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A cooperative game theory systems approach to the value analysis of (innovation) ecosystems

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ABSTRACT

This paper presents a new approach for analyzing ecosystems in general, be they technical and/or economic, or even biological, with special reference to innovation ecosystems. The approach is grounded in systems theory and game theory, especially cooperative game theory. Theory approaches to ecosystem analysis in previous research are reviewed. Procedures are presented for assessing value creation and value capture in an ecosystem structured by complementary and substitute relations in an artifact subsystem and analogous cooperative and competitive relations in an actor system, CS-relations for short. Some measures of structural importance in an ecosystem structured by such CS-relations are presented and compared, especially the Shapley value. A number of simple but illustrative examples and applications are given. The paper finally proposes a formal representation of an ecosystem as being a pair of cooperative games linked by a map between them, representing ownership and control relations between artifacts and actors. The article is kept at a moderate level of formalism.

1. Introduction

1.1. Systems approach to ecosystems

Increasingly complex technological and economic interdependencies among artifacts and actors involved in innovation processes have since long called for various systems approaches in managing these processes, e.g. in the design of large and costly communication, transportation, defense and energy systems. The response by scholars studying innovation processes has also adopted various systems approaches, using concepts such as technological systems and innovation systems and more recently the concept of innovation ecosystems. The systems approach in studies of innovations have become well established with a rapidly growing scholarly literature, in recent years especially on innovation ecosystems. The ecosystem concept has also become used more generally in empirical studies of various entities such as industries, businesses, entrepreneurs, regions, knowledge bases, technologies, platforms, products etc. (see e.g. Thomas and Autio 2020; Daymond et al., 2022; Holgersson et al., 2022). Thus it is fair to say that an ecosystems approach has by and large become established in studies of a variety of entities. However, as so often is the case in management studies, scholarly practice has run ahead of scholarly theory and the theoretical underpinnings of an ecosystems approach have so far been thin and tentative with few attempts to formulate and formalize a more general basis for it.

1.2. Concept of ecosystem and innovation ecosystem

In an earlier article in this journal Granstrand and Holgersson (2020) reviewed existing conceptualizations of innovation systems and innovation ecosystems in the literature. A synthesis of the review showed that actors, artifacts, and activities are all common conceptual elements in an innovation ecosystem, linked together through relations, in particular complementary and substitute relations among artifacts and analogous cooperative and competitive relations among actors. The synthesis also pointed at the importance of institutions and the evolving nature of innovation ecosystems over time. Based on the conceptual review the article proposed the following definition: “An innovation ecosystem is the evolving set of actors, activities, and artifacts, and the institutions and relations, including complementary and substitute relations, that are important for the innovative performance of an actor or a population of actors."

Innovative activities include R&D activities and the introduction in an economy of new (to the world) and useful artifacts (products, processes, services etc.) but also the introduction of new actors (start-ups, new entrants), giving rise to new structures in the innovation ecosystem.

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Other types of new structural changes could also be considered as innovations, like new types of contractual or collaborative agreements (such as open sourcing and creative commons) or new institutional arrangements (such as new legislation or new government measures).

Innovation activities create value to some actors by definition but also typically destroy value to some other actors. Innovation activities carried out by a group or coalition of actors jointly create value in a way that can be expressed by a value creation function as described further below. Each actor in the coalition can then capture a share of this value which can be expressed by a value capture or value sharing function, also to be described below. The innovative performance of the system is then reflected by the value creation function at collective level and the value sharing or capture function at individual level. A value creation function typically displays economies of scope or synergies or complementarities in the loose sense (to be made precise below) that the whole is more valuable than the sum of the values of its parts. This incentivizes integrating complementary activities among actors as well as among artifacts. A value sharing or capture function typically reflects the outcome of cooperative agreements ('sharing'), often invoking some principle of fairness, or competing interests ('capture').[1] A value creation function and a value sharing function also defines a cooperative game and are in that context labelled value function and value allocation rule respectively.[2] As will also be shown below the specification of a value creation function and a value sharing function could be done both for actors in the actor system and for artifacts in the artifact system.

What distinguishes an innovation ecosystem definition from a general business or industrial ecosystem definition, is its explicit reference to innovative activities and innovative performance. In fact if 'innovation' and 'innovative' is removed from the above definition, it could define any ecosystem, be it economic, technical or technological, or even biological.

The definition of an ecosystem, generalized in this way, is accordingly: "An ecosystem is the evolving set of actors, activities, and artifacts, and the institutions and relations, including complementary and substitute relations, that are important for the performance of an actor or a population of actors." This general ecosystem definition is illustrated in Fig. 1 which could be referred to as a simple "3-A model" of a general ecosystem.[3]

This definition covers many if not most of the defining characteristics used in other definitions of ecosystems, e.g. in the definition in Baldwin (2012) and in Bogers et al. (2019), as well as in definitions of other types of ecosystems, e.g. the definition of business ecosystem in Baldwin (2020) and the definitions of platform ecosystem in Daymond et al. (2022) and Kretschmer et al. (2020). In fact the general definition of an ecosystem could by and large be adapted to a specific x-type of an ecosystem and its x-performance by using a suitable qualifier, x, such as business, platform, industrial, knowledge, technological etc.

1.3. Purpose and outline

Granstrand and Holgersson (2020) also outlined how the proposed definition of an innovation ecosystem could be further developed conceptually and theoretically. Complementary and substitute relations among artifacts and complementary (i.e. cooperative) and substitute (i.e. competitive) among actors – CS-relations for short – are prevalent in any ecosystem. A sharper focus on CS-relations would not only provide a more comprehensive and precise picture of what is going on in an innovation ecosystem such as various forms of coopetition, but would also enable operationalizations through use of established concepts in economics and industrial organization such as economies and dis-economies of scope. A focus on CS-relations would also enable theorizing along the lines of cooperative and competitive game theory, as well as along the lines of value creation through "growing the pie" across complements and complementors and value sharing ("slicing the pie") among them.

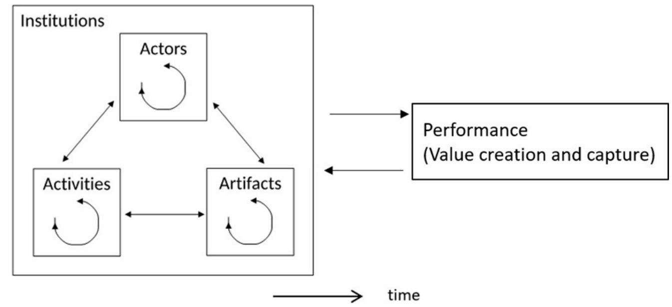


Fig. 1. Illustration of the general ecosystem definition.
(Adapted from Granstrand and Holgersson 2020.).

The purpose of this article is threefold. First, to follow up and substantiate some of the possibilities for further developments as claimed in Granstrand and Holgersson (2020). Second, to develop an original theoretical approach to analyzing ecosystems in general, including innovation ecosystems, grounded in systems theory and cooperative game theory and in so doing also review theory approaches in previous research on ecosystem analysis as well as to introduce a formal notion of coopetitive games. Third, to propose a formal representation of an (innovation) ecosystem. The article is kept at a moderate level of formalism.

The article first offers a general definition of an ecosystem, then reviews theory approaches to ecosystem analysis and then outlines the general features of the article's formal approach to ecosystem analysis. Algebraic and graphical models will be provided for assessing value creation and value capture in an ecosystem structured by strong and weak CS-relations. Some measures of structural importance in an ecosystem structured by CS-relations will also be presented and compared with special reference to the Shapley value. Ownership and control relations linking the actor and the artifact systems are thereafter described. A number of stylized but illustrative examples and applications will be given, which also illustrate coopetition as a mix of cooperation and competition. The article finally proposes a formal definition of an ecosystem as a combination of cooperative games in the actor and artifact system.

2. Previous research on theory approaches to ecosystem analysis

Several structured literature searches have been performed, using Web of Science and Google Scholar and using the search profile ecosystem* with various qualifiers (innovation, business, platform, technology etc.) AND ("game theory*" OR model* OR analytic* OR framework*) with some additional variants and the search profile "innovation ecosystem*" AND theor*. The literature searches were explorative in the sense that assessing existence or absence of literature items rather than frequencies of them was the main objective. The literature search was also made in order to exploit contributions in previous research for possible theory grounding and further developments. Several filters were used, excluding non-English works and pseudo-theoretical and apparently irrelevant works, and including only

[1] A value proposition, as used in some ecosystem literature (e.g. in Adner (2017) and Kapoor (2018)), refers to the case when a focal supplier proposes how potential buyers or partners will benefit from an offering, based on how value is to be created and shared.

[2] There are other labels for a value allocation rule as well in the context of cooperative game theory such as imputation, pay-off function, appropriation function and value distribution function. A value function in a cooperative game is also called a characteristic function. See also Section 3.

[3] As seen the three key entities actors, activities and artifacts and their relations remain the same as in Granstrand and Holgersson (2020).

Table 1
Theory approaches to ecosystem analysis in previous research with relevance scores (RS) to this article.[a]

Article	Theory approach	Application	Key concepts
Cha (2020) (RS = Low)	Strategy theories of multinational firms and of business ecosystems	Development of a framework for theory-building and normative global strategy research.	Business ecosystem Global strategy Ownership advantages
Dubina (2015) (RS = High)	Competitive game theory Optimization theory	Design of a business simulation model of an innovation ecosystem of the Triple Helix type for optimal resource allocation in R&D projects and their implementation.	Innovation ecosystem Triple Helix Simulation R&D Resource allocation
Faissal Bassis and Armellini (2018) (RS = Low)	Systems theory of innovation ecosystems and systems of innovation	Design of a framework and a method for a comparative analysis of the systems theories of innovation based on a literature review.	Innovation ecosystem Systems of innovation
Garnsey and Leong (2008) (RS = Medium)	Resource based theory of the firm Evolutionary economic theory	Explaining the emergence of new business ecosystems and innovative networks created by new biopharma firms through collaborative business models.	Business ecosystem Innovative networks Technology commercialization Resource asynchronies
Hannah and Eisenhardt (2018) (RS = Medium)	Multiple case based theory building for explaining firm management in and of emerging ecosystems	Developmental history of five young firms in the US solar industry during 2007-2014 and their competitive and cooperative relations and strategies.	Ecosystem (of firms) Competition Cooperation Strategy Bottlenecks
Hao et al. (2022) (RS = Medium)	Evolutionary game theory	Three-person evolutionary game model of an innovation ecosystem of Triple Helix type with reference to recycled resource industry for simulating the evolutionary dynamics of strategy and policy decisions.	Innovation ecosystem Stakeholder strategy Evolutionary stable strategies
Jacobides et al. (2018) (RS = High)	New theory	Emergence of ecosystems as evolving sets of organizations with interactions enabled by modularity and different types of non-generic complementarities.	Ecosystem Modularity Complementarity Strategy
Jiang et al. (2023) (RS = Medium)	Evolutionary game theory	Modelling a subsidized online crowdsourcing platform in an open innovation ecosystem with issuers and receivers of work tasks, subjected to costs, synergies, revenues and externalities. Simulation studies of stability and equilibrium.	Open innovation ecosystem Crowdsourcing Synergy mechanism Network externalities
Lou et al. (2004) (RS = Medium)	Competitive game theory	Industrial system with two plants interacting through material exchange.	Industrial ecosystem Emergy analysis Sustainability
Mantovani and Ruiz-Aliseda (2016) (RS = High)	Competitive game theory Industrial organization theory	Four value chains with two complementary products (e.g. hardware and software), each produced by two firms, investing in adaptive collaborative R&D for competitive selling to a unit mass of consumers at differentiated prices.	Systems competition Complementarity Coopetition Two-sided platforms
Mêgnigbêto (2018) (RS = High)	Cooperative game theory	Comparison of Triple Helix innovation systems in South Korea and West Africa, based on publication counts.	Synergy indicators Core Shapley value Nucleolus
Nishino et al. (2018) (RS = High)	Competitive game theory	Modelling cooperative network formation in business ecosystems for multi-agent simulation, with competitive strategies to cooperate or not.	Business ecosystem Cooperative relation Ability Strategy imitation
Tavalaei and Cennamo (2021) (RS = Low)	Industrial organization theory	Analysis of complementor specialization/diversification strategies at product and ecosystem level based on multivariate analysis of a panel dataset of mobile app developers.	Platform ecosystem Complementors Specialization Value co-creation
Xu et al. (2023) (RS = Medium)	Evolutionary game theory	Modelling an innovation ecosystem with two populations – focal companies and non-focal innovation subjects – each having the two strategies value co-creation or opportunistic behavior, for equilibrium analysis and numerical simulation of parameter influences.	Digital innovation ecosystem Value co-creation
Zhu et al. (2023) (RS = Low)	Competitive game theory Optimization theory	Modelling four platform competition scenarios with and without investments under two pricing regimes for analysis of competitive equilibrium pricing, market share and platform profit.	Platform ecosystem Platform competition Two-sided market Third party participant
Zou and He (2021) (RS = Medium)	Evolutionary game theory	Modelling the technology sharing propensity of two firms mixing two pure strategies, to share or not, for equilibrium and stability analysis and simulation.	Innovation ecosystem Platform Technology sharing Competition Cooperation
Zou et al. (2022) (RS = Medium)	Evolutionary game theory	Modelling a three-person game with a firm, a research organization and an intermediary, each mixing the two pure strategies to cooperate or not, for equilibrium and stability analysis and simulation.	Digital innovation ecosystem

[a] The relevance of an article has been based on the number of key features of this article appearing in the article and the centrality of each of these features in the article as assessed by ChatGPT with additional reliability runs and then checked manually for validity. The selected key features of this article are (somewhat abbreviated): Activities/strategies, Actors and Artifacts, Systems approach, Game theory type, Complementary and/or substitute relations, Value Creating and/or Value Capture functions, Boolean or Binary functions, Legal relations, Analytical vs Simulation modelling, Qualitative vs Quantitative, Empirical vs Theoretical.

articles. The restriction of the literature search to the term 'ecosystem' excludes theory approaches using other but conceptually related terms such as innovation networks or systems, or open collaborative innovation and open innovation markets spanning over organizational boundaries, which could also be conceived of as innovation ecosystems (see e.g. Jiang et al., 2023).

Table 1 summarizes the relevant articles found.[4] All these articles are relevant to theory approaches to ecosystem analysis in general. The relevance of each article to the theory approach to ecosystem analysis presented in this article in particular has been assessed by a basic relevance score ranging from 1 to 10, subdivided into Low (1-4), Medium (5-7) and High (8-10), as shown in the table. No article scored lower than 3 and those four articles with low relevance by and large did not use game theory at all.

A number of articles theorized more generally in analyzing ecosystems of various types, as could be expected, some outlining or building new theories (e.g. Hannah and Eisenhardt 2018; Jacobides et al., 2018; Cha 2020; Tavalaei and Cennamo 2021), some combining existing theories (e.g. Garnsey and Leong 2008; Faissal Bassis and Armellini 2018). As to the use of game theory for ecosystem analysis, quite a few used evolutionary game theory concepts for modelling and simulating the evolution of an ecosystem or a collaboration network (e.g. Zou and He 2021; Hao et al., 2022; Jiang et al., 2023; Xu et al., 2023) and some used competitive game theory for simulations (e.g. Dubina 2015; Nishino et al., 2018). Most articles used concepts from competitive game theory (e.g. Lou et al., 2004; Zhu et al., 2023) and some used concepts also from cooperative game theory for modelling coopetition as mixes of competition and cooperation (e.g. Mantovani and Ruiz-Aliseda 2016). Several articles focused on cooperation but then as a particular strategy out of several in a competitive context which is different from using a cooperative game theory approach.

Of the five articles with high relevance to this article, one was not using game theory at all but had a highly relevant discussion of complementarities and modularity (Jacobides et al., 2018). Three used competitive game theory, but with a focus on cooperation (Dubina 2015; Mantovani and Ruiz-Aliseda 2016; Nishino et al., 2018). Only Mêgnigbêto (2018) used a clear cooperative game approach and then for an empirical comparative analysis of two innovation systems, each with three players. The article used an interesting method to estimate the characteristic value creation function, however with some idiosyncratic interpretations of the Shapley values and the nucleolus in the results.

In summary, most articles were applied in nature, not developing any new game theoretic concepts or tools specific for ecosystem analysis, and almost all applications involved a small number of actors, typically two, three or four, often with a small number of strategies as well. In particular no article was found that attempted to formally model value creation and value capture functions explicitly based on the relational structure of an ecosystem with an arbitrary number of actors or artifacts, nor any article that attempted to actually define an ecosystem in game theory terms, as done in this article.

3. General features of a formal approach to ecosystems analysis

Below the basic concepts of a so called state space approach in systems thinking and theory are introduced, followed by basic conceptualizations in cooperative game theory and some elementary examples, which then will be generalized to form the gist of the theory approach to ecosystem analysis.

[4] Some theory approaches to innovation systems or networks also appear in books rather than in articles, e.g. transaction cost theory in Williamson (1975) and Granstrand (1982) and network theory in Powell (1990).

3.1. Entities and state variables

A standard formal definition of a system $\mathscr{S}$ is that it consists of a set of entities $\mathscr{A}$ and a set of relations $\mathscr{R}$, i.e. formally $\mathscr{S} = (\mathscr{A}, \mathscr{R})$. The entities could in principle be anything but in an ecosystem context typically actors (or agents, players, individuals, organizations, or sets of them) or artifacts (e.g. resources, products, processes, technologies, patents, institutions, or sets of them). The relations could as well be anything in principle. In the context here focus will be on complementary or cooperative and substitute or competitive relations, CS-relations for short, to be formally described below. Actors as well as artifacts perform activities, e.g. they cooperate, compete, perform R&D, and trade. Activities are typically interdependent across the entities, in which case the activities are reflected in the relations between the entities. Actors also adopt certain activities as a result of their adopted strategies or strategic decisions. Actors as well as artifacts could also be considered as being in different states or operational modes, corresponding to different activities or exercised strategies. The state of entities also typically depend on conditions in their environment, including legal and other institutional conditions. The relations across entities at any point in time constitute the structure of the system. Entities and relations finally evolve over time with various types of state dependencies and trajectory characteristics.

To start off as simple as possible, consider simple systems with the following structures as depicted in Fig. 2. Fig. 2a and b shows a system with only two entities and one relation between them, being a complementary relation in Fig. 2a and a substitute relation in Fig. 2b.

The entities, be they artifacts or actors, can as mentioned be in varying conditions or states – they can work or not, be present or absent in a portfolio or an organization and so on. To describe in which state or condition an entity is at a certain time point or a certain period t we associate a state variable x_A to each entity A. To simplify as much as possible we consider each entity to be in one of only two possible states here – either the entity is operational (i.e. it works, functions, participates or employs a certain activity or strategy) at time t, denoted $x_A(t) = 1$, or it is not, denoted $x_A(t) = 0$. If it is uncertain in which state an entity is, the state variable can be considered a random variable. Thus a number p_A between the values 0 and 1 can be chosen to represent the likelihood or probability that entity A is operational, denoted $\Pr(x_A = 1) = p_A$, and that it fails to be operational with probability $1 - p_A = \Pr(x_A = 0)$.

In a general case a state variable could take on any real number or be a vector of real numbers.

3.2. Ecosystem value creation function

A value creation function specifies how value is produced by various sub-sets of artifacts and actors.[5] When only the sheer number of actors or amounts of artifacts that come into play matters for value creation, a value creation function could be seen as a kind of utility function when applied to consumable artifacts, or a production function with valorized output, such as the classical Cobb-Douglas function, when applied to

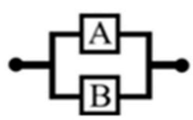
[5] This type of value (creation) function is a defining characteristic of a cooperative game, absent in a competitive (non-cooperative) game, which in turn has a set of strategies (or actions or activity levels) as a defining characteristic, absent in cooperative game theory. The strategies chosen individually by the actors determine their pay-offs or values captured in a competitive game, while in a cooperative game a value sharing function chosen jointly by the actors determines their pay-offs or values captured. Nevertheless value creation and value capture in a cooperative game result from activities, albeit these are not explicitly modelled. As shown in Section 5, a cooperative game can be extended with explicit activities, subsumed in a strategy set, thereby making the value creation function dependent upon strategies, thereby turning the game into a coopetititve game.

	Graphical representation	Description in words	Value creation function
a)		Two complementary entities A and B jointly create positive additional value.	$V(x_A, x_B) = V_A \cdot x_A + V_B \cdot x_B + V_{AB} \cdot x_A \cdot x_B$ where $V_{AB} > 0$. If the stand-alone values V_A and V_B are zero, then by definition A and B are *strong complements*, i.e. $V(x_A, x_B) = V_{AB} \cdot x_A \cdot x_B$

Examples:

Actor system: A simple value chain with one supplier and one buyer.

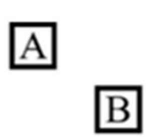
Artifact system: A product with two components or a simple portfolio with two assets.

b)		Two substitute entities A and B jointly create negative additional value.	$V(x_A, x_B) = V_A \cdot x_A + V_B \cdot x_B + V_{AB} \cdot x_A \cdot x_B$ where $V_{AB} < 0$. If the stand-alone values V_A and V_B are equal and $= -V_{AB}$, then by definition A and B are *strong substitutes*, $V(x_A, x_B) = V_A(x_A + x_B - x_A \cdot x_B)$

Examples:

Actor system: Two competing firms.

Artifact system: Two technological substitutes or alternative solutions.

c)		Two independent entities which do not add (subtract) value to (from) each other	$V(x_A, x_B) = V_A \cdot x_A + V_B \cdot x_B$ (i.e. $V_{AB} = 0$)

Examples:

Actor system: Two independent firms.

Artifact system: Two products or assets with no interaction effects.

Fig. 2. Elementary systems with two entities, being complementary, substitutes and independent.

resources, and the truck-loading example when applied to actors. The value sharing function then reflects how the value of this output is allocated among actors or artifacts through e.g. market mechanisms and externalities. However, often the structural relations among artifacts and actors matter, especially in settings with small numbers of interrelated artifacts and actors.

In what follows we will again simplify as a start and specify a value creation function as simple as possible. Either a fixed positive value is created or no value is created, depending on whether the state variables x_A are operational or not. Now with a set of interrelated entities A,B,C, … in different states x_A, x_B, x_C, … we want to know if the whole system $\mathscr{S}$ is operational or not, i.e. that the state variable $x_{\mathscr{S}}$ for the system is = 1 or 0. The system state variable $x_{\mathscr{S}}$ is then a function of all the entity state variables, called the system function for systems in general. Here we will refer to this function as the value creation function, since a functioning (operational) ecosystem can be considered as value creating.

The value creation function of an ecosystem is denoted $V(x_A,\ x_B, x_C, \ldots)$ and abbreviated $V(\boldsymbol{x})$, where $\boldsymbol{x}$ is a string or vector of entity state variables. Thus if all state variables are binary 0, 1-variables $\boldsymbol{x}$ is a bit string and V is defined on the set of all possible bit strings, representing all possible combinations of entity states. The value creation function could in this case just as well be defined on all subsets of the set of entities, where a given subset denotes the set of all entities which are operational in a given situation. $V(\{A, C\})$ then denotes the value created when entity A and C but not B functions, i.e equals $V(\boldsymbol{x})$ for $x_A = x_C = 1$ and $x_B = 0$. Thus the function V could be equivalently expressed as a function of a state vector $\boldsymbol{x}$ or as a function of subsets and the symbol V is used for both cases, which should not create any confusion. Also, with a slight abuse of notation, the set brackets could be dropped, i.e. $V(A, C) := V(\{A, C\})$. The value function defined on subsets of entities then correspond to the value function defined on subsets of actors or players in cooperative game theory, while the value function defined on state vectors $\boldsymbol{x}$ corresponds to the system function in system reliability theory in the case its values are 1 or 0.

3.3. Value concepts, economies of scope and CS-relations

With the help of a value creation function defined on sets, a number of value concepts for sets of entities can be defined. If the value of an empty set (or empty coalition of actors or empty portfolio of artifacts) is set to zero as a reference point, then the *stand-alone value* of an entity A is $= V(A)$ which usually is assumed to be $\geq\ 0$, and the value *added by A to B* $= V(A,B)–V(B)$, which in other words is the marginal value of A in the set (coalition, portfolio) $\{A,B\}$.

The (bivariate) *economies of scope* or *synergies* or *complementarities* between entities A and B is defined to be $= V(A,B)–V(A)–V(B)$, i.e. it is the joint value of the entities $V(A,B)$ minus their stand-alone values.[6] Economies of scope could be positive or negative, and in case there are positive economies of scope, the two entities are *complementary*, i.e. they have a complementary relation, and in case there are negative economies of scope (dis-economies of scope) the two entities are *substitutes*, i.e. they have a substitute relation. These value concepts are symmetric while the marginal value or added value is an asymmetric value concept. The value concepts apply to entities, regardless if they are actors or artifacts. In case of actors one usually uses the terms cooperative rather than complementary and competitive rather than substitutes, however (See further Section 3.6.).

In summary so far, we have conceptually outlined a systems theoretic approach for general ecosystems with artifacts and actors being operational or not for value creation as expressed by a value creation function or an ecosystem value function. This function corresponds to the value function in cooperative game theory and when it is binary to the system function in system reliability theory.

3.4. Elementary system examples

Next, we turn to two basic examples of the simplest possible systems with two entities and a system structure (configuration) with only one relation, being complementary or substitute. Systems like these could be described by text, graphics or formulas as shown in Fig. 2. Expressing the value creation function as a function of state variables is particularly useful as will be shown below, since it turns out that any value function $V(\boldsymbol{x})$ can be expressed as a general multilinear polynomial in the state variables.[7] This means that in the case of two entities (actors or artifacts) A and B: $V(x_A, x_B) = V_A{\cdot}x_A + V_B{\cdot}x_B + V_{AB}{\cdot}x_A{\cdot}x_B$ where V_A and V_B are the stand-alone values of A and B and V_{AB} denotes the economies of scope between them (still with the convention that empty coalitions or portfolios have zero value).

If A and B are complements and their stand-alone values are equal to zero, A and B are defined as being *strong complements*, i.e. neither entity has a value without the other. This case could be represented graphically as the entities being linked to each other in series, and algebraically as a *multiplication* of their state variables, i.e. $V(x_A, x_B)\ =\ V_{AB}{\cdot}x_A{\cdot}x_B$, corresponding to the statement in words that both x_A <u>and</u> x_B must be = 1 to create any value.

If the stand-alone values of A and B are equal and positive and in addition equal their joint value $V(A,B)$, then A and B, are defined here as being *strong substitutes*, i.e. neither entity adds any value to the other. This case could be represented graphically as the entities being linked to each other in parallel, and expressed algebraically as $V(x_A, x_B) = V_A(1-(1-x_A)(1-x_B)) =: V_A{\cdot}(x_A \sqcup x_B)$ and expressed in words by stating that the system is not operational, i.e. does not create value, i.e. $V(x_A, x_B) = 0$, if both x_A and $x_B = 0$, i.e. both entities are non-operational. Linking two entities in parallel corresponds to what is called here a *substication* of their state variables, an operation for which the so called ip-symbol $\sqcup$ is used, i.e. $x_A \sqcup x_B := (1 - (1-x_A)(1-x_B))$ for x_A and $x_B = 0$ or 1.

Now consider the case with three entities and both substitute and complementary relations as depicted in Fig. 3.

In case of strong complementary and substitute relations, the value creation function or ecosystem value function becomes binary, assuming only the value 0 or some positive value V_0 if the system as a whole is operational (the system state variable $x_{\mathscr{S}}$ above equals 1). The value creation function is then easy to compute by using multiplication for entities linked in series and *substication* for entities linked in parallel.

In principle any such system could as well also be described logically in words using only the conjunction '<u>and</u>' and the disjunction '<u>or</u>', representing strong complementary and substitute relations.[8] The conjunction 'and' in 'A and B' is then analogous to linking A and B in series and the disjunction 'or' in 'A or B' analogous to linking A and B in parallel in a circuit diagram.

Thus all systems with strong complementary and substitute relations between entities with binary state variables can be modelled graphically analogous to a circuit design and with a binary value creation function expressed algebraically as a multilinear polynomial of state variables.

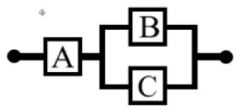
[6] This definition is analogous to the standard definition of economies of scope based on production functions, see e.g. Teece (1982). If there are (positive) economies of scope for any pair of sets A and B, the value creation function is said to be superadditive. Superadditivity is a more general concept than supermodularity or convexity, other concepts used to characterize value functions in cooperative games. There is a rich literature on supermodularity and complementarities of different types, applicable to different types of game settings, see Topkis (1998) and Milgrom and Roberts (1990) for seminal works and Amir (2005) for a survey.

[7] This is a general result in Fourier analysis of real-valued Boolean functions, see e.g. O'Donnell (2014).

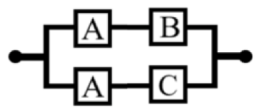
[8] This is a general result in computer science and Boolean function analysis.

Graphical representation | Description in words | Value creation function

a)



or equivalently



A and then B or C have to be operational to create value

$$V(x_A, x_B, x_B) =$$
$$= V_0(x_A \cdot (x_B \sqcup x_C)) =$$
$$= V_0(x_A \cdot x_B + x_A \cdot x_C -$$
$$- x_A \cdot x_B \cdot x_C) =$$
$$= V_0(x_A \cdot x_B \sqcup x_A \cdot x_C)$$

where V_0 is some positive constant

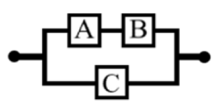
Examples:

Actor system: A single buyer A with two competing suppliers.

Artifact system: A product A with two alternative production processes B and C.

Note: The graphical representation does not need to be unique. The lower one equivalently but more clearly represents two substitute value chains or value paths in the system.

b)



A and B together or C have to be operational to create value

$$V(x_A, x_B, x_B) =$$
$$= V_0\big((x_A \cdot x_B) \sqcup x_C\big) =$$
$$= V_0(x_A \cdot x_B + x_C - x_A \cdot x_B \cdot x_C)$$

Examples:

Actor system: A vertically integrated firm C competes with two dis-integrated seller/buyer firms A and B.

Artifact system: A production process with two stages competes with a fully integrated process.

Fig. 3. Elementary ecosystems with three entities and strong CS-relations.

Also the calculation of certain value capture functions and measures of structural importance is facilitated in case of strong complements and substitutes, as will be shown below. The general case with weak (= non-strong) complementary and substitute relations will also be dealt with below.

3.5. Modularization

One can also consider a subset of entities as a *module* entity, consisting of entities with similar characteristics or structural positions. Modules could be in different operational states and linked in series or in parallel to other entities or entity modules, as illustrated in Fig. 4. The module M is a subsystem and as such it has a system state variable $x_M = 1$ or 0, depending upon the state of the entities in the module and their configuration. The value creation function for the whole system can for proper modularizations then simply be derived by plugging in x_M as shown in Fig. 4. This requires the module to be proper in the sense that no entity in M at the same time appears outside M in the whole system configuration, i.e. a proper modularization truly partitions the set of entities, be they artifacts or actors. Proper modularizations may not exist, however, and if they do they are typically not unique.

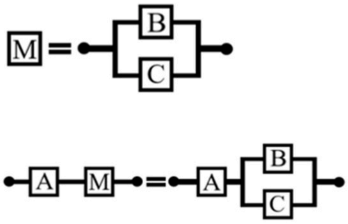
Thus modularization gives an opportunity to reduce the complexity of a system with strong (or approximately strong) CS-relations. Modularization could also be used to reduce complexity of a general system as well as creating value therefrom, e.g. through compatibility standards that enable modules to operate more freely with each other.[9] At the same time modularizations could be done in many ways and it is often not easy to find useful modularizations, e.g. of a large computer program or of a patent portfolio where many patents are relevant for ("read on") many products, which in turn are components in a larger technical system.[10] Modularization ideas are also useful and used in decentralizing large organizations or decomposing large populations (e.g. of specimens) into sub-populations (e.g. of species) in general. Thus modularization may be used both in the artifact system (as in engineering design) and in the actor system (as in organizational design).

[9] In fact, modularization has been hailed as a key driver of innovation and growth in Baldwin and Clark (2000).

[10] See Baldwin and Henkel (2015) for an example of the latter.

Graphical representation	Description in words	Value creation function

a)



Two substitute entities are taken as a module, linked as a complement to another entity

$$x_M = (x_B \sqcup x_C)$$

$$V(x_A,\, x_M) = V_0 \cdot x_A \cdot x_M =$$

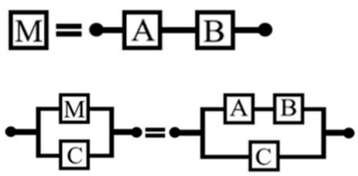
$$= V_0 \cdot (x_A \cdot (x_B \sqcup x_C)) =$$

$$= V_0(x_A \cdot x_B + x_A \cdot x_C - x_A \cdot x_B \cdot x_C)$$

where V_0 is some positive constant

Examples: Same as in Fig. 3a

b)



Two complementary entities are taken as a module, linked in parallel to another entity

$$x_M = x_A \cdot x_B$$

$$V(x_A,\, x_M) = V_0(x_M \sqcup x_C) =$$

$$= V_0 \cdot ((x_A \cdot x_B) \sqcup x_C) =$$

$$= V_0(x_A \cdot x_B + x_C - x_A \cdot x_B \cdot x_C)$$

Examples: Same as in Fig. 3b

Fig. 4. Elementary modularizations.

Fig. 5 shows some general system structures with C-modules, i.e. modules with only complements and S-modules, i.e. modules with only substitutes, linked in series or in parallel, i.e. linked by complementary and substitute relations, i.e. CS-relations. In case there are many entities it is simpler to label them by numbers and label their state variables x_i as done in Fig. 5. To simplify notation further, the module state variables for a C-module is denoted x^M where the superscript denotes multiplication of the m state variables for the entities in the module, and the module state variable for an S-module is denoted y_N, where the subscript denotes substication of the n state variables for the entities in the module.

The general structures in Fig. 5 can be used to generate any structure of an ecosystem with strong CS-relations, since any such system could be shown to be equivalent to (i.e. have the same value creation functions as) a system configuration with a number of C-modules of varying sizes, linked in parallel, or equivalent to a system configuration with a number of S-modules of varying sizes, linked in series. However, a given entity may then occupy several positions in the system structure.[11] Finally, one can note that the system in Fig. 5a with $m=2$ and $n=1$ is structurally the same as the system in Fig. 3b, and that the systems in Fig. 5b and c with $m=1$ and $n=2$ and the system in Fig. 3a have the same structure.

As seen these examples apply to general ecosystems, be they industrial, economic, technical, biological etc.

3.6. *Systems with general complementary and substitute relations*

So far the analysis of ecosystems has been confined to strong complements and strong substitutes, resulting in value created being either zero or one (or some positive constant). This is a very special case of a cooperative game. What one wants is to allow for non-strong CS-relations to come into play and let the value creation function assume any value. The next step is therefore to analyze systems of entities that involve complements and substitutes in a more general sense, as defined below. The ecosystem value creation function $V(S)$ now has to assume any real value, positive or negative, for each subset of entities S in the whole set of entities $N = \{1, 2, \dots, n\}$.[12] The interpretation is that if $V(S)$ is positive (negative) for a coalition of actors or portfolio of artifacts S, that particular coalition or portfolio or mix jointly creates (destroys) value. A food and drink metaphor might be helpful to conceptualize a general value creation function. A mix of oil, vinegar, salt, bread and wine might be priceworthy to consume, while a mix of oil and wine only probably isn't, unless at least some bread is added to the mix, given that it is not toxically moldy, which could destroy any value. However, it is not only a matter of presence or absence of ingredients and their general CS-relations but also a matter of amounts of ingredients and their interactions. A more refined analysis, left aside here, would then need to go beyond an ordinary value creation function of binary variables, and specify one in form of a utility function or production function of e.g. Leontief type.

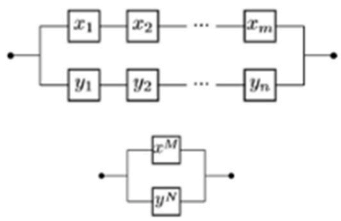
Any value creation function could be generally expressed in the state variables $x_i = 0, 1, i = 1, \dots, n$ in the following way, where the value contribution of each coalition S is summed up for all coalitions (sub-sets) of N and R is any subset of S:

[11] Note that proper modularization requires that no element belongs to more than one module.

[12] Obviously collecting data for each subset is a challenging task, since the number of subsets is 2^n, which quickly grows with n, as does the computational task, calling for approximate methods.

Graphical representation	Description in words	Value creation function

a)



A module with n entities in series is linked in parallel to a module with m entities in series

$$V(x,y) = V_0 \cdot (x_1 \cdot x_2 \cdot \ldots \cdot x_m) \sqcup \sqcup (y_1 \cdot y_2 \cdot \ldots \cdot y_m) =: V_0 \cdot (x^M \sqcup y^N)$$

where $x^M := \prod_{i \in M} x_i$

and $y^N := \prod_{j \in N} y_j$

with $M := \{1,2,\ldots,m\}$

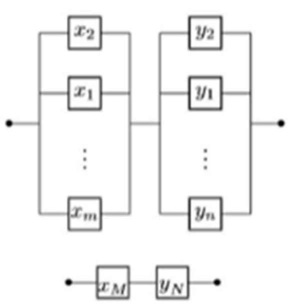
and $N := \{1,2,\ldots,n\}$

Examples:

Actor system: Two competing value chains or supply chains, or value paths.

Artifact system: Two completely different multi-stage processes performing the same function.

b)



A module with m substitutes is linked in series to a module with n substitutes i.e. two complementary modules with substitutes

$$V(x,y) = V_0 \cdot x_M \cdot y_N$$

where $x_M := \coprod_{i \in M} x_i$

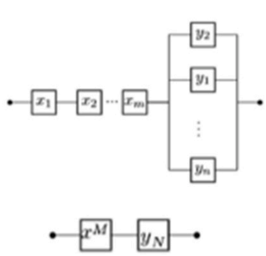
and $y_N := \coprod_{i \in N} y_j$

Examples:

Actor system: A value chain with two competitive markets (with unit demand and supply).

Artifact system: A set of left hand gloves and a set of right hand gloves (with unit demand).

c)



A module with m complements is linked in series (i.e. is complementary) to a module with n substitutes

$$V(x,y) = V_0 \cdot x^M \cdot y_N$$

Examples:

Actor system: A value chain with only one competitive market.

Artifact system: A technical platform with several applications.

Fig. 5. CS-relations between general C- and S-modules.

$V(x) = \sum_{S \subseteq N} V_S \cdot x^S$, where the coefficients $V_S = \sum_{R \subseteq S} (-1)^{|S|-|R|} \cdot V(R)$ with the notation $x^S = \prod_{i \in S} x_i$ for simplicity. [13]

The coefficients V_S could now be used to define general complementary and substitute relations for each subset S of entities, where the entities in subset S are defined to be *complementary in S* if $V_S > 0$ and *substitutes in S* if $V_S < 0$ and *independent in S* if $V_S = 0$. In case the entities are actors on a market they could correspondingly be defined as being cooperative and competitive in S respectively. Thus with this definition a complementary or cooperative and substitute or competitive relation between artifacts or actors is always defined with regard to a special subset of artifacts or actors.

3.7. *Some innovation related examples*

For the case $n = 2$ (and zero value for the empty subset as before) $V(x_1, x_2) = V_1 \cdot x_1 + V_2 \cdot x_2 + V_{12} \cdot x_1 \cdot x_2$. The general definition of bivariate CS-relations then depend on the sign of V_{12} with the definition earlier, while the definitions of strong CS-relations depend on V_{12} as well as on V_1 and V_2. Here x_1 could be a basic new product with stand alone value V_1 and some extra gadget or spare part x_2 with stand-alone value V_2 and then a joint extra benefit V_{12}. x_1 could also be a basic innovation with a cumulative improvement x_2 without stand-alone value, i.e. $V_2 = 0$. x_1 and x_2 could also be two workers or an inventor and an entrepreneur with complimentary skills or two complementary firms (complementors) or two competing firms and so on. Thus, here as well as in a general case, some or all of the n entities could be innovations or innovators or entrepreneurs without affecting the nature of the modelling approach and its CS-relations and value creation function. What decides whether an ecosystem is an innovation ecosystem or not is the interpretation of what the state variables represent, be it innovations or actors with innovative activities or not, as described in a textual representation, not in the algebraic or graphical representation or structure of the model.[14]

Although an algebraic representation of a simple system easily extends to general systems, it is not so easy to give a graphic representation of a system with general CS-relations. A few examples with $n = 3$ may illustrate further.

Ex 1. Two cumulative sequential improvements – x_2 of an original invention x_1 and x_3 of x_2 in turn – add incremental or marginal (positive) values V_{12} and V_{123} without having any stand-alone values. Then the value function is:

$$V(\mathbf{x}) = V_1 x_1 + V_{12} x_1 x_2 + V_{123} x_1 x_2 x_3$$

This structure could be illustrated graphically as in Fig. 6.

A longer value chain of cumulative sequential improvements could be expressed in a similar way. Now the value chain is not a simple one with only strong complements and a binary value creation function, but a cumulative value chain with arbitrary positive values added sequentially. The entities x_1, x_2 and x_3 could also represent patent modules with separate binary value creation functions in turn.

Ex 2. An improvement x_2 of an original invention x_1 has an alternative (substitute) design x_3 invented around x_2. Either or both x_2 and x_3 adds value V_{123}. Now the general value function is:

$$V(\mathbf{x}) = V_1 x_1 + V_{123} x_1 (x_2 \sqcup x_3) = V_1 x_1 + V_{123} x_1 x_2 + V_{123} x_1 x_3 - V_{123} x_1 x_2 x_3$$

Graphically the structure could be depicted as in Fig. 7.

Ex 3. This is a variant of Example 2. x_1 needs either x_2 or x_3 to create value but now x_3 is somewhat superior to its alternative x_2 by a factor $1 + \delta$ where $\delta > 0$ as in the example of standard setting in Layne-Farrar et al. (2007). The value creation function is now given by the only non-zero values being $V(1, 1, 0) = 1$ and $V(1, 0, 1) = 1 + \delta = V(1, 1, 1)$ and else $= 0$. Its algebraic expression then becomes:

$$V(\mathbf{x}) = x_1 x_2 + (1 + \delta) x_1 x_3 - x_1 x_2 x_3 = x_1 \cdot (x_2 \sqcup x_3) + \delta x_1 \cdot x_3$$

and graphically the structure could be represented as in Fig. 8 with a constant factor δ.

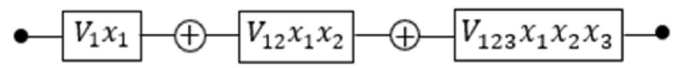


Fig. 6. System structure in Example 1.

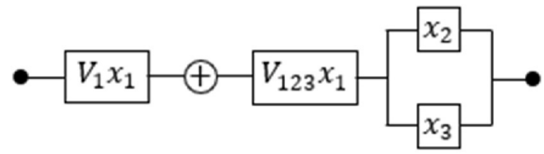


Fig. 7. System structure in Example 2.

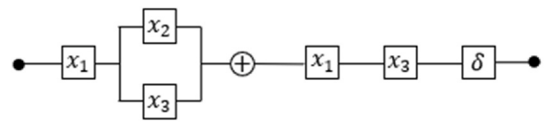


Fig. 8. System structure in Example 3.

More general structures with complementary and substitute actors or artifacts could be represented algebraically and graphically in similar ways, e.g. the actor or artifact structures in the examples given in Brandenburger and Stuart (1996, 2007), Gans and Ryall (2017) and Gilles (2010, p. 110). These representations facilitate various value calculations, e.g. of Shapley values, as illustrated in the next section.

3.8. *Measures of structural importance*

Thus we have tools for analyzing any ecosystem with any CS-relations. It is now of key interest to analyze the problem how the value created in a given system could and should be attributable or allocated to its various entities. In case the entities are actors, this problem could be rephrased as how the various individual actors appropriate or capture or share their jointly created value. In case the entities are artifacts the problem could be rephrased as a valuation or pricing problem.

One solution approach to this problem is to analyze the importance of each entity. In Granstrand and Holgersson (2020) the more precise meaning of "importance" was left to operationalizations when called for. It is now time to call for one, namely the structural importance of an entity. This refers to the entity's importance for value creation in terms of its position in the system structure with its CS-relations to other entities and their CS-relations in turn, rather than in terms of the nature of the entity per se. For example, each entity A and B in Fig. 2a is critical for value creation in the sense that no value is created if the entity is non-operational. In other words both A and B are critical entities for system performance, i.e. value creation. On the other hand, neither A nor B is critical in Fig. 2b. In Fig. 3a entity A is critical but not B and C, and in Fig. 3b there is no critical entity.[15]

There are several ways to operationalize a measure of structural

[13] See O'Donnell (2014). The coefficients V_S are sometimes called Harsanyi dividends in cooperative game theory (see e.g. Gilles 2010).

[14] This being said, nothing prevents using a notation that indicates the innovative nature of an entity, for instance using the notation x_1^+ for an inventive or patented improvement of a product x_1.

[15] In case the entities are patents, critical patents are also referred to as strategic patents or essential patens, e.g. for a standard, in which case they are labelled standard essential patents (SEPs).

importance of an entity, an SI-measure for short. There are also several related measures or characteristics of a system (or network) structure, such as network centrality, strong and weak ties, structural holes, and power indices. These notions will be left aside here.[16]

The simplest approach to allocating value is to simply ignore the structure of the system and treat all entities as equals, in which case each is attributed V_C/n, where V_C is the total jointly created value. This way is primitive but nevertheless often used due to its simplicity (e.g. when simply counting each actor's essential patents in a jointly created patent pool to support a standard, see Holgersson et al., 2018).

Another simplifying and often useful way is to modularize and first look at the structure between modules and based on that structure assign values to modules and then look at the inner structure of each module and based on that share the module's value among its entities. The module M in Fig. 3a is critical as is entity A (which could be considered a trivial module in itself), and thus M and A have the same structural importance and should each have a half of their jointly created total value V_C. Then the entities in M have the same structural position and importance, giving them half of the value assigned to M, which means each get $V_C/4$.[17]

A more sophisticated approach is the so-called Birnbaum's measure of structural importance of an entity. For a given entity, all combinations of the states of the other entities are enumerated and Birnbaum's SI-measure is the number of these combinations for which the entity is critical, divided by the total number of them. In other words for an entity A in a system with n entities, its structural importance $SI_A =$ the sum of A's marginal values to each subset of all the other entities divided by $2^{n-1} =$ the number of all such subsets.

Thus, Birnbaum's measure of SI for an entity is its average marginal value, averaged over all the 2^{n-1} combinations of the two states – operational and non-operational – of the other $n-1$ entities. In terms of the value functions $V(\boldsymbol{x})$ with $\boldsymbol{x} = (x_1,\ldots,x_n)$ and $x_i = 0$ or 1, the Birnbaum SI-measure for entity i then is: $SI_i = \left(\sum_{x:x_j=0,1} V(\boldsymbol{x}; x_i = 1) - V(\boldsymbol{x}; x_i = 0)\right) \Big/ 2^{(n-1)}$.

For the system in Fig. 3a with 3 entities the Birnbaum's SI-measures turn out to be:

$$SI_A = 3V_C/4,\ SI_B = V_C/4 = SI_C$$

This is because A adds value V_C to the three subsets $\{B\}$,$\{C\}$, and $\{B,C\}$ of operational entities but not to the empty subset, corresponding to the case when neither B, nor C is operational, while B as well as C adds value V_C only to the one subset $\{A\}$.

For the system in Fig. 3b Birnbaum's SI-measures are: $SI_A = V_C/4 = SI_B$, $SI_C = 3V_C/4$.

Thus C in Fig. 3b has the same structural importance as A in Fig. 3a, although A is critical while C is not.

A most sophisticated approach to measure structural importance is to use the Shapley value, named after Lloyd Shapley, Nobel Prize winner in economics 2012. The Shapley value for an entity i in a system with a set N of n entities, labelled $Sh_i(N)$, is a weighted average of the marginal values added by entity i to each subset of entities, be they actors or artifacts.[18] It could be computed by first calculating the average marginal value added by the entity to all subsets of a certain size, say with k entities, which do not contain the entity. There are in total $\binom{n-1}{k} = \frac{(n-1)!}{k!(n-1-k)!}$ such subsets in the system. Then one takes the average of these averages over all n possible sizes $k = 0, 1, \ldots, n-1$ and gets $Sh_i(N)$, i.e.:

$$Sh_i(N) = \frac{1}{n}\sum_{k=0}^{n-1}\sum_{\substack{|S|=k\\ i\notin S}}(V(S\cup i) - V(S))\Big/\binom{n-1}{k}$$

A virtue of the Shapley value is that it is derived from a set of axioms, i.e. natural requirements or assumptions, that are easy to comply with, e.g. that an entity that contributes just as much as another entity for any given subset of other entities should receive equally much, and if they do not contribute anything to any others, they should not receive anything. These two requirements ensure that the value distribution or value sharing is fair or egalitarian, regardless of anything else like the wealth distribution among actors. A third requirement is that the total jointly created value is fully distributed, i.e. nothing is left to distribute ("nothing is left on the table"). If in addition it is assumed that the values received by an entity from systems with different value functions simply could be added, then it can be shown that a unique value distribution exists, namely the Shapley value distribution.[19] Now it turns out that there is a simple way to compute the Shapley value from any value function (not just one for strong CS-systems).[20] As mentioned earlier any value function $V(\boldsymbol{x})$ could be expressed as a multilinear polynomial in the entity state variables x_i:

$$V(\boldsymbol{x}) = \sum_{S\subseteq N} V_S \cdot x^S$$

where V_S are coefficients, which turn out to be $V_S = \sum_{R\subseteq S}(-1)^{|S|-|R|}\cdot V(R)$, and x^S denotes $\prod_{i\in S} x_i$ for simplicity. Then the Shapley value simply becomes the sum of these coefficients normalized by the size of the subset S, i.e. $Sh_i(N) = \sum_{\substack{S\subseteq N\\ i\in S}} V_s/|S|$.

Using this way to compute the Shapley values for the entities in Fig. 3a gives: $Sh_A = 1/2 + 1/2 - 1/3 = 2/3$, $Sh_B = 1/2 - 1/3 = 1/6 = Sh_C$ since $V(\boldsymbol{x}) = x_A \cdot x_B + x_A \cdot x_C - x_A \cdot x_B \cdot x_C$ with $V_C = 1$. And for the entities in Fig. 3b: $Sh_A = 1/2 - 1/3 = 1/6 = Sh_B$, $Sh_C = 1 - 1/3 = 2/3$, since $V(\boldsymbol{x}) = x_A \cdot x_B + x_C - x_A \cdot x_B \cdot x_C$.

Thus although their value functions differ, the Shapley value distribution is the same (apart from labelling) in these two examples, but quite different from the value distribution based on Birnbaum's measure of structural importance.

In summary, there are several ways to operationalize a measure of structural importance, the Shapley value being one which is rooted in cooperative game theory. A measure of structural importance could then be used as a basis for allocating or sharing the value created by a coalition among its members. This would require the members to make binding commitments to stick to the basis in their bargaining process, however.

3.9. Value appropriation/capture and value sharing functions in general

So far we have shown how value creation in an ecosystem can be modelled by means of a value creation function and its dependence upon the structure and states of the ecosystem in terms of CS-relations. The question now is how the value created (or destroyed) is or should be

[16] See e.g. Burt (1995) and Powell (1990) for further readings.

[17] The use of this approach has been observed in behavioral experiments, see Granstrand et al. (2020).

[18] The Shapley value is typically used when entities are actors or players and subsets of them are coalitions as in cooperative game theory, in which context the value originated in the early 1950s, see Shapley (1953) and Roth (1988). Nothing prevents the Shapley value in the abstract to be used for artifacts as well, however.

[19] There are several axiomatizations and proofs in the literature, see Shapley (1953) for the original one and Gilles (2010) for more examples.

[20] A new proof is given in Granstrand (2014).

allocated (≈ appropriated, captured, shared, distributed, divided) across actors and artifacts.[21] This brings in competitive game theory alongside cooperative (non-competitive) game theory in the analysis. The value created at system level over time results in or from pay-offs for the various actors directly involved, say as users and producers of innovations, and typically also for third parties subjected to positive and negative externalities.[22] Given a value function that specifies the value each coalition of actors can create for themselves there are many ways or allocation rules to derive a pay-off function that specifies the pay-offs attainable for each individual actor in a coalition.[23] Conversely, given a pay-off function specifying attainable value for each individual actor as a function of the actions or strategies of all actors, there are many ways or aggregation rules to aggregate these pay-offs and transform them into a value function for coalitional values, see e.g. Gilles (2010).[24] There are also many ways to combine cooperative and competitive games e.g. in layers or stages.[25]

Thus a cooperative game can not only be combined with but also be extended to a competitive game by tagging on a rule (function, mapping) for sharing the value attainable in some way by each coalition and in addition letting the choice of this rule depend upon the actions (activities, strategies) available for each actor. Conversely, a competitive game can be extended to a cooperative game by tagging on a rule (function, mapping) for transforming individual values (pay-offs) to coalitional values. A game, be it cooperative or competitive, extended in these ways can be referred to as a *coopetitive game*.[26] Formally, a coopetitive game in normal form is defined here as a tuple (N, A, π, V) where N is a set of actors or artifacts, A an action, strategy or activity set, π an individual pay-off or value function, and V a coalitional pay-off or value function. Note that π and V here are extended functions differing from the ones in pure competitive and cooperative games.

Coopetitive game modelling in general should reflect the nature of bargaining or the bargaining regime involved in the social choice of allocation and aggregation rules. In a pure competitive game competitive bargaining would reign, while in a pure cooperative game cooperative bargaining would reign. Loosely speaking both bargaining regimes involve a mixture of individual and coalitional power and fairness considerations, albeit in different proportions. There are also different fairness types and principles, some of which take bargaining power into account, as done in the search for a bargaining solution in axiomatic bargaining, or in the search for a cooperative solution in coalitional bargaining.[27] The most important cooperative solution is the Shapley value, which also could be seen as a generalization of a bargaining solution.

Another important solution concept in cooperative game theory is the core, which is the set of allocations of coalitional value to individuals for which no coalition has an incentive to deviate from, i.e. those allocations that constitute an equilibrium for coalitions, similar to the Nash equilibrium for individuals in a competitive game.

Nevertheless, explication of the different measures and the fairness principles behind them offers the opportunity for actors to decide beforehand in a pre-contractual (pre-play) phase to specify the fairness principle to be used, e.g. in a collaborative open innovation project or in technology licensing under so called FRAND terms, i.e. fair, reasonable and non-discriminatory terms. However, what is fair or not is clouded with substantial ambiguity, which is a well-recognized problem in licensing businesses. A specific fairness principle could also be invoked by a court, e.g. in damage calculations in case of infringements of intellectual property rights or contract breaches. Different fairness principles and measures of structural importance give different bargaining outcomes in general, in turn different from competitive bargaining.

3.10. Modelling ownership and control relations in an innovation ecosystem

The modelling approach of a general system presented in the preceding sections applies to an actor system as well as to an artifact system alike, as the examples given also illustrate. This section presents how the actor and artifact system can be joined together as a full ecosystem, especially a full innovation ecosystem, together with relations between actors and artifacts, like legal and organizational or managerial relations. Usually the literature on innovation ecosystems focuses either on actors or artifacts or on a mixture of them, with a less clear exposition of the actor-artifact relations.[28] Actor-artifact relations such as ownership or organizational boundaries in the actor and artifact system typically change as a result of activities, be they innovative activities, such as R&D, or not such as ordinary trade by actors on markets for artifacts.

Property rights or more generally rights for ownership and control play a pivotal role for governance of markets and management of firms and other organizations and arguably also for innovations. An allocation to various actors of ownership rights over various artifacts (excluding joint ownership) essentially induces a partition of the set of artifacts into a disjoint union of subsets or modules of privately owned artifacts, one subset for each actor plus possibly one subset of publicly owned artifacts. Thereby an ownership structure is created in the artifact system with ownership relations between the artifact and the actor system. Allocation of non-exclusive usage rights e.g. licensing rights, does not induce a partition of the artifact system into owner modules, however, since an artifact can have many users.

These ownership relations exemplify how institutionally imposed legal relations can be looked upon as a function or mapping L from the artifact system to the actor system. In other words $L : M \rightarrow N$ if we label the artifacts $i = 1, \ldots, m$ and M is the set of them and the actors $j = 1, \ldots, n$ and N is the set of them and moreover assume that each artifact has exactly one owner. (One could also include a specific actor or representative for publicly owned artifacts as well as for a group of owners.) The different subsets of artifacts owned by different actors could then be looked upon as modules and an allocation of ownership rights could be looked upon as a modularization. The structure of CS-relations, be they strong or not, in the artifact system is then inducing a new structure of CS-relations among these modules through the ownership modularization. Any CS-relations among owner modules in turn induce CS-relations

[21] Terms in parenthesis could be taken as synonyms or near synonymous in this context.

[22] Value sharing approaches could as well be used for cost sharing, damage calculations and the like.

[23] Allocation rules or procedures are also referred to as splitting (division) rules, sharing rules or sharing functions, and are closely related to imputations in game theory and to bargaining solutions and cooperative solutions in axiomatic bargaining approaches.

[24] Such aggregation rules are closely related to social welfare functions (defined on utility spaces) and more remotely related to social welfare aggregators or functionals (defined on preference relations).

[25] See e.g. Karlin and Peres (2017). See also Brandenburger and Stuart (2007) for a pioneering approach to combine a pre-play game with subsequent cooperative or competitive games into what is named as "biform games".

[26] One could also extend the structure of a game, e.g. with an information and communication structure among actors or artifacts, in which case the tuple is extended. However, one seldom thinks about games as tuples in practice.

[27] See Mas-Colell et al. (1995) for formal definitions of these concepts. The search for bargaining and cooperative solutions is guided by a number of desirable properties of solutions as rules for value sharing such as being Paretian, invariant under permutations and consistent under changes in the actor or artifact set, properties that could be taken as axioms in an axiomatic solution approach.

[28] See Granstrand and Holgersson (2020) for a review. An early example, although without the label innovation ecosystem, is the seminal work Porter (1980) focusing with its qualitative "5-forces model" on actors (competitors, buyers, suppliers) together with artifacts (technological substitutes, products, and processes) but without explicit actor-artifact relations.

among actors, in turn reflected in a value creation function for them, being the value creation function defined on the set of owner modules.

Fig. 9a illustrates how ownership relations map the artifacts onto the actors and give rise to a modularization of the artifact system, and thereby partition the artifact system. At the same time the value creation function defined on subsets of artifacts is transformed into a value creation function defined on subsets of owner modules which could be taken as a value creation function defined on subsets (coalitions) of owners.

Fig. 9b illustrates with an example how a structure of CS-relations, strong in this case, are transformed by ownership relations to CS-relations in the actor system. At the same time the value creation function for the artifact system is transformed into a value creation function for the actor system. The CS-relations implies that there is competitive or substitute relations between actors A and B and between actors D and C, while there are complementary or cooperative relations between A and D, B and C, and B and D. Finally, actors A and C have mixed cooperative and competitive relations, i.e. they are in *coopetition* with each other. The value creation function for the actor system also displays bivariate economies of scope for the actors A and C, A and D, and B and C, and trivariate diseconomies of scope for the triples A, B and C, and A, C and D.

Measures of structural importance of artifacts, such as Shapley values, are also transformed by ownership relations into measures of structural importance of actors, as Fig. 9b also shows.[29] As seen the Shapley values for actors do not equal the sum of Shapley values for their owned artifacts, which is a rule rather than an exception.

Fig. 9b could also be taken as an illustration of changes in ownership structure. Consider an initial situation where each artifact is owned by a separate actor along a value chain with 4 suppliers, 3 firms and two buyers. (This is the situation in example 2 in Brandenburger and Stuart (1996, pp. 15-16).) The owner of artifact 5 now integrates forward and acquires artifact 8, while the owner of 4 acquires 9, and the owner of 1 integrates horizontally and acquires 2 and 3, while the owner of 6 acquires 7. These ownership changes (happening over time) result in the actor relations to the right in the figure. The value creation function and the measures of structural importance of actors change accordingly.

The impact of ownership changes on value creation and capture functions in general raises important issues regarding incentives for actors to trade, and optimality and equilibria of ownership structures. Other mappings or correspondences between artifacts and actors might also be considered, e.g. allocation of non-exclusive usage rights through licensing. However, then the modularization is no longer proper since any artifact may have many users. These issues fall outside the scope of this article, however.

4. Innovation ecosystem evolution

An ecosystem evolves over time as a result of endogenous activities and exogenous events among artifacts and actors.[30] Structural changes in the artifact system then induce structural changes in the actor system through the ownership and control relations and vice versa. These relations are changed by trade and exchange among actors operating on various markets for resources, products, services and equity. As for an innovation ecosystem, new artifacts are created through R&D, some of which are protected by temporary and transferable IPRs, some of which give rise to newly created firms in the actor system, some of which are being acquired by other firms and so on. Thus entries and exits and trade occur and interact at both artifact and actor levels. In fact much of the dynamics in the evolution of an ecosystem derives from such interaction as the following stylized example tries to illustrate in qualitative rather than quantitative terms for brevity.

Consider a new radical product innovation x_1 developed and patented by a firm or actor A in stage I.

Another firm B develops in stage II a complementary process innovation x_2 and keeps it secret. Both x_1 and x_2 are necessary for value creation and neither has a stand-alone value. A buys a usage control right in form of a know-how license from B for x_2 and pays a royalty as a percentage of product sales derived from the innovations x_1 and x_2. This makes the actor system and the artifact system structurally different regarding usage control rights (but not regarding ownership rights) with different value creation and capture functions. Fig. 10 shows the artifact structure with usage control rights in stage II after A has gained usage control rights to x_2 from B. (Note that the figure just partially illustrates the ecosystem for clarity.)

In stage III a new start-up firm C enters with a substitute process innovation x_3. The artifact system now has the same structure as in Fig. 3a, see Fig. 11. Each actor owns one artifact but actor A has in addition user rights to B's innovation x_2 and A and B shares the value created thereby. In stage IV A acquires C and switches process technology to x_3. The structure of CS-relations in the artifact system is unaltered hereby but the structure of the actor system is changed. B is now left out and unable to capture any value (and is also left out of the value creation equation for the actor system) and A captures all value (some of which was used to pay for C).

In stage V a large diversifying firm D enters from another industry (ecosystem) with a generic scalable improvement x_2^+ of the process technology x_2 adding value δ where $\delta > 0$, as shown in Fig. 12. D acquires the ailing B. After some time stage V evolves into stage VI as actor A's patent x_1 expires (which corresponds to setting $x_1 = 1$ in the value creation function), as shown in Fig. 13. A still has the know-how of B's process technology but D denies A access to its improvement x_2^+.

D can now capture all value, while A is marginalized and exits.

As seen from this stylized example of ecosystem evolution the structures of the artifact and actor systems change in different but interrelated ways. The structure of the artifact system has evolved with entries due to R&D activities and contractual relations and exits due to economic conditions and institutional rules, patent expiry in this example. The actor system has completely changed due to entry and exit strategies and trade activities on technology as well as on product and equity markets, spurred by changes in the artifact system. These changes in turn could be seen as resulting from R&D and IP strategies deployed by actors in response to value creation and capture consequences of each others' moves in a compounded game on technology, product and equity markets.

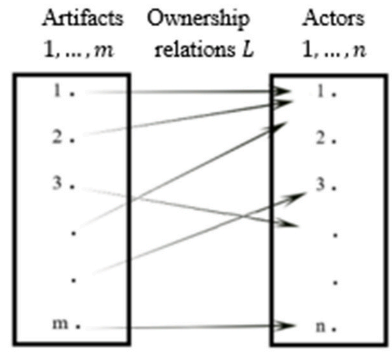
5. Formal game theoretic representation of an (innovation) ecosystem

The examples in the preceding sections show how both the actor system and the artifact system in an ecosystem could be characterized as a cooperative game with a value creation function which assigns values to each coalition of actors or subsets of artifacts, and an individual payoff function which assigns values to each individual actor or artifact. Thus these two functions assign coalitional values and individual values respectively to both actors and artifacts. In order to align the terminology more with the management strategy literature the game-theoretic terms value function and pay-off function could be referred to as a collective value creation function and an individual value capture

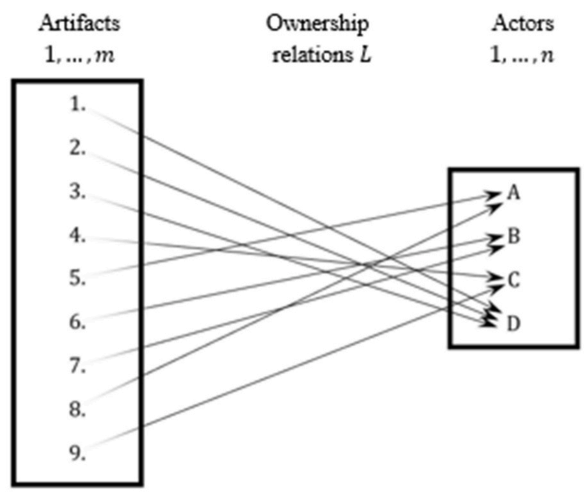
[29] The Shapley values for artifacts are straightforward but tedious to compute and the figure only gives the final results.

[30] For theoretical works on emergence and evolution of innovation ecosystems, see Section 2.

a) Ownership relations as a bipartite many-to-one graph in principle

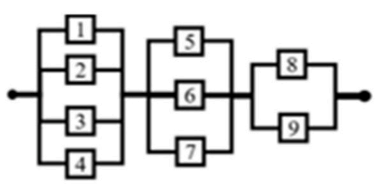


b) Example of ownership relations:



Thus $L(1,2,3,4,5,6,7,8,9) = (D,D,D,C,A,B,B,A,C)$

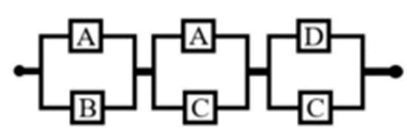
<u>Artifact system with given CS-relations</u>



Value creation function: $V(\boldsymbol{x}) = V_0(\coprod_{i=1}^{4} x_i)(\coprod_{i=5}^{7} x_i)(\coprod_{i=8}^{9} x_i)$

Value capture with Shapley values:
$Sh_1 = Sh_2 = Sh_3 = Sh_4 = 65/1260$
$Sh_5 = Sh_6 = Sh_7 = 128/1260$
$Sh_8 = Sh_9 = 308/1260$

<u>Actor system with CS-relations induced by ownership relations</u>



Value creation function: $W(A,B,C,D) =$
$= V_0(A \sqcup B)(A \sqcup C)(D \sqcup C) = \cdots =$
$= V_0(AC + AD + BC - ABC - ACD)$

Value capture with Shapley values
(as shares of V_0):
$Sh_A = 1/3 = Sh_C$,
$Sh_B = 1/6 = Sh_D$.

<u>Induced CS-relations in the actor system:</u>
Competition between: A and B, D and C
Cooperation between: A and D, B and C, B and D
Coopetition between: A and C

Fig. 9. Illustrations of an ecosystem with artifact and actor ecosystems with ownership relations and CS-relations.

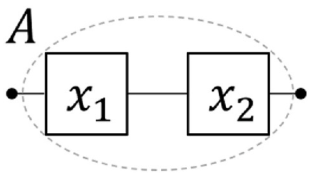


Fig. 10. Product innovation x_1 (invented by actor A) with a strongly complementary process innovation x_2 (invented by actor B) in Stage II after actor A has gained control rights (dashed line) to x_2 from actor B.

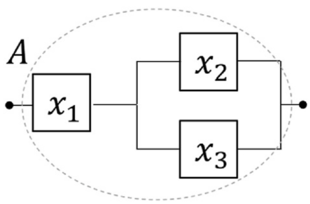


Fig. 11. Artifact system structure with control rights (dashed line) in Stage IV after actor A has acquired actor C, the inventor of x_3, a strong substitute to x_2.

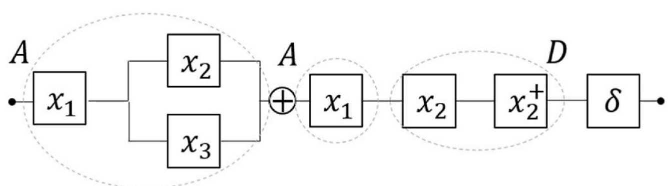


Fig. 12. Artifact system structure with control rights (dashed lines) in Stage V evolving from Stage IV with the new entrant D's process improvement x_2^+ of the process innovation x_2.

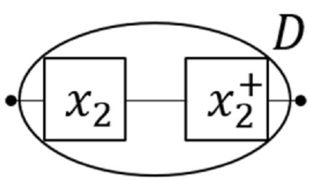


Fig. 13. Artifact system structure with ownership rights (solid line) in Stage VI after expiration of actor A's patent in x_1 and A's exit.

function respectively.[31] Complementary and substitute relations, strong as well as weak, typically exist and vary across actors and artifacts and change over time. These relations are reflected in the value creation function as well as in the value capture function. Ownership and control relations between actors and artifacts are represented by a mapping that maps artifacts to actors. If property rights in artifacts are exclusive the mapping will be many-to-one. If usage rights in artifacts are non-exclusive the mapping will be many-to-many, that is a correspondence between artifacts and actors that is more general than a many-to-one mapping.

Thus an ecosystem can be formally represented as a pair of cooperative games with the game (M,V,π) for the artifact system and the game (N,W,r) for the actor system, together with a mapping or correspondence L from M to N that links artifacts to actors. Here M is the set of artifacts $i = 1,..,m$ and N is the set of actors $j = 1,...,n$; V and W are the value (creation) functions defined for the various subsets of artifacts and the various coalitions of actors respectively; and π and r are the individual pay-off functions for actors and artifacts respectively.[32] The value creation functions and the value capture functions then reflect the performance of the ecosystem at collective and individual levels. A key feature of the representation is a state space approach, that is expressing the value creation and value capture functions in terms of state variables x_i for artifacts and y_j for actors. Such variables generally describe the state of each entity, in particular whether or not they are part of a given subset or coalition at a given point in time. This approach allows the value creation functions V and W to be expanded into multilinear polynomials for easier modelling and calculations, for example of CS-relations, modular structure, measures of structural importance and the value capture functions π and r. This is not the least useful in analysis of more complex ecosystems. Both the artifact game and the actor game could be made dynamic by introducing a time variable in the value functions. These games could also be made stochastic by introducing probabilities for the state variables used for expressing the value (creation) functions in multilinear polynomial forms as described in Section 3.1. Finally, more games could be introduced and combined, e.g. on different levels of dis-aggregation or in different stages.

The alert reader might ask where the activities and institutions end up in this formalization. The answer is that activities enter as strategies in the games and institutions enter as rules of the games and how the games are linked by the correspondence L reflecting the allocation of ownership and control rights. The value creation functions V and W then become dependent upon activities or strategies as well as upon institutional rules.[33] In case this dependency needs to be made explicit one can let the value creation functions V and W depend on activity levels or strategies a in an action set A, i.e. V and W become $V(a)$ and $W(a)$, $a \in A$. The cooperative games then formally become coopetitive games.[34]

It must be noted that this formal definition applies to any ecosystem in general. There is no formal requirement that the activities should be innovative for example. Thus, to repeat, any ecosystem could be formally defined as being constituted by a cooperative actor game, a cooperative artifact game, and a correspondence linking artifacts and actors. The value creation functions and the value capture functions could then be expressed in terms of the state variables for the actors and artifacts. Both cooperative and competitive relations among actors and complementary and substitute relations among artifacts and the ensuing economies and diseconomies of scope are captured in this way.[35] Since any game can be decomposed into subgames this definition allows for breaking up a complex ecosystem into interrelated subgames and view it as a system of games. Reversely, the analysis of a complex ecosystem may be simplified by modularization.

As seen, the definitional elements of the qualitative definition of an innovation ecosystem as given in Section 1, are covered by the formal representation or definition above, and the two definitions are compatible. Representing an innovation ecosystem as a linked pair of

[31] An individual pay-off function also goes by names as value sharing function or scheme or value distribution or allocation rule or appropriation function. If the individual pay-off function is efficient, in the sense that all collectively created value is distributed among individual entities, and individually rational, in the sense that no individual entity gets less than its stand-alone value, then the individual pay-off function is called an imputation.

[32] The individual pay-off functions are here taken as part of the defining characteristics of a cooperative game, although a standard definition of a cooperative game only refers to the value (creation) function.

[33] See the pioneering works by Hart and Moore (1990) and Brandenburger and Stuart (2007) for early examples in the economics and management literature of how value creation functions are made dependent upon activities or strategies.

[34] Arguably only actors may have conscious strategies while artifacts have not, although having various levels of activities, possibly decided automatically (let alone consciously) on their own. If need arises to make the latter explicit, e.g. for autonomous multi-agent systems, one can introduce a strategy set in the artifact system as well (cf. Lou et al., 2004).

[35] The term cooperative game is somewhat misleading since it may wrongly suggest that there are only cooperative and complementary relations and that no competition or substitution takes place. This is even more so when a cooperative game is referred to as a non-competitive game.

cooperative games provides several theoretical as well as practical advantages. First, the formal representation of an ecosystem and the use of state variables ties into game theory and systems theory which provide a solid theoretical basis for studies of innovation ecosystems. This basis can be drawn upon also in further theoretical developments and cross-fertilizations. Second, the state space approach provides analytical and computational AI tools for decision support in assessing value creation and capture in various practical applications, such as fair division of returns from open innovation projects, licensing in ecosystems on FRAND terms, and valuation of patent strategies and structured patent portfolios. The approach also facilitates structural equation modelling in statistical analysis of ecosystems in general. Third, the qualitative and quantitative approach complement each other and enables a number of issues to be addressed in further research with a proper mix of methods. Examples of such issues are determinants behind structural changes in ecosystems and ecosystem evolution, dynamic interaction between different ecosystems, and ecosystem design.

6. Summary and conclusions

The purpose of this article has been three-fold – to follow up and substantiate some of the possibilities for further developments as claimed in Granstrand and Holgersson (2020), to develop an original theoretical approach grounded in systems theory and cooperative game theory to analyze ecosystems in general, including innovation ecosystems, and finally to propose a formal representation and definition of an ecosystem.

With regard to the first purpose, the article elaborated on how the definition of an innovation ecosystem could be generalized to any ecosystem. The focus on the complementary or cooperative and substitute or competitive relations, dubbed CS-relations here, between artifacts and actors in this definition enabled the use of established concepts in economics and industrial organization, such as economies and diseconomies of scope. The focus enabled theorizing along the lines of both cooperative and competitive game theory and thereby also led to the introduction of coopetitive games as an extended combination of both.

As for the second purpose, the article first reviewed theory approaches to ecosystem analysis in previous research to assess the originality of the approach thereafter presented. The article then illustrated how to graphically model strong (or approximately strong) CS-relations as entities linked in series and in parallel and how to express value creation and value capture functions in an ecosystem. With the use of modularization, ecosystems could more easily be structured and analyzed. Further, the article illustrated how the value creation function of a system can be expressed as a function of state variables, and why this is particularly useful for analysis, as the CS-relations can then be represented algebraically as multiplication and substication. The use of game theory further helped illustrate how the Shapley value lends itself particularly useful for calculating and modelling structural importance and value capture in various ecosystems with general CS-relations, strong or weak.

In conclusion, and with regard to the third purpose of the article, a formal definition and representation of a general ecosystem was proposed as being a pair of cooperative, or more generally coopetitive, games linked by a map between them, representing ownership and control relations between artifacts and actors. This mapping can be many-to-many or many-to-one, depending on whether or not property and usage control rights are exclusive or non-exclusive. This formal approach is then not limited to innovation ecosystems.

Along the presentation the article provided a number of simple stylized examples, illustrating how the key concepts and theoretical framework could be useful in analyzing value creation and capture in various ecosystems, be they technical and/or economic, or even biological, with particular reference to value chains and evolving innovation ecosystems. Although these simple examples fall short of depicting the intricacies and complexities of actual ecosystems, the presented conceptual and theoretical framework hopefully proves useful for future research and applied ecosystems analysis. More research is then needed, both theoretical research on properties of coopetitive games in various cooperative, competitive and also evolutionary settings, and applied research on various types of ecosystems – technological, innovation and industrial etc. as well as biological ones.

Declarations of competing interest

None.

Data availability

No data was used for the research described in the article.

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