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Phase-locked, pre-amplified optical injection locking at low input powers

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Abstract: Optical injection locking generally occurs when light from a master laser is unidirectionally injected into a slave laser, such that the injected light overcomes spontaneous emission inside the cavity, and forces the slave laser to behave as a frequency copy of the master. Here, we study the limits of stability for optically pre-amplified optical injection locking in the case of large added noise on the input field and in the presence of a phase locked loop which minimizes the frequency offset between master and slave lasers. We present a set of modified rate equations which we use to describe the physics of the system near the limit of stable injection locking, and report on phase slips which occur due to injected noise momentarily destabilizing the system. We then provide experimental evidence to support the behavior seen in simulation, and are able to successfully recover a CW wave at -80 dBm black box input power (-70 dBm for phase slip free operation), providing 20 dBm of output power from the injection locked slave laser.

1. Introduction

Optical injection locking (OIL) [1] is the process by which two or more optical cavities exchange energy in such a way that the coupled system begins to oscillate at a single frequency. Typically, coherent light from a master laser is uni-directionally injected into a slave laser which lacks isolation, either in reflection mode (via a circulator) or transmission mode. If there is a small enough frequency offset between the injected optical field and the unperturbed frequency of the slave laser, and if the amplitude of the injected field is strong enough to overcome spontaneous emission in the cavity, then the final output of the complete optical system will act as a frequency copy of the master laser with a fixed phase offset which depends on the frequency difference between the injected field and the slave laser's unperturbed frequency [1].

Optical injection locking has a variety of uses, notably i) its ability to force noisy slave lasers to act as high quality frequency copies of a low noise master laser for optical reference distribution [1], ii) its ability to reduce chirp in intensity modulation schemes [2,3], iii) its ability to extend the modulation bandwidth of the slave laser [4,5], and iv) its ability to simultaneously amplify and filter light for applications such as tone selection, pump recovery and narrowband optical amplification [6,7].

As seen in Fig. 1 (shown here for illustrative purposes, with details provided later in the text), and using the standard approximations [1], the conditions for optical injection locking are determined by two parameters: first by the injection ratio (IR), defined as the power ratio between the input field strength and internal field strength of the slave laser, and second by frequency offset between the injected master and unperturbed slave laser optical fields, $\Delta\omega$. For the case of simultaneously low IR and $\Delta\omega$, a laser with finite spectral bandwidth may contain some frequency components with intensity inside the locking bandwidth and some frequency components outside. In this case, it is possible for noise on the input field to momentarily unlock the slave laser and induce periods of punctuated equilibrium. This instability often sets the lower limit on pre-amplified optical injection locking, and is the subject of this work.

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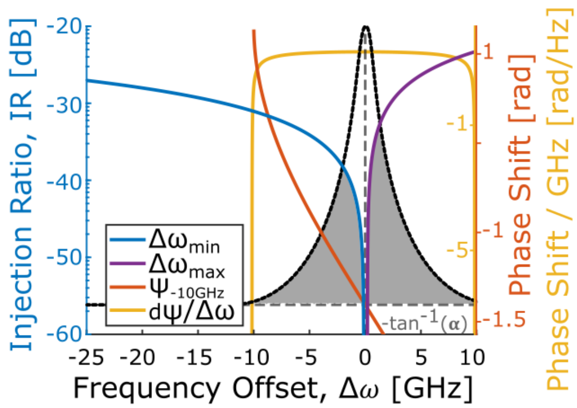

Fig. 1. Simulations of the injection locking bandwidth, $\Delta\omega_{LB} = \Delta\omega_{max} - \Delta\omega_{min}$, the fixed phase shift, Ψ between the injected master field and slave laser output, and the derivative of said phase shift with respect to $\Delta\omega$. The asymmetric locking bandwidth is set by the maximum allowable negative ($\Delta\omega_{min}$) and positive ($\Delta\omega_{max}$) frequency detunings between master and slave lasers, as well as the ratio of injected field strength to the slave laser's unperturbed field strength, IR. Calculations of Ψ and $\frac{d\Psi}{d\Delta\omega}$ assume a $\Delta\omega_{min}$ of 10 GHz and are poorly defined outside the locking bandwidth. A laser with finite linewidth is shown underneath the locking stability curves to illustrate the fact that a finite linewidth input field may provide power at some frequency components within the locking bandwidth and some frequency components outside the bandwidth, keeping in mind that ω_{LB} is a function of the input power via the injection ratio.

Here, we introduce the full set of equations and corresponding 4th order Runge-Kutta simulations for optical injection locking of a noisy slave laser (OIL rate equations which include β terms corresponding to the spontaneous emission factor of the slave laser), and then extend these to investigate the impact of added white phase / frequency noise [8–10]. This is done by modifying the $\Delta\omega$ term to include an additional white noise term, $\partial\omega$, discussed in detail later.

In particular, we are interested in the system behavior at the limits of stable optical injection locking and investigating the role of injected white noise in the case of low signal power, low static offset frequency and strong injected noise. We report on phase slips in the system that occur due to competing signal and noise terms (and which are present in both simulation and experiment) at the boundaries between the locked and unlocked states. Moreover, we also show that by carefully managing the frequency difference between the two lasers using a fast operating phase locked loop (PLL) and significantly pre-amplifying a low noise master laser, injection locking can be achieved at ultra low black box input power to recover low power CW waves [11–13]. Experimentally, we demonstrate recovery of continuous wave (CW) input signals by locking a commercial distributed feedback laser (DFB) slave laser to an ultra-low noise (ULN) fiber laser at -80 dBm (-70 dBm for phase slip free operation) black box input power, producing 20 dBm of power at the output. To our knowledge, this represents the lowest power recovery of a CW signal to date via injection locking.

2. Theory and simulation

The optical setup used is shown in Fig. 2: light from the master laser is pre-amplified and split in two, after which one arm is used as a reference, while the other arm is used to injection lock the slave laser. The reference arm is modulated to produce side bands which are then beat against the output of the slave laser for phase difference detection. When locked, the slave laser will act a frequency copy of the master laser which has picked up a phase shift that depends on the

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frequency difference between the injected field and unperturbed slave laser, while the reference arm has experienced *no* phase shift due to locking. This means that by stabilizing the detected phase difference, we stabilize the offset frequency between master and slave lasers and therefore the stability of the lock in turn. Moreover, because the phase shift which is picked up by the slave laser is least sensitive to frequency offset at the point of zero frequency offset, which is also the point of lowest required input power for injection locking, then by locking the detected phase difference to this point using the PLL, ultra low power injection locking with minimal added noise can be achieved.

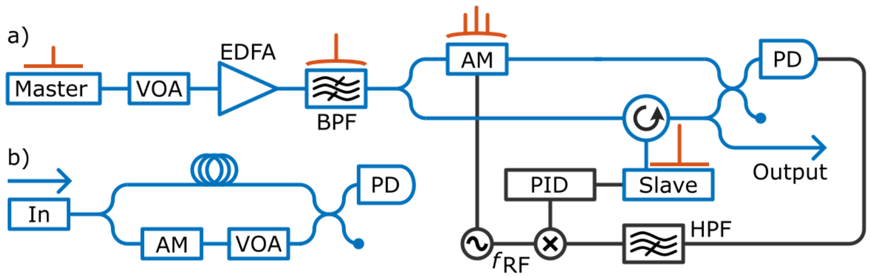

Fig. 2. a) The optical setup (blue) used in experiments to demonstrate pre-amplified, phase locked, optical injection locking. Light is sent from the master laser into a variable optical attenuator (VOA), an erbium doped fiber amplifier (EDFA), and finally a band pass filter (BPF) before being split in two. The amplitude modulator (AM) and slave laser lie on separate arms, but these arms are recombined before reaching the photodiode (PD). The output electrical signal (black) is sent through a high pass filter (HPF). This signal is then mixed with the RF modulation frequency, f_{RF}, and is sent to the proportional, integral, and differential (PID) control electronics which feed back on the slave laser current. b) The delayed self heterodyne interferometer used for phase noise measurements of the master laser and slave laser output.

Mathematically, optical injection locking (OIL) of a noisy slave laser can be described using the standard laser rate equations for a semiconductor laser [14], modified to include injection ($\Delta\omega$ and $|E|_{\text{inj}}$) and spontaneous emission (β) terms [8–10]:

$$\frac{d|E|}{dt} = \frac{1}{2}\{[G_n(N-N_0)/(1+s|E|^2)] - \frac{1}{\tau_p}\}|E| + \kappa|E|_{\text{inj}}\cos(\Psi) + \frac{2\beta N}{|E|} + \sqrt{\beta N}\xi_{|E|}, \quad (1)$$

$$\frac{d\Psi}{dt} = \frac{\alpha}{2}\{[G_n(N-N_0)/(1+s|E|^2)] - \frac{1}{\tau_p}\} - \Delta\omega - \kappa\frac{|E|_{\text{inj}}}{|E|}\sin(\Psi) + \frac{\sqrt{\beta N}}{|E|}\xi_\phi, \quad (2)$$

$$\frac{dN}{dt} = R_p - \frac{N}{\tau_r} - [G_n(N-N_0)/(1+s|E|^2)]|E|^2, \quad (3)$$

where $|E|(t)$ from $\mathcal{E}_s(t) = |E|(t)e^{i\omega_s t}$ represents the slowly varying internal electric field of the semiconductor slave laser normalized to mode volume and photon count, $\Psi(t) = \phi_s(t) - \Delta\omega t$ represents the phase difference between slave laser and the injected field, and $N(t)$ represents carrier concentration inside the cavity. Furthermore, Table 1 provides details of notation, and values which, in simulation, produce stable optical injection locking via fourth-order Runge-Kutta simulations over a period of 10 microseconds, with a step size of 1 picosecond.

Generally speaking, we say that the system is "locked" when we find steady state solutions to $\Psi(t)$, and "unlocked" when $\Psi(t)$ is either periodic, is trivially affected by the injected field and steadily advances to $\pm\infty$, or has devolved into chaos, none of which behaviors are within the scope of this paper. Broadly speaking, the locking condition on Ψ is satisfied when the slave

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Table 1. Simulation parameters (unless otherwise specified)

ω_s	unperturbed slave laser frequency	$= 2\pi \cdot 192.5 \cdot 10^{12}\ \mathrm{s}^{-1}$
$\Delta\omega$	fixed frequency detuning between fields	$= 1.0 \cdot 10^{-9}\ \mathrm{s}^{-1}$
G_n	normalized gain coefficient	$= 5.0 \cdot 10^{-13}\ \mathrm{m}^3/\mathrm{s}$
N_0	carrier concentration at transparency	$= 1.0 \cdot 10^{+24}\ \mathrm{m}^{-3}$
s	gain compression coefficient	$= 1.0 \cdot 10^{-24}\ \mathrm{m}^3$
τ_p	photon lifetime in the cavity	$= 3.0 \cdot 10^{-12}\ \mathrm{s}$
κ	coupling coefficient of the injected field	$= 2.0 \cdot 10^{+12}$
$\lvert E\rvert_{\mathrm{inj}}$	amplitude of the injected optical field	$= 1.0 \cdot 10^{+6}\ \mathrm{m}^{-3/2}$
β	spontaneous emission factor	$= 1.0 \cdot 10^{-4}$
$\xi(t)$	dimensionless noise term	with mean 0 and standard deviation 1.0
α	linewidth enhancement factor	$= 6$
τ_R	electron-hole recombination time	$= 2.0 \cdot 10^{-9}\ \mathrm{s}$
R_p	normalized pump parameter	$= 5.0 \cdot 10^{+33}\ \mathrm{m}^{-3}/\mathrm{s}^{-1}$

laser's amplitude-to-phase conversion (α) terms are counteracted by the injection ($\Delta\omega$ and $|E|_{\mathrm{inj}}$) terms, without the noise (β) terms interfering with the lock. That is to say, locking is achieved when the injected field amplitude is strong compared to the frequency offset between master and slave laser, the unperturbed inter-cavity power of the slave laser, and the noise terms which might interfere at the boundaries of stable locking.

To modify these rate equations such that they correspond to the pre-amplified, phase locked scheme studied here, we can replace the frequency offset term ($\Delta\omega$) with $\Delta\omega_{tot} = \Delta\omega + \partial\omega$, where $\partial\omega$ is a stochastic white noise term that represents the time dependent frequency difference between master and slave in the presence of phase / frequency noise with standard deviation σ and mean 0. Importantly, this is a quite general term and may represent frequency difference fluctuations due to either an imperfect PLL, random walk, or added phase / frequency noise due to amplified spontaneous emission (ASE) from pre-amplification. As such, $\partial\omega$ does not have a strict, one-to-one correspondence with the optical signal to noise ratio (OSNR). Thermal noise, or noise from an imperfect PLL, for instance, may drastically affect $\partial\omega$ without changing the OSNR - which is normally defined using the peak signal power and some recovered noise power that is well outside of the laser's finite bandwidth. Here, we are typically interested only in noise which falls within the injection locking bandwidth (on the order of a few to a few tens of GHz), and which is therefore still within the finite bandwidth of the coherent signal being injected, such that OSNR is poorly defined. Furthermore, we neglect intensity noise terms due to the typically small impact of input laser relative intensity noise (RIN) on injection locking when compared to phase / frequency noise due to the lack of restoring force on $\Psi(t)$ [10,15]. In simulation, light from the noiseless master signal ($|E|_{\mathrm{inj}}$) is injected along with white frequency noise ($\partial\omega$) at a constant rate while the slave laser is turned on at time $t = 0$.

Interestingly, we find that when frequency noise on the injected signal (i.e., $\partial\omega$) becomes large enough, uncorrelated photons in the slave laser are able to momentarily unlock the system. During this brief interval of instability, Ψ picks up a phase shift depending on the sign of $\frac{d\Psi}{dt}$ before settling down again around a constant value when the instantaneous frequency difference has returned to a point within the locking bandwidth. In simulation, the magnitude of this phase slip depends on the amplitude of $\frac{d\Psi}{dt}$ (typically dominated by $\Delta\omega_{total} = \Delta\omega + \partial\omega$), the length of the step size, and the number of simulation time steps spent unlocked.

This behavior is demonstrated in Fig. 3, which shows time domain simulations of both the activation (left) and steady state behavior (right) of the electric field magnitude, phase offset, and carrier density as a function of time for the case of typical optical injection locking at $IR \approx -45$

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dB, and is compared against the case of phase locked OIL with strong added white noise, using values provided in Table 1, and a $\partial\omega$ term with standard deviation $\sigma = 1$ GHz. Simulations assume that the injected field power is constant, and that the slave laser is turned on at time $t = 0$.

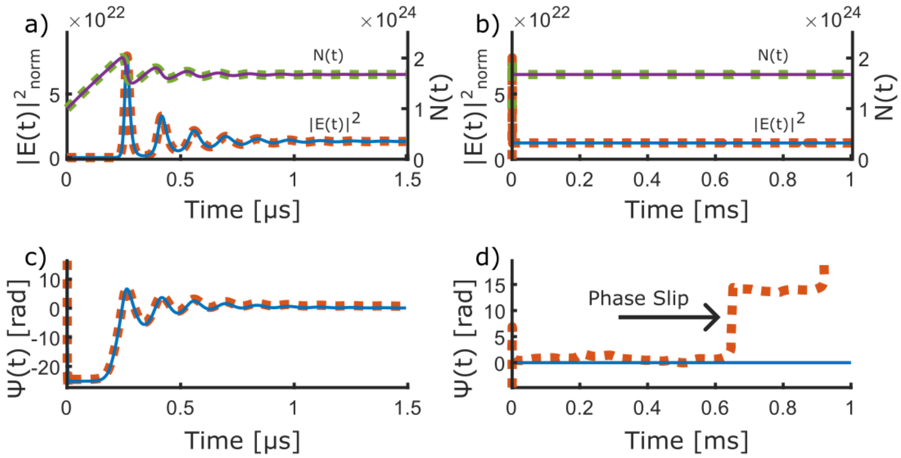

Fig. 3. Activation (left, 1.5 microseconds) and long term stability (right, 1 millisecond) of the electric field magnitude, the phase difference between slave and master laser, and the carrier count as a function of time. In figures a) and b), the blue trace and orange (dotted) traces corresponded to the mode and photon count normalized electric field, while the green and purple (dotted) traces correspond to the carrier count in the cavity. In figures c) and d), the orange and blue dotted trace correspond to the fixed phase offset between master and slave lasers while locked. In all cases, the solid line represents 4th order Runga Kutta simulations of the unmodified rate equations, while the dotted trace corresponds to simulations of the modified rate equations.

Figures 3(a) and (b) show that both the activation behavior and long term stability of both the electric field magnitude and carrier concentration inside the slave laser are largely unaffected by $\partial\omega$, even in the presence of strong noise. This makes sense, given that the slave laser is assumed to maintain constant output power regardless of intensity fluctuations on the master input. However, we do see the strong impact of added white phase / frequency noise on $\Psi(t)$ in figures c) and d). Again, this makes sense, given that the slave laser is assumed to perfectly follow the injected signal (when locked), and in the case of high $\partial\omega$, the injected signal contains a significant amount of noise which is transferred onto the slave laser output. In particular, we take note of the previously mentioned phase slips (Fig. 3(d)) which occur more often as additional white noise is added to the system, and which induce periods of punctuated equilibrium in the time domain trace of Ψ.

In order to better understand the interplay of noise and signal power in the phase slipping process, and as seen in Fig. 4, we next run two sets of simulations, independently varying the strength of the injection ratio (IR) and the injected white frequency noise ($\partial\omega$), while holding all other values at the fixed, non-zero values given in Table 1, unless otherwise stated. In each case, we allow for 5 ms of stabilization to avoid activation effects of the laser, and in each case, we calculate the standard deviation (STD) of phase noise on $\Psi(t)$ between 5 ms and 10 ms after activation.

Figure 4(a) investigates the role of increasing injected signal strength by fixing both β and $\partial\omega$, and varying $|E|_{\mathrm{inj}}$ (and thus the injection ratio, $IR = |E|_{\mathrm{inj}}/|E|_0$, with $|E|_0$ coming from the unperturbed slave laser electric field magnitude at steady state). The first two curves (blue and orange) correspond to the case of moderate to strong amplification ($\partial\omega$ with $\sigma = 10$ MHz)

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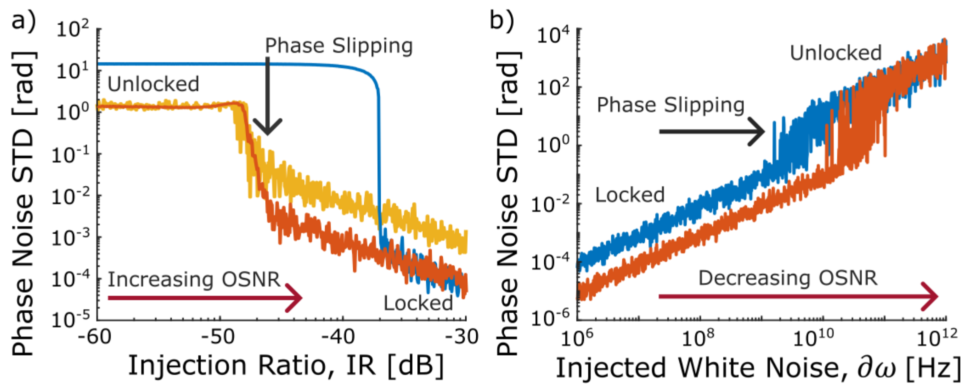

Fig. 4. The simulated standard deviation (STD) of phase noise on $\Psi(t)$ calculated over 5 ms, after the system is given 5 ms to stabilize. Unless otherwise stated, all parameters used correspond with those seen in Table 1. (a) Phase noise STD on $\Psi(t)$ as a function of increasing injection ratio, IR. The blue curve corresponds to a $\Delta\omega$ of -10 GHz, while the orange and yellow traces correspond to a $\Delta\omega$ of -1 GHz. Furthermore, the blue and orange traces correspond to a $\partial\omega$ with σ = 10 MHz while the yellow curve is simulated with σ = 100 MHz. (b) Phase noise STD as a function of increasing σ, with $IR = -40$ and -30 dB for the blue and oranges traces, respectively.

before (blue) and after (orange) the PLL is activated to reduce the mean frequency offset ($\Delta\omega$) from 10 GHz for the blue curve to -1 GHz for the orange curve, and demonstrate how a $\Delta\omega$ offset increases the required injected power for locking. Once locked, however, both the blue and orange curves can be said to follow the injected field perfectly (including the injected white noise on $\partial\omega$). The final curve of Fig. 4(a) (yellow) corresponds to the case of very strong amplification ($\partial\omega$ with σ = 100 MHz) with a PLL which fixes the static frequency offset, $\Delta\omega$, to -1 GHz.

In general, and for all curves in Fig. 4(a), we find that as the injected signal power increases while injected noise power stays constant, the system remains unlocked until the threshold power for injection locking is reached. At this point, we enter a transition region, where injected white noise, if strong enough, is able to momentarily unlock the system and induce periods of punctuated equilibrium, as indicated by variable length plateaus in the time trace of $\Psi(t)$ (as seen in Fig. 3(d)). As the injected signal strength increases relative to the injected noise, phase slips become less common and the phase noise STD on $\Psi(t)$ continues to decrease with increasing OSNR. As suggested here, the frequency and likelihood of phase slipping increases drastically as a function of injected white noise power, an effect investigated in depth later.

Figure 4(b) shows the impact of increasing the amount of injected white noise (corresponding to ASE due to pre-amplification [as opposed to ASE native to the slave laser] or to an imperfect PLL), while holding the injected signal strength constant, corresponding to a decrease in OSNR. Unlike β terms, $\partial\omega$ does have a strong impact on the final output phase noise STD of $\Psi(t)$, given that $\partial\omega$ corresponds to injected white frequency noise on the signal, and the slave laser is assumed to follow the injected signal perfectly. As the injected white noise power increases, the final output phase noise STD on $\Psi(t)$ increases while maintaining lock until the threshold for slipping is reached when the noise power is large enough to temporarily unlock the system. Phase slipping becomes more and more frequent over time, until phase slips are so frequent that periods of stability become rare, and the system can no longer be regarded as truly locked.

The entirety of this process can be visualized by calculating the locking bandwidth of the slave laser, and considering how noise on the frequency offset may impact these locking conditions. We make the standard set of assumptions for convenience [1], and consider white frequency noise on $\Delta\omega$ as the only perturbation to the system, such that we can define the optical locking

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bandwidth as follows:

$$\Delta\omega_{min} = (-\kappa\frac{|E|_{\mathrm{inj}}}{|E|_0})\sqrt{1+\alpha^2} < \Delta\omega + \partial\omega < \kappa\frac{|E|_{\mathrm{inj}}}{|E|_0} = \Delta\omega_{max}. \tag{4}$$

Furthermore, we define the locking bandwidth, $\Delta\omega_{LB}$, as being equal to the frequency difference between $\Delta\omega_{min}$ and $\Delta\omega_{max}$. Using similar math, under steady state conditions, and assuming static noise, we find a fixed phase offset between injection locked master and slave laser equal to [1]

$$\Psi = \sin^{-1}\{\frac{\Delta\omega + \partial\omega}{\Delta\omega_{min}}\} - \tan^{-1}(\alpha). \tag{5}$$

In order to illustrate these findings, simulations of Eqs. (4) and 5 were shown in Fig. 1 to serve as a visual introduction to the concept, with details being provided here. First, we vary the injection ratio between -60 and -20 dB, with fixed κ and α in order to illustrate the asymmetry in locking bandwidth. Then, in the orange trace of Fig. 1, the evolution of Ψ is shown as a function of $\Delta\omega$, where we assume a $\Delta\omega_{min}$ of -10 GHz. An important point to note here is that Ψ both becomes nearly linear, and its second derivative changes sign at $\Delta\omega = 0$.

Assuming that $\Delta\omega \gg 0$ and $\langle\partial\omega\rangle = 0$, then differentiating Ψ with respect to $\Delta\omega$ yields

$$\frac{d\Psi}{d\Delta\omega} = -(\Delta\omega_{min}^2 - \Delta\omega^2)^{-1/2}. \tag{6}$$

The value of $\frac{d\Psi}{d\Delta\omega}$ is always negative, but its absolute value reaches a minimum at $\Delta\omega = 0$, indicating the point of minimal detuning to phase offset noise transfer (i.e., the $\Delta\omega$ for which fluctuations stemming from $\partial\omega$ effect the final slave laser output the least). The actual value at the minimum value depends on $\Delta\omega_{min}$, which in turn depends on the injection ratio. That being said, $\frac{d\Psi}{d\Delta\omega}$ does become quite flat towards the center of the locking bandwidth (as seen in Fig. 1), and as such, can be used to ensure the system does not fall out of lock while IR is reduced.

For any given injection locked system then, we can minimize the residual phase noise between the master and locked slave laser by minimizing their detuning. Importantly, however, $\Delta\omega = 0$ is not only the point of lowest detuning to phase offset noise transfer, it is also the point where the required injection ratio for successful locking is lowest, allowing for ultra low power OIL. In experiments, this is accomplished by manually adjusting the PID set point to minimize the recovered phase noise at the output, while continuously reducing input signal power into the black box device using the VOA. That is to say, while decreasing power into the EDFA (and thus into the slave laser), the injection locking bandwidth decreases. If the locking bandwidth decreases to the point where the frequency offset between master and slave lasers, $\Delta\omega$, is near the boundary of locking, then the frequency noise to phase offset noise transfer will be very high. At this point, the PID set point which controls Ψ (and therefore $\Delta\omega$ in turn), can be changed to minimize phase noise, indicating that you are near the center of the injection locking bandwidth, and the injected power can be further decreased while maintaining a lock.

Importantly, despite the fact that the system's input frequency noise to output phase noise transfer, and its point of lowest required input power are fixed at $\Delta\omega = 0$, this is not actually the point at which the system is most robust to noise, due to the asymmetry induced by α. One can therefore reduce or induce white noise induced phase slips near the boundary by moving away from or towards $\Delta\omega_{min} + \Delta\omega_{LB}/2$, the center of the injection locking bandwidth. Although not exact due to the various approximations made along the way, we can approximate the boundary of locking as follows:

$$\partial\omega_{\mathrm{boundary}} \approx \frac{\kappa}{2}(1 + \sqrt{1+\alpha^2})IR. \tag{7}$$

If we use the estimation of the locking boundary provided by Eq. (7), and solve for $IR = -40$ and -30 dB, corresponding to the blue and orange curves seen in Fig. 4(b), we find a boundary at

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of ~ 0.7 GHz and 7 GHz, respectively. As evident in the same figure, this is approximately the amount of white noise on $\partial\omega$ required to induce phase slips, thereby validating the approach.

3. Experimental results

Experimentally, we implement a phase locked, pre-amplified, optical injection locking scheme, and measure the output optical phase noise PSD of the slave laser using a delayed self heterodyne interferometer as we vary the attenuation of the master laser before pre-amplification. This is done using the same optical setup as described in Fig. 2(b), using a delayed self heterodyne interferometer with delay line length 25 km, an amplitude modulator to shift the recovered signal from base band, and a VOA to balance power between the two arms of the interferometer. Importantly, this corresponds to a variation in injected OSNR created by fixing the noise power and varying the signal power (roughly equivalent to the plot seen in 4(a)).

The assumption of fixed noise power out of the EDFA comes from the fact that any EDFA followed by a BPF produces noise power, P_{ASE} equal to $(G-1)NFh\nu B$, where G is the amplifier gain, NF is the amplifier noise figure, $h\nu$ is the photon energy, and B is the bandwidth of the filter. In the small signal regime, it is reasonable to assume that both the noise figure and gain of the EDFA are fixed, resulting in constant noise power, although this assumption of constant noise power out of the EDFA breaks down in the gain saturated region of the EDFA. The BPF used is a tunable optical filter with a flat-top and 0.1 nm bandwidth. The EDFA used has a rated NF of approximately 5 dB and gain of approximately 30 dB.

Experimental results are shown in Fig. 5. So long as a) the in-band signal power is greater than the in-band noise power, and that b) the signal power is sufficiently strong to lock the slave laser in the presence of all frequency noise sources (ASE, imperfect PLL, etc.), the slave laser can be locked at very low power. Strong amplification, however, degrades the optical signal considerably which corresponds to an increase of the white noise floor at the both the input and output.

For instance, as seen in 5(a), although the device can be said to be locked at -80 dBm black box input power (measured after the VOA / before the EDFA, and the lowest input power to which any locking was possible), we see the white noise level has increased to the point of inducing phase slips – a behavior typical of injection locking at the white noise limit and predicted by our models. This corresponds to a signal which is only marginally improved relative to the DFB slave laser when unlocked (black).

At -70 dBm input power, phase slipping is essentially nonexistent, but can be easily induced by even a slight misalignment of the PID set point (shown). Otherwise, the output signal is stable over long time scales, and its phase noise performance between 1 kHz and 1 MHz is considerably improved compared to the DFB slave laser, although still worse at high offset frequencies.

At -60 dBm of black box input power, phase slipping has become essentially nonexistent, the device is stable over very long time scales, and the phase noise performance of the output is equal to or better than the DFB at all frequencies above 1 kHz.

Additional peaks are seen above 10 kHz and correspond to technical noise from the photoreceiver. For reference, the black curve corresponds to the unlocked DFB slave laser and the olive-brown curve represents the case of OIL with no pre-amplification and an input power of -10 dBm from the low noise master laser.

Figure 5(b) shows the optical spectrum after the amplifier and filter, but before injection into the slave laser, as measured on an optical spectrum analyzer with 0.02 nm resolution. The signal after amplification is still visible down to -70 dBm black box input power, but is buried in noise at powers lower than -80 dBm. Between -80 dBm and -50 dBm of input power, the measured white noise floor is nearly fixed, supporting the assumptions made previously regarding a fixed noise power and variable signal power in these experiments.

The blue curve of Fig. 5(c) shows the white noise floor of the phase noise PSD (measured at 10 kHz) for each curve in Fig. 5(a), and can be roughly equated to simulations from Fig. 4(a). Below

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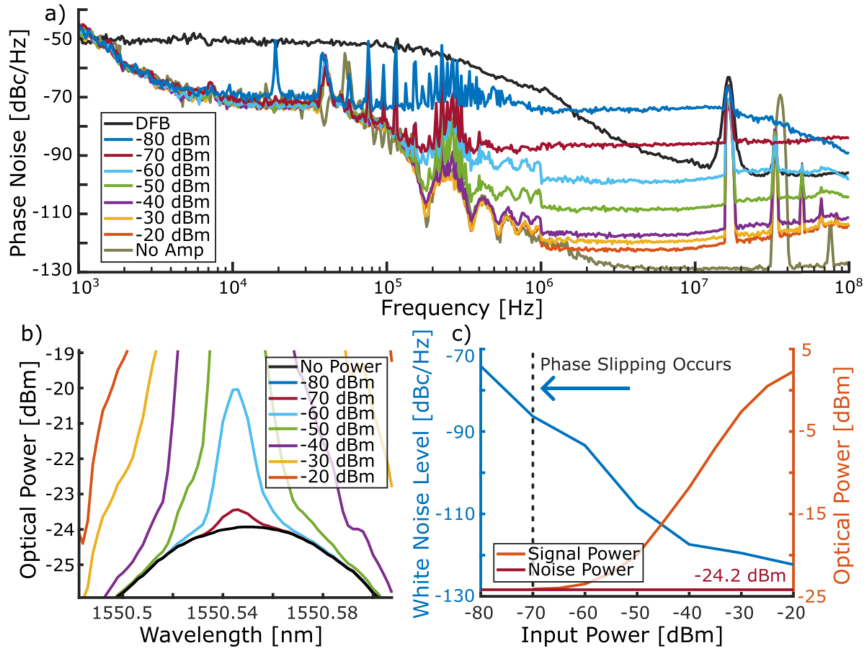

Fig. 5. a) The phase noise PSD of the injection locked slave laser output for various black box input powers, as measured by the delayed self heterodyne interferometer seen in Fig. 2(a). b) The corresponding optical spectra of light injected into the slave laser, after pre-amplification and filtration. c) The white noise floor of each curve seen in a), omitting the DFB and amplifier bypass, as measured at 10 kHz offset frequency (blue). Assuming a static noise power and variable signal power (valid in the weak signal regime), this curve roughly corresponds to Fig. 4(a). The signal power (orange) is extracted from b) and is shown to demonstrate the breakdown in EDFA linearity in the gain saturated regime.

-80 dBm of input power, the system unlocks, and between -80 dBm and -70 dBm of input power, we observe phase slipping which becomes less and less common as signal power increases. As the injected power continues to increase while white noise power remains constant, the output phase noise reduces. This matches well with our predictions, and supports the model presented above. The linearity of this curve breaks down along with the EDFA's linearity, seen as gain saturation in the orange curve. Moreover, the linear growth of phase noise in the PSD seen in Fig. 5(a) further supports the assumptions regarding linear behavior of the EDFA when far away from the gain saturation region.

4. Conclusions

In conclusion, we have demonstrated how pre-amplificiation and phase offset locking via a PID controller can be used in conjunction with OIL to recover CW signals at very low power. We then presented a thorough analysis of what physical processes limit that ability, and reported on how the system behaves at the limit of stable injection locking, with a focus on phase slips which occur due to the temporary unlocking of the slave laser.

In simulation, we began with the standard noisy optical injection locking rate equations which were modified to simulate the conditions of phase locking with added white phase noise (due to EDFA driven ASE, an imperfect PLL, etc.). The injected signal strength, $|E|_{\mathrm{inj}}$, static frequency offset, $\Delta\omega$, injected noise strength, $\partial\omega$, and injection ratio, IR were identified as the key parameters governing system behavior, and the impact of each is studied in detail. These rate equation solutions were then related back to the standard short hand approximations, and used to generate an approximation of the noise boundary for OIL.

Experimentally, we demonstrate optical injection locking of pre-amplified signals as low at -80 dBm (-70 dBm for phase slip free operation), with an output power of 20 dBm. Although the device can be said to be frequency locked at such low input signal power, white noise at this level does become a competitive term and may momentarily unlocks the system. This generates periods of punctuated equilibrium in the slave laser offset phase, corresponding to sharp peaks in the phase/frequency noise power spectral density. As the noise power is fixed and signal power is increased, these phase slips become less and less common and the phase noise is improved, matching predictions made by our simulations.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

References

1. Z. Liu and R. Slavík, "Optical injection locking: From principle to applications," J. Lightwave Technol. **38**(1), 43–59 (2020).
2. S. Mohrdiek, H. Burkhard, and H. Walter, "Chirp reduction of directly modulated semiconductor lasers at 10 gb/s by strong cw light injection," J. Lightwave Technol. **12**(3), 418–424 (1994).
3. Y. Okajima, S. Hwang, and J. Liu, "Experimental observation of chirp reduction in bandwidth-enhanced semiconductor lasers subject to strong optical injection," Opt. Commun. **219**(1-6), 357–364 (2003).
4. E. K. Lau, H.-K. Sung, and M. C. Wu, "Frequency response enhancement of optical injection-locked lasers," IEEE J. Quantum Electron. **44**(1), 90–99 (2007).
5. E. K. Lau, L. J. Wong, and M. C. Wu, "Enhanced modulation characteristics of optical injection-locked lasers: A tutorial," IEEE J. Sel. Top. Quantum Electron. **15**(3), 618–633 (2009).
6. J. C. Skehan, C. Naveau, J. Schroder, *et al.*, "Widely tunable, low linewidth, and high power laser source using an electro-optic comb and injection-locked slave laser array," Opt. Express **29**(11), 17077–17086 (2021).
7. J. C. Skehan, Ó. B. Helgason, J. Schröder, *et al.*, "Widely tunable narrow linewidth laser source based on photonic molecule microcombs and optical injection locking," Opt. Express **30**(13), 22388–22395 (2022).
8. S. P. Ó Dúill, P. M. Anandarajah, R. Zhou, *et al.*, "Numerical investigation into the injection-locking phenomena of gain switched lasers for optical frequency comb generation," Appl. Phys. Lett. **106**(21), 211105 (2015).
9. M. C. Wu, C. Chang-Hasnain, E. K. Lau, *et al.*, "High-speed modulation of optical injection-locked semiconductor lasers," in *Optical Fiber Communication Conference*, (Optica Publishing Group, 2008), p. OThK3.
10. G. S. Samarakkody, D. Munasinghe, S. Alahakoon, *et al.*, "The behavioral study of an optical injection-locked semiconductor laser under the influence of intensity and phase noise," in *2019 IEEE 5th International Conference on Wireless and Telematics (ICWT)*, (IEEE, 2019), pp. 1–6.
11. R. T. Ramos, P. Gallion, D. Erasme, *et al.*, "Optical injection locking and phase-lock loop combined systems," Opt. Lett. **19**(1), 4–6 (1994).
12. A. Bordonalli, C. Walton, and A. J. Seeds, "High-performance phase locking of wide linewidth semiconductor lasers by combined use of optical injection locking and optical phase-lock loop," J. Lightwave Technol. **17**(2), 328–342 (1999).
13. R. Kakarla, J. Schröder, and P. A. Andrekson, "Optical injection locking at sub nano-watt powers," Opt. Lett. **43**(23), 5769–5772 (2018).
14. J. C. Cartledge and R. Srinivasan, "Extraction of DFB laser rate equation parameters for system simulation purposes," J. Lightwave Technol. **15**(5), 852–860 (1997).
15. A. Fragkos, A. Bogris, D. Syvridis, *et al.*, "Amplitude noise limiting amplifier for phase encoded signals using injection locking in semiconductor lasers," J. Lightwave Technol. **30**(5), 764–771 (2011).