# Sensitivity analysis of beamforming techniques for periodic broadcast V2V communication

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# Sensitivity Analysis of Beamforming Techniques for Periodic Broadcast V2V Communication

Chouaib Bencheikh Lehocine, Fredrik Brännström, and Erik G. Ström, *Fellow, IEEE*

*Abstract*—In this work, we extend the results of two previously proposed transmit beamforming techniques for periodic, broadcast, vehicle-to-vehicle communication with a common fixed broadcast period, to the scenario where vehicular users (VUs) use different, and potentially varying broadcast periods. The two techniques, analog beamforming network (ABN) of phase shifters and antenna switching network (ASN), were previously developed in accordance with a multiple antenna receiver that employs an analog combining network (ACN) of phase shifters. To accommodate the use of multiple broadcast periods, we propose the design of phase shifter parameters of ABN-ACN and ASN-ACN systems using a design period $T_\mathrm{d}$. Then, we analytically derive sets of broadcast periods that sustain optimality, in the sense that the sum signal-to-noise ratio (SNR) of $K$ consecutive packets for any receiving VU is maximized. Next, we provide guidelines on how to set $T_\mathrm{d}$ to ensure a sufficient granularity of the sets of optimal broadcast periods. Finally, we investigate using numerical computations the effect of certain design choices on the sensitivity of ABN/ASN-ACN systems to small variations of the broadcast period.

*Index Terms*—Broadcast Vehicle-to-Vehicle communication, periodic communication, Multiple antennas.

## I. Introduction

Cooperative intelligent transportation systems (C-ITS) rely on the exchange of periodic, broadcast cooperative awareness messages (CAMs). CAMs require an antenna system with good reception in all azimuth directions. Due to antenna pattern distortions caused by factors like vehicle body, antennas placement, and antennas housing [1], the cooperative service can suffer from deterioration due to alignment of the received signal with angles where the antenna system has very low gain. In [2], an analog combining network (ACN) of phase shifters that combines multiple antenna signals in analog domain has been proposed to ensure robustness of the system against the aforementioned events. This has been extended to the transmit side in [3], where, assuming receiving vehicular users (VUs) that are equipped with ACN, two transmit multiple antenna schemes: analog beamforming network (ABN) and antenna switching network (ASN), have been proposed. ABN/ASN are fully analog, low-cost, schemes, which set them apart from the typical multiple transmit antenna schemes, e.g., Alamouti [4], cyclic delay diversity (CDD) [5], etc., which require digital

processing. In addition, ACN at the receiver (Rx) is both a low-cost and low-complexity solution, compared to standard digital schemes like maximal ratio combining (MRC) [6], since ACN does not rely on channel state information.

To design ABN/ASN systems, a performance metric labelled burst error probability (BrEP), and more specifically its equivalent—under certain assumptions, the sum of signal-to-noise ratio (SNR) is used. BrEP is the probability of losing $K$ consecutive CAM packets. The metric is related to age-of-information (AoI) [7], defined as the the age of the status information contained in the last correctly received CAM. AoI and BrEP take into account the reliability and latency as perceived by a C-ITS application that relies on the content of CAMs, and thus are deemed more suitable metrics than packet error rate in this scenario of periodic vehicle-to-vehicle (V2V) communication. In absence of channel state information, phase shifters parameters for ABN/ASN systems were derived in [3] to maximize the sum-SNR for the worst receiving VU. Those phase slopes were found to ensure optimal performance for all receiving VUs, and not only the worst one. The proposed networks parameters in [2], [3] were derived in a scenario where all vehicles broadcast their CAMs using the same, fixed period. In this work, we show how to design ABN/ASN systems in a scenario where different VUs use different, and potentially varying, broadcast periods. Our main contributions can be summarized as follows.

- We design the ABN/ASN phase shifters for all VUs to be optimal for a fixed design broadcast period $T_\mathrm{d}$, and analytically derive sets of broadcast periods that VUs can choose their CAMs repetition interval from and still retain optimal performance. The sets are derived for any number and type of antennas used by the VUs.
- We give design guidelines on how to set $T_\mathrm{d}$ to ensure dense sets of optimal broadcast periods.
- Using numerical computation and example antenna radiation patterns, we demonstrate the effect of the choice of $T_\mathrm{d}$ on the sensitivity of the ABN/ASN to small variations of the broadcast periods around the optimal values.

## II. System Model

### A. Data Traffic Model

VUs broadcast CAMs that include their status information like position, speed, heading, etc. The generation interval of CAMs is in the range of $0.1 \leq T \leq 1$ s, and it is set following the originating vehicle dynamics [8]. A VU checks its current speed, position and heading, at least every 0.1 s,

(a) Single, fixed broadcast period



(b) Multiple broadcast periods

Fig. 1. CAM broadcast period scenarios.



(a) Analog beamforming network of phase shifters (ABN)



(b) Transmit antenna switching network (ASN)
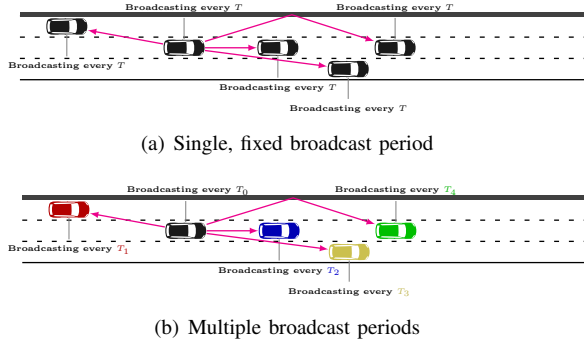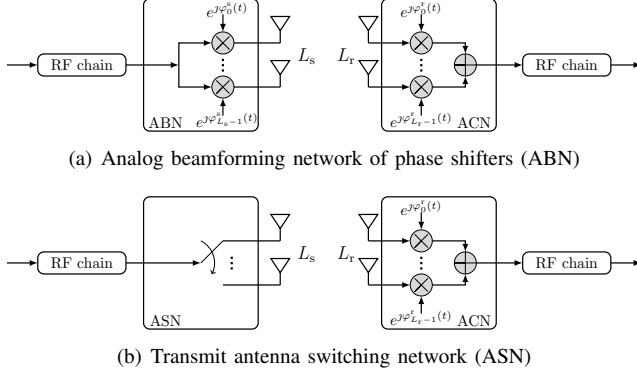
Fig. 2. Multiple antenna schemes.

and compares it with respect to the information included in the last generated CAM. A new CAM is generated whenever the change of dynamics exceeds certain thresholds (position difference $\geq 4$ m, absolute speed difference $\geq 0.5$ m/s, or heading difference $\geq 4$ deg). Besides that, a C-ITS application can request a particular CAM generation interval overriding the previously mentioned trigger conditions [8]. Whether an application requests a broadcast interval, or it is set depending on the vehicles dynamics, it has to be greater than, or equal to the minimum period defined according to the channel load status [8].

In our study, we consider two scenarios that are shown in Fig. 1. The first one is a simplified scenario where all VUs use a common fixed broadcast period $T \in [0.1, 1]$ s. The second one corresponds to a practical scenario where VUs broadcast their CAMs using different, and potentially varying periods $T_0, T_1, T_2, \ldots, \in [0.1, 1]$ s. Note that for the communication to be periodic, we assume that the same value of periods is sustained for for at least $K$ consecutive packets before it is allowed to change[1].

### B. Multiple Antenna Schemes: ABN, ASN and ACN

The multiple antenna schemes considered in this work are shown in Fig. 2. Let VUs employ $L_r$ antennas and an ACN of phase shifters [2] at the Rx. Phase shifters are modeled as

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[1]This can be enforced by setting a parameter that ensures sustaining the same generation interval for consecutive CAMs [8].

affine functions of time, and the ACN combining vector is given by

$$[\boldsymbol{w}]_l = \mathrm{e}^{-\jmath \overbrace{(\alpha_l^{\mathrm{r}} t + \beta_l^{\mathrm{r}})}^{\varphi_l^{\mathrm{r}}(t)}}, \quad 0 \leq l \leq L_{\mathrm{r}} - 1, \qquad (1)$$

where $\alpha_l^{\mathrm{r}} \in \mathbb{R}$ is a phase slope, and $\beta_l^{\mathrm{r}} \in [0, 2\pi)$ is initial, unknown phase offset. At the transmitter (Tx), VUs are equipped with $L_{\mathrm{s}}$ antennas, and either an ABN of phase shifters or an ASN.

(i) ABN: Similarly to ACN, every antenna output is phase shifted by $\varphi_m^{\mathrm{s}}(t) = (\alpha_m^{\mathrm{s}} t + \beta_m^{\mathrm{s}})$, where $\alpha_m^{\mathrm{s}} \in \mathbb{R}$, and $\beta_m^{\mathrm{s}} \in [0, 2\pi)$ denote the phase slope, and the initial unknown phase offset, respectively. We can model the beamforming vector as

$$[\boldsymbol{b}]_m = \frac{1}{\sqrt{L_{\mathrm{s}}}} \mathrm{e}^{\jmath \overbrace{(\alpha_m^{\mathrm{s}} t + \beta_m^{\mathrm{s}})}^{\varphi_m^{\mathrm{s}}(t)}}, \quad 0 \leq m \leq L_{\mathrm{s}} - 1. \quad (2)$$

(ii) ASN: This scheme alternates between the transmitting antennas in a periodic manner. For every packet transmission, one antenna is used while the remaining antennas are silent. The beamforming vector can be modeled as

$$\begin{aligned} [\boldsymbol{b}]_m &= 1, \quad m = \mathrm{mod}(k, L_{\mathrm{s}}) \\ [\boldsymbol{b}]_i &= 0, \quad \forall i \neq m \end{aligned} \qquad (3)$$

where $k$ is the packet index.

### C. Channel Model for Worst-Case Propagation

Consider a scarce multipath (MP) propagation scenario, where there exist a dominant component with a certain direction, i.e., an angle of departure (AOD) and an angle of arrival (AOA); and few MP components with low angular spread. Such propagation scenario occurs especially on roads that are not surrounded by buildings, e.g., highways [9]. In such conditions, the V2V link is in a precarious condition, since in case this dominant component coincides with a direction where the transmit and/or receive antennas have very low gain the packet may be lost. Furthermore, if the dominant component experiences negligible change in direction over few consecutive packets, then an outage may occur. Thus, to develop a robust antenna system we assume this bad, worst-case propagation scenario. Since the dominant component carries most of the received power, we model the channel between $L_{\mathrm{r}}$ and $L_{\mathrm{s}}$ antennas solely based on it, following

$$h_{l,m}(t) = a(t) g_l^{\mathrm{r}}(\phi^{\mathrm{r}}) g_m^{\mathrm{s}}(\phi^{\mathrm{s}}) \mathrm{e}^{-\jmath \Omega_l^{\mathrm{r}}} \mathrm{e}^{\jmath \Omega_m^{\mathrm{s}}}, \qquad (4)$$

where $g_l^{\mathrm{r}}$, $g_m^{\mathrm{s}}$ are the azimuth plane far-field functions of the $l^{\mathrm{th}}$ receive and $m^{\mathrm{th}}$ transmit antennas, respectively, $\phi^{\mathrm{r}}$ and $\phi^{\mathrm{s}}$ are the azimuth plane AOA and AOD, respectively, and $\Omega_l^{\mathrm{r}}$ and $\Omega_m^{\mathrm{s}}$ are the receive and transmit relative phase shifts with respect to antennas with indices $l = 0$ and $m = 0$, respectively. The complex amplitude of the signal is given by $a(t) = |a(t)| \mathrm{e}^{-\jmath 2\pi f_c \tau(t)}$, where $f_c$ is the carrier frequency and $\tau(t)$ is the propagation delay. As part of worst-case propagation modelling, we assume that the AOA, AOD are approximately

the same for a duration of $K > 1$ consecutive packets. Since, $\Omega_l^{\mathrm{r}}$ and $\Omega_m^{\mathrm{s}}$ depend on both the array geometry—which is fixed, and on AOA, AOD, they can be assumed to remain the same over $K$ consecutive packets as well, and so does the average received power of the $K$ consecutive packets.

### D. Performance Metric: Sum-SNR

Assume that a C-ITS application that relies on the status information carried by CAMs has a requirement on the maximum AoI (i.e., the age of status information carried by the last correctly received CAM) for proper functionality, $A_{\max}$. The application is declared in outage if $A_{\max}$ is exceeded. Assuming that the latency between the generation of a CAM, and its reception at the receiving node is negligible, and recalling that the broadcast period is assumed to be sustained for at least $K$ consecutive packets (periodic communication), we can relate exceeding the maximum AoI to having a burst of $K$ consecutive CAMs in error, following $A_{\max} = KT$ s. Thus, by minimizing the BrEP of $K$ consecutive packets we minimize the probability of exceeding $A_{\max}$.

As $T$ varies, we assume that $A_{\max}$ varies too, implying that the maximum tolerable AoI of a C-ITS application is proportional to a fixed $K$. This is a reasonable consideration, since *(i)* in absence of radio channel congestion, $T$ varies if it is set according to vehicles dynamics. Following that, a low value of $T$ reflects a faster change of vehicle dynamics and hence a stricter AoI deadline is needed. A high value of $T$, reflects a slow change of vehicle dynamics, which implies that available status information are valid for a longer span of time, and accordingly the AoI is relaxed. On the other hand, *(ii)* in case of congestion on the radio channel, $T$ may be set higher than the value prescribed by VUs dynamics, or higher than a preset value by C-ITS application. However, since radio congestion occur in traffic conditions that allow for many vehicles to be close to each other (e.g., traffic congestion), it implies that vehicles dynamics change slowly, and hence it is reasonable to relax $A_{\max}$ in this case too. Note that the arguments *(i)* and *(ii)* serve as a good motivation to support the assumption in Section II-C regarding the negligible change of the AOA and AOD of the dominant propagation component over the duration $KT$ s, when $0.1 \leq T \leq 1$ s.

We related AoI to BrEP. Then, assuming exponentially distributed packet error probability, and mutually independent packet errors, we can derive [2, Section III], [10, Section III.B] that minimizing the BrEP of $K$ consecutive packets is equivalent to maximizing the sum of the SNR of $K$ consecutive packets. That is referred to as sum-SNR and it is our main performance metric.

Now, consider the received signal at a reference VU,

$$r(t) = a(t)x(t)\boldsymbol{w}^{\mathsf{H}}\boldsymbol{H}\boldsymbol{b} + \boldsymbol{w}^{\mathsf{H}}\boldsymbol{n}, \quad (5)$$

where $x(t) = \tilde{x}(t - \tau(t))$, $\tilde{x}(t)$ is the transmitted baseband signal, $[H]_{l,m} = h_{l,m}(t)/a(t)$, and $\boldsymbol{n}$ is an $L_{\mathrm{r}}$-vector of independent, white, complex Gaussian noise processes, $\mathcal{CN}(0, \sigma_{\mathrm{n}}^2)$. The noise after the combiner $\boldsymbol{w}^{\mathsf{H}}\boldsymbol{n}$ is $\mathcal{CN}(0, L_{\mathrm{r}}\sigma_{\mathrm{n}}^2)$. Taking into account the typical size of a CAM, its transmission time

$T_{\mathrm{m}}$ satisfies $T_{\mathrm{m}} \ll T$ (based on IEEE802.11p) [3]. Following this, and assuming that phase slopes are slowly varying over $T_{\mathrm{m}}$, we can approximate

$$\boldsymbol{w}^{\mathsf{H}}\boldsymbol{H}\boldsymbol{b}|_{t=t_0} \approx \boldsymbol{w}^{\mathsf{H}}\boldsymbol{H}\boldsymbol{b}|_{t=kT}, \quad kT \leq t_0 \leq kT + T_{\mathrm{m}}. \quad (6)$$

Then, the SNR per packet can be expressed as

$$\gamma_k = \frac{P_{\mathrm{r}}}{\sigma_{\mathrm{n}}^2} \frac{|\boldsymbol{w}^{\mathsf{H}}\boldsymbol{H}\boldsymbol{b}|^2}{L_{\mathrm{r}}}, \quad (7)$$

where $P_{\mathrm{r}} = \mathbb{E}\{|a(t)x(t)|^2\}$ is the average received signal power (approximated to be the same for the $K$ consecutive packets). Given that the SNR for the ABN is $\gamma_k^{\mathrm{ABN}}$, and for the ASN is $\gamma_k^{\mathrm{ASN}}$, the normalized sum-SNR with respect to $P_{\mathrm{r}}/\sigma_{\mathrm{n}}^2$ can be expressed as

$$S_{\mathrm{ABN}}(\phi^{\mathrm{r}}, \phi^{\mathrm{s}}, \boldsymbol{\alpha}^{\mathrm{r}}, \boldsymbol{\alpha}^{\mathrm{s}}, \boldsymbol{\beta}^{\mathrm{r}}, \boldsymbol{\beta}^{\mathrm{s}}) = \sigma_{\mathrm{n}}^2/P_{\mathrm{r}} \sum_{k=0}^{K-1} \gamma_k^{\mathrm{ABN}}, \quad (8)$$

$$S_{\mathrm{ASN}}(\phi^{\mathrm{r}}, \phi^{\mathrm{s}}, \boldsymbol{\alpha}^{\mathrm{r}}, \boldsymbol{\beta}^{\mathrm{r}}) = \sigma_{\mathrm{n}}^2/P_{\mathrm{r}} \sum_{k=0}^{K-1} \gamma_k^{\mathrm{ASN}}, \quad (9)$$

where $\boldsymbol{\alpha}^{\mathrm{r}}, \boldsymbol{\beta}^{\mathrm{r}}$ are $L_{\mathrm{r}}$-vectors with elements corresponding to $\alpha_l^{\mathrm{r}}$ and $\beta_l^{\mathrm{r}}$, and $\boldsymbol{\alpha}^{\mathrm{s}}, \boldsymbol{\beta}^{\mathrm{s}}$ are $L_{\mathrm{s}}$-vectors with elements corresponding to $\alpha_m^{\mathrm{s}}$ and $\beta_m^{\mathrm{s}}$.

## III. DESIGN FOR A SINGLE FIXED BROADCAST PERIOD

In this section, we consider the scenario shown in Fig. 1(a), where all vehicles use the same, fixed broadcast period $T \in [0.1, 1]$ s. Broadcast messages are intended for all VUs in the vicinity of the transmitting VU. To achieve robustness, we want to maximize the sum-SNR for the worst receiving VU under the considered propagation conditions. To simplify the derivation of optimal phase slopes, we assume that all receiving VUs have the same antenna system (i.e., $L_{\mathrm{r}}$ antennas with the same far field functions $g_l^{\mathrm{r}}$). Thus, the optimization problem corresponding to maximizing the sum-SNR over $K$ consecutive, periodic packets for the worst receiving VU is equivalent to, maximizing the sum-SNR for worst-case AOA, AOD, and worst-case initial phase offsets $\boldsymbol{\beta}^{\mathrm{r}}, \boldsymbol{\beta}^{\mathrm{s}}$ [3]. The solution to the optimization problem has been formulated in [3] for both ABN and ASN systems. It was found that the optimum sum-SNR of an ABN is achieved when $L_{\mathrm{r}}L_{\mathrm{s}} \leq K$, and $\boldsymbol{\alpha}^{\mathrm{r}}$ and $\boldsymbol{\alpha}^{\mathrm{s}}$ satisfy the following conditions

$$(\alpha_i^{\mathrm{r}} - \alpha_l^{\mathrm{r}})T/2 \in \mathcal{X}_K^{\star}, \quad (10a)$$

$$(\alpha_j^{\mathrm{s}} - \alpha_m^{\mathrm{s}})T/2 \in \mathcal{X}_K^{\star}, \quad (10b)$$

$$\big((\alpha_i^{\mathrm{r}} - \alpha_l^{\mathrm{r}}) \pm (\alpha_j^{\mathrm{s}} - \alpha_m^{\mathrm{s}})\big)T/2 \in \mathcal{X}_K^{\star}, \quad (10c)$$

where $0 \leq l < i \leq L_{\mathrm{r}} - 1$, $0 \leq m < j \leq L_{\mathrm{s}} - 1$, and

$$\mathcal{X}_K^{\star} \triangleq \{q\pi/K : q \in \mathbb{Z}\} \setminus \{m\pi : m \in \mathbb{Z}\}. \quad (11)$$

As for ASN, it was found that optimum sum-SNR is achieved when $K_{\mathrm{r}} = K/L_{\mathrm{s}} \in \mathbb{Z}$, $L_{\mathrm{r}}L_{\mathrm{s}} \leq K$, and $\boldsymbol{\alpha}^{\mathrm{r}}$ satisfies the following conditions instead,

$$L_{\mathrm{s}}(\alpha_i^{\mathrm{r}} - \alpha_l^{\mathrm{r}})T/2 \in \mathcal{X}_{K_{\mathrm{r}}}^{\star}, \quad (12)$$

where $0 \leq l < i \leq L_{\mathrm{r}} - 1$, and $\mathcal{X}_{K_{\mathrm{r}}}^{\star}$ is defined following (11).

Given a reference receiving user that is equipped with an ACN, we can design the phase slopes such that optimality is ensured when communicating with transmitting users equipped with either ABN or ASN (i.e., both (10) and (12) are satisfied). Such construction is given by

$$\alpha_l^{\mathrm{r}} = l\frac{2\pi}{KT}, \quad l = 0, 1, \ldots, L_{\mathrm{r}} - 1. \tag{13}$$

For transmitting users equipped with ABN the phase slopes need to be designed in tandem with (13), such that (10) is satisfied. One optimal phase slopes construction is given by

$$\alpha_m^{\mathrm{s}} = mL_{\mathrm{r}}\frac{2\pi}{KT}, \quad m = 0, 1, \ldots, L_{\mathrm{s}} - 1. \tag{14}$$

For ASN no parameter needs to be adjusted. However, the optimality of phase slopes at Rx is ensured only when $K_{\mathrm{r}} \in \mathbb{Z}$ (a negligibly lower, suboptimal performance is observed otherwise, see [3]).

The phase slopes (13) and (14) maximize the sum-SNR for any direction (i.e., AOA, AOD) and not only for the worst-case one. In addition, they do not depend on the far field functions of antennas, which implies that they are also optimal when receiving VUs have an antenna system with different far field functions. Following this, we can conclude that these phase slopes maximize the sum-SNR for all receiving users and only the worst one. Given the use of (13) and (14), the optimal sum-SNR guaranteed by ABN and ASN is identical, that is [3]

$$\sup_{(\boldsymbol{\alpha}^{\mathrm{r}}, \boldsymbol{\alpha}^{\mathrm{s}})} \inf_{(\boldsymbol{\beta}^{\mathrm{r}}, \boldsymbol{\beta}^{\mathrm{s}})} S_{\mathrm{ABN}}(\phi^{\mathrm{r}}, \phi^{\mathrm{s}}, \boldsymbol{\alpha}^{\mathrm{r}}, \boldsymbol{\alpha}^{\mathrm{s}}, \boldsymbol{\beta}^{\mathrm{r}}, \boldsymbol{\beta}^{\mathrm{s}}) = K\overline{G}(\phi^{\mathrm{r}}, \phi^{\mathrm{s}})$$

$$\sup_{\boldsymbol{\alpha}^{\mathrm{r}}} \inf_{\boldsymbol{\beta}^{\mathrm{r}}} S_{\mathrm{ASN}}(\phi^{\mathrm{r}}, \phi^{\mathrm{s}}, \boldsymbol{\alpha}^{\mathrm{r}}, \boldsymbol{\beta}^{\mathrm{r}}) = K\overline{G}(\phi^{\mathrm{r}}, \phi^{\mathrm{s}}),$$

where, $\boldsymbol{\alpha}^{\mathrm{r}} \in \mathbb{R}^{L_{\mathrm{r}}}, \boldsymbol{\alpha}^{\mathrm{s}} \in \mathbb{R}^{L_{\mathrm{s}}}, \boldsymbol{\beta}^{\mathrm{r}} \in [0, 2\pi)^{L_{\mathrm{r}}}, \boldsymbol{\beta}^{\mathrm{s}} \in [0, 2\pi)^{L_{\mathrm{s}}}$, and $\overline{G}(\phi^{\mathrm{r}}, \phi^{\mathrm{s}})$ is the equivalent radiation pattern given by

$$\overline{G}(\phi^{\mathrm{r}}, \phi^{\mathrm{s}}) = \sum_{l=0}^{L_{\mathrm{r}}-1} \frac{|g_l^{\mathrm{r}}(\phi^{\mathrm{r}})|^2}{L_{\mathrm{r}}} \sum_{m=0}^{L_{\mathrm{s}}-1} \frac{|g_m^{\mathrm{s}}(\phi^{\mathrm{s}})|^2}{L_{\mathrm{s}}}. \tag{15}$$

These results can be generalized to users with different number of antennas statisfying $L_{\mathrm{s}} \leq L_{\mathrm{s,max}}$ and $L_{\mathrm{r}} \leq L_{\mathrm{r,max}}$, where $L_{\mathrm{r,max}} L_{\mathrm{s,max}} \leq K$, and $L_{\mathrm{r,max}}, L_{\mathrm{s,max}}$ are respectively, the maximum Tx and Rx number of antennas a VU can use. Details about that can be found in [3].

## IV. SUPPORTING DIFFERENT BROADCAST PERIODS

We obtained the phase slopes in the simplified scenario where all vehicles use a common fixed period $T$. In the following, we extend this results to the scenario where different VUs use different, and possibly varying, broadcast periods as shown in Fig. 1(b). We recall that despite that the broadcast periods may vary, they are still assumed to be sustained for at least $K$ packets to ensure periodic communication.

Let all VUs use a design period $T_{\mathrm{d}}$, i.e., phase slopes are designed such that they satisfy (10) and (12), when $T = T_{\mathrm{d}}$. Now, we analyze how we can choose $T_{\mathrm{d}}$ such that the designed phase slopes are optimal for several broadcast periods $T_0, T_1, T_2, \ldots \in [0.1, 1]$ s.

### A. Sets of Optimal Broadcast Periods

*1) ABN Transmitters:* Let the designed phase slopes satisfy (10) when $T = T_{\mathrm{d}}$. For a generic CAM period $0.1 \leq T \leq 1$ s, (including $T \neq T_{\mathrm{d}}$) the phase slopes are still optimal if (10) is satisfied. A set of broadcast values over which optimality is retained, independently of the number of antennas and the construction of designed phase slopes, is stated in the following lemma.

For convenience, we define for $k^q$, where $k$ is a prime and $q \in \mathbb{N} = \{0, 1, \ldots\}$, the set

$$\mathcal{N}_{k^q} \triangleq \begin{cases} \{mk : m \in \mathbb{N}\}, & q > 0 \\ \emptyset, & q = 0 \end{cases}. \tag{16}$$

**Lemma 1.** *Suppose the phase slope vectors $\boldsymbol{\alpha}^{\mathrm{r}}, \boldsymbol{\alpha}^{\mathrm{s}}$ satisfy (10) when $T = T_{\mathrm{d}}$. If $K = k_1^p k_2^q > 1$, where $k_1, k_2$ are primes and $p, q \in \mathbb{N}$, then the phase slope vectors $\boldsymbol{\alpha}^{\mathrm{r}}, \boldsymbol{\alpha}^{\mathrm{s}}$ satisfy (10) when $T \in \mathcal{T}_{\mathrm{ABN}}^{\star}$, where*

$$\mathcal{T}_{\mathrm{ABN}}^{\star} = \left\{0.1 \leq nT_{\mathrm{d}} \leq 1 : n \in \mathbb{N} \setminus (\mathcal{N}_{k_1^p} \cup \mathcal{N}_{k_2^q})\right\}. \tag{17}$$

*Proof.* Every condition in (10) with $T = T_{\mathrm{d}}$ can be written in the form $AT_{\mathrm{d}}/2 \in \mathcal{X}_K^{\star}$ where $A \in \mathbb{R}$ is determined by $\boldsymbol{\alpha}^{\mathrm{r}}$ and $\boldsymbol{\alpha}^{\mathrm{s}}$. It follows from the definition of $\mathcal{X}_K^{\star}$ that $AT_{\mathrm{d}}/2 = q_A\pi/K$, where the integer $q_A$ is not a multiple of $K$. If $T = nT_{\mathrm{d}}$, then

$$\frac{AT}{2} = \frac{AnT_{\mathrm{d}}}{2} = nq_A\frac{\pi}{K}.$$

Hence, $AT/2 \in \mathcal{X}_K^{\star}$ if $nq_A$ is not a multiple of $K$. We will show (by contradiction) that this is the case.

Suppose $T = nT_{\mathrm{d}} \in \mathcal{T}_{\mathrm{ABN}}^{\star}$ and $nq_A = lK$ for some integer $l$. We can write $q_A = mK + r$, where $m \in \mathbb{Z}$ and $0 < r < K$, and

$$lK = nq_A = n(mK + r) = nmK + nr,$$

which implies that $n$ is a multiple of $K$ (since $0 < r < K$). A multiple of $K = k_1^p k_2^q$ must be a multiple of either the prime $k_1$ or the prime $k_2$. However, since $n \notin (\mathcal{N}_{k_1^p} \cup \mathcal{N}_{k_2^q})$, $n$ is neither a multiple of $k_1$ nor of $k_2$, and thus $n$ is not a multiple of $K$, and we have a contradiction.

It therefore follows that $AT/2 \in \mathcal{X}_K^{\star}$. Thus, every $\boldsymbol{\alpha}^{\mathrm{r}}, \boldsymbol{\alpha}^{\mathrm{s}}$ that satisfy (10) for $T = T_{\mathrm{d}}$, also satisfy (10) for every $T \in \mathcal{T}_{\mathrm{ABN}}^{\star}$, and this concludes the proof. $\square$

*2) ASN Transmitters:* We recall that ASN-ACN phase slopes were derived for $K/L_{\mathrm{s}} = K_{\mathrm{r}} \in \mathbb{Z}$. Phase shifters are only used at the Rx. A similar result to Lemma 1 can be stated for ASN-ACN system.

**Lemma 2.** *Suppose that $\boldsymbol{\alpha}^{\mathrm{r}}$ satisfies (12) for $T = T_{\mathrm{d}}$. If $K_{\mathrm{r}} = K/L_{\mathrm{s}} = k_{\mathrm{r},1}^p k_{\mathrm{r},2}^q > 1$, where $k_{\mathrm{r},1}, k_{\mathrm{r},2}$ are primes and $p, q \in \mathbb{N}$, then $\boldsymbol{\alpha}^{\mathrm{r}}$ also satisfies (12) for $T \in \mathcal{T}_{\mathrm{ASN}}^{\star}$ where*

$$\mathcal{T}_{\mathrm{ASN}}^{\star} = \left\{0.1 \leq nT_{\mathrm{d}} \leq 1 : n \in \mathbb{N} \setminus (\mathcal{N}_{k_{\mathrm{r},1}^p} \cup \mathcal{N}_{k_{\mathrm{r},2}^q})\right\}. \tag{18}$$

*Proof.* The proof follows the same steps used to demonstrate Lemma 1. $\square$

We make a few remarks about these two results. Firstly, the sets $\mathcal{T}_{\text{ABN}}^{\star}$ and $\mathcal{T}_{\text{ASN}}^{\star}$ are not identical in the general case, e.g., for $K = 6 = 3 \times 2$ and $L_{\text{s}} = 3$, we have $\mathcal{T}_{\text{ABN}}^{\star} = \{0.1 \leq nT_{\text{d}} \leq 1 : n \neq 2l, n \neq 3m, n, m, l \in \mathbb{N}\}$, while $\mathcal{T}_{\text{ASN}}^{\star} = \{0.1 \leq nT_{\text{d}} \leq 1 : n \neq 2l, n, l \in \mathbb{N}\}$. Secondly, $\mathcal{T}_{\text{ABN}}^{\star}$ and $\mathcal{T}_{\text{ASN}}^{\star}$ do not depend on the construction of phase slopes or the number of antennas in the system. More specific sets can be developed if $L_{\text{r}}$, $L_{\text{s}}$, and phase slopes construction are known. Such specific sets are supersets of $\mathcal{T}_{\text{ABN}}^{\star}$ and $\mathcal{T}_{\text{ASN}}^{\star}$. Lastly, all integers from 2 to 29 can be factorized as $k_1^p k_2^q$, which we believe includes all practically relevant values of $K$ or $K_{\text{r}}$.

### B. Design of Phase Slopes

Guided by the results of Lemma 1 and Lemma 2, we deduce that designing the phase slopes according to $T_{\text{d}}$ that corresponds to the smallest possible CAM period, that is $T_{\text{d}} = \min T = 0.1$ s, results in larger sets, $\mathcal{T}_{\text{ABN}}^{\star}$, $\mathcal{T}_{\text{ASN}}^{\star}$ than using $T_{\text{d}} > 0.1$ s. As an example, for $K = 4 = 2^2$, $L_{\text{s}} = 2$, $(K_{\text{r}} = 2)$, and $T_{\text{d}} = 0.1$ s, we get

$$\mathcal{T}_{\text{ABN}}^{\star} = \mathcal{T}_{\text{ASN}}^{\star} = \{0.1, 0.3, 0.5, 0.7, 0.9\}, \qquad (19)$$

implying that five values of broadcast periods are known to retain optimality, when phase slopes are designed based on $T_{\text{d}} = 0.1$ s. To be able to deploy more values of $T$ with optimal performance, we propose reducing $T_{\text{d}}$ to be a fraction of the smallest possible CAM period. An example with $T_{\text{d}} = 0.1/3$ s, $(K = 4)$ results in

$$\mathcal{T}_{\text{ABN}}^{\star} = \mathcal{T}_{\text{ASN}}^{\star} = \{0.1n/3 : n = 3, 5, 7, 9, \ldots, 29\}, \qquad (20)$$

implying a larger set ($|\mathcal{T}_{\text{ABN}}^{\star}| = |\mathcal{T}_{\text{ASN}}^{\star}| = 14$) of broadcast periods with optimal performance compared to the set achieved with $T_{\text{d}} = 0.1$ s.

We point out that the approximation (6), used in Section II-D, holds under the assumption that phase slopes are slowly varying over packet duration $T_{\text{m}} \ll T$. Looking at (13) and (14), with $T = T_{\text{d}}$, we see that a smaller $T_{\text{d}}$ implies increased phase slopes, and thus, faster changes are experienced over a packet duration. Therefore, $T_{\text{d}}$ can not be reduced dramatically, and $T_{\text{m}} \ll T_{\text{d}}$ need to be satisfied for the assumption and the approximation to be be valid.

### V. NUMERICAL RESULTS

In this section, we are interested in validating that optimality holds when $T$ is within $\mathcal{T}_{\text{ABN}}^{\star}$ and $\mathcal{T}_{\text{ASN}}^{\star}$, and observing how performance is affected when $T$ is not within those sets. Consider a reference receiving VU employing an ACN with phase slopes that are designed following (13) with $T = T_{\text{d}}$, and a reference transmitting VU that employs either an ABN with phase slopes that are designed following (14) with $T = T_{\text{d}}$, or an ASN. We quantify the performance of the ABN/ASN systems when $T$ varies in the range $[0.1, 1]s$. We assume that the period $T$ is sustained for at least $K$ packets in a burst. Let $K = 4$, $L_{\text{r}} = 2$, $L_{\text{s}} = 2$, with both Tx and Rx equipped with antennas that have the radiation patterns shown in Fig. 3.
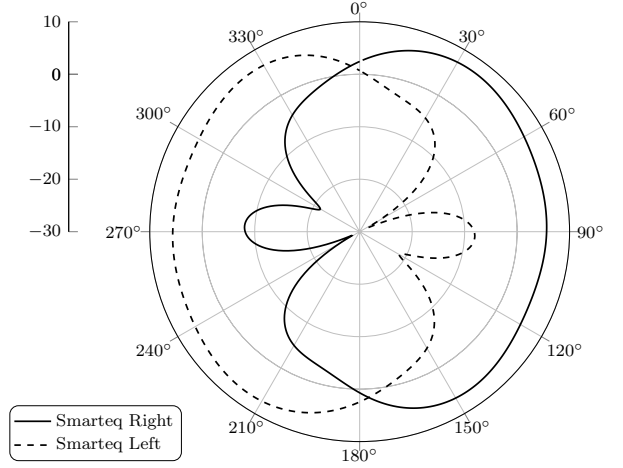


Fig. 3. Antennas radiation patterns, $|g(\phi)|^2$, in dBi. They are back-to-back patch antennas designed by Smarteq[3] for vehicular applications.
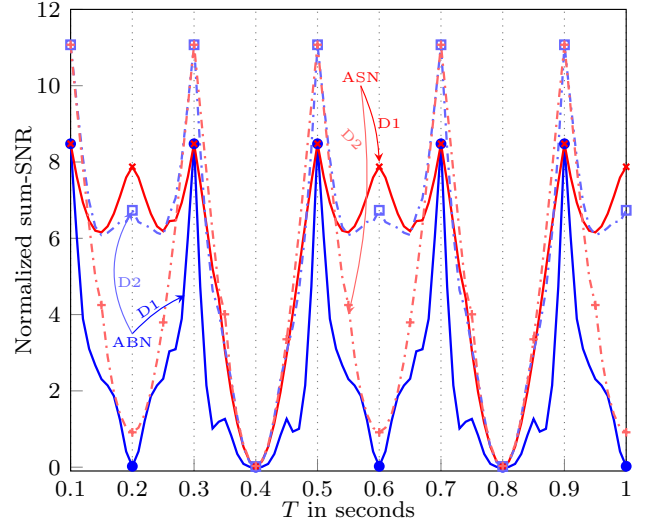


Fig. 4. Normalized Sum-SNR of ABN at the worst-case initial phase offsets $(\boldsymbol{\beta}^{\text{r}}, \boldsymbol{\beta}^{\text{s}})$, and of ASN at worst-case $\boldsymbol{\beta}^{\text{r}}$, as a function of $T$, for $T_{\text{d}} = 0.1$ s and $K = 4$. The sum-SNR is shown at two fixed directions D1 $= (178, 178)$ deg (solid lines) and D2 $= (178, 90)$ deg (dash-dotted lines).

Since the phase slopes are optimal for any direction $\text{D} = (\phi^{\text{r}}, \phi^{\text{s}})$, we select two directions of interest to plot the sum-SNR, namely, (i) D1 $= (178, 178)$ deg, and (ii) D2 $= (178, 90)$ deg. In Fig. 4 we show the normalized sum-SNR of both schemes, $S_{\text{ABN}}$ and $S_{\text{ASN}}$, at worst-case initial phase offsets for the two fixed directions. $T_{\text{d}} = 0.1$ s is assumed for phase slopes for both ABN and ASN systems. In accordance with (19), we see in Fig. 4 that for $T \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ s, the performance is at its highest for both schemes, and it corresponds to the optimum. However, at the excluded values of $T$ in (17) and (18), satisfying $T = 2mT_{\text{d}}$, (i.e., $T \in \{0.2, 0.4, 0.6, 0.8, 1.0\}$) the ABN sum-
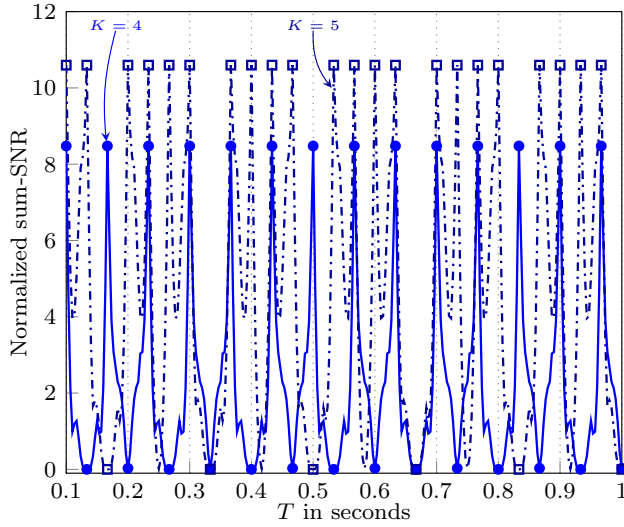
Fig. 5. Normalized Sum-SNR of ABN at the worst-case initial phase offsets $(\boldsymbol{\beta}^{\mathrm{r}}, \boldsymbol{\beta}^{\mathrm{s}})$, as a function of $T$, for $T_{\mathrm{d}} = 0.1/3$ s, $K = 4$ and $K = 5$. The sum-SNR is shown at D1 = $(178, 178)$ deg.

SNR is reduced at both directions, and the reduction is severe at D1. Looking at ASN sum-SNR at D1, we see that the performance reduction is severe at $T = 4mT_{\mathrm{d}}$, however, at $T = 2mT_{\mathrm{d}} \neq 4mT_{\mathrm{d}}$ (i.e., $T \in \{0.2, 0.6, 1.0\}$), the sum-SNR is close to the optimal. That is explained by the fact that both Tx antennas at $\phi^{\mathrm{s}} \approx 178$ deg have similar gain (consult Fig. 3), and therefore, this is a special case where the system is equivalent to $1 \times 2$ system. Using (10), we can confirm that for a system with $L_{\mathrm{s}} = 1$, $L_{\mathrm{r}} = 2$ the designed phase slopes are optimal when $T = 2T_{\mathrm{d}}$. However, looking at D2 where $\phi^{\mathrm{s}} = 90 \deg$, we can see that the $2 \times 2$ ASN system experiences a dramatic reduction of performance.

In Fig. 5 we plot similar curve for the ABN when $T_{\mathrm{d}} = 0.1/3$ s. We can observe that in accordance with (20) we have a denser grid of optimal broadcast period values within $[0.1, 1]$ $s$. The same results apply to ASN. For $K = 5$, which is a prime, we can achieve an even denser set $\mathcal{T}_{\mathrm{ABN}}^{\star}$. However, that is valid for ABN only (since $K/L_{\mathrm{r}} \notin \mathbb{Z}$). Comparing the results of Fig. 4 and Fig. 5, we observe that the system where phase slopes are designed using $T_{\mathrm{d}} = 0.1/3$ s, exhibits faster decays of worst-case sum-SNR when $T$ deviates from the optimal values compared to the decays experienced when designing the phase slopes using $T_{\mathrm{d}} = 0.1$ s. This indicates that if we dramatically reduce $T_{\mathrm{d}}$, we may render the ABN system more susceptible to variations around the optimal values which could occur, for example, due to jitter of the generation interval $T$. The same applies to ASN.

## VI. DISCUSSION AND CONCLUSION

Given that all VUs use ABN/ASN systems with phase slopes that are designed using $T_{\mathrm{d}}$, we have learned that a transmission with any period $T \in \mathcal{T}_{\mathrm{ABN}}^{\star}$, and $T \in \mathcal{T}_{\mathrm{ASN}}^{\star}$ for ABN and ASN equipped VUs, respectively, maximizes the sum-SNR of $K$ consecutive CAMs at any receiving VU. That im-

plies (under the assumptions in Section II-D) that ABN/ASN simultaneously minimize the probability of exceeding varying AoI deadlines $A_{\max} \in \{KT : T \in \mathcal{T}_{\mathrm{ABN}}^{\star}, T \in \mathcal{T}_{\mathrm{ASN}}^{\star}\}$.

We recall that the broadcast period of a VU is set either following the change of the vehicle dynamics, or following a requested value by a C-ITS application. With proper choice of $T_{\mathrm{d}}$ (e.g., $0.1/3$ s), a sufficient granularity of optimal broadcast periods within $[0.1, 1]$ s can be achieved such that the vehicle dynamics can be effectively tracked using values within $\mathcal{T}_{\mathrm{ABN}}^{\star}$ and $\mathcal{T}_{\mathrm{ASN}}^{\star}$. On the other hand, if a requested period by a C-ITS application at certain time is not within $\mathcal{T}_{\mathrm{ABN}}^{\star}$, or $\mathcal{T}_{\mathrm{ASN}}^{\star}$ (for ABN or ASN transmitters, respectively), the C-ITS application can reduce the broadcast period to a smaller one to sustain optimal performance, or increase it, if the load of the radio channel does not allow for a smaller period. An increase, or decrease of broadcast period with a single step (e.g., $0.1/3$ s) than the actual requested period, should not affect severely the channel load, or the overall latency requirement of the communication service. Choosing a small value for $T_{\mathrm{d}}$ has an advantage on the density of the optimal broadcast periods sets. However, we noted that to keep a modelling approximation valid, $T_{\mathrm{m}} \ll T_{\mathrm{d}}$ need to be satisfied. Furthermore, numerical results indicate that a too small $T_{\mathrm{d}}$ may render the ABN/ASN systems susceptible to variations of broadcast period around the optimal values. Thus, $T_{\mathrm{d}}$ should be judiciously chosen such that dense sets $\mathcal{T}_{\mathrm{ABN}}^{\star}$, $\mathcal{T}_{\mathrm{ASN}}^{\star}$ are achieved, while the multiple antenna systems maintain certain robustness to small variations of broadcast periods.

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