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The gyroscopic effect and moment of inertia

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Abstract

This paper describes a lab to help students develop their understanding of rotational motion. The focus is on moment of inertia, which the students investigate by rolling cylinders down a ramp and determine in two different ways for a bicycle wheel. The most important and original part of the lab is the exploration of the gyroscopic effect, where measurements of precession and rotation frequencies are made using the variation of the detected magnetic field, enabling the calculation of the moment of inertia. The lab is received well by the students and can be done with relatively simple equipment easily accessible to them.

Keywords: laboratory exercise, rotational motion, gyroscopic effect, moment of inertia, precession

Supplementary material for this article is available online

1. Introduction

Many teachers who have taught introductory courses in mechanics at university level have experienced difficulties when introducing concepts like moment of inertia, torque, and angular momentum [1–3]. These concepts are often new to the students who are usually familiar with more easily understood concepts with a more concrete meaning such as mass, displacement, velocity, and others from linear motion [4, 5]. The former group of concepts are more abstract and mathematical and must be understood or accepted differently than the latter, more concrete concepts [6]. The mathematical concepts, however, simplify calculations and help explain phenomena in physics. For rotational motion, for example, the calculations become much easier than when using only Newton's second law $\boldsymbol{F} = m\,\boldsymbol{a}$.

In this article we introduce a laboratory exercise (henceforth the lab) that has been designed and used during a mechanics course for engineering chemistry students. The lab consists of three parts that aim to improve student learning of mathematical concepts relating to rotational motion. While the main focus of the measurements is the moment of inertia of a rotating object, the lab also introduces the gyroscopic effect to the students. One reason for choosing the gyroscopic effect is

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that it is fascinating and counter intuitive [7]—almost everyone who observes it for the first time is puzzled by that the rotation of a wheel hanging in a rope makes the system precess (rotate around a vertical axis through the rope) and wants to understand the observed behaviour. The lab shows the students how powerful the mathematical concepts are when it comes to describing such rotational phenomena.

Throughout the article we use the variation theory of learning [8, 9] to pinpoint possibilities for learning opened up for the students as they experience systematic variation created in the lab. The idea behind this is that students learn by noticing variation to a background of invariance, as can be exemplified in the teaching and learning of the economics concept of price, which depends on supply and demand [10]. By first varying supply and demand individually while keeping the other invariant, how price depends on each of them becomes discernible to the students. In a next step, supply and demand could be varied simultaneously so that their joint effect on price can be studied. It has been argued and shown that this approach is useful also for the teaching of physics [11–13], and can be combined with a reflection on how representations afford the students access to physics knowledge [14, 15].

1.1. Rotational motion

When studying rotational motion, students meet new concepts such as moment of inertia ($\boldsymbol{I}$), torque ($\boldsymbol{\tau}$), and angular momentum ($\boldsymbol{L}$).

Moment of inertia can very simply be described as a body's 'resistance' to change its state of rotation. The concept is therefore analogous to a body's mass, which is its resistance to change its velocity—to be accelerated.

Torque is for rotational motion what force is for linear motion, calculated as force times the perpendicular length of the lever. Most students are familiar with the magnitude of torque, but its direction when described as a vector is surprising to many of them.

The concept angular momentum is even more challenging for students to get a feel for. Whereas in linear motion a force is needed to change the linear momentum of a body, in rotational motion a torque most be applied to change the body's angular momentum. Just as with torque, students often find the direction of angular momentum as a vector to be strange at first sight.

Figure 1. The wheel used in the lab with a rope mounted on a handle. A small magnet has been attached to one of the spokes.

For this lab, the relation $\boldsymbol{\tau} = \mathrm{d}\boldsymbol{L}/\mathrm{d}t$ between torque and the time derivative of the angular momentum is needed. The absolute values are $\tau = mgr$ for torque, where m is the total mass of the wheel used and r is the distance from the centre of the wheel to the rope it is hanging in (see figure 1), and $L = \omega I$ for the angular momentum, where ω is the rotation frequency.

For a wheel hanging in a rope as in figure 1, the direction of the angular momentum $\boldsymbol{L}$ is horizontal if the wheel rotates in a vertical plane. The torque $\boldsymbol{\tau}$ also lies in a horizontal plane but is perpendicular to $\boldsymbol{L}$, which implies that $\mathrm{d}\boldsymbol{L}$ is also perpendicular to $\boldsymbol{L}$. Thus, during the time $\mathrm{d}t$ the torque will cause a change in the direction of $\boldsymbol{L}$ by the amount $\mathrm{d}\theta$ (see figure 2). This makes the wheel rotate around a vertical axis around the rope. This rotation is called precession.

From the definition of radians, $\mathrm{d}\theta = \mathrm{d}L/L$ and $\mathrm{d}\theta/\mathrm{d}t = \mathrm{d}L/\mathrm{d}t\ (1/L)$.

The precession frequency $\Omega = \mathrm{d}\theta/\mathrm{d}t$ and since $\mathrm{d}L/\mathrm{d}t$ equals the torque mgr, and $L = I\omega$,

$$\Omega = \frac{mgr}{I}\frac{1}{\omega}. \tag{1}$$

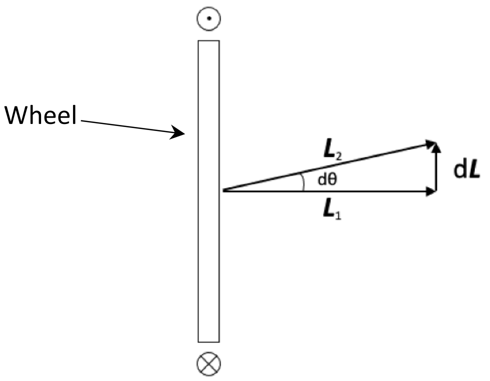

Figure 2. The wheel seen from above with the direction of rotation shown with a circled dot and cross. The angular momentum $\boldsymbol{L}_1$ is transformed to $\boldsymbol{L}_2$ during the time dt.

2. The lab

The lab was designed to deal with rotational phenomena, particularly focusing on moment of inertia, and consists of three parts.

(a) Cylinders rolling along a ramp.
(b) The relation between the rotation and precession frequencies, which gives the moment of inertia of a rotating wheel hanging in a rope.
(c) Determining the moment of inertia of the wheel by using it as a physical pendulum.

The equipment needed for all three parts of the lab is a set of cylinders, a ramp, a wheel with handles and a small magnet attached to a spoke, an iOLab device (can be replaced with a mobile phone), a scalar to weigh the wheel, a ruler, weights, and a computer.

About 120 students have completed the lab. Nine students divided into groups of three did the lab simultaneously. There was one lab assistant present during each lab session. In total three different assistants were involved. Because of the large number of students some must start the lab work before the theory had been covered in the lectures due to limited lab capacity. To partly overcome this problem, the students have had access to a short compendium where moment of inertia, torque, angular momentum, and important relations between these are introduced and the parallel axis theorem is derived. This gives them a background to the theory for rotational motion that is needed for the lab.

The students' observations during the lab hopefully make them desire to become familiar with the mathematical tools that make the observations easier to explain, including the gyroscopic effect. In other words, the lab creates a relevance structure [8].

Below we give more detailed descriptions of the three parts. There is also a supplementary video that describes some of the different parts of the lab.

2.1. Cylinders rolling along a ramp

The purpose of the first part of the lab is to give the students a feel for the concept moment of inertia as cylinders are rolling along a ramp [16]. As the students compare and contrast the speeds of cylinders with different geometries (viz. solid or hollow with a very thin wall), masses and radii they find that all the solid cylinders reach the bottom of the ramp with equal speeds and that all the hollow cylinders too roll equally fast but slower than the solid ones. By opening up this variation to the students they are able to conclude that only the geometry is important for the speed. The derivation of the relation showing that speed is independent of mass and radius is, however, left for the lectures. The lab is rather used to create a relevance structure [8]—a curiosity and desire among the students to see a proper derivation.

2.2. The relation between rotation frequency and precession frequency

The second part of the lab draws on the gyroscopic effect and measurements of precession and rotation frequencies are made using the time variation of the magnetic field detected by an iOLab device (for a more detailed description of what the device can be used for, see [17, 18]). Although the students' task in this part of the lab is to find the moment of inertia of the wheel, there are several aspects that should be interesting to them.

The version of a gyroscope that we have used is a bicycle wheel with two handles, to one of which a rope is attached (figure 1). When the wheel rotates around its hub it precesses around a vertical axis through the rope. While this is surprising to many of the students, the lower precession frequency following from a higher rotation frequency is even more surprising. This part of the lab also makes it possible to show that a weight added to the other handle of the wheel makes the system precess faster despite the rotation frequency of the wheel being constant. When the weight is removed the precession is slower again. Graphs showing the detected total magnetic field as a function of time afford the students with access to the frequencies for the rotation (ω) and precession (Ω). The students determine the precession frequencies for different rotation frequencies.

These counter intuitive behaviours of the wheel can rather easily be explained using the scalar moment of inertia and the vectors torque and angular momentum. To make it easier for the students to detect errors in their calculations, they are asked to calculate the moment of inertia of the wheel if all the mass had been placed in the periphery (mR^2, where R is the radius of the wheel). The proper I for the rotating wheel must be smaller than mR^2 but of the same order of magnitude. For the equipment that we use, mR^2 is 0.15 kg m^2, about twice the proper value for the moment of inertia.

Figures 3 and 4 show the detected total magnetic field as a function of time for two of the different rotation frequencies. The graph can easily be explained to the students; the slower sinusoidal variation (from which they get the precession frequency) is due to the change in the detected magnetic field from the Earth when the wheel precesses. The faster variation[3] (from which they get the rotation frequency) is caused by the detected magnetic field from a magnet attached to one of the spokes.

Figure 5 shows a plot of the precession frequency Ω (vertical axis) as a function of the inverse of the rotation frequency $1/\omega$ (horizontal axis). According to equation (1), dividing mgr by the slope gives the moment of inertia, I. Finding this out is left as an exercise for the students. For this particular wheel with $r = 0.10$ m and $m = 2.2$ kg the moment of inertia becomes 0.07 kg m^2.

[3] Before the students carried out the lab, the frequency of rotation was compared with a LED attached to a spoke and with the small magnet. The two frequencies were identical.

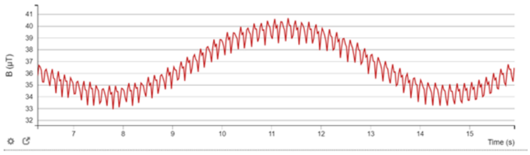

Figure 3. Detected total magnetic field with a higher rotation frequency and lower precession frequency, shown as a function of time.

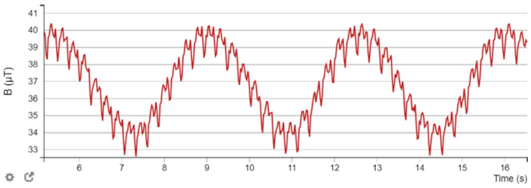

Figure 4. Detected total magnetic field with a lower rotation frequency and higher precession frequency, shown as a function of time.

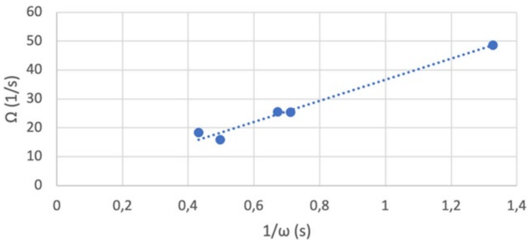

Figure 5. The precession frequency, Ω, (vertical axis) as a function of the inverse of the rotation frequency, $1/\omega$, (horizontal axis). The slope together with mgr gives the moment of inertia, I.

Here, the students get to experience how the variation in one aspect, the rotation frequency, affects another aspect, the precession frequency. Although both of those aspects vary, however, the moment of inertia (which was varied in part 1 of the lab) is constant.

To allow the students to experience more aspects of the gyroscope effect, they also study the relation between the direction of rotation and the direction of precession and explain why the latter changes with the former. Furthermore, the wheel's behaviour is studied when an external weight is added to, and removed from, the side of the wheel opposed from the rope while the frequency of rotation is constant (see figure 6). Figure 7

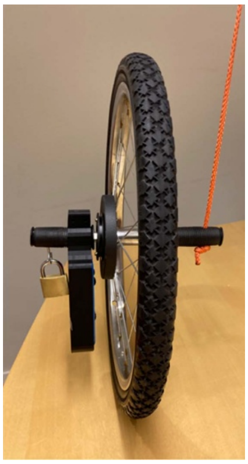

Figure 6. The wheel with an external weight added in the form of a padlock.

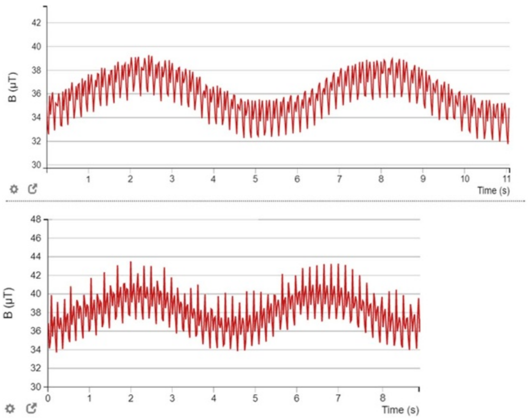

Figure 7. Top: detected total magnetic field without an extra weight added. Bottom: detected total magnetic field with extra weight.

shows the variation in the total detected magnetic field for approximately two precession cycles both without and with an external weight. The rotation frequencies in the two cases are roughly identical while the precession frequency is increased when the extra weight is added. The students are asked to explain this observation using equation (1). Both when varying the direction of rotation and when adding and removing an extra weight, one aspect is varied at a time showing how another aspect depends on it. From a variation theory point of view this suggests these relationships become

Figure 8. The distance d between the wheel's centre and the pivot (at the top) at the maximum swing angle.

noticeable to the students. The graphs also make these aspects clearly visible.

In a simple model of the case of the added extra weight, mgr is replaced by $(mgr + Mgr_2)$ in equation (1) $\left(\Omega = \frac{mgr}{I}\frac{1}{\omega}\right)$, where $M = 0.45$ kg is the mass of the extra weight and $r_2 = 0.235$ m is the distance between the rope and the point on the handle where the weight is attached. In our experimental setup the numerical value of mgr is 2.2 and $mgr + Mgr_2$ is 3.2 with the ratio 0.69. The period without the extra weight is just above 6 s and with the weight about 4 s, giving a ratio of 0.67. This is in good agreement with what is expected from the simple model.

2.3. *Determining the moment of inertia of the wheel by using it as a physical pendulum*

In the third part of the lab, the moment of inertia is calculated when using the wheel as a physical pendulum and timing the period. The pendulum has a different moment of inertia than the wheel rotating with respect to the hub. The pendulum has a pivot just inside the rim of the wheel at a distance d from the hub as shown in figure 8. The period T is measured via timing of for instance ten cycles. With help of the differential equation for a physical pendulum and using the approximation that $\sin\theta = \theta$ for small angles, the physical pendulum's moment of inertia I_p with respect to the pivot is given by the formula

$$I_p = mgd(T^2/4\pi^2) \tag{2}$$

where m is the mass of the wheel.

The parallel axis theorem for two-dimensional objects (equation (3)) is then used to find an approximate value for the moment of inertia with respect to the centre of the wheel, I:

$$I_p = I + md^2. \quad (3)$$

With $m = 2.20$ kg, $d = 0.18$ m and $T = 1.1$ s, the moment of inertia of the wheel is found to be 0.06 kg m^2, which is in rather good agreement with the value found in part 2 of the lab.

3. Conclusion

A lab dealing with concepts describing rotational motion was designed and used during a mechanics course for engineering chemistry students. The nominal time for the lab has been 4 h and almost all student groups were able to complete the different parts in that time.

That all three participating lab assistants have requested to be assigned for the lab the next time it is given indicates that the lab worked well. We have also found that the lab compendium described earlier works well.

The course evaluation survey shows that the general opinion about the course is that it covered too many items, mechanics, wave physics, and modern physics. One question in the course survey asked the students what they think should be maintained for the next year. A 40% of those answering the survey found the lab to be a highlight in the course. They typically found it hard but interesting. No other part of the course was mentioned as a highlight as many times as the lab.

As mentioned above, some of the students did the lab before the necessary theory had been discussed in the lectures. These students found it more difficult to understand part 2 of the lab.

We conclude by a quote from one of the students who evaluated the course:

> I thought that the lab was very good, particularly when it comes to learning the more difficult concepts like angular momentum since they are visualized. The concepts were difficult to learn anyway but had probably been even harder without the lab. The fact that the gyroscopic effect is so 'weird' or 'non-intuitive' made me interested in understanding how it works.

The lab together with the preparation compendium thus gave the students a deeper understanding of rotational phenomena.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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