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# Back-Pressure Traffic Signal Control in the Presence of Noisy Queue Information<sup>\*</sup>

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**Abstract:** In this paper, we consider centralized traffic signal control policies using the max-weight algorithm when the queue size measurement is noisy. We first show analytically that the standard max-weight algorithm is throughput optimal even under noisy queue measurements. However, the average steady-state queue lengths and subsequently the average delays are increased. In order to alleviate the effect of these noisy measurements we add filtering to the max-weight algorithm; more specifically, we propose the Filtered-max-weight algorithm, which is based on particle filtering. We demonstrate via simulations that the Filtered-max-weight algorithm performs better than the standard max-weight algorithm in the presence of noisy measurements.

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**Keywords:** traffic signal control; back-pressure; noisy queue backlog information; throughput region; transportation systems; particle filtering.

## 1. INTRODUCTION

Traffic conditions in major cities have become an important issue, since congestion results in delays, carbon dioxide emissions, higher energy expenditure and accident risks (see, for example, [Bigazzi and Figliozzi 2012] and references therein). Control and coordination of traffic movements has been at the epicenter of intelligent transportation networks from different angles. One approach is to introduce traffic adaptive signaling at intersections, since a fixed-cycle control system might be an inevitable waste of precious green time/phase without incoming traffic demand, i.e., queues. Towards this end it is observed that urban network throughput can be significantly improved if traffic light algorithms are orchestrated to serve major flows. SCATS [Sims and Dobinson 1980] and SCOOT [Robertson and Bretherton 1991] constitute two of the many systems proposed in the literature and are also deployed in many cities across the world. However, none of these approaches have proven to work for all traffic conditions (under- and over-saturated) for which a policy exists, i.e., they are not proven to be *network throughput optimal*. A throughput optimal approach, called the max-weight (or back-pressure) algorithm, for handling traffic in communication networks was proposed in the seminal

paper [Tassiulas and Ephremides 1992]. Since then, the idea has been advanced by many researchers and it has been applied extensively in communication networks and in other areas as well. Recently, there has been an interest in applying the same concept in transportation systems (see, for example, [Varaiya and Pravin 2013, Gregoire et al. 2015, Le et al. 2015, Zaidi et al. 2015, 2016] and references therein). [Ramadhan et al. 2020] shows that the max-pressure approach for a disturbed network has the capability to avoid congestion across road segments in real traffic conditions. The extended backpressure traffic signal control algorithm can also maintain stability of urban traffic network while preventing queue spillback such that the performance of the traffic network is improved [Hao et al. 2020].

For this application, however, the measurement of the queue size is not so trivial. One approach uses video cameras, where some vehicles may be occluded. Another approach is the use of radar sensors, which are more accurate than cameras, but are relatively weak at understanding the precise shape of the object, and as a result, the system might not distinguish the real traffic (even though their speeds can be determined). Finally, LiDARs are promising sensors, but they are very expensive and require very high computing power compared to cameras and radars, making them prone to system malfunctions and software glitches. As a result, in many cases, computing the traffic

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may lead to erroneous values of the queue size. This problem has motivated us to investigate how the back-pressure algorithm performs when it has noisy queue information and what should it be to optimize performance.

The idea of incomplete network state information has been extensively studied, but mainly in the context of topology and channel-state uncertainties (see, for example, [Pantelidou et al. 2007, Ying and Shakkottai 2012] and references therein). In addition, the idea of noisy queue backlog information has also been considered for example, in [Neely 2003] where, a Dynamic Routing and Power Control (DRPC) policy is proposed and it is stated that it still provides stability whenever possible. The impact of delayed queue size and channel-state information on the throughput region was investigated in [Ying and Shakkottai 2011], but the effect of the delayed queue size was not investigated in isolation. More recently, [Le et al. 2015] propose a decentralized traffic signal control policy which is throughput optimal under noisy measurements of the queue size. However, for their proposed policy, each slot is divided among the different phases, some of which would be very small to actually accommodate some of the traffic, while a large number of switchings between different phases would be required resulting in a high loss of service.

In this work, we consider centralized traffic signal control policies in which the queue size measurement is noisy and we study their performance. More specifically, our contributions are as follows:

- We analytically show that the standard back-pressure algorithm is throughput optimal even under noisy queue measurements.
- We show via simulations that filtering improves the performance of the back-pressure algorithm since it reduces the steady-state average queue size and subsequently reduces the average travel delay in the network.

The remainder of the paper is organized as follows. In Section 2, we introduce the notation used throughout the paper, describe the model of the network and provide preliminary results and definitions. In Section 3 we present our main result. Then, in Section 4 we propose a filtering approach based on particle filtering (herein called the Filtered-max-weight algorithm) that is to be used to improve the steady-state average queue size. In Section 5 the numerical simulations are presented to support our algorithm. Finally, conclusions are drawn in Section 6.

## 2. NOTATION AND PRELIMINARIES

### 2.1 Notation and System Model

We let  $\mathbb{R}$ ,  $\mathbb{N}$ , and  $\mathbb{N}_0$  denote the set of real numbers, natural numbers, and natural numbers including zero, respectively. The Euclidean norm is denoted by  $\|\cdot\|$ .  $\mathbb{E}\{\cdot\}$  represents the expectation of its argument. Also, by  $x \wedge y$  we denote the  $\min\{x, y\}$ . The cardinality of a set  $\mathcal{X}$  is denoted by  $|\mathcal{X}|$ .

We follow the standard road network model, where the network is modeled as a directed graph  $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ ; nodes  $i \in \mathcal{N}$  represent the lanes with the capability to queue vehicles, and links  $\varepsilon_{ij} \in \mathcal{E}$  represent the possible flows from node (lane)  $i$  to node (lane)  $j$  connecting the incoming

nodes with the outgoing ones. Note that a road might have more than one lanes; in this case each lane is considered to be one node, and that each junction  $\ell \in \mathcal{J}$  consists of a set of links. All nodes that have links to node  $i$  directly are said to be *in-neighbors* of node  $i$  and belong to the set  $\mathcal{N}_i^- = \{j \in \mathcal{N} \mid \varepsilon_{ji} \in \mathcal{E}\}$ . Similarly, the nodes that receive vehicles from node  $i$  comprise its out-neighbors and are denoted by  $\mathcal{N}_i^+ = \{j \in \mathcal{N} \mid \varepsilon_{ij} \in \mathcal{E}\}$ .

We assume that all controllers at the intersections have a common cycle of length  $T$  and decisions are taken synchronously. Hence, time is considered as a slotted time model with  $t \in \mathbb{N}_0$  being the number of the cycle about to start. The loss of service due to idle times during switches is fixed and since it does not affect the subsequent analysis it is assumed to be zero for simplicity.

### 2.2 Arrivals and service rates

Consider a slotted system with  $n$  lanes (i.e.,  $n$  nodes with queues). Let  $A(t) = (A_1(t) \ A_2(t) \ \dots \ A_n(t))^T$  denote the vector of exogenous arrivals in which  $A_i(t)$  is assumed to be an i.i.d. random variable and takes integer values (i.e., the number of vehicles). The arrival rate  $A_i(t)$  is given by

$$\lambda_i \triangleq \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbb{E}\{A_i(t)\},$$

whereas the second moments  $\mathbb{E}\{A_i^2(t)\}$  are assumed to be finite. Let  $\boldsymbol{\lambda} \triangleq (\lambda_1 \ \lambda_2 \ \dots \ \lambda_n)^T$  denote the vector of arrival rates of all the nodes in the network. Let also  $S(t) = (S_1(t) \ S_2(t) \ \dots \ S_m(t))^T$  denote the vector of services at all links (i.e.,  $m = |\mathcal{E}|$ ) at time (traffic cycle)  $t$ , and  $S(t) \in \mathcal{S}$ , where  $\mathcal{S}$  is the set of all possible service vectors. Note that the service  $S_i(t)$  at any link  $i$  may vary for each traffic cycle and it is bounded, i.e.,  $S_{\max} = \max_{i \in \mathcal{N}} S_i(t) < \infty$  for all times  $t$ .

### 2.3 Queue dynamics

Let  $Q_i(t)$  denote the queue backlog (i.e., the number of cars) at lane  $i \in \mathcal{N}$  and the vector  $Q(t) = (Q_1(t) \ Q_2(t) \ \dots \ Q_n(t))^T$  represent the current state of the system at the beginning of time slot  $t$ . Suppose now that the decision is being made on noisy measurements of the queue backlogs  $\tilde{Q}(t)$  and

$$\tilde{Q}(t) = Q(t) + \nu(t), \quad (1)$$

where  $\nu(t) = (\nu_1(t) \ \nu_2(t) \ \dots \ \nu_n(t))^T$ . Since  $\nu_i(t)$  represents the noise at lane  $i$  at time  $t$  with respect to the number of vehicles seen in the queue, so  $\nu_i(t)$  takes integer values. Note that since we consider roads with finite length,  $Q_i(t)$  cannot be negative and  $\nu_i(t)$  is finite for all  $i$ .

In case node  $i$  is selected to forward vehicles to its out-neighbors  $j \in \mathcal{N}_i^+$ , then the number of vehicles from  $i$  to  $j$  is denoted by  $S_{ij}$  and the actual number of vehicles served is  $S_{i\bullet}(t) \wedge Q_i(t)$ , where  $S_{i\bullet}(t) = \sum_{j \in \mathcal{N}_i^+} S_{ij}(t)$  (note that  $S_i(t)$  refers to the service rate on link  $i$ , whereas  $S_{i\bullet}$  refers to the service rate from node  $i$  to all its out-going nodes). In the text, we will often substitute  $S_{ij}(t)$  by  $S_{i\bullet}(t)p_{ij}(t)$ , where  $p_{ij}(t)$  is the proportion of the traffic from node  $i$  to

node  $j$ . The service rates at each traffic cycle are assumed to be known to the traffic light controller.

Let  $a(t) = (a_1(t) \ a_2(t) \ \dots \ a_n(t))^T$  denote the vector of decision variables representing the decision taken at time step  $t$ , where  $a_i(t) \in \{0, 1\}$ , i.e., if node  $i$  is selected at time slot  $t$  then  $a_i(t) = 1$ , otherwise it is zero. This decision depends on the current noisy measurement of the queue backlogs  $\tilde{Q}(t)$  and services  $S(t)$ , but we use  $a(t)$  for brevity (instead of  $a(\tilde{Q}(t), S(t))$ ).

For a given policy  $\{a(t)\}_{t=0}^\infty$ , for node  $i$  the evolution of the queue size is as follows

$$Q_i(t+1) = Q_i(t) - a_i(t)(S_{i\bullet}(t) \wedge Q_i(t)) + A_i(t) + \sum_{j \in \mathcal{N}_i^-} a_j(t)(S_{ji}(t) \wedge Q_j(t)). \quad (2)$$

#### 2.4 Network stability and throughput region

We recall the definitions of *network stability* and the *throughput region* [Tassiulas and Ephremides 1992]. We begin with the definition of the *queue stability* with respect to a generic backlog  $Q_i(t)$ ; A queue with stochastic arrival and departure processes is called stable if its average backlog is bounded, i.e.,

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbb{E}\{Q_i(t)\} < \infty.$$

A network is called stable if all individual queues in the network are stable. Given the vector of queues backlogs we define the total queue size of the network to be  $Q^\Sigma(t) = \sum_{i \in \mathcal{N}} Q_i(t)$ . Equivalently, the network with stochastic arrival and departure processes is called stable if its average backlog is bounded, i.e.,

$$\limsup_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^T \mathbb{E}\{Q^\Sigma(t)\} < \infty.$$

The throughput region  $\Lambda$  of a network is the set of all traffic arrival rate vectors for which there exists a scheme that stabilizes the network. Such a scheme is known as a *throughput-optimal scheme*.

#### 2.5 Traffic Phases

At every slot  $t \in \mathbb{N}$ , junction  $\ell$  activates a signal phase  $\sigma \in \mathcal{P}_\ell$ , where  $\mathcal{P}_\ell$  denotes the possible phases at the junction. Once  $\sigma$  is chosen, a set of non-overlapping links are activated for transfer of vehicles. When a phase  $\sigma \in \mathcal{P}_\ell$  is activated at junction  $\ell$ , the number of vehicles transferred by lane  $i$  to  $j$  at time instant  $t$  is given by  $S_{ij}(t)$ , as defined in Section 2.3. Once a phase is selected, if lane  $i$  belongs to the roads activated, then  $a_i = 1$ .

#### 2.6 The max-weight (or back-pressure) algorithm

For each lane  $i$  going to lane  $j$ , the *pressure weight*  $W_{ij}$  is computed as follows:

$$W_{ij}(Q(t)) = Q_i(t) - Q_j(t). \quad (3a)$$

For each phase  $\sigma \in \mathcal{P}_\ell$ , the junction computes the pressure release

$$w_\sigma(Q(t)) = \sum_{(i,j) \in \sigma} W_{ij}(Q(t)) S_{ij}(t), \quad \forall \sigma \in \mathcal{P}_\ell, \quad (3b)$$

and chooses the one with the highest pressure release, i.e.,

$$\sigma^*(t) = \arg \max_{\sigma \in \mathcal{P}_\ell} w_\sigma(Q(t)). \quad (3c)$$

### 3. THE NOISY-MAX-WEIGHT (OR NOISY-BACK-PRESSURE) ALGORITHM

For the case for which the queue length measurements are noisy, for each lane  $i$  going to lane  $j$ , the *pressure weight*  $W_{ij}$  is computed as follows:

$$W_{ij}(\tilde{Q}(t)) = \tilde{Q}_i(t) - \tilde{Q}_j(t). \quad (4a)$$

For each phase  $\sigma \in \mathcal{P}_\ell$ , the junction computes the pressure release

$$w_\sigma(\tilde{Q}(t)) = \sum_{(i,j) \in \sigma} W_{ij}(\tilde{Q}(t)) S_{ij}(t), \quad \forall \sigma \in \mathcal{P}_\ell, \quad (4b)$$

and chooses the one with the highest pressure release, i.e.,

$$\tilde{\sigma}^*(t) = \arg \max_{\sigma \in \mathcal{P}_\ell} w_\sigma(\tilde{Q}(t)). \quad (4c)$$

For considering the evolution of individual queues, we can more conveniently represent  $(i, j) \in \sigma$  by  $i \in \ell$  and  $j \in \mathcal{N}_i^+$ . Then, using (4b) the total pressure release at a junction  $\ell \in \mathcal{J}$  can be written with respect to the individual nodes as follows:

$$\begin{aligned} \sum_{\sigma \in \mathcal{P}_\ell} w_\sigma(\tilde{Q}(t)) &= \sum_{i \in \ell} \sum_{j \in \mathcal{N}_i^+} (S_{ij}(\tilde{Q}_i - \tilde{Q}_j)) \\ &= \sum_{i \in \ell} S_{i\bullet} \tilde{Q}_i - \sum_{j \in \mathcal{N}_i^+} S_{ij} \tilde{Q}_j = \sum_{i \in \ell} S_{i\bullet} \underbrace{\left( \tilde{Q}_i - \sum_{j \in \mathcal{N}_i^+} p_{ij} \tilde{Q}_j \right)}_{\triangleq w_i(\tilde{Q}(t))}. \end{aligned}$$

The relation between the noisy and non-noisy pressure releases can be easily shown to be

$$w_\sigma(\tilde{Q}(t)) = w_\sigma(Q(t)) + w_\sigma(\nu(t)). \quad (5)$$

Our main finding with respect to erroneous measurement of queue lengths equation (4b), is formally stated in the following theorem

**Theorem 1.** The noisy-max-weight algorithm given by (4) is throughput optimal for any  $\lambda \in \Lambda$ .

**Proof.** The proof is to be included in extended journal version [Charalambous et al. 2023]

Theorem 1 essentially states that even though the measured queue length is noisy, and the decisions are taken based on noisy information, the noisy-max-weight algorithm is still throughput optimal. In other words, noise does not affect the throughput region of the algorithm.

### 4. THE FILTERED-MAX-WEIGHT ALGORITHM

The magnitude of the noise affects the performance of the max-weight algorithm, even in a simple network. In order to suppress the noise level it is necessary to estimate online the state of queues, that evolves in time, as accurately as possible using sequence of noisy observations. In this section, we propose the use of a Bayesian sequential estimator to deal with the noise in the queue measurements.

Recalling that  $Q_i(t)$  denotes the state of the queue of node  $i$  at time  $t$ , then equations (2) and (1) provide a description of the models for the process and measurements, respectively; in its general form, for each node  $i$ , it is given by

$$Q_i(t+1) = f_{a_i(t), a_{j \in \mathcal{N}_i^-(t)}}(Q_i(t), Q_{j \in \mathcal{N}_i^-(t)}, A_i(t)), \quad (6a)$$

$$\tilde{Q}_i(t) = Q_i(t) + \nu_i(t), \quad (6b)$$

where  $Q_{j \in \mathcal{N}_i^-(t)}$  denotes the queue states of the the neighbors of node  $i$ , and  $f : \mathbb{R} \times \mathbb{R}^{|\mathcal{N}_i^-|} \times \mathbb{R} \mapsto \mathbb{R}$  is, in our case, a nonlinear function. Note that when a decision is taken, the services and actions are known. Hence, if there are no endogenous arrivals or they are known exactly, the only randomness in (6a) stems from the arrivals  $A_i(t)$ . Both the process and measurement noise are generally assumed to be independent, identically distributed iid stochastic processes, but not necessarily additive or Gaussian. We also assume that the initial state of the queues is independent of the noise processes and its distribution is given through a Probability Density Function  $p(Q(0))$ . If the pdf of the noise processes are known, system (6) can be equivalently represented by

$$Q_i(t+1) \sim p(\cdot | Q_i(t), Q_{j \in \mathcal{N}_i^-(t)}), \quad (7a)$$

$$\tilde{Q}_i(t) \sim p(\cdot | Q_i(t)). \quad (7b)$$

In our case, it is obvious from equation (2) that the system is nonlinear with discrete queue states and the noise processes are non-Gaussian and bounded. Thus, we can resort to either a hidden Markov model [Rabiner 1989] or a particle filter [Gordon et al. 1993, Doucet et al. 2001, Arulampalam et al. 2002]. We have selected a particle filter, due to its ease of implementation and flexibility.

The idea of adopting particle filtering to provide estimates in traffic control problems has appeared is several occasions, such as, in [Mihaylova et al. 2007] for real-time estimation of traffic state in freeway networks and in [Pascale et al. 2013] for density estimation of a road.

By combining all nodes together, (7) becomes

$$Q(t+1) \sim p(\cdot | Q(t)), \quad (8a)$$

$$\tilde{Q}(t) \sim p(\cdot | Q(t)). \quad (8b)$$

Let  $\tilde{Q}_i^{1:t-1}(t)$  denote the sequence of noisy observations at node  $i$  from time 1 up to and including time  $t-1$ . The particle filter now involves 2 steps:

- 1) *Prediction*: At time  $t$ , we have particle representation of  $p(Q(t) | \tilde{Q}^{1:t-1})$  in the form<sup>1</sup>

$$\left\{ \mu^{(p)}(t-1), Q^{(p)}(t) \right\}_{p=1}^{N_p},$$

where  $N_p$  is the number of particles (for simplicity, we assume that all nodes use the same number of particles), the weights are such that  $\sum_{p=1}^{N_p} \mu^{(p)}(t-1) = 1$ . It is often impossible to sample from the posterior pdf and hence, to obtain  $Q^{(p)}(t)$  we use the transition prior as the proposal distribution, which is the traffic state model (6a);  $Q_i^{(p)}(t)$  is obtained locally

by (an intersection associated with) road  $i$  by (6a) provided road  $i$  is informed about  $Q_{j \in \mathcal{N}_i^-(t-1)}^{(p)}$ .

- 2) *Correction*: Now the weight associated with  $Q^{(p)}(t)$  is computed as

$$\begin{aligned} \mu^{(p)}(t) &\propto \mu^{(p)}(t-1) \times p(\tilde{Q}(t) | Q^{(p)}(t)) \\ &= \mu^{(p)}(t-1) \times \prod_{i \in \mathcal{N}} p(\tilde{Q}_i(t) | Q_i^{(p)}(t)), \end{aligned}$$

which can be computed through average consensus for directed graphs on  $\log p(\tilde{Q}_i(t) | Q_i^{(p)}(t))$  over  $i$  for each particle  $p$ . After such a consensus, each road will know  $\mu^{(p)}(t)$  for all  $p$ . Note that the likelihood function  $p(\tilde{Q}_i(t) | Q_i^{(p)}(t))$  is calculated from (6b) using the predicted state of  $Q_i^{(p)}(t)$  and the known pdf of the measurement process. The weights are normalized and then if weightage of the insignificant weights  $N_{eff}$  is greater than a threshold  $N_t$ , then resampling is performed.

We note that each road  $i$  only keeps track of

$$\left\{ \mu^{(p)}(t), Q_i^{(p)}(t) \right\}_{p=1}^{N_p},$$

but must be informed about  $Q_{j \in \mathcal{N}_i^-(t-1)}^{(p)}$  and must perform a consensus operation to compute  $\mu^{(p)}(t)$ , which are the same for all nodes. The Filtered-max-weight algorithm (PF-FMW) is outlined in Algorithm 1.

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#### Algorithm 1 PF-FMW

---

```

1: for  $t = 1, 2, \dots$  do
2:   for  $i = 1, 2, \dots, N$  do
3:     Compute Noisy Queue Length
4:      $Q_i(t+1) \leftarrow f_{a_i(t), a_{j \in \mathcal{N}_i^-(t)}}(Q_i(t), Q_{j \in \mathcal{N}_i^-(t)}, A_i(t))$ 
5:      $\tilde{Q}_i(t)^{(p)} \leftarrow Q_i(t)^{(p)} + \nu_i(t)$ 
6:     Central Particle Filtering Estimation
7:     for  $j = 1, 2, \dots, N$  do
8:       Prediction
9:      $\left\{ \mu_i^{(p)}(t-1), Q_i^{(p)}(t) \right\}_{p=1}^{N_p}$ 
10:    Correction
11:     $\mu_i^{(p)}(t) \propto \mu_i^{(p)}(t-1) \times p(\tilde{Q}_i(t) | Q_i^{(p)}(t))$ 
12:    Normalize weights s.t.  $\sum_{p=1}^{N_p} \mu_i^{(p)} \leftarrow 1$ 
13:    if  $N_{eff} \leq N_t$  then
14:      Resample
15:    end if
16:  end for
17:  Determine most probable queue state
18:   $\tilde{Q}_i(t) \leftarrow \arg \max_{Q_i(t)} p(Q_i(t) | \tilde{Q}_i^{0:t})$ 
19:  end for
20: end for
```

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## 5. NUMERICAL EVALUATIONS

### 5.1 Network and Simulation Parameters

We evaluated the performance of the proposed algorithm using SUMO, a microscopic road traffic simulator. For collecting the metrics of interest from the network and

<sup>1</sup> This process is initialized at time  $t = 1$  with a particle list of all zeros, if the system starts in an empty state.

for making the scheduling decisions of the proposed algorithm, we developed a MATLAB script which interacted with SUMO via TraCI4Matlab. The chosen network is composed of a 4x4 grid with 9 signalized interactions and 40 two-way roads. All roads are of the same size. Each controlled junction is regulated with four phases shown in Fig. 1.

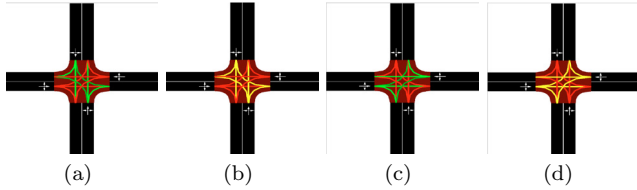


Fig. 1. Phases of a four-way junction: (a) Phase 1, (b) Phase 2, (c) Phase 3, (d) Phase 4.

For both fixed time schedule control (FT) and Back Pressure (BP) control we assume that the time period for each cycle equals to 90s at all intersections according to the signal plan (phase distribution) given in Table. 1. All the traffic measurements are taken after every 45 sec in order to update the signal phases according to back pressure controller. In order to assess the effectiveness of the algorithm, the routes of the vehicles entering the network are generated randomly. The effectiveness of the algorithm is shown through comparison of aggregate queue lengths of all the roads in the network. The simulations were performed for FT and BP controller. BP controller was also investigated for noise and using particle filter estimation for different noise levels.  $N_p$  is chosen to be 300 for simulations. The noise  $v_i$  is assumed to have a uniform distribution. The arrival rate and route of the vehicles are generated randomly for all cases.

Table 1. Phase Distribution.

Type	Phase1	Phase2	Phase3	Phase4
Four-way	42	3	42	3

## 5.2 Simulation Results and Discussions

The simulations were performed for high traffic scenario where the traffic is generated for 800 vehicles entering in the network at the rate of 5700 vehicles/hr. The simulations were run for 1000 secs. The controllers used to compare results are Fixed Time (FT), Back Pressure (BP) with no noise, unfiltered BP (noisy-max-weight NBP), filtered BP (Filtered-max-weight FBP) with noise levels [-3,3] and [-5,5] respectively. We first show the filter performance by showing mean square error (MSE) of queue size estimation for three different lanes in the network for multiple noise levels. The filter is used to estimate actual queue size and attenuate the noise. It can be seen in Fig. 2, that the MSE for estimated queue size are closer to the true queue sizes as compared with the observed after filtering, which is one of reasons of improved performance of FBP controller.

Fig. 3 shows the cumulative queue lengths accumulated for each controller. It can be seen that FT controller is not converging and results in a deadlock. Back Pressure (BP) controller is able to clear the traffic in all cases. However, it

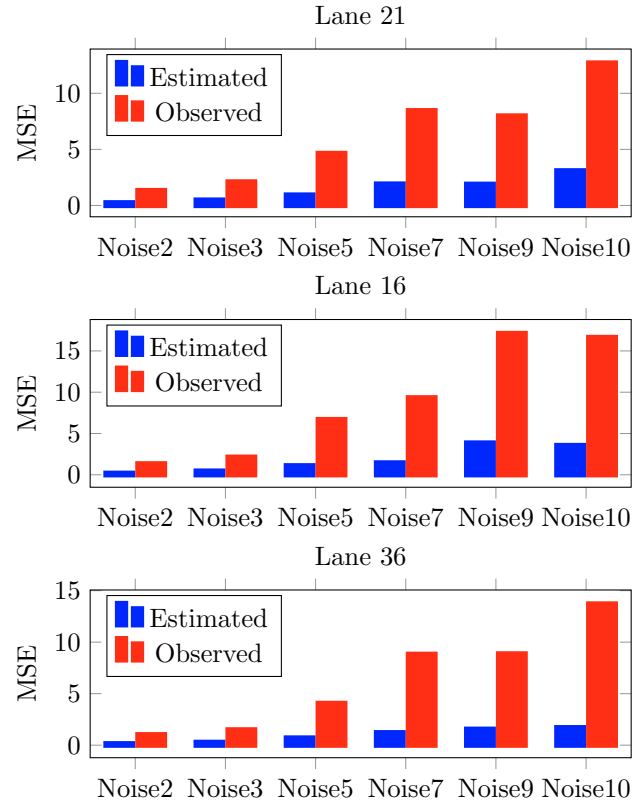


Fig. 2. Filter Performance Analysis: Mean Square Error of Queue Size for Multiple lanes for different noise levels.

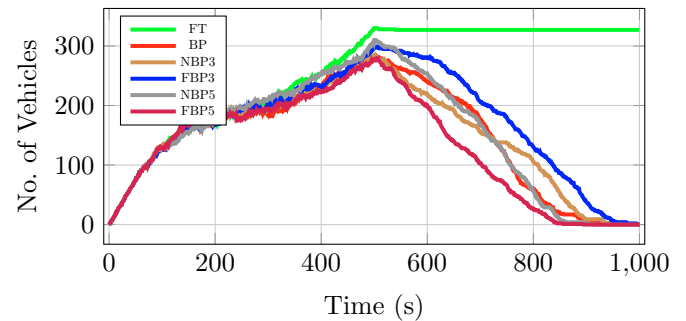


Fig. 3. Cumulative Queue Length of the Network for FT, BP, Filtered-max-weight and Noisy-max-weight Controllers.

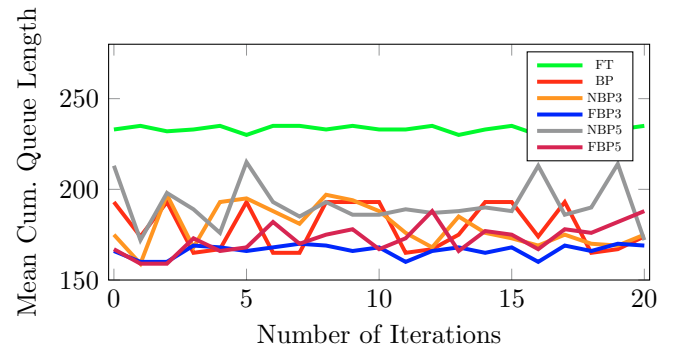


Fig. 4. Steady State Performance Analysis for Multiple Controllers.

is also observed that in the presence of noise, Filtered-max-weight performs better than noisy-max-weight for all test

cases and supports that Filtered-max-weight algorithm is indeed throughput optimal.

Analysis of steady-state performance of all controllers is presented in Fig. 4. The simulations were performed for 20 different scenarios and average queue lengths in steady state were recorded in each case. It can be observed that Filtered-max-weight shows improved performance under noisy observations both in terms of convergence and steady state response.

## 6. CONCLUSIONS

In this paper we considered the max-weight algorithm when the queue size measurement is noisy for centralized traffic signal control. We first proved analytically that the max-weight algorithm is throughput optimal even under noisy queue measurements. Then, we proposed the Filtered-max-weight algorithm, which is based on particle filtering, in order to alleviate the effect of the noisy measurements. We demonstrate via simulations that by incorporating the Filtered-max-weight algorithm the performance of the network is somewhat improved with respect to the average queue length. The work is done under the assumption that the queue lengths are infinite. In future, it is important to study the effect of the queue lengths on the throughput region of the network.

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