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Abstract *We propose a novel tracking algorithm for optical channels suffering from fast state of polarization (SOP) rotations and polarization-dependent loss (PDL). Unlike gradient descent-based algorithms that require step size adjustment when the channel conditions change, our algorithm performs similarly or better without parameter tuning.*

Introduction

In modern optical communication systems, advanced digital signal processing (DSP) utilizes polarization-division multiplexing and high-order modulations to reach high spectral efficiency (SE)[1]. However, high SE comes at the expense of a lower tolerance to impairments such as state of polarization (SOP) drift and polarization-dependent loss (PDL)[2]. In a fiber link, the SOP drifts randomly due to external perturbations (e.g., mechanical/thermal stress, weather conditions, etc.) of the fiber. Previous studies have reported that the SOP drift can be extremely slow (days) in buried fibers[3] and very fast (microseconds) in aerial fibers[4]–[6]. The SOP drift combined with PDL results in a time-varying power imbalance between the polarizations making the polarization tracking challenging at the receiver.

The gradient descent (GD)-based algorithms are the most commonly used tracking algorithms in the literature. The most popular ones are the constant modulus algorithm (CMA)[7], modified CMA (MCMA)[8], and decision-directed least mean squares (DDLMS)[9],[10]. However, these algorithms suffer from phase ambiguity and the singularity problem[11] which might be resolved by pilot-assisted[12] and hybrid[13] algorithms. However, GD-based algorithms are also step size sensitive[14], implying that they may require adjusting their step size as the SOP drift speed changes. However, in some applications, an estimate of the SOP drift speed is not available at the receiver.

In this paper, considering both SOP drift and PDL, we propose a novel tracking algorithm that shows a higher polarization drift tolerance than the conventional GD-based algorithms. Moreover, unlike the GD-based algorithms that require step size adjustment, the proposed algorithm requires no parameter tuning, making it robust to varying channel conditions.

System Model

Assuming negligible (or already compensated) nonlinearities, chromatic dispersion, and polarization-mode dispersion (PMD), we consider memoryless dual-polarization transmission in the presence of PDL, SOP drift, and amplified spontaneous emission (ASE) noise at the receiver. The 2-dimensional random vector $\underline{\mathbf{s}}_k$ denotes the transmitted signal at time k, taking values from a set $\mathcal{S} = \{\underline{c}_1, \underline{c}_2, ..., \underline{c}_M\}$ of complex, zero-mean, equiprobable constellation points. The vector of received complex samples is

$$\underline{\mathbf{x}}_k = \mathbf{H}_k \underline{\mathbf{s}}_k + \underline{\mathbf{z}}_k, \tag{1}$$

where $\mathbf{H}_k$ is a 2×2 complex channel matrix, and $\underline{\mathbf{z}}_k \sim \mathcal{CN}(\underline{\mathbf{0}}, \sigma_z^2 \mathrm{I}_2)$ is a vector of complex, random ASE noise samples. The considered dual-polarization PDL (DP-PDL) channel is described by a combination of the SOP drift model in[15] and N concatenated PDL elements. In particular, the DP-PDL channel matrix is

$$\mathbf{H}_k = \mathbf{\Gamma}_N \mathbf{J}_{k,N} \cdots \mathbf{\Gamma}_1 \mathbf{J}_{k,1} = \prod_{n=1}^{N} \mathbf{\Gamma}_n \mathbf{J}_{k,n}, \tag{2}$$

where n is the segment index and $\mathbf{J}_{k,n}$ is a random 2×2 unitary matrix accounting for SOP drift. It is defined in[15] as

$$\mathbf{J}_{k+1,n} = \exp(-j \underline{\boldsymbol{\alpha}}_{k,n} \cdot \overrightarrow{\sigma}) \mathbf{J}_{k,n}, \tag{3}$$

where $\exp(\cdot)$ is the matrix exponential and $\underline{\boldsymbol{\alpha}}_{k,n} \sim \mathcal{N}(0, \sigma_p^2 \mathrm{I}_3)$, where $\sigma_p^2 = 2\pi \Delta p \cdot T$ and Δp is the *polarization linewidth* determining the speed of the SOP drift and T is the symbol duration. The total polarization linewidth scales with N and can be defined as $\Delta p_{\text{tot}} = N \cdot \Delta p$. Finally, $\overrightarrow{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ is a tensor of the Pauli spin matrices[16]. Moreover, $\mathbf{\Gamma}_n = \mathrm{diag}(\sqrt{1+\gamma_n}, \sqrt{1-\gamma_n})$ is a 2×2 positive real-valued diagonal ma-

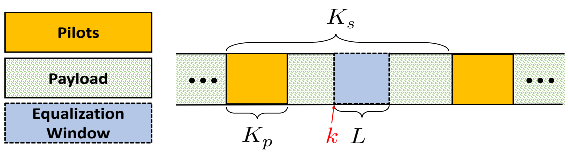

Fig. 1: Data frame with inserted pilot and payload symbols.

trix modeling the power imbalance induced by PDL[17], where $0 \leq \gamma_n \leq 1$ is each segment's PDL ratio. The segment-wise PDL is defined as $\varphi_n = (1+\gamma_n)/(1-\gamma_n)$, and the average aggregated PDL over K transmitted symbols in dB is defined as

$$\bar{\rho} = 10\log_{10}\left(\mathbb{E}_{\mathbf{H}}\left[\frac{1}{K}\sum_{k=0}^{K-1}\frac{\|\boldsymbol{\lambda}_k^{\max}\|^2}{\|\boldsymbol{\lambda}_k^{\min}\|^2}\right]\right), \quad (4)$$

where $\boldsymbol{\lambda}_k^{\max}$ and $\boldsymbol{\lambda}_k^{\min}$ are the eigenvalues of $\mathbf{H}_k$.

Proposed Algorithm

We propose a pilot-aided adaptive algorithm called sliding window least squares (SW-LS). The idea is to use a length L decision-directed sliding window called the *equalization window* and update the channel estimate for each window.

The data frame construction is shown in Fig. 1, where a pilot sequence of length K_p is inserted at the beginning of each transmission block of length K_s. The $2 \times K_p$ matrix of pilots is defined as $\mathrm{D} = [\underline{d}_0, \underline{d}_1, \ldots, \underline{d}_{K_p-1}]$, where the pilots are chosen from the quadrature phase-shift keying (QPSK) constellation. The pilots are orthogonal with respect to each polarization (i.e., $\mathrm{D}\mathrm{D}^\dagger = \delta\mathrm{I}_2$ where $\delta \in \mathbb{R}$ is a constant). This coincides with the optimal pilot selection suggested in[18],[19].

Using the previous estimate of the channel $\hat{\mathbf{H}}_k$ and the minimum Euclidean distance criterion, the transmitted symbols in an equalization window can be estimated as

$$\underline{\hat{\mathbf{s}}}_k = \arg\min_{\underline{c}\in\mathcal{S}}\left\|\hat{\mathbf{H}}_k^{-1}\underline{\mathbf{x}}_k - \underline{c}\right\|^2 \quad (5)$$

and the $2 \times L$ matrix of estimated symbols can be formed as $\hat{\mathbf{S}}_k = [\underline{\hat{\mathbf{s}}}_k, \underline{\hat{\mathbf{s}}}_{k+1}, \ldots, \underline{\hat{\mathbf{s}}}_{k+L-1}]$. Defining the $2 \times L$ matrix of received symbols $\mathbf{X}_k = [\underline{\mathbf{x}}_k, \underline{\mathbf{x}}_{k+1}, \ldots, \underline{\mathbf{x}}_{k+L-1}]$, the channel estimation problem can be written as

$$\arg\min_{\hat{\mathbf{G}}_k}\left\|\mathbf{X}_k - \hat{\mathbf{G}}_k\hat{\mathbf{H}}_k\hat{\mathbf{S}}_k\right\|^2. \quad (6)$$

The optimal solution of Eq. (6) is given by the least squares (LS) algorithm as

$$\hat{\mathbf{G}}_k = \mathbf{X}_k\hat{\mathbf{S}}_k^\dagger\hat{\mathbf{H}}_k^\dagger\left(\hat{\mathbf{H}}_k\hat{\mathbf{S}}_k\hat{\mathbf{S}}_k^\dagger\hat{\mathbf{H}}_k^\dagger\right)^{-1}. \quad (7)$$

Initializing the estimated channel matrix as $\hat{\mathbf{H}}_0 = \mathrm{I}_2$, the estimated channel $\hat{\mathbf{H}}_k$ then is updated by

$$\hat{\mathbf{H}}_{k+\nu} = \hat{\mathbf{G}}_k\hat{\mathbf{H}}_k, \quad (8)$$

where ν is the sliding stride of the equalization window, meaning that the equalization window slides ν symbols at a time.

Algorithm 1 describes the SW-LS algorithm.

Algorithm 1 SW-LS

Input: $\mathbf{X}_k, \hat{\mathbf{H}}_k, k, \nu$
Output: $\hat{\mathbf{H}}_{k+\nu}, \tilde{\mathbf{S}}_k, k$

for $l = 0, \ldots, L-1$ **do**
  $i = (k + l \mod K_s)$
  if $i \leq K_p - 1$ **then**
    $\underline{\hat{\mathbf{s}}}_{k+l} = \underline{d}_i$
  else
    $\underline{\hat{\mathbf{s}}}_{k+l} = \arg\min_{\underline{c}\in\mathcal{S}}\left\|\hat{\mathbf{H}}_k^{-1}\underline{\mathbf{x}}_{k+l} - \underline{c}\right\|^2$ // Eq. (5)
end
$\hat{\mathbf{G}}_k = \mathbf{X}_k\hat{\mathbf{S}}_k^\dagger\hat{\mathbf{H}}_k^\dagger\left(\hat{\mathbf{H}}_k\hat{\mathbf{S}}_k\hat{\mathbf{S}}_k^\dagger\hat{\mathbf{H}}_k^\dagger\right)^{-1}$ // Eq. (7)
$\hat{\mathbf{H}}_{k+\nu} = \hat{\mathbf{G}}_k\hat{\mathbf{H}}_k$
$\tilde{\mathbf{S}}_k = [\underline{\hat{\mathbf{s}}}_k, \underline{\hat{\mathbf{s}}}_{k+1}, \ldots, \underline{\hat{\mathbf{s}}}_{k+\nu-1}]$
$k = k + \nu$

Simulation Setup

Polarization-multiplexed 16 quadrature amplitude modulation (PM-16-QAM) at a symbol rate of $R_s = 28$ Gbaud (i.e., $T = 1/R_s$) is considered. A random sequence of $K = 10^5$ symbols is transmitted on each polarization, initialized by random, independent, unitary matrices $\mathbf{J}_{0,1}, \ldots, \mathbf{J}_{0,N}$. The presented results are averaged over 10^5 such sequences. The link has $N = 20$ segments with identical polarization linewidth Δ_p and segment-wise PDL $\varphi_n = 0.70$ dB, resulting in an average aggregated PDL $\bar{\rho} = 3.0$ dB. The launch power in each polarization is $P = \mathbb{E}[\underline{\mathbf{s}}_k^\dagger\underline{\mathbf{s}}_k]$. The symbol error rate (SER) is evaluated for various setups in the presence of SOP drift, PDL, and ASE noise quantified by the signal-to-noise ratio (SNR) per polarization, defined as $\mathrm{SNR} = P/\sigma_z^2$. The pilot and transmission block lengths are set to $K_p = 16$ and $K_s = 1016$ symbols, yielding 1.6% overhead. The SW-LS algorithm is set to use an equalization window of $L = 24$ with a sliding stride of $\nu = 6$ to get a fair performance while keeping the complexity low.

We compare to MCMA[8] (blind), LS (pilot-based), and LS-DDLMS (hybrid) benchmarks. In LS-DDLMS, the DDLMS[9],[10] algorithm is initialized with a coarse estimate of the channel obtained by applying LS to the pilot sequence in each transmission block. The maximum likelihood

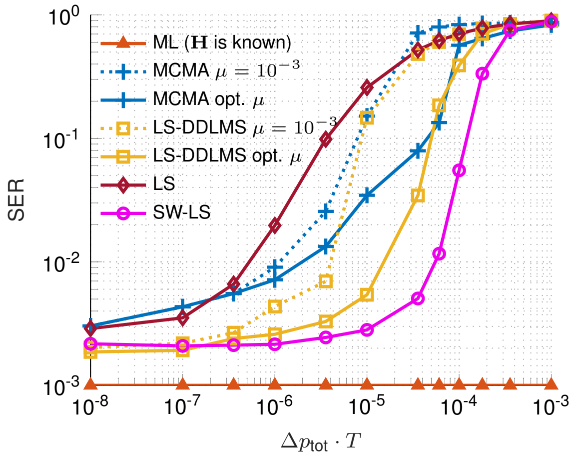

Fig. 2: SER of various algorithms for a fiber with $N = 20$ segments at SNR $= 18$ dB where $\bar{\rho} = 3.0$ dB is obtained by setting the $\varphi_n = 0.7$ dB for all the segments.

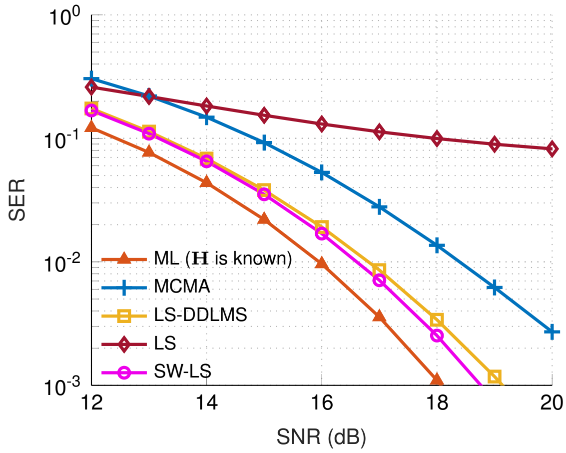

Fig. 3: SER of various algorithms for a DP-PDL channel with $\bar{\rho} = 3.0$ dB and $\Delta p_{\text{tot}} \cdot T = 3.57 \cdot 10^{-6}$.

(ML) detection for a known channel at the receiver is also used as a benchmark. The step size μ of the MCMA and DDLMS algorithms is optimized as a function of SOP drift speed, SNR, and PDL. For MCMA, the phase ambiguity is resolved by coherent differential coding and the singularity problem is assumed to be ideally mitigated.

Results

Polarization Drift Tolerance: We start by evaluating the channel tracking ability of SW-LS and the conventional algorithms for the DP-PDL channel. The polarization drift sensitivity is measured by sweeping Δ_p while the SNR is fixed.

Fig. 2 shows the SER versus $\Delta p_{\text{tot}} \cdot T$ for an average aggregated PDL $\bar{\rho} = 3.0$ dB, where the SNR is set such that SER $= 10^{-3}$ is achieved with ML detection. It can be seen that the LS-DDLMS and SW-LS algorithms behave roughly the same at low SOP drifts ($\Delta p_{\text{tot}} \cdot T < 10^{-6}$); however, at higher drift speeds ($\Delta p_{\text{tot}} \cdot T \geq 10^{-6}$), SW-LS shows the best polarization drift tolerance at the expense of higher complexity.

Moreover, to show the sensitivity of the GD-based algorithms to proper step size μ adjustment, the SER of MCMA and LS-DDLMS for a fixed μ is also plotted (see blue and yellow dotted curves). In particular, we set $\mu = 10^{-3}$, which gives the best performance of MCMA and DDLMS for slowly varying channels. Clearly, the performance of MCMA and LS-DDLMS is highly dependent on the proper adjustment of μ. However, the proposed algorithm requires no parameter tuning, which might be advantageous in some applications. For instance, in bursty channels, the SOP drift speed does not remain constant during the whole transmission. Thus, an algorithm that is tuned for a specific SOP drift speed may fail to track sudden changes in the channel.

Additive Noise Tolerance: Fig. 3 shows the SER versus SNR of various algorithms for a DP-PDL channel with $\bar{\rho} = 3.0$ dB and $\Delta p_{\text{tot}} \cdot T = 3.57 \cdot 10^{-6}$. Results show that the proposed SW-LS algorithm outperforms the benchmarks for the range of considered SNRs. An even more significant performance gap between SW-LS and the benchmarks has been observed for faster channels; however, those results are omitted from the paper to save space.

The superior performance of SW-LS comes at the expense of approximately L/ν times higher complexity than the benchmarks. Despite the higher tracking complexity, SW-LS has no parameter adjustment complexity and is robust for a large range of drift speeds where the benchmarks behave poorly. Besides, lower complexity can be obtained by reducing the L/ν ratio, but it also degrades the performance.

Conclusion

We have proposed the SW-LS algorithm to track memoryless DP-PDL channels. Numerical simulations show that the proposed algorithm is more robust to polarization drift than the benchmarks, including MCMA as the most popular tracking algorithm in the literature. While parameter adjustment is required for the GD-based algorithms (e.g., the CMA, MCMA, DDLMS, etc.), the proposed algorithm outperforms the benchmarks without needing such adjustments. The effect of polarization-mode dispersion is left for future work.

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