rm st} = \kappa L^{-1}(0)\vec{e}_3 + \mathcal{O}(\xi)\vec{R}^{(1)}$, here $\|\vec{R}^{(1)}\| = 1$. The stability of this solution can then be investigated in a standard way, see for example reference [29]. However, to simplify this procedure, we will consider the adiabatic case when $\alpha \ll 1$, which corresponds to a typical experimental situation [16] and reduces the problem to one that allows the use of Poincare analysis. More specifically, this inequality allows one to find a solution of equation (13) to the accuracy α,

$$\vec{R}(x,t) = \kappa L^{-1}(x(t))(1 + \alpha\dot{x}\partial_x L^{-1}(x(t)) + \mathcal{O}(\alpha^2))\vec{e}_3, \tag{15}$$

and then substituting this solution into equation (12) gives (to accuracy α) the following nonlinear differential equation for $x(t)$,

$$\ddot{x} - \eta(x,\phi)\dot{x} + x = F(x,\phi), \tag{16}$$

the solution of which may be analyzed via Poincare's theory. Here, the nonlinear force $F(x,\phi)$ and friction coefficient (in what follows we refer to it as a pumping coefficient) $\eta(x,\phi)$ generated by interaction with the nonequilibrium electronic environment, take the form,

$$F(x,\phi) = \kappa\xi\frac{\tilde{\Delta}_{\rm d}}{2\tilde{\mathcal{D}}^2}\left[\sin\,\phi - \tilde{\varepsilon}_{\rm d}\,\sinh\,(2x)\right], \tag{17}$$

$$\eta(x,\phi) = \kappa\alpha\xi\frac{\tilde{\Delta}_{\rm d}}{2\tilde{\mathcal{D}}^6}\left[\tilde{\Delta}_{\rm d}^2\,\sinh(2x)\left(\tilde{\mathcal{D}}^2 + 4\right)\left\{\sin\,\phi - \tilde{\varepsilon}_{\rm d}\,\sinh(2x)\right\} + 8\tilde{\varepsilon}_d\tilde{\mathcal{D}}^2\left\{\sin^2(\phi/2) + \sinh^2 x\right\}\right]. \tag{18}$$

Here $\tilde{\mathcal{D}}^2 \equiv \mathcal{D}^2(x,\phi)/\Gamma_{\rm n}^2 = \tilde{\Delta}_{\rm d}^2\left[\sinh^2 x + \cos^2(\phi/2)\right] + \tilde{\varepsilon}_{\rm d}^2 + 1$.

3. Self-sustained oscillations

In order to find the stationary solutions $x_{\rm c}(t)$ in equation (16), it is natural to use the smallness of the parameter ξ and look for such solutions in the form $x_{\rm c}(t) = x_{\rm st} + \sqrt{A}\,\sin\,(t + \varphi(t)) + \mathcal{O}(\xi)$, where $x_{\rm st}, \dot{A}(t), \dot{\varphi}(t) \sim \xi$ (see appendix for details). Then, with the accepted accuracy ξ, one can get the following equations:

$$\dot{A} = A\bar{\eta}(A,\phi), \tag{19}$$

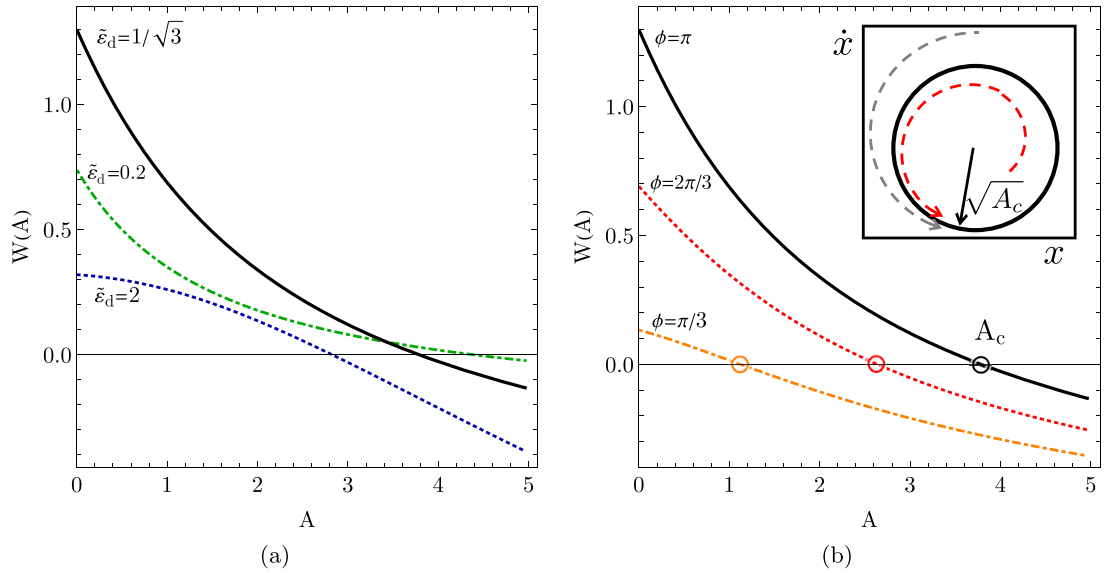

Figure 3. Plots of the function $W(A)$, proportional to the pumping coefficient $\bar{\eta}(A,\phi)$, for different values of (a) the relative position of the dot energy level $\tilde{\varepsilon}_d = 0.2; 1/\sqrt{3}; 2$ for $\phi = \pi$, and (b) the superconducting phase difference $\phi = \pi/3, 2\pi/3, \pi$ for $\tilde{\varepsilon}_d = 1/\sqrt{3}$. The zeroes of the functions correspond to the amplitude of the limiting cycle (see the inset in (b)), which strongly depends on the superconducting phase difference and reaches its maximum at $\phi = \pi$. The other parameters are $\tilde{\Delta}_d = 1, \kappa = +1$.

$$\dot{\varphi} = -A^{-1/2}\bar{F}(A,\phi), \tag{20}$$

where

$$\bar{\eta}(A,\phi) = (\pi)^{-1}\int_0^{2\pi} d\psi \cos^2(\psi)\eta\left(\sqrt{A}\,\sin\,\psi,\phi\right) \equiv \kappa\xi\alpha W(A,\phi), \tag{21}$$

$$\bar{F}(A,\phi) = (2\pi)^{-1}\int_0^{2\pi} d\psi\,\sin(\psi)F\left(\sqrt{A}\,\sin\,\psi,\phi\right). \tag{22}$$

The pumping coefficient $\bar{\eta}(A,\phi)$ has an obvious physical meaning: it gives the ratio between the energy supplied into the mechanical degree of freedom for one period of mechanical oscillation with amplitude $\sqrt{A}$ and the total mechanical energy.

It is evident from equation (19) that stationary regimes $\dot{A} = 0$ are given by equations $A = 0$ ($x(t) = x_{st}$) and $\bar{\eta}(A,\phi) = 0$. The first one is a static state of the nanotube, and the second one corresponds to periodic oscillations with the amplitude $\sqrt{A_c}$, where $W(A_c,\phi) = 0$. The static regime is stable when $\bar{\eta}(0,\phi) < 0$ and unstable otherwise. The stability of the periodic solution is defined by the sign of the derivative $\partial_A\bar{\eta}(A,\phi)|_{A=A_c}$: if it is negative (positive), then the periodic regime is stable (unstable). Analyzing equations (16) and (21), one can conclude that the pumping coefficient $\bar{\eta}(A,\phi) \propto \kappa$ is an odd function of ε_d and takes the following limit values,

$$\bar{\eta}(0,\phi) = \kappa\alpha\xi W(0,\phi) = +\kappa\alpha\xi\frac{4\tilde{\varepsilon}_d\tilde{\Delta}_d}{\tilde{D}^4(0,\phi)}\sin^2\left(\frac{\phi}{2}\right), \tag{23}$$

$$\bar{\eta}(A\to\infty,\phi) = \kappa\alpha\xi W(A\to\infty,\phi) = -\kappa\alpha\xi\frac{\tilde{\varepsilon}_d}{2\tilde{\Delta}_d}, \tag{24}$$

from which follows that at $\phi \neq 0$, the solution of the equation $W(A_c,\phi) = 0$, corresponding to the stationary periodic regime, exists at any values of the other parameters. However, at small $\phi \ll 1$ the pumping coefficient, $\eta(x,\phi)$, determined by interaction with the electronic subsystem, is quite small and is equalized by the friction coefficient ($\propto Q^{-1}$, where Q is the quality factor determined by interaction with the thermodynamic environment [12]). The case when $\phi = 0$ is very unstable with respect to the small asymmetry parameter $|a| \ll 1$ (see below equation (7)) and will be analyzed elsewhere. The function $W(A)$ and A_c at different $\phi \geqslant 1$ and $\tilde{\varepsilon}_d > 0$ are presented in figures 3 and 4.

From equation (23), it follows that if $\kappa\varepsilon_d > 0$ and $\phi \neq 0$, the static mechanical state $x = x_{st} \ll 1$ is unstable with respect to the appearance of bending oscillations with amplitudes that exponentially increase in time with the increment $\gamma = \kappa\alpha\xi W(0,\phi)$. The latter takes its maximum at $\phi = \pi$ for the fixed values of

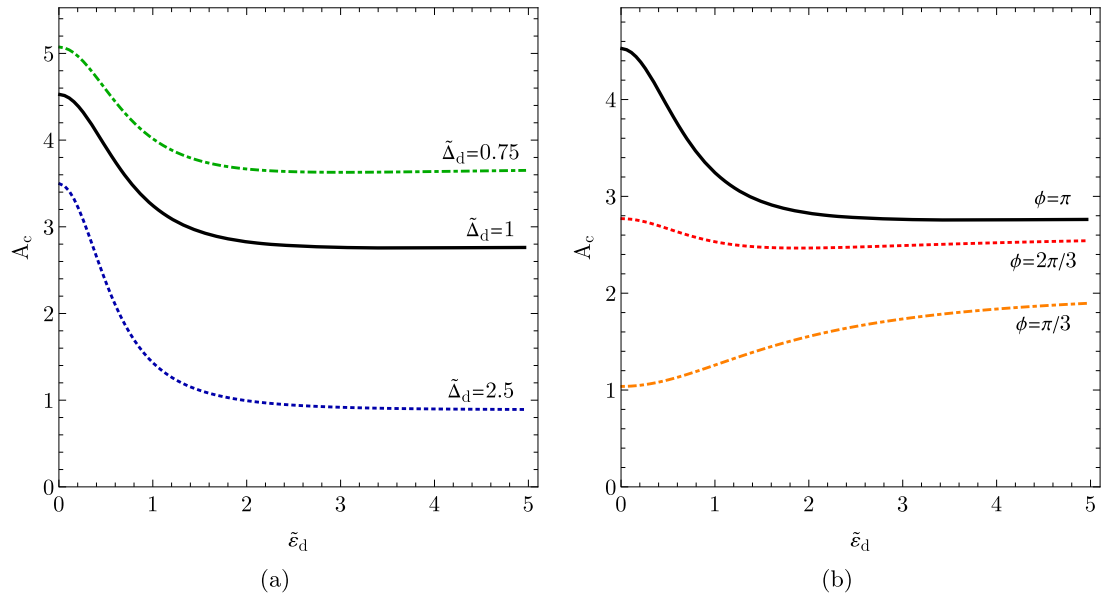

Figure 4. Dependencies of A_c of the limiting cycles on the relative position of the dot energy level $\tilde{\varepsilon}_d$ (counted from the Fermi energy) for different values of (a) $\tilde{\Delta}_d = 0.75; 1; 2.5$ for $\phi = \pi$, and (b) the superconducting phase difference $\phi = \pi/3, 2\pi/3; \pi$ for $\tilde{\Delta}_d = 1$.

all other parameters (notice that $x_{st}(\pi) = 0$). However, the increase saturates at the amplitude $\sqrt{A_c}$, resulting in self-sustained oscillations at this amplitude. It should be noted that the amplitude saturation in the system under consideration is a completely internal effect and still takes place when the friction caused by interaction with a thermodynamic environment is zero. A 'self-saturation' effect was also reported in [8] where a special magnetic NEM system was considered.

4. Electric current

The self-sustained oscillations considered above have a very specific transport signature. This raises the possibility of detecting the mechanical instability through electric current measurement. To explore such a possibility, let us consider the electric current through the system I_n, determined in a standard way, $I_n = e\kappa\ \mathrm{Tr}\left\{\hat{\dot{N}}\hat{\rho}\right\}$, where $\hat{\dot{N}} = \mathrm{i}[\hat{H}, \hat{N}]$ and $\hat{N}$ is the operator of the number of electrons in the normal electrode. In the deep subgap regime considered above, one can neglect quasiparticle current since it is exponentially small. As a result, in the high bias voltage regime at $\kappa = +1$, where electron tunneling from the QD to the normal leads is forbidden, an expression for I_n can be easily obtained by analyzing figure 2. From those diagrams, one can see that a decrease in the number of electrons in the normal electrode is defined by two different processes. The first one is the tunneling of an electron with spin up or down into the empty dot. The rate of this process is $2\Gamma_n\rho_0$, where $\rho_0 = (R_0 + R_3)/2$ is the probability that the dot is empty. The second one is the tunneling of an electron into the dot occupied by a single electron with spin up or down. The rate of this process is $2\Gamma_n\rho_e$. Taking into account the normalization condition $2\rho_e + R_0 = 1$, and using a similar speculation for $eV_b < 0$, one gets from equation (15) the following equation for I_n,

$$I_n(t) = \kappa I_0\,(1 + \kappa R_3) = \kappa I_0\left[\frac{|\Delta_d(x,\phi)|^2}{\mathcal{D}^2(x,\phi)} + \alpha\dot{x}f(x) + \mathcal{O}(\alpha^2)\right], \tag{25}$$

where $I_0 = e\Gamma_n$. In the stationary regime corresponding to the generation of self-sustained oscillations with amplitude $\sqrt{A_c}$, the average electric current $\bar{I}_n$ is defined by the equation

$$\bar{I}_n(\kappa,\varepsilon_d) = \kappa I_0\left[\frac{\Delta_d^2\cos^2(\phi/2)}{\Delta_d^2\cos^2(\phi/2) + \Gamma_n^2 + \varepsilon_d^2} + \theta(\kappa\varepsilon_d)\delta\bar{I}(A_c)\right], \tag{26}$$

where the first term corresponds to the static dc current, which crucially depends on the superconducting phase difference ϕ. In particular, the first term is equal to zero at $\phi = \pi$, in contrast to the second term,

$$\delta\bar{I}(A_c) = \frac{1}{2\pi}\frac{\Delta_d^2(\Gamma_n^2 + \varepsilon_d^2)}{\mathcal{D}^2(0,\phi)}\int_0^{2\pi} d\psi \frac{\sinh^2\left(\sqrt{A_c}\,\sin\,\psi\right)}{\mathcal{D}^2\left(\sqrt{A_c}\,\sin\,\psi,\phi\right)} > 0, \tag{27}$$

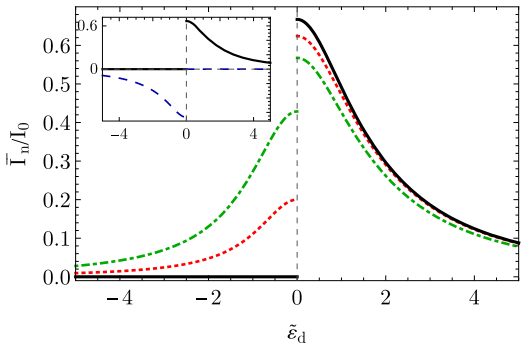

Figure 5. Dependencies of the dc electric current $\bar{I}_n$ (normalized to $I_0 = e\Gamma_n$) on the relative position of the QD energy level $\tilde{\varepsilon}_d$ for different values of superconducting phase difference $\phi = \pi/3$ (green dot–dashed curve), $2\pi/3$ (red dotted), and π (black solid) for $\tilde{\Delta}_d = 1$ and $\kappa = +1$. The maximum effect occurs at $\phi = \pi$ when the dc current is absent in the static regime, while it is close to the maximum one in the stable stationary regime of the self-sustained oscillations. Inset: dependencies of the time-averaged electric current for $\kappa = +1$ (black solid curve, associated with the same one in the main plot) and for $\kappa = -1$ when the bias voltage is applied in the opposite direction (blue dashed curve), representing a diode-like behaviour of the current.

which emerges exclusively due to the self-sustained oscillations and equals zero if the static state is stable, as indicated by the Heaviside step function $\theta(\kappa\varepsilon_d)$. Plots of $\bar{I}_n$ as a function of $\tilde{\varepsilon}_d$ at $\tilde{\Delta}_d = 1$ and $\phi = \pi/3, 2\pi/3, \pi$ are presented in figure 5. These graphs show that the nanomechanical instability discussed in this article leads to the emergence of significant diode and transistor effects. The effects are most pronounced at $\phi = \pi$ when in the static regime $A_c = 0$ where the Cooper pair exchange between the dot and the superconducting leads is completely blocked. In such a situation, a jump in the average current from zero to a finite value $\sim I_0$ (or vice versa) occurs if the direction of the bias voltage changes (diode effect) or if the position of the level ε_d controlled by the gate voltage passes zero (transistor effect). Note that the discontinuity of the average current as a function of ε_d must be treated to the accuracy ξ accepted in this paper.

5. Conclusions

In summary, we considered a NEMS comprising a CNT suspended above a trench in a normal metal electrode that undergoes bending vibrations in the gap between two superconductors. The nanotube was treated as a movable single-level QD in which the superconducting order parameter is induced as a result of Cooper pair exchange with the superconductors. The latter essentially depends on the bending of the nanotube and the phase difference between the superconductors. We have shown that in such a system, the static, straight configuration of the nanotube is unstable regarding the occurrence of self-sustained bending vibrations in a wide range of parameters if a bias voltage is applied between the normal and superconducting leads. It was demonstrated that the occurrence of this instability crucially depends on the direction of the bias voltage and the relative position of the QD level. This makes it possible to govern the operating mode of the system by changing the bias and gate voltages. We have also shown that the appearance of self-sustained mechanical vibrations strongly affects the dc current through the system, leading to transistor and diode effects. The latter can be used for the direct experimental observation of the predicted phenomena.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Appendix. Krylov–Bogoliubov averaging method

Here we present details of the derivation of equations (19) and (20). The equation (16) can be treated as the one describing the harmonic oscillator slightly perturbed by the interaction with the electronic subsystem,

$$\ddot{x} + x = \xi f(\dot{x}, x). \tag{A.1}$$

In such a case of small electromechanical coupling $\xi \ll 1$, one can use the Krylov–Bogoliubov method of averaging [30] (akin to the Van der Pol averaging method) to find an approximate solution of the equation (16). Following it, in the stationary regime of developed self-oscillations the dot displacement x_c takes a form,

$$x_c(t) = x_{st} + \sqrt{A}\,\sin\left(t + \varphi(t)\right) + \mathcal{O}(\xi), \tag{A.2}$$

where the amplitude $\sqrt{A(t)}$ and the phase $\varphi(t)$ vary slowly in time. Then, by substituting the ansatz (A.2) into equation (16) and assuming the same functional form for the derivative $\dot{x}_c$, one can find following equations for $A(t)$ and $\varphi(t)$,

$$\dot{A} = 2A\cos^2\psi\eta\left(\sqrt{A}\,\sin\,\psi, \phi\right), \tag{A.3}$$

$$\dot{\varphi} = -A^{-1/2}\,\sin\,\psi F\left(\sqrt{A}\,\sin\,\psi, \phi\right). \tag{A.4}$$

Since the amplitude and the phase change scarcely during a period, the rhs of equations (A.3) and (A.4) can be replaced by their average over the period of oscillations, and as a result, one obtain equations (19) and (22).

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