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***Abstract*— We study the effect of phase noise on the achievable spectral efficiency (SE) of a distributed massive multi-input multiple-output (DM-MIMO) network. We obtain a closed-form expression for this network considering the phase noise, independent Rayleigh fading channel, and minimum mean square error (MMSE) channel estimation. In this network, Access points (APs) under the time-division duplex (TDD) operation estimate the channels via the uplink training phase and transmit the downlink data. The equations and simulation results present the effect of phase noise introduced by oscillators in APs. Results show the impact of phase noise at APs on the achievable SE. Consequently, we observe that in the DM-MIMO network, with increasing the number of UEs, PN causes some degradation in the SE. On the other hand, with increasing the variance of the PN, the SE decreases. Through simulations, we verify our analytical results and closed-form equations.**

***Keywords*— phase noise, DM-MIMO, SE, MMSE, TDD.**

I. Introduction

Massive multi-input multiple-output (M-MIMO) systems are cellular networks with numerous antennas and can significantly improve spectral efficiency. They are deployed on collocated M-MIMO (CM-MIMO), where the antennas are physically placed on an array in the cell center or base station (BS), and Distributed M-MIMO (DM-MIMO) where contains many geographically distributed single or multiple antennas access points (APs) and connected with a high-speed backhaul [1]. APs in a DM-MIMO system communicate with fewer user equipment (UEs) over the same time/frequency resources, causing a high degree of freedom to provide spatial multiplexing. Some of the advantages of DM-MIMO compared to CM-MIMO are better coverage, decreased transmit power, higher spectral efficiency (SE), and energy efficiency [2]-[3].

The impact of hardware impairments on M-MIMO networks at both the APs (equipped with large antenna arrays) and the single-antenna UEs have been observed [4]. Since most impairments such as power amplifier distortion, phase noise, and quantization noise are highly dependent on the transmit waveform, the impact needs more clarified analyses for using simplified stochastic models that assume a correlation between the transmit waveform and the hardware impairments [5]. Many calibration schemes and compensation algorithms can be used in transceivers to remove these hardware impairments, but a specific amount of distortions remains. These residual distortions are modeled either by additive Gaussian noises at the transmitter and receiver sides as the aggregate effect of many impairments or multiplicative coefficients to the channels containing the phase noise (PN), the phase drifts in the local oscillators (LOs) [6].

The PN caused by imperfect oscillators is one of the most crucial hardware impairments in DM-MIMO systems. Each antenna needs an independent oscillator in these systems because of the geographical distance between the antennas. Using cheap hardware may cause phase noise during the up-conversion of the baseband signal to bandpass and vice versa [7]. In the case of DM-MIMO systems, each AP and UE has an oscillator that works independently from others because they are located at long distances from each other [8]. Working these independent oscillators in APs and UEs besides timing offset, carrier frequency, and phase offset cause some phase noises.

This paper analyzes the effect of phase noise in a DM-MIMO network. We analyze pilot transmission from UEs to the APs in uplink operation and then precoded data transmission in downlink operation. Each AP estimates the channel in uplink operation through minimum mean square error (MMSE) estimation. The DM-MIMO precoding matrix can be evaluated by a central processor (CP) in the downlink operation. In the end, all the APs, simultaneously send their data to the UEs in the same time slot. This paper aims to show the effect of phase noise on SE. The main contribution is deriving the closed-form expression for the downlink capacity of a DM-MIMO system by considering the channel estimate errors. Our result shows the importance of phase noise compensation in DM-MIMO networks. The organization of this paper is as follows. Section I and II describe the introduction and system model parts, respectively. Section III introduces the spectral efficiency and problem formulation. Finally, the simulation results and their analysis are shown in section IV, followed by the conclusion in section V.

II. System Model

We assume a distributed massive MIMO (DM-MIMO) network that operates in time division duplex (TDD) mode with M AP and K UE equipped with a single antenna. Each AP is connected to a central processing unit (CPU) via a backhaul network and simultaneously serves all K UE on the same time resources. In addition, all UE and AP are distributed randomly in a large area. We consider perfect timing and frequency synchronization.

We consider the channel vector between AP m and UE k, indicated by $h_{m,k}$, during the transmission in each coherence

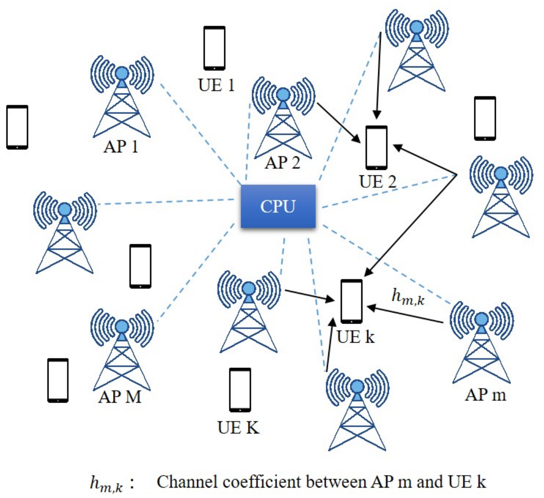

Fig. 1. Distributed Massive MIMO system model

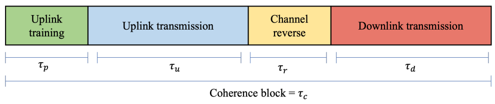

Fig. 2. The coherence block used in our system model

block is fixed and exhibits flat-fading. This channel vector is represented through an independent correlated Rayleigh fading distribution as $h_{m,k} \sim \mathcal{CN}(0, \beta_{m,k})$. The complex Gaussian distribution models the small-scale fading, and $\beta_{m,k}$ shows the large-scale fading.

We consider three stages for our network: (i) uplink (UL) training with pilots for channel estimation, (ii) UL data transmission, and (iii) downlink (DL) data transmission. Based on the standard M-MIMO TDD protocol, each coherence block τ_c is divided into τ_p channel uses for UL pilots, τ_u for UL data, and τ_d for DL data such that $\tau_c = \tau_p + \tau_u + \tau_d + \tau_r$. In addition, we assume τ_r as a delay between the end of UL transmission and the start of DL transmission depicted as channel reverse in Fig.2.

In this paper, we analyze the SE for downlink mode that focuses on the stages of the UL training with pilots for channel estimation and the DL data transmission.

A. Phase noise

We assume that the oscillator phase noise ϕ has a Wiener phase noise process caused by the oscillator at the transmitter. Due to imperfect local oscillators, PN causes random rotations of the transmitted and received signals in a DM-MIMO system. In general, PN happens at all AP and users. We consider a discrete-time Wiener PN model for the mth AP at symbol n as

$$\phi_m(n) = \phi_m(n-1) + \Delta_m(n), \ \Delta_m(n) \sim \mathcal{N}(0, \sigma^2_{\Delta_m}), \quad (1)$$

where $\sigma^2_{\Delta_m}$ is the PN increment variance at the mth AP, ϕ_m is the phase noise in the mth AP, and Δ_m is the innovation.

We define $\phi^u_m(n)$ as phase noise in UL mode in mth AP and nth symbol and $\phi^d_m(n)$ as phase noise in DL mode in mth AP and nth symbol that u and d show the UL and DL modes, respectively. We assume the DL block starts τ_r symbols after the end UL block means $\phi^d_m(n) = \phi^u_m(n - \tau_r)$. We define the average value of the phase noise on all symbols in UL training mode as $\bar{\phi}^u_m = \frac{1}{\tau_p}\sum_{n=1}^{\tau_p} \phi^u_m(n)$ and as the same for DL data transmission mode.

B. Uplink Training

We assume there are τ_p mutually orthogonal pilot sequences with the length of τ_p that all UEs send to all APs in the UL training stage. The pilot sequence used by UE k to send to all APs is $\sqrt{\rho_k}\boldsymbol{\psi}_k \in \mathbb{C}^{\tau_p \times 1}$, in which we assumed $\tau_p \geq K$. The training sequence $\boldsymbol{\psi}_k$ is transmitted by a user and is orthogonal to other training sequences, then $\boldsymbol{\psi}_k^H \boldsymbol{\psi}_i = 0$ if $k \neq i$ and $\boldsymbol{\psi}_k^H \boldsymbol{\psi}_i = \tau_p$ if $k = i$. In addition, ρ_k is the UL transmit power. The received signal in uplink mode at the AP m in symbol n is

$$y_m(n) = \sum_{k=1}^{K} \sqrt{\rho_k} h_{m,k} e^{j\phi^u_m(n)} \psi_k(n) + w_m(n), \quad (2)$$

So, for a block of pilots, we can write it as $\mathbf{y}_m = \mathbf{\Phi}^u_m \sum_{k=1}^{K} \sqrt{\rho_k} h_{m,k} \boldsymbol{\psi}_k + \mathbf{w}_m$, where $\mathbf{y}_m = [y_m(1), \cdots, y_m(\tau_p)]^T \in \mathbb{C}^{\tau_p \times 1}$, $\mathbf{w}_m = [w_m(1), \cdots, w_m(\tau_p)]^T \in \mathbb{C}^{\tau_p \times 1}$ is a Gaussian noise matrix whose elements are i.i.d. with $\mathcal{CN}(0, \sigma^2_{w_m})$. The PN matrix is as $\mathbf{\Phi}^u_m \in \mathbb{C}^{\tau_p \times \tau_p}$ that is a diagonal matrix equal to $\mathbf{\Phi}^u_m \triangleq \text{diag}\big(e^{j\phi^u_m(1)}, \cdots, e^{j\phi^u_m(\tau_p)}\big)$.

C. Uplink Data Transmission

During uplink data transmission, the received signal in uplink mode at the AP m for a block of pilots is $\mathbf{y}_m = \mathbf{\Phi}^u_m \sum_{k=1}^{K} \sqrt{\rho_k} h_{m,k} \boldsymbol{s}_k + \mathbf{w}_m$,
where the transmit signal is $\mathbf{s}_k = [s_k(1), \cdots, s_k(\tau_u)]^T \in \mathbb{C}^{\tau_u \times 1}$.

D. Downlink Data Transmission

In our DM-MIMO network, we consider that all the AP simultaneously serve all the UE. The AP attempt to send the data symbol q_k to user k, while $E\{|q_k|^2\} = 1$, $k = 1, \cdots, K$. In addition, we assume that the data symbols are uncorrelated, i.e., $E\{q_k q_t^*\} = 0$ for any $t \neq k$ and have zero mean. The transmitted signal from the AP m to all the UE in symbol n that $n \in \{1, \cdots, \tau_d\}$, using the maximum ratio precoding scheme that depends on its local channel estimate, is

$$x_m(n) = \sum_{i=1}^{K} \sqrt{p_{m,i}} v^*_{m,i} q_i(n) \quad (3)$$

where the transmitted signal is x_m and the precoding coefficient is $v_{m,k}$. The transmit power is $p_{m,k}$, satisfying a per-AP power constraint $p_{m,k} \leq p_{max}, \forall m$. We can write the vector of transmitted signals for a block of pilots as $\mathbf{x}_m = \sum_{i=1}^{K} \sqrt{p_{m,i}} v^*_{m,i} \mathbf{q}_i$. where $\mathbf{x}_m = [x_m(1), \cdots, x_m(\tau_d)]^T \in \mathbb{C}^{\tau_d \times 1}$ and $\mathbf{q}_k = [q_k(1), \cdots, q_k(\tau_d)]^T \in \mathbb{C}^{\tau_d \times 1}$. Based on our consideration in the system model, when $\tau_p \geq K$, there is no parallel estimated channels and all the estimated channels are orthogonal. Hence, the matrix of the channel estimates,

$\hat{\mathbf{G}}_m = [\hat{g}_{m,1}, \cdots, \hat{g}_{m,K}] \in \mathbb{C}^{M\times K}$, is full-rank. Then, we can write the received data signal at UE k as

$$\begin{aligned} y_k(n) &= \sum_{m=1}^{M} h_{m,k} e^{j\phi_m^d(n)} x_m(n) + w_k(n) \\ &= \sum_{m=1}^{M}\sum_{i=1}^{K} \sqrt{p_{m,i}} h_{m,k} v_{m,i}^* e^{j\phi_m^d(n)} q_i(n) + w_k(n) \end{aligned} \tag{4}$$

III. Performance Analysis

A. Phase Noise Analysis

According to the block fading, we can split the phase noise into two parts of the block's average value and a Gaussian random variable. Then, we can use the Taylor first-order approximation $e^x = 1 + x$ as below.

$$e^{j\phi_m^u(n)} = e^{j(\bar{\phi}_m^u + \delta_m^u(n))} \approx e^{j\bar{\phi}_m^u}(1 + j\delta_m^u(n)) \tag{5}$$

where $\delta_m^u(n) = \phi_m^u(n) - \bar{\phi}_m^u$ is the difference between instantaneous and average phase noises in UL mode and $\delta_m^u(n)$ is the random Gaussian variable with $\mathcal{CN}(0, \sigma^2_{\delta^u_m})$. We obtain

$$\boldsymbol{\Phi}_m^u = \mathrm{diag}(e^{j\bar{\phi}_m^u}(1 + j\delta_m^u(n))) = e^{j\bar{\phi}_m^u}(I_{\tau_p} + j\boldsymbol{\delta}_m^u) \tag{6}$$

where $\boldsymbol{\delta}_m^u = \mathrm{diag}(\delta_m^u(n)) \in \mathbb{C}^{\tau_p\times\tau_p}$. As the same of UL stage, we approximate the phase noise in DL data transmission mode with superscript d instead of u. In addition, we suppose there is a correlation for parameter δ_m between the AP of m and m' as $E\{\delta_m^d \delta_{m'}^d\} = \eta^d_{m,m'}$, if $m \neq m'$.

B. MMSE Channel Estimation

For estimating $h_{m,k}$, the channel to UE k, the AP m first correlates the received signal with the associated pilot signal $\boldsymbol{\psi}_k$ to obtain $\hat{y}_{m,k} \triangleq \frac{1}{\sqrt{\tau_p}}\boldsymbol{\psi}_k^H \mathbf{y}_m$, which is given by

$$\hat{y}_{m,k} = \sqrt{\rho_k \tau_p} h_{m,k} e^{j\bar{\phi}_m^u} + w_{m,k} \tag{7}$$

where the second part is equal to

$$\begin{aligned} w_{m,k} &= j\frac{\sqrt{\rho_k}}{\sqrt{\tau_p}} h_{m,k} e^{j\bar{\phi}_m^u} \boldsymbol{\psi}_k^H \boldsymbol{\delta}_m^u \boldsymbol{\psi}_k \\ &+ j\sum_{i\neq k}^{K} \frac{\sqrt{\rho_i}}{\sqrt{\tau_p}} h_{m,i} e^{j\bar{\phi}_m^u} \boldsymbol{\psi}_k^H \boldsymbol{\delta}_m^u \boldsymbol{\psi}_i + \frac{1}{\sqrt{\tau_p}}\boldsymbol{\psi}_k^H \mathbf{w}_m \end{aligned} \tag{8}$$

where $w_{m,k} \sim \mathcal{CN}(0, \sigma^2_{w_{m,k}})$ and $E\{|\delta_m^u|^2\} = \sigma^2_{\delta^u_m}$. We define $g_{m,k} = h_{m,k} e^{j\bar{\phi}_m^u}$ then, $E\{|h_{m,k}|^2\} = E\{|g_{m,k}|^2\} = \beta_{m,k}$. Using standard results from estimation theory, the minimum mean square error (MMSE) estimate of $g_{m,k}$ is

$$\hat{g}_{m,k} = \frac{\sqrt{\rho_k\tau_p}\beta_{m,k}}{A_1 + \sigma^2_{w_m}}(\sqrt{\rho_k\tau_p} g_{m,k} + w_{m,k}) \tag{9}$$

where $A_1 = \rho_k\tau_p\beta_{m,k} + \rho_k\tau_p\beta_{m,k}\sigma^2_{\delta^u_m} + \sum_{i\neq k}^{K}\rho_i\tau_p\beta_{m,i}\sigma^2_{\delta^u_m}$. The estimation error is given by $\tilde{g}_{m,k} = g_{m,k} - \hat{g}_{m,k}$. The estimate and estimation error are independent and distributed as $\hat{g}_{m,k} \sim \mathcal{CN}(0, \gamma_{m,k})$, and $\tilde{g}_{m,k} \sim \mathcal{CN}(0, (\beta_{m,k} - \gamma_{m,k}))$, respectively, where $\gamma_{m,k} = \frac{\rho_k\tau_p\beta^2_{m,k}}{A_1 + \sigma^2_{w_m}}$ is the mean-square of the estimate.

C. Maximum Ratio Transmission (MRT) Precoding

We manifest the closed-form expression for the achievable downlink SE considering phase noise and independent Rayleigh fading channel through the maximum ratio transmission (MRT) scheme for AP. The MRT precoding coefficient constructed by AP m towards UE k, denoted by $v_{m,k} = \frac{\hat{g}_{m,k}}{\sqrt{E\{|\hat{g}_{m,k}|^2\}}} = \frac{\hat{g}_{m,k}}{\sqrt{\gamma_{m,k}}}$.

D. Spectral Efficiency

We can write the received data signal at UE k as

$$\begin{aligned} y_k(n) = &\sum_{m=1}^{M} \sqrt{p_{m,k}} h_{m,k} v_{m,k}^* e^{j\bar{\phi}_m^d} q_k(n) \\ &+ \sum_{m=1}^{M}\sum_{i=1, i\neq k}^{K} \sqrt{p_{m,i}} h_{m,k} v_{m,i}^* e^{j\bar{\phi}_m^d} q_i(n) + w_k'(n) \end{aligned} \tag{10}$$

In equation (10), the first term shows the desired signal for the UE k, the second term includes all UE' signals for each UE i, $i \neq k$ that is equal to multi-user interference. Moreover, the third term is i.i.d. Gaussian noise at the receiver, $w_k' \sim \mathcal{CN}(0, \sigma^2_{w_k'})$ that can be written as

$$w_k'(n) = j\sum_{m=1}^{M}\sum_{i=1}^{K} \sqrt{p_{m,i}} h_{m,k} v_{m,i}^* e^{j\bar{\phi}_m^d} \delta_m^d(n) q_i(n) + w_k(n) \tag{11}$$

The downlink ergodic channel capacity of UE k can be written as below.

$$\mathrm{SE}_k = \frac{\tau_d}{\tau_c} E\{\log_2(1 + \mathrm{SINR}_k)\} \quad \text{[bit/s/Hz]}, \tag{12}$$

Expression (12) is correct despite the precoding scheme used. We write the effective SINR of the achievable downlink SE as equation (13) at the top of the page. and the closed form is as below

$$\mathrm{SINR}_k = \frac{|\sum_{m=1}^{M}\sqrt{p_{m,k}\gamma_{m,k}}|^2}{A_2 + \sigma^2_{w_k'}} \tag{14}$$

where

$$A_2 = \sum_{i=1}^{K}\sum_{m=1}^{M} p_{m,i}\beta_{m,k} + \sum_{i=1, i\neq k}^{K} |\sum_{m=1}^{M}\sqrt{p_{m,i}\gamma_{m,i}}|^2 \tag{15}$$

IV. Simulation Results and Discussions

This section represents our simulation results for evaluating the achievable sum SE in a DM-MIMO network considering the effect of phase noise. We assume a simulation scenario with $M = 25$ single antenna AP and K UEs distributed in a 1×1 km square. In our system model, the variance of the phase noise is equal to $Q = 10^{-2}$, $\tau_c = 200$, $\tau_p = 20$, $\tau_r = 20$ and $B = 20$ MHz. In addition, we assume that the transmit power for each UE is 100mW and for each AP is 200mW. We use $\tau_u = 80$ and $\tau_d = 80$ for evaluation of UL and DL, respectively. In Fig.3, we plot the average downlink SE per UE as a function of the number of UEs K to observe the effect of phase noise. We consider the number of users to be

$$\mathrm{SINR}_k = \frac{|\sum_{m=1}^{M} E\{\sqrt{p_{m,k}} h_{m,k} v_{m,k}^{*} e^{j\bar{\phi}_m^d}\}|^2}{\sum_{i=1}^{K} E\{|\sum_{m=1}^{M} \sqrt{p_{m,i}} h_{m,k} v_{m,i}^{*} e^{j\bar{\phi}_m^d}|^2\} - |\sum_{m=1}^{M} E\{\sqrt{p_{m,k}} h_{m,k} v_{m,k}^{*} e^{j\bar{\phi}_m^d}\}|^2 + \sigma_{w_k'}^2} \tag{13}$$

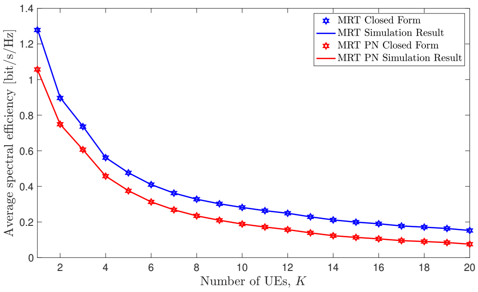

Fig. 3. The average downlink SE per UE as a function of the number of UEs K

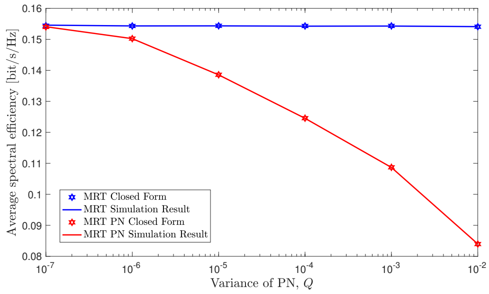

Fig. 5. The average downlink SE per variance of PN as Q

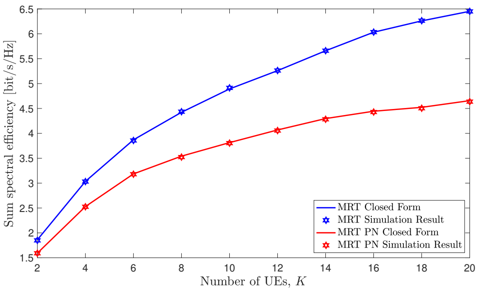

Fig. 4. Sum spectral efficiency per UE in downlink mode as a function of the number of UEs K

between 1 to 20, and the variance of the phase noise is equal to $Q = 10^{-2}$. This figure shows closed-form and simulation results for a DL DM-MIMO system with MRT precoding. PN decreases the average spectral efficiency with the same amount for each number of UEs. It means that for more number of UEs, the effect of PN on sum spectral efficiency is so high. Because we considered observing the effect of PN only on the APs, so with the constant number of APs, the average SE has the same degradation for each number of the user. Fig.4 shows sum spectral efficiency (SE) per UE for MRT precoding in downlink as a function of the number of UEs K to observe the effect of phase noise. We see the effect of PN on a higher number of users is so much in this figure and according to the analysis for Fig.3. In Fig.5, the effect of variance of PN as $\log Q$ on the average downlink SE with a network with $M = 25$ and $K = 20$ are shown, respectively. The figures show that with increasing the amount of variance of PN, the average downlink SE and sum spectral efficiency decrease.

V. Conclusion

In this paper, we obtained the closed form for the sum SE of a DM-MIMO network considering the effect of phase noise. Simulation results demonstrated how much the performance would worsen if the phase noise effect were not compensated. We demonstrated the average downlink SE and sum spectral efficiency of a DM-MIMO network considering PN decrease. This degradation in spectral efficiency increases with increasing the number of UEs. Finally, we observed that the variance of PN has an inverse effect on spectral efficiency, which means that, with increasing the variance of PN, the spectral efficiency decreases.

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