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INQUIRY-BASED INFORMAL MATHEMATICS LEARNING IN A SCIENCE CENTER

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Abstract

This study examines how visitors engage with two game-based mathematics exhibits to investigate inquiry-based informal mathematics learning in a science center. Inquiry-based learning originates from Dewey's theory of inquiry, is related to the scientific method of inquiry, and is seldom used in mathematics due to its perceived inaccessibility. We investigate visitors' interactions and conversations while playing two new game-based exhibits: a two-player game about the Nobel prize-winning Nash equilibrium, and a problem-solving puzzle finding a hidden arithmetic computation. Both exhibits are designed to inspire inquiry by providing hints and probing questions during gameplay. A small-scale field experiment using video recordings of visitors' engagement with the exhibits was conducted. The findings show that discoveries of mathematical strategies emerged in all groups (5 for each exhibit) indicating self-directed inquiry, that collaboration was essential and preferably with equal contributions within the groups, and that the games scaffolding with hints and feedback seemed essential to inject inspiration for inquiry. We conclude that inquiry-based informal mathematics learning is feasible in a science center.

Keywords: Informal learning, inquiry, mathematics, science center, exploration, exhibit, evaluation.

1 INTRODUCTION

Mathematics learning normally takes place in a classroom setting, following a structured curriculum with a predetermined set of learning objectives. Informal learning environments, such as science centers, offer different opportunities for students and the public to engage with mathematics through exploration and experimentation. Science centers are educational institutions specifically designed to encourage the public to learn through self-directed learning and discovery, making them unique settings for exploring the concepts and applications of mathematics.

In a science center context, the assumed learning by exhibit designers is different to school learning and the goal is often to provide opportunities for self-driven and active informal learning [1]. This means that exhibits need to inspire visitors to be interested, intrinsically motivated, and engaged in self-directed discovery to support inquiry-based learning. Educational interactive games are promising for informal learning, as they are often motivational, engaging, and social. They can also support self-regulated learning [2][3], by providing informative direct feedback to the players. Moreover, play and collaboration are essential features of informal learning, play because it motivates engagement and is a way to explore content [1]. Games are a common approach to mathematics learning, and when gameplay is cooperative it has also been shown to promote positive attitudes towards mathematics [4]. Cooperation versus competition is a hot topic within game design. The impact of cooperation compared to competition related to problem-solving in mathematics was studied by [5], showing that cooperation outperformed competition independent of student age, knowledge levels, and problem type. In a study with a mathematics game affording both collaboration and competition, it was revealed that students most often combined the two approaches and invented several ways to collaborate and compete beyond the activities directly supported by the game's interface [6]. Hence, the nature of inquiry-based learning makes educational games a reasonable approach to create intriguing and challenging activities; yet limited enough to allow for exploration and discovery within a given topic. Games are also suitable for informal learning at science centers.

This paper examines the informal learning of mathematics through inquiry-based exploration in science centers using two interactive and collaborative games. The study focuses on types of inquiries, forms of collaboration, and scaffolding that spontaneously emerge during visitors' engagement with the exhibits.

2 THEORETICAL FRAME

The theoretical frame of this study is a socio-cultural, situated view of learning. Learning is understood as an essentially social process where meaning is created and negotiated through actions and collective experiences with other humans and with the surrounding environment. The science center visitors situated, social actions and interactions with two mathematics exhibits are the focus of this study. The purpose of these exhibits is to stimulate and invoke mathematical inquiry among the visitors by engaging them in collaboration and gameplay. Mathematical inquiries in this context will be enacted from visitors' motivation to play well, to discover productive strategies in the game, or to solve the puzzle within a limited number of steps.

Inquiry-based learning is a broad pedagogical approach supported by many educators and education systems [7]. It is a learner-centred approach closely connected to project-based and experiential learning. Inquiry in education originates from John Dewey theory of inquiry [8], in which inquiry is seen as the basis of discovery as well as of learning. Inquiry-based learning utilizes critical and creative thinking and relies on learners' ability to ask questions, experiment, invent and test hypotheses, and communicate ideas, arguments, and findings. Hence, it is related to the scientific method of inquiry and according to Dewey "close to the attitude of the scientific mind" [9]. The method is less common in mathematics compared to science education, due to its perceived inaccessibility.

Dewey sees inquiry as a situated process where the interplay between known and unknown become crucial for driving the investigation [7], such as when a group of individuals engage in a challenge. Therefore, inquiries are often organized as collaborative tasks where learners are stimulated by challenges or probing questions, for example, collaborative games in a public exhibition as in this study. Through reflective inquiry, learning can emerge as an adaptive process connecting experiences with sensations and ideas. However, as Dewey points out; not all experiences are "genuinely or equally educative" [8], and therefore we must investigate how inquiry-based experiences unfold in practice.

2.1 Informal learning

Informal learning has fundamentally different prerequisites for learning than formal education [1]. By informal learning, we rely on the following definition related to STEM as "lifelong learning in science, technology, engineering, and math (STEM) that takes place across a multitude of designed settings and experiences outside of the formal classroom" [10]. Learning experiences in informal settings are described as "guided by learner interests, voluntary, personal, ongoing, contextually relevant, collaborative, nonlinear, and open-ended" [1] [11]. Normally there is no predefined task, no obligation, no teacher, and no assessment of the activity promoting informal learning. Informal learning can occur in many ways and at many places; at museums, during after-school activities, as hobbies or in leisure time activities. Informal learning is organized similarly whether addressing children or adults and is characterized by [11] as follows: it is interactive and embedded in a meaningful activity, guidance is provided through social interaction with other participants and by structures around the activity, the intention of talk is conversational and not didactic, the involvement is voluntary and based on interest and own initiatives, participants utilize their knowledge and develop new skills and ideas, and if assessment occurs it is for the sake of assessing the activity, not the learner.

Despite what the term informal suggests, the lack of rigour does not concern the learning content but the setting in which the learning occurs. Still, informal learning can motivate learners to achieve high levels of expertise in certain domains, as evidenced by experienced hobbyists or citizen scientists [11]. However, many informal learning experiences are opportunistic and have low visibility and are therefore immensely hard to evaluate [11]. Learning outcomes in informal settings are often beyond traditional discipline knowledge and skills, targeting curiosity, engagement, and attitudes. These types of constructs are more challenging to operationalize and evaluate, and it is also hard to avoid violating the informal nature. The reasons evaluation is so difficult are several; the informal setting means that most traditional evaluation methods such as experimental designs, direct inquiries and assessments of knowledge intrude on the very nature of informal learning. Moreover, voluntariness makes recruitment of participants uncertain, and parameters normally predetermined in studies such as age range, gender distribution, the duration of the studied activity, and what the activity consists of, are not in the hands of the researchers if the informal learning setting is taken seriously. Therefore, non-traditional methods for evaluating informal learning are called for [11].

3 RESEARCH APPROACH

Our research approach is to study informal mathematics learning in a science center context, where we try to minimize the interference of the study to maintain the sensation of an authentic informal learning experience as much as possible.

3.1 Informal learning at science centers

Science centers are a type of museum providing hands-on activities, interactive exhibits, and direct experience with scientific phenomena. Recently, the science center Exploratorium in San Francisco arranged a conference devoted to learning issues in science museums and other venues for informal science education. A challenge discussed at the conference among leading researchers in the area concerned the affordances of the museum as a learning space and how to make exhibits relevant and relatable to visitors. Science center interactive exhibits have the potential to engage visitors, but there are many obstacles on the route towards learning. It is often assumed that interactive exhibits engender engagement and social interactions which is productive for science learning [12], but we know little about how these processes unfold. Scaffolding by accompanying visitors and exhibit resources seems key to guiding social interactions and conversations in interesting directions. An early study of how visitors engaged with a simulated Tornado exhibit in the Exploratorium [13], revealed that the identified learning opportunities all originated from one prompted question by a researcher: why does the tornado spin? Without such scaffolding, no noteworthy explorations and learning opportunities may have emerged. This example illustrates the essence to stimulate inquiry.

There are different types of scaffolding. Besides inquiries as in the Tornado example, transfer-talk to other contexts or previous experiences can support the learning process. For example, [14] examined different scaffolding variations when students tried to learn from an interactive game-based exhibit with scientific content in a science museum. They found that fact-based rhetorical yes and no questions neither stimulated reasoning nor inquiry. The authors stressed the challenge of scaffolding to provide appropriate support for learning. A study of an augmented reality mathematics game deployed at a science center in Sweden provided evidence of an interactive game-based exhibit supporting mathematical reasoning among special education students at a level they had never reasoned in mathematics before [15]. The involved teachers ascribed this progress to the game since it provided a safe place to experiment with a tangible representation of arithmetic without the risk of failure and is a computer game with a motivating reward system. Besides mathematical learning, there were gains in peer-to-peer communication and collaboration skills, evident from the science center observation.

3.2 The empirical case study

The current study was conducted at Sweden's national science center in Gothenburg, Universeum. The most recent development is a mathematic exhibition "Mathrix" that opened to the public in February 2023. The purpose of Mathrix is to present mathematics in relation to the everyday activities and interests of a typical teenager as the main target group: the self, nature, the world, and social relations. The overall design idea of the exhibits is to activate visitors through explorations and discoveries related to the mathematics involved in daily activities. The goal is to stimulate engagement with the exhibits and their mathematical content and to foster a more positive attitude towards the subject. Two mathematics exhibits are chosen as the primary study objects in this research, for the following reasons: They are game based and designed to stimulate mathematical reasoning and strategic thinking and they address important mathematical areas (arithmetic and game theory).

The game theory exhibit (see Fig 1, left) is a two-player game called *Jaget eller Laget (Me or We in English)*. It targets the Nobel prize-winning Nash equilibrium and is a psychologically intriguing and strategic game. The goal of the game is two-fold and is an open question to the players: to optimize for yourself and try to win or to optimize for both of you and collaborate. However, collaboration is only beneficial for you if your co-player also collaborates, so trust and cheating become part of the game. The arithmetic exhibit (see Fig 1, right) called Nerdle (*Nördlig*) is a problem-solving puzzle where the challenge is to guess a hidden arithmetic computation like 12+24=36, within six guesses. The set of ten digits, four arithmetic operations and the equal sign is used to form an expression which must be a valid mathematical computation. Feedback is provided after each guess: a correct symbol at the correct position yield green, a correct symbol at the wrong position yield yellow and an incorrect symbol at any position yields red, see Fig 1. (right). This feedback is vital to have a fair chance to guess the solution since the number of possible combinations is immense. Both exhibits are iteratively developed, user-tested, and designed to inspire inquiry by providing hints and probing questions during gameplay.

Figure 1. The two mathematics exhibits; the Nash Equilibrium game (left) and the Nerdle game (right).
Photos by Universeum, www.universeum.se.

3.3 Method

The study is framed as a field experiment, where visitors' voluntary, unguided, and unscripted exhibit interactions and conversations with each other during gameplay are examined. The goal is to capture an authentic visit as much as possible, by minimizing the intrusion caused by the study. Participants are recruited from ordinary visitors, and their interactions are documented using video observations capturing discussions, gameplay, and reactions to hints and probing questions from each other or the exhibit design. The visitors' inquiry actions will be identified and categorized. The purpose is to understand and compare the two exhibits regarding the types of inquiries that emerge, how collaboration unfolds, and the role of scaffolding support and guidance in the inquiry process.

Participants were recruited in dyads or groups of three during regular opening hours by asking visitors that entered the Mathrix by their own choice and spent some time in the exhibition. The study coincided with an event for students in their last year of upper secondary school, regarding career options. A crowd of simultaneous visitors within the exhibition's target group of 13-18 years is unusual for the science center so the study benefitted from this opportunity.

The recruitment proceeded as follows. Couples or groups were approached by one of the researchers, who explained the purpose of the study. If they were willing to participate in the study, they were provided written information about the study (same as the verbal explanation) and were asked to fill in a consent form. We clearly informed that their interactions and conversations will be video-recorded and that the cameras were positioned to capture as little of them as possible, while still covering the exhibit interaction and the displays of interest. See Fig. 2 for the camera view of the recordings. There was information on how to withdraw their consent in case they change their minds in the future.

Figure 2. View of the video recordings; the Nash Equilibrium game (left), and the Nerdle game (right).
Photos by Universeum, www.universeum.se

After the recruitment procedure, participants were asked to play the chosen game as they normally would have, and for as long as they wish. The data produced were video recordings augmented with audio recordings to assure good sound quality. After the gameplay, the researcher shortly asked the players about the strategies used, and this mini-interview was also recorded. The strength of video-recorded data is that repeated views are possible and allow several analytical perspectives [16]. Audio-visual research is suitable for informal settings since it does not interfere with the actions (such as thinking-aloud protocols) and is an advocated method for science centers [17]. Repeated viewing makes it possible to explore how participants interact with each other and their embodied interaction with the exhibits and the information around the exhibit. The video-recorded data are examined focusing on three analytical perspectives:

1. *Type of inquiry: what types of inquiry arise during gameplay and what learning opportunities can be associated with these inquiries?*
2. *Form of collaboration: how the cooperation among players unfolds during gameplay, and whether an inquiry process is supported by their behaviours.*
3. *Role of scaffolding from the exhibits and from the co-players:* how the players utilize scaffolding such as explanations and guidance from co-player(s), and scaffolding such as instructions, feedback and hints from the exhibits.

There is an ethical aspect of using video recordings regarding ensuring anonymity and confidentiality. The study was conducted following the recommendations in the Rules and Guidelines for Research established by the Swedish Research Council. No participants under the age of 15 were recruited, and no sensitive personal information about the subjects was collected.

4   RESULTS

The below results are based on the video recording data produced in the field experiment and were analysed and discussed by two researchers (the first two authors).

4.1   Participation

Ten groups in total, five per exhibit, were recruited. Each group consisted of 2 or 3 persons. Two of these groups participated in both the Nash Equilibrium game and the Nerdle game experiments.

4.1.1   Exhibit 1: The Nash Equilibrium game

Group 1 consisted of two male students 2 from upper secondary school, who speak Swedish. Group 2 was a male-female couple in their thirties, also speaking Swedish. Group 3 was two students from upper secondary school, one male and one female, both Swedish speaking. Group 4 consisted of three students from upper secondary school, one female playing alone and a male and a female playing together. The 5th group was two females from Latvia, aged around 30-40 years. In total, there were 5 males and 6 females, so an even distribution despite the recruitment procedure.

These five groups played the Nash Equilibrium game 21 times, for a total of 40 minutes. The number of played games per group varied between 1 and 8, and the duration of play varied between 4 and 12 minutes yielding an average of 8 minutes of play and 4,2 games/group. There was a winning pattern across all groups: at most one tie and only victories to the same player in each group.

4.1.2   Exhibit 2: The Nerdle game

Group 1 consisted of a young English couple in their twenties. Group 2 was formed by three Turkish people, two males and one female in their thirties. The 3rd group consisted of three students in upper secondary school, one male and two females and is the same constellation as group 4 above. Group 4 was another young English couple in their twenties, one male and one female. Finally, group 5 consisted of the same Swedish couple in their thirties as Group 2 for the Nash game. In total 6 males and 6 females participated in the experiment.

The Nerdle game normally takes some time to play, and the 5 groups played in total for 70 minutes (average 14 minutes) and during that time they solved 9 puzzles altogether. Groups solved between 1 and 3 puzzles, which took between 8 and 18 minutes. Time varied; the 18-minute group solved 1 puzzle.

4.2 Types of inquiry

The analytic perspective of Inquiry investigates what types of inquiry the informal learning situation stimulates, by analysing the participants' reasoning and strategic thinking in the recorded conversations.

In the Nash Equilibrium game, the only challenge is strategic, since each round in the game is only to choose between a green button signalling collaboration (the "we"-choice), or a red button signalling competition (the "me"-choice). The scores of each round are determined by both players' choices and are hidden until both players have decided. The scores are set as follows: two green gives 2 points each, two reds give no points, one red and one green give 3 points to the player choosing red and -1 to the player choosing green. Hence both players can benefit if they collaborate simultaneously. But it is tempting to cheat the other player since it gives a 4-point advantage to the cheating party. This creates tension and psychological intrigue between the players. For more information on Nash Equilibrium, see https://en.wikipedia.org/wiki/Nash_equilibrium.

In the Nerdle game, the challenge is two-fold: to construct a proposal that is a possible solution at each row based on the current knowledge gathered from game feedback so far, and the strategy to limit the possible solution space by excluding digits and operators so that the solution is found within the game limits of six rows. To construct a possible solution a correct computation equation must be provided matching the positions on the row.

4.2.1   Exhibit 1: The Nash Equilibrium game

In Group 1, player 1 discovered a strategy rather soon. We refer to the strategy as a "distrustful winning strategy" since one player tries to convince the other that they should collaborate (play green), but then cheats the co-player and competes anyway (play red). If both players use this strategy they end up in a situation where no one loses but no one receives any points either, a type of deadlock which is called a Nash equilibrium. Player 1's distrustful winning strategy worked most of the time since Player 2 was more inclined to collaborate, as evidenced by 7 victories for Player 1. Their respective strategies were confirmed in the post-play interview, where player 2 stated "*I tried to trust him, but I couldn't*". Both players tried to maximize their points and win simultaneously, a goal not feasible for both.

In Group 2, player 2 quickly discovered the distrustful winning strategy but she tried hard to disguise her strategy. Player 1's strategy was more inclined towards collaboration and getting good points together. Player 1 stated: "If *I continue to play red, I will not win*". Player 2 admitted her distrustful strategy: "*I pretended I wanted to collaborate in the beginning, like we are buddies, and I will do as you want but then, in the end, I tried to win anyway*". She also succeeded with her strategy 3 of 4 games.

In Group 3, player 1 realized immediately that the game was the prisoners' dilemma and had a clear strategy from the start to compete. He was trying to convince Player 2 to collaborate throughout the game, using the distrustful winning strategy. Player 2 seemed not to really grasp what the game was about and mainly did what she was told, hence a rather uneven game. They played only one game in 7 minutes because player 2 had a hard time making up her mind about what to choose.

In Group 4, there were two participants acting as Player 2 and one acting as Player 1. The male player was very talkative and claimed that they would play collaborate but were not, i.e., the distrustful winning strategy. He explained the strategies he had discovered: "*If both play green all the time, we need one red to wi*n" and "*If we get ahead once, we can play red the remaining rounds to secure our victory*". This group inquire about general strategies and explores a lot and sometimes disagrees, e.g., the male argues: "*Now we are only trying to win, not collects points as well*", and his partner replies: "*Yes, a victory is a victory, it doesn't matter with how much*". In the post-play interview, they commented: "*To win, you only had to beat the other player once*" and "*If the goal was to collect as many points as possible, we should have collaborated more. But it was fun*".

In Group 5, there was not much talk perhaps because they were asked to speak English, not their mother tongue. Player 1 seemed to use the distrustful winning strategy enough to win but also to collaborate in between to distract/or cheat the other player to do the same. In the post-game interview, Player 1 explained: "*First I tried out, then I was trying to win*" and Player 2 repeated several times: "*and I lost*" but she appeared not to try winning.

To summarize, in all five groups one of the players discovered and used the distrustful winning strategy more or less explicitly, which was also confirmed in the post-game interviews. Their strategy was successful, and they won most of their games. Some groups raised more inquiries than others and explored different strategies, in particular Group 4. The three groups that were more even between the players, i.e., groups 1,2 and 4, seemed to enjoy the game more than the uneven games.

4.2.2 Exhibit 2: The Nerdle game

In Group 1 there was mainly one active player, and he solved three puzzles. In the first, he was experimenting a bit, in the second he tried the same equation as in the first and found they were different. He used the same strategy in all games. The uneven collaboration was confirmed: "*I solved all three*".

Group 2 first tested a random possible expression, and then evaluated the guess to proceed. They were making draft solutions in the current row, but if the proposed expression did not work, they erased all and started over. They applied a systematic and cautious strategy of searching for solutions with few errors but time-consuming. After the game, they looked at the game statistics and compared their number of guesses with everybody else. They expressed the puzzle was fun and were laughing a lot.

Group 3 started with a rather ad-hoc hypothesis: "*We must start with 12*" and then tried to solve the puzzle from there. Their strategy was to test as many digits and operators as possible in each row, and thereby limit the solution space for the next round. They failed a couple of times to create valid expressions but could adjust one number to make it correct. In the second game, they tried to simplify and use fewer digits and operators, which took longer time in the end. Hence, they combined an ad-hoc (and wrong) starting hypothesis and a solution-oriented strategy. After some time, they refined their strategy to explore fewer digits each turn.

Group 4 started with a random possible equation and based on the feedback from the game on the first and second rows they deliberately tried wrong solutions for the third and fourth rows just to exclude some digits and solved the puzzle on row 5. Hence, they applied a solution-oriented strategy to explore many digits (to limit the solutions space), that even overruled providing possible solutions every step.

Group 5 started by exploring the game "*Let's start by guessing a bit*", and then counted the rows and columns in the matrix. Their ambition was to exclude digits not part of the solution. They tried to plan ahead and only include the digits 0-4 in the first row. Then successively included more digits on the following rows and solved the puzzle after lots of thinking on the last row. Hence, their strategy was a solution-oriented strategy as group 4. They concluded: "*It was fun but hard, we are not so good at math*".

In summary, the game involves a local strategy for each row, and a global to solve the puzzle. The local strategy involves 1) a correct computation, and 2) utilizing feedback from previous rows. 3 out of 5 groups applied a local strategy to solve each row. 2 groups used a global strategy to limit the solution space from the start, and one group involved the global strategy later. One group deliberately violated 2) in favour of the global strategy, and one group had an ad-hoc premise that was unproductive. Groups that discussed, negotiated and planned more spent more time solving each puzzle.

4.3 Collaboration

The analytic perspective of collaboration seeks to investigate how the cooperation between players unfolded within the different groups and between the two games.

4.3.1 Exhibit 1: The Nash Equilibrium game

In Group 1, both players experiment in the beginning, then one player tries to convince the other to collaborate alone (he cheats), but the other adopts this behaviour. In the second game, both compete for most of the time. Then they start to discuss their strategies, that they know it is safe to play red, so they officially change the goal to collaboration: "*Play green now, then we get more points. Play green, just once*", but despite his pretended collaborative attempt the player still cheat, creating distrust. They learn from each other's strategies and adapt accordingly.

Group 2 talks a lot and explains verbally what they do. Player 2 claims she has a strategy. They experiment and discover what happens, and focus on interpreting the interface in the first game. They play again, experiment and switch between collaboration and competition, resulting in a tie next round. Then they explain the points rules out loud, which they can see on the large display above them. Player 2 wants to win and is cheating sometimes, and she gets very pleased when she wins.

In Group 3, Player 1 is trying hard to convince Player 2 to collaborate while he competes himself. Player 2 doesn't seem to be able to focus on the game, as she is split between the play partner and a friend standing beside wanting them to leave. An uneven game with little collaboration and probably little learning as player 1 already knew the Nash Equilibrium beforehand and player 2 was unfocused.

In Group 4 they play two against one. Both players start collaborating, and then both compete. Then player 1 alternates, while player 2 still competes. Both players alternate during the next games, but

player 2 is more inclined to compete and chose red a lot. Still, player 2 tries to negotiate to collaborate, but both players are distrustful and cheat the other. The group is rather even gameplaywise, and there is a lot of reasoning out loud concerning interesting and insightful strategies.

Group 5 also experimented with red and green choices, and the first game ended up a tie. In the second game player 2 mainly collaborates and player 1 wins making player 2 surprised. In the third game, player 1 won again by alternately between red and green but she made sure she was ahead. She had a clear intention to win, and it was a rather uneven game.

To summarize: two groups were rather uneven, played fewer games and seemed less engaged in the gameplay than the other groups. This can be interpreted as the psychological intrigue relying on equally strong players to be a rewarding experience. The other three groups were even in the ways they experimented and discovered strategies, tried to outsmart each other and searched for winning strategies. One player was always more successful—the player who discovered a winning strategy first.

4.3.2 Exhibit 2: The Nerdle game

In Group 1 the cooperation was uneven, the male played alone only mumbling to himself first, successively he started explaining to his partner more. During the second game, she makes suggestions, but he is still in the lead and solves the puzzle solitary and she goes "Wow!". He solves the game a third time and she is impressed. One player is clearly scaffolding the other, which may result in learning opportunities for both players according to the learning-by-teaching paradigm.

Group 2 apply a group consensus strategy to cooperation: One person makes a guess, and then all three participants negotiate and agree on the next row. They discuss every input carefully, and everybody needs to agree before they press the guess button. There are lots of constructive discussions in this group. There are equal contributions, constructive discussions and consensus group collaboration which is a good ground for learning opportunities.

In Group 3, all three participants found the game fun but hard and they talked a lot about every step. All participants contributed equally, and they laughed at their mistakes. The group was enthusiastic when solving the first puzzle and relieved when solving the second. A collaboration based on equal contributions, constructive discussions, and a positive spirit has a high potential to create learning opportunities.

Group 4 only solved one puzzle. They started out with the female interacting with the exhibit, but they switched after a while when she had trouble finding the solution and the male took over. The cooperation started out equal but turned into an unequal contribution in the end.

Group 5 Discussed livelily and collaborated on every step, an equal collaboration.

To summarize: 3 out of 5 groups collaborated, discussed, and negotiated continuously to solve the puzzle. The other two groups (Groups 1 and 4) collaborated unequally, where one person was leading the work and the other was more of a bystander. The uneven groups appeared slightly less engaged.

4.4 Scaffolding

The analytic perspective of scaffolding intends to gain insight into exhibit design and to investigate the scaffolding that emerges within the visitor groups. It is well known from previous science center research that parents scaffold their children during visits, and we expect that also teenagers and adults with different knowledge and skills can scaffold each other successfully.

4.4.1 Exhibit 1: The Nash Equilibrium game

In the first group, there was no apparent use of the game's hints and guidance that was observable from the video camera view, but they may have used these without our awareness. It was evident that both players understood the idea of the game anyway, according to the way they played and their comments. No scaffolding between the players occurred.

In Group 2, the players noticed and acted on the game-playing hints provided by the game; Player 2 often reads them out loud. A misunderstanding occurred regarding the number of games won by each player which is displayed on the large screen on the wall, the reason being that the game was not restarted but accumulated results from a previous game session.

Group 3 reads the instructions out loud; green for collaboration and red for competing. Player 1 explains the point rules to Player 2.

In Group 4 there were many discussions that can function as scaffolding, since the participants explains and interprets the instructions together, for example, all three are involved in interpreting the point system. The male participant explains all along how he thinks, and explicitly inquires to find general strategies. After 3-4 games they also read from the rather extensive amount of information on the wall.

The only indication that Group 5 noticed the hints, was when Player 2 asked about what distrust means, a word used in one of the hints. Neither player talked very much nor explained anything to the other, perhaps because they were uncomfortable speaking in English.

In summary, no scaffolding from the researcher was needed. Participants seem to read the hints and instructions in three groups, and the players clearly explain rules and/or strategies to each other.

4.4.2   Exhibit 2: The Nerdle game

Three groups understood the instructions without help, the other two with a little help from the researcher: "This is like wordle but with numbers". In the first group, one player clearly scaffolded the other, but in the other groups, the conversations were more of mutual and conversational character.

5   CONCLUSIONS

Our findings show that all groups were involved in mathematical inquiry about winning strategies or strategies to solve a puzzle. In the Nash Equilibrium game, at least one of the players in each group discovered the "distrustful winning strategy" and some groups discovered several other strategies. In the Nerdle puzzle, all groups discovered local strategies to propose mathematically correct equations and most groups discovered global strategies to limit the solution space. These types of strategy discoveries originate from inquiries. Collaboration was essential and a balance of equal contributions seemed positive for both engagement and enjoyment and created more discussions, known to be favourable for learning. However, constellations of visitor groups are beyond our control in informal settings, and hence the exhibits should be designed for unequal collaborations as well. Regarding scaffolding, the game hints and feedback seemed essential to inject inspiration for inquiry, as was the scaffolding provided by other group members.

Despite the well-known Hawthorne effect that people try harder when observed, our results are still promising concerning inquiry-based informal mathematics learning being feasible in a science center context. However, the exhibits must inspire inquiry and bring opportunities to collaborate, explore and discuss mathematical strategies. We argue that games are suitable candidates for inquiry-based informal learning as they can provide relevant rules and immediate feedback to allow for self-regulated behaviours crucial for self-directed learning, in addition to being engaging and fun. Being a small-scale study, these results are preliminary and should be validated in an extensive study with more exhibits and a larger population of visitors, which is future work.

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