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On Optimization-Based Coordination of Automated Vehicles in Confined Sites

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Cover page photo obtained from Volvo Autonomous Solutions illustrating a machine and truck operating in a confined site.

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Abstract

Confined sites like ports, mines, and quarries present opportunities for early deployment of automated vehicles (AVs), as they provide controlled environments with reduced safety risks from external factors. Effective coordination of fully automated vehicles in such settings is crucial, as it can increase productivity, improve safety, and possibly reduce the number of operating vehicles needed.

Optimization-based control methods are useful for planning AV operations, considering key operational constraints. However, these methods can be slow for real-time applications due to the complexity of solving the optimization problems involved, especially for coordinating multiple vehicles. This thesis introduces a method using optimization-based heuristics to simplify and approximate these problems.

The method involves a two-stage optimization approach for AV coordination in confined sites. Specifically, the combinatorial part of the coordination problem that is related to the occupancy orders of the conflict zones is formulated as a Mixed Integer Quadratic Program (MIQP). In the second stage, the optimal control commands for each vehicle are found under a fixed crossing order by solving a Nonlinear Program (NLP). To improve the computational demand we propose a decomposition strategy based on graph theory, where the centralized NLP is decomposed into multiple, parallelly solvable NLPs. Utilizing the Lagrange dual variables we propose a method that can further decompose the NLP and can be used to find a trade-off between improved computation time and optimality.

Finally, we adapted the optimization-based method to be able to handle the scenarios when human-driven vehicles (HDVs) are present in the confined site. Specifically, the heuristic predicts the HDV behavior using a model that accounts for various human reactions. The NLP is modified to capture HDV movements and establish safety constraints between AVs and HDVs. Through closed-loop receding horizon control, we demonstrate how the occupancy order for the zones can be dynamically adapted to current conditions and HDV motion predictions. Furthermore, it is shown how the method can be used to control the HDVs using the AVs to improve site productivity.

Keywords: Automated vehicles, cooperative systems, graph theory, motion control, multi-agent systems, optimal scheduling, optimal control.

List of Publications

This thesis is based on the following publications:

[A] **Stefan Kojchev**, Robert Hult, Jonas Fredriksson, Maximilian Kneissl, "A Two-Stage MIQP-Based Optimization Approach for Coordinating Automated Electric Vehicles in Confined Sites". *Published in IEEE Transactions on Intelligent Transportation Systems (2023).*

[B] **Stefan Kojchev**, Robert Hult, Maximilian Kneissl, Jonas Fredriksson, "A Computational Decomposition Strategy for Optimization-Based Coordination of Automated Vehicles in Confined Sites". *Under a second review in IEEE Transactions on Intelligent Transportation Systems (2023).*

[C] **Stefan Kojchev**, Robert Hult, Maximilian Kneissl, Jonas Fredriksson, "Optimization-Based Coordination of Automated and Human-Driven Vehicles in Confined Sites". *Submitted to IEEE Special Issue on Transactions on Control Systems Technology (2023).*

Other publications and patent applications by the author, not included in this thesis, are:

[D] **S. Kojchev**, E. Klintberg and J. Fredriksson, "A safety monitoring concept for fully automated driving". *Published in IEEE 23rd International Conference on Intelligent Transportation Systems (ITSC)*, pp. 1-7. IEEE, 2020.

[E] **S. Kojchev**, A. Gupta, R. Hult and J. Fredriksson, "An iterative algorithm for volume maximization of N-step backward reachable sets for constrained linear time-varying systems". *Published in the 60th IEEE Conference on Decision and Control (CDC)*, pp. 5027-5032. IEEE, 2021.

[F] **S. Kojchev**, R. Hult and J. Fredriksson, "Optimization based coordination of autonomous vehicles in confined areas". *Published in IEEE 25th International Conference on Intelligent Transportation Systems (ITSC), pp. 1957-1963. IEEE, 2022.*

[G] **S. Kojchev**, R. Hult and J. Fredriksson, "Quadratic approximation based heuristic for optimization-based coordination of automated vehicles in confined areas". *Published in IEEE 61st Conference on Decision and Control (CDC), pp. 6156-6162. IEEE, 2022.*

[H] **S. Kojchev**, R. Hult and J. Fredriksson, "Energy Efficient Optimization-Based Coordination of Electric Automated Vehicles in Confined Areas". *In 2023 62nd IEEE Conference on Decision and Control (CDC), pp. 3433-3440. IEEE, 2023..*

[I] **S. Kojchev**, J. Hellgren, R. Hult and J. Fredriksson, "Combined Path and Trajectory Planning for Energy-Efficient Coordination of Automated Vehicles in Confined Areas". *Accepted to the 2024 European Control Conference (ECC) IEEE, 2024.*

[J] Oskar Wigstöm, R. Hult and **S. Kojchev**, "Method and Device for Coordinating Vehicle Routes in Confined Areas". *Patent pending, European Patent Office, Application no. 21205593.3, October 2021.*

[K] R. Hult and **S. Kojchev**, "Method and Device for Coordinating Vehicle Routes in Confined Areas". *Patent pending, European Patent Office, Application no. 22165524.4, March 2022.*

[L] M. Kneissl, R. Hult and **S. Kojchev**, "Efficient Trajectory Planning for a Fleet of Vehicles". *Patent pending, European Patent Office, Application no. 23171302.5, May 2023.*

[M] Jonas Hellgren and **S. Kojchev**, "A Traffic Planning Method for a Vehicle Fleet". *Patent pending, European Patent Office, Application no. 23173049.0, May 2023.*

[N] Jonas Hellgren and **S. Kojchev**, "A Traffic Planning Method for a Vehicle Fleet". *Patent pending, European Patent Office, Application no. 23173047.4, May 2023.*

[O] **S. Kojchev**, "Efficient Trajectory Planning for a Fleet of Loadable Vehicles". *Patent pending, European Patent Office, Application no. 24166979.5, April 2024.*

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Stefan Kojchev
Gothenburg, May, 2024

Acronyms

AV:	Automated Vehicle
BnB:	Branch and Bound
CBF:	Control Barrier Function
CLF:	Control Lyapunov Function
DOC:	Direct Optimal Control
GNSS:	Global Navigation Satellite System
HDV:	Human-Driven Vehicle
IMU:	Inertial Measurement Unit
MINLP:	Mixed Integer Nonlinear Program
MIQP:	Mixed Integer Quadratic Program
MPC:	Model Predictive Control
MUTEX:	MUTually EXclussive
NLP:	Nonlinear Program
OCP:	Optimal Control Problem
QP:	Quadratic Program
RTK:	Real-Time Kinematic positioning
SQP:	Sequential Quadratic Program
SVO:	Social Value Orientation

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Part I

Introductory Chapters

CHAPTER 1

Background and Outline

1.1 Introduction

Automated vehicles (AVs) represent a significant advancement in transportation technology. Defined by their capacity to operate without human input, AVs offer numerous potential benefits, including improved traffic efficiency, reduced emissions, and increased safety [1]. Traffic accidents, predominantly caused by human error, could be significantly reduced through the deployment of AVs [2]. Furthermore, AVs are expected to greatly enhance mobility for individuals who are unable to drive, such as the elderly or those with disabilities [3].

The current state of automated vehicle technology is rapidly evolving with ongoing research and development in multiple areas, such as perception, control, and verification. While these developments are promising, most current AVs operate at Level 2 or 3 automation, indicating a need for occasional human intervention [4]. One of the persisting challenges in deploying AVs is ensuring reliability and safety in diverse and unpredictable conditions [5], [6].

Confined sites like mines, quarries, and ports offer a unique and highly suitable environment for the deployment of AVs. The primary advantage of

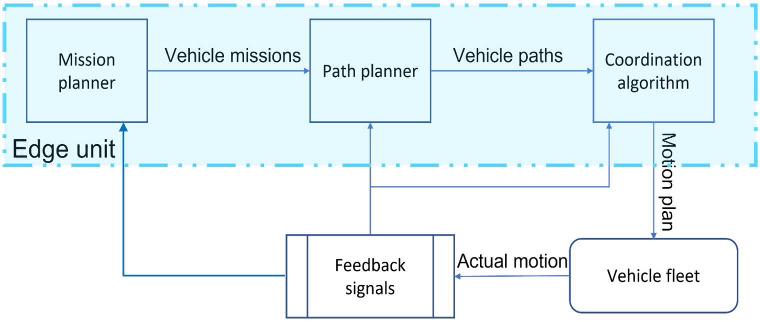


Figure 1.1: Architecture of the confined site AV system.

these sites is that by construction, they are devoid of unpredictable agents such as pedestrians, bicycles, human-driven vehicles, etc., typically found in urban or public road settings. This aspect significantly reduces the complexity the AVs must handle, making these sites ideal for early adoption and testing.

Similarly to public roads, the deployment of automated vehicles in confined sites offers potential advantages. In particular, AVs can enhance the safety of the site, as confined sites often involve hazardous tasks and harsh conditions where human operators are at risk. In mines, for example, AVs can undertake deep underground operations where the risk of incidents, such as gas explosions or cave-ins, is high. Furthermore, the deployment of AVs could increase operational efficiency since the vehicles can operate continuously without the need for breaks.

In general, in confined sites, the main objective is defined by the site owner, involving tasks like ore extraction or goods transportation over extended periods. This abstract mission necessitates strategic decisions regarding the number of vehicles required, the specific role of each vehicle, the efficient routing to the destination, and the computation of motion profiles that enhance overall performance, notably in terms of energy efficiency. Solving all these problems simultaneously poses a considerable computational challenge, often leading to large-scale problems that may be computationally intractable.

A common strategy for managing such large-scale problems is to decompose the problem into multiple smaller and computationally tractable problems. An architecture illustrating a proposed decomposition of a site control system is depicted in Figure 1.1, which outlines a framework comprising four key components: *Mission planner*, *Path planner*, *Coordination algorithm*, and *Vehicle fleet*. The *Mission planner* component determines the number of vehicles necessary to complete the site mission and assigns each available vehicle a transport mission. A transport mission is a vague description of what should be achieved and which control points (loading/unloading zones, charging zones, etc.) should be visited, for example, Vehicle 1 should load a specific amount from mining point A, Vehicle 2 should charge at charging station C, etc. Subsequently, the *Path planner* develops routes that respect these control points and align with the road network. Using the road network, the *Coordination algorithm* is tasked to compute the state and input trajectories for all vehicles such that joint utilization of the control points and other inter-vehicle conflicts are avoided. The assembled motion plans are communicated to the *Vehicle fleet*, which executes them and provides feedback.

This thesis primarily focuses on the *Coordination algorithm* component. It also investigates a closed-loop formulation of the proposed method for this component, which includes developing a strategy for the *Vehicle fleet* component and examining the interaction between the *Vehicle fleet* and the *Coordination algorithm*.

1.2 Related work

The coordination of automated vehicles, particularly on public roads, has drawn significant interest from the research community [7], [8]. The main challenge in the coordination scenarios is the computational complexity of generating collision-free motion profiles and managing MUTually EXclusive (MUTEX) zones, i.e., areas where vehicles share resources or where conflicts may arise [9]. Each MUTEX zone implies that there should be an order in which the vehicles use the zone. The combinatorial challenge of these problems increases with the number of interacting vehicles and zones. Other challenges involve communication limitations [10], architecture design choices, and managing uncertainties like sensor and measurement perturbations.

An early proposal to solve the intersection coordination problem is presented in [11], where a reservation request protocol is introduced to ensure a deadlock-free crossing. However, the approach does not directly consider the vehicle dynamics. Alternative approaches for obtaining the zone occupancy orders are through mixed-integer optimization [12]–[14], scheduling [15]–[17] or tree-based search methods [18], [19]. In these approaches, once the occupancy orders have been computed, i.e., the potential conflicts have been resolved, there is a subsequent component that computes the vehicle motion profiles.

Recent advancements have introduced optimal control-based methodologies, utilizing Model Predictive Control (MPC) [20]–[22], direct optimal control [23]–[25] and trajectory optimization methods [26]–[28]. Alternatively, consensus-based methods [29]–[31] have emerged as a solution to the problem where AVs communicate and collaborate to make joint decisions. Each vehicle shares information about its state (like position and speed) and intentions with others, which is then used by all participating vehicles to collectively plan their trajectories, ensuring efficient traffic flow and safety. The goal is to reach a "consensus" on the best course of action for all vehicles, thereby optimizing travel time, reducing congestion, and enhancing safety at intersections and other traffic scenarios without the need for traditional traffic signals. This cooperative strategy relies on real-time data exchange between vehicles.

While it may be reasonable for confined sites to assume the absence of uncontrolled agents like Human-Driven Vehicles (HDVs), some site owners may specifically demand the coexistence of AVs and HDVs on their sites. For public roads, it is believed that the road will be shared between the AVs and HDVs for a substantial period [1]. The mixed-traffic coordination of vehicles, i.e., coordination of AVs and HDVs, on the other hand, has received less attention from the research community and currently is a popular topic, [32]. The authors of [11] extended their reservation request protocol to incorporate and control HDVs, where the route of the HDVs is required to be known [33]. The coordination of mixed traffic in intersections has been investigated in [34], [35], where [34] plans the vehicle maneuvers based on a probabilistic, multi-modal prediction with a decision tree representation, and [35] proposes a sensitivity-based heuristic for dynamically changing the occupancy order for an intersection scenario between one AV and HDV. In [36], reinforcement learning is used for cooperative behavior planning for mixed-traffic coordination at intersections.

The coordination of mixed traffic has also been investigated for merging scenarios. The authors in [37] propose a hierarchical control framework where a high-level controller determines the merging sequence while a lower-level controller based on trajectory optimization is tasked with computing the motion profiles of the vehicles. The paper considers all possible triplet combinations of

AVs and HDVs. However, it only focuses on the low-level control part, meaning that the triplet computation and its recomputation are excluded from the analysis. In [38], a safety-critical decentralized approach based on Control Barrier Functions and Control Lyapunov Functions is proposed for the merging scenarios in a mixed-traffic environment. The approach, depending on the position of the HDV, either uses a free-flow model or a car-following model for estimating the motion of the HDV and is capable of switching between the models. The paper relies on a specific instance of the driver reaction parameters. Other research efforts in this field have utilized the Social Value Orientation (SVO) [39], [40], to quantify the human drivers' behavior and predict how the human drivers will interact and cooperate with other vehicles. The framework presented in [39] is implemented in a receding horizon control manner for robustness against stochastic human driving behavior. However, this paper considers only the interaction between one AV and HDV. The authors in [41] propose a robust mixed-traffic platoon control framework using tube MPC to account for the uncertainties of the HDVs. However, the approach is bounded by the use of linear models.

1.3 Research gaps

Coordination of AVs in confined sites has some distinct differences compared to public road scenarios. Specifically, as the road network is known, it is possible to plan the motion of the vehicles from the start of a transport mission to its end. Planning the motion over long horizons is particularly beneficial in terms of energy efficiency [42]. Furthermore, confined areas have additional MUTEX zones besides intersections, as mentioned earlier. A consequence of long-horizon planning is that a vehicle can experience multiple combinations of the MUTEX zones along its route. The authors in [43] and [44] propose approaches to multiple intersection coordination. However, they consider a "cut-out" around the intersections with vehicles arriving at speed in comparison to the desired full route motion planning that considers all MUTEX zones at the planning stage.

In the context of autonomous mining, a dynamic fleet planning method has been proposed in [45] employing a modified genetic algorithm to resolve conflicts and minimize delays and waiting times. However, the method does not fully consider vehicle dynamics and necessitates predetermined standstill locations and times, potentially leading to suboptimal behavior.

The coordination of vehicles in valet parking and the coordination of industrial robots share several characteristics with confined site scenarios, with both involving a restricted area where vehicles navigate through various MUTEX zones, [46]–[51]. In these applications, vehicles typically operate at lower speeds, aiding in safety assurance and the modeled dynamics. Conversely, in confined sites, higher vehicle speeds are preferred to enhance productivity. In addition, the valet parking applications have laxer energy efficiency and productivity goals.

In light of these research gaps, this thesis explores the following research questions:

- **RQ1:** How can the vehicle coordination problem in confined sites be formulated as an optimization problem that is not bounded by a specific choice of model, constraints, and objective function?
- **RQ2:** How can the specific confined site application be exploited with decomposition schemes to decouple or distribute most of the computations when solving the optimization problem formulated in **RQ1**?
- **RQ3:** How can the proposed optimization-based approach formulated in **RQ1** be adapted to include human-driven vehicles that have various operating behaviors?

1.4 Contributions

This thesis focuses on developing an optimization-based method for the vehicle coordination problem in confined sites. As such, the mission and path planning components are outside the scope of this thesis. We assume that the number of vehicles, their starting and end destinations, and their paths in the confined site are computed. Furthermore, communication delays between the components are left outside the scope of this thesis.

The main contributions of this thesis are:

- A two-stage centralized optimization-based approach for the vehicle coordination problem in confined sites. The proposed approach provides a high-level motion plan for the vehicles and is not dependent on a specific choice of vehicle model, constraints, and objective function.

- A decomposition strategy that identifies subproblems from the original coordination problem. These subproblems can be solved in parallel to ease the computational demand of the approach.
- Integrating the charging process and charging zone occupancy constraints in the problem for electric vehicles. The approach can be extended to other dwelling zones, such as loading and unloading.
- A receding horizon closed-loop formulation of the problem capable of satisfying the occupancy constraints under perturbations such as model mismatch, sensor, and measurement noise.
- Adapting the proposed approach to scenarios where human-driven vehicles are present on the site and interact with the automated vehicles.

1.5 Outline

This thesis is divided into two parts, where Part I provides a context and background for the papers that are appended in Part II. Part I consists of five chapters, where Chapter 1 introduces the topic and contribution of the thesis. Chapter 2 formulates the motion models and provides fundamental concepts in optimal control, numerical optimization, and graph theory. The confined site coordination problem is defined in Chapter 3 as well as the optimization-based approach that is utilized. Chapter 4 provides a summary of the included papers in Part II. Finally, Chapter 5 presents some concluding remarks as well as suggestions for future research directions.

CHAPTER 2

Preliminaries

This chapter gives an overview of some of the concepts used in the thesis. In particular, Section 2.1 introduces optimal control and concepts such as direct optimal control, model predictive control, and mixed-integer problems. Section 2.2 provides fundamentals related to graph theory and different types of graphs relevant to this thesis.

2.1 Optimal Control

Optimal control theory is a mathematical framework aimed at finding control policies that optimize a certain performance criterion for dynamical systems. An *Optimal Control Problem* typically involves a dynamical system described by differential equations, a performance index (objective function) to be optimized, and constraints on the controls and states.

In specific, we focus on OCPs of the type

$$\min_{u(t)} \; V(x(t_f)) + \int_{t_0}^{t_f} l\left(x(t), u(t)\right) dt \tag{2.1a}$$

$$\text{s.t} \quad \text{initial states} \;\; x_0 = \hat{x}_0 \tag{2.1b}$$

$$\text{system dynamics} \;\; \dot{x}(t) = f\left(x(t), u(t)\right) \tag{2.1c}$$

$$\text{state and input constraints} \;\; h\left(x(t), u(t)\right) \leq 0, \tag{2.1d}$$

where $x(t), u(t)$ denote the state and control input trajectories, with t_0 indicating the initial time and t_f the final time, V is the terminal cost, l is the stage cost, x_0 is the initial state, and f is the system dynamics, and h are the state and input constraint functions. The solution of OCP (2.1) is the control policy $u^*(t)$.

Direct Optimal Control

An approach to solve the OCP (2.1) and obtain $u^*(t)$ is to solve a discretized, finite-dimension problem whose optimal solution approximates $u^*(t)$. This method is known as *Direct Optimal Control* and is also known as the "first discretize, then optimize" approach. A parametrized approximation of the control input trajectory is usually searched for the discretized problem. A standard input parametrization choice is a piecewise constant input on a uniform time grid $t_0, t_1, \ldots, t_{M_k}$, where M_k is the number of grid elements. This in essence leads to $u(t) = u(k), t \in [t_k, t_{k+1}[$, with $u = (u_0, \ldots, u_{M_{k-1}}$ and $t_k = k\Delta t$. The input discretization can be related to $x(t)$ by for example multiple shooting [52], which in essence leads to finding $x = (x_1, \ldots, x_{M_k})$ such that

$$x_{k+1} - F_k(x_k, u_k) = 0, \quad k = 0, \ldots, M_k - 1, \tag{2.2}$$

where $x_0 = \hat{x}_0$ and $F_k(x_k, u_k)$ denotes the numerical solution to (2.1c) at t_{k+1}, when $x(t_k) = x_k$ and $u(t) = u(k),\ t \in [t_k, t_{k+1}[$. Numerical integration is necessary to express the stage cost (2.1a) and the dynamics (2.1c) in the discretized problem. Numerical integrators can be either explicit or implicit, and there are many different methods available, the most common of which are Euler methods, Runge-Kutta methods, and collocation methods [53]. Regardless of which numerical integrator is chosen, a discretized version of OCP

(2.1) is:

$$\min_{x(k),u(k)} \quad V(x_{M_k}) + \sum_{k=0}^{M_k-1} l_k\left(x(k), u(k)\right) \tag{2.3a}$$

$$\text{s.t} \quad \text{initial states} \;\; x_0 = \hat{x}_0 \tag{2.3b}$$

$$\text{system dynamics} \;\; x_{k+1} - F_k(x_k, u_k) = 0, \;\; k = 0, \ldots, M_k - 1 \tag{2.3c}$$

$$\text{state and input constraints} \;\; h\left(x(k), u(k)\right) \leq 0, \tag{2.3d}$$

where $l_k\left(x(k), u(k)\right)$ is the numerical integration of $l\left(x(t), u(t)\right)$ over $[t_k, t_{k+1}[$. When the optimization problem (2.3) has a nonlinear objective function and nonlinear constraints, it is classified as a *Nonlinear Program*. A special case is quadratic positive-definite l, linear f, and affine h, making the problem a convex *Quadratic Program*. Details of algorithms for solving NLPs and QPs can be found in [54].

Model Predictive Control

Once we have computed a solution to an optimization problem such as (2.3) we might try to control the real process with the obtained trajectory. This approach is known as *open-loop control*. However, this approach often leads to poor performance, as the real process will typically not coincide with the model that is used in the optimization problem. Furthermore, additional process and measurement disturbances and uncertainties will further influence the mismatch between the real process and the modeled problem. Closed-loop control or feedback is often used to cope with model mismatches and disturbances. A practical realization of optimal control that includes feedback is MPC.

By solving the optimization problem (2.3a) an MPC uses the model (2.3c) to obtain a prediction of the optimal action u^* over a future time window. However, since this prediction is inaccurate due to the model mismatch and process noise, only the first part of u^* is applied. Subsequently, the system response is evaluated, and the process is repeated. This introduces *feedback* and allows MPC to compensate for the mismatches and perturbations. The time window is commonly known as the prediction horizon. The length of the prediction horizon, denoted as M_{MPC}, remains constant, allowing its endpoint

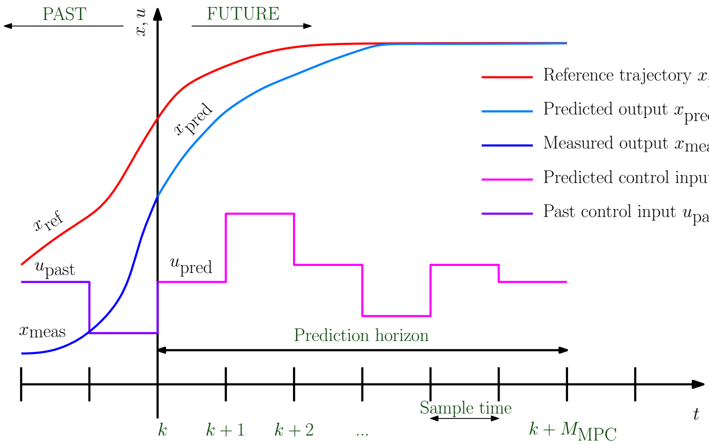


Figure 2.1: Illustration of the receding horizon principle.

to progress with the real-time process. Due to this movement of the prediction horizon, MPC is also known as *receding horizon control*.

The implementation of MPC can, in discrete time, roughly be formulated as follows:

1. At time $t(k)$, observe the current state of the system $\hat{x}$.
2. Solve an open-loop optimization problem such as (2.3a) starting at the state $\hat{x}(k)$ for the optimal control input u^*.
3. Implement the first control action u_0^* at the real process.
4. Wait until $t(k) + \Delta t$ and then repeat the procedure, where Δt is known as the sample time.

Figure 2.1 illustrates the operational concept of MPC. Further details on MPC variants and the theory behind stability and feasibility can be found in [55].

Mixed-Integer Problems

A Mixed-Integer Problem is a problem with both real and integer decision variables. A general form of an MIP where the integer variables are binary can be stated as

$$
\begin{aligned}
\min_{x,b} \quad & f(x,b) && (2.4a) \\
\text{s.t} \quad & g(x,b) = 0 && (2.4b) \\
& h(x,b) \leq 0 && (2.4c) \\
& b \in \{0,1\}^{n_b} , && (2.4d)
\end{aligned}
$$

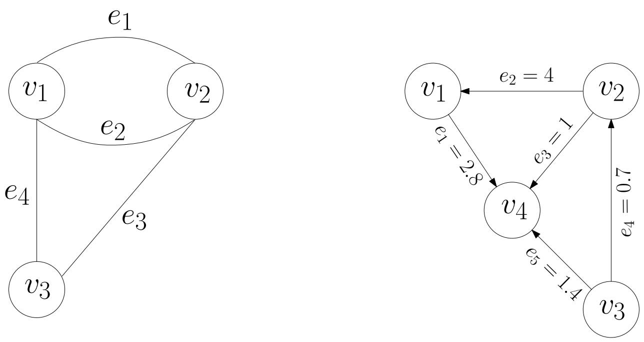
where $x \in \mathbb{R}^n$ denotes the vector of continuous variables and b denotes the vector of binary variables, with n, n_b being the number of continuous and binary variables, respectively. The objective function is denoted as f, g is the vector of equality constraints, and h is the vector of inequality constraints. If f, g, h are twice differentiable in x and b, we speak of a *Mixed-Integer Non-Linear Program*. Generally, MINLPs are very hard to solve due to the combinatorial nature of the binary variables.

A popular approach for solving MIPs is the Branch and Bound method. It involves dividing the problem into smaller subproblems, bounding their potential solutions, and intelligently exploring the solution space to converge toward the global optimum. The method consists of key components like node selection, branching, and bounding, and it is characterized by its ability to efficiently explore large solution spaces, making it a fundamental technique in solving complex optimization problems. Numerous variants of the Branch and Bound method exist, primarily distinguished by their search strategies. Nevertheless, this fundamental procedure remains the core of numerous widely used solvers for mixed-integer problems.

2.2 Graph Theory

Graph theory is a fundamental field of mathematics and computer science, focusing on the study of graphs—mathematical structures used to model pairwise relations between objects [56], [57]. A graph $G = (\mathcal{V}, \mathcal{E})$ is composed of vertices $\mathcal{V}$ (also called nodes) and edges $\mathcal{E}$ (links or lines) that connect pairs of vertices. Graph theory has applications in various areas, including computer networks, social networks, biology (to model and study relationships between genes, proteins, etc.), transportation networks, and many more.

In general, graphs can be directed or undirected, where a directed graph is a graph where the edges have a direction, indicating a one-way relationship



(a) Undirected multigraph (b) Directed weighted graph

Figure 2.2: Examples of an undirected multigraph and a weighted directed graph.

between two vertices. In undirected graphs, the edges do not have a direction, and the relationship between two vertices is considered bidirectional. Furthermore, the undirected and directed graphs can be multigraphs where the graph contains multiple edges (also known as parallel edges) between the same set of vertices, and they can be weighted graphs where each edge is assigned a weight or cost. Figure 2.2 depicts an undirected multigraph and a directed weighted graph.

An adjacency matrix A_{adj} is a square matrix used to represent a finite graph. The matrix elements indicate whether pairs of vertices are adjacent or not in the graph. For undirected graphs, the adjacency matrix is an $n \times n$ matrix, where $A_{\text{adj}}[i, j] = 1$ if there is an edge between vertex i and vertex j, and $A_{\text{adj}}[i, j] = 0$ otherwise. For a weighted graph, instead of 1, $A_{\text{adj}}[i, j]$ would represent the weight of the edge between vertices i and j. For directed graphs, the adjacency matrix is not necessarily symmetric. That is, $A_{\text{adj}}[i, j]$ can be different than $A_{\text{adj}}[j, i]$, reflecting the direction of edges. The adjacency matrix is useful for analyzing graph properties and implementing graph algorithms in computer programs.

Beyond the traditional adjacency matrices and linear interactions, graph theory expands to include bond, dual, and nonlinear graphs offering different ways to model complex systems. Bond graphs provide a framework for depicting energy flow in mechanical, electrical, and hydraulic systems by representing energy components as edges in a graph, capturing system dynamics effectively. This method is particularly useful for modeling systems with nonlinear dynamics, focusing on energy transfer mechanisms. Dual graphs offer a perspective where the roles of nodes and edges are inverted, facilitating the analysis of spatial relationships and network topology. This concept is valuable in geographical mapping and optimizing network structures, providing alternative analytical approaches. Nonlinear graphs extend the application of graph theory to model complex relationships that are not linear, crucial for fields like neural networks and biological systems.

CHAPTER 3

Confined Site Vehicle Coordination

This section discusses the modeling aspects of the coordination problem, focusing on the vehicle motion models and conditions for collision avoidance. Furthermore, it defines the high-level vehicle coordination problem as an Optimal Control Problem and outlines the heuristic strategy employed in the included papers. Finally, the low-level receding horizon controller is defined, and the closed-loop framework is presented.

3.1 Motion Models

For the vehicle coordination task, it is sufficient to only consider the longitudinal dynamics as it is often assumed that the given path is known and that the vehicles follow the assigned path perfectly.

General Form

The vehicle motion along the path can, without restriction from the above-mentioned assumption, be described as

$$\dot{p}_i(t) = v_i(t) \tag{3.1}$$
$$\dot{x}_i(t) = f_i(p_i(t), x_i(t), u_i(t)) \tag{3.2}$$
$$0 \leq h_i(p_i(t), x_i(t), u_i(t)). \tag{3.3}$$

where $p_i(t) \in \mathbb{R}$ is vehicle i's position, $x_i(t) \in \mathbb{R}^{n_i}$ is the vehicle's state, $u_i(t) \in \mathbb{R}^{m_i}$ is the control input, with $i \in \{1, \dots, N_{\text{AV}}\}$ where N_{AV} is the total number of AVs. The state is subdivided as $x_i(t) = (v_i(t), z_i(t))$, with the speed along the path $v_i(t) \in \mathbb{R}$ and $z_i(t) \in \mathbb{R}^{n_i - 1}$ collecting possible other states. The functions f_i and h_i, both assumed smooth, describe the dynamics and constraints that capture, e.g., actuator and state limitations.

Remark 1. *Note that the remaining possible states $z_i(t)$ directly depend on the choice of a vehicle model. For example, if the vehicle is modeled as a triple integrator $\dddot{x}(t) = u(t)$, the remaining vehicle states are $z_i(t) = a_i(t)$, with $a_i(t)$ being the acceleration. The state variables are thus $p_i(t)$, $x_i(t) = (v_i(t), a_i(t))$.*

General Form in Space

For confined site optimization, it is beneficial to optimize the trajectories of the vehicles over their full paths, as the time it takes a vehicle to traverse a path is dependent on the solution and not known *a-priori*. Consequently, it is inappropriate to plan the vehicle's motion with time as the independent variable. The motion model (3.1) can be reformulated in the spatial domain using that $\frac{\mathrm{d}p_i}{\mathrm{d}t} = v_i(t)$ and $\mathrm{d}t = \mathrm{d}p_i / v_i(t)$:

$$\frac{\mathrm{d}t_i}{\mathrm{d}p_i} = \frac{1}{v_i(p_i)} \tag{3.4}$$
$$\frac{\mathrm{d}x_i}{\mathrm{d}p_i} = \frac{1}{v_i(p_i)} f_i(p_i, x_i(p_i), u_i(p_i)) \tag{3.5}$$
$$0 \leq h(p_i, x_i, u_i). \tag{3.6}$$

This leads to that position is now the independent variable and that travel time t_i is a state variable.

Remark 2. *Note that equation* (3.4) *imposes that the velocity must be strictly positive.*

Human-driven vehicle models

Similarly to the AVs, for confined sites, it can be assumed that the routes of human-driven vehicles are known since they can be required to follow certain pre-defined routes. It is thus possible to formulate the motion of the HDVs along the path with the position as an independent variable. However, HDVs have an uncertain motion profile due to the human factor. Accounting for the uncertainty, the HDV motion along the path can be formulated as

$$\frac{\mathrm{d}t_j}{\mathrm{d}p_j} = \frac{1}{v_j(p_j, q_j(p_j))} \tag{3.7}$$

$$\frac{\mathrm{d}x_j}{\mathrm{d}p_j} = \frac{1}{v_j(p_j, q_j(p_j))} f_i(p_j, x_j(p_j), u_j(p_j), q_j(p_j)) \tag{3.8}$$

$$0 \leq h(p_j, x_j, u_j, q_j(p_j)), \tag{3.9}$$

where q_j is the system parameter uncertainty, which describes the human factor, and $j \in 1, \ldots N_{\text{HDV}}$ where N_{HDV} is the total number of HDVs.

Car-following model

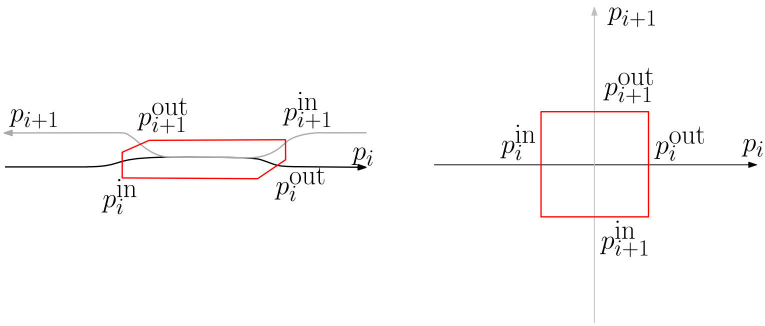
The model defined with (3.7)-(3.9) represents the motion of the HDV in free-flow, i.e., when there are no other interacting vehicles that would influence the motion of the HDV. However, in special cases when there is a vehicle in front of the HDV, the motion of the HDV can be defined using a car-following model

$$\frac{\mathrm{d}t_j}{\mathrm{d}p_j} = \frac{1}{v_j(p_j, x_i^{\text{lead}}(p_i))} \tag{3.10}$$

$$\frac{\mathrm{d}x_j}{\mathrm{d}p_j} = \frac{1}{v_j(p_j, x_i^{\text{lead}}(p_i))} f_i(p_j, x_j(p_j), u_j(p_j), x_i^{\text{lead}}(p_i)) \tag{3.11}$$

$$0 \leq h(p_j, x_j, u_j, x_i^{\text{lead}}(p_i)), \tag{3.12}$$

where $x_i^{\text{lead}}(p_i)$ is the lead vehicle's states. Consequently, when the HDV operates behind an AV, it is no longer uncertain since its motion depends on the lead AV.



(a) Narrow road (b) Intersection-like

Figure 3.1: Narrow road and intersection-like MUTEX zones.

3.2 Collision Avoidance Conditions

The collision avoidance conditions should ensure a conflict-free occupancy of the MUTEX zones. The mutual exclusion condition depends on the type of zone. There are multiple types of MUTEX zones that the vehicles encounter during their operation, like intersections, narrow roads, dwelling zones, etc. Furthermore, the vehicles can have multiple combinations of different MUTEX zones for one operating mission.

A MUTEX zone is defined by the entry and exit position $[p_i^{in}, p_i^{out}]$ on the path of each vehicle. From the known positions, the entry and exit times of Vehicle i are obtained from (3.4) and are $t_i^{in} = t_i(p_i^{in})$ and $t_i^{out} = t_i(p_i^{out})$, respectively.

Narrow roads and intersection-like zones

In the narrow road MUTEX zones, Figure 3.1-(a), meeting oncoming vehicles is not possible. From a safety perspective, this translates to “reserving” the zone for one or more vehicles coming from the same direction. The vehicles coming from the opposite direction are not allowed to occupy the zone until it is vacated. The intersection-like MUTEX zone, Figure 3.1-(b) is similar to the narrow road regarding its safety requirement, i.e., Vehicle a cannot enter the MUTEX zone before Vehicle b exits the MUTEX zone, or vice-versa.

Let $\mathcal{I} = \{I_1, I_2, ..., I_{r_{\text{tot}}}\}$ denote the set of all intersections and narrow roads

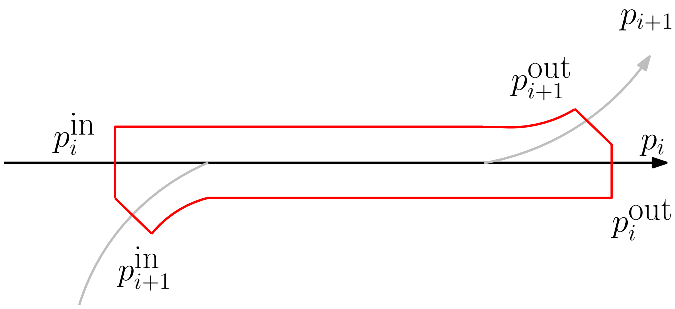


Figure 3.2: Merge-split MUTEX zone.

in the confined area, with r_{tot} being the total number of intersections and narrow roads, and $\mathcal{Q}_r = \{q_{r,1}, q_{r,2}, ..., q_{r,l}\}$ denote the set of vehicles that cross an intersection or narrow road I_r. A sufficient condition for collision avoidance for the r-th intersection or narrow road MUTEX zone can be formulated as

$$t_i^{\text{out}} \leq t_{i+1}^{\text{in}} \ \text{ or } \ t_{i+1}^{\text{out}} \leq t_i^{\text{in}}, \quad i \in \mathbb{I}_{[1,|\mathcal{Q}_r|-1]}, \tag{3.13}$$

where t is determined from (3.4).

Merge-splits

In the merge-split zone depicted in Figure 3.2, two vehicles coming from different roads but moving in the same direction of travel join together on a common patch of road. After some distance, the roads separate. For this type of MUTEX zone, let $\mathcal{MS} = \{MS_1, MS_2, ..., MS_{w_{\text{tot}}}\}$ denote a set of all merge-split zones, with w_{tot} being the total number of merge-split zones in the site and $\mathcal{Z}_w = \{z_{w,1}, z_{w,2}, ..., z_{w,h}\}$ denote the set of vehicles that cross the merge-split zone MS_w. For efficiency, it is desirable to have several vehicles in the zone simultaneously instead of blocking the whole zone. This requires imposing rear-end collision constraints once the vehicles have entered the merge-split zone. The collision avoidance requirement for the w-th merge-split zone is described with the following constraints:

$$t_i^{k_i} + \Delta t \leq t_{i+1}(p_i^{k_i} - p_i^{\text{in}} + p_{i+1}^{\text{in}} - c), \quad k_i^{\text{in}} \leq k_i \leq k_i^{\text{out}} \tag{3.14a}$$

or

$$t_{i+1}^{k_{i+1}} + \Delta t \leq t_i(p_{i+1}^{k_{i+1}} - p_{i+1}^{\text{in}} + p_i^{\text{in}} - c), \quad k_{i+1}^{\text{in}} \leq k_{i+1} \leq k_{i+1}^{\text{out}}, \tag{3.14b}$$

$$i \in \mathbb{I}_{[1,|\mathcal{Z}_w|-1]},$$

where k_i is an index of the position vector $p_{s_{w,i}}$.

The constraints (3.14) ensures that while in the MUTEX zone, the vehicles must be separated by at least a time-period Δt and a distance c, depending on if vehicle i is in front of vehicle $i+1$ or vice versa. This is equivalent to the standard offset and time-headway formulation often used in automotive adaptive cruise controllers [58].

Dwelling zones

Dwelling zones such as charging stations and loading and unloading stations are also zones that are characteristic for confined sites. In these zones, the vehicles are absorbed for some time while they perform an action such as charging or loading and unloading goods or materials. The mission planner components assigns a vehicle to visit a dwelling zone. The zones consist of a road patch that leads to the charger/loading/unloading location and a road patch after the charger/loading/unloading location until a merge point with the remainder of the road. When a vehicle visits a dwelling station, it is required to make a full stop at the charger/loading/unloading location, and after some time, t_i^{absorb} the vehicle leaves the station with an increased state of charge or increased/decreased vehicle mass. The advantage of utilizing a spatial model is that time is a state variable. To account for the absorption time, we can modify the time state constraint by adding the duration of the charging/loading/unloading process. Essentially, the time state after the charger/loading/unloading location is:

$$t_i^{\text{DZ}+1} = t_i^{\text{DZ}+1} + t_i^{\text{absorb}}, \tag{3.15}$$

where DZ is the position of the charger/loading/unloading location. Note that the charging/loading/unloading time is communicated and decided by the mission planner component. The increase in the state of charge or increase/decrease of vehicle mass depends directly on the charging/loading/unloading time and the capacity of the charging/loading/unloading stations.

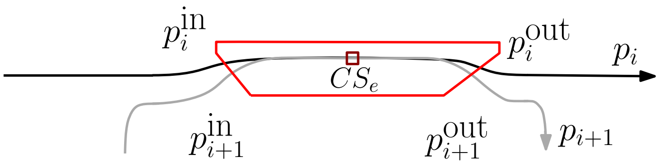


Figure 3.3: Charging station MUTEX dwelling zone.

In the case when two or more vehicles are assigned to the dwelling zone, the algorithm needs to enforce rear-end constraints for collision avoidance. We formalize the constraints by first defining a set of all dwelling zones as $\mathcal{DZ} = \{DZ_1, DZ_2, ..., DZ_{e_{\text{tot}}}\}$, with e_{tot} being the total number of dwelling zones in the site. Furthermore, let $\mathcal{G}_e = \{g_{e,1}, g_{e,2}, \dots, g_{e,y}\}$ denote the set of vehicles that utilize the dwelling zone DZ_e. The collision avoidance constraints for the e-th dwelling zone are thus stated as

$$t_i^{k_i} + \Delta t \leq t_{i+1}(p_i^{k_i} - p_i^{\text{in}} + p_{i+1}^{\text{in}} - c), \quad k_i^{\text{in}} \leq k_i < k_i^{\text{DZ}} \tag{3.16a}$$

$$v_i^{k_i} = \underline{v}_i, \quad k_i = k_i^{\text{DZ}} \tag{3.16b}$$

$$t_i^{k_i} = t_i^{k_i} + t_i^{\text{absorb}}, \quad k_i = k_i^{\text{DZ}} + 1 \tag{3.16c}$$

$$t_i^{k_i} + \Delta t \leq t_{i+1}(p_i^{k_i} - p_i^{\text{in}} + p_{i+1}^{\text{in}} - c), \quad k_i^{\text{DZ}} + 1 \leq k_i \leq k_i^{\text{out}} \tag{3.16d}$$

or

$$t_{i+1}^{k_{i+1}} + \Delta t \leq t_i(p_{i+1}^{k_{i+1}} - p_{i+1}^{\text{in}} + p_i^{\text{in}} - c), \quad k_{i+1}^{\text{in}} \leq k_{i+1} < k_{i+1}^{\text{DZ}} \tag{3.16e}$$

$$v_{i+1}^{k_{i+1}} = \underline{v}_{i+1}, \quad k_{i+1} = k_{i+1}^{\text{DZ}} \tag{3.16f}$$

$$t_{i+1}^{k_{i+1}} = t_{i+1}^{k_{i+1}} + t_i^{\text{absorb}}, \quad k_{i+1} = k_{i+1}^{\text{DZ}} + 1 \tag{3.16g}$$

$$t_{i+1}^{k_{i+1}} + \Delta t \leq t_i(p_{i+1}^{k_{i+1}} - p_{i+1}^{\text{in}} + p_i^{\text{in}} - c), \quad k_{i+1}^{\text{DZ}} + 1 \leq k_{i+1} \leq k_{i+1}^{\text{out}}, \tag{3.16h}$$

$$i \in \mathbb{I}_{[1, |\mathcal{G}_e| - 1]},$$

where k_i^{DZ} is the index where the vehicle position is at the location of the charger or loading/unloading station and $\underline{v}_i$ is the lower bound on the velocity.

Remark 3. *The vehicles are required to make a full stop at the location of the charger or loading/unloading station, however, as noted by Remark 2, this is restricted when using a model in the spatial domain. This restriction imposes that the low-level controller must take the vehicles to a full stop and form a queue where the preceding vehicles wait behind the vehicle that utilizes the charger/loading/unloading station at a sufficient distance.*

MUTEX zones with HDVs

When an AV has a MUTEX zone with an HDV, it primarily needs to account for the variety of reactions and motion profiles the human-driven vehicle can have due to its uncertain system model (3.7), (3.8). We consider the two extreme cases of human driving, i.e., conservative and aggressive driving. From the HDV system model we can obtain upper and lower bounds for the entry and exit times, where the upper bound $(\bar{t}_j^{\text{in}}, \bar{t}_j^{\text{out}})$, i.e., higher entry and exit times, corresponds to a conservative driver and the lower entry and exit times $(\underline{t}_j^{\text{in}}, \underline{t}_j^{\text{out}})$ correspond to an aggressive driver.

The AV safety constraints must consider the entry and exit times range. Consequently, we can reformulate the constraints in (3.13) as

$$t_i^{\text{out}} \leq \underline{t}_{i+1}^{\text{in}} \text{ or } \bar{t}_{i+1}^{\text{out}} \leq t_i^{\text{in}}, \quad i \in \mathbb{I}_{[1,|\mathcal{Q}_r|-1]}. \tag{3.17}$$

and (3.14) as

$$t_i^{k_i} + \Delta t \leq \underline{t}_{i+1}(p_i^{k_i} - p_i^{\text{in}} + p_{i+1}^{\text{in}} - c), \quad k_i^{\text{in}} \leq k_i \leq k_i^{\text{out}} \tag{3.18a}$$

$$\text{or}$$

$$\bar{t}_{i+1}^{k_{i+1}} + \Delta t \leq t_i(p_{i+1}^{k_{i+1}} - p_{i+1}^{\text{in}} + p_i^{\text{in}} - c), \quad k_{i+1}^{\text{in}} \leq k_{i+1} \leq k_{i+1}^{\text{out}} \tag{3.18b}$$

$$i \in \mathbb{I}_{[1,|\mathcal{Z}_w|-1]},$$

and (3.16) as

$$\begin{aligned}
t_i^{k_i} + \Delta t &\leq \underline{t}_{i+1}(p_i^{k_i} - p_i^{\text{in}} + p_{i+1}^{\text{in}} - c), \quad k_i^{\text{in}} \leq k_i < k_i^{\text{DZ}} && \text{(3.19a)}\\
v_i^{k_i} &= \underline{v}_i, \quad k_i = k_i^{\text{DZ}} && \text{(3.19b)}\\
t_i^{k_i} &= t_i^{k_i} + t_i^{\text{absorb}}, \quad k_i = k_i^{\text{DZ}} + 1 && \text{(3.19c)}\\
t_i^{k_i} + \Delta t &\leq \underline{t}_{i+1}(p_i^{k_i} - p_i^{\text{in}} + p_{i+1}^{\text{in}} - c), \quad k_i^{\text{DZ}} + 1 \leq k_i \leq k_i^{\text{out}} && \text{(3.19d)}\\
&\text{or}\\
\bar{t}_{i+1}^{k_{i+1}} + \Delta t &\leq t_i(p_{i+1}^{k_{i+1}} - p_{i+1}^{\text{in}} + p_i^{\text{in}} - c), \quad k_{i+1}^{\text{in}} \leq k_{i+1} < k_{i+1}^{\text{DZ}} && \text{(3.19e)}\\
v_{i+1}^{k_{i+1}} &= \underline{v}_{i+1}, \quad k_{i+1} = k_{i+1}^{\text{DZ}} && \text{(3.19f)}\\
t_{i+1}^{k_{i+1}} &= t_{i+1}^{k_{i+1}} + t_i^{\text{absorb}}, \quad k_{i+1} = k_{i+1}^{\text{DZ}} + 1 && \text{(3.19g)}\\
\bar{t}_{i+1}^{k_{i+1}} + \Delta t &\leq t_i(p_{i+1}^{k_{i+1}} - p_{i+1}^{\text{in}} + p_i^{\text{in}} - c), \quad k_{i+1}^{\text{DZ}} + 1 \leq k_{i+1} \leq k_{i+1}^{\text{out}}, && \text{(3.19h)}
\end{aligned}$$

$$i \in \mathbb{I}_{[1,|\mathcal{G}_e|-1]},$$

where the vehicle with index $i+1$ in (3.17), (3.18) and (3.19) corresponds to an HDV.

Remark 4. *Note that each MUTEX zone implies a crossing order that in this paper is denoted as* $\mathcal{O}^{\mathcal{I}}$, $\mathcal{O}^{\mathcal{MS}}$, $\mathcal{O}^{\mathcal{DZ}}$, *where* $\mathcal{O}^{\mathcal{I}} = \left\{\mathcal{O}_1^{\mathcal{I}}, \ldots, \mathcal{O}_{r_{\text{tot}}}^{\mathcal{I}}\right\}$ *is the collection of all crossing order sets for all intersections and narrow roads and* $\mathcal{O}^{\mathcal{MS}} = \left\{\mathcal{O}_1^{\mathcal{MS}}, \ldots, \mathcal{O}_{w_{\text{tot}}}^{\mathcal{MS}}\right\}$ *is the collection of all crossing order sets for the merge-split zones, and* $\mathcal{O}^{\mathcal{DZ}} = \left\{\mathcal{O}_1^{\mathcal{DZ}}, \ldots, \mathcal{O}_{e_{\text{tot}}}^{\mathcal{DZ}}\right\}$ *collects all crossing order sets for the dwelling zones.*

3.3 General Optimal Vehicle Coordination Problem

With the defined motion models and constraints, we can now assemble the vehicle coordination problem as an OCP. The optimal coordination of N_{AV} fully automated vehicles in confined sites is the solution to the following OCP, given the initial state $\mathcal{X}_0 = \{x_{1,0}, ..., x_{N_{\text{AV}},0}\}$

$$\min_{x_i, u_i, \mathcal{O}^{\mathcal{I}}, \mathcal{O}^{\mathcal{MS}}, \mathcal{O}^{\mathcal{DZ}}} \sum_{i=1}^{N_{\text{AV}}} J_i\left(x_i, u_i\right) \tag{3.20a}$$

$$\text{s.t.} \quad \text{initial states } \; x_{i,0} = \hat{x}_{i,0}, \forall i \tag{3.20b}$$

$$\text{system dynamics } \; (3.4), (3.5)\ \forall i, \tag{3.20c}$$

$$\text{state and input constraints } \; (3.6),\ \forall i, \tag{3.20d}$$

$$\text{safety constraints with AVs } \; (3.13), (3.14), (3.16)\ \forall i, \tag{3.20e}$$

$$\text{safety constraints with HDVs } \; (3.17), (3.18), (3.19)\ \forall i. \tag{3.20f}$$

The solution of (3.20) provides the state and input trajectories $\mathcal{X}^* = \left\{x_1^*, ..., x_{N_{\text{AV}}}^*\right\}$, $\mathcal{U}^* = \left\{u_1^*, ..., u_{N_{\text{AV}}}^*\right\}$.

Remark 5. *The cost function of the OCP* (3.20) *can take different forms and could depend on the vehicle type and the goal of the specific site. The framework itself is not constrained by a specific choice of a cost function. For coordination problems, some common choices are minimizing the deviation from the reference velocity and minimizing end time, both of which are related to productivity. Furthermore, for electric vehicles, it is beneficial to improve* *energy efficiency by minimizing the consumed energy. Minimizing the squares of the vehicle's acceleration and jerk is typically related to improved comfort and reduction of component wear.*

Remark 6. *Note in particular that this involves finding the crossing orders* $\mathcal{O}^{\mathcal{I}}$, $\mathcal{O}^{\mathcal{MS}}$, $\mathcal{O}^{\mathcal{DZ}}$ *which makes the problem combinatorial and expensive to solve. The reason is that different crossing orders correspond to different state and input trajectories. The solution space rapidly grows with the number of vehicles and MUTEX zones.*

Remark 7. *OCP* (3.20) *can be stated as a mixed integer nonlinear program, where the crossing order corresponds to the "integer part" and the state and control trajectories correspond to the "NLP part".*

3.4 A two-stage optimization-based heuristic

Due to the combinatorial nature of the OCP (3.20), finding a solution can, in the worst case, require a full exploration of the solution space. In most practical cases, directly solving (3.20) is not a reasonable, viable option. Therefore, a common approach is to rely on heuristics to decompose and approximate the problem.

The heuristic approach taken in this thesis is to split the OCP into two stages, wherein the first stage we obtain the crossing order sets, followed by solving an NLP for the state and input trajectories, using the obtained crossing order. The heuristic utilizes the formulation of OCP (3.20) as a MINLP. We can, therefore, state the OCP (3.20) in the general form as

$$\min_{\mathcal{W},b} \quad J(\mathcal{W}) \tag{3.21a}$$

$$\text{s.t.} \quad g(\mathcal{W}) = 0 \tag{3.21b}$$

$$h(\mathcal{W}) \leq 0 \tag{3.21c}$$

$$c(\mathcal{W}, b) \leq 0, \tag{3.21d}$$

where $\mathcal{W} = \{\mathcal{X}, \mathcal{U}\}$ is a set containing the state and input variables, $J(\mathcal{W}) = \sum_{i=1}^{N_{\mathrm{AV}}} J_i(w_i)$, $g(\mathcal{W}), h(\mathcal{W})$ gather all equality and inequality constraints, and $c(\mathcal{W}, b) = c_w(\mathcal{W}) + Cb$ are the integer constraints for the combinatorial part of the problem with C being a matrix that captures the influence of the integer variables.

Practical reformulation of the collision avoidance conditions

A common way to handle constraints such as (3.13), (3.14), (3.16), (3.17), (3.18) and (3.19) is to introduce auxiliary binary variables and use the "big-M" technique [59]. For example, an equivalent representation to the constraint (3.13), is

$$t_i^{\text{out}} - t_{i+1}^{\text{in}} \leq b_{i,i+1} M, \tag{3.22a}$$

$$t_{i+1}^{\text{out}} - t_i^{\text{in}} \leq (1 - b_{i,i+1}) M. \tag{3.22b}$$

where $b_{i,i+1} \in \{0,1\}$, $i \in \mathbb{I}_{[1,|\mathcal{Q}_r|-1]}$ and M a sufficiently large positive number. In the case where $b_{i,i+1} = 0$, the vehicle $i+1$ is constrained to cross the MUTEX zone after the vehicle i, with the opposite being true if $b_{i,i+1} = 1$. We collect all integer variables for all MUTEX zones in $b_{\text{MUTEX}} \in \mathbb{Z}_2^{r_{\text{tot}}+w_{\text{tot}}+e_{\text{tot}}}$.

MIQP-based crossing order heuristic

One approach of obtaining the crossing order sets is through an approximation of the MINLP (3.21). In this thesis, we propose a *Mixed Integer Quadratic Program* that is assembled as a quadratic approximation of (3.21). The way the quadratic approximation is formed is similar to how the QP sub-problems are formed in *Sequential Quadratic Programming* methods [54]. In essence, we can reformulate (3.21) as

$$\begin{aligned}
\min_{\Delta\mathcal{W}, b} \quad & \frac{1}{2} \begin{bmatrix} \Delta\mathcal{W} \\ b \end{bmatrix}^T \mathbf{H}(\mathcal{W}, \lambda, \mu) \begin{bmatrix} \Delta\mathcal{W} \\ b \end{bmatrix} + \\
& \nabla_{\mathcal{W}} J(\mathcal{W})^T \begin{bmatrix} \Delta\mathcal{W} \\ b \end{bmatrix} + J(\mathcal{W}^{**}) && \text{(3.23a)} \\
\text{s.t.} \quad & g(\mathcal{W}^{**}) + \nabla_{\mathcal{W}} g(\mathcal{W}^{**})^T \begin{bmatrix} \Delta\mathcal{W} \\ b \end{bmatrix} = 0 && \text{(3.23b)} \\
& h(\mathcal{W}^{**}) + \nabla_{\mathcal{W}} h(\mathcal{W}^{**})^T \begin{bmatrix} \Delta\mathcal{W} \\ b \end{bmatrix} \leq 0 && \text{(3.23c)} \\
& c_w(\mathcal{W}^{**}) + \nabla_{\mathcal{W}} c_w(\mathcal{W}^{**})^T \begin{bmatrix} \Delta\mathcal{W} \\ b \end{bmatrix} + Cb \leq 0, && \text{(3.23d)}
\end{aligned}$$

where $\mathbf{H}(\mathcal{W}, \lambda, \mu) = \text{blkdiag}\left(\{H_i\}_{i=1}^{N_{\text{AV}}}, \mathbf{0}_{n_{\text{tot}}, n_{\text{tot}}}\right)$ is a block diagonal matrix with positive definite $H_i(w_i, \lambda_i, \mu_i) = \nabla_{w_i}^2 \mathcal{L}(w_i, \lambda_i, \mu_i) = \nabla_{w_i}^2 J_i(w_i) -$

$\nabla^2_{w_i} \lambda_i^T g(w_i) - \nabla^2_{w_i} \mu_i^T h(w_i)$, where λ_i, μ_i are the dual variables, $b = [b_{\text{MUTEX}};\ b_{\text{HDV}}]$ is a vector comprised of binary variables related to the safety constraints and binary variables related to if a car-following model should be used for the HDV motion profile, and $\mathbf{0}_{n_{\text{tot}},n_{\text{tot}}}$ zeros of appropriate size for the integer variables, $n_{\text{tot}} = r_{\text{tot}} + w_{\text{tot}} + e_{\text{tot}} + hdv_{\text{tot}}$ being the total amount of MUTEX zones and total amount of binary variables used for the HDVs, and $\Delta\mathcal{W} = \mathcal{W} - \mathcal{W}^{**}$, with a solution guess $\mathcal{W}^{**}$. The MIQP problem (3.23) can be compactly written as

$$\min_{\mathcal{W},b} \ \frac{1}{2} \begin{bmatrix} \mathcal{W} \\ b \end{bmatrix}^T \mathbf{H} \begin{bmatrix} \mathcal{W} \\ b \end{bmatrix} + \mathbf{J}^T \begin{bmatrix} \mathcal{W} \\ b \end{bmatrix} + \alpha \tag{3.24a}$$

$$\text{s.t.} \quad A_{\text{eq}} \begin{bmatrix} \mathcal{W} \\ b \end{bmatrix} = b_{\text{eq}} \tag{3.24b}$$

$$A_{\text{ineq}} \begin{bmatrix} \mathcal{W} \\ b \end{bmatrix} \leq b_{\text{ineq}}, \tag{3.24c}$$

where $\mathbf{J}$ now contains all the first order terms, α contains the linear terms and where the constraints (3.23b)-(3.23d) are grouped into equality constraints $A_{\text{eq}}, b_{\text{eq}}$ and inequality constraints $A_{\text{ineq}}, b_{\text{ineq}}$, respectively. The solution to the MIQP problem provides crossing orders $\hat{\mathcal{O}}^{\mathcal{I}}$, $\hat{\mathcal{O}}^{\mathcal{MS}}$, $\hat{\mathcal{O}}^{\mathcal{DZ}}$ that is obtained from the values of the integer variables b and are optimal for the approximated problem.

Remark 8. *The crossing orders obtained from the MIQP* (3.24) *are optimal solutions for the given problem. However, as the MIQP is an approximation of the MINLP, the crossing orders resulting from the MIQP could be sub-optimal for the MINLP.*

As mentioned, the HDV switches to a car-following model if there is an AV ahead of it in the merge-split zones. In the MIQP (3.24), the relation captured with the binary variables determines if this condition is satisfied and if the car-following model should be used. The car-following model is activated if any of the merge-split constraints are violated by the uncertain free flow HDV model, i.e., if the time-headway between the lead AV and the following HDV is less than a defined threshold. When these conditions are fulfilled, the HDV should switch to a car-following model in the merge-split zone.

Remark 9. *We make the simplification that the dual variables* (λ_i, μ_i) *are*

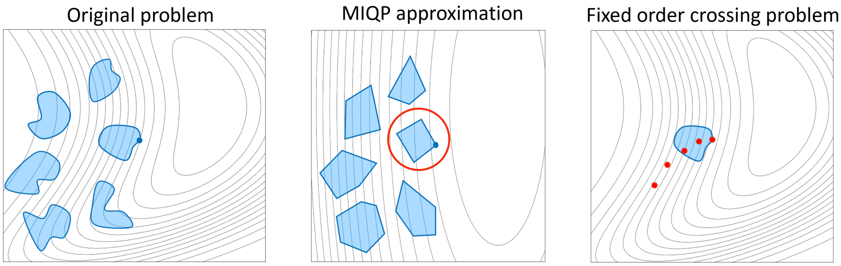


Figure 3.4: Illustration of the two-stage heuristic used for the vehicle coordination problem.

equal to zero. This results in that H_i only includes the second order expansion of the cost function, i.e., $H_i(w_i) = \nabla^2_{w_i} J_i(w_i)$.

Remark 10. *In practice, there is no restriction on the solution guess $\mathcal{W}^{**}$ as long as it is a feasible solution. A solid solution guess can be obtained, for example, by solving the optimization problem* (3.20) *without safety constraints* (3.20e), (3.20f), *or through a forward simulation of the vehicles with, for example, a simple feedback controller. It is also important to note that the heuristic is more sensitive to poor solution guesses if the vehicles have limited control authority, for example, if the vehicles are in close proximity to a MUTEX zone.*

Fixed-order NLP

With the obtained crossing order sets from the MIQP (3.24), the OCP (3.20) is reduced to an NLP since all other integer solutions are removed. Obtaining the optimal state and control trajectories is thus found through solving the *fixed-order coordination problem*

$$\min_{x_i, u_i} \quad J(\mathcal{W}) \tag{3.25a}$$

$$\text{s.t} \quad (3.20\text{b}) - (3.20\text{f}), \ \forall i \tag{3.25b}$$

$$b = b^*. \tag{3.25c}$$

Furthermore, using the values of the binary variables b_{HDV}, it can be determined if, in a merge-split MUTEX zone, there are conditions for the HDV to switch to the car-following model. In the cases where those conditions are satisfied, the motion of the HDV during the merge-split zone in the fixed-order NLP is defined using the car-following model. This allows for the NLP to have more accurate estimates of the HDV motion and to utilize the partial controllability of the HDV by adjusting the motion of the lead AV.

The two-stage approximation approach that solves (3.20) is summarized in Algorithm 1 and the process is illustrated through Figure 3.4 and the flow chart in Figure 3.5.

Algorithm 1 Two-stage approximation algorithm

Input: $N_{\mathrm{AV}}, \mathcal{I}, \mathcal{Q}_r, \mathcal{MS}, \mathcal{Z}_w, \mathcal{DZ}, \mathcal{G}_e$, vehicle paths
Output: $\mathcal{X}^*$, $\mathcal{U}^*$

1: $\forall i$: Obtain a solution guess by, e.g., solving NLP (3.20) w/o the safety constraints (3.20e), (3.20f).
2: Calculate and form the terms $\mathbf{H}, \mathbf{J}, \alpha$.
3: Solve the MIQP (3.24) to obtain b^*.
4: Solve the fixed-order NLP (3.25) using $b = b^*$ to obtain $\mathcal{X}^*$, $\mathcal{U}^*$.

3.5 Receding horizon control

In a practical setting, the available measurements are typically uncertain, there are additional external disturbances, and the motion model that is used is not exact to the real process. For that reason, we need to introduce feedback to the control framework. A closed-loop formulation of the coordination problem can be achieved by having a low-level controller that tracks the high-level motion plan in a receding horizon fashion. The low-level controller in this thesis is set up as a *Model Predictive Controller* that tracks the motion plan in time and sends the actuation signals to the vehicles. A brief introduction is given in Section 2.1. We assume that the AVs and HDVs are equipped with communication equipment, inertial measurement units, and RTK-GNSS receivers. Figure 3.6 illustrates the proposed control structure.

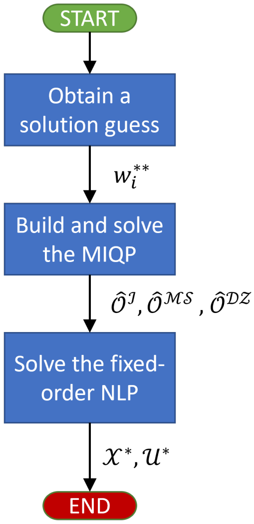


Figure 3.5: Flow chart illustrating the two-stage approach.

Robustness aspects

The high-level control formulation requires that the vehicle is at a prescribed position at a given time. However, satisfaction of such constraints in practice is difficult due to the presence of perturbations, such as measurement errors and/or process noise. Consequently, the safety constraints are at risk of being violated, leading to undesired occupancy of the MUTEX zones. Robust coordination could be achieved through constraint tightening in the high-level controller, similarly as done in [25]. In particular, we can replace constraints (3.13) with

$$t_i^{\text{out}} \leq t_{i+1}^{\text{in}} + \sigma_{r,1} \quad \text{or} \quad t_{i+1}^{\text{out}} \leq t_i^{\text{in}} + \sigma_{r,2}, \quad i \in \mathbb{I}_{[1,|\mathcal{Q}_r|-1]}, \tag{3.26}$$

where $\sigma_{r,1} \geq 0$ and $\sigma_{r,2} \geq 0$ are slack variables.

Recomputation algorithm

The high-level optimization-based approach is invoked at the beginning of the mission and when the plan needs to be recalculated due to changes, such as

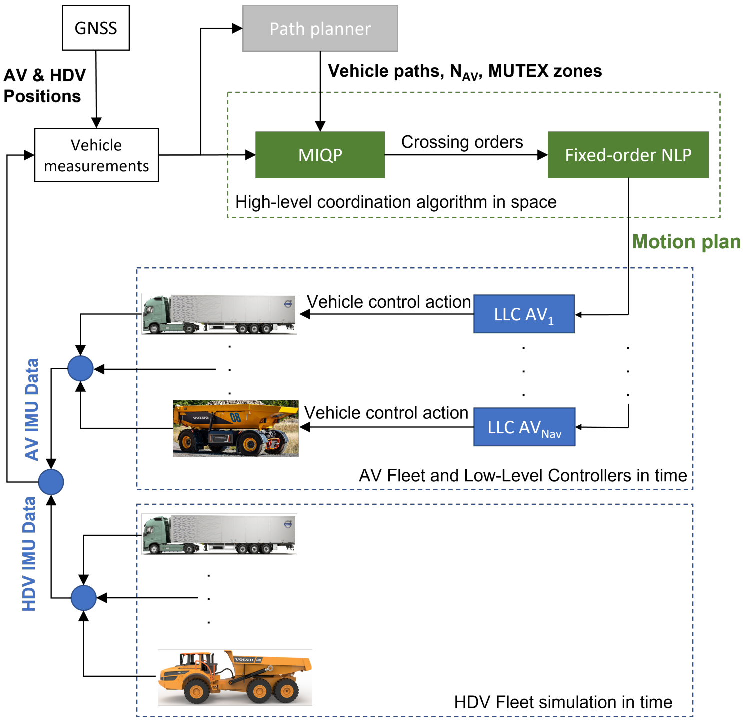


Figure 3.6: Schematic illustration of the proposed control structure. The low-level controller (LLC) translates the motion plan to appropriate vehicle control actions.

vehicles deviating from the plan, a certain amount of time has elapsed, or a new vehicle entering the confined area. A new vehicle can, for example, enter the confined site optimization problem when more transportation resources are required to accomplish the productivity goal. When the recomputation is triggered, the initial step involves identifying environmental changes since the last high-level algorithm iteration. This encompasses tracking zones traversed by vehicles and identifying any new MUTEX zones. The responsibility for this lies with the *Path planner* component, which is beyond the scope of this thesis.

Remark 11. *The HDV can have a large disparity between the entry and exit* *times due to the uncertain motion profile. However, through receding-horizon optimization, the range of occupancy time is reduced with each recomputation as the vehicle moves closer to the zone.*

The high-level controller receives the number of vehicles, their paths, and the MUTEX zones as input from the *Path planner* component. Each recomputation yields a potential new set of crossing orders for vehicles. There exists a potential scenario where a crossing order is issued for a zone that vehicles cannot adhere to, especially when one or more vehicles are close to the zone. This situation can lead to an infeasible NLP problem as the control ability of the vehicles is limited. To address this, if a vehicle is positioned at a predefined distance before the MUTEX zone, the crossing order for that vehicle in that zone remains fixed. Consequently, the vehicle is excluded from the MIQP problem for that zone. Nonetheless, the safety constraints for other interacting vehicles are maintained at the fixed-order NLP level. This approach allows the occupancy order of other interacting vehicles to adapt if needed while ensuring compliance with safety constraints involving the vehicles deemed too close to the zone.

3.6 Summary

In this chapter, we introduced the system models and vehicle and MUTEX zone constraints that define the vehicle coordination problem. Using the models and constraints, we introduced an optimal control formulation of the vehicle coordination problem. The proposed OCP formulation is general, meaning that it does not depend on a specific choice of objective function, vehicle models, and constraints. We also introduced a two-stage optimization-based heuristic that is used to solve the optimal vehicle coordination problem in a practical setting. Furthermore, we stated a closed-loop formulation of the problem through a receding horizon approach that tracks the state trajectories resulting from the vehicle coordination problem. The appended papers elaborate on the two-stage heuristic, its use in mixed-traffic vehicle coordination, and a decomposition strategy for the heuristic that can be used as a tunable trade-off between solution optimality and computational demand.

CHAPTER 4

Summary of included papers

This chapter provides a summary of the included papers. The thesis author was responsible for developing the problem formulation in collaboration with the co-authors, implementing the algorithms, and authoring the papers. The co-authors contributed with support to the analysis of the results and the writing process.

4.1 Paper A

Stefan Kojchev, Robert Hult, Jonas Fredriksson, Maximilian Kneissl
A Two-Stage MIQP-Based Optimization Approach for Coordinating Automated Electric Vehicles in Confined Sites
IEEE Transactions on Intelligent Transportation Systems (2023)
©845 IEEE DOI: 10.1109/TITS.2023.3320168 .

This paper proposes a high-level optimization-based approach for the coordination of automated vehicles in confined sites. The paper motivates why the vehicle coordination problem is suitable to be formulated as an optimization problem. However, the stated optimization problem is difficult to solve due to its combinatorial nature that is related to the utilization of the site's mutually exclussive zones. To address this issue, we propose a two-stage optimization-based heuristic. The first stage of the approach consists of defining and solving a mixed integer quadratic program that obtains the occupancy orders for all MUTEX zones between the vehicles. The subsequent component utilizes the obtained occupancy orders and computes the optimal state and input trajectories for the AVs by solving a nonlinear program. The paper focuses on electric vehicles prioritizing energy efficiency. Furthermore, we include charging MUTEX zones and the charging process into the optimization algorithm. We demonstrate the efficacy of our approach through simulation and compare it with alternative optimization-based heuristics.

Research Objective: This work addressed **RQ1** and proposed a method not bounded by a specific choice of model, constraints, and objective function. The analysis focused on electric vehicles and the necessity to include the charging process in the vehicle coordination problem. The method is a base for the extensions proposed in Paper B and Paper C

4.2 Paper B

Stefan Kojchev, Robert Hult, Maximilian Kneissl, Jonas Fredriksson
A Computational Decomposition Strategy for Optimization-Based Coordination of Automated Vehicles in Confined Sites
Under a second review in IEEE Transactions on Intelligent Transportation Systems (2023) .

In this paper, we present a computation decomposition approach for the optimization-based heuristic presented in Paper A. The method reduces the computational demand of the second step of the heuristic by decomposing the nonlinear program into multiple smaller, parallelly solvable, NLPs. Leveraging graph theory, the approach models the connections between the vehicles through the mutually exclusive zones that they share. Utilizing the vehicle positions and the confined site road network the method identifies non-significant MUTEX relations, i.e., MUTEX zones where there cannot be a conflict between the vehicles, and identifies independent subproblems. We show that the results from our proposed method are equivalent to those obtained by solving the non-decomposed NLP. Furthermore, by utilizing the dual variables connected to the MUTEX constraints, our approach can further subdivide the problem. Specifically, the method can be used to balance optimality and computational effort. We demonstrate how this method can be applied both for creating initial motion plans and for updating existing ones. Simulation examples highlight the computational advantages of our method compared to the non-decomposed problem.

Research Objective: This work aimed to utilize the confined site application to reduce the computational load associated with solving the vehicle coordination problem. Specifically, it focused **RQ2** and developed a strategy to manage dynamic situations, such as the introduction of a new vehicle into the coordination problem.

4.3 Paper C

Stefan Kojchev, Robert Hult, Maximilian Kneissl, Jonas Fredriksson
Optimization-Based Coordination of Automated and Human-Driven Vehicles in Confined Sites
Submitted to IEEE Special Issue on Transactions on Control Systems Technology (2023) .

This paper builds on the two-stage optimization-based heuristic from Paper A by adding human-driven vehicles and applying the approach in a closed-loop receding horizon framework. We propose a model for HDVs that accounts for different driving styles and uncertainties. Furthermore, the method captures the possibility for the HDV to operate in car-following mode, in particular when there is a lead vehicle in the merge-split MUTEX zones. This situation offers an opportunity for the framework to influence the HDV's motion through the lead AV. In particular, the algorithm could control the HDV motion for the benefit of the overall AV fleet. Additionally, the HDVs exhibit more predictable motion in car-following mode, reducing uncertainty about occupancy times in upcoming MUTEX. Closed-loop control is achieved through a receding-horizon approach that tracks the high-level motion plan. We demonstrate the efficacy of our methodology through simulation scenarios. These simulations show our system's ability to exert partial control over HDVs and dynamically adjust the coordination strategy based on observed HDV motion profiles. This adaptability underlines the potential of our framework to improve the integration and management of mixed-traffic environments involving both HDVs and AVs.

Research Objective: This study aims to integrate human-driven vehicles into the coordination problem by modifying the methodology presented in Paper A. The primary focus is on addressing **RQ3** and on developing a receding horizon closed-loop formulation.

CHAPTER 5

Conclusion

In this chapter, the thesis is concluded by addressing the research goals and possible directions for future research.

Concluding remarks

This thesis presented an optimization-based framework for coordinating automated vehicles in confined environments. The framework leverages the confined site requirements and the presence of a communication and computation center. Consequently, the coordination problem is formed and computed centrally for all vehicles. However, this results in a complex and computationally challenging problem, particularly due to the combinatorial aspects related to occupancy constraints in MUTEX zones. To address this, we propose a two-stage optimization-based heuristic where the combinatorial part of the problem is considered by solving an MIQP formed as an approximation of the coordination problem. The second stage uses an NLP to determine optimal vehicle states and inputs, respecting the established occupancy order. Importantly, the proposed framework is not dependent or bounded by a specific choice of confined site layout, models, constraints, or objective functions and can be adapted to various confined site layouts and conditions.

Additionally, the heuristic is adapted for mixed-traffic situations with human-driven vehicles. It incorporates a vehicle model to estimate the motion of the HDV, accounting for driver reaction uncertainty. The framework effectively utilizes these estimates to organize occupancy orders and influence HDV behavior, particularly when an automated vehicle leads an HDV. In such scenarios, a car-following model is employed, wherein the HDV motion responds to the leading AV. We applied the optimization algorithms to closed-loop control through MPC, showcasing the framework's ability to dynamically adjust occupancy orders and motion profiles in response to the observed conditions and estimation.

However, the computational demand of the framework remains a significant challenge, especially for long optimization horizons. In particular, the centralized NLP that computes the motion plan for all automated vehicles is the predominant component in the computational demand of the approach. This thesis addresses this challenge by proposing a decomposition strategy that segments the centralized NLP into smaller, parallel-solvable NLPs. The decomposition strategy is based on a graph-theoretical interpretation of the coordination problem and utilizes the dual variables related to the MUTEX constraints as graph weights. The approach can be used to determine the trade-off between optimality and computation since as the problem is increasingly decomposed, there is a corresponding decrease in the level of optimality achieved.

Future work

In this section, several possible directions for future research are presented.

Safety guarantees

The presented work incorporates safety constraints and ensures inter-vehicle collisions and undesired MUTEX zone occupancy, even in the presence of measurement and sensor perturbances. Nevertheless, this approach depends on models and assumptions that only partially reflect actual conditions. For the presented algorithm and others similar to it to be implemented and delivered to customers, they must consistently adhere to safety constraints and provide guarantees. This issue is commonly addressed by integrating an additional component that monitors and, if required, overrides the decisions and actions of the automated system. One proposed framework and approach is presented in [60], which proposes a safety monitoring strategy utilizing backward reachability. An alternative approach involves forward reachability, ensuring collision-free vehicle states by considering the projected movements of traffic participants and environmental uncertainties [61]–[63]. Control Barrier Functions (CBFs), as detailed in [64]–[66], are a prevalent method for ensuring the safety of automated functions. However, the application of reachability-based methods is constrained by their computational demands, while the challenge for CBF-based methods lies in calculating barrier functions for general classes of control systems.

Improved prediction of the HDV motion

In Paper C, a model for predicting human-driven vehicle motion in free flow is presented. The presented model could be further extended by including the driver reaction delay, for example as done in [67]. Additionally, alternative methodologies for HDV motion prediction exist, with a comprehensive review available in [68]. Recent advancements in learning-based techniques, as exemplified in [69], [70], could also be employed for more accurate HDV action estimation. Furthermore, in Paper C, the AVs robustly account for the uncertain motion of HDVs, potentially leading to a more conservative motion profile for the AVs. Alternatives to this approach include the scenario-based method outlined in [71] or a stochastic model that incorporates fail-safe trajectories, as suggested in [72].

Uncertain HDV path

In Paper C, it is assumed that the path of the HDV is predetermined. While this assumption is viable for scenarios involving confined sites, situations, where the HDV's path is not explicitly known, are also relevant. Particularly in environments with multiple road splits (forked roads), the HDV might choose any available route. In such instances, the AVs must initially account for all potential MUTEX zones that could be shared with the HDV. Through successive recomputations of the framework, the path of the HDV becomes more certain. The AVs can adapt to the HDV path by eliminating now-irrelevant MUTEX zones.

Experimental implementation and testing

A logical extension of the presented framework is its implementation and testing in real-world experimental setups. With ongoing advancements in optimization solvers [73], the availability of a computational and communication center on the site, and improved onboard computing capabilities, solving the optimization problem within real-time computational constraints is becoming more feasible. However, challenges that in this thesis have been omitted, such as communication and actuation delays, could present a difficulty in the implementation of the framework [74]. Additionally, further practical improvements are required especially on the computational demand of the NLP. Implementing these improvements would allow for the implementation and assessment of the algorithms from this thesis in practical test scenarios, potentially leading to their future integration into commercial systems, and contributing to their safety and energy efficiency.

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