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Power-Efficient Voronoi Constellations for Fiber-Optic Communication Systems

Shen Li , Ali Mirani , Magnus Karlsson, Senior Member, IEEE, and Erik Agrell, Fellow, IEEE

Abstract—Voronoi constellations (VCs) are considered as an effective geometric shaping method due to their high power efficiencies and low complexity. In this paper, the performance of 16- and 32-dimensional VCs with a variety of spectral efficiencies transmitted in the nonlinear fiber channel are investigated. Both single-channel and wavelength-division multiplexing systems are considered for the transmission of the VCs, as well as different signal processing schemes, including chromatic dispersion compensation and digital backpropagation. Multiple performance metrics including the uncoded bit error rate, mutual information (MI), and generalized mutual information (GMI) of VCs are evaluated. Compared with quadrature amplitude modulation (QAM) formats, the VCs provide 1.0–2.4 dB launch power gains, up to 0.50 bits/symbol/dimension-pair MI gains, up to around 30% potential reach increase at the same MI, and up to 0.30 bits/symbol/dimension-pair GMI gains in a limited launch power range. The observed performance gains over QAM are found higher than in the back-to-back case. Moreover, a general GMI estimation method for very large constellations using importance sampling is proposed for the first time.

Index Terms—Fiber-optic communication, generalized mutual information, geometric shaping, information rates, lattices, multidimensional modulation formats, Voronoi constellations.

I. INTRODUCTION

GEOMETRIC shaping is a way to gain power efficiency by adjusting the position of constellation points with respect to uniform quadrature amplitude modulation (QAM). In recent years, great interests have been shown in performing shaping in fiber-optical communications, partially because no theoretical limits have been found for the maximum shaping gain of the nonlinear fiber channel yet, which could potentially be more than the ultimate 1.53 dB shaping gain in the linear additive white Gaussian noise (AWGN) channel. Actually, there have been indications showing that geometrically shaped modulation formats might achieve gains higher than 1.53 dB [1]. On the other hand, some works show that probabilistically shaped constellations introduce a modulation-dependent nonlinear interference for fiber transmission, which reduces the shaping gain achieved in the AWGN channel [2], [3].

Coherent fiber communication inherently consists of four dimensions: two orthogonal quadratures in two orthogonal polarizations. More dimensions can be realized utilizing time slots, wavelengths, and spatial dimensions, and performing shaping over higher dimensions jointly is expected to achieve larger performance gains. A comprehensive review of several geometrically-shaped modulation formats can be found in [4, Table I]. Much work has been devoted to design geometrically shaped 4-dimensional constellations [2], [5], [6], [7], [8]. Among the works in the literature, usually look-up tables storing all coordinates of constellation points are needed, which makes the detection complexity and storage requirement increase exponentially when extending to higher dimensions.

Voronoi constellations (VCs) based on lattices, inherently performing a joint shaping of multiple dimensions, can be a good trade-off between shaping gain and complexity. VCs have a shaping lattice providing the shaping gain and a coding lattice providing the coding gain, which were first proposed by Conway and Sloane in [9], together with their low-complexity encoding and decoding algorithms, and then generalized by Forney [10]. No look-up tables are needed, neither no dramatic complexity increase in high dimensions. VCs were used for shaping in some wireless network applications [11], [12], [13], [14], [15], [16]. For the AWGN channel, uncoded BER gains of VCs over QAM were reported in [17], [18], and mutual information (MI) gains of VCs were demonstrated in [19].

VCs were first studied for fiber communications in [18], where Mirani et al. reported significant uncoded bit error rate (BER) gains of VCs over QAM transmitted in a wavelength-division multiplexing (WDM) system. Later in [20], power gains of VCs were demonstrated in experiments for a 80 km single-channel transmission.

In this paper, different from the work in [18] and [20], we investigate the performance of another type of VC with a cubic coding lattice in both single-channel and WDM simulations. The considered VCs have different numbers of dimensions, shaping lattices, and spectral efficiencies. Several important performance metrics for both hard- and soft-decision forward error correction (FEC) are evaluated for the VCs, including the uncoded BER, MI, and generalized mutual information (GMI), among which the MI and GMI performances for VCs in the nonlinear fiber channel are demonstrated for the first time.
knowledge. Consistently with [1], the observed power, MI, and 
GMI gains of VCs over QAM formats in fiber transmission are 
found higher than in the AWGN channel. The data rates 
and transmission distances of the studied systems are comparable to 
400ZR [21] and the upcoming 800 Gbps and 1.2 Tbps standards. 
The target systems of VCs could be data center interconnects, 
etg., campus and metro data centers, which require rather high 
throughput with limited complexity for short distances (usually 
less than 100 km), or metro optical links with up to hundreds of 
kilometers. Moreover, a general GMI estimation method for very 
large constellations is proposed, using importance sampling, 
extending the MI estimation method in [19].

Notation: Bold lowercase symbols denote row vectors and 
bold uppercase symbols denote random vectors or matrices. The 
sets of integer, real, complex, and natural numbers are denoted 
by \(\mathbb{Z} \), \(\mathbb{R} \), \(\mathbb{C} \), and \(\mathbb{N} \), respectively. Other sets are denoted 
by calligraphic symbols. Rounding a vector to its nearest integer 
vector is denoted by \(\lceil \cdot \rceil \), in which ties are broken arbitrarily.

II. BASICS OF VCs

VCs are structured multidimensional lattice-based constella-
tions. An \(n\)-dimensional lattice is spanned by \(n\) linearly 
independent basis vectors which are the rows of its \(n \times n\) generator 
matrix \(G\). All linear combinations of these basis vectors with integer 
coefficients form the lattice, i.e.,
\[
\Lambda \triangleq \{uG : u \in \mathbb{Z}^n\}. 
\]

From the definition, a lattice must contain the all-zero point \(0\). 
The Voronoi region of a lattice \(\Lambda\), denoted as \(\Omega(\Lambda)\), contains all 
points in \(\mathbb{R}^n\) having the all-zero point \(0\) as their closest lattice 
point in \(\Lambda\).

A VC has a coding lattice \(\Lambda\) and a shaping lattice \(\Lambda_s\), and \(\Lambda_s\) 
is a sublattice of \(\Lambda\), i.e., \(\Lambda_s \subset \Lambda\). Forney defined a VC based on 
the lattice partition \(\Lambda/\Lambda_s\) as a set of translated lattice points of 
\(\Lambda\) enclosed by the Voronoi region of \(\Lambda_s\) [10], i.e.,
\[
\Gamma \triangleq (\Lambda - \alpha) \cap \Omega(\Lambda_s), 
\]

where \(\alpha \in \mathbb{R}^n\) is the offset vector (see Fig. 1 for an example 
VC). The offset vector avoids lattice points falling on the 
boundary of \(\Omega(\Lambda_s)\), and can be optimized to minimize the 
average symbol energy of the VC. The number of points in a 
VC is \(M = |\det G_s| / |\det G|\), where \(G_s\) and \(G\) are the generator 
matrices of \(\Lambda_s\) and \(\Lambda\), respectively. The length of the bit labels 
is \(m = \log_2 M\) bits and the spectral efficiency [22], [23], [24] 
of a VC per dimension-pair is defined as
\[
\beta = 2m/n \text{ [bits/symbol/dimension-pair]}. 
\]

A dimension-pair could refer to any two real dimensions for a 
multidimensional AWGN channel, and for fiber-optic channels, 
it could be an in-phase and quadrature (I/Q) pair of a single 
polarization component. The average symbol energy of a VC is
\[
E_s = \frac{1}{M} \sum_{x \in \Gamma} ||x||^2. 
\]
The amount of information per symbol that a certain channel can transmit with an arbitrarily small error probability, using any (optimal or suboptimal) encoder/decoder pair, is known as the achievable information rate (AIR). The maximum AIRs over any (optimal or suboptimal) encoder/decoder pair, is known as the capacity of the channel indicating the fundamental limits of a coded modulation scheme, are the MI and GMI, of which the former is optimal for binary codes or memoryless discrete Markovian channels, while the latter applies to nonbinary codes or multilevel codes, and the latter applies to binary interleaved coded modulation (BICM) [27]. The MI and GMI of VCs transmitted in the nonlinear fiber channel are definitely worth investigation. However, their calculation in general cases, and then specifically to VCs.

### A. GMI Estimation Method Based on Importance Sampling

We consider a modulation format $\mathcal{X}$ consisting of $M$ equally probable symbols with distinct binary labels of length $m = \log_2 M$. For a memoryless discrete channel with the conditional probability $f_{Y|X}(y|x)$, where $x \in \mathcal{X}$ denotes the transmitted symbol and $y \in \mathbb{R}^n$ denotes the received noisy symbol, the GMI can be written as [27, (15)]

$$\text{GMI} \doteq \frac{1}{M} \sum_{k=1}^{M} \sum_{b=0,1} \log_2 \left( f_{k,b}(y) / \left( f_{k,0}(y) + f_{k,1}(y) \right) \right) dy,$$

where $\mathcal{X}_b^k \subset \mathcal{X}$ is the set of constellation points with a bit $b$ at position $k$ in their $m$-bit binary label, and

$$f_{k,b}(y) = \frac{2}{M} \sum_{x \in \mathcal{X}_b^k} f_{Y|X}(y|x).$$

If $N_s$ samples are uniformly and independently drawn from $\mathcal{X}$ and transmitted through the simulated channel $f_{Y|X}(y|x)$, then the GMI in (6) can be approximated using these $N_s$ channel realization pairs $(x^{(i)}, y^{(i)})$, where the superscripts represent the time index. Let the set $\mathcal{X}_b^k$ denote all the time samples of input symbols $x^{(i)}$ that have a bit $b$ at position $k$. Then (6) can be approximated by

$$\text{GMI} \doteq \frac{1}{N_s} \sum_{k=1}^{M} \sum_{b=0,1} \sum_{i \in \mathcal{X}_b^k} \log_2 \left( f_{k,b}(y^{(i)}) / \left( f_{k,0}(y^{(i)}) + f_{k,1}(y^{(i)}) \right) \right).$$

(8)

It can be noted that enumerating all constellation points in $\mathcal{X}$ to calculate (7) is practically impossible when $M$ it too large. Extending the idea from [19], instead of enumerating every point in $\mathcal{X}$, we consider a much smaller importance set $\mathcal{I}(y)$, such that the contribution to the sum $f_{k,b}(y)$ from constellation points in $\mathcal{X} - \mathcal{I}(y)$ is negligible. Then $f_{k,b}(y)$ can be approximated by enumerating all constellation points in $\mathcal{I}(y) \cap \mathcal{X}_b^k$. 

---

### TABLE I

<table>
<thead>
<tr>
<th>Name</th>
<th>$\Lambda/\Lambda_s$</th>
<th>$M$</th>
<th>$m$</th>
<th>$\beta$</th>
<th>$\alpha$</th>
<th>ASG [dB]</th>
</tr>
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<tbody>
<tr>
<td>$D_2^5$</td>
<td>$\mathbb{Z}^2/4D_2$</td>
<td>32</td>
<td>5</td>
<td>5</td>
<td>$(-0.5, 0)$</td>
<td>0</td>
</tr>
<tr>
<td>$D_4^4$</td>
<td>$\mathbb{Z}^4/16D_4$</td>
<td>131072</td>
<td>17</td>
<td>8.5</td>
<td>$0.500, 0, -0.168, 0.334$</td>
<td>0.37</td>
</tr>
<tr>
<td>$E_2^{14}$</td>
<td>$\mathbb{Z}^2/8E_4$</td>
<td>1677216</td>
<td>24</td>
<td>6</td>
<td>$\in \Omega(28)$</td>
<td>0.65</td>
</tr>
<tr>
<td>$\Lambda_{16}^{28}$</td>
<td>$\mathbb{Z}^{16}/8\Lambda_{16}$</td>
<td>$1.2 \times 10^{18}$</td>
<td>60</td>
<td>7.5</td>
<td>$\in \Omega(16)$</td>
<td>0.86</td>
</tr>
<tr>
<td>$\Lambda_{16}^{28}$</td>
<td>$\mathbb{Z}^{16}/16\Lambda_{16}$</td>
<td>$7.6 \times 10^{22}$</td>
<td>76</td>
<td>9.5</td>
<td>$\in \Omega(16)$</td>
<td>0.86</td>
</tr>
<tr>
<td>$\Lambda_{16}^{28}$</td>
<td>$\mathbb{Z}^{16}/32\Lambda_{16}$</td>
<td>$5.0 \times 10^{23}$</td>
<td>92</td>
<td>11.5</td>
<td>$\in \Omega(16)$</td>
<td>0.86</td>
</tr>
<tr>
<td>$L_{123}^{42}$</td>
<td>$\mathbb{Z}^{32}/8L_{32}$</td>
<td>$1.1 \times 10^{37}$</td>
<td>123</td>
<td>7.6875</td>
<td>$\in \Omega(32)$</td>
<td>0.94</td>
</tr>
<tr>
<td>$L_{123}^{42}$</td>
<td>$\mathbb{Z}^{32}/16L_{32}$</td>
<td>$4.6 \times 10^{46}$</td>
<td>155</td>
<td>9.6875</td>
<td>$\in \Omega(32)$</td>
<td>0.94</td>
</tr>
<tr>
<td>$L_{123}^{42}$</td>
<td>$\mathbb{Z}^{32}/32L_{32}$</td>
<td>$2.0 \times 10^{55}$</td>
<td>187</td>
<td>11.6875</td>
<td>$\in \Omega(32)$</td>
<td>0.94</td>
</tr>
</tbody>
</table>
The importance set \( \mathcal{I}(y) \) can be further divided into \( D \) disjoint subsets \( \mathcal{I}_d(y) \) for \( d = 1, \ldots, D \). From each subset, \( K_d \) random samples \( x_{d,j} \) for \( j = 1, \ldots, K_d \) are drawn uniformly if \( K_d < |\mathcal{I}_d(y)| \), unless \( |\mathcal{I}_d(y)| \) is small enough so that enumerating all constellation points is more computationally efficient, then \( x_{d,j} \) for \( j = 1, \ldots, K_d \) are all constellation points in \( \mathcal{I}_d(y) \), where \( K_d = |\mathcal{I}_d(y)| \). If we call the subscript \( j \) “sample indices,” and let the set \( \mathcal{T}_{d,k,b}(y) \subseteq \{1, \ldots, K_d\} \) denote all the sample indices of the random samples (or all constellation points in small subsets \( \mathcal{I}_d(y) \)) \( x_{d,j} \) that have a binary label \( b \) at position \( k \), then (7) can be approximated as

\[
\tilde{f}_{k,b}(y) \approx \frac{2}{M} \sum_{x \in \mathcal{I}_k \cap \mathcal{I}(y)} f_{Y|X}(y|x) 
\approx \frac{2}{M} \sum_{d=1}^{D} \frac{|\mathcal{I}_d(y)|}{K_d} \sum_{j \in \mathcal{T}_{d,k,b}(y)} f_{Y|X}(y|x_{d,j}).
\]

(10)

As \( D \) and \( K_d \) increase, (10) should converge to (7). So far, combining (8) and (10), the expressions for estimating the GMI of very large constellations are obtained. These expressions are applicable to any analytical channel (or an analytical auxiliary channel if the real channel is not analytically known), and any structured modulation formats as long as the importance set can be well defined. Generally, the important set depends on the modulation format \( X \) and the channel law \( f_{Y|X}(y|x) \). If the constellation points in the important set cannot be enumerated, nor sampled, then it is infeasible to estimate the GMI by the proposed method.

B. GMI Estimation for the VCs

Since an analytical channel law of the nonlinear fiber channel is not known, an auxiliary channel law \( q_{Y|X}(y|x) \) is usually used to replace \( f_{Y|X}(y|x) \) in (7), generating a lower bound on the GMI for the fiber channel. A common and reasonable choice of the auxiliary channel is the Gaussian channel, which has the conditional distribution

\[
q_{Y|X}(y|x) = \frac{1}{(2\pi\sigma^2/n)^{n/2}} \exp \left( -\frac{\|y-x\|^2}{2\sigma^2/n} \right),
\]

(11)

where \( \sigma^2 \) is the total noise power for \( n \) real dimensions. Then the SNR is defined as \( E_b/\sigma^2 \).

Upon receiving a noisy symbol \( y \), according to (11), the constellation points that are close to \( y \) have the most impact on the GMI estimation. The nice structure of the cubic coding lattice makes these constellation points easy to enumerate in a “Euclidean ball” centered at \( y + a \) with radius \( R \)

\[
\mathcal{B}(y, R^2) \triangleq \{ x : \| x + a - y + a \|^2 \leq R^2, \ x + a \in \mathbb{Z}^n \},
\]

(12)

where the squared radius \( R^2 \in \mathbb{N} \). The ball can be further divided into \( R^2 + 1 \) “shells” defined as

\[
\mathcal{S}(y, r^2) \triangleq \{ x : \| x + a - y + a \|^2 = r^2, \ x + a \in \mathbb{Z}^n \},
\]

(13)

>for \( r^2 = 0, 1, \ldots, R^2 \).

The importance set in this specific application is not only a function of \( y \), but also depends on its squared radius \( R^2 \), and can be defined as

\[
\mathcal{I}(y, R^2) = \mathcal{B}(y, R^2) \cap \Gamma,
\]

(14)

which consists of \( D = R^2 + 1 \) disjoint subsets \( \mathcal{I}_d(y) = \mathcal{S}(y, d - 1) \cap \Gamma \) for \( d = 1, \ldots, D \).

In analogy with (8) and (10), the lower bound of the GMI can be approximated as

\[
\tilde{\text{GMI}} \approx \frac{1}{N_s} \sum_{k=1}^{m} \sum_{b \in \{0,1\}} \sum_{i \in \mathcal{Z}_k} \log_2 \left( \frac{\tilde{f}_{k,b}(y(i))}{\tilde{f}_{k,1}(y(i))} \right) \]

(15)

where

\[
\tilde{f}_{k,b}(y) \approx \frac{2}{M} \sum_{d=1}^{D} \frac{|\mathcal{I}_d(y)|}{K_d} \sum_{j \in \mathcal{T}_{d,k,b}(y)} q_{Y|X}(y|x_{d,j}).
\]

(16)

In our simulations, we found that the subsums in \( \mathcal{I}_d^{(k,b)}(y) \) for all \( d = 1, \ldots, D \) in (16) can be approximated very well with \( K_d = \min \{|\mathcal{I}_d(y)|, 10^5\} \) samples. With this number of samples in each subset, we increase \( D \) from 1, until the condition

\[
\max_{i=1,2,\ldots,N_s} \left( \frac{\tilde{f}_{k,b}^{(D+1)}(y(i)) - \tilde{f}_{k,b}^{(D)}(y(i))}{\tilde{f}_{k,b}^{(D)}(y(i))} \right) < 0.5\%
\]

(17)

is met for all \( k = 1, \ldots, m \) and \( b \in \{0,1\} \), where the super- scripts of the estimated \( \tilde{f}_{k,b}(y(i)) \) denote the number of subsets being used. Then we believe that with no less than \( D \) subsets, \( \tilde{f}_{k,b}(y(i)) \) converges to the exact value (7). Finally, accurate approximation of \( \tilde{g}_{k,b}(y(i)) \) for all \( i = 1, \ldots, N_s \) yields accurate estimation of GMI.

Example 2: We again take \( D_8^2 \) as an example due to the simplicity for illustration. Fig. 2 shows the importance region \( \mathcal{I}_1(y,8) \) and its subsets when \( D = 9 \). Given a noisy \( y \), the importance region is centered at \( y + a \) \( - a \), which itself forms the first subset \( \mathcal{I}_1(y) \). In this example, \( \mathcal{I}_1(y) = \mathcal{I}_7(y) = \mathcal{I}_8(y) = \emptyset \).

Example 3: We transmit \( D_8^2 \) over the Gaussian channel. For random \( k \) and \( y \), Fig. 3(a) shows the convergence of the estimated \( \tilde{f}_{k,b}^{(D)}(y) \) using (10) to the benchmark values \( f_{k,b}(y) \) computed using (7) for different SNRs. In this example, \( K_d = |\mathcal{I}_d(y)| \) for all \( d = 1, \ldots, D \).

Example 4: We transmit \( E_b^2 \) over the Gaussian channel. For random \( k \) and \( y \), Fig. 3(b) shows the convergence of the estimated \( \tilde{f}_{k,b}^{(D)}(y) \) using (10) to the benchmark values \( f_{k,b}(y) \) computed using (7) in the medium SNR range. In this example, \( K_d = 10^5 \) uniform samples from \( \mathcal{I}_d(y) \) are used for subsets with \( d > 8 \) and \( K_d = |\mathcal{I}_d(y)| \) for subsets with \( d \leq 8 \).

IV. PERFORMANCE ANALYSIS

In this section, we study the BER, MI, and GMI performance of 16-dimensional and 32-dimensional VCs in Table II over the
Fig. 2. The importance region for $D_{2}^{2}$ in Example 2. The constellation points inside the gray shaded region (including those falling on the boundary) form the importance region for GMI estimation. The constellation points falling on each “shell” form the subsets $I_d(y)$.

Fig. 3. The estimated value $f_{k,b}^{(D)}(y)$ as a function of $D$ in Examples 2 and 3. The black dashed lines are the corresponding benchmark values $f_{k,b}(y)$ calculated using (7). Accurate estimation is validated by the convergence of all curves to the benchmark values.

Fig. 4. The mapping of VCs to one wavelength. Integers from 1 to $n$ represent the information carried by the $n$ dimensions of VCs. All spectral efficiencies given in this paper are normalized to a single I/Q plane (2 dimensions).

Fig. 5. The BER as a function of the OSNR for VCs compared with QAM constellations in the B2B scenario. Dashed lines represent QAM constellations. Lines with the same colors represent the same spectral efficiencies.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol rate</td>
<td>45 Gbaud</td>
</tr>
<tr>
<td>Pulse shape</td>
<td>RRC, rolloff 0.1</td>
</tr>
<tr>
<td>Fiber attenuation</td>
<td>0.2 dB/km</td>
</tr>
<tr>
<td>Fiber nonlinear coefficient</td>
<td>1.27 W$^{-1}$ km$^{-1}$</td>
</tr>
<tr>
<td>Dispersion parameter</td>
<td>17 ps/μm/km</td>
</tr>
<tr>
<td>EDFA noise factor</td>
<td>5 dB</td>
</tr>
<tr>
<td>Span length</td>
<td>80 or 100 km</td>
</tr>
<tr>
<td>Center frequency</td>
<td>1550 nm</td>
</tr>
<tr>
<td>Channel spacing</td>
<td>50 GHz</td>
</tr>
<tr>
<td>Oversampling factor in SSFM</td>
<td>32</td>
</tr>
<tr>
<td>Oversampling factor in DBP</td>
<td>32</td>
</tr>
<tr>
<td>Step size in SSFM</td>
<td>1 km</td>
</tr>
<tr>
<td>Pilot overhead</td>
<td>2%</td>
</tr>
<tr>
<td>Number of symbol time slots transmitting VC symbols</td>
<td>$8 \times 10^4$</td>
</tr>
</tbody>
</table>

TABLE II

The Simulation Parameters

nonlinear fiber channel, since their ASGs are high, which implies large potential shaping gains in the nonlinear fiber channel. The benchmarks for comparison are the QAM formats with the same spectral efficiencies. For transmission of an $n$-dimensional VC, we consider (a) a single-channel system in which $n/4$ time slots are needed to transmit one VC symbol, and (b) a WDM system with $n/4$ wavelengths in which each wavelength carries one VC symbol in $n/4$ time slots. The mapping of VCs to one wavelength is depicted in Fig. 4. For the WDM case, many
other different ways of assigning VCs to physical dimensions are possible. Nevertheless, our simulations show no big performance difference using these different ways. The experimental results of comparing different physical realizations for another type of VC draw the same conclusion [29]. Therefore, we choose the mapping in Fig. 4 because this scheme does not need many coherent receivers to decode one VC symbol.

VCs can have non-integer spectral efficiencies from the definition of (3), as \( \beta \) is not guaranteed to be an integer. The performance of VCs should be compared with QAM constellations at the same spectral efficiencies. Thus, for QAM transmission, in order to have a non-integer spectral efficiency, two two-dimensional QAM formats with different cardinalities \( M_1 \) and \( M_2 \) can be assigned to different I/Q pairs, resulting in a spectral efficiency of

\[
\beta_{\text{QAM}} = \frac{d_1 \log_2 M_1 + d_2 \log_2 M_2}{d_1 + d_2} \text{[bits/symbol/dimension-pair]},
\]

where \( d_1 \) and \( d_2 \) are the number of I/Q pairs in Fig. 4 transmitting \( M_1 \)-QAM and \( M_2 \)-QAM, respectively, and \( d_1 + d_2 = n/2 \). The QAM constellation having the same spectral efficiency as \( \Lambda_{16}^{10} \) has parameters \( d_1 = 4, \ d_2 = 4, \ M_1 = 1024, \) and \( M_2 = 512 \); for the QAM corresponding to \( L_{32}^{155} \), we should set \( d_1 = 11, \ d_2 = 5, \ M_1 = 1024, \) and \( M_2 = 512 \). The two constituent QAM constellations are scaled to the same minimum distance, which maximizes the minimum distance of the resulting hybrid-QAM constellation for a given \( n \)-dimensional symbol energy \( E_s \) [30, Ch. 4.3]. For modeling the fiber channel, the Manakov equation [31] is adopted and the split-step Fourier method (SSFM) [32] is used to simulate the channel, which is sufficiently accurate to capture the nonlinearities in a real fiber. In the digital signal processing (DSP) chain at the receiver side, a one-time chromatic dispersion (CD) compensation is performed. Alternatively, a full-field digital backpropagation (DBP) can be implemented to increase the transmission distance of VCs with
Fig. 8. The BER as a function of the launch power for VCs in the single-channel case with a small step size DBP. Dashed lines with the same colors represent QAM having the same spectral efficiencies $\beta$ as VCs.

Fig. 9. The maximum AIRs as a function of the launch power of VCs in the single-channel case. In (a), a one-time electronic CD compensation is performed. In (b), a full-field 1-km step size DBP is used. Solid and dashed lines without markers represent the MI and GMI of QAM, respectively. Lines with the same colors have the same spectral efficiencies $\beta$. For each VC, two horizontal dash-dotted lines are drawn at 80% and 87% of $\beta$, corresponding to an FEC overhead of 25% and 15%, respectively.

A. Back-to-Back Case

A back-to-back (B2B) scenario reflects the performance of VCs in the absence of fiber nonlinearities. The BER performance of the considered VCs depends on the spectral efficiencies, number of dimensions, and the labeling scheme. Fig. 5 presents the BER performance of the considered VCs compared with QAM in the B2B scenario. The optical signal-to-noise ratio (OSNR) is calculated assuming that it is measured in a reference optical bandwidth of 12.5 GHz (0.1-nm wavelength). VCs with higher spectral efficiencies have larger OSNR gains. Up to 1 dB power gains are observed at a BER of $10^{-3}$ for $L_{16}^{76}$. The 32-dimensional VCs do not show higher gains than 16-dimensional VCs, since the bit labeling scheme is more efficient in 16-dimensional VCs. To compare the labeling performance for these very large VCs, one could calculate the Gray penalty [34], [35] using Algorithm 5 in [17]. For example, $L_{16}^{76}$ has a Gray penalty of approximately 1.33, which is smaller than that of $L_{16}^{155}$ (1.40).

Fig. 6 shows the estimated MI and GMI as a function of OSNR in the B2B scenario. The MI is estimated using the method in [19, Section V-B] and parameters are chosen as suggested therein. The GMI is estimated using the method and parameters proposed in Section III-B. The results show that VCs outperform QAMs throughout the whole OSNR range in the B2B scenario. The optical signal-to-noise ratio (OSNR) is calculated assuming that it is measured in a reference optical bandwidth of 12.5 GHz (0.1-nm wavelength). VCs with higher spectral efficiencies have larger OSNR gains. Up to 1 dB power gains are observed at a BER of $10^{-3}$ for $L_{16}^{76}$. The 32-dimensional VCs do not show higher gains than 16-dimensional VCs, since the bit labeling scheme is more efficient in 16-dimensional VCs. To compare the labeling performance for these very large VCs, one could calculate the Gray penalty [34], [35] using Algorithm 5 in [17]. For example, $L_{16}^{76}$ has a Gray penalty of approximately 1.33, which is smaller than that of $L_{16}^{155}$ (1.40).

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Fig. 10. The maximum AIRs as a function of the transmission distance for the VCs in the single-channel case, with a span length of 80 km. Solid and dashed lines without markers represent the MI and GMI of QAM, respectively. Lines with the same colors have the same spectral efficiencies $\beta$.

Terms of MI. Up to 0.33 bits/symbol/dimension-pair MI gains over QAMs can be observed. However, as for the GMI, VCs only outperform QAM in a limited OSNR range, and VCs with higher spectral efficiencies tend to have larger GMI gains. A 0.21 bits/symbol/dimension-pair GMI gain is observed for $\Lambda_{92}^{16}$ when the GMI is close to $\beta$. Compared with 16-dimensional VCs, the 32-dimensional VCs have almost no GMI gains due to the worse labeling performance.

B. Single-Channel Case

Fig. 7 shows the BER performance of the considered VCs over a nonlinear fiber channel. Only CD compensation is performed at the receiver. The BER is presented between $10^{-4}$ and $10^{-2}$, which covers most commonly-used hard-decision forward error correction limits for fiber communications [36]. The simulated transmission distances are short, since the spectral efficiencies are high. It shows that the considered VCs reduce the minimum BER over QAM by 25%–58%, and show launch power gains ranging from 1.0–2.3 dB over QAM at the minimum BER achieved by QAM, larger than the OSNR improvements realized in the B2B case, which are less than 1 dB.

The considered VCs are shown to be suitable for short-distance transmission. Despite this, we also study their performance when a full-field DBP with a rather small step size (1 km) is used to support the high SNR needed by these VCs for longer transmission distances. As an example, $\Lambda_{76}^{16}$ and $L_{32}^{155}$ still maintain good launch power gains as shown in Fig. 8 even when most of the nonlinearities are compensated in the DBP. This extreme case shows the advantage of transmitting VCs over long distances if high complexity is allowed in the DSP.

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somewhat higher when the SNR is higher. If we assume an FEC code with 20% overhead, which reduces the spectral efficiency to 83% of the uncoded spectral efficiency $\beta$, 0.17 and 0.11 bits/symbol/dimension-pair GMI gains are observed for $\Lambda^{76}_{16}$ and $L^{155}_{32}$, respectively. VCs only outperform QAM when the GMI is above 80% and 85% of $\beta$ for $\Lambda^{76}_{16}$ and $L^{155}_{32}$, respectively. This means that there exists a BICM scheme with an FEC overhead smaller than around 25% and 18% for $\Lambda^{76}_{16}$ and $L^{155}_{32}$, respectively, in which using VCs as modulation formats achieves better performance than QAM.

Fig. 10 shows the maximum AIRs as a function of the transmission distance, in which the launch power is optimized for each distance. For medium distance range, VCs can achieve up to 0.50 bits/symbol/dimension-pair MI gains over QAM at the same transmission distance. Without any compensation of nonlinearities, at around 310 km, $\Lambda^{76}_{16}$ and $L^{155}_{32}$ increase the reach by 96 km achieving the same MI. No significant gains are shown in terms of GMI. With DBP at the receiver (10-km step size), $\Lambda^{76}_{16}$ realizes larger MI and GMI gains, or reach increase at the same MI and GMI, compared with the case without DBP. For example, at the same GMI value, $\Lambda^{76}_{16}$ increases the transmission distance of QAM from 198 to 240 km. For short transmission distances (40–120 km) without DBP, VCs maintain large MI gains and reach increase at the same MI, and realize up to 0.30 bits/symbol/dimension-pair GMI gains. The maximum observed MI and GMI gains over QAM are up to 0.50 and 0.30 bits/symbol/dimension-pair, respectively, which are larger than the maximum observed MI and GMI gains in the B2B case (0.33 and 0.21 bits/symbol/dimension-pair), respectively.

C. WDM Case

The BER performance of VCs compared with QAM in WDM systems without and with a small step size DBP are shown in Fig. 11 and Fig. 12, respectively. The shown BER is the average BER over all wavelengths, not from one of the two center channels as usual, which however is shown to not affect the performance analysis significantly, since there is no big difference among the BER of each single channel; see the dotted
MI gains and reach increase are larger than without DBP. For short distances, VCs achieve higher maximum AIR gains than for medium distances, and increase the transmission distance by 10%–20% at the same MI. Overall, the AIR gains in WDM are marginally smaller than in the corresponding single-channel case. This might be due to that the fiber nonlinearities, especially the cross-phase modulation, might have a slightly stronger impact on VCs than QAM formats. However, the maximum observed MI gain of VCs over QAM is still found higher than in the B2B case (0.39 > 0.33 bits/symbol/dimension-pair), which is consistent with the observation in the single-channel case.

V. CONCLUSION

We have simulated transmission of 16- and 32-dimensional Voronoi constellations with a cubic coding lattice over the nonlinear fiber channel and studied their performance in both single-channel and WDM systems. The BER, MI, and GMI performance of VCs are compared with QAM constellations at the same spectral efficiencies. Extended from the MI estimation method from our previous work [19], a GMI estimation method for very large constellations is proposed and applied to the considered VCs. The MI and GMI performance of VCs in fiber-optic communications are first demonstrated to our knowledge. The launch power gains of VCs over QAM imply that they can achieve better performance than QAM in systems with hard-decision FEC decoding. However, in systems with BICM and soft-decision decoding, where the GMI is the right predictor for the post-FEC BER, the performance gains of VCs might be limited. However, their good MI gains over QAM imply that designing a multilevel code specifically for the considered VCs might achieve these potential rate gains and reach increases for fiber communications, which remains as future work.

REFERENCES


