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Editors' Suggestion

Single-Shot Measurements of Phonon Number States Using the Autler-Townes Effect

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We present a single-shot method to measure motional states in the number basis. The technique can be applied to systems with at least three nondegenerate energy levels which can be coupled to a linear quantum harmonic oscillator. The method relies on probing an Autler-Townes splitting that arises when a phonon-number changing transition is strongly coupled. We demonstrate the method using a single trapped ion and show that it may be used in a nondemolition fashion to prepare phonon number states. We also show how the Autler-Townes splitting can be used to measure phonon number distributions.

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The common approach to trapped ion quantum information processing is to use electronic states to store information while the motional modes shared by a chain of ions enable entangling operations [1]. However, the motional modes can play a more active role. For instance, the motional degree of freedom can be used for storing quantum information [2], allowing for continuous variable quantum information processing with trapped ions. The motional modes are also a very important tool in quantum logic spectroscopy [3], which enables the implementation of precise atomic clocks [4]. Also, in metrology an advantage can be achieved via nonclassical states of ion motion [5–7]. On the more fundamental side, trapped ion motion acts as the working medium in studies of quantum thermodynamics [8–10]. Investigations of the dynamics of phonon pair creation upon trap potential changes are a possibility to simulate particle creation, thus creating a link between quantum information processing and cosmology [11]. Finally, the measurements of local phonons and their tracking enables quantum simulations in motional degrees of freedom [12,13].

Trapped ion motion can be measured in various ways [8,12,14–19], including via cross-Kerr nonlinearity [18,20,21] and composite pulse sequences [12]. There are also schemes using rapid adiabatic passage (RAP) [22,23] and stimulated Raman adiabatic passage (STIRAP) [24] sequences or polychromatic amplitude modulated beams [25]. However, most detection methods destroy the state throughout the measurement process.

Schemes for resolving Fock states in a nondemolition manner have been proposed and measured using the dispersive ac Stark shift on Rydberg atoms interacting with photons in a cavity [26,27]. Here, the measurement relies on a Ramsey type experiment to measure the phase shift introduced by the cavity photons. The same dispersive interaction has been used in superconducting circuits [28,29], where a qubit is coupled to the mode of a microwave cavity in a strong dispersive interaction regime. The ac Stark shift resulting from the coupling splits the qubit spectrum, turning it into an anharmonic ladder where the dressed states energies are proportional to the dispersive coupling rate and, consequently, to the number of photons in the system.

In this work, we introduce a novel technique that is based on the Autler-Townes effect [30] and can measure in a single shot a motional mode in the number (Fock) basis. Our approach also allows for quantum non-demolition measurement of motional Fock states and can be used to determine the phonon distribution. We demonstrate this method using a single $^{88}Sr^{+}$ ion trapped in a linear Paul trap.

The Autler-Townes effect is commonly probed in three-level systems and in this work we denote these levels as $|S\rangle$, $|D\rangle$, and $|D'\rangle$. The coupling of the two levels, $|S\rangle$ and $|D'\rangle$, via a laser field is described by the following Hamiltonian

$$H = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega_C \\ \Omega_C & 2\Delta_C \end{pmatrix}, \tag{1}$$

where Ω_C is the coupling strength and $\Delta_C = \omega_L - \omega_0$ is the detuning of the laser field frequency ω_L from the transition

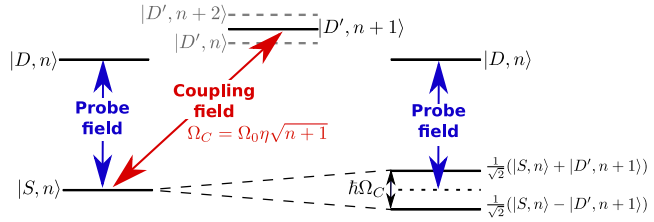

FIG. 1. Autler-Townes scheme for phonon number detection. (left) The $|D, n\rangle \leftrightarrow |S, n\rangle$ resonance is weakly probed while levels $|S, n\rangle$ and $|D', n+1\rangle$ are strongly coupled via a laser on a blue sideband transition. (right) The system described in terms of dressed states. The splitting between the dressed states reveals the $|S, n\rangle \leftrightarrow |D', n+1\rangle$ coupling strength Ω_C. The splitting behavior is similar when coupling to a red sideband transition.

resonance ω_0. When the laser field is resonant ($\Delta_C = 0$) the dressed eigenstates with eigenenergies $\pm\hbar\Omega_C/2$ are $(|S\rangle \pm |D'\rangle)/\sqrt{2}$, respectively.

These two eigenstates can be probed from the third level $|D\rangle$. Namely, if level $|S\rangle$ is strongly coupled to the level $|D'\rangle$ the spectral resonance for the transition from level $|D\rangle$ to level $|S\rangle$ is split into a doublet. By driving the system resonantly to one of the doublet peaks at $\pm\hbar\Omega_C/2$ it is possible to excite the respective dressed eigenstate $(|S\rangle \pm |D'\rangle)/\sqrt{2}$. This is called the Autler-Townes effect [30].

Here, we show that if the internal levels are coupled to a motional degree of freedom the Autler-Townes effect can be used to probe motional number states. When $|S, n\rangle \leftrightarrow |D', n \pm 1\rangle$ is a phonon-number-changing transition, with n the respective phonon number states, Ω_C becomes sensitive to the population of the motional mode [31], as it is depicted in Fig. 1. The splitting between the Autler-Townes doublet peaks is proportional to the strength of the $|S, n\rangle \leftrightarrow |D', n \pm 1\rangle$ coupling strength, $\Omega_C(n)$. This allows us to use the Autler-Townes doublet to probe the phonon number. In Fig. 1 the Autler-Townes effect is plotted for the coupling to the blue sideband transition (BSB) of a motional mode, where the coupling Ω_C scales with $\sqrt{n+1}$. An analog scheme can be used for a red sideband transition (RSB) with a scaling of $\sqrt{n}$.

We demonstrated this phonon number-dependent Autler Townes effect by preparing a single trapped $^{88}Sr^+$ ion in electronic state $|D, n\rangle$, where n describes the number state of one of the radial modes. For the measurements presented we employed states $|D\rangle \equiv 4^2D_{5/2}$, $m_J = -\frac{1}{2}$, $|S\rangle \equiv 5^2S_{1/2}$, $m_J = -\frac{1}{2}$ and $|D'\rangle \equiv 4^2D_{5/2}$, $m_J = -\frac{3}{2}$. We probed the $|D, n\rangle \leftrightarrow |S, n\rangle$ carrier resonance while strongly coupling the BSB transition, $|S, n\rangle \leftrightarrow |D', n+1\rangle$. The resultant spectrum displays an Autler-Townes doublet with a phonon-number dependent splitting, see Fig. 2(a).

When applying the strong coupling field to a RSB transition $|S, n\rangle$ to $|D', n-1\rangle$, the resultant spectrum also displays an Autler-Townes doublet, however, with a different coupling dependent scaling than the BSB transition, see

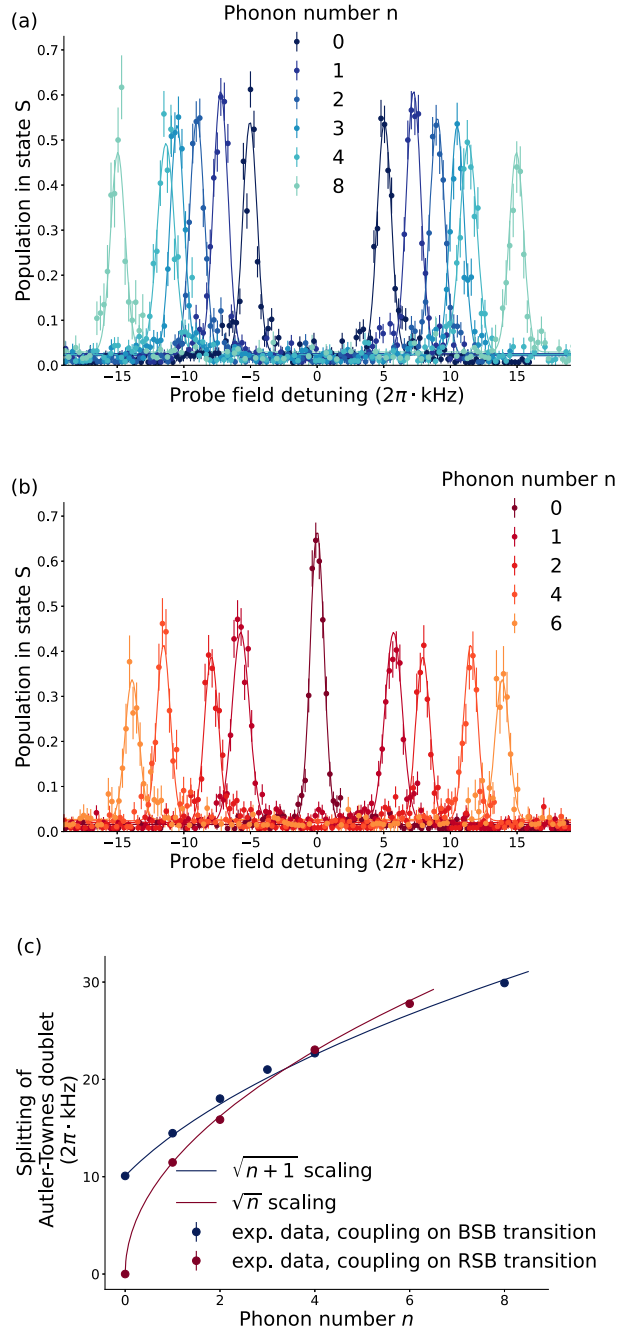

FIG. 2. Splitting of the Autler-Townes doublet. When the coupling field is resonant to a phonon-number-changing transition the splitting of the Autler-Townes doublet depends on the number of phonons in the system. (a) For the coupling field resonant to a BSB transition the splitting scales with the phonon number n as $\sqrt{n+1}$. (b) For the coupling field resonant to a RSB transition the splitting scales with $\sqrt{n}$. Error bars represent quantum projection noise (68% confidence intervals). The lines are fits to the data with the amplitude and the splitting as a fitting parameters. (c) Splitting of the Autler-Townes doublet extracted from the fitting curves in (a) and (b). The scalings of the splitting are described by $\sqrt{n+1}$ (BSB transition) and $\sqrt{n}$ (RSB transition), respectively. Note that the phonon number presented only represents a subset and not all the consecutive Fock states.

Fig. 2(b). For both experiments the peak height of the Autler-Townes doublet decreases with an increasing phonon number because the coupling strength of the probe field changes proportional to $(1-\eta^2 n)$.

We extract the doublet splittings by fitting the data presented in Figs. 2(a) and 2(b). The fit confirms the respective $\sqrt{n+1}$ and $\sqrt{n}$ scalings, as shown in Fig. 2(c). The scalings of the doublet splittings are consistent with the strengths of the phonon-number changing transitions for laser-ion interaction in the Lamb-Dicke regime [1]. For both Jaynes-Cummings type interactions, the coupling to the RSB and the BSB transition, the scaling of the splitting is proportional to the initial coupling strength Ω_0. In Fig. 2, the splitting of the RSB takes larger values for increasing phonon number because of the higher value of Ω_0 for this scan.

The phonon-state dependent Autler-Townes splitting can be used to detect the occupation of a particular phonon number state with almost unit efficiency. The phonon detection pulse sequence is shown in Fig. 3(a). To prepare phonon number states we cooled the ion to its motional ground state and initialized the state $|S, n=0\rangle$ before iteratively adding phonons one by one. The phonon addition is performed by applying π pulses first on the BSB transition to state $|D, n+1\rangle$ and then on the carrier transition back to state $|S, n+1\rangle$, as described in [14]. To achieve highly efficient preparation of phonon number states we included state-dependent fluorescence detection steps, performed on the $|S\rangle$ to $|f\rangle \equiv 5^2P_{1/2}$ transition, after the BSB π pulse and before the next carrier pulse [32]. Absence of fluorescence indicates successful addition of a phonon.

Before applying the phonon detection sequence, we used a π pulse to initialize the ion in state $|D, n\rangle$. A π pulse then drives the transition to one of the Autler-Townes doublet states for the phonon number state to be detected. Other phonon number states are off resonant and will not be excited. At the end of the Autler-Townes phonon detection sequence the ion is in a superposition of the states $|S, n\rangle$ and $|D', n+1\rangle$, however, the fluorescence detection only shows a signal if the ion is in the S manifold. Therefore, to increase the signal strength from the S manifold at the end of the phonon detection sequence as a transfer step a π pulse was applied on the transition from $|D', n+1\rangle$ to $|S', n+1\rangle$ (with $|S'\rangle \equiv 5^2S_{1/2}$, $m_J = \frac{1}{2}$). Fluorescence detection was used to distinguish population in $|S\rangle$ and $|S'\rangle$ from population in $|D\rangle$ and $|D'\rangle$.

Using this pulse scheme [Fig. 3(a)], the ions's motional state can be detected by probing discrete frequencies corresponding to the phonon-number-dependent peak position. We prepared and probed the ion in different phonon number states, ranging from 0 and 8, as shown in Fig. 3(b). If the probe field is not resonant to the probed phonon-dependent Autler-Townes transition, the ion remains undisturbed in $|D, n\rangle$. If the probe field drives the

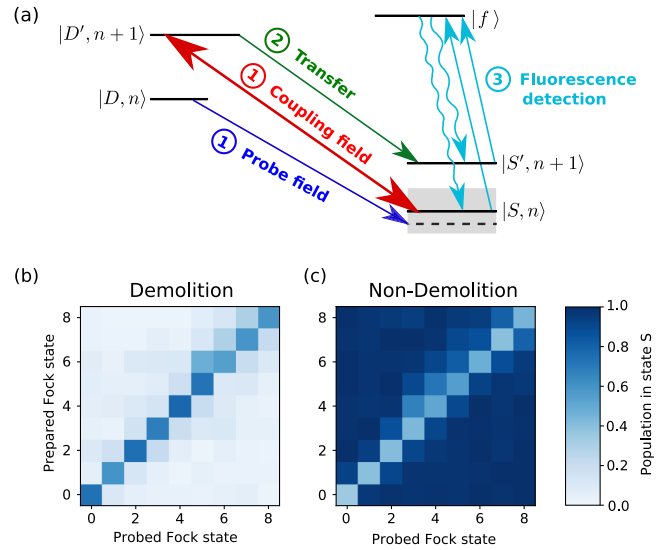

FIG. 3. (a) Measurement sequence: The ion is initialized in $|D, n\rangle$. In the first step, a π pulse is applied on the $|D\rangle \leftrightarrow |S\rangle$ carrier transition while a phonon-number-changing transition is strongly coupled (in the figure a BSB transition is shown). The probe frequency is set to an Autler-Townes peak resonance, such that the $|D\rangle \rightarrow |S\rangle$ transfer only occurs if the prepared Fock state $|n\rangle$ equals the probed Fock state $|m\rangle$. In the second step, any population in $|D', n+1\rangle$ is transferred to $|S', n\rangle$ before both S states are coupled to the fluorescing state $|f\rangle$. Finally, detection of fluorescence indicates $n = m$. (b) Experimental demonstration of the sequence in (a). The ion was prepared in different Fock states, and when the probed Fock state matched the prepared one, fluorescence was detected. (c) Experimental results when the transfer step is instead applied to the $|S, n\rangle \leftrightarrow |D, n\rangle$ carrier transition, moving all the population into the D manifold, and enabling a nondestructive measurement of the ion motion in the Fock basis.

phonon-dependent Autler-Townes transition the final state is a superposition of the states $|S, n\rangle$ and $|S', n+1\rangle$. This outcome is detected by fluorescence. However, fluorescence detection involves the scattering of many photons and, therefore, changes the motional state, corresponding to a destructive (demolition) detection of the phonon state. The method may be repeated (without previous reinitialization) to test for different phonon numbers sequentially until a positive result is achieved. Each particular test relies on *a priori* knowledge of the expected peak positions. These can be determined from spectra as in Fig. 2 or by measuring the frequency of Rabi oscillations on the BSB transition for a ground-state-cooled ion. Off-diagonal excitation in Fig. 3 can be explained by laser intensity fluctuations resulting in ac Stark shift on the Autler-Townes doublets, or imperfect Fock state preparation.

In the method described above the nature of the detection scheme prevents any recovery of the previous state of the system once the correct phonon number state has been identified. By changing the transfer step [Fig. 3(a)] to a π pulse on the $|S\rangle \leftrightarrow |D\rangle$ carrier transition, the absence of

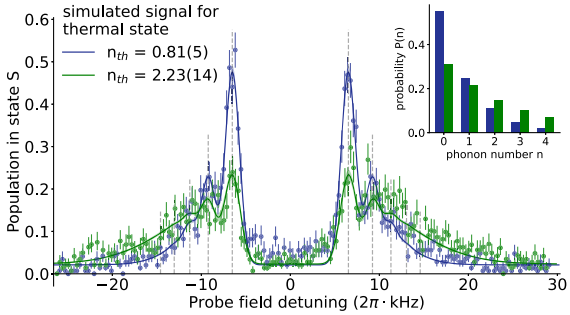

FIG. 4. The Autler-Townes splitting shows multiple peaks for a thermal distribution of the ion motion. The peak positions (indicated by dashed lines) are defined by the phonon number and the amplitudes by the population probability of the respective phonon number. By scanning the Autler-Townes spectrum the thermal distribution of the ion motion can be obtained.

fluorescence indicates a positive result, as shown by the experimental data in Fig. 3(c) and this particular Fock state is detected without heating the ion motion. In case the probed and prepared Fock state mismatch, the population will stay in $|D, n\rangle$ during the coupling sequence and the transfer pulse will induce the transition from $|D\rangle \leftrightarrow |S\rangle$, resulting in fluorescence and motional heating. This alternate method enacts a nondemolition measurement in the phonon-number basis. The final state of the system after detection is therefore a superposition of $|D, n\rangle$ and $|D', n+1\rangle$. From here the ion could be prepared in the measured phonon number state using the scheme presented in the Supplemental Material [33].

When the ion is prepared in a thermal state, the ion's motional state is a distribution of phonon number states. Each phonon number leads to a different splitting for the Autler-Townes doublet. Therefore, the thermal state manifests as a set of peaks at the respective phonon coupling strengths. The amplitudes of the different Autler-Townes peaks can be used to determine the probability of the respective phonon number to be occupied, and thus the phonon distribution of the thermal state can be characterized as shown in Fig. 4.

The thermal distribution in Fig. 4 was created by shortening the cooling cycle before the Autler-Townes splitting scan, such that the ion did not reach its motional ground state. With shorter cooling cycles, the ion is more likely to populate higher number states, leading to a thermal state ρ_{th} with higher average phonon number. To determine the temperature, 24 Gaussian peaks on each side, each corresponding to one Autler-Townes doublet, were fitted to the spectrum. The positions are fixed to the expected splittings and the amplitudes are chosen to be proportional to the populations of the individual phonon number states for a thermal state $P(n) = \langle n|\rho_{\mathrm{th}}|n\rangle$. The first five peak positions are indicated by gray dashed lines, with the corresponding phonon number state populations shown in the inset.

The resolving power of our technique is determined by the resonance linewidths and by the splitting between resonances. The splitting between neighboring resonances decreases as $\sim n^{-1/2}$, making the technique less powerful for larger n. This drawback is common to other phonon measurement methods [8,12,14–19]. Larger splittings between resonances can be achieved by using a stronger coupling field, though this can also cause a higher background signal due to unwanted excitation of other levels. Furthermore, as the strength of the coupling field is increased, the ac-Stark shifts increase due to coupling to other levels and coupling field intensity fluctuations can cause broadening of the resonances [34]. The ac Stark shift can be compensated by adding an off-resonant field with the opposite detuning from the resonance [35].

The spectral linewidth may be limited by the laser linewidth, magnetic field noise or Fourier broadening. Longer probe times are required to reduce Fourier broadening, but this comes at the expense of increasing the sensitivity to anomalous heating which changes the ion's motional state [36].

A further point to note is that the first-order description of the coupling strength scaling $\sim\sqrt{n+1}$ for BSB transitions and $\sim\sqrt{n}$ for RSB transitions breaks down when $\eta^2(2n+1) \ll 1$, where η is the Lamb-Dicke parameter. In this regime the coupling strengths are best described using Bessel functions [5].

However, an advantage of our method for resolving Fock states using the ac stark effect in comparison to the ones already measured for Rydberg atoms and superconducting circuits [26–29] might be the applicability in systems with weaker couplings. Schemes in superconducting circuits often require an ultrastrong coupling, due to the large detuning Δ with $g^2/\Delta \gg \kappa$, Γ, where $g = i\eta\Omega_P$ (in our case) with the coupling strength of the probe beam Ω_P, in order to dispersively resolve single photons. The decay rate is defined as Γ and the field dissipation by κ. The strong coupling used in our scheme is achievable even in cavity QED systems [37], requiring only that $g \gg \kappa$, Γ.

We introduced and demonstrated a measurement method for trapped ion motion in the number basis. It relies on the Autler-Townes effect where the splitting is dependent on the motional Fock state when coupling to a phonon number changing transition. We showed the expected scaling depending on BSB or RSB coupling and demonstrated that this method can be used to detect Fock states. The method can be repeatedly applied in a single experimental run until a positive result is achieved. Otherwise the method can enact a nondemolition measurement, and prepare trapped ions in phonon number states. A scan of the Autler-Townes spectrum allows one to determine the phonon number distribution of a thermal state. The method is scalable for longer ion crystals and measurement of common modes, as long as the motional modes are individually resolvable and the Autler-Townes doublets

do not overlap with other motional modes. The information obtained then describes the motional quanta in the probed mode. Assuming that the excitation efficiency of the individual Autler-Townes doublet is high enough, one can also create a Fock state from a thermal distribution by using the Autler-Townes effect. The phonon number then defines the frequency of the laser, probing a well-defined peak from the Autler-Townes separation, as done in Figs. 3(a) and 3(b). We have shown that the measured scaling matches the theoretical model which scales as $\sqrt{n+1}$ for coupling of a BSB transition and $\sqrt{n}$ for coupling to a RSB transition. For using the method, a system in which a quantum harmonic oscillator is coupled to a three level quantum system is needed. Hence, it can also be used in different systems such as superconducting circuits coupled to microwave cavities, or ions coupled to optical cavities, or atoms.

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[33] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.131.223603 for further details on the experimental system and procedure for generating Fock states, as well as the complete analysis of resonance lineshapes are included here.

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