THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

# Opinion Dynamic Models of Decision-Making in Traffic

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*For Ian*

# Abstract

Autonomous vehicles (AVs) have the potential to improve both the efficiency and the safety of road traffic. Vehicles that can plan their routes, anticipate accidents and communicate open the door to transportation that is nearly free of human error. However, in the transition toward fully autonomous transportation systems, AVs must handle the dangers of human road users (HRUs).

One of the challenges faced by AVs is to accurately interpret and predict human behavior and decision-making. Due to the vast number of factors that influence every single individual, precise, deterministic models of decision-making between humans are practically infeasible. Moreover, while AVs may exchange explicit, technical information about each other's decisions, such communication might be difficult between them and HRUs. As a result, HRUs introduce an element of uncertainty in traffic scenes.

While many methods for estimating HRU decision-making are based on data-driven machine-learning methods, model-based approaches that use data for calibration are advantageous for simulation and prediction due to their relatively low parameter complexity. However, such models need to describe decision-making using stochastic abstractions that also capture the effect that interaction between HRUs has on their decision-making process. In this thesis, a framework based on *Markovian opinion dynamics* is suggested for modeling human decision-making in traffic as a network of continuous-time Markov chain agents that randomly switch between decisions. Interaction is expressed as social forces that modulate the rates at which agents change their own decisions depending on the decisions of others. The probability of intuitive effects such as group-wise agreement and disagreement can be predicted based on the modeled interaction within and between groups of agents.

The model can be used to anticipate how traffic scenario probabilities evolve from an initial observation to a stationary prediction. This thesis suggests how such a transition can be derived over the horizon of a predictive controller that determines the acceleration of an AV based on the expected HRU decision-making process.



**Keywords:** Autonomous vehicles, human road users, decision-making, opinion dynamics, continuous-time Markov chains, model predictive control

## List of Publications

This thesis is based on the following publications:

[A] **Carl-Johan Heiker**, Paolo Falcone, "Decision Modeling in Markovian Multi-Agent Systems". *2022 IEEE 61$^{st}$ Conference on Decision and Control*, Cancun, Mexico, Dec. 2022.

[B] **Carl-Johan Heiker**, Elisa Gaetan, Laura Giarré, Paolo Falcone, "Repulsive Markovian Models for Opinion Dynamics". *Systems & Control Letters*, Vol. 185, March 2024.

[C] **Carl-Johan Heiker**, Paolo Falcone, "Trajectory Planning Among Interactive Markovian Multi-Modal Obstacles using Scenario-MPC". Submitted to *22$^{nd}$ European Conference on Control (ECC)*, Stockholm, Sweden, June 2024.

Other publications by the author, not included in this thesis, are:

[D] E. Gaetan, L. Giarré, S. Sacone, P. Falcone, **C.-J. Heiker**, "Modelling Traffic Scenarios via Markovian Opinion Dynamics". Presented at *26$^{th}$ International Conference on Intelligent Transportation Systems (ITSC)*, Bilbao, Spain, Sept. 2023.

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## Acronyms

| | |
|---|---|
| RU: | Road User |
| HRU: | Human Road User |
| AV: | Autonomus Vehicle |
| ADAS: | Advanced Driver Assistance System |
| SFM: | Social Force Model |
| POMDP: | Partially Observable Markov Decision Process |

| | |
|---|---|
| HMM: | Hidden Markov Model |
| DTMC: | Discrete-Time Markov Chain |
| CTMC: | Continuous-Time Markov Chain |

# Contents

# Part I

# Overview

# CHAPTER 1

# Introduction

## 1.1 Motivation

From 2010 to 2019, the number of road deaths in traffic has been on the decline. However, the eight percent reduction during this period is significantly lower than the 50% goal set by the UN in 2010 [1]. While the decrease in fatal accidents among human road users (HRUs) traveling in cars is relatively sharp, the downward trend is less pronounced when it comes to pedestrians and cyclists, and road deaths among HRUs on powered two-wheelers has increased [1]. Traffic scenarios involving HRUs are thus still dangerous, and finding how to navigate in them safely is one of the main challenges in the development of autonomous vehicles (AVs).

The difficulty lies in predicting the behavior of HRUs despite the wide range of factors that influence their actions and may lead to dangerous scenarios. For instance, [2] lists distractions, misjudgements and fatigue as common causes of human error among drivers. However, human drivers are but one class of HRUs that may be present in a traffic scene. As highlighted by [3], it is also important to anticipate the behavior of other types of HRUs, such as pedestrians and cyclists, who are arguably even more unpredictable than drivers as they frequently disobey traffic rules.

A popular approach for identifying subtle traits that may cause accidents in diverse HRU traffic is machine learning methods. These are data-driven and can, often accurately, capture the effect of more complex behavioral aspects, such as the effects of interaction between HRUs [4]–[6]. However, [7] lists high parameter complexity, changing environments, processing of big data, accountability, and transparency as challenges for machine learning approaches. The alternative to machine learning is model-based methods, which are preferred in simulation applications as their parameters are usually few and physically meaningful. However, predicting HRU behavior from models of their physical dynamics cannot always be motivated, as accuracy gains often come at the price of dramatically increasing the size and complexity of the model [4]. The question is: Can complex, interactive HRU behavior be captured in a simple and easily interpretable modeling framework?

## Modeling decision-making in traffic

Instead of modeling the physical dynamics of HRUs, there are approaches that focus on describing HRU decision-making and the effects of social interaction. In [8], pedestrian behavioral models are divided into three categories: interaction-free, pedestrian interaction, and game-theoretic. Interaction-free models cover obstacle avoidance and basic navigation, which can be affected by factors such as age, gender, and social aspects. Models of pedestrian interaction range from describing microscopic interactions using social forces, to macroscopic descriptions of crowd dynamics. Game-theoretic models describe interactive and strategic decision making among HRUs in specific traffic scenario applications, such as lane changing. For instance, game theory has been used to model decision-making processes between AVs in intersections and on-ramps in [9] and [10], respectively. Game theoretic approaches commonly use agent-based descriptions of HRUs, which makes it possible to model decision-making that follows rules based on social factors, such as in [11]. In [12], HRU behavior is modeled using an agent based approach that is divided into three layers. The first layer takes care of obstacle avoidance, the second layer covers basic behavior using so-called social force models (SFMs), and the third layer describes complex interaction using game theory. As shown in [11] and [13], agent based methods can also be calibrated using data, which is important to achieve accuracy.

Inspired by the game-theoretical multi-agent modeling approach to traffic problems, this thesis focuses on how an entire traffic scenario involving different classes of HRUs can be described as a decision-making process between agents. Such a model should capture two important aspects of HRU behavior. First, the model should describe the influence that *interaction* has on HRU decision-making. Second, the decision-making process of each agent needs to be represented by a *stochastic abstraction* in order to handle the vast number of factors that determine their behavior.

### Interaction

In a recent survey [14], various forms of pedestrian interaction are suggested to have a significant effect on decision-making. When a pedestrian decides to cross a street, studies have shown that drivers are more likely to yield if the pedestrian is part of a larger group. Other social factors, such as how social norms affect the interpretation of intentions, imitation between drivers and pedestrians, and nonverbal communication are also important.

Interaction in behavioral models is sometimes expressed using social forces, which usually describe how agents affect each other's motion as a form of magnetic potential. As in the three-layered model in [12], [15] and [16] describe basic interaction using SFMs, while decision-making between HRUs is modeled using game-theory. However, SFMs can also represent more intricate forms of interaction, such as in [17] and [18], where it is shown how the kinematic behavior of pedestrinas can be affected by both vehicle-to-pedestrian and pedestrian-to-pedestrian interaction. Moreover, an attractive feature of SFMs is that their interaction parameters can be tuned to data, as in the model of driver interaction in shared spaces presented in [19]. Because they are versatile, interpretable and learnable, SFMs are thus suitable for modeling interaction between different HRUs.

### Stochastic decision makers

Due to the many factors that influence HRU behavior, an enormous amount of information would be required to represent HRUs as deterministic decision-makers. Instead, complex HRU behavior can be represented by stochastic abstractions. For example, [20] introduces stochastic terms into a microscopic driver model, and [21] and uses a dynamic Bayesian network to describe and predict the intentions of HRUs. In [22], driver intentions near an intersection are modeled by estimating the so-called time for action as Gaussian distributed variables.

However, stochastic behavior can also be modeled as discrete event systems using Markov chains. For example, [23] partitions the continuous state space of a kinematic vehicle model into grids that serve as states in a discrete-time Markov chain (DTMC), so that the probabilities with which HRUs transition in the grid can be learned and simulated. While state transitions occur at fixed time intervals in DTMCs, transitions in continuous-time Markov chains (CTMCs) can take place at any point in continuous time, a property that is suitable for modeling events that occur in traffic. In [24] and [25], for instance, traffic scenes are described as CTMC queueing systems that can be used to predict effects based on arrivals and departures of HRUs.

Markovian models are also popular for representing pedestrian intentions and decision-making, such as in [26], where partially observable Markov decision processes (POMDPs) are used for this purpose. Here, the probability of pedestrian intentions change depending on which predefined goal they are currently heading towards. Because HRU intentions cannot be measured directly, they are often modeled using Hidden Markov Models (HMMs), see for example [27] and [28]. The benefit of HMMs is that they assume that an underlying (hidden) Markov chain generates measurable signals that are observed in continuous time. This implies that the parameters of a Markov chain model of HRU decision-making in traffic can be learned from observations by using it in an HMM structure.

## 1.2 Research questions

To investigate how interactive decision-making among HRUs can be modeled, the following research questions are posed.

- **RQ1** How can the effect of different interaction forms be captured in a model of decision-making between uncertain HRUs?
- **RQ2** How can large-scale traffic scenarios be modeled as decision-making processes?
- **RQ3** How can models of HRU decision-making be used in the control of AVs?

## 1.3 Contributions

This thesis proposes a framework for modeling decision-making between HRUs as a multi-agent system of interactive Markovian agents who influence each other through social force-like functions. Specifically, the following contributions are made.

- Three social force-like functions describing decision influence between agents are presented in papers A and B. These functions modulate the rates at which CTMC agents change their decisions, based on the decisions of others. *Attraction* raises the probability of agreement within agents subgroups, while two forms of *repulsion* increase the probability of disagreement between agents from different groups.
- A *network model* describing the transitions between all possible agent decision configurations is presented. However, this model does not scale well with the number of agents and decisions. Therefore, papers A and B show that a *marginalized* model of significantly lower dimension can be derived also in the presence of the decision influence forces.
- In papers A and B, disagreement is modeled in two different ways, leading to different network models. However, two different models of the same effect should, by intuition, be able to *similarly* describe a decision-making process. A definition of *similarity* between models of decision-making processes is therefore suggested in Paper B.
- A method for obtaining two similar decision repulsion models through constrained optimization is formulated in Paper B.
- Lastly, Paper C shows how a model of HRU decision-making can be used to predict the evolution of decision probabilities over the horizon of a model predictive controller (MPC). The decision probabilities are used by the MPC to shape the acceleration of an AV that approaches an intersection that may be occupied by HRUs.

## 1.4 Thesis outline

This thesis is organized as follows. In Chapter 2, a unified walk-through of the modeling framework, originally presented in Papers A and B, is presented. In

Chapter 3, a summary of how the three research questions in Section 1.2 are answered in the included papers is provided. Chapter 4 summarizes the included papers, and conclusions and suggestions for future work are formulated in Chapter 5.

# CHAPTER 2

# Modeling

In Chapter 1, stochastic representations of how HRUs make decisions in traffic were suggested, as exact deterministic models that capture the vastly varying HRU behavior are unrealistic. At the same time, HRU interaction plays a key role in determining the probability of every possible HRU decision. To capture both the stochastic and the interactive aspects of decision-making in traffic, the model suggested in this thesis is based on *Markovian opinion dynamics*.

## 2.1 Markovian opinion dynamics

Classic opinion dynamics describe how agents influence each other to change opinion [29], [30]. The basic DeGrootian model [31] is, however, a deterministic linear system in which the opinion of the agents at the next time instance is a weighted average of the opinions that neighboring agents have currently. The Friedkin-Johnsen model [32] extends the DeGrootian model by considering stubborn agents, and [33] describes agents that, while changing their opinions randomly, are influenced by the opinions of their neighbors if they are within a confidence interval of their own opinion. Some models describe different forms of interaction, such as antagonism [34], [35] and mixed antagonistic and friendly relationships [36].

Different from the classic, deterministic opinion dynamic models, the approach which is presented in [37]–[40] and based on [41]–[43] describes opinion dynamics in which agents change their opinions stochastically. Specifically, each agent changes its opinion in continuous time as a CTMC over a set of possible opinions. At the same time, the rate of opinion change depends on the current opinions of their neighbors, describing the interaction. Based on this modeling principle, this thesis discusses how traffic scenes involving HRUs can be modeled as decision-making processes between CTMC agents that interact through agreement and disagreement, described as SFMs adapted for the Markovian opinion dynamic setting.

A suitable starting point when summarising the model developed in papers A and B is to describe how agents make decisions in isolation.

## 2.2 Isolated agent model

In its nominal form, the model of a decision-making agent is a CTMC description of its transitions between decisions in isolation, assuming that it does not interact with other agents.

### Decision states and transitions

An agent $\alpha$ can choose between $M$ decisions in the set

$$\mathcal{S} = \{s_1, s_2, \dots s_M\}. \tag{2.1}$$

A change of decision is modeled as a transition between the elements in $\mathcal{S}$. In a CTMC, a transition $s_i \to s_j$ can occur at any point in continuous time and is assumed to be an event that occurs in zero time. In Fig. 2.1, a CTMC description of an agent $\alpha_n$ is shown with arrows representing transitions between

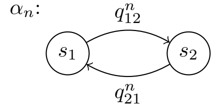



**Figure 2.1:** Decision-making agent $\alpha_n$ as a CTMC.

two possible decision states, $s_1$ and $s_2$.

## State holding time

The time until $\alpha$ transitions from a decision state $s_i$ is called the *state holding time* and is denoted $V_i$. As described in [44], this is an exponentially distributed random variable, so that the probability that $\alpha_n$ stays in $s_i$ for a time interval of at most length $\tau$ is

$$P[V_i \leq \tau] = 1 - e^{\Lambda_i \tau}, \tag{2.2}$$

where $P$ denotes probability and $\Lambda_i$ is the distribution parameter. It follows that the expected decision state holding time is

$$\mathbb{E}[V_i] = -\frac{1}{\Lambda_i}. \tag{2.3}$$

## Transition rates

The state holding time parameter $\Lambda_i$ is connected to the *instantaneous rates* $q_{ij}$ at which transitions from the decision state $s_i$ to any other state $s_j$ occur. Specifically, $\Lambda_i$ is equal to the sum of the rates of all possible transitions out of $s_i$. All transition rates are collected in the transition rate matrix $Q \in \mathbb{R}^{M \times M}$, defined as

$$Q[i,j] = \begin{cases} q_{ij} \geq 0 & \text{if } i \neq j, \\ q_{ii} = -\sum_{j=1}^{M} Q[i,j] & \text{otherwise.} \end{cases} \tag{2.4}$$

Hence, $\alpha$'s isolated behavior in terms of decision-making is determined by the transition rates in $Q$.

### Time-homogeneity

A CTMC with transition rates that are independent of time is called *time-homogeneous*, which is the case for isolated agents. This is also the case for the agent $\alpha_n$ seen in Fig. 2.1 which has two transitions, $s_1 \to s_2$ and $s_2 \to s_1$, that both have constant transition rates denoted $q_{12}^n$ and $q_{21}^n$.

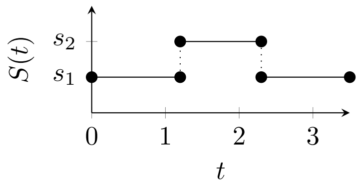



**Figure 2.2:** State changes in continuous time.

## 2.3 Properties of the isolated agent model

In the following, three important aspects of the isolated agent model are discussed, beginning with the *Markov property*.

### The Markov property

A CTMC is a class of Markov processes and thus maintains the Markov property. As an example, consider the plot in Fig. 2.2 which shows how the CTMC $\alpha_n$ from Fig. 2.1 may transition between its decision states. The state of the process at time $t$, denoted $S(t)$, is a random variable that can be observed in one of the two states, $s_1$ and $s_2$. Starting in $t = 0$ with $S(0) = s_1$, a transition to $s_2$ occurs at $t = 1.2$. Finally, a transition back to $s_1$ is made at $t = 2.3$. This process is *Markovian* if the probability of observing $S(t)$ assuming a specific state in the future is conditioned only on the latest observation, which summarizes the entire history of the previous states. Formally, the Markov property can be written as

$$\begin{aligned} &P\big[S(t_{k+1}) = x_{k+1} | S(t_k) = x_k, S(t_{k-1}) = x_{k-1}, \dots S(t_0) = x_0\big] \\ &= P\big[S(t_{k+1}) = x_{k+1} | S(t_k) = x_k\big], \end{aligned} \tag{2.5}$$

where $x_i$ denotes a specific state that $S(t)$ assumes at a point in time $t_i$.

### State probabilities

Thanks to the Markov property, the probability of observing a CTMC agent in each of its states can be determined from the transition rate matrix (2.4) and an initial condition. Specifically, the $M \times 1$ vector of all *decision state probabilities*, denoted $\Pi(t)$, is found by solving the first-order differential equation

$$\dot{\Pi}(t) = Q^T \Pi(t) \tag{2.6}$$

from the initial state probabilities $\Pi(0)$. Here, $\dot{\Pi}(t)$ denotes the first time derivative of $\Pi(t)$. The decision state probability is $\Pi(t)$ thus a probability distribution that evolves with time and has positive real elements that sum to one for any $t$.

### Ergodicity

An important property for the CTMC models used in this work is *ergodicity*. An ergodic CTMC can visit all of its states with a nonzero probability and is guaranteed to converge to a stationary, unique state probability distribution *independent* of the initial probability $\Pi(0)$ [38]. Formally,

$$\lim_{t \to \infty} \Pi(t) = \bar{\Pi}, \tag{2.7}$$

where $\bar{\Pi}$ denotes the stationary state probability distribution.

## 2.4 Group-wise agent interaction

In decision-making processes, such as political campaigns [40], the social nature of the decision-makers can lead them to organize in groups characterized by internal agreement. Conversely, agents from different groups tend to disagree. To this end, papers A and B explore how these effects can be emulated through models of group-wise interaction between HRUs in traffic. First, however, the concept of groups and the topology of agreement and disagreement

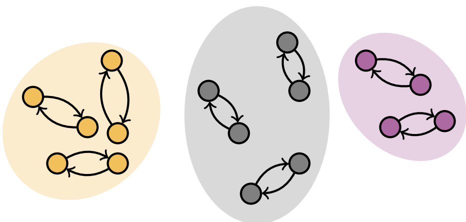


**Figure 2.3:** Three groups of agents with binary decision sets.

is formally defined.

## Agent groups

In a set $\mathcal{N}$ of $N$ agents, groups are defined as subsets $\mathcal{A} \subseteq \mathcal{N}$, intuitively visualized in Fig. 2.3. These sets are disjoint, meaning that agents are part of exactly one group. Paper A considers two primary types of group-wise agent interaction.

### Internal agreement

The feature that defines each group is that its members are drawn towards making the same decisions as one another. Because each agent is part of only one group, a single graph

$$G_{\mathcal{A}} = \big(\mathcal{N}, \mathcal{E}_{\mathcal{A}}, \Lambda_{\mathcal{A}}\big), \tag{2.8}$$

over $\mathcal{N}$ can describe the topology of all internal interaction between agents in each group simultaneously. $A$ denotes "attraction", and $\mathcal{E}_{\mathcal{A}}$ represents the edges that divide agents into groups. Within each group, agents can have varying levels of influence on each other, and $\Lambda_{\mathcal{A}}$ is introduced as an $N \times N$ row normalized adjacency matrix of weights describing the influence strength between separate agents within every group.

### External disagreement

Contrary to internal agreement, agents disagree with those from opposing groups. Agents in one or more groups $\mathcal{R}_\ell$ from the set of opposing groups $\mathcal{R}$ may induce a subject group $\mathcal{A}$ into making decisions that are different from theirs. Moreover, agent groups can both induce other groups (thus assuming the role of $\mathcal{R}_\ell$) and be induced by other groups (assuming that of $\mathcal{A}$) at the same time.

The topology of the repulsion that a group $\mathcal{R}_\ell$ exerts on $\mathcal{A}$ and can be described by a graph

$$G^{\mathcal{A}}_{\mathcal{R}_\ell} = \big(\mathcal{N}, \mathcal{E}^{\mathcal{A}}_{\mathcal{R}_\ell}, \Gamma^{\mathcal{A}}_{\mathcal{R}_\ell}\big), \tag{2.9}$$

where $\mathcal{E}^{\mathcal{A}}_{\mathcal{R}_\ell}$ represents the edges defining the repulsion between agents from different groups. The influence strength of the repulsion between each pair of conflicting groups is described by the row normalized $N \times N$ adjacency matrix $\Gamma^{\mathcal{A}}_{\mathcal{R}_\ell}$.

## 2.5 Transition rate modulation

While the transition rates of the isolated agent model can be chosen offline to describe a desired behavior, they are constant and do not describe the agent's response to the observed decisions of other agents. To achieve this, transition rates that are dependent on the states of other agents, and therefore on time, are introduced. This principle is called *transition rate modulation* [37], [38], and is in papers A, B and C used to model group-wise agreement and disagreement as social force-like functions.

### Agreement as an attraction force

Introduced in Paper A, the *attraction force* induces agreement within a group $\mathcal{A}$. The force is an additive increase

$$\psi_j^n(t) = \lambda(n, \mathcal{A}) \sum_{\alpha_k \in \mathcal{A}} \Lambda_{\mathcal{A}}[n, k] \mathbf{1}_j^k(t) \tag{2.10}$$

of the $n$:th agent $\alpha_n$'s transition rate toward a decision state $s_j$ depending on which agents $\alpha_k$ from $\mathcal{A}$ are in $s_j$ at time $t$. The *indicator function* $\mathbf{1}_j^k(t)$ is one if $\alpha_k$ has decided $s_j$ at time $t$ and zero otherwise. Hence, the summation over $k$ gives the weighted fraction of agents in $\mathcal{A}$ who are in $s_j$ at each time instance. Finally, $\lambda(n, \mathcal{A})$ determines the strength of the attraction towards the decision $s_j$. By default, this parameter is assumed to be dependent on passive traits, such as agent index $n$ or its group membership $\mathcal{A}$. However, it can in special cases be state-dependent, which implies that it changes with the time-dependent indicator function.

### Disagreement as a repulsion force

Opposite to attraction, the *repulsion force* models disagreement between agents in different groups. While an *indirect* form of repulsion is introduced in Paper A, Paper B also presents the alternative *direct* form.

### Direct form

For an agent $\alpha_n$, the *direct repulsion force* reduces the transition rates towards the decision $s_j$ depending on the agents $\alpha_l$ in the conflicting groups $\mathcal{R}_\ell$ who are also in $s_j$ at time $t$. The reduction is formally expressed as

$$^{-}\xi_j^n(t) = \gamma(n, \mathcal{A}, \mathcal{R}_\ell) \sum_{\alpha_l \in \mathcal{R}_\ell} \Gamma_{\mathcal{R}_\ell}^{\mathcal{A}}[n, l] \mathbf{1}_j^l(t). \tag{2.11}$$

As $\lambda(n, \mathcal{A})$ does for attraction, $\gamma(n, \mathcal{A}, \mathcal{R}_\ell)$ sets the repulsion strength, and is also assumed by default to be a function of the same passive traits $n$ and $\mathcal{A}$. Additionally, as members from $\mathcal{A}$ can be repulsed by several groups $\mathcal{R}_\ell$, $\gamma$ can be dependent on this group as well. Also like $\lambda$, $\gamma$ can be state-dependent in special cases.

As the direct repulsion form is a decrease in transition rates, it needs to be limited to avoid negative rates in the CTMC. As a result, a transition rate can only be reduced to a degree set by the nominal transition rate, which may have a limited effect. However, repulsion can also be expressed in an indirect form, which eliminates this restriction.

### Indirect form

Instead of reducing transition rates toward decisions that are popular among other groups, Paper A presents the *indirect repulsion force*, which induces disagreement by *increasing* rates toward decisions that are unpopular among agents from other groups. For the agent $\alpha_n$, the transition rate increase toward $s_j$ is

$$^{+}\xi_j^n(t) = \gamma \sum_{\alpha_l \in \mathcal{R}_\ell} \Gamma_{\mathcal{R}_\ell}^{\mathcal{A}}[n, l]\big(1 - \mathbf{1}_j^l(t)\big), \tag{2.12}$$

where the negated indicator function $1 - \mathbf{1}_j^l(t)$ is used to denote when $\alpha_l \in \mathcal{R}_\ell$ has decided $s_j$.

## 2.6 Agent networks

When agents change their transition rates over time due to observations of the decisions of their neighbors, their CTMC representations are no longer time-homogeneous. However, by defining the decision process as a network of CTMCs, a time-homogeneous CTMC representation of the entire network can be obtained.

## Nominal network CTMC

The nominal single-agent CTMC is a description of agent behavior in isolation. Similarly, the nominal network model is a single-CTMC description of several isolated agents that are observed simultaneously.

### State-space

A state $S_{\mathbb{X}}(t)$ in a network formulation $\mathbb{X}$ is a random variable that can assume values denoted as the tuple

$$s^{\mathbb{X}} = \langle s^1, s^2, \dots s^N \rangle \tag{2.13}$$

collecting the states of all $N$ agents in the set $\mathcal{N}$. The state space of the entire network is the cartesian product of the sets of each agent

$$\mathcal{S}_{\mathbb{X}} = \mathcal{S}_1 \times \cdots \times \mathcal{S}_n \times \dots \mathcal{S}_N. \tag{2.14}$$

Here, each state set $\mathcal{S}_n$ is the decision set $\mathcal{S}$ that is shared among all agents, but it is indexed to distinguish between the individual agents in a network state in (2.14). As $\mathcal{S}$ contains $M$ decisions, the network has $M^N$ decision states.

### Transitions

Because each single agent is a CTMC, the probability that two agents transition at the same point in continuous time is zero. Hence, every possible transition between two network states is defined by the transition of a single agent, in the context of an otherwise static network. This makes it possible to define the network $\mathbb{X}$ as a continuous-time Markov chain over $\mathcal{S}_{\mathbb{X}}$. Furthermore, since every single agent is ergodic, the network CTMC is ergodic, too [38].

### Isolation

Following the standard definition of a CTMC in 2.2, the probabilities of each possible network decision configuration in $\mathbb{X}$ are collected in the $M^N \times 1$ vector $\Pi_{\mathbb{X}}(t)$ and are found by solving the first-order differential equation

$$\dot{\Pi}_{\mathbb{X}}(t) = Q^T \Pi_{\mathbb{X}}(t) \tag{2.15}$$

from the initial condition $\Pi_{\mathbb{X}}(0)$. Since the network is a CTMC, its $M^N \times M^N$ transition rate matrix $Q$ in (2.15) follows the definition (2.4), albeit for the network state space, and can be constructed from the isolated transition rate matrices $Q_n$ of each agent $\alpha_n$ as

$$Q = \sum_{n=1}^{N} I_{M^{n-1}} \otimes Q_n \otimes I_{M^{N-n}}. \tag{2.16}$$

Here, $I_{M^{n-1}}$ and $I_{M^{N-n}}$ denote identity matrices of dimension $M^{n-1}$ and $M^{n-1}$, respectively, while $\otimes$ represents the Kronecker product.

## Interaction

For every possible network transition $s_i^{\mathbb{X}} \rightarrow s_j^{\mathbb{X}}$, the state of all agents is known. Hence, the indicator functions in each of the forces (2.10), (2.11) and (2.12) can be evaluated deterministically. This implies that each force can be represented as a constant, additional transition rate matrix in the network model.

### Attraction

The network form of the attraction force is represented in Paper A as a constant, real-valued transition rate matrix

$$A[i,j] = \begin{cases} \psi_b^n\left(t|s_i^{\mathbb{X}}, s_j^{\mathbb{X}}\right) & \text{if } i \neq j \\ -\sum_{j \neq i} A[i,j] & \text{otherwise.} \end{cases} \tag{2.17}$$

Each network transition $s_i^{\mathbb{X}} \rightarrow s_j^{\mathbb{X}}$ is defined by the state transition $s_a \rightarrow s_b$ of some agent $\alpha_n$, for which the attraction force $\psi$ defined in (2.10) is evaluated.

### Direct repulsion

Similar to the attraction case, a network transition rate matrix for the direct repulsion force is derived in Paper B as

$$R^-[i,j] = \begin{cases} -\,{}^-\xi_j^n\left(t|s_i^{\mathbb{X}}, s_j^{\mathbb{X}}\right) & \text{if } i \neq j \\ -\sum_{j\neq i} R^-[i,j] & \text{otherwise.} \end{cases} \tag{2.18}$$

Notably, this matrix represents transition rate reductions resulting from the direct repulsion force ${}^-\xi$ defined in (2.11) and thus follows the definition of a negated CTMC transition rate matrix.

### Indirect repulsion

Lastly, Paper A defines the network transition rate matrix for the indirect repulsion form as

$$R^+[i,j] = \begin{cases} {}^+\xi_j^n\left(t|s_i^{\mathbb{X}}, s_j^{\mathbb{X}}\right) & \text{if } i \neq j \\ -\sum_{j\neq i} R^+[i,j] & \text{otherwise,} \end{cases} \tag{2.19}$$

where ${}^+\xi$ is the indirect repulsion force defined in (2.12). Opposite to the direct repulsion, this matrix follows the definition of a standard CTMC transition rate matrix.

## Interactive network CTMC

To describe interaction in the network, the isolated network model (2.15) is extended by adding one or several network transition rate matrix force representations $A$, $R^+$ or $R^-$ to $Q$. The state probabilities of the interactive network decision process $\mathbb{I}$ are found by solving

$$\dot{\Pi}_{\mathbb{I}}(t) = (Q + I)^T \Pi_{\mathbb{I}}(t) \tag{2.20}$$

from an initial probability vector $\Pi_{\mathbb{I}}(0)$. As an example, interaction in the form of attraction and indirect repulsion is expressed by setting the matrix $I = A + R^+$.

**Remark:** In the case of direct repulsion, the transition rates of each network transition are reduced. To ensure a network formulation (2.20) with positive transition rates, rate reductions must be upper-limited by the transition rates in $Q$. The effects that this has on the behavior of the network are investigated in Paper B.

## 2.7 Similarity

When two interaction forces describe the same effect, such as when direct and indirect repulsion are two ways of modeling disagreement, intuition says that both models should describe *similar* decision processes. It is assumed that strict equivalence between the transient solutions of two CTMC models of the same dimension, but with different rate matrices, is not necessarily possible. Therefore, *similarity* is introduced in Paper B as two conditions for when two CTMC models $\mathbb{X}_a$ and $\mathbb{X}_b$ are close to describing the same decision-making process in practice.

1. The difference between $\mathbb{X}_a$'s and $\mathbb{X}_b$'s decision state holding times (2.2) should be minimized according to some objective function.

2. $\mathbb{X}_a$ and $\mathbb{X}_b$ should reach identical stationary decision probabilities.

These conditions can be used to formulate a constrained optimization problem that, if solved, finds the transition rates of an unknown model, such that it is *similar* to a given model. In Paper B, a form of direct repulsion is found so that it is similar to a given indirect repulsion formulation, and the procedure is summarized next.

### Minimal difference in decision state holding time

In the following, it is assumed that two networks $\mathbb{D}$ and $\mathbb{I}$ are defined with direct and indirect repulsion, respectively, but have the same isolated behavior specified by the network transition rate matrix $Q$. The rate matrix $R^+$ representing the indirect repulsion in $\mathbb{I}$ is given, and the objective is to find a network transition rate matrix $R^-$ for $\mathbb{D}$ such that $\mathbb{D}$ and $\mathbb{I}$ become similar according to the two criteria in Section 2.7.

As the expected value of the state holding time (2.3) is the inverse of the distribution parameter, the rate increase from $R^+[i,i]$ decreases $\mathbb{I}$'s state holding time. Conversely, $R^-[i,i]$ can only reduce transition rates in $\mathbb{D}$, increasing the state holding time. Hence, to produce a $\mathbb{D}$ similar to $\mathbb{I}$, the diagonal elements of $R^-[i,i]$ should be minimized. To achieve this, a quadratic objective function

$$f(r,H) = r^T A^T H A r, \tag{2.21}$$

where $r$ is a vector of unknown decision variables containing the off-diagonal, nonzero elements of $R^-$, is chosen in Paper B. $A$ is a matrix that collects the diagonal elements of each row of $R^-$ in terms of the corresponding off-diagonal elements, and $H$ is a diagonal matrix that can be used for weighting the minimization.

## Identical stationary decision probabilities

When the stationary network decision probabilities $\bar{\Pi}$ of $\mathbb{D}$ and $\mathbb{I}$ are equal, (2.20) can be used to formulate $(R^-)^T\bar{\Pi} = -(R^+)^T\bar{\Pi}$. The left-hand side of this expression can be reformulated as a function of $r$ and a matrix $M_{\bar{\Pi}}$ of elements in $\bar{\Pi}$, resulting in

$$M_{\bar{\Pi}} r = (R^+)^T\bar{\Pi}, \tag{2.22a}$$

$$0 < r < q. \tag{2.22b}$$

With the vector $q$ containing the off-diagonals of $Q$ taken by the same indexing method as $r$, the inequality constraint ensures that $R^-$ has positive off-diagonals that do not exceed those of $Q$.

Minimizing the objective function 2.21 under the constraints in 2.22 defines a constrained optimization problem

$$\underset{r}{\text{minimize}} \quad f(r,H) \tag{2.23a}$$

$$\text{subject to} \quad M_{\bar{\Pi}} r = -(R^+)^T\bar{\Pi}, \tag{2.23b}$$

$$0 < r < q, \tag{2.23c}$$

for obtaining a network $\mathbb{D}$ similar to $\mathbb{I}$. This method is one of the main contributions of Paper B. While it is described for direct and indirect repulsion here, it could be generalized for any two forms of network transition rate modulation by modifying (2.23b) and (2.23c) depending on the structure of the modulation.

## 2.8 Marginalization

The CTMC description of a decision-making process in a network of $N$ agents with $M$ decisions has $M^N$ states representing the probability of every decision configuration in the network. Hence, this model does not scale well with the number of agents and the number of decision states that they can assume. Moreover, the resolution of this model is unnecessarily high in cases when only the decision probabilities of each agent are of interest.

In general, finding the probability distribution of a single variable from a joint distribution is called *marginalization* and can be done by summation over the joint distribution. For example, the decision probabilities of the individual agents can always be obtained by summation over the network's decision-state probabilities. However, this still requires deriving and operating on the network CTMC. Fortunately, linear models describing the concatenated individual decision state probabilities of each agent in the network can be derived *analytically*, even in the presence of the three different force functions introduced in papers A and B.

### Marginalized model

The marginalized decision probability state space

$$\Pi_{\mathbb{M}}(t) = \begin{bmatrix} \Pi_1(t) & \Pi_2(t) & \dots & \Pi_N(t) \end{bmatrix} \tag{2.24}$$

consists of the concatenated state probability vectors $\Pi_n(t)$ of each agent $\alpha_n$. Similar to the network CTMC descriptions, the marginalized model describes the first time derivative of the probabilities, denoted $\dot{\Pi}_{\mathbb{M}}(t)$, as a linear system. Due to ergodicity of the individual agents, this system also converges to a stationary solution [37]. Next follows a description of how marginalized linear models can be derived for networks of agents that interact through the forces defined in Section 2.5.

#### Derivation from the infinitesimal definition

In papers A and B, the marginalized model is as in [37] derived from the *infinitesimal definition* of the transition probabilities in a CTMC. The probability that the agent $\alpha_n$ is in state $s_i$ at some time $t$ can be described as the expected value of an indicator function, such that $\pi_i^n(t) = \mathbb{E}[I_i^n(t)]$. The infinitesimal definition of a CTMC then states that the probability that the agent transitions to $s_j$ after an infinitesimally small time interval $\delta t$, given the state of the agent network $S_{\mathbb{X}}(t)$ at time $t$ is

$$\mathbb{E}\big[I_j^n(t+\delta t)|S_{\mathbb{X}}(t)\big] = \Big(\mathbb{E}\big[I_j^n(t)\big] \cdot \big(1 - \mathcal{Q}_{\text{out}}\big) + \\ + \mathbb{E}\big[\big(1 - I_j^n(t)\big)\big] \cdot \mathcal{Q}_{\text{in}}\Big)\delta t. \tag{2.25}$$

Here, $\mathcal{Q}_{\text{in}}$ symbolizes the $n$:th agent's total rates into $s_j$ from other states $s_i \neq s_j$. Conversely, $\mathcal{Q}_{\text{out}}$ represents the total rates out of $s_j$ to other states $s_i \neq s_j$. Dividing this expression by $\delta t$ and subtracting $\mathbb{E}\big[I_j^n(t+\delta t)\big]$ from both sides of the equality yields

$$\frac{\mathbb{E}\big[I_j^n(t+\delta t)|S_{\mathbb{X}}(t)\big] - \mathbb{E}\big[I_j^n(t)\big]}{\delta t} = \mathbb{E}\big[-I_j^n(t) \cdot \mathcal{Q}_{\text{out}} + \big(1 - I_j^r(t)\big) \cdot \mathcal{Q}_{\text{in}}\big]. \tag{2.26}$$

Letting $\delta t$ approach zero in this expression yields the definition of $\dot{\pi}_j^n(t)$ on the left-hand side, which is the probability rate of the $n$:th agents transition into state $s_j$.

The previous expression corresponds to one row of the marginalized model, which is the time derivative of the entire vector (2.24). Deriving the marginalized model consists of evaluating (2.26) for all agents and states for both the isolated rates and the rates originating from the interaction forces.

### Isolation

From the definition (2.6), the probability rate of $\alpha_n$'s transition towards $s_j$ is

$$\dot{\pi}_j^n(t) = \sum_{i=1}^{M} Q_n[i,j]\pi_i^n(t). \tag{2.27}$$

Hence, the marginalized form of the entire isolated network model, which defines the nominal marginalized model, is

$$\dot{\Pi}_{\mathbb{M}}(t) = Q_{\mathbb{M}}^T \Pi_{\mathbb{M}}(t), \tag{2.28}$$

where

$$Q_{\mathbb{M}} = \text{blkdiag}\big(Q_1, Q_2, \ldots Q_N\big), \tag{2.29}$$

a block diagonal matrix consisting of the transition rate matrices of each agent.

### Attraction force

In Paper A, the tuning parameter $\lambda$ determines the magnitude with which the attraction force (2.10) acts on $\alpha_n$, and is assumed to be dependent on $\alpha_n$'s group $\mathcal{A}$, but independent on the state that $\alpha_n$ is in. The row corresponding to the $n$:th agent and the $j$:th state in the marginalized attraction force model can be found by developing the total in-and outgoing attractive forces in the right-hand side of (2.26), which yields

$$\dot{\pi}_j^n(t) = \lambda(n, \mathcal{A})\Big( \sum_{\alpha_k \in \mathcal{A}} \Lambda_{\mathcal{A}}[n, k]\pi_j^k(t) - \pi_j^n(t)\Big). \tag{2.30}$$

In vector form, the contribution that 2.30 makes to $\dot{\Pi}_{\mathbb{M}}(t)$ is expressed as a product between a $MN \times MN$ matrix $A_{\mathbb{M}}$ constructed from 2.30 and the concatenated decision probabilities $\Pi_{\mathbb{M}}(t)$.

### Indirect repulsion force

The parameter for determining the influence strength of the indirect repulsion force, denoted $\gamma$ in (2.12), is assumed to be state-independent but can depend on the agent index $n$ and the conflict pair $\big(\mathcal{A}, \mathcal{R}_\ell\big)$. In this case, evaluation of the right-hand side of (2.26) leads to

$$\dot{\pi}_j^n(t) = \sum_{\mathcal{R}_\ell \in \mathcal{R}} \gamma(n, \mathcal{A}, \mathcal{R}_\ell)\Big(1 + (M-1)\pi_j^n(t) - \sum_{\alpha_l \in \mathcal{R}_\ell} \Gamma_{\mathcal{R}_\ell}^{\mathcal{A}}[r, l]\pi_j^l(t)\Big). \tag{2.31}$$

Because the indirect repulsion is a function of a negated transition rate function $1 - I_j^l(t)$, a constant term appears in each row of the marginalized model. Hence, the vector form of (2.31) contributing to $\dot{\Pi}_{\mathbb{M}}(t)$ consists of a product between a $MN \times MN$ matrix $R_{\mathbb{M}}^{+}$ and $\Pi_{\mathbb{M}}(t)$, and the addition of a constant vector $E_{\mathbb{M}}$.

### Direct repulsion force

Finally, with the same assumptions regarding state independent tuning parameters as in the indirect case, Paper B shows that the right-hand side of

(2.26) can be evaluated also with direct repulsion to find

$$\dot{\pi}_j^n(t) = \sum_{\mathcal{R}_\ell \in \mathcal{R}} \gamma(n, \mathcal{A}, \mathcal{R}_\ell)\Big(\pi_j^n(t) - \sum_{\alpha_l \in \mathcal{R}_\ell} \Gamma_{\mathcal{R}_\ell}^{\mathcal{A}}[n,l]\pi_j^l(t)\Big). \tag{2.32}$$

The contribution from the direct repulsion to all rows of the marginalized model $\dot{\Pi}_{\mathbb{M}}(t)$ can be expressed using $\Pi_{\mathbb{M}}(t)$ and a $MN \times MN$ matrix $R_{\mathbb{M}}^{-}$.

### Marginalizable network formulations

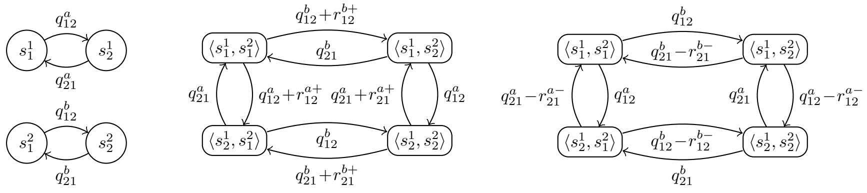
A necessary condition for marginalization is state-independent parameters. If this is fulfilled, the strength of the influence exerted on an agent toward each decision is determined only by the configuration in which other agents are in agreement or disagreement with the transitioning agent's decision. Hence, the same configuration can be constructed for every possible transition that the agent can make, resulting in identical network transition rates. Therefore, one way of judging if a network is *marginalizable* is to determine if its transition rate matrices fulfill the necessary equality conditions.

## 2.9 Simultaneous similarity and marginalizability

The central question in Paper B is whether or not *similarity* between two *marginalizable* models can be achieved. In other words, does the linear problem formed by similarity have a solution under the additional equality constraints from marginalizability? However, the number of unknowns in the problem is dependent on the interaction topology, and this makes it difficult to formulate general conditions that hold for all possible network formulations. Instead, Paper B presents a counterexample showing that the two properties do not hold at the same time *in general.* In the following, the main results of the counterexample are summarized.

### Counterexample

For a minimally sized binary decision-making process between agents $a$ and $b$, two networks $\mathbb{I}$ and $\mathbb{D}$ are formulated with indirect and direct repulsion, respectively. The CTMCs representing $\mathbb{I}$ and $\mathbb{D}$ are presented in Fig. 2.4. These networks are assumed to have identical isolated behavior, described by




**(a)** $a$ and $b$. **(b)** Network $\mathbb{I}$. **(c)** Network $\mathbb{D}$.

**Figure 2.4:** Agents $a$ and $b$ and network models formulated between them using indirect and direct repulsion, respectively.

the known rate matrix $Q$. Moreover, the transition rates $r^{a/b+}_{ij}$ representing the indirect repulsion in $\mathbb{I}$ are assumed to be known. The task is to choose the four unknown network transition rates $r^{a/b-}_{ij}$ representing the direct repulsion in $\mathbb{D}$ so that $\mathbb{D}$ and $\mathbb{I}$ are similar. In Paper B, this problem is studied under the additional constraints of marginalizability.

### No network is marginalizable

For $\mathbb{D}$ to be similar to $\mathbb{I}$ without necessarily being marginalizable, the constraints

$$0 < r^{b-}_{21} = \frac{\bar{\pi}_1\, r^{b+}_{12} - \bar{\pi}_4\, r^{b+}_{21} + \bar{\pi}_3\, x}{\bar{\pi}_2} < q^b_{21}, \tag{2.33a}$$

$$0 < r^{a-}_{12} = \frac{\bar{\pi}_4\, r^{a+}_{21} + \bar{\pi}_4\, r^{b+}_{21} - \bar{\pi}_3\, x}{\bar{\pi}_2} < q^a_{12}, \tag{2.33b}$$

$$0 < r^{a-}_{21} = \frac{\bar{\pi}_1\, r^{a+}_{12} + \bar{\pi}_4\, r^{b+}_{21} - \bar{\pi}_3\, x}{\bar{\pi}_3} < q^a_{21}, \tag{2.33c}$$

$$0 < r^{b-}_{12} = x < q^b_{12} \tag{2.33d}$$

need to hold for some shared stationary network decision state probabilities $\bar{\pi}_i$ where $i = 1, \dots 4$. This is possible since the rate $x$, can be chosen freely on an interval.

### $\mathbb{D}$ is marginalizable

If $\mathbb{D}$ is marginalizable, its state-independent force parameters imply $r_{21}^{a-} = r_{12}^{a-}$ and $r_{21}^{b-} = r_{12}^{b-}$, reducing (2.33) to

$$0 < r_{21}^{b-} = \frac{\bar{\pi}_1\, r_{12}^{b+}}{\bar{\pi}_2} + \frac{\bar{\pi}_4\, r_{21}^{a+}}{\bar{\pi}_2} + \frac{\bar{\pi}_1\, r_{12}^{a+}}{\bar{\pi}_2 - \bar{\pi}_3} - \frac{\bar{\pi}_4\, r_{21}^{a+}}{\bar{\pi}_2 - \bar{\pi}_3}, \tag{2.34a}$$

$$0 < r_{21}^{a-} = \frac{\bar{\pi}_4\, r_{21}^{a+}}{\bar{\pi}_2 - \bar{\pi}_3} - \frac{\bar{\pi}_1\, r_{12}^{a+}}{\bar{\pi}_2 - \bar{\pi}_3}, \tag{2.34b}$$

$$0 = \frac{\bar{\pi}_1\, r_{12}^{a+}}{\bar{\pi}_3} - \frac{\bar{\pi}_1\, r_{12}^{b+}}{\bar{\pi}_2} - \frac{\bar{\pi}_4\, r_{21}^{a+}}{\bar{\pi}_2} + \frac{\bar{\pi}_4\, r_{21}^{b+}}{\bar{\pi}_3}. \tag{2.34c}$$

The existence of a solution is now determined only by $\mathbb{I}$ and $\bar{\Pi}$ according to (2.34c). Hence, similarity to $\mathbb{I}$ cannot be guaranteed by designing $\mathbb{D}$, if $\mathbb{D}$ is marginalizable.

### $\mathbb{D}$ and $\mathbb{I}$ are marginalizable

Lastly, also enforcing marginalizability of $\mathbb{I}$ implies introducing the constraints $r_{12}^{a+} = r_{21}^{a+}$ and $r_{12}^{b+} = r_{21}^{b+}$ in (2.34). With this addition, no valid stationary probability vector or indirect repulsion rates can be chosen to avoid contradictions in (2.34). Hence, this counterexample shows that similarity between two marginalizable networks is not always possible.

# CHAPTER 3

## Decision-making in traffic

Using the modeling framework from Chapter 2, this chapter addresses the three research questions formulated in Section 1.2. First, agreement and disagreement in a decision-making process among individual HRUs approaching an intersection is modeled using two different forms of interaction: attraction and indirect repulsion. After this, a busy intersection is modeled as a macroscopic decision-making process between parts of the intersection. Using this approach, a controller that uses the predicted development of HRU behavior in the intersection to determine the acceleration of an AV is designed.

## 3.1 Predicting decisions of individual interacting HRUs

The first research question asks how the effect of different interaction forms can be expressed in models of decision-making among uncertain HRUs. In Paper A, this is examined in an intersection scenario.

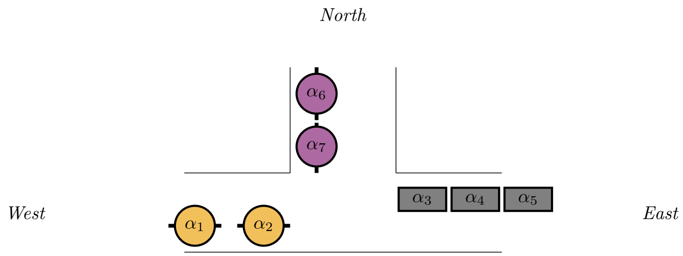



**Figure 3.1:** Intersection with cyclists $\alpha_1$, $\alpha_2$, $\alpha_6$ and $\alpha_7$ and drivers $\alpha_3$, $\alpha_4$ and $\alpha_5$.

## Problem formulation

Paper A considers three narrow roads heading *North*, *East*, and *West* that meet in an unsignaled intersection depicted in Fig. 3.1. In total, seven different HRUs are approaching the center. To be safe, it is assumed that they cannot all pass the intersection at the same time. The objective is to predict the probability that each HRU decides to *go* through the intersection, or *yield* for the others.

### Modeling assumptions

In this scenario, the number of HRUs is assumed to be fixed, and their isolated behavior is known beforehand. Additionally, the time interval between approaching and leaving the intersection is long enough for any decision-making process to converge to a stationary solution.

## Model

The scenario is modeled as a decision-making process between HRUs who decide to either *yield* or *go* through the intersection.

### Agents

Specifically, Fig. 3.1 shows two cyclists $\alpha_1$, $\alpha_2$ arriving from the *West*, and two cyclists $\alpha_6$ and $\alpha_7$ approaching from the *North*. Drivers $\alpha_3$, $\alpha_4$ and $\alpha_5$ come from the *East*. With a finite number of road users, each of them can be described as a separate CTMC decision-making agent. The set of all agents

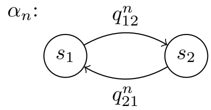



**Figure 3.2:** CTMC model of a road user $\alpha_n$ with transition rate matrix $Q_n$.

in the scenario is thus

$$\mathcal{N} = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7\}. \tag{3.1}$$

### Decision states

The two interesting decisions in the problem are represented as the states $s_1 =$ yield and $s_2 =$ go, collected in the set

$$\mathcal{S} = \{s_1, s_2\}. \tag{3.2}$$

### Transitions

In isolation, each agent $\alpha_n$'s preferences toward either decision are determined by transition rates $q_{ij}^n$ in the $2{\times}2$ transition rate matrix $Q_n$. Realistically, most HRUs approaching an empty intersection lean toward *go* but could sometimes decide *yield* as a safety precaution, albeit at a lower frequency. The CTMC structure describing each agent is shown in Fig. 3.2.

### Groups

Decision-making between cyclists and drivers may not be the same as decision-making between drivers only, especially if they arrive from different directions. Hence, the HRUs are divided into three groups, formulated as disjoint subsets of $\mathcal{N}$, depending on their type of vehicle and direction of origin. The groups

$$\mathcal{C}_1 = \{\alpha_1, \alpha_2\}, \tag{3.3}$$
$$\mathcal{C}_2 = \{\alpha_6, \alpha_7\}, \tag{3.4}$$
$$\mathcal{D} = \{\alpha_3, \alpha_4, \alpha_5\}, \tag{3.5}$$

represent cyclists from the west and the north, and drivers from the east, respectively.

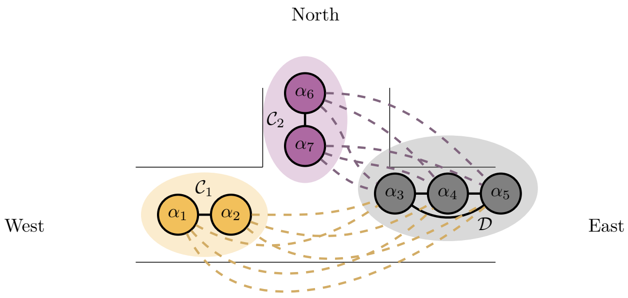



**Figure 3.3:** Interaction topology for the attraction force within each group (solid) and indirect repulsion between members of different groups (dashed).

### Interaction forces

Within each group, agents are assumed to *agree* on decisions. This is modeled as attraction forces, summarized in Section 2.5, between the agents. Agent-to-agent influence strength is equal between agents within each group, and tuning parameters are state-independent.

Between groups, agents are assumed to *disagree*, which is modeled as indirect repulsion forces. Cyclists in $\mathcal{C}_1$ and $\mathcal{C}_2$ exert decision repulsion on drivers in $\mathcal{D}$, but not on each other, and drivers are more susceptible to change compared to cyclists. As in the attraction force, all tuning parameters are state-independent. The topology of the different interaction forces in the intersection scenario is shown in Fig. 3.3.

### Network models and marginalization

In Paper A, the effect of different forms of interaction in the intersection problem is investigated. Three alternative $M^N = 128$ state network CTMC models of the decision-making process are formulated.

1. $\mathbb{X}$ is the nominal isolated network, defined by a rate matrix $Q$ constructed using (2.16).

2. $\mathbb{A}$ adds attraction forces to the nominal behavior of $\mathbb{X}$. Its transition matrix is $Q + A$, where $A$ is obtained through (2.17).

3. $\mathbb{S}$ extends $\mathbb{A}$ with indirect repulsion. Its total network transition rate matrix is $Q + A + R^+$ where $R^+$ is obtained through (2.19).

Due to the large scale of the network models, marginalized models with $MN = 14$ states describing the individual agent decision probabilities are derived for each network. Specifically, $Q_{\mathbb{M}}$ describes the isolated behavior according to (2.29), attraction is represented by the matrix $A_{\mathbb{M}}$ found by formulating (2.30) for every agent and state, and repulsion is described by $R_{\mathbb{M}}^+$ and $E_{\mathbb{M}}$ obtained through (2.31). For the three networks, the decision probabilities of each road user are given by solving

$$\dot{\Pi}_{\mathbb{M}}^{\mathbb{I}}(t) = Q_{\mathbb{M}}^T \Pi_{\mathbb{M}}^{\mathbb{I}}(t), \tag{3.6}$$

$$\dot{\Pi}_{\mathbb{M}}^{\mathbb{A}}(t) = \left(Q_{\mathbb{M}} + A_{\mathbb{M}}\right)^T \Pi_{\mathbb{M}}^{\mathbb{A}}(t), \text{ and} \tag{3.7}$$

$$\dot{\Pi}_{\mathbb{M}}^{\mathbb{S}}(t) = \left(Q_{\mathbb{M}} + A_{\mathbb{M}} + R_{\mathbb{M}}\right)^T \Pi_{\mathbb{M}}^{\mathbb{S}}(t) + E_{\mathbb{M}}, \tag{3.8}$$

respectively. Each model is simulated numerically, and while details can be found in Paper A, the main results are summarized next.

## Results

As a baseline, Fig. 3.4 shows each road user's decision probability in isolation according to $\mathbb{I}$. For every road user, the left- and right-hand bars represent the stationary probability to *yield* and *go* in red and green, respectively. For comparison, arbitrary initial decision probabilities are shown as grey bars. Most agents increase their probability to *go*, which is solely due to their isolated rate matrices.

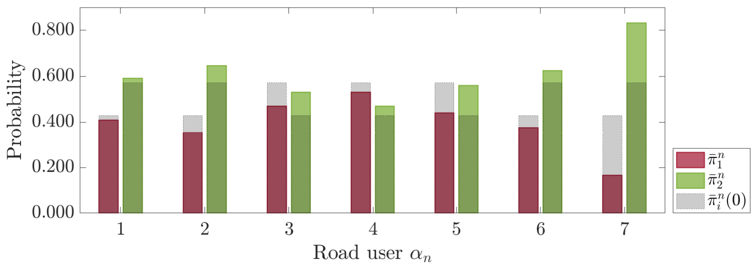



**Figure 3.4:** Network $\mathbb{I}$, isolated behavior.

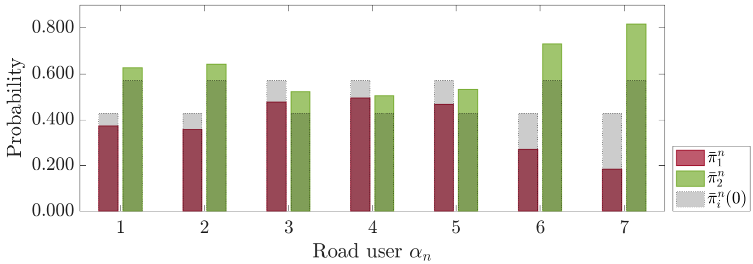



**Figure 3.5:** Network $\mathbb{A}$, isolated behavior with added attraction.

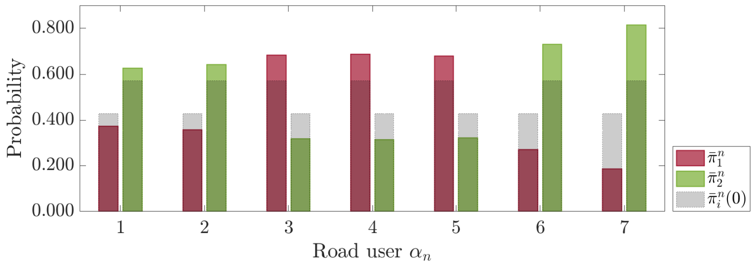



**Figure 3.6:** Network $\mathbb{S}$, isolated behavior with added attraction and repulsion.

Compared to this isolated case $\mathbb{I}$ in Fig. 3.4, a slight change in the stationary decision probabilities of $\mathbb{A}$ can be observed in Fig. 3.5. Agents $\alpha_1$ and $\alpha_6$ increase their probabilities toward *go*, the preferred decision of their respective group members, $\alpha_2$ and $\alpha_7$, whose decision probabilities remain unchanged. This effect is obtained by the high intensity of the attraction force exerted on $\alpha_1$ and $\alpha_6$. The force intensity is also high in $\mathcal{D}$, and a slight decrease in the probability to *go* can be observed for $\alpha_3$ and $\alpha_5$. However, a more prominent increase to *go* is seen in $\alpha_4$. This result shows that attraction can be used to emulate different levels of an agreement effect between certain decision-makers.

Decision probabilities according to $\mathbb{S}$ are shown in Fig. 3.6. With the inclusion of indirect repulsion, HRUs in $\mathcal{D}$ converge to the same high probability to *yield*, whereas the probability distributions for those in $\mathcal{C}_1$ and $\mathcal{C}_2$ remain unchanged. This disagreement effect is achieved by making $\mathcal{D}$ highly reactive to decisions made by $\mathcal{C}_1$ and $\mathcal{C}_2$, but not vice versa.

The results from Paper A summarized here answer the first research question posed in Section 1.2. In a Markovian opinion dynamic model of a decision-making process between HRUs, interaction can be modeled as social forces to emulate effects such as agreement and disagreement between agents. However, while marginalized models can be derived to describe traffic scenarios with increasing numbers of HRUs as decision-making processes, there are traffic scenarios in which individual HRUs cannot be modeled as separate decision-making agents. Such a case is covered in the next section.

## 3.2 Modeling large-scale traffic scenarios as decision-making processes

Consider the intersection depicted in Fig. 3.7. Compared to the intersection example in Section 3.1, the vast and varying number of HRUs in the scene makes it impossible to model the scene as a decision-making process between individual HRUs. The second research question in Section 1.2 addresses how decision-making processes in such large-scale traffic scenarios can be modeled. In Paper C, a method for doing so is suggested, and the approach is summarized in the following.

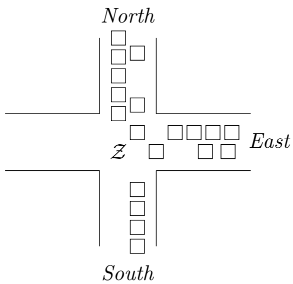



**Figure 3.7:** Road users from three directions *North*, *East* and *South* in a four-way intersection, and a shared zone $\mathcal{Z}$ in the center.

## Problem formulation

In the scenario shown in Fig. 3.7, HRUs continuously enter and exit an intersection in three different directions, *North*, *East*, and *South*. As they pass, they must all access the shared zone $\mathcal{Z}$, and the task is to predict the probability that the zone is occupied. However, modeling this scenario as a decision-making process among individual road is difficult for two reasons.

- At any moment, the scale of any network model becomes too great due to the number of road users.
- The frequency at which changes in decisions can be observed is lower than the duration in which road users stay in the intersection.

### Modeling assumptions

The exact number of HRUs in the traffic scene is assumed to be unknown, but varying and high on average. Similar to the scenario in Section 3.1, it is assumed that HRUs have to consider the actions of others as they pass the shared zone and that the process can be observed until it becomes stationary.

## Model

To handle a scenario with a varying number of HRUs, the approach suggested in Paper C is to widen the definition of a decision-making agent.

### Obstacle-generating agents

Instead of defining individual HRUs as agents, each HRU direction of origin, *North*, *East* and *South* is considered an agent in the set

$$\mathcal{N} = \{\alpha_1, \alpha_2, \alpha_3\}. \tag{3.9}$$

These agents interactively decide whether or not to occupy the shared zone $\mathcal{Z}$ with HRUs, defining the set of decisions

$$\mathcal{S} = \{s_1, s_2\}. \tag{3.10}$$

In Paper C, these agents are referred to as *obstacle-generating agents*, defined as CTMCs over $\mathcal{S}$. The nominal behavior of each agent in $\mathcal{N}$ is determined by a $2 \times 2$ transition rate matrix $Q_n$. In this low-dimensional example, the concept of groups is disregarded, and interaction is described as social forces between individual agents.

#### Interaction forces

As a simplified representation of the interactions in an intersection, it is assumed that HRUs from each direction prefer to enter $\mathcal{Z}$ when road users from other directions are not there. This is modeled using the indirect repulsion force (2.12). For $\alpha_n$, the transition rate increase toward a decision state $s_j$ is

$$\xi_j^n(t) = \gamma(n,l) \sum_{\alpha_l} \Gamma_{\mathcal{R}_\ell}^{\mathcal{A}}[n,l]\big(1 - \mathbf{1}_j^l(t)\big), \tag{3.11}$$

depending on the state of another agent $\alpha_l$. The agent representation of the intersection is shown in Fig. 3.8, in which each agent's transition rates are sums of an isolated rate and indirect repulsion.

#### Network model

Given the low number of agents, the network model has only nine states. Hence, marginalization is not directly necessary, and a network CTMC model $\mathbb{I}$ defined by the transition rate matrix $I = Q + R^+$ is constructed according to the procedure in Section 2.6. The matrices $Q$ and $R^+$ describe the isolated behavior of $\alpha_1$, $\alpha_2$ and $\alpha_3$ and the influence of indirect repulsion forces, respectively.

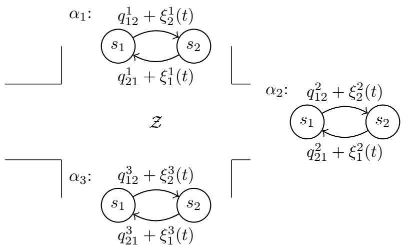



**Figure 3.8:** Abstract agent representation of the intersection. Through indirect repulsion, the agents $\alpha_1$, $\alpha_2$, and $\alpha_3$ influence each other's decisions on whether or not to occupy $\mathcal{Z}$ with HRUs.

To summarize, Paper C answers the second research question in Section 1.2 by suggesting that traffic scenarios in which the number of HRUs is too high or varying to be modeled as decision-making processes between individual HRUs can instead be modeled as decision-making between abstract, geographically static agents. In the intersection case in Fig. 3.7, these represent HRU directions of origin that decide whether or not to occupy a shared zone in the intersection. In Paper C, this modeling approach is used to construct a controller for an AV, addressing the final research question of this thesis.

## 3.3 Decision probabilities in model predictive control

Finally, the third research question in Section 1.2 addresses how models of HRU decision-making can be used in the control of AVs. Paper C demonstrates how a model of decision-making can be applied to a predictive controller that determines the acceleration of an AV. This control strategy, and the results from applying it to the intersection scenario from Section 3.2, are summarized next.

### Problem formulation

Assume that an autonomous ego vehicle is approaching the intersection scenario described in Section 3.2. This scenario is depicted in Fig. 3.9, and the objective is now to design a controller for the acceleration of the ego vehicle

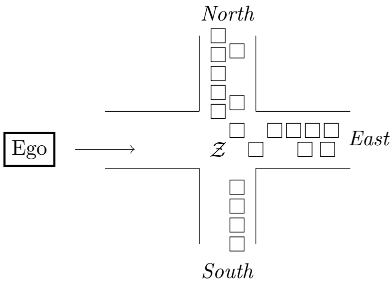



**Figure 3.9:** Autonomous ego vehicle approaching the intersection from Section 3.2.

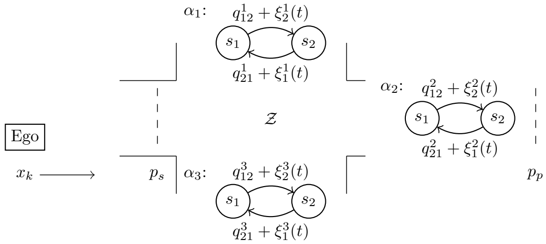



**Figure 3.10:** The ego vehicle approaching the intersection scene, described using the obstacle-generating agents $\alpha_1$, $\alpha_2$ and $\alpha_3$.

based on the predicted behavior of the intersection. Importantly, collisions with HRUs need to be avoided, but overly conservative behavior is undesired.

### Modeling assumptions

The intersection is assumed to behave according to the network model $\mathbb{I}$ defined in Section 3.2, and the traffic scenario described as a decision-making process between agents can be seen in Fig. 3.10. The position and velocity of the ego vehicle in the direction of the intersection is described as a discretized double integrator system. Moreover, the ego vehicle is assumed to follow a predefined path defined by an onboard system. In Fig. 3.10, $p_s$ and $p_p$ denote positions before and after the intersection, respectively.

## Controller

Paper C presents an MPC for the acceleration of the ego vehicle. Specifically, a scenario approach (S-MPC) similar to the method in [45] for avoiding HRUs described as uncertain moving obstacles is considered. In this strategy, a set of environment scenarios are formulated, each corresponding to a possible obstacle mode that may occur with some probability. With knowledge of that probability, an expected value of the cost function for each scenario can be formulated. The S-MPC then finds the control action that minimizes the expected cost of all scenarios simultaneously, subject to the constraints from each scenario.

In [45], however, the probabilities of each scenario are assumed to be constant. By instead describing the traffic scenario as a CTMC decision-making process, a prediction of how the obstacle mode probabilities evolve from an initial observation over the prediction horizon of the controller can be achieved. If these probabilities are used to generate the expected cost, the acceleration of the ego vehicle is determined based on the behavior of the complete traffic scene.

### Decision modes as scenarios

While the network $\mathbb{I}$ describes the probability of every decision configuration in the intersection, the primary concern is the number of agents that are generating obstacles in $\mathcal{Z}$ simultaneously. Four different modes collected in the set

$$\mathcal{M} = \{m_0, m_1, m_2, m_3\}, \tag{3.12}$$

where $m_\ell$ corresponds to having $\ell$ different agents generating obstacles in $\mathcal{Z}$ simultaneously. Table 3.1 shows how the probability of each mode can be obtained by summation over the network decision probabilities $\Pi_{\mathbb{I}}(t)$.

As the risk of collision with HRUs changes depending on the number $\ell$ of active obstacle-generating agents, every mode $m_\ell$ is associated with a reference $r_\ell$ for the ego vehicle's position and velocity and a constraint on its maximal position. In $m_0$ there are no HRUs in $\mathcal{Z}$, and the maximal position is a point after the intersection, denoted $p_p$ in Fig. 3.10. In $m_1$, $m_2$ and $m_3$ there are HRUs in $\mathcal{Z}$, and the maximal position lies before the intersection, which is denoted $p_s$ in Fig. 3.10. Additionally, the velocity reference decreases and becomes more conservative as the number of active cluster-generating agents increases. The position reference is always equal to the position constraint.

| Mode $m_\ell$ | Mode probability $\pi^{m_\ell}(t)$ |
|---|---|
| $m_0$ | $\pi_1^{\mathbb{I}}(t)$ |
| $m_1$ | $\pi_2^{\mathbb{I}}(t) + \pi_3^{\mathbb{I}}(t) + \pi_5^{\mathbb{I}}(t)$ |
| $m_2$ | $\pi_4^{\mathbb{I}}(t) + \pi_6^{\mathbb{I}}(t) + \pi_7^{\mathbb{I}}(t)$ |
| $m_3$ | $\pi_8^{\mathbb{I}}(t)$ |

**Table 3.1:** Decision modes as scenarios in the S-MPC.

### Optimal control problem

The S-MPC solves the following control problem.

$$\min_{x^\ell,u^\ell} \sum_{h=k}^{k+H} \sum_{\ell=1}^{L} \pi^{m_\ell}_{h|k} \Big( (x^\ell_{h|k} - r_\ell)^T Q_c (x^\ell_{h|k} - r_\ell) + u^{\ell,T}_{h|k} R_c u^\ell_{h|k} \Big) \tag{3.13}$$

$$\begin{aligned}
\text{s.t. } & x^\ell_{h+1|k} = A_{\text{ego}} x^\ell_{h|k} + B_{\text{ego}} u^\ell_{h|k}, && \forall\, m_\ell \in \mathcal{M}, && (3.14)\\
& f(x^\ell_{h|k}, u^\ell_{h|k}) \leq 0, && \forall\, m_\ell \in \mathcal{M}, && (3.15)\\
& g(x^\ell_{h|k}, u^\ell_{h|k}, m^\ell_{h|k}) \leq 0, && \forall\, m_\ell \in \mathcal{M}, && (3.16)\\
& u^\ell_{h|k} = u^o_{h|k}, && \forall\, m_\ell \neq m_o \in \mathcal{M}, && (3.17)\\
& x^\ell_{k|k} = x_k, && \forall\, m_\ell \in \mathcal{M}. && (3.18)
\end{aligned}$$

The objective function (3.13) describes the expected stage cost over the prediction horizon of length $H$ as the quadratic cost of reference deviations and control for each scenario $m_\ell$, weighted by the predicted mode probability $\pi^{m_\ell}_{h|k}$. The evolution of the mode probabilities over the prediction horizon is derived by simulation of $\mathbb{I}$ from an initial observation at $t_k$, and the total expected cost is obtained by summation over all stages and scenarios.

Minimization of the expected cost is done under several constraints. The constraint (3.14) specifies the ego vehicle dynamics in each scenario, while (3.15) represents the state- and input limitations and (3.16) represents the constraints for each scenario. Equality between inputs and initial states across all scenarios is ensured by (3.17) and (3.18).

## Numerical simulation

Two S-MPCs, $C_s$ and $C_t$, are compared in two simulations of 6 seconds each. $C_s$ uses the stationary solution of the decision-making process over the entire prediction horizon, while $C_t$ uses the transient mode probabilities from an observation of the intersection at each time step. During the simulation, four events occur that change the initial mode probability $\Pi^{\mathcal{M}}_{k|k}$. These are summarized in Table 3.2. Initially, the intersection is in $m_3$ which implies that agents from all directions send HRUs into $\mathcal{Z}$. As time goes on, the number of agents who have HRUs in the intersection is decremented until $t_k = 3$, when the intersection is empty.

| Event | Time $t_k$ $[s]$ | Observation | $\Pi_{k\|k}^{\mathcal{M}}$ |
|---|---|---|---|
| $e_0$ | 0 | All agents in $s_2$. | $\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}^T$ |
| $e_1$ | 1 | Two agents in $s_2$. | $\begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}^T$ |
| $e_2$ | 2 | One agent in $s_2$. | $\begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix}^T$ |
| $e_3$ | 3 | All agents in $s_1$. | $\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}^T$ |

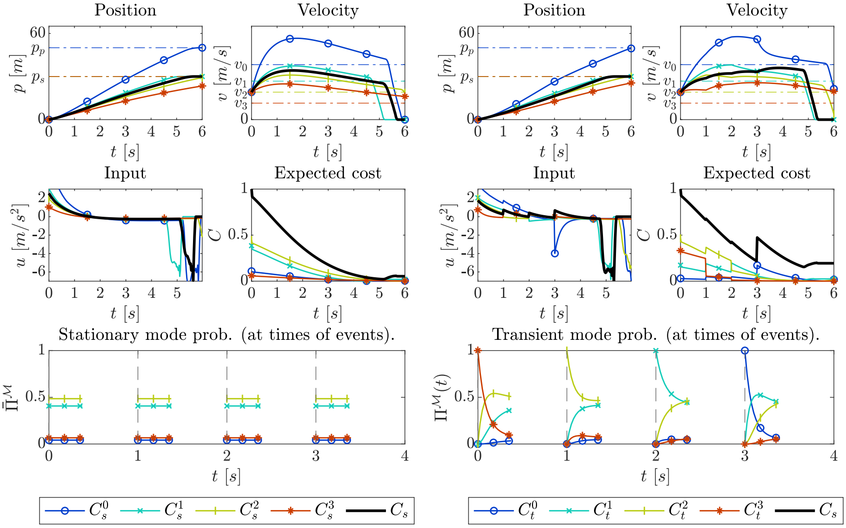
**Table 3.2:** Events in the simulation.

## Results

In Figures 3.11a and 3.11b, the black trajectories are produced by applying $C_s$ and $C_t$, respectively, to control the AV. Moreover, these are compared to controllers generated from each separate scenario, denoted $C_{s/t}^{\ell}$.

As a new event in Table 3.2 occurs, the mode probabilities used by $C_t$ evolve from a different initial value to the stationary solution, affecting the performance. Comparing the expected costs of $C_t$ in Fig. 3.11b to those of $C_s$ in Fig. 3.11a, the initial cost of the most dangerous scenario, $m_3$, has a higher influence on $C_t$ than on $C_s$. This is because $C_t$ takes the initial observation of $m_3$ into account in the transient scenario probabilities. Similar effects on $C_t$'s behavior can be seen for all other events. This induces a slow ramp-up in the velocity of the ego vehicle. In contrast, $C_s$ produces a high initial velocity before ramping down.

Answering the third and final research question posed in this thesis, one control application for models of decision-making processes in traffic is scenario-based model predictive controllers that minimize the expected cost of uncertain traffic scenarios. When the expected cost is based on probabilities from a model of the behavior of a complete traffic scene, the effect that different forms of HRU behavior and interaction have on the controller can be simulated. Moreover, the results of Paper C show that the use of transient scenario probabilities in an S-MPC approach has a significant effect on the performance of the controller compared to using static probabilities.




**(a)** $C_s$, using stationary mode probabilities.

**(b)** $C_t$, using transient mode probabilities.

**Figure 3.11:** Performance of the controllers $C_s$ and $C_t$, constructed using stationary and transient scenario probabilities, respectively.

# CHAPTER 4

## Summary of included papers

This chapter provides a summary of the included papers.

## 4.1 Paper A

> **Carl-Johan Heiker**, Paolo Falcone
> Decision Modeling in Markovian Multi-Agent Systems
> *2022 IEEE 61$^{st}$ Conference on Decision and Control (CDC)*,
> Cancun, Mexico, Dec. 2022.
> ©2022 IEEE, DOI: 10.1109/CDC51059.2022.9993134.

This paper considers a traffic scenario in which multiple, interactive HRUs approach a three-way intersection. The objective is to determine a model predicting which of the HRUs will decide to go through the intersection, and who will decide to yield for the others. However, an assumption is that deterministic models of the future decisions of HRUs are unobtainable. A stochastic abstraction of the decision-making process is necessary to derive every agent's decision probability. Moreover, this model needs to capture how interaction affects decision-making. The decision-making process between the HRUs is modeled as a Markovian multi-agent opinion dynamic network. In this framework, each HRU is an agent that chooses between decisions as a CTMC. Interaction is described as modulating the transition rate between each agent's decision state, depending on the observed decisions of other agents. As agents may react to the decisions of others in several different ways, this paper introduces the concept of agent groups into the Markovian opinion dynamic framework. Interaction within agent groups is modeled as an attractive rate-modulating social force, drawing group members toward making the same decisions. Conversely, a repulsion force describes the interaction between separate agent groups, pushing them toward different decisions. This paper shows that reducing the network model into a small-scale marginalized form is possible even when defining interaction through group-wise attractive and repulsive forces. Lastly, the intersection example is simulated as a decision process between groups, defined by vehicle type and direction of origin. The model predicts that all drivers will yield for cyclists with high probability, due to simultaneous attraction and repulsion.

## 4.2 Paper B

**Carl-Johan Heiker**, Elisa Gaetan, Laura Giarré, Paolo Falcone
Repulsive Markovian Models for Opinion Dynamics
*Systems & Control Letters*,
vol. 185, March 2024.
©2024 Elsevier, DOI: 10.1016/j.sysconle.2024.105720.

This paper extends the framework introduced in Paper A, which describes a Markovian opinion dynamic model of a decision process in a network of stochastic agents. In Paper A, disagreement between agents is modeled as a repulsion force between agent groups. The force increases the rate at which an agent transitions to each decision state depending on which agents from conflicting groups are currently in different states. Paper B refers to this as indirect repulsion and introduces the direct repulsion force as an alternative. This force instead decreases rates toward a decision state if conflicting agents are in the same decision state. As direct repulsion appears more intuitive, it is necessary to determine technically if the two force models can similarly describe the same decision process. This paper defines the similarity property between two general networks as them reaching the same stationary decision probabilities with minimal difference in expected decision state holding time. Moreover, this paper shows that the marginalization procedure used to reduce the model dimension in Paper A can be performed with the suggested direct repulsion, under the same conditions. However, using a counterexample, it is shown that similarity is not generally possible if the two models can also be marginalized. Moreover, numerical simulations show how direct repulsion can induce a longer convergence time until stationary decisions, compared to indirect repulsion. Lastly, the less restrictive indirect repulsion is recommended as the two disagreement models cannot necessarily describe the same decision process under the same conditions.

## 4.3 Paper C

> **Carl-Johan Heiker**, Paolo Falcone
> Trajectory Planning Among Interactive Markovian Multi-Modal Obstacles using Scenario-MPC
> Submitted to *22$^{nd}$ European Conference on Control (ECC)*,
> Stockholm, Sweden, June, 2024.

In this paper, a traffic scenario in which an autonomous ego-vehicle approaching an intersection is considered. The intersection has a shared zone, and three directions from which HRUs can enter at any point in time. To enable safe navigation through the intersection, the AV needs to plan its acceleration according to the predicted behavior of the complete intersection scene. To describe the behavior of the intersection independently of the number of HRUs in the scene, each HRU direction of origin is modeled as an abstract form of agent that can generate obstacles in the shared zone. Using the framework introduced and developed in papers A and B, the intersection is modeled as a decision-making process between the three agents that decide whether or not to send vehicles into the shared zone. This model thus predicts the evolution of the probabilities of several obstacle modes from an initial observation. Each mode enforces different constraints for the ego-vehicle. The acceleration of the ego-vehicle is determined by an S-MPC that minimizes the expected cost of all possible obstacle modes. At the same time, the constraints from each mode are enforced, ensuring that the vehicle brakes in time. In numerical simulations of the traffic scenario, it is shown that predicting the evolution of the mode probabilities over the controller's prediction horizon induces a less conservative braking behavior for the ego-vehicle, compared to static probabilities.

# CHAPTER 5

# Concluding remarks and future work

## 5.1 Discussion and conclusion

To increase AV's abilities to interpret and predict their surroundings, this thesis presents a framework for how traffic scenarios can be modeled as decision-making processes between HRUs. The approach relies on two main assumptions of how HRUs behave. First, the amount of information necessary for describing and predicting HRU behavior using deterministic models is assumed to be practically infinite, and any model for how HRUs make decisions needs to be stochastic. Second, the decisions made by HRUs are assumed to be affected by both their motives and different forms of interaction with other HRUs.

### Summary of the modeling approach

To capture both the stochastic and interactive aspects of HRU behavior, a Markovian opinion dynamic framework is suggested for modeling how decisions are made in traffic. Corresponding to the first aspect, decision-makers are described as CTMC agents that make stochastic transitions between decisions in a set. To handle the second aspect, two forms of interaction between agent subgroups are modeled as social forces that modulate the rates at which agents transition between decisions. Within agent groups, *attraction* increases the transition rate toward popular decisions to emulate agreement. Conversely, two forms of *repulsion* are proposed to emulate disagreement between agents in different groups.

A CTMC model of how the network of agents transitions between decision configurations due to personal motives and interaction is formulated. Moreover, methods for assessing how the dimension of this model can be reduced, how two different forms of interaction can achieve similar effects, and how the approach can be applied to MPC are presented in the included papers.

## Answers to research questions

In Chapter 1, three research questions were formulated to assess how decision-making processes between HRUs can be modeled and utilized. Beginning with a recap of the research questions, this section discusses how they are answered in the thesis.

- **RQ1** How can the effect of different interaction forms be captured in a model of decision-making between uncertain HRUs?
- **RQ2** How can large-scale traffic scenarios be modeled as decision-making processes?
- **RQ3** How can models of HRU decision-making be used in the control of AVs?

### Modelling the effect of interaction in decision-making processes

A challenge for the development of AVs is the difficulty of predicting the behavior of surrounding traffic. The lack of information required to construct deterministic models, and the interactive nature of HRUs, make describing their decision processes a challenge. This thesis therefore suggests a Markovian opinion dynamics framework for modeling how HRUs change decisions stochastically while influencing each other through interaction. With this method, the probability of HRU decisions can be predicted.

The effects of interaction between HRUs in a traffic scene are assumed to vary. For example, cyclists may see other cyclists pass an intersection and follow, while drivers collectively yield for them. Paper A therefore focuses on modeling two primary effects in a decision-making process in traffic: agreement and disagreement between groups of HRUs, represented as agents.

In Paper A, the agreement effect is achieved through the *attraction force*, which increases an HRU's transition rates toward decisions that are currently popular in its group. Using a similar form of transition rate modulation, the effect of disagreement is modeled indirectly by increasing the HRU's transition rates toward decisions that are *unpopular* among other HRU groups. However, disagreement can also be modeled as a direct decrease of rates toward decisions that are popular in other groups and is explored in Paper B. While this introduces time-dependent transition rates in each agent, making them non-homogeneous, a single CTMC over the decision configurations in the entire network can be constructed. For this structure, all transition rates can be evaluated as constants, making the network a time-homogeneous CTMC that can be used to predict the decision probabilities for all HRUs.

The results of Paper A, summarized in Section 3.1, thus answer the first research question by showing that the stationary HRU decision probabilities predicted by the model change depending on the interaction type. In the intersection example, groups of cyclists and drivers diverge from their nominal behavior and approach similar decision probabilities if the attraction force is present within their groups. Moreover, repulsion is required to describe an increased probability that HRU groups disagree. However, the basic forms of interaction explored here cannot be guaranteed to have a unique effect. As is shown in Paper B, effects such as disagreement can be achieved by several forms of interaction. Hence, while any form of interaction can be used to explain outcomes in traffic scenes, methods for learning the actual interaction are required to predict them.

### Large scale traffic scenarios

In the intersection scenario presented in Paper C, new HRUs are arriving at a crossing in relatively large numbers, from three different directions. At the same time, HRUs depart from the scene at a rapid pace. In this scenario, modeling a decision-making process between individual HRUs as in papers A and B is difficult, as the approach assumes a constant number of decision-makers. Moreover, the number of decisions and HRUs may be too much even for a marginalized version of the network model.

Throughout this thesis, and in papers A and B, decision-making processes have been described between *agents*. In Paper C, it is shown that the concept of a decision-making agent is not necessarily synonymous with a single HRU. Instead, the intersection itself can be seen as a decision-making process between abstract agents. Specifically, Paper C defines each direction in which HRUs enter and exit as an agent that decides whether or not to send vehicles into a shared zone in the center. Interaction in the form of forces between the abstract agents represents the effect of interaction between the HRUs in the scene. In this approach, HRUs are seen as actors that manifest the decision-making of a collective.

Using this principle, the framework described in this thesis can be applied to large-scale traffic scenarios, thus answering the second research question. Section 3.2 summarizes how the intersection scenario from Paper C can be modeled as three agents that decide whether or not to occupy a shared zone with HRUs. This scenario only requires three agents, which suggests that it is possible to describe even larger scenarios as decision-making between agents. As a generalization, this process can be seen as a way of modeling how uncertain agents in infrastructure negotiate shared resources.

### Control applications

This thesis aims to improve the capability of AVs to interpret and anticipate the behavior of their surrounding HRUs. In line with this, the third and final research question addresses how models of decision-making processes can be used in the control of AVs.

Continuing with the large-scale intersection scenario, Paper C considers an AV that approaches the traffic scene and presents an S-MPC determining the AV's acceleration based on the decision-making process description of the HRUs in the intersection. Summarized in Section 3.3, the controller predicts the evolution of the agent decision probabilities using the intersection model over a time horizon. As agents decide whether or not to send HRUs into the intersection, the probability of several obstacle scenarios can be derived for the AV. Using the scenario probabilities, an optimal control problem for minimizing the expected cost of the AV's behavior in all scenarios over the horizon is constructed. In each time step, the S-MPC then solves this problem to derive an acceleration that abides by the constraints of all scenarios.

The result of Paper C is a comparison between using stationary versus transient scenario probabilities in the S-MPC. While the stationary, constant probabilities result in an acceleration of the AV that does reflect the intersection, transient scenario probabilities express the predicted change in the probability that each scenario is observed after an initial observation. Hence, the transient probabilities induce a trade-off in the cost function between the current observation and the scenarios that will be likely in the future. In the example presented in Paper C, the intersection model predicts that the most probable scenarios are when either one or two agents generate obstacles in the shared zone, whereas it is unlikely that the intersection is either full or empty. Each possible scenario occurs at some point in the simulation, and it is shown that the transient probabilities result in a less conservative AV acceleration compared to when the stationary solution is used.

## Challenges

Applying the model presented in this thesis to real traffic scenarios presents several challenges, and some of them are briefly discussed in the following.

### Availability of data

While machine learning methods are among the more popular modern methods for learning the behavior of HRUs in a complete traffic scene, the approach described in this thesis is model-based. While the model can be used to simulate the effect of different behaviors using relatively few parameters, the model needs to be calibrated using data. This is an issue, as HRU decisions are not directly measurable and must be inferred from available data. In the most optimistic case, motorized vehicles may share detailed data recorded on the vehicles themselves. Information on how drivers operate the vehicles that is intuitively linked to decision-making, such as the use of indicator lights, may also be available. However, the same information is unavailable for other important HRUs, such as pedestrians and cyclists. Hence, the model needs to be simple enough to tune using data that is available for all HRUs, such as position and velocity.

### Location dependent models

Parts of the model are dependent on location. While HRUs may have different overall driving styles and interaction patterns that are less dependent on external factors, separating the effect of interaction from the effect of road geometry and other local features appears more difficult. While data of isolated and interacting HRUs could be obtained from a single location and used to learn the effect of interaction compared to isolation, the validity of the model is then limited to the particular location.

#### Several valid interaction models

Paper B deals with the fact that several interaction forms could lead to the same stationary decision probabilities. This also influences how an interaction model can be learned from data. For instance, an unrealistic interaction topology could be assumed such that it describes an observed effect for some set of tuning parameters. To avoid a misleading description of HRU behavior, identifying the characteristics of the interaction itself is thus preferred over tuning the model parameters to merely obtain the correct stationary decision probabilities.

#### The effect of external disturbances

Lastly, as the model is based on a CTMC, there is no way of describing the influence that external events may have on the decision-making process. For instance, it is as of now impossible to describe the influence that an AV has on a decision-making process as it approaches an intersection. However, this could be achieved if the CTMC model of the decision-making process is translated into a controlled Markov chain such as a discrete-time Markov decision process [46].

## 5.2 Future work

Inspired by the challenges presented previously, this section suggests some of the possible research directions for applying the presented model to real traffic scenarios.

### Learning

One of the main challenges with applying this model to traffic scenarios is how to accurately learn its parameters from data. As discussed earlier, one issue is that decisions need to be inferred from available data, a problem that can be approached in several ways. For instance, by assuming that measurements are the output of an HMM, a Markov chain representing the decision-making process could be learned from measurable HRU data. This would however require that the CTMC model is converted into a DTMC, which introduces additional assumptions. By instead interpreting the CTMC transition rates as distribution parameters for decision holding time, it could alternatively be possible to learn parameters without first converting the CTMC to discrete time.

## Feedback

When a single AV approaches a traffic scenario in which there are many HRUs present, such as in Paper C, it can be argued that the AV's influence on the decision process is negligible. However, there are also traffic scenarios in which an AV has a greater effect on the behavior of surrounding HRUs, which will likely influence the decision-making process between them. To model this, one approach is to introduce AVs as decision-makers who are stubborn to the point that their decisions approach deterministic ones. Using interaction forces, their behaviors could be predefined conditionally on the actions of the HRU agents. A second approach is, as mentioned previously, to construct a Markov decision process from the model.

## Control

While an introductory example of how the model can be used in scenario-based MPC for AVs navigating uncertain environments is shown in Paper C, the control strategy is a simplified version of that in [45]. Hence, a more realistic use case in which a non-simplified scenario MPC utilizing transient scenario probabilities is suggested. This control strategy could serve as an automatic tuning of the optimization problem after how probable each scenario is after an initial observation, and adjust the necessary level of conservative AV behavior accordingly.

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