THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN SOLID AND STRUCTURAL MECHANICS

3-D rate dependent micromechanical model for polymer composites

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Cover:
Schematic representation of numerical models and simulation results of different material systems and layups validated using the developed dynamic material model. Experimental result from [21].

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Abstract

Fibre reinforced polymeric composites are in high demand in automotive and aviation industries to improve fuel efficiency. However, the dynamic behaviour of composites is not very well understood. Furthermore, dynamic loading together with the anisotropic nature and complex nonlinear behaviour of polymer composites results in a complex failure behaviour. This behaviour is of significant importance to account for in automobile crash simulation and impact modeling of aircraft structures.

In this thesis, a micromechanics based constitutive model is developed to predict the nonlinear behaviour and failure of unidirectional fibre reinforced polymer composites subjected to compressive dynamic loading. The carbon fibres are assumed to be hyperelastic transversely isotropic. For the matrix, a viscoelastic-viscoplastic constitutive model with hardening enhanced by continuum damage is advocated. A three parameter Maxwell model is used for the linear viscoelastic behaviour of the matrix. The nonlinear viscoplastic behaviour is introduced by coupling a Perzyna-type Bingham/Norton model with an intralaminar matrix continuum damage model. The pressure dependence of the onset of plastic yielding in matrix shear dominated response under compressive loading is also considered. The proposed model is formulated in a geometrically nonlinear description that separates the fibre and the matrix contributions. The model draws from computational homogenization of the unidirectional ply level response, with the matrix and the fibres as subscale constituents. A major feature is that the subscale constituents are coupled via *isostrain* and *isostress* assumptions parallel and transverse to the fibres, respectively. An improved *isostress* formulation is proposed to include in a better way longitudinal fibre shear response. The elastic response is improved by considering a non-uniform stress distribution in the matrix. For intralaminar damage growth, a continuum damage enhanced formulation of Lemaitre type is proposed. This model is combined with a surface based cohesive model that describes interlaminar delamination.

Based on the model, the shear induced failure behaviour in compression of the composite material is characterized. Finite element simulations are conducted to validate observed rate dependent properties of off-axis loaded unidirectional composites and angle-ply laminates. The predictions of the finite element simulations are compared to published experimental results of different material systems under compression loading at different strain rates. The results obtained are in reasonable agreement with the experiments. Typical applications are carbon/epoxy composites, where unidirectional carbon fibres are embedded in a polymer matrix. In the future, the model is possible to extend to orthotropic plies and textile reinforced composites. The model is micromechanically motivated, hence it is also possible to extend for rate dependent fibres, e.g. glass fibres.

Keywords: viscoelasticity, viscoplasticity, continuum damage, finite element, cohesive surface, carbon/epoxy composites, angle-ply laminates.

"The only way to learn mathematics is to do mathematics."

- Paul Halmos -

Preface

The work in the thesis was carried out at the Department of Polymers, Fibres and Composites, RISE AB, Sweden in collaboration with the Division of Material and Computational Mechanics, Department of Industrial and Materials Science, Chalmers University of Technology. The research was financially supported by the ICONIC project under the Marie Skłodowska-Curie grant agreement No 721256 of the European Union Horizon 2020 research and innovation programme and the Swedish Foundation for Strategic Research (SSF) under FID16-0041. The computations in Paper C and D were performed on resources provided by Chalmers Centre for Computational Science and Engineering (C3SE). The support is gratefully acknowledged.

Acknowledgements

Completing the enclosed work has been a remarkably enjoyable and enlightening experience. First and foremost, I express my sincere gratitude and thanks to the many individuals who have impacted my learning over the past few years.

First, I would like to start my acknowledgements by thanking Dr. Robin Olsson and Dr. Erik Marklund, my research supervisor and co-supervisor, for their useful guidance and warm encouragement, which helped me all the time during my research and writing of this thesis. Even when the road got tough, they convincingly guided and encouraged me to be professional and do the right thing. Without their persistent help, the goal of this project would not have been realized. I am sincerely grateful to my other co-supervisor, Professor Ragnar Larsson for his constant support and invaluable guidance. This project could have never been possible without the suggestions and help provided by him. The exchange of ideas with him was a constant source of inspiration and certainly improved the quality of my research. I think this is the right place to thank you all for all the advice and patience in my supervision. It has been a true honour and a privilege to work alongside such eminent researchers and mentors. They have been exceptionally supportive, and always been available to discuss issues related to the project as well as to discuss general matters. Beyond these guiding figures, I am also grateful for the support from my friends and colleagues at the Department of Polymers, Fibres and Composites at RISE who directly or indirectly have supported me or have constituted a positive stimulus for me in any form while writing this dissertation.

Lastly, and most importantly, I express my forever gratitude to my wife Mamta Rana, son Ayaansh Chauhan and my family and friends for their support and guidance. I thank them for their love, emotional support and for all they have done for me. This thesis is dedicated to them. In addition to the above mentioned, there are many others who have contributed to the work, and I owe my deepest gratitude to everyone who has made the outcome of this thesis possible.

Vivekendra Singh
Gothenburg, December 2022

Thesis

This thesis consists of an extended summary and the following appended papers.

Paper A	Larsson, R., **Singh, V.**, Olsson, R., Marklund, E. A micromechanically based model for strain rate effects in unidirectional composites. *Mechanics of Materials*, Vol. 148, 2020, 193–212.
Paper B	Larsson, R., **Singh, V.**, Olsson, R., Marklund, E. A micromechanically based model for dynamic damage evolution in unidirectional composites. *International Journal of Solids and Structures*, Vol 238, 2022, 111368.
Paper C	**Singh, V.**, Larsson, R., Olsson, R., Marklund, E. A micromechanics based model for rate dependent compression loaded unidirectional composites. *Composites Science and Technology*. https://doi.org/10.1016/j.compscitech.2022.109821.
Paper D	**Singh, V.**, Larsson, R., Olsson, R., Marklund, E. Rate dependent material model for progressive failure and delamination growth in multidirectional composite laminates. *Submitted for publication.*

The appended papers in Part II of the thesis were prepared in collaboration with the co-authors.

Paper A: Ragnar Larsson proposed the approach and methodology from discussions among the authors. Vivekendra Singh was responsible for the major progress of the work associated with data curation, numerical implementation and validation. Vivekendra Singh wrote the original draft with supervision from Robin Olsson, Ragnar Larsson and Erik Marklund.

Paper B: Ragnar Larsson suggested the approach. Vivekendra Singh and Ragnar Larsson were responsible for software development. Vivekendra Singh wrote the original draft with supervision from Robin Olsson, Ragnar Larsson and Erik Marklund. Vivekendra Singh performed data curation, numerical implementation and validation.

Paper C: Vivekendra Singh suggested the approach. Vivekendra Singh formulated the theory together with Ragnar Larsson, Robin Olsson and Erik Marklund. Vivekendra Singh performed software development, numerical implementation and simulations. Vivekendra Singh led the planning and writing of the paper with supervision from Robin Olsson, Ragnar Larsson and Erik Marklund.

Paper D: Vivekendra Singh suggested the approach. Vivekendra Singh performed data curation, numerical implementation and simulations. Vivekendra Singh led the planning and writing of the paper with supervision from Robin Olsson, Ragnar Larsson and Erik Marklund.

Conference papers:

- **Singh, V.**, Larsson, R., Olsson, R., Marklund, E. Effect of strain rate at compressive and tensile loading of unidirectional plies in structural composites. In: Proceedings of the ECCOMAS, 2019, Girona, Spain.

- **Singh, V.**, Larsson, R., Olsson, R., Marklund, E. Application of a rate dependent model on a unidirectional non-crimp carbon/epoxy composite. In: Proceedings of the ECCOMAS, 2021, Gothenburg, Sweden.

Additional publications by the author:

- **Singh, V.** Literature survey of strain rate effects on composites. *Technical report - TR18-001 (open)*, 2018, Swerea SICOMP (now RISE), Mölndal, Sweden.

- **Singh, V.** A micromechanically based model for strain rate effects in unidirectional composites. *Technical report - CR19-017 (confidential)*, 2019, Swerea SICOMP (now RISE), Mölndal, Sweden.

Contents

II Appended papers A-D I

Part I

Extended Summary

1. Introduction

1.1 Background

The employment of carbon fibre-reinforced polymers (CFRP) has been extensively developed in the automotive and aeronautical industries during the last 40 years. This is mainly due to the desire to optimise the stiffness and strength-to-weight properties. Particularly, in the aeronautics industry, composite materials are introduced to increase efficiency without compromising safety.

In the past, the application of composites has remained limited mainly to military applications due to lack of confidence for safe design, limited technology readiness and the comparatively high cost for the material and manufacturing. However, in the last decades, this perspective has begun to change due to the pressing need for the industry to reduce the time and cost of bringing new components to the market. This calls for the continuous development of improved design methods. The reduction in prices within the last decade together with the improved understanding of the material behaviour and the rising awareness of the need for reduced weight and lower fuel consumption have allowed for composites to be employed in civil aerospace applications.

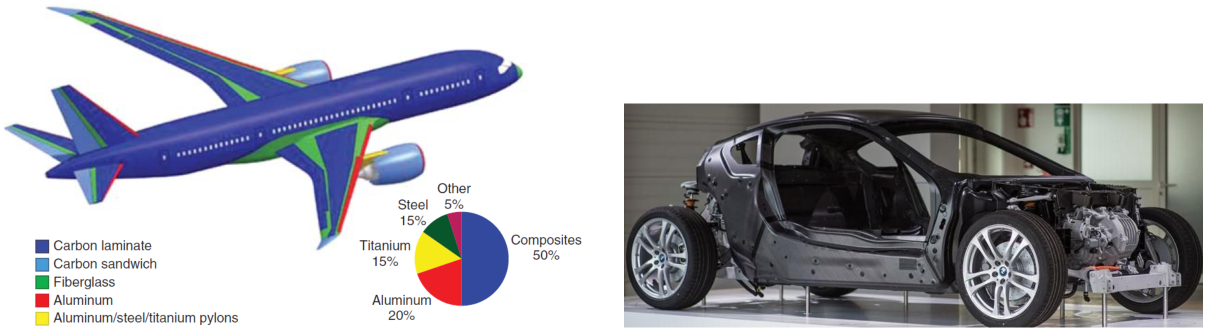

Figure 1.1: Boeing 787 Dreamliner structural material distribution [1] and Life module passenger cell made from carbon-fibre-reinforced plastic [2].

A good example of successful substitution of metallic components by composite or hybrid materials is the Boeing 787 Dreamliner, see Fig 1.1. The reason is that composite materials provide lower weight and improved fatigue life and, therefore, may require lower maintenance and fewer inspections. Another cost driven sector is automotive and transportation where rising fuel prices, lower costs for the material and comparatively less time in manufacturing have pushed the sector for developing light weight and fuel-efficient vehicles. One such example is the cockpit of BMW i8 (Fig 1.1) made of CFRP to improve passenger safety in the high-end car [2].

CFRP used in the above applications are frequently subjected to dynamic loads such as automotive crashes or bird strike on aeroplanes. The performance of CFRP under such complex applications requires detailed understanding of the material behaviour, which often mark the critical loading event for design. Impact induced loading poses various difficulties for structural design. First, impact events often result in complex three-dimensional stress states, which require advanced theories for damage prediction. In addition to that, a realistic analysis of CFRP composites requires, both at the micro- and macro-scales, a proper account of the non-linear stress-strain relationships. Even the modeling of a single lamina appears to be quite complex since phenomena like fracture, delamination, microbuckling and large deformations have to be considered together with their interactions.

Furthermore, composite materials, especially with polymeric matrices, are well known for their strain rate dependent material behaviour. As a consequence, special constitutive models for the numerical simulation of impact events are required. The current predictive capability of composite constitutive models is often insufficient to master the above mentioned challenges. Many available models are limited to plane stress and the representation of the complex composite failure behaviour often lacks a sound physical basis. Therefore, predictive modelling of the composite response for impact loading requires constitutive models which incorporate the strain rate dependent material behaviour for realistic prediction of the stresses induced due to impact loading.

1.2 Aim and scope of research

The research is presented in this thesis is inspired by the need to introduce composite materials in parts of automobiles and aircrafts subjected to dynamic events such as automotive crashes or bird strikes on aeroplanes.

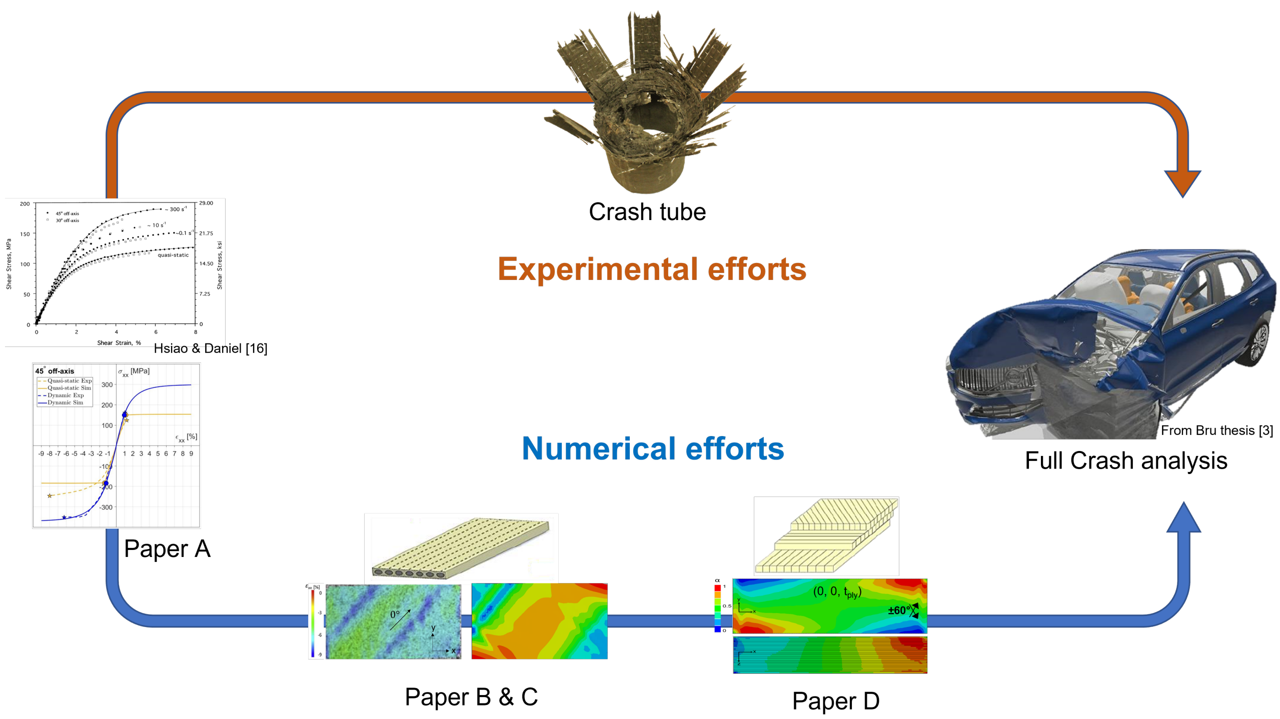

Figure 1.2: A schematic of numerical and experimental efforts to develop a framework for full-scale crash analysis.

Dynamic events involve crushing or yielding of a material due to elastic or plastic deformation with damage takes place within a fraction of second. In addition, the thermoset polymer composites considered in the thesis are strain rate sensitive in nature. Therefore, material models for such events can not be validated with coupons tested under quasi-static loading.

The aim of this thesis is to develop an advanced constitutive model including strain rate effects and continuum damage to predict the dynamic response for unidirectional carbon fibre reinforced polymer composites subjected to compressive loading. Hence, focus is on the material modeling, calibration and validation against available experimental testing of the dynamic response of carbon/epoxy polymer composites in compression. In order to properly account for the constituents of the composite, micromechanics is employed for a full-scale structural analysis of dynamic events. It was an ambitious aim. However in this thesis work, the modeling and 3D simulation of an angle-ply laminate of carbon/epoxy composite under dynamic loading has been achieved. The flow of the thesis development is shown in Fig 1.2, from **Paper A** to **D** alongside the ambition to further extend the proposed approach for full crash analysis. The experimental efforts are not a part of this thesis work and shown in 1.2 only for the sake of completeness.

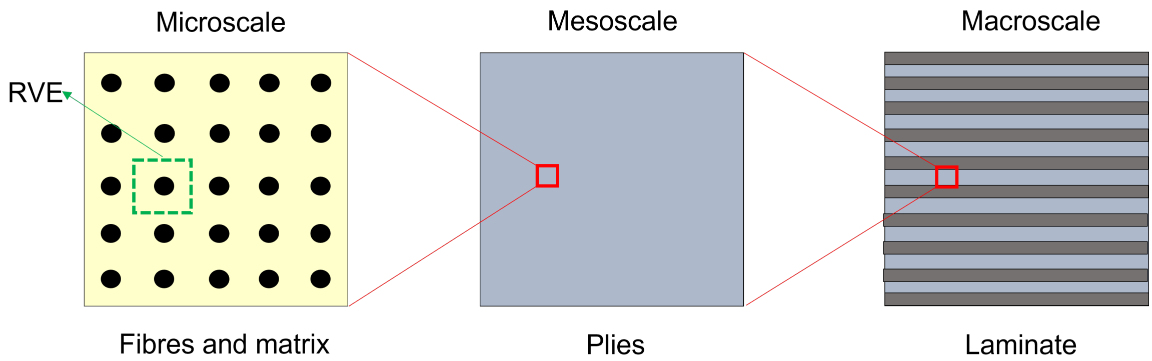

Figure 1.3: The three scales as identified in UD composites.

The proposed constitutive model is based on a thermodynamically consistent framework and micromechanics approach. In this approach, we assume that the composite at the mesoscale has nonlinear rate-dependent characteristics due to the interaction between the constituents at microscale. The schematic of the current approach is shown in Figure 1.3. The constitutive model at microscale consists of tranversely isotropic carbon fibres and viscoelastic-viscoplastic matrix with continuum damage. To get the response at mesoscale, a simplified but efficient *homogenization* approach based on simplified assumption of *isostress* and *isostrain* is implemented. For the macroscale, the homogenized plies are stacked together based to their fibre orientations with cohesive surfaces between them.

The advantage of a micromechanics based approach is that it has the ability to predict a dynamic non-linear response for various material systems e.g. CFRP, GFRP etc and architectures such as angle-ply and unidirectional laminates. The proposed constitutive model is developed for explicit Finite Element (FE) analysis which has been shown to be a powerful tool for the simulation of dynamic events. The key for successfull modelling to dynamic events is a strain rate dependent constitutive model which considers three-dimensional stress states for strength prediction and subsequent damage evolution.

Reaching this objective requires the following tasks to be addressed within the thesis:

- Incorporation of strain rate dependent material behaviour into the constitutive equations based on small strain theory. In **Paper A**, a novel rate dependent constitutive model based on micromechanics, i.e. modeling fibres and the matrix constituents separately, is proposed.
- Extension of the model to include continuum damage and to allow for nonlinear kinematics. The original formulation described in **Paper A** is extended to account for intralaminar dynamic damage growth and evolution in the matrix in **Paper B**. Numerical predictions (utilizing the developed framework) are made and compared with experimental data.
- Improvement of the constitutive model and comparison to small off-axis loading experiments. In **Paper C**, the formulation is modified by an improvement for viscoelastic behaviour and the *Homogenization* technique related to longitudinal shear. The model is also compared with in-house experimental results.
- Model application to laminate analysis. In **Paper D**, the model implementation is used to predict the rate dependent intra- and interlaminar damage growth in angle-ply laminates.

The current state of research represents novel contributions to each and one of the tasks stated above. Focus of the constitutive model development lies in the model implementation for the prediction of the damage initiation and the subsequent damage evolution.

1.3 Thesis Outline

The thesis comprises **7 chapters**. **Chapter 2** presents an extensive literature survey of published strain rate dependent experimental data of various composite materials including strain rate data of pure fibres and the matrix material to set up the background for the research. In **Chapter 3** of the thesis, an introduction to available rate dependent constitutive models for composites is given. Both meso and microscale based approaches are described. **Chapter 4** describes the methodology and derivation of the rate-dependent constitutive equations/model for a unidirectional carbon/epoxy composite. Dynamic damage initiation and propagation for a unidirectional carbon/epoxy composite is presented. It also presents the numerical implementation and validation of the rate dependent non-linear constitutive model into an ABAQUS user-defined material subroutine and applications to predict rate dependent material response of a unidirectional carbon/epoxy composite under compressive dynamic loading. A short summary of all appended papers is presented in **Chapter 5**. Concluding remarks are given in **Chapter 6** and suggestions for further research in **Chapter 7**.

2. Rate dependent mechanical response - experimental observations

In many structural applications composite materials are exposed to dynamic loadings, particularly use of composites in high speed aircraft, missiles, automotive and other transportation industries. Therefore, it is important to characterize and understand the dynamic behaviour of CFRP in order to develop numerical models which can capture the dynamic response of these materials in high technology applications.

The focus of this thesis work was not on performing experiments for high speed testing of carbon/epoxy composite specimen. Instead, focus is placed on developing an efficient computational model for simulating dynamic behaviour of carbon/epoxy composite in compression. Therefore, the literature survey played a major role to find out the relevant experimental data on dynamic behaviour of polymer composites for the validation of material model from **Paper A - D**, except in **Paper C** where the dynamic experimental data is provided by one of our project partners. Strain rate studies found in the literature are based on different types of composite material systems such as unidirectional (UD), woven fabrics, metal-matrix composites and many more. In this thesis work, we have considered only UD composites with an emphasis on carbon/epoxy or glass/epoxy systems. In this chapter, relevant rate dependent experimental observations for fibres, matrix and unidirectional composites are provided.

2.1 Rate dependent behaviour of fibres

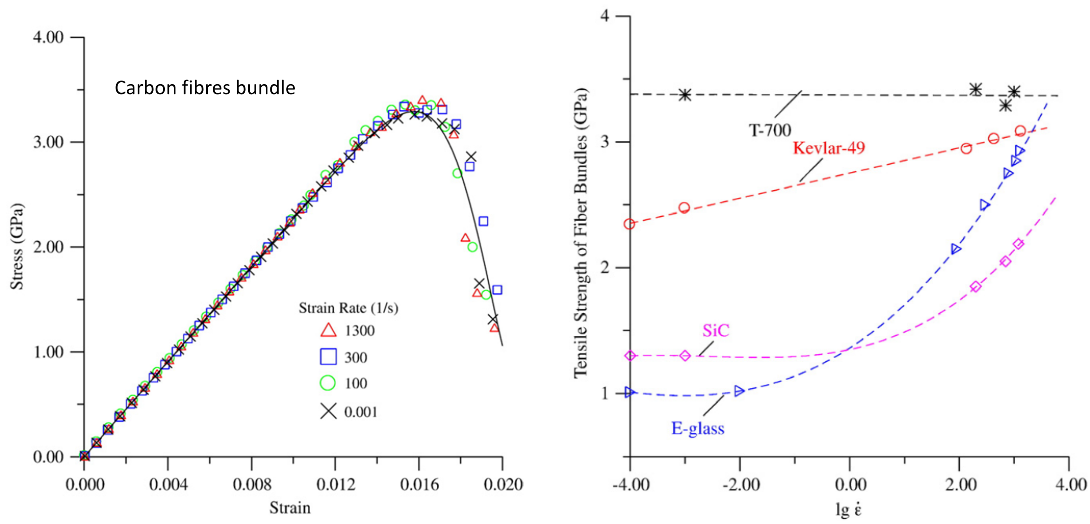

Figure 2.1: Comparison of mechanical properties of different fibres at different strain rates. [6]

In general glass and polymer fibres are rate sensitive, while carbon fibres are not. However, only limited experimental data exist regarding the strain rate effect on the mechanical properties of fibres or fibre bundles.

Xia et al. [4] observed a significant strain rate effect on the modulus, strength and failure strain of glass fibre bundles as shown in Figure 2.1. Wang and Xia [5] later investigated rate dependent behaviour of the Kevlar® aramid fibre bundles and found that they are sensitive to strain rate, but the degree of rate sensitivity is not as pronounced as found for glass fibre bundles.

Regarding carbon fibres, experimental studies are performed by Zhou et al. [6] on the tensile behaviour of T700 carbon fibre bundles and they reported that strain rate has no effect on the mechanical properties of carbon fibres (Figure 2.1). The quasi-static tensile tests were performed on a conventional testing machines. For the impact tensile tests, a tension split-Hopkinson bar was used. This thesis work is focussed on carbon/epoxy composite systems. Based on the above observations i.e. carbon fibres are rate independent, an elastic transversely isotropic model is used for the carbon fibres throughout the thesis i.e. **Paper A - D**.

2.2 Rate dependent behaviour of polymers

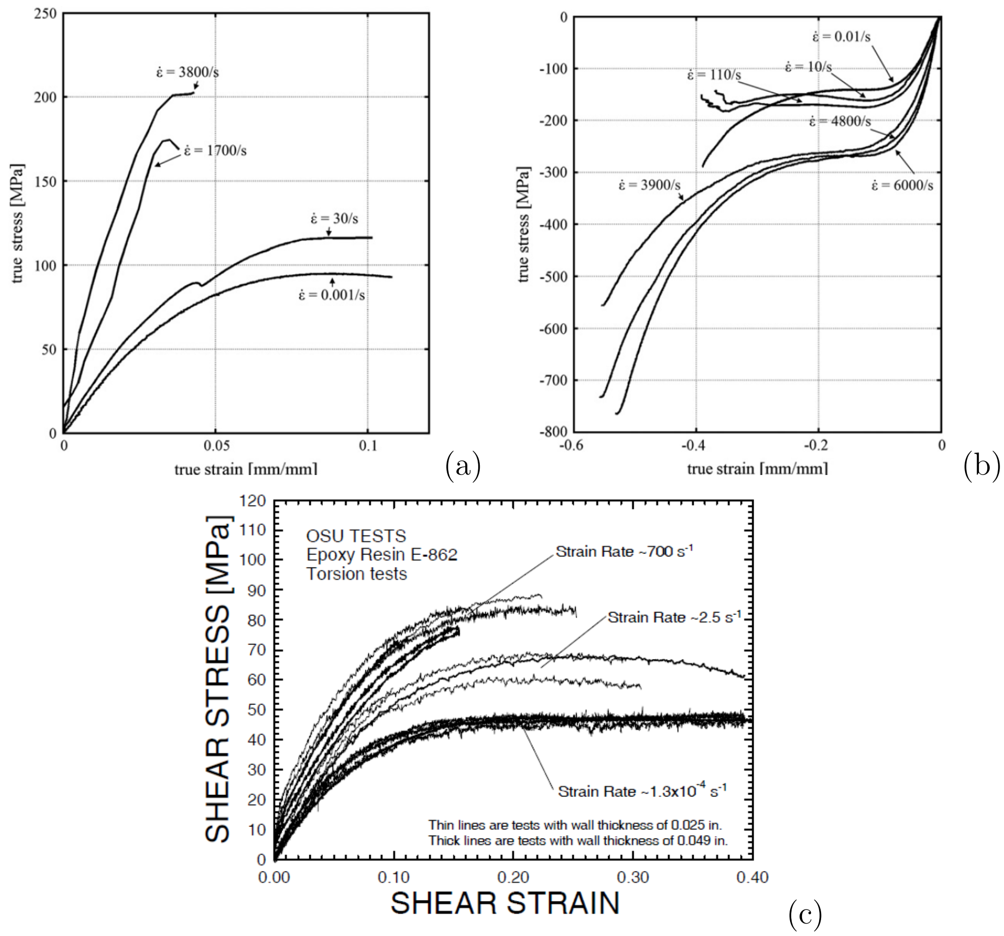

Figure 2.2: Strain rate dependence of RTM-6 epoxy resin in (a) tension and (b) compression [9]. Rate dependent shear behaviour of (c) E-862 epoxy [8].

In tension, polymers show an increase in stiffness and strength, while failure strain decreases with strain rate [7, 8, 9]. This is observed by Gilat et al. [7] who investigated the strain rate behaviour of E-862 and Cytec PR-520 epoxy resin. A transition from ductile to brittle response was observed when progressing from low strain rate to high strain rate. This may be due to the effects of stress relaxation and plastic deformation that can occur during the relatively long time scale of the lowest rate tests. Therefore, the failure strain was significantly lower for the dynamic tests. The failure strength was found to increase moderately with increasing strain rate. Similar observations were found by Gerlach et al. [9] for a thermoset RTM-6 epoxy resin in tension.

In compression, an increasing stiffness and strength have been reported by authors, e.g. in [9, 10], while no effect was reported in [11, 12]. A conventional testing machine for a quasi-static testing and a compressive SHB for a dynamic testing was used by Gerlach et al. [9] to perform a mechanical testing on a thermoset resin. A significant increase of yield stress and compressive stiffenss was reported at high strain rates (Figure 2.2(b)). Buckley et al. [12] investigated the compressive properties of three thermosetting resins namely bisphenol A (BPA) epoxy (Ciba CT-200), a BPA epoxy resin and a bismaleimide (BMI) epoxy over strain rates from 10^{-3} to 5×10^{3} /s. It was reported that the yield strength increases with increasing strain rate, but the compressive modulus is not affected much. However, significant ductility was observed for all three resin systems.

Under pure shear loading, an increasing stiffness and strength with strain rate has been observed by the authors in [8, 13]. Hou et al. [13] tested three commercially available thermosetting epoxy systems under shear and found that the shear modulus and shear strength increase while failure strain decreases with increasing strain rate. Gilat et al. [8] studied the shear response of an epoxy system at different strain rates and reported that shear stiffness and strength increases with increasing strain rate. A ductile material behaviour in the stress-strain response was observed for all strain rates as shown in Figure 2.2.

It is observed from the above discussion that the matrix has a non-linear rate dependent behaviour. Therefore a viscoelastic-viscoplastic (**Paper A**) with continuum damage (**Paper B - C**) based constitutive model is proposed for the matrix. This model is further extended to include an isotropic hardening in **Paper D**.

2.3 Rate dependent behaviour of UD composite

For carbon/epoxy composites, in tension along the fibre direction, no effect on modulus, strength and failure strain was reported by authors in [14, 15, 16]. However, in the transverse direction and other off-axis angles, an increase in modulus and strength and no effect on the failure strain was reported by authors [15, 16] using different test machines. Daniel et al. [16] found a moderate increase in tensile modulus but yield strength and failure strain does not vary significantly with strain rate. They performed the dynamic characterization of carbon/epoxy (SP288/AS) at various strain rate ranging from quasi-static to over 500 /s.

More experimental data exists on the strain rate effect on mechanical properties of polymer composites in compression. This is partly due to that such experiments are more convenient to perform, and highly important for crash modeling. Under longitudinal compression, a slight

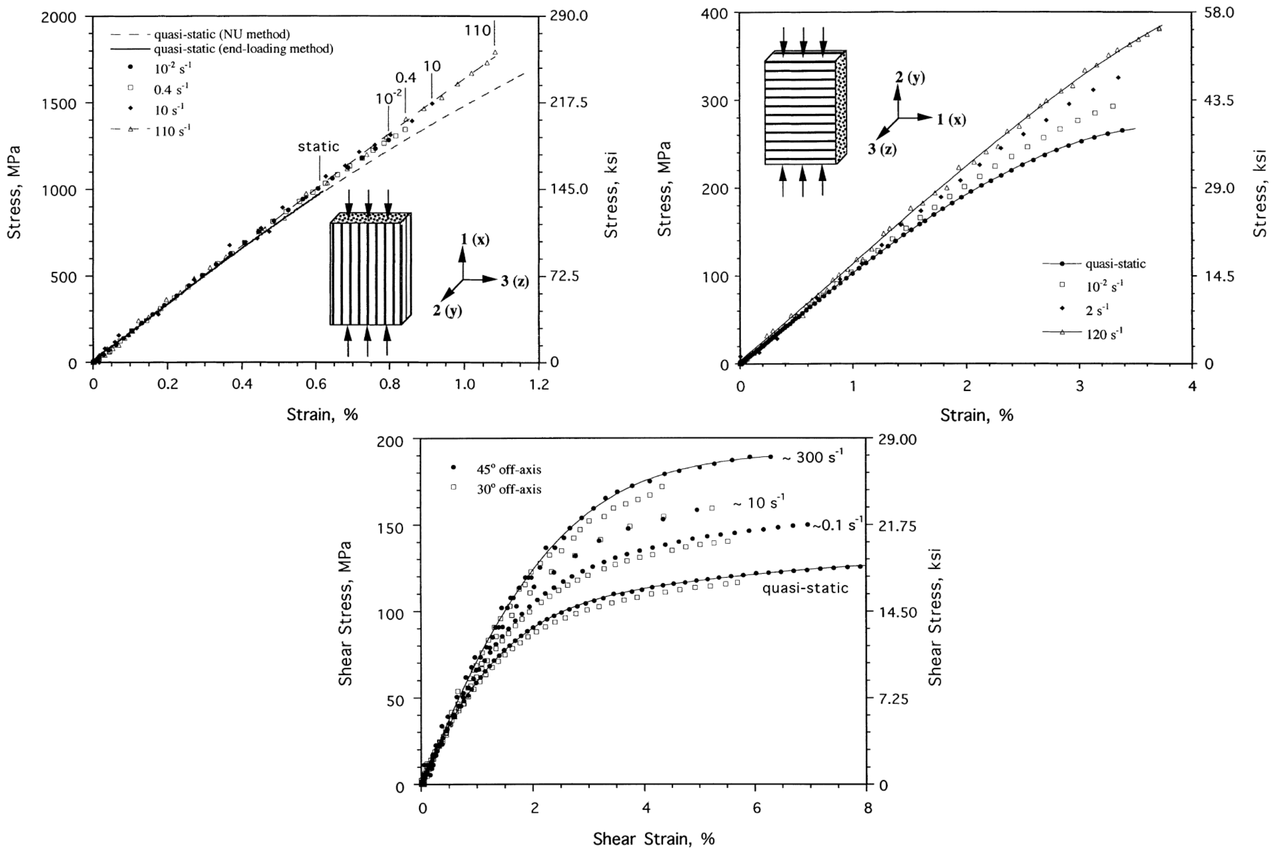

Figure 2.3: Measured quasi-static and dynamic response of unidirectional carbon/epoxy composite in longitudinal (upper left) and transverse (upper right) compression [17] and shear (lower center) [20].

increase in the modulus and a significant increase in the strength was reported in [17, 18, 19]. An increase in failure strain was also reported in [17, 19]. Hsiao and Daniel [17] investigated the dynamic compressive behaviour of unidirectional carbon/epoxy laminates made of Hexcel IM6G/3501-6 and found that longitudinal compressive strength and failure strain increases with increasing strain rate, but no strain rate effect was found on the longitudinal compressive modulus. Recently, Koerber and Camanho [19] also investigated the strain rate effect on unidirectional carbon/epoxy prepreg (Hexply IM7-8552) loaded in longitudinal compression. The quasi-static tests were performed on a conventional hydraulic test machine at a strain rate of 3.6×10^{-4} /s and a split Hopkinson pressure bar (SHPB) setup was used to carry out dynamic tests at strain rates between 63 /s and 118 /s. It was found that the longitudinal compressive modulus was not strain rate sensitive; however longitudinal compressive strength and failure strain increased with strain rates.

In transverse compression, an increase in transverse stiffness and strength was reported in [18, 20, 21]. Koerber et al. [21] performed quasi-static and dynamic tests on an UD carbon/epoxy composite in transverse compression. Quasi-static tests were carried out in a standard test machine and dynamic tests using a compressive SHB at strain rates between 90 and 350 /s. An increase in transverse modulus and strength is observed.

The investigation of the strain rate effect on the shear response of polymer composites has received significant attention over the past decades. For dynamic shear testing however, no common standard exists and different approaches have been used in previous experimental studies. From the commonly used test methods, good agreement exists regarding the accuracy of shear stiffness measurements. Huge discrepancies are found however when determining the shear strength, which can be attributed to reasons such as: edge effects, imperfect stress distributions, in-situ effects and perhaps most importantly, the presence of normal stresses.

Authors, e.g [15, 16, 20, 21], reported an increase in shear modulus and strength and a decreasing failure strain under in-plane shear loading. Koerber et al. [21] tested quasi-static and dynamic loadings under in-plane shear loading and found an increase in the stiffness and yield strength. Shokrieh and Omidi [22] studied in-plane shear behaviour of unidirectional E-glass /ML-506 epoxy resin composite under quasi-static and intermediate strain rate loading conditions and found that under dynamic loading, the failure shear strength increased with increasing the strain rate but the shear modulus decreased. A more through review of rate dependent behaviour of UD carbon/epoxy and other material systems such as glass/epoxy is available in a technical report by Singh [23].

Based on these observations, a simple but an efficient *Homogenization* technique is developed combining the constituents response to simulate the dynamic response of a UD ply and used in **Paper A - B**. An improved version of this technique is used in **Paper C - D**.

3. Rate dependent mechanical response - constitutive models

Macroscopic localization generally result from the damage and failure events at the microstructure level, including matrix localization under shear bands, matrix cracking, fibre breaking and delamination. As a result, the damage failure modeling of composites requires a multiscale approach incorporating the microscale mechanisms. In this chapter, a review of dynamic constitutive models for composite are presented. A variety of approaches have been applied to model the rate dependent response of polymer matrix composites. From the knowledge of the material and structural response of polymer composites, researchers developed dynamic constitutive equations on both micro (constituent) level and meso (ply) level. For developing the models on micro level, different models were developed for constitutents under dynamic loadings and explained in the upcoming sections.

3.1 Constitutive models for fibres

Fibres can be rate-independent and rate dependent. Based on the literature review, carbon fibres are considered to be rate-independent while glass and other polymer fibres are clearly rate sensitive. Carbon fibres are therefore modeled as a transversely isotropic and linearly elastic material [24, 25] and glass fibres are assumed as a rate dependent elastic material [26]. A coupled viscoelastic – viscoplastic model is also developed for the aramid and polyester fibres for short loading times [27, 28].

3.2 Constitutive models for the matrix

Polymers are known to have a rate dependent deformation response. Traditionally, for very small strain analyses, linear viscoelasticity has been used to simulate the material behaviour phenomelogically [29]. In linear viscoelastic models, combinations of springs and dashpots have been used to capture the rate dependent behaviour. For cases where the strains are large enough that the response is no longer linear, nonlinear viscoelastic models have been developed.

Numerous experimental observations and studies have shown that the responses of pure polymers are rate- and pressure dependent and exhibit both recoverable (elastic/viscoelastic) and irrecoverable (plastic/viscoplastic) deformations, even under relatively low stress levels. As the stress increases, the rate dependent responses become more pronounced [30]. Therefore, to better model the complex constitutive behaviour of polymers the combined effects of viscoelasticity, viscoplasticity, and viscodamage (i.e. delay or time-dependent damage) have to be considered in modeling of these materials. Macroscopic phenomenological models used to capture the viscosity of a polymer are usually linear viscoelastic models (Maxwell, Kelvin, standard linear solid model by Zener, Multiple-elements models), non-linear viscoelasticity models and viscoplastic models. Goldberg et al. [31] developed a viscoplastic state variable model based on the Bodner–Partom model to describe the strain-rate-dependent deformation of the polymer. Praud et al. [32] proposed a phenomenological model for semicrystalline thermoplastic polymers, which accounts for viscoelasticty, viscoplasticity and ductile damage.

Schapery [33, 34] has developed a viscoelastic–viscoplastic model with continuum damage for the matrix based on the laws of thermodynamics. However, Schapery's damage model has some limitations as it can be used only for predicting viscoplasticity and damage evolution in tensile stresses. Zhang et al. [35] included damage evolution in the nonlinear-viscoelastic behaviour of the polymer, but this model did not consider the effects of viscoplasticity and does not have the capability to distinguish between rate-, and temperature - dependent behaviours.

3.3 Homogenization approaches

The effective (homogenized) elastic properties at a mesoscale (ply) are estimated from microscale models using a Numerical homogenization techniques. These techniques were based on analyzing the behaviour of a Representative Volume Element (RVE) of the composite. The RVE is the smallest portion of the composite which is considered to be a representative of the response of the composite as a whole.

The simplest types of technique are equations based on various uniform stress and uniform strain assumptions utilized within the composite RVE to compute the effective properties and response of the material. Examples of this type of approach include the traditional Voigt [36] and Reuss [37] based "rule of mixtures" equations and Halpin-Tsai based closed-form approximations for transverse and shear modulus. These models incorporate limited use of microstructural information like fibre volume fraction and gives direct formulation for the effective mechanical properties which can later be used to form the stiffness matrix of the composite. While this approach involved a great deal of approximation and simplification, the resulting equations were very simple in form, very easy to implement within a computer code, and very computationally efficient.

A more sophisticated method to compute the effective properties of composite materials is based on solving the governing constitutive equations in an average sense within the RVE. Examples of this methodology include the Concentric Cylinders Model [38], the Self Consistent Method [38] and the Mori-Tanaka Method [39]. These scheme based models give direct estimation of the stiffness matrix of the composite by a continuum mechanics approach. This method completely satisfy the field equations of mechanics, resulting in a more accurate representation of the physics of the problem, in comparison to the later approach. More recently these methods, in particular the Mori-Tanaka Method, were used to model the rate dependent behaviour of polymer composites [40].

3.4 Constitutive models at micro level

Based on the literature review, it was found that some fibres are rate sensitive and some are not and almost all matrix material, on the other hand, are known to be rate sensitive. Therefore, when developing strain rate dependent models for composites care must be taken to separate the effect of the constituents on the composite. For this purpose, micromechanics based models in which the effective properties and response of the composite are computed based on the properties and response of the individual constituents are suitable for dynamic application. Most of the three dimensional material models presented in the literature for dynamic simulations consider the entire lamina to be rate sensitive [41, 42] with no attention being paid to the specific mechanisms which cause failure. In reality, failure in a composite is a result of specific local mechanisms such as fibre failure, matrix cracking or delamination. The accurate modeling of these local failure mechanisms requires a detailed microscale model. Therefore, this thesis work is based on developing a micromechanics based model to capture the rate effects experienced by the UD carbon/epoxy composites under compressive dynamic loading.

There are some who developed micromechanics based approaches to simulate the dynamic response of polymer matrix composites but without damage. For example, Wang et al. [43] developed a finite strain elastic-viscoplastic self-consistent model for polycrystalline materials. Schapery [44] proposed a nonlinear viscoelastic-viscoplastic model and considered a Concentric Cylinder Assembly model to predict the viscoplastic behaviour of a glass fibre composite. Goldberg et al. [45] developed a nonlinear, strain rate dependent deformation and strength model for the analysis of polymer matrix composites. More recently, a constitutive model considering viscous effects in the mechanical behaviour of a UD carbon/epoxy system using a fully 3D viscoelastic-viscoplastic material model at the ply scale was developed by Gerbaud et al. [46]. Recently physically based three dimensional failure theories started to emerge. These novel theories aim at representation of the failure mechanisms and enable a more realistic prediction of the various composite failure modes. Gutkin et al. [47] presented a model accounting for damage growth during fibre kinking in the UD composites. Camanho et al. [48] proposed a smeared crack model to predict the ply failure. Costa et al. [49, 50] developed a physically based model and Larsson et al. [51] proposed a set of CDM models for fibre kinking under compression in UD composites. A three dimensional CDM model was proposed by Pinho et al. [52].

A micromechanical model with progressive failure was developed by Tabiei et al. [24] where strain rate (via viscoplasticity) is accounted for the matrix model and a CDM based failure model is used to incorporate the progressive post-failure behaviour under impact. Another model was proposed by Nguyen et al. [25] to predict nonlinear behaviour of UD composites under quasi-static loading. The fibres are assumed to be transversely hyperelastic isotropic and the matrix obeys a hyperelastic viscoelastic/plastic constitutive model enhanced by a nonlocal damage model. Recently, Tan and Liu [53] developed a micromechanical model to capture the matrix shearing and fibre rotation of CFRPs at different strain rates. The carbon fibre composite is homogenised, based on various inelastic slip systems identified from the fibre architecture. Based on the current state of art on micromechanics based dynamic models for composites, a need has been found to develop a reliable micromechanical model to capture the strain rate effects and local progressive damage growth for crash/dynamic applications. For such an application, a simple but efficient material model is needed for computational efficiency and therefore a linear elastic and transversely isotropic model for the fibres and linear viscoelastic-viscoplastic with continuum damage model for the matrix is used. To get the response at ply level, an improved homogenization approach based on *constant stress* and *constant strain* is developed.

3.5 Constitutive models at ply (meso) level

In this approach, the composite material is modeled as an anisotropic, homogeneous material, without any attention being paid to the individual constituents. However, this approach has not been taken into consideration in this thesis work but is included for the sake of completeness.

Xia and Xing [54] developed a one-dimensional constitutive equation for glass fibre reinforced epoxy using a statistical analysis of impact test data under strain rate from 300 to 2000 /s. The model can describe the macroscopic mechanical behaviour of unidirectional glass-fibre-reinforced epoxy. Chen and Sun [55] proposed a 3-D plastic potential function to describe the nonlinear behaviour in anisotropic fibre composites. Following [55], Weeks and Sun [56] developed a mesomechanical, rate dependent constitutive model based on a one-parameter plasticity model and a modified Johnson rate dependent model for different strain rates. Rate dependency is incorporated via a quadratic plastic potential function. This model was further developed by Thiruppukuzhi and Sun [57] for general loading conditions in unidirectional and woven fabric composites. Later they [58] proposed a rate-dependent failure criterion based on the viscoplasticity model for glass/epoxy composites and found an equivalent plastic strain rate. A constitutive model was proposed by Gates et al. [59] to describe the elastic/viscoplastic behaviour of composites under plane stress conditions. This model was formulated for a quasi-static plasticity and time dependent viscoplasticity. Eskandari et al. [60] developed a mesoscale model by considering a viscoelastic-viscoplastic model coupled with CDM for composites at high strain rates.

The overall conclusion from this portion of the review is that the dynamic response of composite materials has been modeled using a variety of methods. In microscale approaches, the rate dependence and nonlinearity of the polymer matrix were modeled at the constituent level. The homogenization techniques then compute the effective deformation response of the composite based on the response of the individual constituents. However, the nonlinearity and rate dependence of the composite were accounted for at the ply level in mesoscale approaches.

4. Numerical framework for non-linear dynamic behaviour of UD composites

Composite materials are heterogenous materials and the constitutive model therefore depends on the scale on which modelling is performed. Thus, in the current research, the material behaviour of fibre-reinforced composites is investigated on different scales. A multiscale modeling strategy is used since it can align with the multiscale nature of composite materials and aid in assessing damage evolution and failure of CFRP materials. The material response at various length scales can be assessed, including the microscale where individual fibres and the surrounding matrix are considered, the mesoscale and macroscale where homogenized plies or a laminate containing multiple plies are represented, respectively, and the structural scale where components or an assembly of components are accounted for (Figure 4.1).

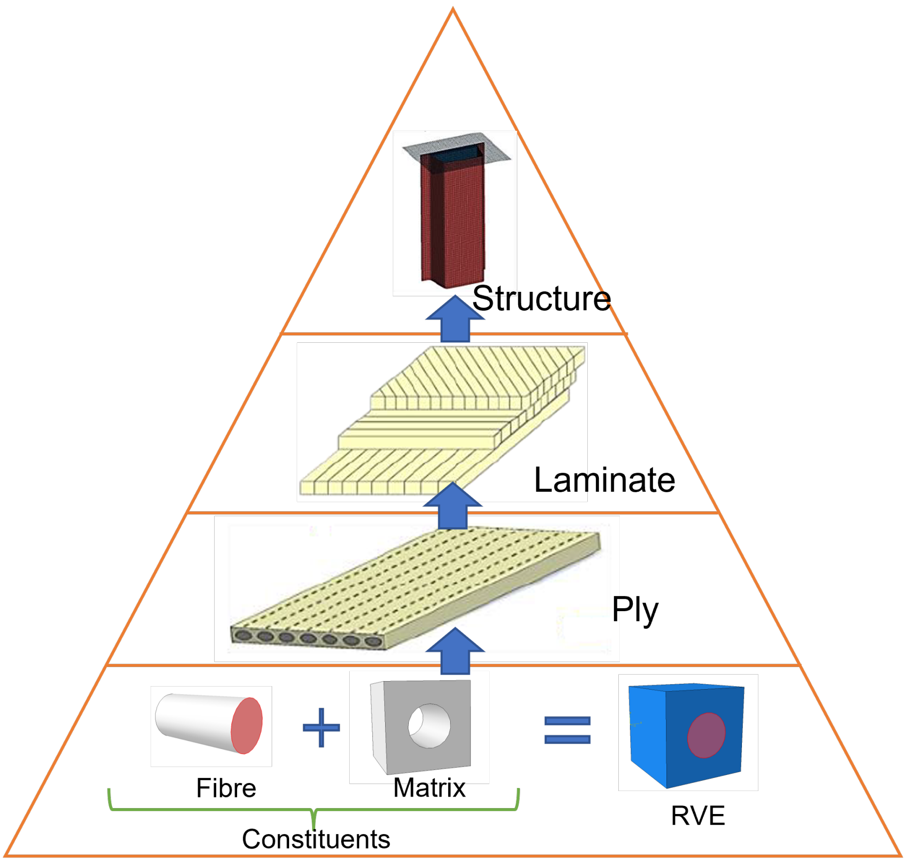

Figure 4.1: Schematic of a bottom-up (hierarchical models) multiscale modeling approach.

There are two main approaches for the computational multiscale analysis of heterogeneous materials: the top-down (global-to-local) approach and the bottom-up (hierarchical) approach. For the top-down approach, normally, a finite-element analysis is first used to analyze the entire structure of interest and identify the local critical regions where the damage may occur. Further detailed analyses are then performed on those regions using lower length scale models. The limitations of this method are the requirement of a large number of costly experiments and limited capabilities to predict different damage states.

On the other hand, the bottom-up approach begins the analysis from the scale of the material microstructure by considering the properties of the fibres and matrix. The homogenized constitutive response of the microstructure is passed up to the next length scale model to predict the mechanical response of composite materials (Figure 4.1). This bottom-up approach provides a better connection between different scales, allowing better physical representation and fewer physical experiments. It is also a preferred method toward the virtual testing of composite materials.

To utilize a bottom-up multiscale approach, reliable constituent mechanical properties, including the matrix, are critical for developing a ply-level model. For the crashworthiness analyses of CFRP structures, the strain rate dependent response of the polymer matrix is important to incorporate into a micromechanics based model.

4.1 Problem Formulation

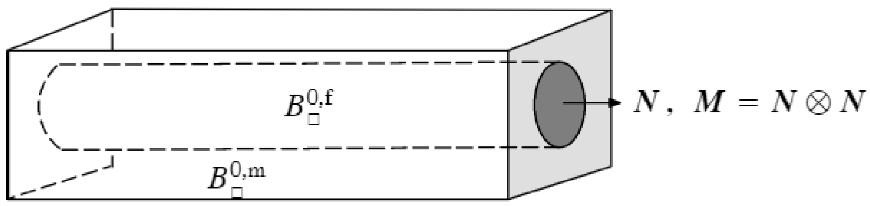

Figure 4.2: RVE of the micromechanics based constitutive model.

Following the bottom-up approach, a dynamic constitutive model is developed herein considering the microscale of a UD-ply. The model is used to validate experiments performed at different scales. Micromechanics based models can estimate the properties of a UD composite at the ply level by changing the fibre and matrix properties. These models are therefore effective in simulating the response of any laminate with an arbitrary layup. To do that, an RVE has been established, where fibres and the matrix modeled separately (see Figure 4.2). The material response is homogenised by smearing the behaviour of fibres and matrix over the RVE. In this thesis, the RVE represents a single unidirectional reinforced lamina where fibre orientation is defined by the **M** tensor.

The RVE is shown in Fig 4.2 has the region $B_{\square}^{0}$, where the fibre region is denoted $B_{\square}^{0,f} \in B_{\square}^{0}$ and the polymer matrix region is denoted $B_{\square}^{0,m} \in B_{\square}^{0}$. A carbon/epoxy composite is embedded in the RVE, where carbon-fibres are considered rate-independent whereby all rate dependency of the composite is assumed to occur in the resin, as explained in section 2.2. Therefore, the carbon fibres are assumed to be linear elastic transversely isotropic and the epoxy is considered non-linear viscoelastic-viscoplastic with damage.

4.2 Research objectives

The objective of the research is to develop a rate dependent constitutive model to predict the mechanical properties of a UD carbon/epoxy composite under compressive dynamic loading.

Hence, a main incentive is to improve the available constitutive model capabilities for UD carbon/epoxy composites subjected to dynamic loading. In order to reach these objectives, the following tasks are addressed in the thesis:

- *incorporation of strain rate dependent material behaviour into the constitutive equations.*
- *extension of the constitutive model to include an intralaminar damage algorithm which predicts the initiation and propagation of damage.*
- *calibration/validation of the proposed model with conducted experiments for the dynamic model parameters.*
- *implementation/validation of the proposed constitutive model into an explicit FE environment.*

The current state of research requires a novel contribution to each of the tasks stated above. The focus of the constitutive model development lies in the implementation of a rate dependent model and to couple it with continuum damage for the prediction of the initiation and subsequent damage evolution. Micro- and meso-scales are considered in the model development.

4.3 Material models for constituents

Most experimental results on carbon fibre-reinforced polymers indicate that irrespective of the mode of loading, the mechanical properties such as stiffness, strength and failure strain are dependent on the applied strain rates (see section 2.2). Since the same rate-dependent behaviour has been observed for unreinforced polymeric materials, it is assumed that the rate-dependent characteristics of the composite can be described by the viscoelastic-viscoplastic property of the polymer. Rate-dependent constitutive models, first developed for nonlinear behavior of metals, based on plasticity and viscoplasticity have been developed for polymer matrix composites in the recent past [55, 56, 57, 58]. However, the deformation mechanisms of a polymer compared to a metal are different. Metals are isotropic, crystalline structures. Elasticity in metals is governed by intermolecular forces between atoms, while inelasticity is caused by yield or dislocation mechanisms when the distortion energy is above a critical value. Thermoset polymers, on the other hand, are amorphous long molecules with covalent cross-links and weaker van der Waals bonds. Cross-links give polymers elasticity, but viscous flow always accompanies the deformation of a polymer. Polymers are generally viscoelastic but some exhibit viscoplasticity, i.e., they undergo permanent deformation upon loading. Rheological models for the carbon fibres and rate-dependent matrix developed in this thesis will be discussed in the following sections.

4.3.1 Model for carbon fibres

As mentioned in the introduction, in contrast to glass and Kevlar fibres, carbon fibres do not show any rate dependency. The stiffness of carbon fibres is assumed to be rate insensitive. Therefore, they are assumed as an elastic transversely isotropic material. Note that the transverse and shear moduli in the UD composite show rate dependent behaviour as the matrix material significantly contributes to the response.

The total Kirchhoff stress in the carbon fibre is defined as

$$\boldsymbol{\tau}^{\mathrm{f}} = \boldsymbol{F} \cdot \frac{\partial \psi^{\mathrm{f}}}{\partial \boldsymbol{E}} \cdot \boldsymbol{F}^{t} = \boldsymbol{\tau}_{d}^{\mathrm{f}} + \boldsymbol{\tau}_{v}^{\mathrm{f}} + \boldsymbol{\tau}_{s}^{\mathrm{f}} + \boldsymbol{\tau}_{a}^{\mathrm{f}} \tag{4.1}$$

where $\boldsymbol{\tau}_{d}^{\mathrm{f}}$ and $\boldsymbol{\tau}_{v}^{\mathrm{f}}$ are respectively the deviatoric and the volume change energies and $\boldsymbol{\tau}_{s}^{\mathrm{f}}$ and $\boldsymbol{\tau}_{a}^{\mathrm{f}}$ refer to the longitudinal fibre shear and the axial fibre actions.

4.3.2 Model for matrix under compression

The experimental evidence in Chapter 2 demonstrates that strain rate dependent material behaviour is pronounced in composite materials, especially for compressive off-axis loading. For a carbon fiber composite system, the strain rate effect occurs in the matrix material as discussed in section 2.2. The dynamic behaviour is modeled by introducing viscoelastic-viscoplastic behaviour for the polymer matrix.

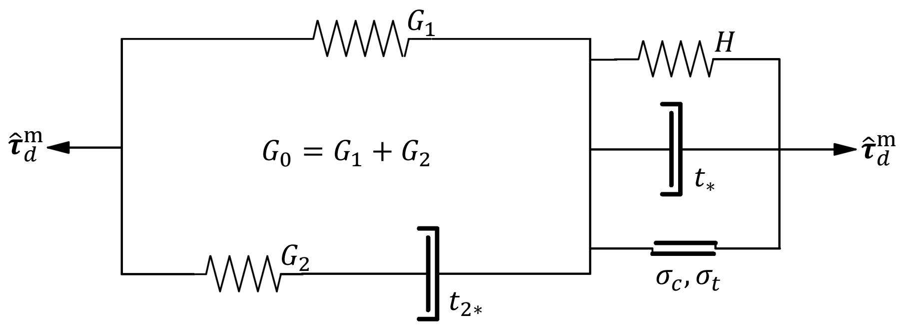

Figure 4.3: Adapted rheological model for the viscoelastic-viscoplastic shear stress response of the polymer matrix of the composite.

From the rheology in figure 4.3, viscoelastic material behaviour of the deviatoric (or shear) stress response $\hat{\boldsymbol{\tau}}_{d}^{\mathrm{m}}$ is described by a spring-damper system. Commonly used viscoelastic models are the Kelvin-Voigt (spring and damper in parallel), the Maxwell model (series of spring and damper) and the Zener model (Kelvin-Voigt with spring in series or Maxwell with spring in parallel). For the viscoplastic response, the Bingham and the Norton model are commonly used.

Herein, the Zener viscoelastic model is combined with a Bingham viscoplastic model as shown in the figure 4.3. This choice enables efficient model implementation alongside relatively few parameters to be determined. These are: the quasi-static and dynamic shear moduli, G_1 and G_2, and the relaxation times t_{2*} and t_* associated with the viscoelastic and viscoplastic response. The pressure dependency of the polymer matrix is considered by defining σ_t, σ_c describing the quasi-static yield stress of the polymer matrix in tension and compression.

4.4 Intralaminar continuum damage model

Experimental studies have shown that the failure of a composite is sensitive to microstructure variations. Therefore, the failure of composite materials is a multi-scale phenomenon with damage initiation and evolution at different length scales.

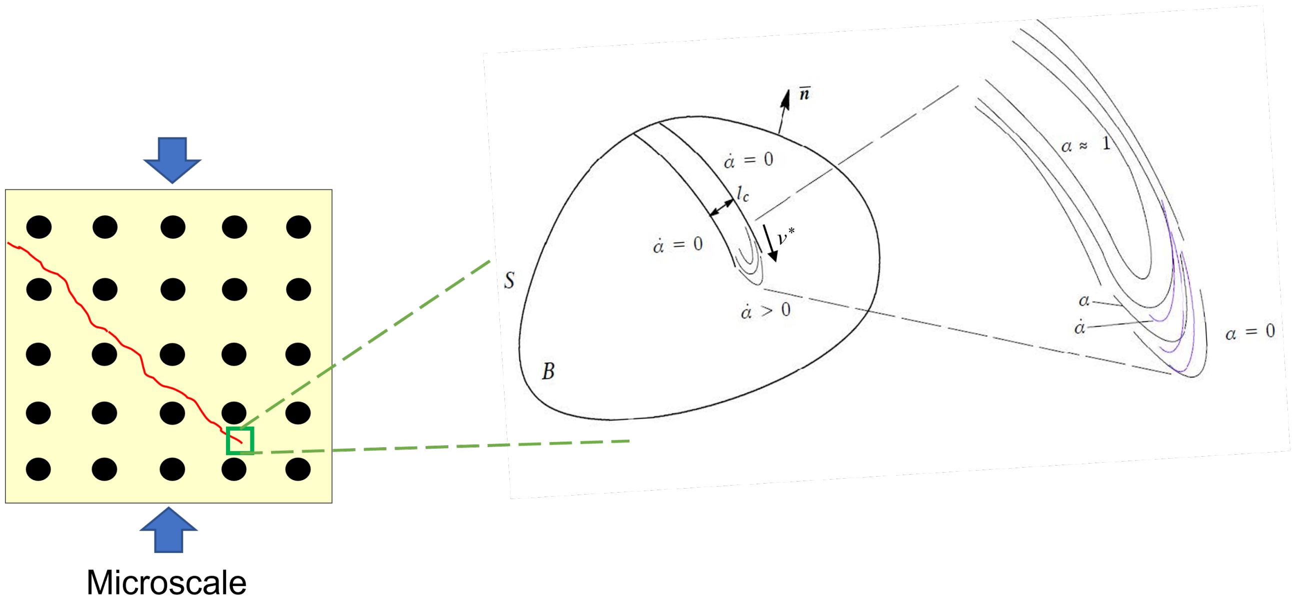

Figure 4.4: A damage degrading solid in reference configuration B_0 with a diffusive localized distribution of the damage field representing the separation of crack surfaces.

In this thesis, a micromechanics based approach is adopted where the polymer matrix is degraded based on a continuum damage mechanics (CDM) model. This model couples the microscale of the polymer matrix to the damage field, defined at the ply-level. The stress degradation of the matrix may be described in terms of the total Kirchhoff stress written as

$$\boldsymbol{\tau}^{\mathrm{m}} = f[\alpha]\hat{\boldsymbol{\tau}}_d^{\mathrm{m}} + \tau_v^{\mathrm{m}}\mathbf{1} \text{ with } \hat{\boldsymbol{\tau}}_d^{\mathrm{m}} = \hat{\boldsymbol{\tau}}_{1,d}^{\mathrm{m}} + \hat{\boldsymbol{\tau}}_{2,d}^{\mathrm{m}} \tag{4.2}$$

where $f[\alpha] = (1-\alpha)^2 + r$ is the damage degradation function (that only degrades the matrix).

The CDM model predicts the onset and evolution of damage ($\dot{\alpha}$) using the Bingham type damage evolution law as shown in the following equation

$$l_c\dot{\alpha} = v^*\langle\alpha^{\mathrm{s}}[\alpha] - \alpha\rangle \text{ with } \alpha^{\mathrm{s}} = \frac{\mathcal{A}_{\mathrm{T}}[\alpha]l_c}{\mathcal{G}_{\mathrm{c}}} \tag{4.3}$$

where α^{s} represents the static damage, the internal length parameter l_c describes the diffusive character of the fracture area, v^* is the fracture area progression speed parameter that controls the damage evolution $\dot{\alpha}$, $\mathcal{A}_{\mathrm{T}}$ is the (total) damage driving energy and $\mathcal{G}_{\mathrm{c}}$ is the fracture energy for the matrix [51]. A total continuum dissipation is introduced used to define the coupling between elastic and, in particular, viscoplastic contributions to the damage driving force $\mathcal{A}_T = \mathcal{A} + \mathcal{B}$, cf. paper B. Various formulations are conceivable based on this formulation. In paper C the elastic $\mathcal{A}-$contribution is omitted, whereby damage evolution does not commence until the onset of viscoplasticity.

4.5 Interlaminar damage growth

The rate dependent material model described in the previous sections is used to simulate an intralaminar damage growth in the polymer composites at different strain rates and under compressive loads. To use this material model for a laminate analysis, an interlaminar damage model is needed to simulate dynamic failure such as delamination in a composite structure. The cohesive method based on fracture mechanics has been frequently utilized for the prediction of delamination propagation. To capture the ply-to-ply delamination in dynamic loading conditions, the surface-based cohesive method with a bilinear traction-separation law is used. It is primarily chosen because the interface thickness is negligibly small as compared to the ply thickness.

In this cohesive law, the evaluation of delamination damage was divided into two stages: the delamination initiation stage and the delamination propagation stage. A maximum nominal stress failure criterion is adopted to describe the initiation of the delamination, while the fracture mechanics based 'B-K criterion' [61] is adopted to describe the propagation of the delamination. This model is readily available in ABAQUS software. A complete derivation of the surface based cohesive model can be found in the ABAQUS Manual.

4.6 Computational homogenization for a UD ply

This section summarises how strain the rate dependent material behaviour is homogenized from the fiber and the matrix constituents. The fiber response is assumed elastic, whereas the matrix material governs the quasi-brittle failure process in compression under dynamic loading. Concepts of computational homogenization are then used to connect the microscale with the (ply-level) mesoscale. The homogenized response at the ply level is used to calibrate the dynamic parameters for the model.

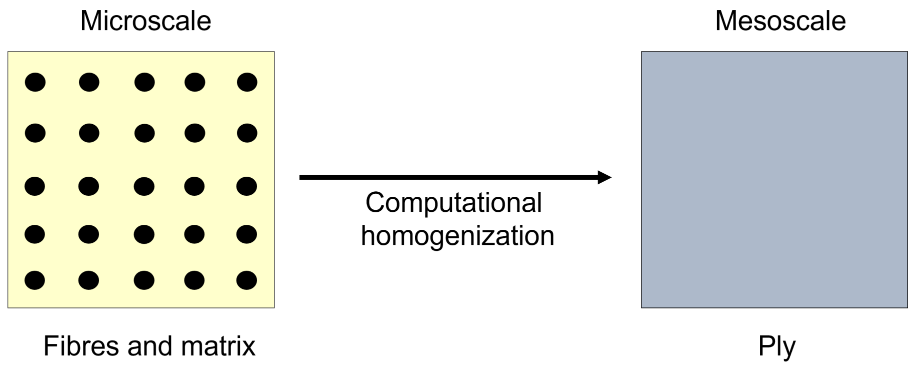

Figure 4.5: Schematic of computational homogenization technique linking microscale behaviour to mesoscale response.

As to the homogenization, the ply level response is obtained by volume averaging the mechanical response of constituents i.e. carbon fibres and matrix over an RVE using a computational homogenization *technique*. An efficient and simplified homogenization approach is used in this work for numerical efficiency. The homogenization is based on an *isostrain* assumption along the fibre direction, i.e. fibres and matrix experience strains equal to the strain on the composite and an *isostress* assumption is used for the transverse direction i.e. fibres and matrix experience stresses equal to the stress on the composite. We also assumed a perfect bond between fibres

and matrix. This is used in **Paper A - B** to simulate dynamic behaviour of a ply with different fibre orientation.
The homogenized stress response at ply level used in **Paper A - B** is obtained explicitly as

$$\bar{\boldsymbol{S}} = v^{\mathrm{m}}\,\boldsymbol{S}^{\mathrm{m}} + v^{\mathrm{f}}\,\boldsymbol{S}^{\mathrm{f}} + a^{\mathrm{m}}\,v^{\mathrm{m}}\,\hat{\boldsymbol{I}} : \left(\boldsymbol{S}^{\mathrm{m}} - \boldsymbol{S}^{\mathrm{f}}\right) \tag{4.4}$$

where $\boldsymbol{S}$ is the 2nd Piola Kirchhoff stress, a^{m} is the "equilibrated" counterpart of the scalar piecewise constant fluctuation field a in the matrix (cf. **Paper B**), $\hat{\boldsymbol{I}}$ defines a projection tensor projecting macroscopic strain onto the transverse direction of the fibres.

This homogenization technique is not accurate enough to predict in-plane shear failure and fibre kinking for small fibre angles due to the lack of *piecewise constant* scalar field in fibre shear direction. The technique is improved and implemented in **Paper C - D**. The improved homogenized stress response at ply is

$$\bar{\boldsymbol{S}} = v^{m}\boldsymbol{S}^{m} + v^{f}\boldsymbol{S}^{f} + v^{m}\left(a^{m}\mathbb{F} + b^{m}\mathbb{G}\right) : \left(\boldsymbol{S}^{m} - \boldsymbol{S}^{f}\right) \tag{4.5}$$

where a^{m} and b^{m} are the "equilibrated" counterpart of the scalar piecewise constant fluctuation fields a and b in the matrix (cf. **Paper C**) for representing a split of microscopic strain into transverse and in-plane shear strains. It results in an *isostress* assumption in the transverse and in-plane shear direction to the fibres. In equations 4.4 and 4.5, $\bar{\boldsymbol{S}}$ represents the homogenized 2nd Piola Kirchhoff stress and $\boldsymbol{S}^{f}$ and $\boldsymbol{S}^{m}$ are the 2nd Piola Kirchhoff stresses of the fibre and matrix constituents, respectively. v^{m} is the volume fraction matrix material and v^{f} is the fibre volume fraction. Homogenized response of the constituents model gives complete insight into the damage evolution at the microscale.

4.7 Estimation of dynamic model parameters

The calibration of composite damage models has been widely discussed in the composite research community. Ideally, the dynamic and damage parameters should be directly measured from experiments. The brittle nature of most composites, however, makes this a difficult task. Therefore, an approach based on numerical optimization, such as the least square method, is used to identify dynamic model parameters. This relies on obtaining best guesses for dynamic parameters for a given set of experimental results.

In this approach, a calibration routine finds the dynamic model parameters by minimizing the error between homogenized material response at a material point and the uniaxial stress-strain response from the experiments. For the homogenized material response, assumed fibre and matrix properties are used as an input to the material model (see figure 4.6). Both quasi-static and dynamic experimental results are used for the calibration. Once the model parameters are obtained from the calibration of material response, they are used for structural simulation of the UD composite as shown in figure 4.6.

This technique has been used in this thesis for predicting the dynamic parameters for the matrix model from off-axis experimental responses under quasi-static and dynamic loading. As shown in figure 4.7, experimental results for 45° and a 30° off-axis specimens were used to find out dynamic model parameters in **Paper B** and **D**. Introduction of an isotropic hardening and

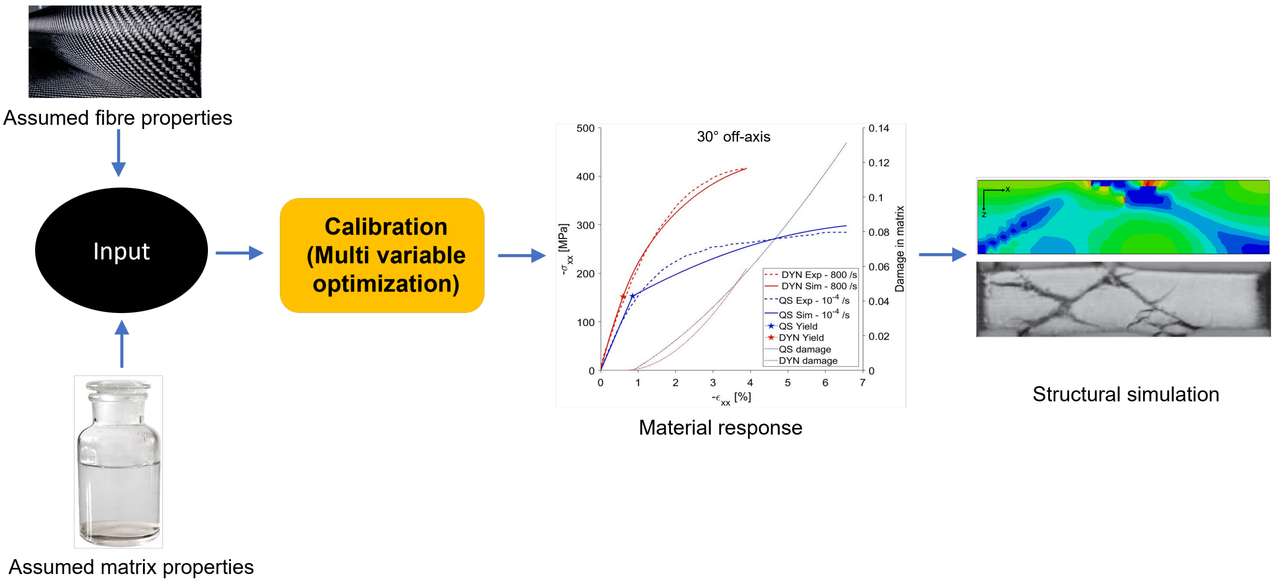

Figure 4.6: Flowchart describing main steps in calibration routine. Experimental result from [21].

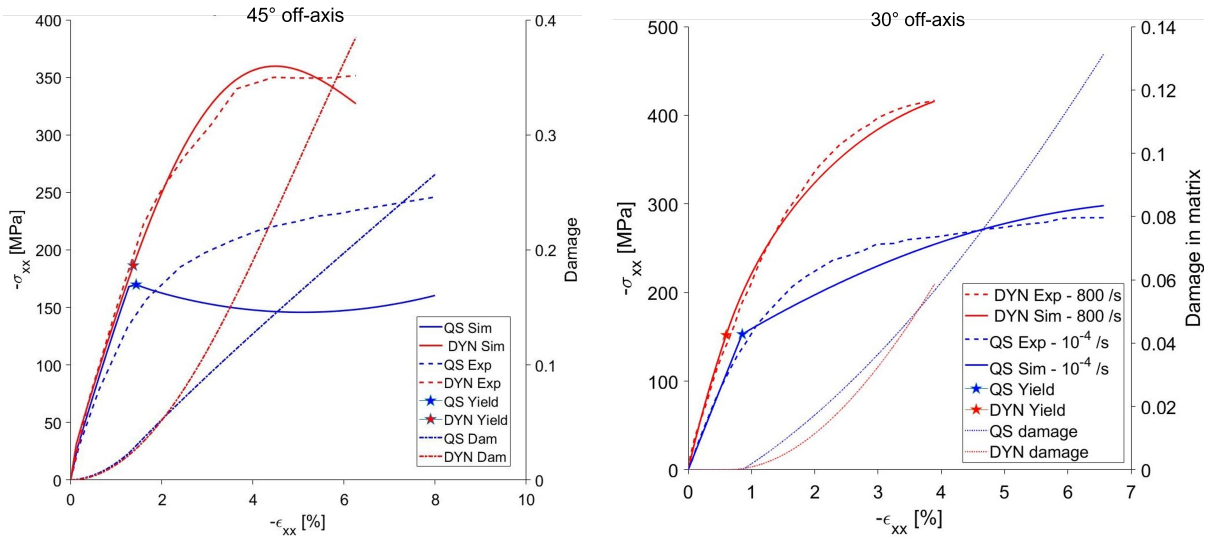

Figure 4.7: Calibration results for **Paper B** (45° off-axis) and **Paper D** (30° off-axis).

a better homogenization technique improved the calibration results from **Paper B** to **Paper D** (see figure 4.7). Currently, the calibration is implemented within MATLAB, which then provides the required parameters. For future applications, it would be possible to implement the calibration algorithm as a preprocessing routine, thus avoiding the external calibration procedure.

4.8 Numerical implementation and validation

This section explains the implementation of the proposed constitutive model in the explicit finite element solver Abaqus/Explicit. For dynamic analyses, an explicit dynamic FE has already proven to be a powerful tool. The non-linear problem is solved incrementally with the calculation increment (time step) being defined by the material properties and the spatial discretization. For each time step, an increment of stress is predicted for a given increment of strain. The assessment of the current state of stress (at time t) is usually performed by an incremental stress predictor or trial stress. The type of element used is the 8-node brick, C3D8R, with reduced integration and enhanced hourglass control.

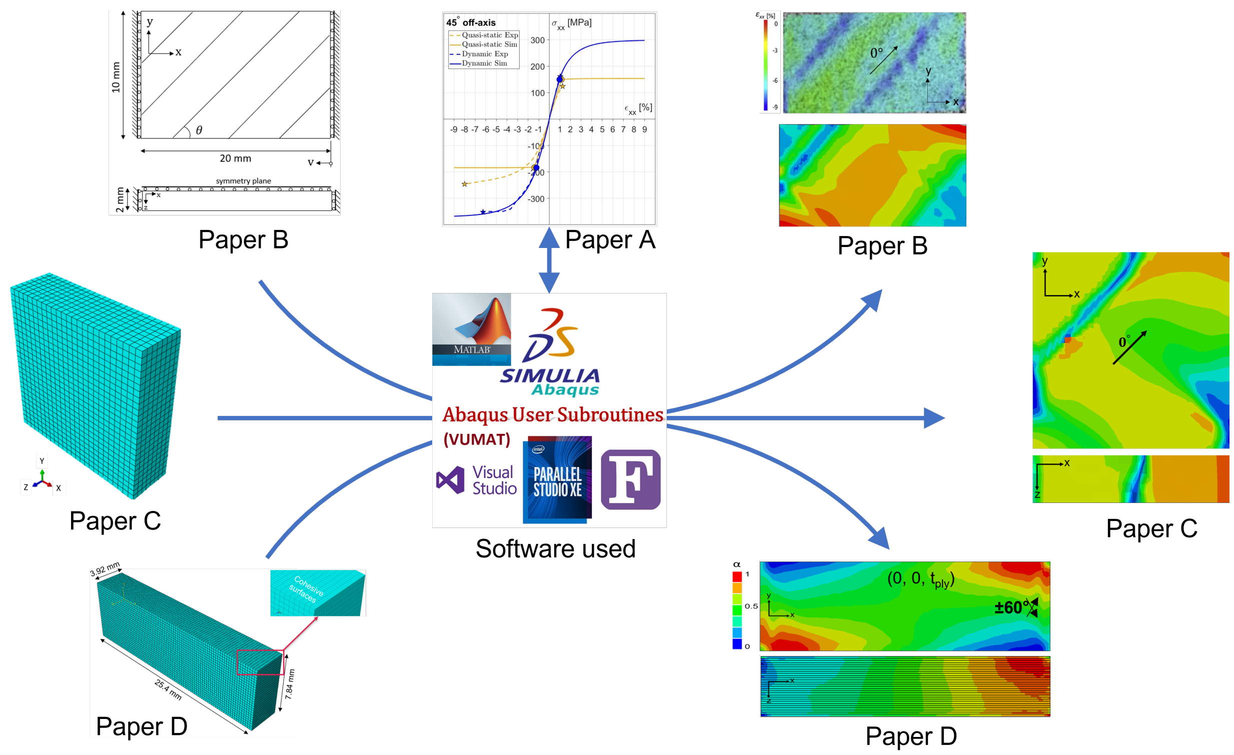

Figure 4.8: Representation of numerical models and simulation results of different material systems and layups tested in various papers. Experimental result from [21].

Different material systems and layups are simulated to demonstrate the robustness of the constitutive model. For example, in **Paper A**, the homogenized material model response was successfully verified by numerical simulations of the IM7/8552 material system subjected to a variety of uniaxial off-axis tests using a MATLAB subroutine.

For **Paper B**, **C** and **D** the constitutive model is implemented in ABAQUS/Explicit solver using the user material subroutine VUMAT. The model is validated by comparing the finite element predictions with the published experimentally measured stress-strain responses of a UD polymer composite in compression and subjected to quasi-static and dynamic loading [21]. This allows simulation of a realistic multilayered and multidirectional composite laminate with microscopic damage growth.

In **Paper B**, an IM7/8552 material system is again investigated to simulate the dynamic damage growth observed in the experiments. Another material system known as unidirectional NCF (non-crimp fabric) is validated for different off-axis tests in **Paper C**. After validating two material systems successfully in **Paper A**, **B** and **C**, the ambition was to test the material model for a laminate analysis. Therefore, **Paper D** was focused on validating a rate dependent response of an angle-ply laminate of IM7/8552 polymer composite. A schematic of different numerical models tested and validated in this thesis work is shown in figure 4.8.

5. Summary of appended papers

Paper A : A micromechanically based model for strain rate effects in unidirectional composites.
In this paper, a novel 3D constitutive model for capturing rate dependent response of a UD carbon/epoxy composite is developed. The model is micromechanics based, consisting of a rate dependent polymer matrix and an elastic fibre material. For the matrix, a viscoelastic-viscoplastic model with pressure dependency and a linear elastic transversely isotropic model for the fibres is implemented, based on small deformation theory. A *simplified* computational homogenization is used to connect constituent models based on *isostrain* assumption along the fibre direction and an *isostress* assumption transverse to the fibre direction. The material model is validated through numerical simulations of an IM7/8552 UD composite subjected to various uniaxial off-axis tests in compression at the material point using a MATLAB subroutine. Fairly good agreement is obtained for the homogenized response in the validation.

Paper B : A micromechanically based model for dynamic damage evolution in unidirectional composites.
In this paper, **Paper A** was extended by proposing a rate dependent continuum damage model coupled to a viscoelastic-viscoplastic formulation for the matrix model. A formulation including nonlinear kinematics is developed to capture the in-plane (intralaminar) failure mechanism of UD composites. The rate dependent constitutive model at ply level is based on simple constitutive models for the polymer matrix and fibres with the intention to apply for dynamic analyses of large scale structural composites. The model was validated by experimental results of the IM7/8552 UD polymer composite in quasi-static and dynamic compression loading. Relatively good correlations were achieved between the numerical models and experimental results under dynamic loading.

Paper C : A micromechanics based model for rate dependent compression loaded unidirectional composites.
In this paper, the *simplified* homogenization technique formulated and applied in **Paper A** and **B** is improved by a novel splitting of the microscopic strain into transverse strain and in-plane shear strain with parameters a and b which represent variation in the microscopic strain. The constituents are therefore coupled via an *isostrain* assumption along the fibres and *isostress* transverse and in-plane shear to the fibres direction. This allows prediction of shear failure and fibre kinking for small fibre angles, not captured by previous versions of the model. A modification is also implemented to improve the elastic response of the model by considering a non-uniform stress distribution in the matrix. This is achieved by accounting for Halpin-Tsai based predictions in the formulation. The model is applied to validate experimental results of NCF material system under quasi-static and dynamic loading and in compression.

Paper D : Rate dependent material model for progressive failure and delamination growth in multidirectional composite laminates.
In this paper, the rate dependent constitutive model developed (**Paper A** and **B**), improved (**Paper C**) and applied at a ply level is used to simulate quasi-static and dynamic compressive failure processes of angle-ply laminates from IM7/8552 carbon/epoxy. A standard traction-separation law based on cohesive surface from ABAQUS is applied to describe the interlaminar damage growth. The strain hardening phenomenon of polymer matrices is modelled by including an isotropic hardening to the viscoplastic response of the matrix. The model is applied to

simulate experimental results of angle-ply laminates of IM7/8552 under compressive quasi-static and dynamic loading. The predicted intralaminar (in-plane) and interlaminar (out-of-plane) damage growth of the composite laminates is consistent with the experimental observations.

6. Concluding remarks

At the start of this work, an extensive literature review of available data on rate dependent behaviour of unidirectional composite material was carried out. The review underlined the need to include strain rate effects in material models for dynamic applications e.g. crash modeling. In addition to that, studies are available in the literature about the development of dynamic models based on continuum damage mechanics for modeling the rate effects in polymer composites and also discussed in section 3. Most of these studies did not consider explicit models for constituents together with explicit formulation for damage evolution which makes such models limited to a certain class of polymer composites. To the author's knowledge, few studies have focused on modeling the dynamic damage evolution with micromechanics based approach for the constituents. For dynamic applications, the material model must be tested under dynamic loading conditions and based on a nonlinear kinematic formulation. Additionally, a computational efficient modeling approach is needed for applications such as e.g. crash analyses.

Therefore, the main goal and the focus of this work was to develop a novel computationally efficient 3D constitutive model for unidirectional carbon/epoxy composite to capture strain rate effects in compression. Since the focus was on developing an efficient model for dynamic applications, a simplified modeling approach was adopted. The developed material model is micromechanics based and consists of separate models for the constituents i.e. fibre and the matrix material. A main point with the micro-mechanical approach is that simplified modeling of the constituents suffices for an accurate result, i.e. an elastic model for the carbon fibres and an inelastic model with damage for the matrix for numerical efficiency. To get the response at the (ply) mesoscale, a reliable micromechanical coupling between constituents is needed and therefore an efficient homogenization technique was developed. This technique was based on the assumption of *isostrain* along the fibre direction and *isostress* transverse and in the in-plane shear direction to the fibres. Again, a simplified homogenization technique was developed that was needed for cost-efficiency of the large structural simulations.

In the material model, relatively well defined elastic parameters are involved in the fibre model. However, the matrix model requires several model parameters for the non-linear behavior. The viscoelastic model is based on a 3-parameter viscoelastic model, consisting of quasi-static and dynamic shear moduli, G_1 and G_0 and a viscoelastic damper with the relaxation time t_{2*}. A Perzyna-type viscoplastic model that obey the Bingham type model is used and consist of a pressure dependent quasi-static yielding σ_c and σ_t, a hardeninig parameter H and a visco-plastic damper with relaxation time t_*. This is coupled to a continuum damage model to predict the post-peak softening and shear failure of the matrix. The damage evolution is driven by the viscoplastic dissipation of the matrix and controlled by the parameters such as fracture energy G_c for the nest resin, the internal length parameter l_c and the damage progression velocity parameter v^*. Some of these parameters controls the dynamic stiffness response e.g. G_0 and t_{2*}. The dynamic nonlinear response prior to the damage onset is controlled by the t_* parameter and the dynamic post-peek softening response is influenced by l_c and v^* parameters. These parameters are important particularly for dynamic analysis.

Out of these model parameters some are estimated from experiments and others are identified by using a calibration method. For the calibration, a multivariable optimization code based on least square method is implemented in MATLAB. This code together with FORTRAN VUMAT performed a calibration by comparing model response with the experimental data at a material point level. The calibration is essentially a back calculation to get the dynamic parameters of the matrix from the uniaxial stress-strain data of the composite. The experimental data is based on a composite with a certain off-axis fibre angle and therefore the calibrated model parameters are not well suited for simulating UD composites with off-axis angles significantly larger than the one used for calibration. The damage parameters are realistically estimated from **Paper B** - **D** due to the lack of post-peak data in the experiments.

The focus was on dynamic applications, therefore an explicit material model was developed as a user defined subroutine VUMAT and implemented in Abaqus/Explicit for numerical simulations. Small time steps are needed both in FE and material point analysis to solve the local equilibrium problem associated with the homogenization in terms of a projected stress balance between the fibre and the matrix constituents in the transverse and in-plane shear fibre plane. This is particularly important to get better convergence while resolving dynamic damage evolution in small off-axis cases. For the 3D Explicit FE-analysis, the test specimens were discretized with 8-node reduced integration C3D8R solid elements with enhanced hourglass control throughout this thesis. For the laminate analysis, the model with 3D solid elements together with cohesive surfaces for delamination was used. This approach is computationally expensive particularly for thin walled applications. The model is never tested with shell elements in this thesis work and therefore needs to be considered in the future.

To validate the proposed material model, in **Paper A** the model was used to simulate dynamic behaviour of IM7/8552 material system prior to the onset of damage and evolution at a material point level. The homogenized model response was based on *isostrain* assumption along the fibres and *isostress* transverse to the fibre direction. The model was based on small deformation theory. At this point, the model was lacking the damage model for the matrix and the subsequent FE predictions. These limitations were addressed in **Paper B**, where the model was extended to include a continuum damage model coupled with viscoplastic formulation for the matrix and implemented in Abaqus/Explicit software using a user defined material subroutine (VUMAT). The model was developed based on non-linear kinematics for dynamic applications. This means that the homogenized stress response at mesoscale is described in terms of Lagrange strain for the inelastic matrix and transverse and in-plane elastic fibre responses. For the validation, the model was used to simulate in-plane quasi-static and dynamic in-plane failure growth observed in the experiments of IM7/8552 material system under different off-axis fibre angle specimens in compression. An overestimation of strength prediction were observed for small fibre off-axis cases with the current version of the model for **Paper C**. Therefore, the model was extended to include an improvement in the homogenization technique. In **Paper C**, a novel split of microscopic strain into transverse strain and in-plane shear strain is introduced and extend the *isostress* assumption to in-plane shear direction to the fibres. Additionally, a modification is also implemented in the model to consider non-uniform stress distribution in the matrix. The model was tested to simulate the off-axis specimens of NCF material system under dynamic compressive loadings and significant improvements were observed in the results. In **Paper D**, the model was applied to simulate angle-ply laminates under dynamic compressive loading conditions. To do that, the ply (meso) model based on continuum damage (in-plane failure) is combined with the surface-based cohesive behaviour to describe the out-of-plane interlaminar delamination failure for the IM7/8552 material specimens. Overall, the FE results of the UD carbon/epoxy composite tested under quasi-static and dynamic experiments from **Paper A - D** showed promising results for the prediction of rate dependent stiffness, strength and damage evolution in compression for off-axis specimens and angle-ply laminates. Computational robustness of the model is also demonstrated by simulating various off-axis test cases and certainly decrease the computational cost for the material failure analysis of large structures.

7. Future research

The current application of the micromechanics based material model is for UD carbon/epoxy composite structures. However, in the future, an obvious extension is to include a rate dependent model for the fibres too, e.g. glass fibres, to expand the scope of the current constitutive model for future applications based on glass/epoxy composites. As seen from section 2.1 glass fibres are rate sensitive due to viscoelasticity.

In the current version of the material model, a quasi-brittle continuum damage model is implemented and extensively tested to simulate the rate dependent response of UD carbon/epoxy composite system in compression. An extension is needed to include a brittle damage model for simulating the post-peek response as observed in tensile experiments of UD carbon/epoxy composites.

The material model is developed for simulating the rate effects in UD ply based composites and therefore a simple linear elastic model for the fibres and an inelastic model for the matrix is used. The model has shown promising validation results for UD composites even with different fibre off-axis angles. In the future, the aim is to extend the applicability of the model to more complex fibre architectures e.g. woven or other textile reinforced composites. Currently the model can be applied to RVE sized scale, however, to extend the application to larger structures would require developing strategies for efficient multiscale analysis.

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