A technical note on modelling the effect of source motion on amplitude and Doppler shift using resampling

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Abstract

Modelling of the sound field due to a simple monopole source moving at subsonic speed is investigated. It is shown that both the Doppler shift and the influence of the Doppler factor on the amplitude and the directivity can be modelled by a single resampling process demanding area conservation. This is the main novel contribution of the paper. Here, the Doppler factor is defined as $(1 - u_r/c)^{-1}$, where u_r is the component of the moving source velocity in the direction of the receiver and c is the sound speed. The paper may contribute to an increased understanding of the reasons for the Doppler factor in relation to higher order convective factors.

Keywords: Moving sound sources, Doppler factor, Resampling

1 Introduction

For an acoustic source in motion relative to a listener position, the well-known Doppler shift may be perceived by the listener as a time-varying change in pitch; for a tonal source, such as an ambulance siren, the pitch is higher when the ambulance is approaching and lower when it has passed and is retreating. There is also an alteration of the amplitude due to the motion, which amplifies an approaching source and weakens a retreating source. The amplification comes in addition to the distance depending effects, e.g., of spherical spreading and air attenuation, and it is linked with the same Doppler factor as governs the Doppler shift.

Here, the sound field due to an omni-directional, simple monopole source moving at subsonic speed is investigated in a theoretical modelling approach. The topic has been dealt with in previous literature (e.g. [1, 2, 3, 4, 5, 6, 7]) and the finding of the current paper concerns the link between the Doppler amplification factor and a resampling process that uses area conservation, at the same time as the Doppler shift is captured. The used theory is described in the next section whereafter implementation, results, and conclusion follow.

2 Theory

The theoretical description of the current paper largely follows the notation of [4, Ch. 9.1] and starts by repeating the main elements of the theory, for clarity. The equation for an acoustic pressure field in 3D space and time, $p(\mathbf{x}, t)$, caused by a source of strength $s(\mathbf{x}, t)$, can be written as

$$p(\mathbf{x}, t) = \int \frac{s(\mathbf{y}, \tau)}{4\pi|\mathbf{x} - \mathbf{y}|} \mathrm{d}^3\mathbf{y} \tag{1}$$

or, equivalently, using a delta function, as

$$p(\mathbf{x}, t) = \int \frac{s(\mathbf{y}, \tau)}{4\pi|\mathbf{x} - \mathbf{y}|} \delta(t - \tau - |\mathbf{x} - \mathbf{y}|/c) \mathrm{d}^3\mathbf{y} \mathrm{d}\tau \tag{2}$$

where $|\mathbf{x} - \mathbf{y}|$ is the distance between the source (at $\mathbf{x}$) and the receiver (at $\mathbf{y}$) at time τ, which is the emission time of the source, often called the retarded time; $\tau = t - |\mathbf{x} - \mathbf{y}|/c$, where t is the time of the received pressure (the reception time) and c is the speed of sound in the medium. Supposing that the source is concentrated at a single moving point, $\mathbf{x} = \mathbf{x}_s(t)$, and rewriting the source strength as $s(\mathbf{x}, t) = S(t)\delta(\mathbf{x} - \mathbf{x}_s(t))$, the sound field can be written

$$p(\mathbf{x}, t) = \int \frac{S(t)\delta(\mathbf{x} - \mathbf{x}_s(\tau))\delta(t - \tau - |\mathbf{x} - \mathbf{y}|/c)}{4\pi|\mathbf{x} - \mathbf{y}|} \mathrm{d}^3\mathbf{y} \mathrm{d}\tau. \tag{3}$$

Using the delta function $\delta(\mathbf{x} - \mathbf{x}_s(\tau))$ to evaluate the integral over space gives

$$p(\mathbf{x}, t) = \int_{-\infty}^{\infty} \frac{S(\tau)\delta(t - \tau - |\mathbf{x} - \mathbf{x}_s(\tau)|/c)}{4\pi|\mathbf{x} - \mathbf{x}_s(\tau)|} \mathrm{d}\tau. \tag{4}$$

The next step, using the remaining delta function, $\delta(t - \tau - |\mathbf{x} - \mathbf{x}_s(\tau)|/c)$, to evaluate the integral over time, is less simple since the argument of the delta function is a more involved function of τ, i.e. $\delta(g(\tau))$, where $g(\tau) = t - \tau - |\mathbf{x} - \mathbf{x}_s(\tau)|/c$, which results in a factor $|\mathrm{d}g/\mathrm{d}\tau|^{-1}$ [4],

$$p(\mathbf{x}, t) = \frac{S(\tau^*)}{4\pi|\mathbf{x} - \mathbf{x}_s(\tau^*)|} \left| \frac{\mathrm{d}g}{\mathrm{d}\tau} \right|^{-1} \tag{5}$$

where the emission time, now denoted τ^*, is a zero of $g(\tau) = 0$, i.e. fulfilling $t - \tau^* - |\mathbf{x} - \mathbf{x}_s(\tau^*)|/c = 0$. (It should be noted that for supersonic motion, there may be multiple zeros of $g(\tau)$ resulting in multiple terms on the right hand side of Eq. (5) [4].) Performing the derivative $\mathrm{d}g/\mathrm{d}\tau$ can be shown to result in

$$\left|\frac{\mathrm{d}g}{\mathrm{d}\tau}\right| = |1 - M_r| \tag{6}$$

where M_r is the Mach number in the direction of the receiver, i.e. $M_r = u_r/c$ where u_r is the component of the source velocity in the direction of the receiver. We can define this velocity component as $u_r = u\cos\theta$ where u is the speed of the source and θ is the angle to the receiver from the direction of the source [4], as depicted in Figure 1. Inserting Eq. (6) in Eq. (5) shows the resulting Doppler effect on the amplitude:

$$p(\mathbf{x}, t) = \frac{S(\tau^*)}{4\pi|\mathbf{x} - \mathbf{x}_s(\tau^*)||1 - M_r|} = \frac{S(\tau^*)}{4\pi|\mathbf{x} - \mathbf{x}_s(\tau^*)|\left|1 - \frac{u}{c}\cos\theta\right|}. \tag{7}$$

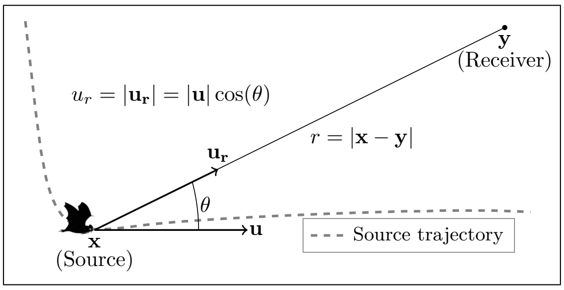

Figure 1: Geometry of source at $\mathbf{x}$ moving with velocity $\mathbf{u}$ and stationary receiver at $\mathbf{y}$ in 3D space.

To study the effect of source motion on the frequency of the sound at the receiver, one may look at the ratio of the received pressure and its time derivative [4]. For a stationary situation, with a non-moving source, assuming $S(t) = S_0\exp(\mathrm{j}\omega t)$, the ratio $p'(t)/p(t)$ equals $\mathrm{j}\omega$. On the other hand, for a moving source, the time derivative of $S(\tau^*)$ produces an inner derivative $\mathrm{d}\tau^*/\mathrm{d}t$ that can be shown to equal $(1 - M_r)^{-1}$ [4]. This causes a shift of the frequency from ω to $\omega/(1 - M_r)$, which is the well-known Doppler shift. It should be noted that this frequency estimate is a first-order approximation valid for $u/\omega r(1 - M_r) \ll 1$, where r is the source–receiver distance [4], which could be seen as a low-speed or far-field approximation. In [5, Chapter 14] the Doppler shift due to $\mathrm{d}\tau^*/\mathrm{d}t = (1 - M_r)^{-1}$ is derived without assuming uniform motion, i.e. the shown Doppler shift is generally valid for source motion with varying speed and direction. This is due to that the essential element of the Doppler shift comes from the varying difference between the emission time, τ^*, and the reception time, t,

$$t - \tau^* = |\mathbf{x} - \mathbf{x}_s(\tau^*)|/c \tag{8}$$

which is constant only for $|\mathbf{x} - \mathbf{x}_s(\tau^*)|/c = const.$, i.e. only for a fixed source position (under the assumptions of fixed receiver position and constant sound speed).

3 Implementation and results

From a signal processing point of view, e.g., when synthesising a sound coming from a moving source, the audio signal may be a discretised version of the sound pressure $p(t)$, as derived above, by using a fixed sample rate. That is, the audio signal is given at discrete time steps, $t_1 = 0$, $t_2 = \Delta t$, $t_3 = 2\Delta t$, ..., $t_N = (N-1)\Delta t$, where Δt is a constant. With this starting point, it is needed, for each discrete time step, t_i $(i = 1 \ldots N)$, to find the emission time τ_i^* fulfilling Eq. (8), i.e.,

$$\tau_i^* = t_i - r_i/c \tag{9}$$

where $r_i = |\mathbf{x} - \mathbf{x}_s(\tau_i^*)|$. Assuming that we know the trajectory of the source, the values of r_i $(i = 1 \ldots N)$, can be pre-calculated and used for resampling of the source signal. Thereby, the capturing of the Doppler shift in $p(t_i)$ can be translated into a sampling of the source signal S at time steps $t_i - r_i/c$.

Such a sampling process is exemplified in Figure 2 (upper plot) for a source that moves toward the receiver. Looking at for instance the fourth sample of the source signal (see arrows in plot), it is seen that the resampling (here made using linear interpolation) is later in time compared with the original sampling; since the source has moved closer to the receiver compared with the previous sample, the emission time of the sample will be later. The lower plot in Figure 2 shows the result of the resampling process, where the resampled values of the source signal are moved to the prescribed time instances of the fixed sample rate. Looking at a whole period of the signal, it can be seen that it is shortened as a result of the resampling process (here, by about 1.5 samples), i.e. the pitch is increased, as expected for a source that moves toward the receiver. (The calculated increase in pitch, given by the factor $(1 - u/c)^{-1}$, is about 8 % for this example.)

The sampling process described above only shifts the chosen samples in time. Thereby the time shift due to the moving source is captured, and hence also the change in pitch. However, the change in amplitude is not captured in the process. In order to find a way to model the amplitude change, we can start by noticing that a time step of the emission, i.e. $\tau_{i+1}^* - \tau_i^*$, does not equal the time step at the receiver, Δt, when the source is moving toward or away from the receiver. This is exemplified in Figure 3, again for the fourth sample, but here zoomed in. One can see that the time step $\tau_5^* - \tau_4^*$, i.e. the width of the red rectangle, is larger than the time step $t_5 - t_4 = \Delta t$, i.e. the width of the blue rectangle. The length of the time step is governed by $\mathrm{d}\tau^*/\mathrm{d}t$, so an area conserving resampling, capturing that the red rectangle is wider than the blue rectangle in Figure 3, should use a multiplying factor of $\mathrm{d}\tau^*/\mathrm{d}t = (1 - M_r)^{-1}$ on the amplitude. (The effect of this is included in the lower plot of Figure 3.) Hence, the factor for an area conserving resampling is identical to the amplitude Doppler factor known from literature, i.e., $(1 - M_r)^{-1}$, as e.g. in Eq. (7). (The area conservation is like the rectangle method for numerical integration, or Riemann integral, in that, for a well-behaved function like an acoustic signal, the error can be made arbitrarily small by reducing the integration step, i.e. here the sampling time step.)

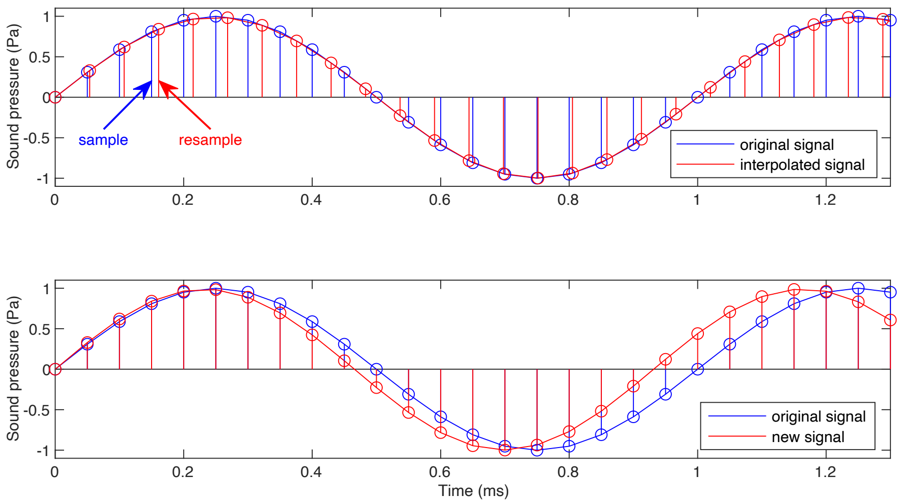

Figure 2: Example of capturing Doppler shift via resampling of source signal. Here, the source moves straight toward the receiver at 25 m/s (90 km/h), starting from a distance of $R_0 = 100$ m. The source emits sound at a single frequency of 1 kHz and the sample rate is 20 kHz. The constant amplitude of the source signal is chosen to produce a received pressure amplitude of 1 Pa at the start position of the source. Before the sampling, the total propagation delay of R_0/c is omitted from the received signal. A sound speed of 340 m/s is used here.

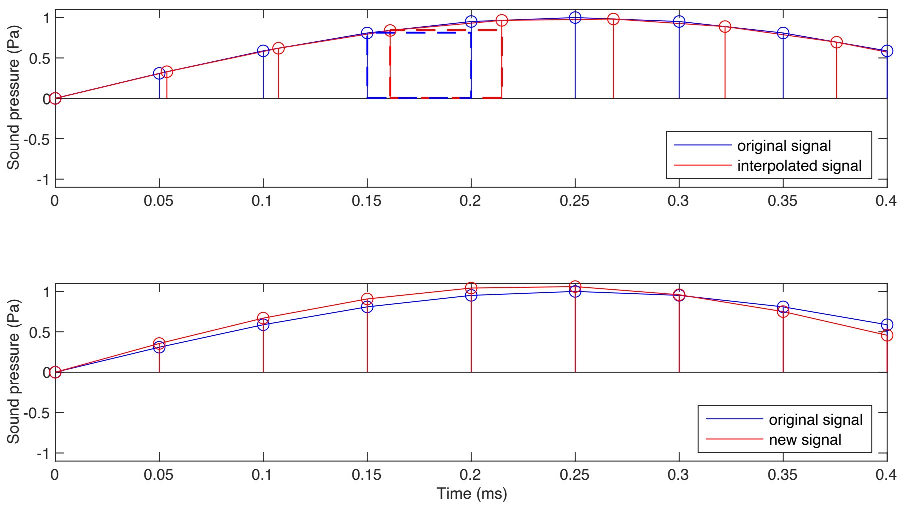

Figure 3: Zoomed version of Figure 2 with amplitude corrected in the resampling.

4 Discussion

For the sound field of a moving source at subsonic speed, it is shown here that a resampling process using area conservation gives the correct Doppler factor on the amplitude (and hence on the directivity), in addition to the Doppler shift, which is the main result of the paper. The area conservation may be seen as a conservation of momentum. It could also be noted that the results of the present paper are valid for a source moving with varying speed and direction even though derivations of the Doppler effects presented in literature often assume uniform motion of the source.

The Doppler factor on the amplitude can be seen as coming from the derivative $|\mathrm{d}g/\mathrm{d}\tau|$ in Eq. (5), originating from the non-constant argument of the delta function $\delta(t - \tau - |\mathbf{x} - \mathbf{x}_s(\tau)|/c)$ when the source location, $\mathbf{x}_s$, is not stationary. This factor, i.e. $(1 - M_r)^{-1}$, can be seen as essential and unavoidable for any moving source. The source description used here, i.e. $s(\mathbf{x}, t) = S(t)\delta(\mathbf{x} - \mathbf{x}_s(t))$, sometimes called *simple source*, is simplistic in that it does not model any physics based source type. However, the physics based *mass source* and *force source* (often denoted as monopole and dipole, respectively) also contain the same Doppler factor on the amplitude, as shown e.g. in [5], where the source term has a time derivative for the mass source and a spatial derivative for the force source, which results in further factors on the amplitude.

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