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# Safety-Based Speed Control of a Wheelchair Using Robust Adaptive Model Predictive Control

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**Abstract**—Electric-powered wheelchairs play a vital role in ensuring accessibility for individuals with mobility impairments. The design of controllers for tracking tasks must prioritize the safety of wheelchair operation across various scenarios and for a diverse range of users. In this study, we propose a safety-oriented speed tracking control algorithm for wheelchair systems that accounts for external disturbances and uncertain parameters at the dynamic level. We employ a set-membership approach to estimate uncertain parameters online in deterministic sets. Additionally, we present a model predictive control scheme with real-time adaptation of the system model and controller parameters to ensure safety-related constraint satisfaction during the tracking process. This proposed controller effectively guides the wheelchair speed toward the desired reference while maintaining safety constraints. In cases where the reference is inadmissible and violates constraints, the controller can navigate the system to the vicinity of the nearest admissible reference. The efficiency of the proposed control scheme is demonstrated through high-fidelity speed tracking results from two tasks involving both admissible and inadmissible references.

**Index Terms**—Model predictive control (MPC), robotic wheelchair, safety constraints, speed tracking.

## I. INTRODUCTION

WHEELCHAIRS are essential devices in providing mobility for elderly and physically impaired people including patients with a spinal cord injury and stroke patients [1], [2], [3]. Among various types of wheelchairs,

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electric-powered wheelchairs have gained increasing popularity due to their convenience compared to manual wheelchairs. According to a survey study in [4], around 80% of electric wheelchair users rely on the joystick to maneuver the wheelchair. However, the unmodified reference generated by the joystick may result in unexpected high speed and acceleration with safety issues, and advanced speed control is generally required.

As a differential robot with a clear mechatronics system model, the tracking control of wheelchairs has been widely studied by many researchers [5], [6], [7]. Depending on the type of model used and the level of control implemented, the speed control of wheelchairs can be divided into two categories. The first category is related to the kinematic model of the system where the motion of interest fails to satisfy Brockett's necessary conditions [8]. Given the desired Cartesian position and orientation of the wheelchair, the control objective is to design the linear and angular velocities such that the wheelchair tends to the given position with the required orientation. In [8], a finite-time tracking of the wheelchair based on cascaded control architecture and sliding model control was presented. In [9], an adaptive tracking controller was designed for a wheelchair with the input-to-state stability guarantee.

Although some existing algorithms, such as the timed elastic band-based method, can minimize the execution time of trajectory while considering kinodynamic constraints, the tracking control at the kinematic level is under the assumption of perfect velocity tracking at the dynamics level, which may not be realistic in practical applications [10], [11]. The ignored disturbances and system uncertainties can cause severe constraint violation, even if the velocity and acceleration tolerances are considered at the kinematic level when conducting both planning and control.

The second class of tracking control for differential wheeled robots involves the dynamics of actuators. The objective of this type of control is to design the current or voltage at the dynamics level to ensure the convergence of the system state to the desired Cartesian position or wheel velocities. In [10], the trajectory tracking of the differential wheeled robot is achieved by force control using cascaded and backstepping techniques. Later, Fu et al. [12] extended their controller for motion tracking of a mobile robot at the voltage level with a simplification based on a linear relation between velocity and voltage. Based on Lyapunov and backstepping methods, Do et al. [13] proposed an adaptive control law for stabilizing and

tracking of differential wheeled robots with unknown system parameters.

However, none of the methods discussed above guarantee the safety-related constraints of wheelchairs in the presence of disturbances. This is a significant and common problem encountered when the wheelchair is required to operate within specific speed and acceleration constraints while daily-life tasks introduce external disturbances, such as driving the wheelchair on inclined ramps. Furthermore, with the rise of rentable or sharing wheelchairs at big-city hospitals, there is a growing need for a controller that ensures safety-related constraints at the dynamics level, while considering the variation of system parameters due to the change of users [14].

Model predictive control (MPC), as an optimal control strategy capable of explicitly handling the operation constraints, has been widely used in applications with strict demands on state and input constraints [15], [16], [17], [18]. To ensure robust constraint satisfaction for systems with parameter variations, in [19], a robust tube MPC is designed for a linear parameter varying (LPV) system and the online computation load is reduced by constructing a terminal set involving the norm bounds of tube parameters. In [20], a recursive least square-based adaptive MPC is developed for constrained systems with unknown model parameters. The tube-based adaptive MPC offers less conservative performance compared to the robust tube MPC. For systems with both parameter variations and external disturbances, robust adaptive MPC with the set-membership approach is adopted for robust constraint satisfaction with online parameter update [21], [22], [23], [24].

The main contribution of this work is the development of a safety-based constrained tracking control algorithm for wheelchair systems. Unlike most existing works in the field, which primarily focus on the regulation problem, this work addresses the tracking problem of systems with constraints while considering the presence of external disturbances and uncertain parameters. The proposed algorithm incorporates a set-membership approach to handle unknown-but-bounded disturbances and update the set of uncertain parameters for subsequent MPC design. To ensure safe operation, the algorithm includes an optimization-based method to compute the closest admissible reference for tracking in cases where infeasible references violate safety constraints. The proposed MPC framework robustly satisfies state and input constraints using reference-dependent and tube-based constraints in vertex representation of states. The work guarantees recursive feasibility and input-to-state stability of the wheelchair system with the proposed MPC controller. The effectiveness of the safety-based control algorithm is validated through two speed tracking tasks conducted on a high-fidelity model of a practical wheelchair. In summary, the key novelty of this work lies in its safety-based approach to address the tracking problem of wheelchair systems, considering constraints, external disturbances, and uncertain parameters, with the validation of its effectiveness on practical tasks.

The remainder of this article is organized as follows: In Section II, the dynamics of the wheelchair system at the actuator level is described and the system in the LPV form with unknown parameters is discussed. The proposed

robust adaptive tracking controller with details in parameter estimation, feasible reference generation, robust constraint satisfaction, and tracking MPC formulation are presented in Section III. The tracking results of the proposed safety-based control algorithm are demonstrated in Section IV. Section V concludes this work.

*Notations:* When defining the variables, we follow the rule that capitalized letters are for matrices and small letters are for vectors or scalars.  $\mathbb{R}$  and  $\mathbb{Z}$  are the sets of real and integer numbers. Given two integers  $a, b \in \mathbb{Z}$ ,  $\mathbb{Z}_{a+} \triangleq \{i \in \mathbb{Z} \mid i \geq a\}$  and  $\mathbb{Z}_{[a,b]} \triangleq \{i \in \mathbb{Z} \mid a \leq i \leq b\}$ . The  $i$ th row of matrix  $X$  and the  $i$ th element of vector  $x$  are represented by  $X^{[i]}$  and  $x^{[i]}$ , respectively. The  $i$ th vertex of  $x \in \mathcal{X}$  is denoted by  $x^{(i)}$ . A non-negative matrix is denoted by  $X \geq 0$ . The positive definite and semidefinite matrices are represented as  $X > 0$  and  $X \geq 0$ , respectively. For a matrix  $X \in \mathbb{R}^{n \times n}$ , the smallest eigenvalue is denoted by  $\underline{\lambda}(X)$ . The identity matrix of dimension  $n$  is denoted by  $I_n$  and an  $m$ -dimensional vector with all elements as 1 is denoted by  $\mathbf{1}_m$ . The  $m \times n$  matrix with all elements as zero is denoted by  $\mathbf{0}_{m,n}$ . A diagonal matrix with main diagonal elements  $a_1, \dots, a_n$  is denoted by  $\text{diag}(a_1, \dots, a_n)$ . The following sets are defined:  $\mathbb{S}^n = \{X \in \mathbb{R}^{n \times n} : X = X^\top\}$ ,  $\mathbb{S}_{>0}^n = \{X \in \mathbb{S}^n : X > 0\}$ , and  $\mathbb{S}_{\geq 0}^n = \{X \in \mathbb{S}^n : X \geq 0\}$ . A convex polyhedral set of  $x$  is defined as  $\mathcal{P}_x(F_x, b_x) = \{x \mid F_x x \leq b_x\}$ . With  $P \in \mathbb{S}_{>0}^n$ , an ellipsoidal set of  $x$  is defined as  $\mathcal{E}(P, 1) = \{x \mid x^\top P x \leq 1\}$ . For a vector  $x \in \mathbb{R}^n$  with a matrix  $Q$ ,  $\|x\|$  denotes the 2-norm and  $\|x\|_Q$  stands for  $\sqrt{x^\top Q x}$ . The vector  $x_{i|k}$  represents the predicted value of  $x$  at a sampling time instant  $k+i$  based on the measurement at  $k$ . The vector  $x(k)$  stands for the measured value of  $x$  at a sampling time instant  $k$ . A continuous function  $f : [0, a) \rightarrow [0, \infty)$  belongs to class  $\mathcal{K}_\infty$  if  $a = \infty$  and  $f(r) \rightarrow \infty$  as  $r \rightarrow \infty$ .

## II. SYSTEM DESCRIPTION AND PROBLEM FORMULATION

In this section, we first introduce the modeling methodology of the wheelchair. Next, a control-oriented model of wheelchair in the form of LPV system can be obtained. Then, we address the problem considered in this article.

### A. Modeling of Wheelchair

With implicit variable change from rotary to translational movement, the electrical and mechanical subsystems of the electric-powered wheelchair are given as follows [13], [25]:

$$L\dot{i}_c + R i_c + K_e v = u \quad (1a)$$

$$M\dot{v} + Dv + w_f = K_t i_c \quad (1b)$$

where  $v = [v_1, v_2]^\top$  is the vector of linear velocities,  $u = [u_1, u_2]^\top$  is the vector of motor voltages,  $i_c = [i_{c1}, i_{c2}]^\top$  is the vector of currents, and  $w_f = [w_1, w_2]^\top$  is the vector of lumped disturbance torques on the right and left wheels, respectively.  $M$  and  $D$  are the equivalent mass and damping coefficient matrices, which can be expressed as

$$M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, D = \text{diag}(d_1, d_2)$$

where  $m_{11}$ ,  $m_{12}$ ,  $m_{21}$ , and  $m_{22}$  are scalars. The nonzero matrix  $M$  couples the dynamics of the left and right wheels, and  $d_1$  and  $d_2$  are the damping coefficients for the right and left wheels, respectively. Furthermore,  $L = \text{diag}(l_1, l_2)$  and  $R = \text{diag}(r_1, r_2)$  are the inductance and resistance matrices of the system, respectively.  $K_e = \text{diag}(k_{e1}, k_{e2})$  and  $K_t = \text{diag}(k_{t1}, k_{t2})$  are the back electromotive force constant and torque constant matrices, respectively.

To reduce the hardware costs of wheelchairs, motors without current sensing are widely used. For systems without current feedback  $i_c$ , the dynamics in (1a) and (1b) can be reformulated as

$$\ddot{v} + (M^{-1}D + L^{-1}R)\dot{v} + \Gamma v = M^{-1}L^{-1}K_t u + w \quad (2)$$

where  $w = -M^{-1}\dot{w}_f - M^{-1}L^{-1}Rw_f$  and  $\Gamma = M^{-1}L^{-1}RD + M^{-1}L^{-1}K_t K_e$  are the lumped terms.

Let  $x \triangleq [v^\top, \dot{v}^\top]^\top$  be the state vector. By using the Euler forward approximation, the discrete-time state-space model with a sampling time interval  $T_s$  can be formulated as

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Ew(k) \\ &= \begin{bmatrix} I_2 & T_s I_2 \\ -T_s \Gamma & I_2 - T_s(M^{-1}D + L^{-1}R) \end{bmatrix} x(k) \\ &\quad + \begin{bmatrix} \mathbf{0}_{2,2} \\ T_s M^{-1} K_t L^{-1} \end{bmatrix} u(k) + \begin{bmatrix} \mathbf{0}_{2,2} \\ T_s I_2 \end{bmatrix} w(k). \end{aligned} \quad (3)$$

All the states  $x(k)$  and inputs  $u(k)$  are required to satisfy the following constraints to guarantee the safety of the wheelchair:

$$Gx(k) + Hu(k) \leq b \quad (4)$$

with given matrices  $G \in \mathbb{R}^{n_c \times 4}$ ,  $H \in \mathbb{R}^{n_c \times 2}$ , and  $b \in \mathbb{R}^{n_c}$ .

*Remark 1:* The safety constraints in (4) are defined in a general form, where  $b$  is not necessarily a vector with all elements of 1. This form offers more degrees of freedom when configuring state and input sets, and indicates that the coordinate origin may not be inside both state and input constraint sets.

*Assumption 1:* For the wheelchair dynamics (3), the disturbance vector  $w(k)$  is unknown but can be bounded by a convex polyhedral set as

$$w(k) \in \mathcal{P}_w(\bar{F}_w, \bar{b}_w) \quad \forall k \in \mathbb{Z}_{0+} \quad (5)$$

with  $\bar{F}_w \in \mathbb{R}^{n_w \times 2}$  and  $\bar{b}_w \in \mathbb{R}^{n_w}$ .

### B. Problem Formulation

Due to the fact that the wheelchair would be used by diverse users, there exist time-varying uncertain parameters in the wheelchair dynamics (3). Therefore, the model (3) is reformulated in an LPV form as follows:

$$x(k+1) = A(\theta)x(k) + B(\theta)u(k) + Ew(k) \quad (6)$$

where  $A(\theta)$  and  $B(\theta)$  are parameter-varying affine matrices with unknown parameter  $\theta \in \mathbb{R}^q$  as

$$A(\theta) = A_0 + \sum_{j=1}^q A_j \theta^{[j]}, \quad B(\theta) = B_0 + \sum_{j=1}^q B_j \theta^{[j]} \quad (7)$$

where  $q$  is the number of unknown parameters and the matrices  $A_j, B_j, j \in \mathbb{Z}_{[1,q]}$  are known.

*Assumption 2:* For the wheelchair application, the parameter  $\theta$  may involve the varying of mass and damping coefficient due to the change of operators. The true value of uncertain parameter  $\theta^*$  is unknown, piecewise constant but can be bounded by a polyhedral set  $\theta^* \in \mathcal{P}_\theta(\bar{F}_\theta, \bar{b}_\theta)$ , where  $\bar{F}_\theta \in \mathbb{R}^{n_\theta \times q}$  and  $\bar{b}_\theta \in \mathbb{R}^{n_\theta}$  and can be determined based on the interested range of uncertainties.

With given piecewise constant speed references  $v_d$  representing the desired speeds of right and left wheels, this work is dedicated to solve the following problem.

*Problem 1:* For the wheelchair system in the LPV form of (6) with (7), design a tracking MPC law  $u = \kappa(x, v_d)$  to steer the wheelchair speed to the desired piecewise constant reference  $v_d$  while the state and input constraint (4) are always satisfied in the presence of parameter uncertainties and external disturbances.

## III. CONTROLLER ARCHITECTURE

To handle the problem stated in the previous section, we next propose a robust adaptive MPC for achieving the safety-based speed tracking control of the wheelchair with unknown parameters and external disturbances. The set-membership approach is used to provide an updated polytopic set of unknown parameters and the estimated value is updated based on the Euclidean projection. An admissible state and input are introduced when infeasible reference is given in speed tracking control. It is followed by the construction of polytopic tubes for ensuring the state and input constraints. Finally, an MPC tracking control scheme is presented with terminal cost and terminal constraint. The closed-loop properties of the proposed control algorithm are provided.

### A. Set-Membership Parameter Estimation

To use the wheelchair LPV model in the MPC design, an estimation method for unknown uncertain parameters is required. As suggested by [22] and [26], the set-membership parameter estimation can provide a quantification of associate uncertainties and contribute to the MPC design, which is adopted in this work.

For notation simplicity, we denote the uncertain regressor vector  $\Phi \in \mathbb{R}^{4 \times q}$  and the certain regressor vector  $\phi \in \mathbb{R}^{4 \times 1}$  as

$$\Phi(x, u) = [A_1 x + B_1 u, \dots, A_q x + B_q u] \quad (8a)$$

$$\phi(x, u) = A_0 x + B_0 u. \quad (8b)$$

Let  $\bar{F}_\theta \in \mathbb{R}^{n_\theta \times q}$  be a predefined time-invariant matrix, the estimated parameter at time instant  $k$ , that is,  $\hat{\theta}(k)$ , is bounded in a polytopic set

$$\hat{\theta}(k) \in \Theta(k) = \mathcal{P}_\theta(\bar{F}_\theta, b_\theta(k)). \quad (9)$$

Then, the uncertain parameter bounding set can be iteratively computed by

$$\Theta(k) = \Theta(k-1) \cap \Delta_\Theta(k), \quad k \in \mathbb{Z}_{1+} \quad (10)$$

where  $\Delta_{\Theta}(k) = \{\theta \mid x(k) - A(\theta)x(k-1) - B(\theta)u(k-1) \in \mathcal{P}(\bar{F}_w, \bar{b}_w)\} = \{\theta \mid -\bar{F}_w\Phi(k-1)\theta \leq \bar{b}_w + \bar{F}_w(\phi(k-1) - x(k))\}$  and  $\Theta(0) = \mathcal{P}_{\theta}(\bar{F}_{\theta}, \bar{b}_{\theta})$ .

Since the constant matrix  $\bar{F}_{\theta}$  defines fixed directions of half spaces, the update of  $\Theta(k)$  is equivalent to updating  $b_{\theta}(k)$ , which can be obtained by solving the following optimization problem [27, Proof of Proposition 3.31]:

$$b_{\theta}^{[l]}(k) = \min_{\Lambda^{[l]}} \Lambda^{[l]} \left[ \begin{array}{c} b_{\theta}(k-1) \\ \bar{b}_w + \bar{F}_w(\phi(k-1) - x(k)) \end{array} \right] \quad (11a)$$

$$\text{subject to } \Lambda^{[l]} \left[ \begin{array}{c} \bar{F}_{\theta} \\ -\bar{F}_w\Phi(k-1) \end{array} \right] = \bar{F}_{\theta}^{[l]} \quad (11b)$$

$$\Lambda^{[l]} \geq 0 \quad (11c)$$

for  $l \in \mathbb{Z}_{[1, n_{\theta}]}$ , where  $\Lambda \in \mathbb{R}^{n_{\Lambda} \times n_{\Lambda}}$ ,  $n_{\Lambda} = n_{\theta} + n_w$ . At time instant  $k \in \mathbb{Z}_{1+}$ , the estimated value of uncertain parameters is computed by a projection of  $\hat{\theta}(k-1)$  to the set  $\Theta(k)$

$$\hat{\theta}(k) = \arg \min_{\theta \in \Theta(k)} \|\theta - \hat{\theta}(k-1)\|^2 \quad (12)$$

with a given initial  $\hat{\theta}(0)$  that can be the nominal values from the identified range of data.

*Remark 2:* Other parameter estimation methods, such as Kalman filters or least mean squares filters can be used to generate an intermediate value of uncertain parameter before projecting  $\hat{\theta}(k-1)$  to  $\Theta(k)$  [22]. Here, the direct projection (12) is essential in the closed-loop property analysis of the proposed control algorithm.

## B. Controller Design

1) *Generation of Admissible Reference:* Under certain circumstances, such as operating the wheelchair manually with joystick input, the given reference  $v_d$  may not be admissible since the state and input constraints for safety purpose cannot be satisfied. The proposed controller is required to steer the wheelchair to the closest admissible reference characterized by introducing a pair of steady state and input  $(x_s, u_s)$  as

$$(x_s(k), u_s(k)) = \arg \min_{x_s, u_s} \|v_d(k) - y_s\|_{Q_s}^2 \quad (13a)$$

$$\text{subject to } x_s = A(\hat{\theta}(k))x_s + B(\hat{\theta}(k))u_s \quad (13b)$$

$$y_s = C_s x_s \quad (13c)$$

$$Gx_s + Hu_s \leq b \quad (13d)$$

where  $C_s = [I_2, \mathbf{0}_{2,2}]$ , and the weighting matrix  $Q_s$  can be chosen as  $I_2$  to equally penalize the speed difference.

2) *Synthesis of Local and Terminal Control Gains:* To achieve speed tracking using MPC with  $N$ -step prediction horizon, an error state  $e_{ilk} = x_{ilk} - x_s(k)$  and virtual control input  $\bar{u}_{ilk} = u_{ilk} - u_s(k)$  are introduced, and the error coordinate follows:

$$e_{i+1|k} = A(\hat{\theta}(k))e_{ilk} + B(\hat{\theta}(k))\bar{u}_{ilk}, \quad i \in \mathbb{Z}_{[0, N-1]}. \quad (14)$$

The control sequence at time instant  $k$  is designed below to deal with the open-loop and closed-loop mismatch caused by uncertain parameters and external disturbances

$$\bar{u}_{ilk} = \begin{cases} Ke_{ilk} + \mu_{ilk}, & i \in \mathbb{Z}_{[0, N-1]} \\ K_f e_{ilk}, & i = N \end{cases} \quad (15)$$

where  $\mu_{ilk}$ ,  $i \in \mathbb{Z}_{[0, N-1]}$  are control parameters in the MPC;  $K \in \mathbb{R}^{2 \times 4}$  and  $K_f \in \mathbb{R}^{2 \times 4}$  are local and terminal control gains, respectively. The following assumptions are useful for finding the control gains  $K$  and  $K_f$ .

*Assumption 3 (Local Control Gain  $K$ ):* For matrix  $\bar{F}_e \in \mathbb{R}^{n_e \times 4}$  and all  $e$  that satisfies  $\bar{F}_e e \leq \mathbf{1}_{n_c}$ , there exists a local control gain  $K$  such that  $\bar{F}_e(A(\theta) + B(\theta)K)e \leq \varepsilon \mathbf{1}_{n_c}$  with a  $\varepsilon \in [0, 1)$  for all  $\theta \in \mathcal{P}_{\theta}(\bar{F}_{\theta}, \bar{b}_{\theta})$ .

*Assumption 4 (Terminal Control Gain  $K_f$ ):* For all  $\theta \in \mathcal{P}_{\theta}(\bar{F}_{\theta}, \bar{b}_{\theta})$ , given matrices  $Q \in \mathbb{S}_{\geq 0}^4$  and  $R \in \mathbb{S}_{> 0}^2$ , there exist a positive definite matrix  $P \in \mathbb{S}_{> 0}^4$  and a terminal state feedback gain  $K_f$  such that  $P - A_c(\theta)^{\top} P A_c(\theta) - Q - K_f^{\top} R K_f \succeq 0$  and  $e \in \mathcal{E}(A_c(\theta)^{\top} P A_c(\theta), \varepsilon_f)$  with a  $\varepsilon_f \in [0, 1)$  for all  $e \in \mathcal{E}(P, 1)$ , where  $A_c(\theta) = A(\theta) + B(\theta)K_f$ .

*Remark 3:* The matrix  $\bar{F}_e$  can be chosen such that the polytopic set  $\mathcal{P}_e(\bar{F}_e, \mathbf{1}_{n_c})$  approximating a robust control invariant set of state formed by  $\mathcal{E}(\bar{P}, 1) = \{e \mid e^{\top} \bar{P} e \leq 1\}$ , where  $\bar{P}$  can be chosen by satisfying the condition  $\bar{P} - (A(\theta) + B(\theta)K)^{\top} \bar{P} (A(\theta) + B(\theta)K) \succ 0$ . Then, the value of  $K$  can be computed by minimizing  $\varepsilon$  while subjecting to set inclusion with  $\mathcal{P}_e(\bar{F}_e, \mathbf{1}_{n_c}) \subseteq \mathcal{P}_e(\bar{F}_e(A(\theta) + B(\theta)K), \varepsilon \mathbf{1}_{n_c})$  [27].

*Remark 4:* The terminal control gain  $K_f$  can be chosen by satisfying the condition described in Assumption 4. Alternatively, since the local control gain  $K$  is chosen from a contractive polyhedral set, there may also exist a contractive ellipsoidal set as discussed in Assumption 4 with this gain. So the terminal control gain  $K_f$  can be chosen to be the same as  $K$ .

3) *Tube-Based Constraint Satisfaction:* To ensure the robust constraints satisfaction based on the predicted error state and input, we start by constructing a sequence of time-varying sets along the MPC prediction horizon for the tracking error state

$$\bar{F}_e e_{ilk} \leq \alpha_{ilk}, \quad i \in \mathbb{Z}_{[0, N]} \quad (16)$$

where  $\alpha_{ilk} \in \mathbb{R}^{n_e \times 1}$  are another decision variables in the MPC.

With the given  $\bar{F}_e$ , the set of error state in (16) can be represented using the vertices of each set as

$$e_{ilk}^{(j)} = S_j \alpha_{ilk}, \quad j \in \mathbb{Z}_{[1, p]} \quad (17)$$

where  $p$  is the total number of vertices of the tracking error set. For each vertex with index  $j$ , the following equality holds:

$$\bar{F}_e^{[l]} e_{ilk}^{(j)} = \alpha_{ilk}^{[l]}, \quad j \in \mathbb{Z}_{[1, p]}, \quad l \in \mathcal{R}_j \quad (18)$$

where  $\mathcal{R}_j \in \mathbb{R}^4 \subset \mathbb{Z}_{[1, n_c]}$  is the index set for the vertex with index  $j$  that describes the active rows of inequality in (16).

From (17) and (18), we can see that the  $S_j$  is independent to  $\alpha_{ilk}$  and can be computed based on

$$\bar{F}_e^{[l]} S_j = I_{n_c}^{[l]}, \quad j \in \mathbb{Z}_{[1, p]}, \quad l \in \mathcal{R}_j. \quad (19)$$

For all  $e$  satisfying (16),  $\theta \in \Theta(k)$  and  $w \in \mathcal{P}_w(\bar{F}_w, \bar{b}_w)$ , at the next prediction time step  $i+1$ , the following condition should also hold, for  $i \in \mathbb{Z}_{[0, N-1]}$ :

$$\bar{F}_e \left( A(\hat{\theta}(k)) e_{ilk} + B(\hat{\theta}(k)) \bar{u}_{ilk} \right) + \bar{w} \leq \alpha_{i+1|k} \quad (20)$$

where the vector  $\bar{w}$  has its elements  $w^{[l]} = \max_{w \in \mathcal{P}_w} (\bar{F}_e w)^{[l]}$  for  $l \in \mathbb{Z}_{[1, n_w]}$ .

*Proposition 1 (Polytopic Set Inclusion [27]):* For two polytopic sets  $\mathcal{P}_1(F_1, b_1)$  and  $\mathcal{P}_2(F_2, b_2)$ , the inclusion  $\mathcal{P}_1(F_1, b_1) \subseteq \mathcal{P}_2(F_2, b_2)$  holds if and only if there exists a non-negative matrix  $\Omega \geq 0$  such that

$$\Omega F_1 = F_2 \quad (21a)$$

$$\Omega b_1 \leq b_2. \quad (21b)$$

Based on Proposition 1, for all  $\hat{\theta}(k) \in \Theta(k)$  from (9), the inequality (20) is reformulated as

$$\begin{aligned} & \bar{F}_e \Phi(S_j \alpha_{i|k}, KS_j \alpha_{i|k} + \mu_{i|k}) \hat{\theta}(k) \\ & + \bar{F}_e \phi(S_j \alpha_{i|k}, KS_j \alpha_{i|k} + \mu_{i|k}) + \bar{w} \leq \alpha_{i+1|k}. \end{aligned} \quad (22)$$

By using Proposition 1, the condition (22) for all  $\hat{\theta}(k) \in \Theta(k)$  can be reformulated with vertex representation as follows:

$$\Omega_{j,i|k} \bar{F}_\theta = \bar{F}_e \Phi(S_j \alpha_{i|k}, KS_j \alpha_{i|k} + \mu_{i|k}) \quad (23a)$$

$$\begin{aligned} & \Omega_{j,i|k} b_\theta(k) + \bar{F}_e \phi(S_j \alpha_{i|k}, KS_j \alpha_{i|k} + \mu_{i|k}) \\ & - \alpha_{i+1|k} \leq -\bar{w} \end{aligned} \quad (23b)$$

$$\Omega_{j,i|k} \geq 0, j \in \mathbb{Z}_{[1,p]}, i \in \mathbb{Z}_{[0,N-1]} \quad (23c)$$

where  $\Omega_{j,i|k} \in \mathbb{R}^{n_e \times q}$  for each  $i$ .

To satisfy the original state and input constraints described in (4), the following constraint should be enforced in the MPC formulation:

$$Ge_{i|k} + H\bar{u}_{i|k} \leq \tilde{b}(k), i \in \mathbb{Z}_{[0,N]} \quad \forall k \in \mathbb{Z}_{0+}$$

where  $\tilde{b}(k) = b - Gx_s(k) - Hu_s(k)$ . Then, the above condition is equivalent to

$$(G + HK)S_j \alpha_{i|k} + H\mu_{i|k} \leq \tilde{b}(k). \quad (24)$$

Following a similar procedure described above with (15), the terminal conditions can also be formulated as follows:

$$\Omega_{j,N|k} \bar{F}_\theta = \bar{F}_e \Phi(S_j \alpha_{N|k}, K_f S_j \alpha_{N|k}) \quad (25a)$$

$$\begin{aligned} & \Omega_{j,N|k} b_\theta(k) + \bar{F}_e \phi(S_j \alpha_{N|k}, K_f S_j \alpha_{N|k}) \\ & - \alpha_{N|k} \leq -\bar{w} \end{aligned} \quad (25b)$$

$$\Omega_{j,N|k} \geq 0, j \in \mathbb{Z}_{[1,p]} \quad (25c)$$

$$(G + HK)S_j \alpha_{N|k} + H\sigma(k) \leq \tilde{b}(k). \quad (25d)$$

4) *Optimization Formulation:* For given speed reference  $v_d(k)$ , admissible steady states and inputs  $(x_s(k), u_s(k))$  obtained by (13), the safety-based speed tracking MPC optimization problem can be formulated as follows:

$$\min_{\mu_{i|k}, \alpha_{i|k}, \Omega_{j,i|k}} \sum_{i=0}^{N-1} \ell(e_{i|k}, \bar{u}_{i|k}) + \|e_{N|k}\|_P^2(\hat{\theta}(k)) \quad (26a)$$

$$\text{subject to } e_{0|k} = x(k) - x_s(k) \quad (26b)$$

$$e_{i+1|k} = A(\hat{\theta}(k))e_{i|k} + B(\hat{\theta}(k))\bar{u}_{i|k} \quad (26c)$$

$$\bar{u}_{i|k} = Ke_{i|k} + \mu_{i|k}, i \in \mathbb{Z}_{[0,N-1]} \quad (26d)$$

$$\bar{u}_{N|k} = K_f e_{N|k} \quad (26e)$$

and (23a)–(23c), (24), (25a)–(25d), with

$$\ell(e_{i|k}, \bar{u}_{i|k}) = \|e_{i|k}\|_Q^2 + \|\bar{u}_{i|k}\|_R^2 \quad (27)$$

where  $Q \in \mathbb{S}_{>0}^4$  is the weighting matrix for penalizing velocity errors and  $R \in \mathbb{S}_{>0}^2$  is the weighting matrix for control input penalization. With the offline computed  $K_f$ ,  $P(\hat{\theta}(k))$  is adapted by using the following condition with online updating  $\hat{\theta}(k)$  by (12):

$$P(\hat{\theta}(k)) - A_c(\hat{\theta}(k))^\top P(\hat{\theta}(k))A_c(\hat{\theta}(k)) - Q - K_f^\top R K_f \succeq 0 \quad (28)$$

where  $A_c(\hat{\theta}(k)) = A(\hat{\theta}(k)) + B(\hat{\theta}(k))K_f$ .

After solving (26) at each sampling time instant  $k \in \mathbb{Z}_{0+}$ , the control action is chosen as

$$u(k) = K(x(k) - x_s(k)) + \mu_{0|k}^* + u_s(k) \quad (29)$$

where  $\mu_{0|k}^*$  is the first element of the optimal solution.

### C. Closed-Loop Property Analysis

We next discuss the closed-loop properties of the wheelchair operated by the proposed safety-based speed tracking MPC controller. The theoretical results are summarized in the following theorem.

*Theorem 1 (Closed-Loop Properties):* Consider Assumptions 1–4 hold. Given a speed reference  $v_d$  and a feasible initial state  $x(0)$ , the wheelchair system (6) operated by the proposed robust adaptive tracking MPC in (26) and estimated parameter updated by (12) is recursively feasible and input-to-state stable (ISS) to a neighborhood of admissible steady state  $x_s$  obtained from the optimization problem (13) and the neighborhood is defined with the bounds of uncertainty parameter  $\theta$  and external disturbance  $w$ .

*Proof (Recursive Feasibility):* Consider the MPC problem (26) is feasible at a sampling time  $k \in \mathbb{Z}_{0+}$ . The optimal solution of (26) at time  $k$  can be denoted by

$$\mu^*(k) = \{\mu_{0|k}^*, \dots, \mu_{N-1|k}^*\}$$

$$\alpha^*(k) = \{\alpha_{0|k}^*, \dots, \alpha_{N|k}^*\}$$

$$\Omega^*(k) = \{\Omega_{j,0|k}^*, \dots, \Omega_{j,N|k}^*, j \in \mathbb{Z}_{[1,p]}\}$$

from which, at the next time instant  $k+1$ , a suboptimal solution sequence can be constructed as follows:  $\mu(k+1) = \{\mu_{1|k}^*, \dots, \mu_{N-1|k}^*, \mu_{N-1|k+1}^*\}$ ,  $\alpha(k+1) = \{\alpha_{1|k}^*, \dots, \alpha_{N|k}^*, \alpha_{N|k}^*\}$ , and  $\Omega(k+1) = \{\Omega_{j,1|k}^*, \dots, \Omega_{j,N|k}^*, \Omega_{j,N|k}^*, j \in \mathbb{Z}_{[1,p]}\}$ . Note that from the conditions discussed in Assumptions 3–4, there exists  $\mu_{N-1|k+1}$  such that

$$KS_j \alpha_{N|k}^* + \mu_{N-1|k+1} = K_f S_j \alpha_{N|k}^*, j \in \mathbb{Z}_{[1,p]}.$$

As a special case mentioned in Remark 4, if  $K_f$  is chosen to be the same as  $K$ , the above condition holds with  $\mu_{N-1|k+1} = \mathbf{0}_{2,1}$ . The above suboptimal solution at time  $k+1$  satisfies all the constraints of (26) for  $\Theta(k+1) \subseteq \Theta(k)$  indicated by (10). Therefore, the MPC optimization problem (26) is also feasible at the next time  $k+1$ . Thus, it is recursively feasible.

*ISS Stability:* Since the closed-loop system is recursive feasible, the Lyapunov candidate function for this closed-loop

system is chosen as the optimal MPC cost function that can be denoted by  $V^*(x(k), \hat{\theta}(k), x_s)$ .

From the MPC cost function defined in (26a), we know

$$\begin{aligned} V^*(x(k), \hat{\theta}(k), x_s) &\geq \|e_{0|k}\|_Q^2 = \|x(k) - x_s\|_Q^2 \\ &\geq \underline{\lambda}(Q)\|x(k) - x_s\|^2 \\ &\triangleq \sigma_1(\|x(k) - x_s\|) \end{aligned} \quad (30)$$

where  $\sigma_1(\cdot)$  is a  $\mathcal{K}_\infty$  function. In addition, for quadratic stage cost functions, it can be verified that there exists a  $\mathcal{K}_\infty$  function  $\sigma_2(\cdot)$  such that [28]

$$V^*(x(k), \hat{\theta}(k), x_s) \leq \sigma_2(\|x(k) - x_s\|). \quad (31)$$

Then, we can derive

$$\begin{aligned} &V^*(x(k+1), \hat{\theta}(k+1), x_s) - V^*(x(k), \hat{\theta}(k), x_s) \\ &\leq V(x(k+1), \hat{\theta}(k+1), x_s) - V^*(x(k), \hat{\theta}(k), x_s) \end{aligned}$$

where  $V(x(k+1), \hat{\theta}(k+1), x_s)$  denotes an MPC cost function at time  $k+1$  with a suboptimal solution.

From the optimal solution at time instant  $k$ , a suboptimal input sequence at time  $k+1$  can be chosen to be

$$\begin{aligned} \bar{u}_{i|k+1} &= Ke_{i|k+1} + \mu_{i+1|k}^*, \quad i \in \mathbb{Z}_{[0, N-2]} \\ \bar{u}_{N-1|k+1} &= K_f e_{N-1|k+1} \\ \bar{u}_{N|k+1} &= K_f e_{N|k+1}. \end{aligned}$$

With this suboptimal input sequence, we have that

$$\begin{aligned} &V(x(k+1), \hat{\theta}(k+1), x_s) - V^*(x(k), \hat{\theta}(k), x_s) \\ &= \sum_{i=0}^{N-2} \left( \ell(e_{i|k+1}, \bar{u}_{i|k+1}) - \ell(e_{i+1|k}^*, \bar{u}_{i+1|k}^*) \right) \\ &\quad - \ell(e_{0|k}^*, \bar{u}_{0|k}^*) + \ell(e_{N-1|k+1}, \bar{u}_{N-1|k+1}) \\ &\quad + \|e_{N|k+1}\|_{P(\hat{\theta}(k+1))}^2 - \|e_{N-1|k+1}\|_{P(\hat{\theta}(k+1))}^2 \\ &\quad + \|e_{N-1|k+1}\|_{P(\hat{\theta}(k+1))}^2 - \|e_{N|k}^*\|_{P(\hat{\theta}(k))}^2. \end{aligned}$$

It follows from (28) that at the next time instant  $k+1$ , we have:

$$\begin{aligned} &\ell(e_{N-1|k+1}, \bar{u}_{N-1|k+1}) + \|e_{N|k+1}^*\|_{P(\hat{\theta}(k+1))}^2 \\ &\quad - \|e_{N-1|k+1}\|_{P(\hat{\theta}(k+1))}^2 \leq 0 \end{aligned}$$

which leads to

$$\begin{aligned} &V(x(k+1), \hat{\theta}(k+1), x_s) - V^*(x(k), \hat{\theta}(k), x_s) \\ &\leq \sum_{i=0}^{N-2} \left( \ell(e_{i|k+1}, \bar{u}_{i|k+1}) - \ell(e_{i+1|k}^*, \bar{u}_{i+1|k}^*) \right) \\ &\quad - \ell(e_{0|k}^*, \bar{u}_{0|k}^*) + \|e_{N-1|k+1}\|_{P(\hat{\theta}(k+1))}^2 - \|e_{N|k}^*\|_{P(\hat{\theta}(k))}^2. \end{aligned}$$

From the parameter update rule indicated by the optimization problem (12), it yields

$$\|\hat{\theta}(k+1) - \hat{\theta}(k)\| \leq \|\hat{\theta}(k) - \theta^*\|.$$

Then, for each  $i \in \mathbb{Z}_{[0, N-2]}$ , there exist  $\mathcal{K}_\infty$  functions  $\varrho_i(\cdot)$ ,  $\eta_i(\cdot)$ , and  $\zeta_i(\cdot)$  such that

$$\begin{aligned} &\ell(e_{i|k+1}, \bar{u}_{i|k+1}) - \ell(e_{i+1|k}^*, \bar{u}_{i+1|k}^*) \\ &\leq \varrho_i(\|e_{i|k+1} - e_{i+1|k}^*\|) \\ &\leq \eta_i(\|w\|) + \zeta_i(\|\hat{\theta}(k) - \theta^*\|), \quad i \in \mathbb{Z}_{[0, N-2]}. \end{aligned}$$

Furthermore, for quadratic functions, there exist  $\mathcal{K}_\infty$  functions  $\varrho_{N-1}(\cdot)$ ,  $\eta_{N-1}(\cdot)$ ,  $\zeta_{N-1}(\cdot)$ , and  $\tau(\cdot)$  such that

$$\begin{aligned} &\|e_{N-1|k+1}\|_{P(\hat{\theta}(k+1))}^2 - \|e_{N|k}^*\|_{P(\hat{\theta}(k))}^2 \\ &\leq \varrho_{N-1}(\|e_{N-1|k+1} - e_{N|k}^*\|) + \tau(\|\hat{\theta}(k+1) - \hat{\theta}(k)\|) \\ &\leq \eta_{N-1}(\|w\|) + \zeta_{N-1}(\|\hat{\theta}(k) - \theta^*\|) + \tau(\|\hat{\theta}(k) - \theta^*\|). \end{aligned}$$

From the definition of  $\ell(e_{0|k}^*, \bar{u}_{0|k}^*)$  in (27), there exists a  $\mathcal{K}_\infty$  function  $\sigma_3(\cdot)$  such that

$$\ell(e_{0|k}^*, \bar{u}_{0|k}^*) \geq \sigma_3(\|e_{0|k}^*\|) = \sigma_3(\|x(k) - x_s\|).$$

Finally, by combining the above derived bounds for each term, we can conclude that

$$\begin{aligned} &V^*(x(k+1), \hat{\theta}(k+1), x_s) - V^*(x(k), \hat{\theta}(k), x_s) \\ &\leq -\sigma_3(\|x(k) - x_s\|) + \eta_w(\|w\|) + \zeta_\theta(\|\hat{\theta}(k) - \theta^*\|) \end{aligned} \quad (32)$$

with

$$\begin{aligned} \eta_w(\|w\|) &\triangleq \sum_{i=0}^{N-1} \eta_i(\|w\|) \\ \zeta_\theta(\|\hat{\theta}(k) - \theta^*\|) &\triangleq \sum_{i=0}^{N-1} \zeta_i(\|\hat{\theta}(k) - \theta^*\|) + \tau(\|\hat{\theta}(k) - \theta^*\|). \end{aligned}$$

Based on [29, Definition 7 and Remark 5], the chosen Lyapunov function satisfies the conditions in (30)–(32), which proves that it is an ISS Lyapunov function. Therefore, the wheelchair system (6), operated by the proposed robust adaptive tracking MPC in (26) with estimated parameter updated by (12), is ISS within a neighborhood of  $x_s$  that is close to the given speed reference  $v_d$ . ■

## IV. RESULTS

In this section, we apply the proposed control algorithm to an electric-powered wheelchair. The experiment with the wheelchair has been carried out to collect voltage and velocity data, which are used for system identification with parameter estimation. The results obtained using a high-fidelity wheelchair model demonstrate the effectiveness of the proposed control algorithm.

### A. Experiment Setup

To validate the effectiveness of the proposed method, the speed tracking control is implemented on a high-fidelity model of the wheelchair as shown in Fig. 1. This wheelchair is powered by a battery with a maximum voltage of 24 V. The two wheels are driven by Brushless dc motors with a maximum



Fig. 1. Electric-powered wheelchair in the experiment.

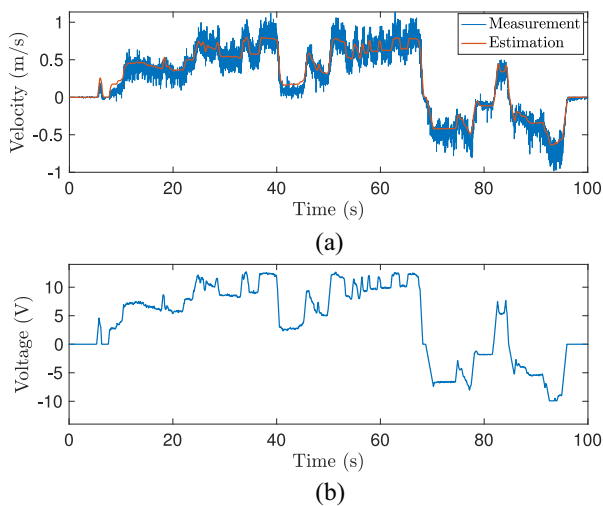


Fig. 2. Model validation result: (a) measured and estimated velocity of right wheel and (b) input voltage.

velocity of 2 m/s. An absolute rotary encoder with 4096 pulses per revolution is used for velocity measurement.

An experiment participant with a weight of 80 kg drives the wheelchair with open-loop voltage input using the joystick to fully excite the system for system identification. Based on the measured voltage input and velocity output, the system parameters are identified using the least square method, and the model validation is conducted by comparing the actual velocity measurement with the estimated velocity based on the proposed model. The identified motor parameters are:  $l_1 = 0.614$ ,  $l_2 = 0.482$ ,  $r_1 = 8.138$ ,  $r_2 = 5.871$ ,  $k_{e1} = 3.471$ ,  $k_{e2} = 2.610$ ,  $k_{t1} = 1.324$ , and  $k_{t2} = 0.827$ . A model validation result for the right wheel is shown in Fig. 2. It can be seen that with the same voltage in model validation, the estimated velocity follows the trend of the measured velocity.

### B. Parameters in the Wheelchair LPV Model

The parameter uncertainty exists when the wheelchair user changes, that is,  $M^{-1} = \bar{M}^{-1} + \beta \tilde{M}$  and  $D = \bar{D} + \gamma \tilde{D}$ , where  $\bar{M}^{-1}$  and  $\bar{D}$  are the nominal matrix of  $M^{-1}$  and  $D$ . The

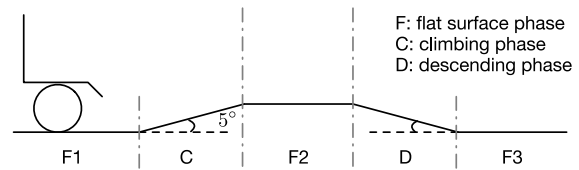


Fig. 3. Task of climbing and descending inclined ramps for wheelchair.

uncertain parameter becomes  $\theta = [\beta \ \gamma \ \beta \gamma]^\top$  for  $q = 3$  and the system model matrices in (7) can be described as follows:

$$\begin{aligned} A_0 &= \begin{bmatrix} I_2 & T_s I_2 \\ -T_s \bar{\Gamma} & I_2 - T_s (\bar{M}^{-1} \bar{D} + L^{-1} R) \end{bmatrix} \\ A_1 &= \begin{bmatrix} \mathbf{0}_{2,2} & \mathbf{0}_{2,2} \\ -T_s (\bar{M} L^{-1} R \bar{D} + \tilde{M} K_t L^{-1} K_e) & -T_s \tilde{M} \bar{D} \end{bmatrix} \\ A_2 &= \begin{bmatrix} \mathbf{0}_{2,2} & \mathbf{0}_{2,2} \\ -T_s \bar{M}^{-1} L^{-1} R \tilde{D} & -T_s \bar{M}^{-1} \tilde{D} \end{bmatrix} \\ A_3 &= \begin{bmatrix} \mathbf{0}_{2,2} & \mathbf{0}_{2,2} \\ -T_s \tilde{M} L^{-1} R \tilde{D} & -T_s \tilde{M} \tilde{D} \end{bmatrix} \\ B_0 &= \begin{bmatrix} \mathbf{0}_{2,2} \\ T_s \bar{M}^{-1} K_t L^{-1} \end{bmatrix}, \quad B_1 = \begin{bmatrix} \mathbf{0}_{2,2} \\ T_s \tilde{M} K_t L^{-1} \end{bmatrix} \\ B_2 &= \mathbf{0}_{4,2}, \quad B_3 = \mathbf{0}_{4,2} \end{aligned}$$

where  $\bar{\Gamma} = \bar{M}^{-1} L^{-1} R \bar{D} + \bar{M}^{-1} K_t L^{-1} K_e$ . After fixing the identified value of motor parameters, the values of nominal matrices  $\bar{M}^{-1}$  and  $\bar{D}$  and the difference matrices  $\tilde{M}$  and  $\tilde{D}$  are computed based on three different participants with weight range from 60 to 85 kg with a detailed value as follows:

$$\begin{aligned} \bar{M}^{-1} &= \begin{bmatrix} 3.822 & 1.776 \\ 1.549 & 4.839 \end{bmatrix}, \quad \bar{D} = \text{diag}(2.034, 1.854) \\ \tilde{M} &= \begin{bmatrix} 1.241 & -0.046 \\ 0.733 & 0.185 \end{bmatrix}, \quad \tilde{D} = \text{diag}(0.048, 0.024). \end{aligned}$$

The uncertain parameter  $\theta$  is considered within the constraint  $[-0.8, -0.8, -0.64]^\top \leq \theta \leq [0.8, 0.8, 0.64]^\top$ . A participant with true system parameters  $\theta^* = [0.5, 0.6, 0.3]^\top$  is involved in the speed tracking. The initial parameter estimation is set as  $\hat{\theta}(0) = [0.55, 0.1, 0.055]^\top$ .

It should be noticed that for a subset of potential users whose weight falls within the range of interest, a more precise model can be obtained by increasing the number of participants in the system identification phase. This model would include values for  $\bar{M}^{-1}$ ,  $D$ ,  $\tilde{M}$ , and  $\tilde{D}$  that can better capture uncertainties resulting from weight changes. The proposed controller is designed to be robust enough to handle uncertainties caused by different users, even when it may not be feasible to gather data from all possible users.

### C. Speed Tracking Results

Based on the fact that speed references can often be represented by combinations of straight lines and circular arcs, we conducted a comprehensive evaluation of our proposed method through two distinct tasks (Task A and Task B). Task A comprises point-to-point movements, while Task B involves sinusoidal speed profiles. The evaluation encompasses both admissible and inadmissible references, showcasing



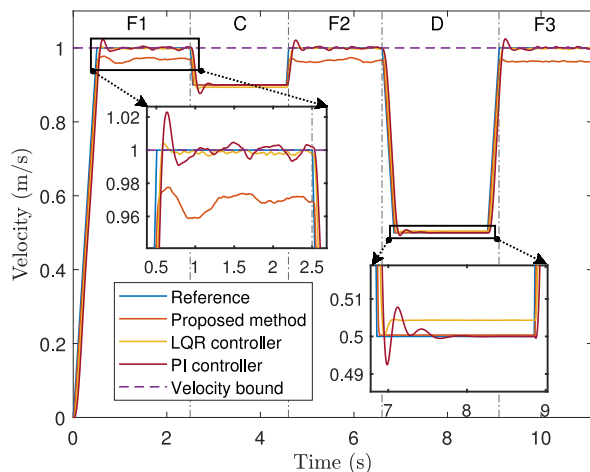


Fig. 4. Speed tracking result of the right wheel of wheelchair in Task A.

the capabilities of the proposed method. Recognizing that wheelchairs may be used by individuals with varying weights, we conducted tests with a potential user whose weight is unknown but falls within the relevant range.

Task A, inspired by the renowned Cybathlon event [30], involves point-to-point movements and incorporates inclined ramps, simulating common challenges faced by wheelchair users in daily activities. Different from the task requirement in Cybathlon, a smaller maximum slope angle is utilized as the design of daily-life ramps considers the capacity of different types of wheelchairs, and the maximum slope for hand-propelled wheelchairs is  $5^\circ$  [31]. The inclusion of inclined ramps introduces unknown external disturbances to the system, making it a challenging and realistic test for showing the effectiveness of our proposed controller in handling such disturbances.

As shown in Fig. 3, the first case (Task A) involves different phases including flat surface, climbing and descending inclined ramps. To generate admissible reference with satisfied constraint ranges, trajectory planning is often adopted [32]. Here, the reference of velocity and acceleration are generated by an available planner where the constraints of velocity and acceleration are considered at the planning level. According to Assumption 1, the disturbance torques are unknown but within the constraint  $-[6.5, 6.5]^T \leq w \leq [6.5, 6.5]^T$  Nm, and the gravity force of operator introduces a constant disturbance on the system when the wheelchair is on the ramps.

To ensure the safety of wheelchair users while providing comfortable transient performance, the wheelchair is required to operate within 1-m/s velocity constraint while respecting  $2\text{-m/s}^2$  maximum acceleration constraint [33]. The speed on the flat surface, climbing, and descending ramps are set as 1, 0.9, and 0.5 m/s, respectively. The sampling time of controller is  $T_s = 0.005$  s. Since the stability and recursive feasibility of the proposed method are not dependent on the length of the prediction horizon, and a longer horizon would result in a greater computational load during real-time implementation, we opt to use a practical horizon length of  $N = 3$  in the experiment. The tuning parameters are set as  $Q = \text{diag}(1000, 1000, 1000, 1000)$  and  $R = \text{diag}(0.001, 0.001)$ .

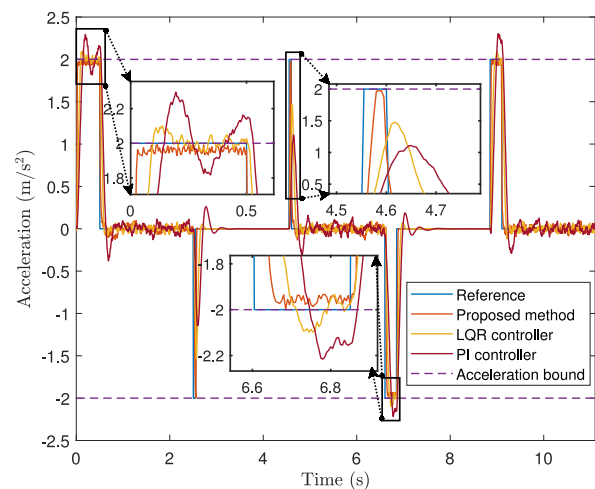


Fig. 5. Acceleration of the right wheel of wheelchair in Task A.

The controller gains  $K$  and  $K_f$  are chosen as the same value following the steps described in Remarks 3 and 4 as:

$$K = \begin{bmatrix} 11.9291 & 0.0019 & -1.9051 & 0.7673 \\ 0.8058 & 12.2490 & 1.1547 & -1.7270 \end{bmatrix}.$$

To further evaluate the safety guarantee property of the proposed controller, a linear-quadratic regulator (LQR) controller and a proportional-integral (PI) controller are utilized as benchmark controllers for comparison. The LQR controller is designed based on the model with the initial estimate of  $\hat{\theta}(0)$  and places greater emphasis on minimizing tracking error than control input. Since the reference for the left and right wheels are same in the Task A, only the tracking results of the right wheel are presented and analyzed here. The velocity and acceleration of the right wheel based on the proposed method, LQR controller, and PI controller for wheelchair riding on ramps are demonstrated in Figs. 4 and 5, respectively.

In the application of wheelchair speed tracking, the speed and acceleration constraints should always be satisfied for safe operation. However, as shown in Figs. 4 and 5, although the reference is generated considering velocity and acceleration constraint satisfaction, with both the LQR and PI controllers, the actual speed and acceleration of wheelchair violate the given safety constraints. Conversely, the proposed MPC controller provides smoother acceleration and deceleration while keeping the maximum acceleration within tolerance during the transient process. Specifically, during the time period between 4.5 and 4.6 s in Fig. 5, the proposed controller strives to approach the maximum acceleration limit, while the LQR and PI controllers operate at lower accelerations with values around 1.47 and 1.1  $\text{m/s}^2$ , respectively, as the wheelchair moves from climbing ramps to flat ground. In the time period between 6.5 and 7 s, the wheelchair is moving from the flat surface to descending ramps. Based on the magnified results in Figs. 4 and 5, the proposed method accelerates and decelerates the wheelchair more smoothly without violating the acceleration constraints, whereas the acceleration exceeds the tolerance and introduces oscillation by using the LQR and PI controller.

The maximum speed errors of the right and left wheels are summarized in Table I. The results show that the proposed

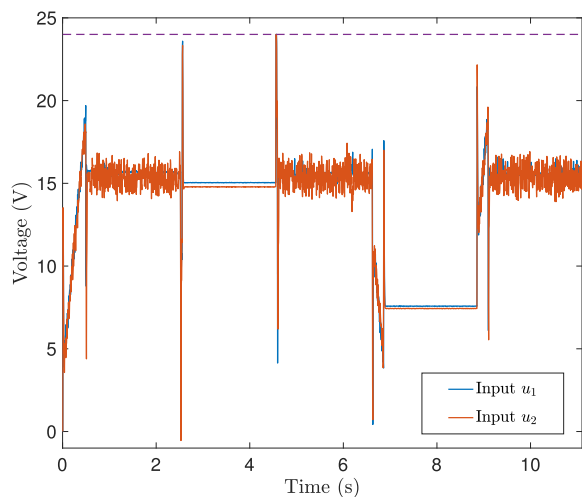


Fig. 6. Input voltage for speed tracking of wheelchairs in Task A.

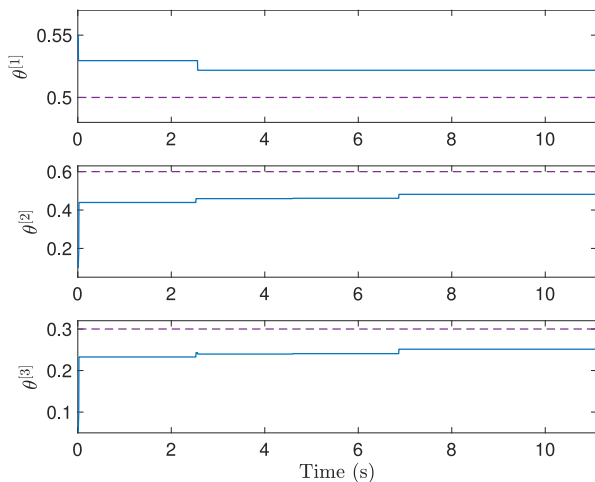


Fig. 7. Evolution of uncertain parameter.

TABLE I  
MAXIMUM SPEED TRACKING ERROR FOR THE WHEELCHAIR IN TASK A

	Proposed method	LQR controller	PI controller
$\ e^{[1]}\ _{\infty}$	0.047	0.094	0.158
$\ e^{[2]}\ _{\infty}$	0.048	0.096	0.149

Note: the unit of numbers is m/s.

method outperforms the benchmark controllers in terms of maximum speed tracking error for both left and right wheel sides. Specifically, the proposed method achieves the smallest tracking error, while the conventional PI controllers exhibit the largest maximum tracking errors for both sides. The input voltage of the right and left wheels are shown in Fig. 6 and both the voltage constraints are satisfied for the left and right wheels. Furthermore, the evolution of estimated uncertain parameters are shown in Fig. 7. The true values of uncertain parameters are identified from a participant, which is unknown to the controller. The estimated value tends to the true value during the process.

In Task B, the wheelchair is required to track sinusoidal references that are inadmissible. The proposed controller must closely follow these references while adhering to maximum

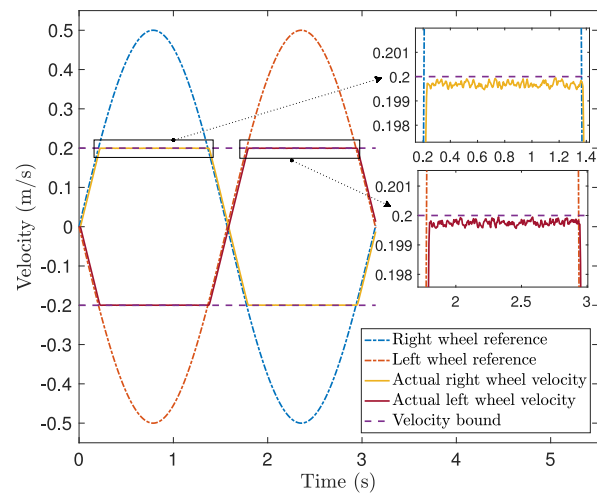


Fig. 8. Speed result for sinusoidal reference tracking.

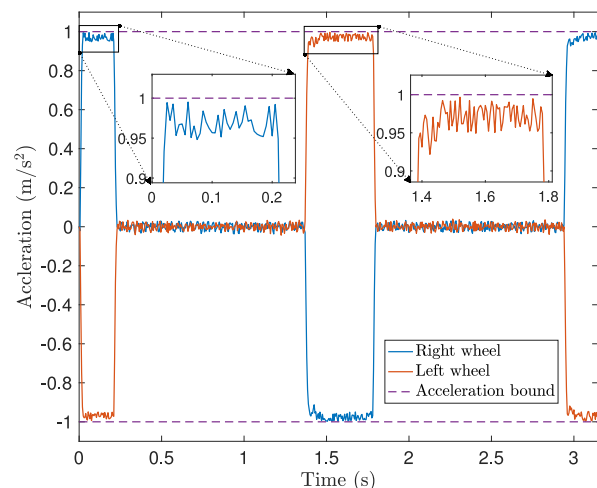


Fig. 9. Acceleration result for sinusoidal reference tracking.

velocity constraint at 0.2 m/s and maximum acceleration at 1 m/s<sup>2</sup>. The speed references for left and right wheel are  $v_{d1}(t) = -0.5 \sin(2t)$  and  $v_{d2}(t) = 0.5 \sin(2t)$ , respectively. This task represents a typical real-life scenario where the given speed reference does not satisfy safety-related constraints, as wheelchair operators may encounter difficulties in effectively controlling the manipulator due to factors, such as lack of experience, advanced age, or physical impairments. These factors render Task B representative of real-life situations, as it replicates scenarios where the controller must tackle demanding references while ensuring compliance with specified speed and acceleration constraints.

The corresponding speed and acceleration during the tracking process are demonstrated in Figs. 8 and 9, respectively. It can be seen from Fig. 8 that, when the reference speeds are inadmissible, the actual speeds of the left and right wheels follow the reference closely. The proposed control algorithm steers the speed of wheelchair to the closest admissible reference instead during 0.2–1.4 s and 1.8–2.9 s when the reference is inadmissible. Meanwhile, it can also be seen from Fig. 9 that the proposed controller ensures the acceleration constraint during the whole process.

It should be noted that the proposed method entails a higher computation load compared to benchmarking controllers. Therefore, when applying this method in high sampling rate applications, careful consideration of the computational demands involved should be taken into account for real-time implementation.

## V. CONCLUSION

In this article, we presented a robust adaptive MPC algorithm for achieving safety-based speed tracking of an electric powered wheelchair. The external disturbances introduced in different scenarios and varying parameters due to change of users have been explicitly considered in the controller design. To ensure recursive feasibility and adherence to constraints in the proposed MPC scheme, we have utilized a set-membership approach. This approach is used for online set propagation, enabling us to bound unknown but constant uncertain parameters effectively. The pair of steady state and input has been introduced when a given reference is inadmissible. Based on the theoretical proof and two representative task results, we have revealed that the proposed method can successfully achieve safety-based speed tracking control of the wheelchair even in the presence of parameter uncertainties and external disturbances. This dual-pronged validation approach, combining rigorous theoretical proofs with practical simulations, helps to ensure the reliability and robustness of our proposed solution.

Moreover, the proposed algorithm exhibits promising potential for application in various domains, including aerospace systems, biomedical systems, etc. These domains often entail safety constraints on joint limits, actuator saturation. Additionally, they often encounter changes in working conditions, resulting in parameter variation over time. The presence of these practical requirements further justifies the motivation behind the development of our proposed method.

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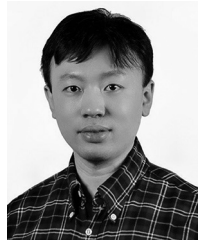


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