# Bounded Error Tracking Control for Contouring Systems with End Effector Measurements



Citation for the original published paper (version of record):

Yuan, M., Manzie, C., Good, M. et al (2019). Bounded Error Tracking Control for Contouring Systems with End Effector Measurements. 2019 IEEE International Conference on Industrial Technology (ICIT). http://dx.doi.org/10.1109/ICIT.2019.8755064



(article starts on next page)

# Bounded Error Tracking Control for Contouring Systems with End Effector Measurements

Meng Yuan[1], Chris Manzie[2], Malcolm Good[1], Iman Shames[2], Farzad Keynejad[3], Troy Robinette[3]
[1]Department of Mechanical Engineering, University of Melbourne, Melbourne, Australia
[2]Department of Electrical and Electronic Engineering, University of Melbourne, Melbourne, Australia
[3]ANCA Motion, Melbourne, Australia

***Abstract*—Many industrial applications of machining require a bounded tracking error during transient and steady processes. Traditional control architectures in machining are unable to explicitly bound tracking errors, and therefore conservative operation is required to ensure satisfactory performance. In this paper, we propose a model-predictive-based approach to guarantee bounded error tracking for the family of systems with available end effector position measurements. The state and input constraints and the bounded error requirements are satisfied by a model predictive controller (MPC) that enforces the system and reference states to remain in a polyhedral robust control invariant (RCI) set. We propose an algorithm for calculating this RCI set in a finite number of computation steps and give the formulation of the MPC. The superiority of the proposed control approach over a conventional tracking controller is demonstrated via simulation of a laboratory machine.**

***Index Terms*—Trajectory tracking, model predictive control, bounded error tracking**

## I. Introduction

In the field of industrial precision motion control, many applications require the manipulated unit to track a time-varying reference or a given path as closely as possible. Tracking error plays an important role in assessing the control performance of a range of machine applications including CNC machines [1], and X-Y tables with ball screw driven servos [2]. This subset of machining applications is the system considered in this paper, which is characterised by the measurements of the end effector position being available to the tracking error controller, either directly or through capability provided by incorporation of additional sensing [3].

Traditionally, the link between the motor and end effector/load is constructed with high stiffness and is treated as rigid while designing the tracking controller. Greater rigidity implies more weight, and hence larger and more costly motors may be required to avoid increasing the time required to perform a given operation [4]. Alternatively, using lighter (but more flexible) manipulators promises faster movements and more efficient utilisation of energy. However, the flexibility of the lightweight structure can cause system vibration which compromises the trajectory tracking performance [5], and may even lead to system instability [6] if not considered in the tracking controller design.

Many control synthesis designs have been proposed to achieve end effector tracking whilst considering the structural flexibility. In the context of the plants being considered in this work (where end effector measurements are available), it has been demonstrated that consideration of the the structural dynamics has led to improved performance in tip tracking [5], X-Y tables [3], [7] and ball screw systems [2], [8], [9]. However, achieving a guaranteed tolerance of tracking error using these approaches, particularly in the presence of system disturbances, is not possible.

In [10], a bounded error tracking model predictive controller (MPC) is designed for a linear time invariant (LTI) system subjected to state and input constraints in the absence of model disturbances. A robust control invariant (RCI) set is utilised in the MPC formulation to ensure the reference is achievable and state, input and performance constraints can be guaranteed.

In this paper, we extend the approach of [10] in two ways. Firstly, the MPC problem is augmented to consider disturbances on the plant model characterised by modelling error or bounded external disturbances. Secondly, an algorithm for estimating the RCI set used in the MPC problem is proposed, that ensures the set can be determined in a fixed number of iterations, thereby alleviating a potential shortcoming of the approach in [10]. The proposed approach is simulated on a representative system to demonstrate adherence to the desired system constraints and to assess the computational tractability of the proposed approach.

*Notation*: $\mathbb{R}$ and $\mathbb{Z}$, $\mathbb{Z}_+$, $\mathbb{Z}_{0+}$ are the sets of real and integer, positive integer, non negative integer numbers, and $\mathbb{Z}_{[0,i]}$ stands for the integer set from 0 to $i$ for $i \in \mathbb{Z}_+$. Consider $a \in \mathbb{R}^{n_a}, b \in \mathbb{R}^{n_b}$, then $(a,b) \triangleq \left[a^T, b^T\right]^T \in \mathbb{R}^{n_a+n_b}$ represents the stacked vector. Set $\mathbb{B}^n(p)$ is the closed ball in $\mathbb{R}^n$ with respect to the infinity norm with radius $p$. For two sets $\mathcal{X}$ and $\mathcal{Y}$, $\mathcal{X}+\mathcal{Y}$ is the Minkowski set sum. Vector $x(k)$ is the value at sampling instant $k$, the time $T_s \cdot k$, where $T_s$ is the sampling period. Vector $x^+$ stands for the value of $x$ at the next time instant, and $x_{i|k}$ represents the predicted value of $x$ at the time sample $k+i$ based on the data at sample $k$.

## II. System Description and Problem Statement

Consider a general two-inertia flexible system with an actuator-driven moving part and a flexibly connected end

978-1-5386-6376-9/19/$31.00 ©2019 IEEE

effector. The dynamics of the mechanical system can be represented as:

$$
\begin{aligned}
x_m^+ &= x_m + T_s v_m \\
v_m^+ &= v_m + \frac{T_s}{M_m}\left(k_t u - k_s\left(x_m - x_e\right) - c_s\left(v_m - v_e\right) - F_d\right) \\
x_e^+ &= x_e + T_s v_e \\
v_e^+ &= v_e + \frac{T_s}{M_e}\left(k_s\left(x_m - x_e\right) + c_s\left(v_m - v_e\right)\right)
\end{aligned} \tag{1}
$$

where $M_m$, $M_e$ are, respectively, the mass of the motor and end effector; $x_m$ and $x_e$ are the position of the motor and end effector separated by the flexible link; $v_m$ and $v_e$ are the velocity of the motor and end effector respectively; $u$ is the current command to the drive system calculated by the controller; $k_s$ and $c_s$ are the spring constant and internal damping coefficient of the flexible link; $k_t$ is the force constant of the motor; $T_s$ is the sampling period.

In this paper, the disturbance force $F_d$ is the sum of the cogging force and the friction force. More generally, $F_d$ can include plant model mismatch and measurement noise, although these are not explicitly considered in this work.

The model of cogging force is adopted from [11]:

$$
F_c(x_m) = \sum_{j=1}^{N}\left(S_{cj}\sin\left(\frac{2\pi j}{\tau}x_m\right) + C_{cj}\cos\left(\frac{2\pi j}{\tau}x_m\right)\right) \tag{2}
$$

where $\tau$ is the pole pitch of the linear motor; $N$ is a positive integer number.

The friction model employed is based on the Lorentzian model of [12] employing the Karnopp remedy [13], [14] to avoid simulation issues around zero velocity:

$$
F_f(x_m, v_m, u) =
\begin{cases}
\left(f_c + f_v|v_m| + \frac{f_s}{1+\left(\frac{v_m}{v_s}\right)^2}\right) sgn(v_m) & |v_m| > \lambda \\
F_e & |v_m| \leq \lambda \\
 & |F_e| \leq |f_c + f_s| \\
\left(f_s + f_c\right) sgn(F_e) & otherwise
\end{cases} \tag{3}
$$

where $F_e \triangleq (k_t u - F_c(x_m))$ is the externally impressed force; $f_c$ is the Coulomb friction coefficient; $f_v$ is the viscous friction coefficient; $f_s$ and $v_s$ are parameters of the Stribeck component in the model; $\lambda$ is the zero velocity interval which is usually chosen as a small number.

The nonlinear mechanical system dynamics (1)-(3) can be written in state space form as:

$$
x^+ = Ax + Bu + Ew \tag{4}
$$

where the state vector is $x \triangleq (x_m, v_m, x_e, v_e)$; $w$ stands for the nonlinear components in (2) and (3). The specific matrices of the LTI model (4) are given as

$$
A = \begin{bmatrix}
1 & T_s & 0 & 0 \\
-\frac{T_s k_s}{M_m} & \frac{M_m - T_s(c_s+f_v)}{M_m} & \frac{T_s k_s}{M_m} & \frac{T_s c_s}{M_m} \\
0 & 0 & 1 & T_s \\
\frac{T_s k_s}{M_e} & \frac{T_s c_s}{M_e} & -\frac{T_s k_s}{M_e} & \frac{M_e - T_s c_s}{M_e}
\end{bmatrix}
$$

$$
B = \begin{bmatrix} 0 \\ \frac{T_s k_t}{M_m} \\ 0 \\ 0 \end{bmatrix},\ E = \begin{bmatrix} 0 \\ -\frac{T_s}{M_m} \\ 0 \\ 0 \end{bmatrix} \tag{5}
$$

The constraints of state, input and disturbance are shown below:

$$
x \in \mathcal{X},\ u \in \mathcal{U},\ w \in \mathcal{W} \tag{6}
$$

This disturbance set $\mathcal{W}$ is determined by the range of function $F_c + \bar{F}_f$ in this paper, and the function $\bar{F}$ is given as:

$$
\bar{F}_f(x_m, v_m, u) =
\begin{cases}
\left(f_c + \frac{f_s}{1+\left(\frac{v_m}{v_s}\right)^2}\right) sgn(v_m) & |v_m| > \lambda \\
F_e - f_v v_m & |v_m| \leq \lambda \\
 & |F_e| \leq |f_c + f_s| \\
\left(f_s + f_c\right) sgn(F_e) - f_v v_m & otherwise
\end{cases} \tag{7}
$$

**Remark 1.** *The LTI model* (4) *can be used to represent a variety of flexibly-connected two-inertia systems with translational and/or rotational degrees of freedom.*

Furthermore, to ensure bounded error tracking, it is necessary to impose constraints on the reference that can be tracked. With this in mind, the reference is assumed to be generated by an external LTI system as

$$
\begin{aligned}
r^+ &= A_r r + B_r u_r \\
x_e^* &= C_r r
\end{aligned} \tag{8}
$$

where $r \in \mathbb{R}^{n_r}$, $u_r \in \mathbb{R}^{m_r}$, $x_e^* \in \mathbb{R}$ are the state, input and output of the reference model, subject to the constraints,

$$
r \in \mathcal{X}^r,\ u_r \in \mathcal{U}^r \tag{9}
$$

Moreover, it is considered that the reference is generated offline, and as such a sufficient feedforward amount of the reference trajectory (denoted by $N$ sampling instants) is available to the controller at each time step.

Ultimately, it is desired to ensure that the tracking error between the reference and the end effector is bounded by some constant, $\epsilon$, to guarantee the tolerance of the machined part. This bounded tracking error requirement can be considered as a performance constraint which is represented as

$$
\|x_e^*(k) - x_e(k)\|_\infty \leq \epsilon, \forall k \in \mathbb{Z}_{0+} \tag{10}
$$

## III. Proposed tracking control synthesis

The control synthesis problem has two stages described in the following subsections. In the first (offline) stage, the maximal robust control invariant set is estimated using a novel algorithm. In the second (online) stage, this set is imposed as a constraint in the MPC problem to ensure recursive feasibility of the implemented controller and subsequently stability of the tracking error.

### A. Estimation of the Maximal Robust Control Invariant Set

The following three definitions are presented for completeness in general setting:

**Definition 1. (RCI and maximal RCI sets)** *Consider the system $x^+ = Ax + Bu + Ew$, where $x \in \mathcal{X} \subseteq \mathbb{R}^n$, $u \in \mathcal{U} \subseteq \mathbb{R}^m$ and $w \in \mathcal{W} \subseteq \mathbb{R}^q$ are the state, input and disturbance vectors. The set $\mathcal{R} \subseteq \mathcal{X}$ is a robust control invariant set (RCI) if*

$$x(k) \in \mathcal{R}, \exists_{u\in\mathcal{U}}, Ax + Bu + Ew \in \mathcal{R}, \forall w \in \mathcal{W}, \forall k \in \mathbb{Z}_{0+}.$$

*Furthermore, the set $\mathcal{R}$ is called a control invariant (CI) set if $w = 0$; the set $\mathcal{R}_\infty$ is the maximal robust control invariant set if any $\mathcal{R} \subseteq \mathcal{R}_\infty$.*

**Definition 2. (Robust admissible input set)** *The robust admissible input set of $\mathcal{R}$ is*

$$\Theta^u(x) = \{u \in \mathcal{U} | Ax + Bu + Ew \in \mathcal{R}, \forall w \in \mathcal{W}\}$$

**Definition 3. (One-step reachable set)** *The one-step reachable set represents the set of states that can be robustly steered to a given set $\mathcal{Y} \subseteq \mathbb{R}^n$ in one step under any possible disturbance $w \in \mathcal{W}$, and can be computed as*

$$D(\mathcal{Y}, \mathcal{W}) \triangleq \{x \in \mathbb{R}^n | \exists_{u\in\mathcal{U}}, Ax + Bu + Ew \in \mathcal{Y}, \forall w \in \mathcal{W}\} \tag{11}$$

With these concepts, we assume the reference is generated offline and stays within the control invariant (CI) set $r(k) \in \mathcal{R}^r_\infty, \forall k \in \mathbb{Z}_{0+}$ by selecting $u_r(k) \in \Theta^{u_r}(r(k)) \subseteq \mathcal{U}^r$. This guarantees the constraints in (9) are satisfied.

Having ensured the invariance of the reference, we now shift attention to ensuring $\|x_e^* - x_e\|_\infty \le \epsilon$ by defining a set of the combined system and reference states, $S^{x,r}$ ,

$$\mathcal{S}^{x,r} = \left\{(x, r) \in \mathbb{R}^{n+n_r} | x \in \mathcal{X}, r \in \mathcal{R}^r_\infty, \|x_e^* - x_e\|_\infty \le \epsilon\right\}$$

At any time $k \in \mathbb{Z}_{0+}$ if $(x(k), r(k)) \in \mathcal{S}^{x,r}$, the controller we developed should ensure $(x(k+1), r(k+1)) \in \mathcal{S}^{x,r}$ for every admissible $r(k+1)$. This requires a robust control invariant set $\mathcal{R}^{x,r} \subseteq \mathcal{S}^{x,r}$, specifically,

$$\begin{aligned}(x, r) \in \mathcal{R}^{x,r}, &\exists_{u\in\mathcal{U}}, (Ax + Bu + Ew, A_r r + B_r u_r) \in \mathcal{R}^{x,r},\\ &\forall (w, u_r) \in (\mathcal{W} \times \Theta^{u_r}(r)).\end{aligned} \tag{12}$$

One method of computing this RCI set $\mathcal{R}^{x,r}$ involves the iteration $\mathcal{R}_{i+1} = \mathcal{R}_i \cap D(\mathcal{R}_i, (\mathcal{W} \times \Theta^{u_r}(r)))$, where $\mathcal{R}_0 = \mathcal{S}^{x,r}$, until $\mathcal{R}_i = \mathcal{R}_{i+1}$. Then $\mathcal{R}^{x,r} = \mathcal{R}^{x,r}_\infty = \mathcal{R}_{i+1}$. This method requires the computed two successive sets $\mathcal{R}_i$ and $\mathcal{R}_{i+1}$ are same, and this maximal RCI set $\mathcal{R}^{x,r}_\infty$ is called finitely determined [15].

In [10], an algorithm is proposed to computer a convex RCI set $\mathcal{R}^{x,r}$ for disturbance-free system $x^+ = Ax + Bu$, which involves the above-mentioned iteration with computation $\mathcal{R}_{i+1} = \mathcal{R}_i \cap D(\mathcal{R}_i, \mathcal{U}^r)$ until $\mathcal{R}_{i+1} = \mathcal{R}_i$. However, the stop criterion is numerically difficult to be satisfied when the finitely determined condition of the maximal RCI set is generally not met for system with external disturbance [16]. Consequently, we propose a modification here based on an *estimate* of the one-step reachable set using an inner approximation [15]:

$$\begin{aligned}\hat{D}(\mathcal{Y}, p) = \{x \in \mathbb{R}^n |\, &\exists_{u\in\mathcal{U}}, Ax + Bu + Ew + d \in \mathcal{Y},\\ &\forall d \in \mathbb{B}^n(p), \forall w \in \mathcal{W}\}\end{aligned} \tag{13}$$

where $d$ is a vector; the parameter $p$ is a tuning parameter that can be used to influence the conservativeness of the estimation. The modified algorithm for computing $\mathcal{R}^{x,r}$ is given as follows:

*Algorithm 1*

1) Initialisation:
Let $i = 0$,
$\mathcal{R}_s \triangleq (\mathbb{R}^n \times \mathcal{R}^r_\infty)$,
$\bar{\mathcal{R}}_0 = \{(x, r) \in \mathbb{R}^{n+n_r} |\, x \in \mathcal{X}, \|x_e^* - x_e\|_\infty \le \epsilon\}$,
and $\mathcal{R}_0 = \mathcal{R}_s \cap \bar{\mathcal{R}}_0$.

2) Iteration: $\mathcal{R}_{i+1} = \mathcal{R}_s \cap \bar{\mathcal{R}}_{i+1}$, where

$$\bar{\mathcal{R}}_{i+1} = \bar{\mathcal{R}}_i \cap \hat{D}\left(\bar{\mathcal{R}}_i, p\right)$$

$$\begin{aligned}\hat{D}\left(\bar{\mathcal{R}}_i, p\right) = \big\{&(x, r) \in \mathbb{R}^{n+n_r} |\, \exists_{u\in\mathcal{U}},\\ &(Ax + Bu + Ew, A_r r + B_r u_r) + d \in \bar{\mathcal{R}}_i,\\ &\forall d \in \mathbb{B}^{n+n_r}(p), \forall (w \times u_r) \in (\mathcal{W} \times \mathcal{U}^r)\big\}\end{aligned}$$

3) if $\mathcal{R}_i \subseteq \mathcal{R}_{i+1} + \mathbb{B}^{n+n_r}(p)$, calculation terminates.

4) Update $i = i + 1$, return to 2).

The following theorem provides a guarantee for the finite termination of the proposed algorithm in estimating the maximal robust control invariant set:

**Theorem 1.** *Given a desired end effector tracking error bound $\epsilon$ and a closed ball with radius $p$, Algorithm 1 terminates in finite number of steps. If the computed set $\mathcal{R}_{i+1} \ne \varnothing$, then $\mathcal{R}^{x,r} = \mathcal{R}_{i+1} \subseteq \mathcal{R}^{x,r}_\infty$ is a polyhedral RCI set for the flexible system* (4) *and reference system* (8) *subjected to the state and input constraints, and robust to $w \in \mathcal{W}$ and $u_r \in \Theta^{u_r}(r)$.*

*Proof.* (*sketch*) The proof follows similar steps to [10], with appropriate modification to allow for the non-empty $\mathcal{W}$ and the use of the approximation of one-step reachable set $\hat{D}(\cdot)$. □

### B. MPC problem formulation

With an RCI set estimated offline as described above, the online task for the controller requires ensuring that the system and reference states within this set at all times to guarantee the end effector position is within the specified tolerance bound for any feasible reference $r \in \mathcal{R}^r_\infty$. At $k \in \mathbb{Z}_{0+}$, $N \in \mathbb{Z}_+$, the future reference $\gamma_k^N = (r(k), \cdots, r(k+N-1))$ is assumed

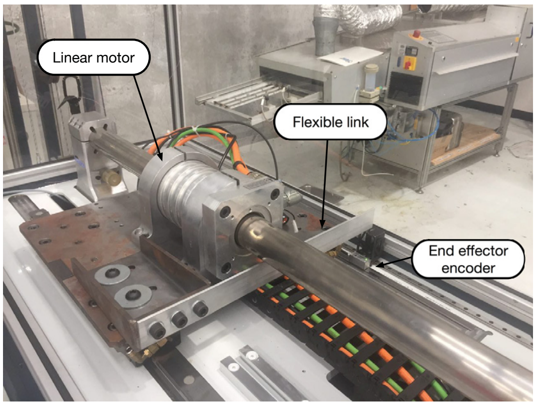



Fig. 1. Linear motor based test bench for system identification.

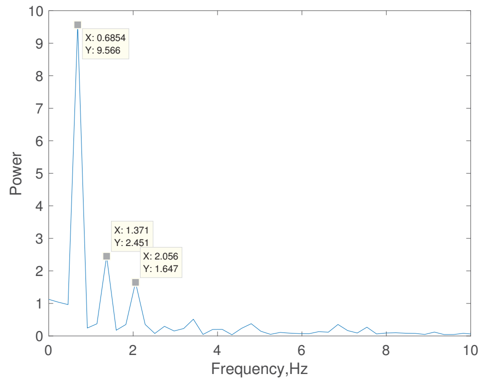



Fig. 2. Spectral analysis of cogging force.

known. The following online optimisation problem is then solved at each time instant:

$$
\begin{aligned}
U^*(k) = \arg\min_{U(k)} \sum_{i=0}^{N-1} & \left( Q \left( x_e^*(k+i) - x_{e(i|k)} \right)^2 + R u_{i|k}^2 \right) \\
s.t.\ & x_{i+1|k} = A x_{i|k} + B u_{i|k} \\
& r_{i|k} = r(k+i) \\
& x_e^*(k+i) = C_r r(k+i) \\
& x_{i|k} \in \mathcal{R}^{x,r}(x, r_{i|k}),\ \forall i \in \mathbb{Z}_{[0,N-1]} \\
& x_{0|k} = x(k)
\end{aligned} \tag{14}
$$

where $U(k) = \left(u_{0|k}, \cdots, u_{N-1|k}\right)$. At each time instant, the optimal control $u(k) = u_{0|k}^*$ is applied to the plant.

**Remark 2.** *The satisfaction of the system states inside the projected RCI set $x_{i|k} \in \mathcal{R}^{x,r}(x, r_{i|k})$ ensures the state, input and performance constraints are guaranteed, specifically, $x \in \mathcal{X}$, $u \in \mathcal{U}$, $\|x_e^* - x_e\|_\infty \leq \epsilon$.*

## IV. Simulation Results

### A. System Identification

In this section, the proposed controller is demonstrated via simulation using a model of the industrial test bench shown in Fig. 1. Model parameters and external disturbances were established from system identification experiments with the linear-motor-based test bench. The LinX® linear motor, designed by ANCA Motion, is a high-precision permanent magnet synchronous tubular motor. Linear encoders are installed on the motor base and the end effector for position measurement.

The friction and cogging force are identified as the disturbance force on the motor base in (1) for high fidelity simulation purposes. The spectral analysis based on fast Fourier transform is used to find the frequency component of the cogging force. The power spectral density with constant velocity at 0.06 m/s is shown in Fig. 2. The pole pitch is identified as $\tau = 87.5$ mm based on the fundamental peak. It can be seen that the first three harmonics are sufficient to approximate the cogging force. The identified model parameters of cogging force are $S_{c1} = 4.07$, $C_{c1} = 9.20$, $S_{c2} = 0.95$, $C_{c2} = -0.26$, $S_{c3} = -0.72$, $C_{c3} = 0.57$.

The identified values of the friction model parameters for the test bench are $f_c = 36.36$, $f_v = 250.9$, $f_s = 22.04$, $v_s = 2.82$ and $\lambda$ is chosen as $10^{-6}$. Fig.3 shows the comparison between the measured friction data and the value calculated based on friction model $F_f$. It indicates that the model provides an excellent representation of the measured data.

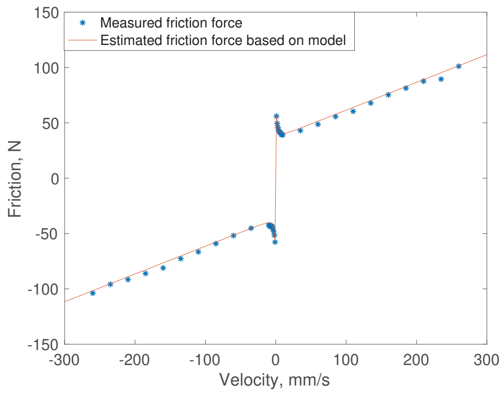



Fig. 3. Friction model fits with the measured friction force data.

To identify the parameters in (5), a point-to-point movement is conducted on the test bench. Then the parameters are identified using a least square method. The identified motor and end effector masses are $M_m = 39.61$ kg, $M_e = 0.38$ kg. The spring constant and internal damping coefficient are $k_s = 2908$ N/m and $c_s = 2.10$ Ns/m.

The models of the disturbance force $w$ is assumed unknown in the controller design process, and only the range of the disturbance force is known. The range of the disturbance force is calculated from the maximum values of (2) and (7) in the domain of interest and found to be $w \in [-70, 70]$ N.

### B. Simulation Results and Comparison

In this section, we desire the end effector to track a reference trajectory with a tracking error bound $\epsilon = 5mm$. The tracking performance using the proposed method is compared with that obtained with a conventional cascaded PI controller. The states of the reference model are the desired position and velocity of the end effector, namely $r \triangleq (x_e^*, v_e^*)$. The reference is generated from a double integrator model whilst considering position and velocity constraints, i.e. $(0, -1) \leq r(t) \leq (0.1, 1)$ [m, m/s]. Acceleration is the input to the reference model, and is subject to the constraints $-10 \leq u_r(t) \leq 10$ [m/s$^2$]. The LTI model of the reference (8) prior to constraint consideration, is given as follows:

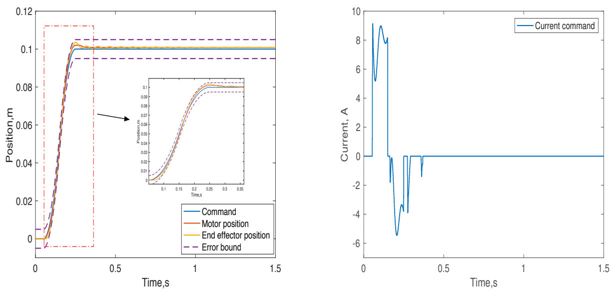
$$r^+ = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} r + \begin{bmatrix} 0 \\ T_s \end{bmatrix} u_r$$
$$x_e^* = \begin{bmatrix} 1 & 0 \end{bmatrix} r \tag{15}$$

The state and input constraints of system are taken to be :

$$-\begin{bmatrix} 0.05 \\ 1 \\ 0.05 \\ 1 \end{bmatrix} \leq x \leq \begin{bmatrix} 0.15 \\ 1 \\ 0.15 \\ 1 \end{bmatrix}$$
$$-20 \leq u \leq 20$$

Yalmip [17] and MPT3 [18] are used to estimate the one-step reachable set, $\hat{D}(\cdot)$ in (13). The RCI set is subsequently calculated using Algorithm 1, and the cost function $J = 10^5 \left(x_e^*(k+i) - x_{e(i|k)}\right)^2 + u_{i|k}^2$ is used in the MPC problem formulation.

In order to demonstrate the performance of the proposed controller, a point-to-point trajectory with 10 m/s$^2$ maximum acceleration, 1 m/s maximum velocity and 0.1 m moving distance is considered for tracking control. The sample period of the proposed controller is 1 ms. Fig. 4(a) shows the tracking performance of the proposed controller, while the current command is given in Fig. 4(b). It can be seen that the trajectory of end effector position stays inside the error bound throughout the process, thus satisfying the tolerance constraint. Furthermore, the input current remains within the acceptable range, demonstrating the efficacy of the proposed approach.




(a) (b)

Fig. 4. Point-to-point tracking performance of proposed controller : (a) trajectory; (b) current input command.

For comparison, the widely applied cascaded PI controller is considered to provide a benchmark for assessing the performance of the proposed algorithm. Since the position and velocity of the end effector are assumed measurable, the difference between the speed of the motor and end effector is used as an additional feedback in the velocity loop. For a fair comparison, the cascaded controller is tuned based on the same cost function used in the MPC formulation.

To provide a fairer comparison between the cascaded PI and the proposed algorithm, two sampling rates of the PI controller are considered to allow for the increased computational demand of the optimisation problem in (14). In the first case, an identical sample rate of 1 kHz is considered for both controllers, whilst a faster rate of 4 kHz is also used to reflect the potentially slower updates available through incorporation of an MPC approach.

The end effector tracking error resulting from the different methods is shown in Fig. 5. The maximum tracking error of the proposed controller, and the 1 kHz and 4 kHz sampling rate cascaded controllers are 4.5 mm, 9.2 mm and 8.0 mm respectively. It can be seen that the proposed controller guarantees the bounded error tracking, whereas the error bound is violated by both cascaded PI controllers. Also, it can be seen that the vibration of the end effector is considerably reduced by the proposed controller. After the reference reaches the desired position, the value of current command calculated by the proposed controller reduces to zero considering the tracking error is already within the bound tolerance. This causes the small vibration and steady state offset of the proposed controller after deceleration - if it was desirable to fully remove the steady state error, this could be tackled by a integrator augmented MPC as discussed in [19].

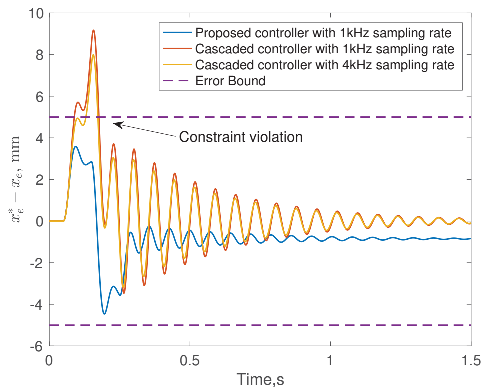
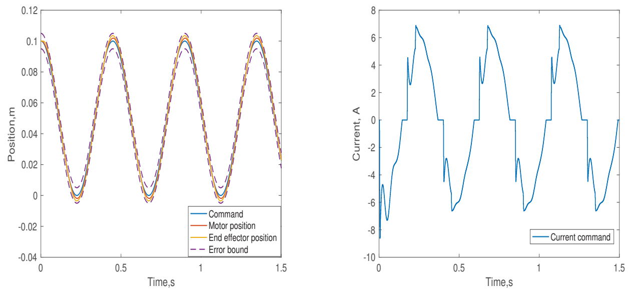



Fig. 5. Point-to-point reference tracking error of end effector based on proposed control method and cascaded PI controllers with different sampling rates.

To further assess the capabilities of the proposed approach, a sinusoidal reference with maximum acceleration of 9.8 m/s$^2$, $x_e^* = 0.05 + 0.05\cos(14t)$, is considered for reference tracking. This form of reference is widely used in laser cutting applications for generating a circular contour.

The sinusoidal reference tracking performance of the proposed controller is shown in Fig. 6(a) with the resulting control input Fig.6(b). As expected, the end effector follows the command closely without violating the error tolerance. The control input also stays within the input constraint.

The end effector tracking error of proposed method and cascaded PI controllers is given in Fig. 7. The error tolerance is achieved by the proposed controller and the 4 kHz sample rate cascaded controller. However, the error bound is violated by using 1 kHz sample rate cascaded controller.




(a) (b)

Fig. 6. Sinusoidal tracking performance of proposed controller : (a) trajectory; (b) current input command.

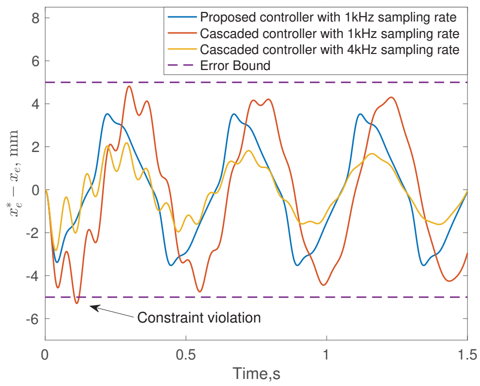



Fig. 7. Sinusoidal reference tracking error of end effector based on proposed control method and cascaded PI controllers with different sampling rates.

The simulation results demonstrate the proposed approach is able to guarantee a given error tolerance and significantly reduce the vibration of the end effector.

## V. Conclusion

A model-predictive-based bounded error trajectory tracking control scheme is proposed in this paper. The estimation of maximal RCI set for systems with external disturbance is considered. Simulation results demonstrate the proposed control scheme can guarantee the desired tracking error tolerance without violating state and input constraints. An improved performance of ensuring tracking error bound and reducing vibration is achieved by the proposed method compared to the conventional cascaded PI controller.

Further work involves implementing this error bound tracking controller on industrial machines, and consider the controller design when the position and the velocity of the end effector are not directly measured.

## References


[1] M. Corapsiz and K. Erenturk, "Trajectory Tracking Control and Contouring Performance of Three Dimensional CNC," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 4, pp. 1–1, 2015.

[2] A. Kamalzadeh and K. Erkorkmaz, "Compensation of Axial Vibrations in Ball Screw Drives," *CIRP Annals*, vol. 56, no. 1, pp. 373–378, 2007.

[3] W. Symens, H. Van Brussel, and J. Swevers, "Gain-scheduling control of machine tools with varying structural flexibility," *CIRP Annals - Manufacturing Technology*, vol. 53, no. 1, pp. 321–324, 2004.

[4] W. He and S. S. Ge, "Vibration Control of a Flexible Beam With Output Constraint," *IEEE Transactions on Industrial Electronics*, vol. 62, no. 8, pp. 5023–5030, Aug. 2015.

[5] L. Lu, Z. Chen, B. Yao, and Q. Wang, "A Two-Loop Performance-Oriented Tip-Tracking Control of a Linear-Motor-Driven Flexible Beam System With Experiments," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 3, pp. 1011–1022, Mar. 2013.

[6] P. J. Serkies and K. Szabat, "Application of the MPC to the position control of the two-mass drive system," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 9, pp. 3679–3688, 2013.

[7] D. Lam, C. Manzie, and M. C. Good, "Model Predictive Contouring Control for Biaxial Systems," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 2, pp. 552–559, Mar. 2013.

[8] C. Zhang and Y. Chen, "Tracking Control of Ball Screw Drives Using ADRC and Equivalent-Error-Model-Based Feedforward Control," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 12, pp. 7682–7692, Dec. 2016.

[9] D. Sepasi, R. Nagamune, and F. Sassani, "Tracking Control of Flexible Ball Screw Drives With Runout Effect and Mass Variation," *IEEE Transactions on Industrial Electronics*, vol. 59, no. 2, pp. 1248–1256, Feb. 2012.

[10] S. Di Cairano and F. Borrelli, "Reference Tracking With Guaranteed Error Bound for Constrained Linear Systems," *IEEE Transactions on Automatic Control*, vol. 61, no. 8, pp. 2245–2250, Aug. 2016.

[11] Chuxiong Hu, Bin Yao, and Qingfeng Wang, "Coordinated Adaptive Robust Contouring Control of an Industrial Biaxial Precision Gantry With Cogging Force Compensations," *IEEE Transactions on Industrial Electronics*, vol. 57, no. 5, pp. 1746–1754, May. 2010.

[12] D. P. Hess and A. Soom, "Friction at a Lubricated Line Contact Operating at Oscillating Sliding Velocities," *Journal of Tribology*, vol. 112, no. 1, p. 147, 1990.

[13] D. Karnopp, "Computer Simulation of Stick-Slip Friction in Mechanical Dynamic Systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 107, no. 1, p. 100, 1985.

[14] H. Olsson, K. Åström, C. Canudas de Wit, M. Gäfvert, and P. Lischinsky, "Friction Models and Friction Compensation," *European Journal of Control*, vol. 4, no. 3, pp. 176–195, Jan. 1998.

[15] M. Rungger and P. Tabuada, "Computing Robust Controlled Invariant Sets of Linear Systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 7, pp. 3665–3670, 2017.

[16] R. Vidal, S. Schaffert, J. Lygeros, and S. Sastry, "Controlled invariance of discrete time systems," in *Hybrid Systems: Computation and Control*, N. Lynch and B. H. Krogh, Eds., pp. 437–451. Berlin, Heidelberg: Springer Berlin Heidelberg, 2000.

[17] J. Lofberg, "YALMIP : a toolbox for modeling and optimization in MATLAB," in *2004 IEEE International Conference on Robotics and Automation (IEEE Cat. No.04CH37508)*, pp. 284–289. IEEE, 2004.

[18] M. Herceg, M. Kvasnica, C. N. Jones, and M. Morari, "Multi-Parametric Toolbox 3.0," in *2013 European Control Conference (ECC)*, pp. 502–510. IEEE, Jul. 2013.

[19] M. A. Stephens, C. Manzie, and M. C. Good, "Model predictive control for reference tracking on an industrial machine tool servo drive," *IEEE Transactions on Industrial Informatics*, vol. 9, no. 2, pp. 808–816, 2013.