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# Integrated Communications and Localization for Massive MIMO LEO Satellite Systems

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## Abstract

Integrated communications and localization (ICAL) will play an important part in future sixth generation (6G) networks for the realization of Internet of Everything (IoE), e.g., cooperative intelligent transportation systems and connected automated vehicles, to support both global communications and seamless localization. Massive multiple-input multiple-output (MIMO) low earth orbit (LEO) satellite systems have great potential in providing wide coverage with enhanced gains, and thus are strong candidates for realizing ubiquitous ICAL. In this paper, we investigate the wideband massive MIMO LEO satellite system to simultaneously support the wireless communications and localization operations in the downlink. In particular, we first characterize the signal propagation properties and derive the localization performance bound. Based on these analyses, we focus on the hybrid analog/digital precoding design to achieve high communication capability and localization precision. Numerical results demonstrate that the proposed ICAL scheme supports both the wireless communication and localization operations for typical system setups.

## Index Terms

Integrated communications and localization, 6G, non-geostationary satellite, LEO satellite, massive MIMO, hybrid precoding, Fisher information matrix, squared position error bound.

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## I. INTRODUCTION

The fifth generation (5G) wireless networks have already been in the process of global deployment and standardization formulation of the related capabilities [2]. However, there still exist many requirements that 5G networks might not satisfy, and the sixth generation (6G) wireless networks are envisioned to provide seamless and ubiquitous coverage, enhanced communication capability and localization precision, and higher intelligence and security level, etc. [2]–[5]. One of the potential application scenarios of 6G networks is the integrated communications and localization (ICAL) on Internet of Everything (IoE), e.g., cooperative intelligent transportation systems and connected automated vehicles, where the signal waveform can be jointly designed for shared spectrum on one hardware platform, to simultaneously perform communications and localization, improving the utilization of resources [2], [5].

One of the common application cases for ICAL on IoE is the terrestrial network [6]–[8]. However, the ICAL performance of the terrestrial network is restricted in some areas where ground infrastructure is infeasible to deploy, or the signals easily get blocked [9]. In these scenarios, the satellite networks can significantly complement the terrestrial networks since they can provide larger coverage, and support higher-capacity communications and more flexible localization for the areas that terrestrial networks might have coverage issues. Thus, the satellite networks are expected to support global communications and seamless localization, and will play an essential part in performing ICAL for 6G networks [5], [10]–[12]. Generally, the satellite networks are divided into two categories, namely geostationary earth orbit (GEO) and non-GEO (NGEO) satellite networks [13]. The GEO satellite networks, including several global navigation satellite systems (GNSSs), e.g., global positioning system (GPS), GLONASS, and BEIDOU, are commonly capable of offering primary navigation with wide coverage [9]. However, the existing GNSSs are susceptible to harsh electromagnetic environment, and easily fail to catch up with the real-time requirements of the intelligent vehicles [9]. The NGEO satellite networks include several low earth orbit (LEO) satellites, which are usually deployed at altitudes of 200 – 2000 km [14], and can be launched with low cost and high flexibility [9]. Moreover, due to lower propagation delay and smaller path loss, the LEO satellite networks can provide better communication capability and localization precision compared with GEO counterparts [9], [15]. So far, several giant LEO satellite systems, e.g., OneWeb, SpaceX, have been launched by governments and corporations, and see a steady reduction in launch costs, which makes it possible

to develop global LEO satellite systems, and complement GNSS.

Massive multiple-input multiple-output (MIMO) can provide numerous degrees of freedom in both temporal and spatial domains [5]. Therefore, it has gained much attention for pure communications and localization, to improve the spectral efficiency (SE) and the precision of localization [15]–[17], which motivates us to adopt the massive MIMO technology for ubiquitous ICAL [5], [7], [18]. However, the implementation of fully digital transceivers in massive MIMO requires a large number of radio frequency (RF) chains, and might lead to high power consumption. Generally, this issue can be circumvented by developing a hybrid precoding strategy in signal transceivers design [19]. Recently, AST SpaceMobile has reported the successful deployment of the 693-square-foot MIMO array on its BlueWalker 3 LEO satellite [20]. Motivated by this, we combine the LEO satellite networks with the employment of massive MIMO in 6G, to support ICAL with the terrestrial user terminals (UTs) in the remote areas [4].

In this work, we propose to implement ICAL in the massive MIMO LEO satellite systems, to trade-off between the communication capability and the localization precision, which are evaluated by the SE and the squared position error bound (SPEB), respectively [5]. Though the precoding designs for the downlink of the ICAL systems have been already investigated in the terrestrial networks [6]–[8], those works can not be directly applied to LEO satellite systems, owing to different signal propagation properties. In particular, due to the long distance between the satellite and the UTs as well as the mobility of the transceivers, there exist a long propagation delay and large Doppler shifts in the considered scenario [15], [21]. Thus, the instantaneous channel state information (iCSI) between the satellite and the UTs is time-varying, which may be difficult to estimate, and moreover, the estimated iCSI might be outdated [22], which makes it challenging to use iCSI for downlink precoding in the considered massive MIMO LEO satellite ICAL system. Motivated by these characteristics, we investigate the precoding design based on the utilization of the statistical CSI (sCSI), which is relatively slow-varying.

Inspired by the above considerations, we propose a hybrid analog/digital transmitter to perform ubiquitous ICAL for wideband massive MIMO LEO satellite systems by exploiting sCSI. The main contributions of the paper are summarized as follows:

- We investigate the upper bound of the ergodic SE expression. Besides, we derive a closed-form Cramér-Rao lower bound (CRLB) for the channel parameters of the considered massive MIMO LEO satellite systems, based on which the SPEB is derived to measure the performance of the downlink localization.

- We investigate the hybrid precoders multi-objective optimization for the considered massive MIMO LEO satellite systems, to trade-off between the communication capability and the localization precision, based on the SE and the SPEB metrics, respectively.
- We develop an hybrid precoding strategy and jointly design the signal waveform based on sCSI, to simultaneously perform communications and localization, and guarantee good performance in terms of both the SE as well as the SPEB metrics, respectively.

#### A. Related Works

*Communications* – So far, the communications for the LEO satellite scenarios have been intensively investigated. In [15], the authors have formulated a massive MIMO communication scheme for both uplink and downlink of the LEO satellite systems based on the average signal-to-leakage-plus-noise ratio (ASLNR) and average signal-to-interference-plus-noise ratio (ASINR) maximization criteria, respectively. The downlink precoding designs for both fully digital and hybrid transmitters have been studied in [11], [22], [23], to maximize the downlink SE or the energy efficiency performance. In [24], the authors focused on the research of the uplink transmit design for the massive MIMO LEO satellite systems.

*Localization* – Wireless localization can be performed under single anchor or multiple anchors cases, both of which have been extensively investigated in the terrestrial systems. In [25], [26], the authors have presented theoretical analyses for multiple anchor localization. The CRLB for single anchor localization has been derived for both two-dimensional (2D) and three-dimensional (3D) scenarios in [16], [27]. In [28], the authors investigated the localization and orientation performance limits for the single anchor scenarios with massive MIMO transmission. In [29], the authors have studied the influence of the synchronization errors and the Doppler effects of the single anchor localization systems.

*ICAL* – The existing ICAL works mainly focus on the terrestrial scenarios. In [6], [7], the authors have designed the beamforming vectors to simultaneously perform communications and localization during data transmission, based on rate maximization, SPEB minimization, or the transmission power minimization criteria. Besides, localization can be also performed along with channel estimation during the pilot transmission. In [8], the authors have proposed a two-stage beamforming scheme, to minimize pilot overhead subject to localization precision constraints in the first stage, and seek for the maximization of the data rate in the second stage with the estimated CSI obtained from stage one.

## B. Organization

The remaining of this paper is organized as follows. Section II formulates the system model for the wideband massive MIMO LEO satellite ICAL system. The performance metrics for both communications and localization, i.e., SE and SPEB, are analyzed in Section III. An algorithmic framework is developed in Section IV to design the hybrid analog/digital precoders for the ICAL system enabling the trade-off between the communication and localization performance. Section V presents the simulation results and the paper is concluded briefly in Section VI.

## C. Notations

The matrices and vectors are denoted by upper and lower case boldface letters, respectively. We define the imaginary unit as  $j = \sqrt{-1}$ .  $\mathbb{C}^{m \times n}$  represents the  $m \times n$ -dimension unitary space. The left-hand side of  $\triangleq$  is defined by the right-hand side.  $\otimes$  denotes the Kronecker product.  $\exp\{\cdot\}$  and  $\log\{\cdot\}$  are the exponential and logarithmic operators, respectively.  $\mathbf{I}_N$  stands for  $N \times N$  identity matrix.  $(\cdot)^T$ ,  $(\cdot)^*$ , and  $(\cdot)^H$  represent the transpose, conjugate, and Hermitian conjugate operations, respectively.  $|x|$ ,  $\angle x$ ,  $\Re\{x\}$ , and  $\lceil x \rceil$  denote the amplitude, the angle, the real part, and the ceiling value of  $x$ , respectively. The circular symmetric complex-valued zero-mean Gaussian distribution with variance  $\sigma^2$  is given by  $\mathcal{CN}(0, \sigma^2)$ .  $\mathbb{E}\{\cdot\}$ ,  $\text{Tr}\{\cdot\}$ , and  $\text{blkdiag}\{\cdot\}$  represent the expectation, the trace, and the block diagonal operators.  $\text{rank}\{\mathbf{X}\}$  stands for the rank of the matrix  $\mathbf{X}$ .  $\|\cdot\|_2$  and  $\|\cdot\|_F$  denote the  $\ell_2$ -norm and Frobenius-norm, respectively. The  $(i, j)$ th element of the matrix  $\mathbf{A}$  is given by  $[\mathbf{A}]_{i,j}$ .  $\mathbf{A} \succeq \mathbf{B}$  refers to the positive semidefinite property of the matrix  $\mathbf{A} - \mathbf{B}$ .  $\partial$  denotes the partial derivative operation.

## II. SYSTEM MODEL

We propose to simultaneously perform communications and localization for the massive MIMO LEO satellite systems, as depicted in Fig. 1. The system is operated at carrier frequency  $f_c$  and the corresponding wavelength is given by  $\lambda_c = c/f_c$ , where  $c$  denotes the speed of the light. We assume  $K$  single-antenna UTs, at unknown position  $\mathbf{p}_k = [p_k^x, p_k^y, p_k^z]^T$  and velocity  $\dot{\mathbf{p}}_k = [\dot{p}_k^x, \dot{p}_k^y, \dot{p}_k^z]^T$ ,  $k = 1, \dots, K$ , are served by a single LEO satellite with known position  $\mathbf{q} = [q^x, q^y, q^z]^T$  and orientation angle  $\mathbf{o} = [\varphi_1, \varphi_2]^T$ <sup>1</sup> where  $\varphi_2$  and  $\varphi_1$  refer to the rotation

<sup>1</sup>The orientation angle can be obtained and pre-compensated by, e.g., programmed tracking, according to the predicted movement of the LEO satellite [30].

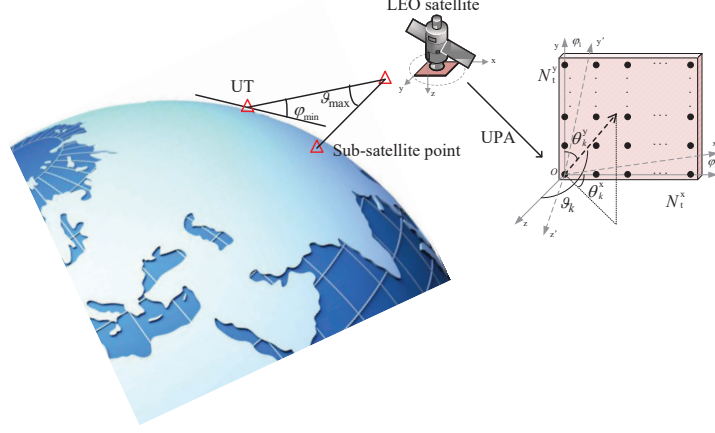


Fig. 1. 3D geometric model of the massive MIMO LEO satellite ICAL system with known satellite position and orientation, unknown UTs position.

around positive  $y$ -<sup>2</sup> and negative  $x'$ -axes, respectively, as depicted in Fig. 1. The positions and velocities of the UTs are assumed to be fixed over the observed interval and can be updated accordingly with large movements. A uniform planar array (UPA) of  $N_t = N_t^x N_t^y$  antennas with half-wavelength separation is applied at the transmitter of the LEO satellite, where  $N_t^x$  and  $N_t^y$  denote the number of antennas at the  $x$ - and  $y$ -axes, respectively. The satellite transmitter is supported by a hybrid precoder with  $N_{\text{rf}}$  ( $K \leq N_{\text{rf}} \leq N_t$ ) RF chains.

To mitigate the inter-symbol interference, the orthogonal frequency division multiplex (OFDM) modulation is employed for the downlink wideband transmission of the LEO satellite ICAL systems [15], [16]. We denote  $B_w$  and  $T_s = 1/(2B_w)$  as the system bandwidth and the sampling period, respectively. In particular, we assume each frame consists of  $M_s$  slots, and there are  $M_{\text{sp}}$  and  $M_{\text{sd}}$  OFDM symbols used for pilot and data transmission in each slot, as depicted in Fig. 2. Thus, in each frame,  $M_p = M_{\text{sp}}M_s$  and  $M_d = M_{\text{sd}}M_s$  OFDM symbols are transmitted through the pilot and data transmission, respectively. Then, we assume  $N_{\text{sc}}$  subcarriers are employed over the system bandwidth  $B_w$ , and the length of the cyclic prefix (CP) is set as  $N_{\text{cp}}$ . Thus, we denote  $f_s$  as the subcarrier separation and the frequency of the  $n$ th subcarrier is given by

$$f_n = \left( n - \frac{N_{\text{sc}} + 1}{2} \right) f_s, \quad n = 1, \dots, N_{\text{sc}}. \quad (1)$$

Subsequently, the OFDM symbol length with and without CP is given by  $T = N_{\text{sc}}T_s + N_{\text{cp}}T_s$

<sup>2</sup>After a rotation by  $\varphi_2$  around positive  $y$ -axis, the  $y$ -coordinate does not change, i.e.,  $y' = y$ , while the  $x$ - and  $z$ -coordinates vary as  $x' = z \sin \varphi_2 + x \cos \varphi_2$  and  $z' = z \cos \varphi_2 - x \sin \varphi_2$ , respectively.

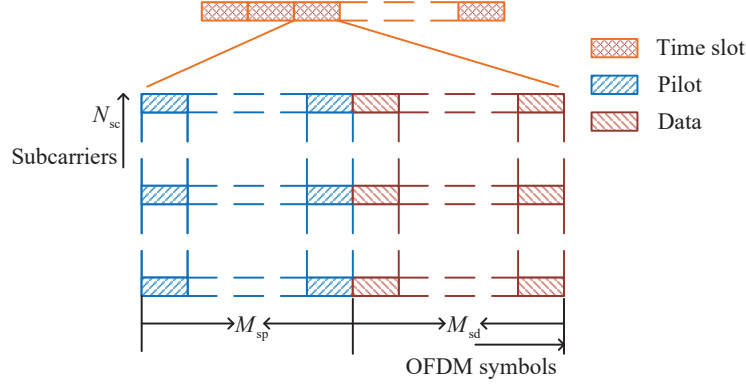


Fig. 2. The time-frequency structure for the transmitted pilot and data signals.

and  $T_{sc} = N_{sc}T_s$ , respectively. In the following, let the subscript and superscript  $g \in \{p, d\}$  represent the pilot and data transmission, respectively.

An ICAL transmission protocol is developed for the LEO satellite systems. First, rough position knowledge of both the satellite and the UTs can be obtained at the LEO satellite side by, e.g., initial access or tracking [31], by exploiting which the satellite transmits precoded pilot and data signals to each UT. Then, the required channel parameters can be evaluated from the received pilot signals at the UT, and more precise localization knowledge can be derived from the estimated channel information [6], [16], [27], to further improve the localization precision and communication capacity.

#### A. Channel Model

In the wideband massive MIMO LEO satellite ICAL systems, the UPA response is not only dependent on the angles-of-departure (AoD) information, but also the frequency. Then, the UPA response  $\mathbf{v}_{k,l}(f)$  for the  $l$ th propagation path of the  $k$ th UT at frequency  $f$  is given by [32]

$$\mathbf{v}_{k,l}(f) = \mathbf{v}_{k,l}^x(f) \otimes \mathbf{v}_{k,l}^y(f) = \mathbf{v}_x(f, \boldsymbol{\theta}_{k,l}) \otimes \mathbf{v}_y(f, \boldsymbol{\theta}_{k,l}) \in \mathbb{C}^{N_t \times 1}, \quad (2)$$

where  $\boldsymbol{\theta}_{k,l} = (\theta_{k,l}^x, \theta_{k,l}^y)$  denotes the AoD pair, as depicted in Fig. 1. In addition, the array response vectors  $\mathbf{v}_x(f, \boldsymbol{\theta}_{k,l}) \in \mathbb{C}^{N_t^x \times 1}$  and  $\mathbf{v}_y(f, \boldsymbol{\theta}_{k,l}) \in \mathbb{C}^{N_t^y \times 1}$  of the x- and y-axes can be expressed as [15], [32]

$$\mathbf{v}_{k,l}^x(f) \triangleq \mathbf{v}_x(f, \boldsymbol{\theta}_{k,l})$$



$$= \frac{1}{\sqrt{N_t^x}} [1 \exp\{-j\varpi \sin \theta_{k,l}^y \cos \theta_{k,l}^x\} \cdots \exp\{-j\varpi(N_t^x - 1) \sin \theta_{k,l}^y \cos \theta_{k,l}^x\}]^T, \quad (3)$$

$$\mathbf{v}_{k,l}^y(f) \triangleq \mathbf{v}_y(f, \boldsymbol{\theta}_{k,l}) = \frac{1}{\sqrt{N_t^y}} [1 \exp\{-j\varpi \cos \theta_{k,l}^y\} \cdots \exp\{-j\varpi(N_t^y - 1) \cos \theta_{k,l}^y\}]^T, \quad (4)$$

where  $\varpi = \pi(1 + f/f_c)$ .

Generally, the LEO satellite is deployed at an altitude much higher than that of the scattering around the UTs, then the AoD of each path for the channel associated with the  $k$ th UT is almost identical, i.e.,  $\boldsymbol{\theta}_{k,l} \triangleq \boldsymbol{\theta}_k, \forall l$ , and thus we have  $\mathbf{v}_{k,l}(f) = \mathbf{v}_k(f)$ ,  $\mathbf{v}_{k,l}^d(f) = \mathbf{v}_k^d(f) \in \mathbb{C}^{N_t^d \times 1}, \forall l, d \in \{x, y\}$ . Let  $\mathbf{v}_{k,n} \triangleq \mathbf{v}_k(f_n)$ ,  $\mathbf{v}_{k,n}^d \triangleq \mathbf{v}_k^d(f_n)$ , and then, with perfect time and frequency synchronization between the satellite and the UTs,<sup>3</sup> the effective channel vector  $\mathbf{h}_{k,m_g,n} \in \mathbb{C}^{N_t \times 1}$  for the  $k$ th UT over the  $n$ th subcarrier of the  $m_g$ th OFDM symbol is given by [22]

$$\mathbf{h}_{k,m_g,n} = \mathbf{h}_{k,m_g,n}^{\text{los}} + \mathbf{h}_{k,m_g,n}^{\text{nlos}}, \quad (5)$$

where  $\mathbf{h}_{k,m_g,n}^{\text{los}}$  and  $\mathbf{h}_{k,m_g,n}^{\text{nlos}}$  denote the line-of-sight (LoS) and non-line-of-sight (NLoS) part of the channel, respectively, and they can be detailed as

$$\mathbf{h}_{k,m,n}^{\text{los}} = g_{k,m,n}^{\text{los}} \mathbf{v}_{k,n}, \quad (6)$$

$$\mathbf{h}_{k,m,n}^{\text{nlos}} = g_{k,m,n}^{\text{nlos}} \mathbf{v}_{k,n}. \quad (7)$$

Define  $g_{k,m_g,n} = g_{k,m_g,n}^{\text{los}} + g_{k,m_g,n}^{\text{nlos}}$ , and then, since there are numerous propagation paths,  $g_{k,m_g,n}$  can be approximated as the composition of a large number of independent and identically distributed components that follow the Rician distribution with Rician factor  $\kappa_k$  and average power  $\gamma_k = \mathbb{E}\{|g_{k,m_g,n}|^2\}$  [35]. In (6), the complex gain  $g_{k,m_g,n}^{\text{los}}$  is given by<sup>4</sup>

$$g_{k,m_g,n}^{\text{los}} = \alpha_k \exp\{j2\pi(\nu_k m_g T - n f_s \tau_k)\}, \quad (8)$$

where  $\nu_k$  and  $\tau_k$  are Doppler shifts and propagation delay of the LoS path with UT  $k$ , respec-

<sup>3</sup>The clock bias/synchronization errors of the different UTs are not considered in the models and algorithms of this work. In particular, synchronization can be assumed to be done by, e.g., a tracking algorithm or a joint localization and synchronization approach [33] for simplicity (Also, the downlink transmission is a second phase of a real-time transport (RTT) protocol, so that the time-of-arrival (ToA) can be used to determine the distance). Besides, in this work, we assume perfect carrier frequency offset synchronization between the UTs and the satellite, which can be obtained and then compensated by e.g., a under-sampling approach [34].

<sup>4</sup>Note that this model is valid for moderate Doppler spreads for which inter-carrier-interference can be negligible [36].

tively.<sup>5</sup> Besides,  $\alpha_k = \sqrt{\frac{\kappa_k \gamma_k}{1+\kappa_k}} \exp\{j\phi_k\}$ , where  $\phi_k \in (0, 2\pi]$  is a random phase. In addition, in (7), the complex gain  $g_{k,m_g,n}^{\text{nos}}$  follows that  $g_{k,m,n}^{\text{nos}} \sim \mathcal{CN}\left(0, \frac{\gamma_k}{1+\kappa_k}\right)$ .

### B. Signal Model

We denote the transmitted pilot signal over the  $m_p$ th OFDM symbol as  $\{\mathbf{s}_{m_p,n}^{\text{p}}\}_{n=1}^{N_{\text{sc}}^{\text{p}}}$ , where  $\mathbf{s}_{m_p,n}^{\text{p}} = \{s_{m_p,n,1}^{\text{p}}, \dots, s_{m_p,n,K}^{\text{p}}\}$ ,  $m_p = 1, \dots, M_p$  and satisfies  $\mathbb{E}\{\mathbf{s}_{m_p,n}^{\text{p}}(\mathbf{s}_{m_p,n}^{\text{p}})^H\} = \mathbf{I}_K$ . Besides, the transmitted data signal over the  $m_d$ th OFDM symbol is denoted as  $\{\mathbf{s}_{m_d,n}^{\text{d}}\}_{n=1}^{N_{\text{sc}}^{\text{d}}}$ , where  $\mathbf{s}_{m_d,n}^{\text{d}} = \{s_{m_d,n,1}^{\text{d}}, \dots, s_{m_d,n,K}^{\text{d}}\}$ ,  $m_d = 1, \dots, M_d$ ,  $\mathbb{E}\{s_{m_d,n,k}^{\text{d}}\} = 0$  and  $\mathbb{E}\{s_{m_d,n,k}^{\text{d}}(s_{m'_d,n',k'}^{\text{d}})^*\} = \delta(m_d - m'_d)\delta(n - n')\delta(k - k')$  [6].

At the  $n$ th subcarrier over the  $m_g$ th OFDM symbol, the transmitted signal  $\mathbf{s}_{m_g,n}^g$  is first processed by a baseband precoder  $\mathbf{W}_{\text{BB},n} \in \mathbb{C}^{N_{\text{rf}} \times K}$  and then by an analog precoder  $\mathbf{W}_{\text{RF}} \in \mathbb{C}^{N_{\text{t}} \times N_{\text{rf}}}$  [38]. Thus, the final transmission signal is given by  $\mathbf{x}_{m_g,n}^g = \mathbf{W}_{\text{RF}}\mathbf{W}_{\text{BB},n}\mathbf{s}_{m_g,n}^g = \mathbf{W}_n\mathbf{s}_{m_g,n}^g$ , where  $\mathbf{W}_n = [\mathbf{w}_{n,1}, \mathbf{w}_{n,2}, \dots, \mathbf{w}_{n,K}]$  is the equivalent hybrid precoding matrix. Then, the received pilot/data signal over the  $n$ th subcarrier of the  $m_g$ th OFDM symbol at UT  $k$  is given by

$$y_{k,m_g,n}^g = \mathbf{h}_{k,m_g,n}^T \mathbf{W}_n \mathbf{s}_{m_g,n}^g + z_{k,m_g,n}^g, \quad (9)$$

where  $z_{k,m_g,n}^g \sim \mathcal{CN}(0, N_0)$ ,  $\forall k, m_g, n$ ,  $\forall g \in \{\text{p}, \text{d}\}$ .

## III. PERFORMANCE METRICS

### A. Communication Spectral Efficiency

In the following, we first omit the OFDM symbol and subcarrier indices  $m_d$  and  $n$  for a clearer description of the channel statistical properties. During data transmission, the ergodic data rate for the  $k$ th UT is given by

$$R_k = \mathbb{E} \left\{ \log \left( 1 + \frac{|\mathbf{h}_k^T \mathbf{w}_k|^2}{\sum_{\ell \neq k} |\mathbf{h}_k^T \mathbf{w}_\ell|^2 + N_0} \right) \right\}. \quad (10)$$

Note that the ergodic data rate can be estimated with Monte Carlo method. However, the computational complexity of the Monte Carlo method is extremely high, and the accurate iCSI is difficult to be obtained at the LEO satellite transmitter, as mentioned in the introduction, which

<sup>5</sup>The Doppler shift related to the mobility of the LEO satellite can be pre-compensated due to its deterministic time variation [37].

motivates us to adopt sCSI. Based on the [39, Lemma 2], it can be concluded that  $R_k$  is concave with respect to the matrix  $\mathbf{h}_k^* \mathbf{h}_k^T$  and thus, is upper bounded by

$$R_k \leq \bar{R}_k = \log \left( 1 + \frac{\mathbf{w}_k^H \mathbb{E} \{ \mathbf{h}_k^* \mathbf{h}_k^T \} \mathbf{w}_k}{\sum_{\ell \neq k} \mathbf{w}_\ell^H \mathbb{E} \{ \mathbf{h}_k^* \mathbf{h}_k^T \} \mathbf{w}_\ell} \right). \quad (11)$$

In (11), the expectation expression  $\mathbb{E} \{ \mathbf{h}_k^* \mathbf{h}_k^T \}$  represents the channel correlation matrix at the satellite side for the  $k$ th UT. Thus, we have  $\mathbb{E} \{ \mathbf{h}_k^* \mathbf{h}_k^T \} = \bar{\mathbf{h}}_k^* \bar{\mathbf{h}}_k^T$  and

$$\bar{\mathbf{h}}_k = \sqrt{\gamma_k} \mathbf{v}_k, \quad (12)$$

which is a sCSI-based channel model. Note that the required sCSI knowledge involves the UPA response  $\mathbf{v}_k$  and the channel gain  $\gamma_k$ , which are assumed to be invariant in a relatively long interval and can be updated accordingly with a large variation of the channel [15]. Subsequently, the upper bound of the ergodic rate with the sCSI knowledge is given by

$$\bar{R}_k = \log \left( 1 + \frac{|\bar{\mathbf{h}}_k^T \mathbf{w}_k|^2}{\sum_{\ell \neq k} |\bar{\mathbf{h}}_k^T \mathbf{w}_\ell|^2 + N_0} \right), \quad (13)$$

where  $\bar{\mathbf{h}}_k$  is given in (12). The tightness of the upper bound has been intensively investigated for the Rician channel assumption in [40], where the upper bound is proved to be even tighter with smaller transmission power and larger Rician factor. Besides, the tightness will also be verified by simulations in Section V. Finally, we can express the SE as<sup>6</sup>

$$R_{\text{sum}} = \frac{1}{B_w} \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} f_s \bar{R}_{k,n} = \frac{1}{B_w} \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} f_s \log \left( 1 + \frac{|\bar{\mathbf{h}}_{k,n}^T \mathbf{w}_{k,n}|^2}{\sum_{\ell \neq k} |\bar{\mathbf{h}}_{k,n}^T \mathbf{w}_{\ell,n}|^2 + N_0} \right), \quad (14)$$

where  $\bar{\mathbf{h}}_{k,n} = \sqrt{\gamma_{k,n}} \mathbf{v}_{k,n}$ .

## B. Localization Accuracy

1) *Received Signal Model:* The received pilot signal at UT  $k$  is given by<sup>7</sup>

$$y_{k,m,n} = (\mathbf{h}_{k,m,n}^{\text{los}})^T \mathbf{W}_n \mathbf{s}_{m,n} + (\mathbf{h}_{k,m,n}^{\text{nlos}})^T \mathbf{W}_n \mathbf{s}_{m,n} + z_{k,m,n}. \quad (15)$$

<sup>6</sup>We focus on each time slot and omit the OFDM symbol index  $m_d$  for brevity.

<sup>7</sup>The superscript and subscript  $p$  is omitted for notation brevity.

For notation convenience, we denote  $\tilde{z}_{k,m,n} = (\mathbf{h}_{k,m,n}^{\text{los}})^T \mathbf{W}_n \mathbf{s}_{m,n} + z_{k,m,n}$ , and then, (15) can be converted into

$$y_{k,m,n} = (\mathbf{h}_{k,m,n}^{\text{los}})^T \mathbf{W}_n \mathbf{s}_{m,n} + \tilde{z}_{k,m,n}, \quad (16)$$

where  $\tilde{z}_{k,m,n}$  follows  $\mathcal{CN}(0, N_{k,n}^{\text{eq}})$ , and the variance  $N_{k,n}^{\text{eq}}$  is given by

$$N_{k,n}^{\text{eq}} = \mathbb{E} \{ |\tilde{z}_{k,m,n}|^2 \} = \frac{\gamma_k}{1 + \kappa_k} \mathbf{v}_{k,n}^T \mathbf{W}_n \mathbf{W}_n^H \mathbf{v}_{k,n}^* + N_0. \quad (17)$$

2) *Fisher Information Matrix (FIM)*: As mentioned before, channel parameters can be estimated from the received signals. The corresponding channel parameters between the satellite and the  $k$ th UT can be characterized by a  $6 \times 1$  vector  $\boldsymbol{\eta}_k = [\theta_k^x, \theta_k^y, \tau_k, \nu_k, \alpha_k^R, \alpha_k^I]^T$ , where  $\alpha_k^R$  and  $\alpha_k^I$  are the real and imaginary parts of  $\alpha_k$ , respectively. Then, we denote  $\hat{\boldsymbol{\eta}}_k$  as an estimate for the parameter vector  $\boldsymbol{\eta}_k$  associated with the  $k$ th UT, the mean squared error (MSE) of which allows a lower bound, given by  $\mathbb{E} \{ (\hat{\boldsymbol{\eta}}_k - \boldsymbol{\eta}_k) (\hat{\boldsymbol{\eta}}_k - \boldsymbol{\eta}_k)^T \} \succeq \mathbf{J}_{\boldsymbol{\eta}_k}^{-1}$ . Note that  $\mathbf{J}_{\boldsymbol{\eta}_k}$  is the FIM for the unknown vector  $\boldsymbol{\eta}_k$  and the  $(i, j)$ th element can be computed from [17], [29], [41], [42]

$$[\mathbf{J}_{\boldsymbol{\eta}_k}]_{i,j} = \sum_{m=1}^M \sum_{n=1}^{N_{\text{sc}}} [\mathbf{J}_{\boldsymbol{\eta}_k}]_{i,j}^{m,n} = \sum_{m=1}^M \sum_{n=1}^{N_{\text{sc}}} \frac{2}{N_{k,n}^{\text{eq}}} \Re \left\{ \mathbb{E} \left\{ \frac{\partial (r_{k,m,n})^H}{\partial [\boldsymbol{\eta}_k]_i} \frac{\partial r_{k,m,n}}{\partial [\boldsymbol{\eta}_k]_j} \right\} \right\}, \quad (18)$$

where  $r_{k,m,n} = (\mathbf{h}_{k,m,n}^{\text{los}})^T \mathbf{W}_n \mathbf{s}_{m,n}$  is the received pilot signal in the absence of noise, and the detailed elements are given in Appendix B.

3) *Transformation for Position Parameters*: It is worth noting that the transformation from channel parameters  $\boldsymbol{\eta}_k$  to  $\bar{\boldsymbol{\eta}}_k = [\mathbf{p}_k^T, \alpha_k^R, \alpha_k^I]^T$  is a bijection. Then, the transformed FIM involved the position information associated with UT  $k$  is given by  $\mathbf{J}_{\bar{\boldsymbol{\eta}}_k} = \boldsymbol{\Gamma}_k \mathbf{J}_{\boldsymbol{\eta}_k} \boldsymbol{\Gamma}_k^T$ . The transformation matrix  $\boldsymbol{\Gamma}_k \in \mathbb{R}^{5 \times 6}$  is given by

$$\boldsymbol{\Gamma}_k \triangleq \frac{\partial \boldsymbol{\eta}_k^T}{\partial \bar{\boldsymbol{\eta}}_k} = \text{blkdiag} \{ \boldsymbol{\Xi}_k, \mathbf{I}_2 \}, \quad (19)$$

where  $\boldsymbol{\Xi}_k$  is detailed in Appendix A.

4) *SPEB*: Localization precision can be measured by the metric squared position error (SPE), which is defined as the MSE between the actual position  $\mathbf{p}_k$  of UT  $k$  and its estimation  $\hat{\mathbf{p}}_k$  [6], i.e.,

$$\rho_k = \mathbb{E} \{ \|\hat{\mathbf{p}}_k - \mathbf{p}_k\|_2^2 \}. \quad (20)$$

Following the information inequality, the bound of the SPE is given by [25]

$$\rho_k \geq \text{Tr} \left\{ [\mathbf{J}_{\tilde{\boldsymbol{\eta}}_k}^{-1}]_{1:3,1:3} \right\}. \quad (21)$$

Then, we define the sum SPEB of the UTs as

$$\rho_{\text{sum}}^{\text{b}} = \sum_{k=1}^K \text{Tr} \left\{ [\mathbf{J}_{\tilde{\boldsymbol{\eta}}_k}^{-1}]_{1:3,1:3} \right\} = \sum_{k=1}^K \text{Tr} \{ \mathbf{E}^T \mathbf{J}_{\tilde{\boldsymbol{\eta}}_k}^{-1} \mathbf{E} \}. \quad (22)$$

where  $\mathbf{E} = [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3]$ , and  $\mathbf{e}_i \in \mathbb{R}^{5 \times 1}$  denotes a vector with the  $i$ th element being one while the others being zero.

#### IV. INTEGRATED COMMUNICATIONS AND LOCALIZATION

##### A. Problem Formulation

The objective of our work is to design hybrid precoders for the considered LEO satellite ICAL systems. To that end, we formulate an optimization problem, to maximize the downlink SE and minimize the sum SPEB. Let  $\mathcal{W} = \{\mathbf{W}_{\text{BB},n}\}_{n=1}^{N_{\text{sc}}}$ , and we define a vector of the objectives as  $f(\mathcal{W}, \mathbf{W}_{\text{RF}}) = [R_{\text{sum}}, -\rho_{\text{sum}}^{\text{b}}]^T$ , then the multi-objective optimization problem is given by<sup>8</sup> [43]

$$\mathcal{P}_1 : \underset{\mathcal{W}, \mathbf{W}_{\text{RF}}}{\text{maximize}} \quad f(\mathcal{W}, \mathbf{W}_{\text{RF}}) \quad (23a)$$

$$\text{s.t.} \quad \sum_{n=1}^{N_{\text{sc}}} \|\mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB},n}\|_F^2 \leq P, \quad (23b)$$

$$\mathbf{W}_{\text{RF}} \in \mathcal{S}. \quad (23c)$$

Note that problem  $\mathcal{P}_1$  is the maximization problem of the vector  $f(\mathcal{W}, \mathbf{W}_{\text{RF}})$  with both communication and localization metrics, which is defined to maximize those two elements simultaneously [44]. In Eq. (23),  $P$  is the transmit power budget, and  $\mathcal{S} \in \{\mathcal{S}_{\text{FC}}, \mathcal{S}_{\text{PC}}\}$  for the fully and partially connected structures, respectively. For the partially connected structure, the antennas are divided in to  $N_{\text{rf}}$  groups and each group allows  $N_{\text{g}} = N_{\text{t}}/N_{\text{rf}}$  antenna element. Thus, the corresponding analog precoder can be expressed in a block diagonal way as  $\mathbf{W}_{\text{RF}} = \text{blkdiag} \{ \mathbf{w}_{\text{RF},1}, \dots, \mathbf{w}_{\text{RF},N_{\text{rf}}} \}$ . It is worth noting that the communication and localization metric components of the objective in Eq. (23a) have different units, and are difficult to handle. To that

<sup>8</sup>Here, we omit the constant  $f_s/B_w$  in the expression of the SE for brevity.

end, we first respectively transform them into dimensionless and easy-to-tackle ones [43]. Then, a weighted sum method is exploited for the transformed multi-objective optimization problem to trade-off between the performance of communications and localization, where a positive weighting coefficient is selected to satisfy the condition for Pareto optimality [43, Definition 1]. Finally, we jointly design the digital precoder for each subcarrier and the analog precoder for all subcarriers.

### B. Spectral Efficiency Maximization

For notation brevity, let the product of the hybrid precoders denoted by  $\mathbf{W}_n = \mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB},n}$ . By utilizing weighted minimum MSE (WMMSE) method, the maximization of SE can be equivalently transformed into minimizing the weighted sum MSE [45]. In particular, we assume that linear combiner  $u_{k,n}$  is incorporated at subcarrier  $n$  of UT  $k$ . Then, by letting  $\mathbf{U} = \{u_{k,n}\}_{k=1,n=1}^{K,N_{\text{sc}}}$  and introducing an auxiliary weight variable  $\boldsymbol{\Omega} = \{\omega_{k,n}\}_{k=1,n=1}^{K,N_{\text{sc}}}$ , the weighted sum MSE is given by

$$\varepsilon^{\text{sum}} = \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \omega_{k,n} \varepsilon_{k,n}(\mathbf{W}_n, u_{k,n}) - \log(\omega_{k,n}), \quad (24)$$

where  $\varepsilon_{k,n}(\mathbf{W}_n, u_{k,n})$  is the MSE between the estimated signal  $\hat{s}_{k,n} = u_{k,n}^* y_{k,n}$  and the transmitted signal  $s_{k,n}$ , given by

$$\begin{aligned} \varepsilon_{k,n}(\mathbf{W}_n, u_{k,n}) &= \mathbb{E} \left\{ (\hat{s}_{k,n} - s_{k,n}) (\hat{s}_{k,n} - s_{k,n})^H \right\} \\ &= |u_{k,n}^* \bar{\mathbf{h}}_{k,n}^T \mathbf{w}_{k,n} - 1|^2 + \sum_{i \neq k} |u_{k,n}^* \bar{\mathbf{h}}_{k,n}^T \mathbf{w}_{i,n}|^2 + N_0 |u_{k,n}|^2. \end{aligned} \quad (25)$$

Note that when the transmitter precoders are known, the minimum MSE (MMSE) receiver can be given by  $u_{k,n}^{\text{MMSE}} = \bar{\mathbf{h}}_{k,n}^T \mathbf{w}_{k,n} \left( \sum_{i=1}^K |\bar{\mathbf{h}}_{k,n}^T \mathbf{w}_{i,n}|^2 + N_0 \right)^{-1}$ .

### C. Sum SPEB Minimization

It is worth noting that the sum SPEB component of the objective in Eq. (23a) is not easy to tackle due to the tightly coupled digital and analog precoders. In this section, we regard the product of the digital and analog precoders as a whole, i.e.,  $\mathbf{W}_n^{\text{loc}} = \mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB},n}$ ,  $\forall n$ , and our

objective is to find the fully digital precoder  $\mathbf{W}_n^{\text{loc}}$  to minimize the sum SPEB, which can be formulated as

$$\mathcal{P}_2 : \underset{\{\mathbf{W}_n^{\text{loc}}\}_{n=1}^{N_{\text{sc}}}}{\text{minimize}} \quad \sum_{k=1}^K \text{Tr} \{ \mathbf{E}^T \mathbf{J}_{\bar{\eta}_k}^{-1} \mathbf{E} \} \quad \text{s.t.} \quad \sum_{n=1}^{N_{\text{sc}}} \|\mathbf{W}_n^{\text{loc}}\|_F^2 \leq P. \quad (26)$$

Thus, the minimization of the sum SPEB is converted into minimizing the sum Euclidean distance between the product of the hybrid precoders and the precoders obtained from problem  $\mathcal{P}_2$ . Then, let  $\mathbf{C}_n \triangleq \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H$  and problem  $\mathcal{P}_2$  can be transformed into a rank-constrained optimization problem

$$\mathcal{P}_3 : \underset{\mathbf{C}_n}{\text{minimize}} \quad \sum_{k=1}^K \text{Tr} \{ \mathbf{E}^T \mathbf{J}_{\bar{\eta}_k}^{-1} \mathbf{E} \} \quad (27a)$$

$$\text{s.t.} \quad \sum_{n=1}^{N_{\text{sc}}} \text{Tr} \{ \mathbf{C}_n \} \leq P, \quad \mathbf{C}_n \succeq \mathbf{0}, \quad \text{rank} \{ \mathbf{C}_n \} \leq K. \quad (27b)$$

Then, we introduce an auxiliary variable  $\mathbf{M}_k \in \mathbb{R}^{3 \times 3}$ , which satisfies the following condition [46]

$$\mathbf{M}_k \succeq \mathbf{E}^T \mathbf{J}_{\bar{\eta}_k}^{-1} \mathbf{E}, \quad \forall k. \quad (28)$$

Note that the FIM  $\mathbf{J}_{\bar{\eta}_k}$  must be a positive semidefinite matrix, and thus, utilizing the property of Schur complement, Eq. (28) can be expressed as [46]

$$\begin{bmatrix} \mathbf{M}_k & \mathbf{E}^T \\ \mathbf{E} & \mathbf{J}_{\bar{\eta}_k}(\mathbf{C}_n) \end{bmatrix} \succeq \mathbf{0}, \quad \forall k. \quad (29)$$

Therefore, problem  $\mathcal{P}_3$  can be converted into [46], [47]

$$\mathcal{P}_4 : \underset{\mathbf{C}_n, \mathbf{M}_k}{\text{minimize}} \quad \sum_{k=1}^K \text{Tr} \{ \mathbf{M}_k \} \quad (30a)$$

$$\text{s.t.} \quad \sum_{n=1}^{N_{\text{sc}}} \text{Tr} \{ \mathbf{C}_n \} \leq P, \quad (30b)$$

$$\mathbf{C}_n \succeq \mathbf{0}, \quad (30c)$$

$$\text{rank} \{ \mathbf{C}_n \} \leq K, \quad (30d)$$

$$\begin{bmatrix} \mathbf{M}_k & \mathbf{E}^T \\ \mathbf{E} & \mathbf{J}_{\bar{\eta}_k}(\mathbf{C}_n) \end{bmatrix} \succeq \mathbf{0}, \quad \forall k. \quad (30e)$$

Note that the rank constraint in (30d) of problem  $\mathcal{P}_4$  is difficult to handle. To that end, we first focus on the relaxed problem by dropping this rank constraint [48]. To further improve computational efficiency, based on [49, Appendix C], the optimal solution for the relaxed problem of  $\mathcal{P}_4$  can be written as

$$\mathbf{C}_n = \mathbf{L}_n \mathbf{Z}_n \mathbf{L}_n^H. \quad (31)$$

The proof is given in Appendix C. In Eq. (31),  $\mathbf{Z}_n$  denotes a  $3K \times 3K$ -dimensional positive semidefinite matrix and  $\mathbf{L}_n = [\mathbf{V}_n \ \mathbf{V}_{n,x} \ \mathbf{V}_{n,y}]$ , where  $\mathbf{V}_n = [\mathbf{v}_{1,n}, \dots, \mathbf{v}_{K,n}]$ ,  $\mathbf{V}_{n,d} = [\mathbf{v}_{1,n,\theta_1^d}, \dots, \mathbf{v}_{K,n,\theta_K^d}]$ , and  $\mathbf{v}_{k,n,\theta_1^d} \triangleq \partial \mathbf{v}_{k,n} / \partial \theta_1^d$  for  $d \in \{x, y\}$ . By utilizing the decomposition in Eq. (31), problem  $\mathcal{P}_4$  can be converted into

$$\mathcal{P}_5 : \underset{\mathbf{Z}_n, \mathbf{M}_k}{\text{minimize}} \quad \sum_{k=1}^K \text{Tr} \{ \mathbf{M}_k \} \quad (32a)$$

$$\text{s.t.} \quad \sum_{n=1}^{N_{sc}} \text{Tr} \{ \mathbf{L}_n \mathbf{Z}_n \mathbf{L}_n^H \} \leq P, \quad (32b)$$

$$\mathbf{Z}_n \succeq \mathbf{0}, \quad (32c)$$

$$\begin{bmatrix} \mathbf{M}_k & \mathbf{E}^T \\ \mathbf{E} & \mathbf{J}_{\bar{\eta}_k}(\mathbf{Z}_n) \end{bmatrix} \succeq \mathbf{0}, \quad \forall k. \quad (32d)$$

To tackle problem  $\mathcal{P}_5$ , since  $\mathbf{J}_{\bar{\eta}_k}(\mathbf{Z}_n)$  is not convex with respect to the variable  $\mathbf{Z}_n$ , we invoke the majorization-minimization (MM) algorithm, the basic philosophy of which is to iteratively handle problem  $\mathcal{P}_5$  through a series of easier problems [50]. In particular, let  $\mathbf{Z}_{n,t}$  denote the solution of the  $t$ th iteration, and then, in  $(t+1)$ th iteration, we substitute  $\mathbf{J}_{\bar{\eta}_k}(\mathbf{Z}_n)$  with its second order Taylor expansion  $\hat{\mathbf{J}}_{\bar{\eta}_k}(\mathbf{Z}_n)$ , the  $(i, j)$ th element of which is given by

$$\begin{aligned} [\hat{\mathbf{J}}_{\bar{\eta}_k}]_{i,j}(\mathbf{Z}_n) &= [\mathbf{J}_{\bar{\eta}_k}]_{i,j}(\mathbf{Z}_{n,t}) + \sum_{m=1}^M \sum_{n=1}^{N_{sc}} \text{Tr} \left\{ \left( \frac{\partial [\mathbf{J}_{\bar{\eta}_k}]_{i,j}^{m,n}(\mathbf{Z}_{n,t})}{\partial \mathbf{Z}_{n,t}} \right)^T (\mathbf{Z}_n - \mathbf{Z}_{n,t}) \right\} \\ &\quad + \frac{L}{2} \|\mathbf{Z}_n - \mathbf{Z}_{n,t}\|_F^2. \end{aligned} \quad (33)$$

In (33), the first order gradient of  $[\mathbf{J}_{\bar{\eta}_k}]_{i,j}^{m,n}(\mathbf{Z}_{n,t})$  with respect to the variable  $\mathbf{Z}_{n,t}$  is given by

$$\frac{\partial [\mathbf{J}_{\bar{\eta}_k}]_{i,j}^{m,n}(\mathbf{Z}_{n,t})}{\partial \mathbf{Z}_{n,t}} = \frac{(\text{Tr} \{ \mathbf{D}_{k,n} \mathbf{Z}_{n,t} \} + N_0) \mathbf{A}_{k,m,n,i,j} - \text{Tr} \{ \mathbf{Z}_{n,t} \mathbf{A}_{k,m,n,i,j} \} \cdot \mathbf{D}_{k,n}}{(\text{Tr} \{ \mathbf{D}_{k,n} \mathbf{Z}_{n,t} \} + N_0)^2}, \quad (34)$$



where  $\mathbf{A}_{k,m,n,i,j} = \mathbf{L}_n^H \frac{\partial \mathbf{h}_{k,m,n}^*}{\partial [\boldsymbol{\eta}_k]_i} \frac{\partial \mathbf{h}_{k,m,n}^T}{\partial [\boldsymbol{\eta}_k]_j} \mathbf{L}_n$  and  $\mathbf{D}_{k,n} = \frac{\gamma_k}{1+\kappa_k} \mathbf{L}_n^H \mathbf{v}_{k,n}^* \mathbf{v}_{k,n}^T \mathbf{L}_n$ . Then, in the  $(t+1)$ th iteration, the corresponding problem can be written as

$$\mathcal{P}_5^{(t+1)} : \underset{\mathbf{Z}_{n,t+1}, \mathbf{M}_{k,t+1}}{\text{minimize}} \quad \sum_{k=1}^K \text{Tr} \{ \mathbf{M}_{k,t+1} \} \quad (35a)$$

$$\text{s.t.} \quad \sum_{n=1}^{N_{\text{sc}}} \text{Tr} \{ \mathbf{L}_n \mathbf{Z}_{n,t+1} \mathbf{L}_n^H \} \leq P, \quad (35b)$$

$$\mathbf{Z}_{n,t+1} \succeq \mathbf{0}, \quad (35c)$$

$$\begin{bmatrix} \mathbf{M}_k & \mathbf{E}^T \\ \mathbf{E} & \hat{\mathbf{J}}_{\hat{\boldsymbol{\eta}}_k}(\mathbf{Z}_{n,t+1}) \end{bmatrix} \succeq \mathbf{0}, \quad \forall k, \quad (35d)$$

which can be handled with semidefinite programming (SDP) solvers [51], [52]. It is worth noting that since the MM method results in a relaxed problem, each stationary point of the sequence with the objective values generated by problem  $\mathcal{P}_5^{(t+1)}$  might be a local sub-optimum of problem  $\mathcal{P}_5$  [53]. Then, with the symmetric positive definite matrix  $\mathbf{Z}_n$  from  $\mathcal{P}_5$ , we have  $\mathbf{C}_n = \mathbf{L}_n \mathbf{Z}_n \mathbf{L}_n^H$ , and the corresponding localization precoder can be reversed through Cholesky decomposition and randomization procedures [31], [54]–[56].

*Remark 1:* Due to the roughness of the prior knowledge obtained at the LEO satellite side for the  $k$ th UT, robust signal design should be formulated with the consideration of the uncertainties in channel parameters, which can be performed by referring to [31], [47].

#### D. Hybrid Precoding for the ICAL Systems

Let  $\bar{\mathbf{W}}_{\text{BB}} = [\mathbf{W}_{\text{BB},1}, \dots, \mathbf{W}_{\text{BB},N_{\text{sc}}}] \in \mathbb{C}^{N_{\text{rf}} \times KN_{\text{sc}}}$  and  $\bar{\mathbf{W}}^{\text{loc}} = [\mathbf{W}_1^{\text{loc}}, \dots, \mathbf{W}_{N_{\text{sc}}}^{\text{loc}}] \in \mathbb{C}^{N_{\text{t}} \times N_{\text{rf}} N_{\text{sc}}}$  for notation brevity. To design the digital and analog precoders, we formulate the following weighted sum problem to minimize the sum MSE of communications and the sum Euclidean distance between the hybrid precoders and the localization-only fully digital precoders, as obtained in Section IV-B and Section IV-C, which is given by [19], [57], [58]

$$\mathcal{Q}_1 : \underset{\mathbf{W}_{\text{RF}}, \{\mathbf{W}_{\text{BB},n}\}_{n=1}^{N_{\text{sc}}}, \mathbf{U}, \boldsymbol{\Omega}}{\text{minimize}} \quad \rho \varepsilon^{\text{sum}}(\mathbf{W}_{\text{RF}}, \bar{\mathbf{W}}_{\text{BB}}, \mathbf{U}, \boldsymbol{\Omega}) + (1 - \rho) d^{\text{sum}}(\mathbf{W}_{\text{RF}}, \bar{\mathbf{W}}_{\text{BB}}) \quad (36a)$$

$$\text{s.t.} \quad \|\mathbf{W}_{\text{RF}} \bar{\mathbf{W}}_{\text{BB}}\|_F^2 \leq P, \quad \mathbf{W}_{\text{RF}} \in \mathcal{S}. \quad (36b)$$

In (36),  $\rho \in [0, 1]$  denotes the weighting coefficient, serving to trade off between the performance of the communications and localization, and  $d^{\text{sum}}$  is defined as  $d^{\text{sum}}(\mathbf{W}_{\text{RF}}, \bar{\mathbf{W}}_{\text{BB}}) =$

$\sum_{n=1}^{N_{\text{sc}}} \|\mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB},n} - \mathbf{W}_n^{\text{loc}}\|_F^2$ . By introducing the auxiliary variables  $\mathbf{G}_{k,n} = [\mathbf{g}_{k,n}^1, \dots, \mathbf{g}_{k,n}^K] = \mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB},n}$ ,  $\forall k$ , problem  $\mathcal{Q}_1$  can be reformulated as [58]

$$\mathcal{Q}_2 : \quad \underset{\mathbf{W}_{\text{RF}}, \{\mathbf{W}_{\text{BB},n}\}_{n=1}^{N_{\text{sc}}}, \mathbf{U}, \boldsymbol{\Omega}}{\text{minimize}} \quad \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \left[ \rho \varepsilon_{k,n}^{\text{w}}(\mathbf{G}_{k,n}, u_{k,n}, \omega_{k,n}) + \frac{1-\rho}{K} d_{k,n}(\mathbf{G}_{k,n}) \right] \quad (37a)$$

$$\text{s.t.} \quad \mathbf{G}_{k,n} = \mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB},n}, \quad \forall k, \quad (37b)$$

$$\sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \text{Tr} \{ \mathbf{G}_{k,n} \mathbf{G}_{k,n}^H \} \leq PK, \quad \mathbf{W}_{\text{RF}} \in \mathcal{S}, \quad (37c)$$

where  $\varepsilon_{k,n}^{\text{w}}(\mathbf{G}_{k,n}, u_{k,n}, \omega_{k,n}) = \omega_{k,n} \varepsilon_{k,n}(\mathbf{G}_{k,n}, u_{k,n}) - \log(\omega_{k,n})$ ,  $d_{k,n} = \|\mathbf{G}_{k,n} - \mathbf{W}_n^{\text{loc}}\|_F^2$  and  $\varepsilon_{k,n}(\mathbf{G}_{k,n}, u_{k,n}) = |u_{k,n}|^2 \bar{\mathbf{h}}_{k,n}^T \mathbf{G}_{k,n} \mathbf{G}_{k,n}^H \bar{\mathbf{h}}_{k,n}^* - u_{k,n} \bar{\mathbf{h}}_{k,n}^T \mathbf{G}_{k,n} \mathbf{e}_k - u_{k,n}^* \mathbf{G}_{k,n}^H \bar{\mathbf{h}}_{k,n}^* \mathbf{e}_k^T + 1 + N_0 |u_{k,n}|^2$ .

In the following, inspired by the alternating direction method of multipliers (ADMM) [59], we adopt the augmented Lagrangian method by introducing dual variables  $\mathbf{Q}_{k,n} \in \mathbb{C}^{N_t \times K}$ , and the corresponding penalty  $\eta_{k,n} > 0$ . Then, let  $\mathcal{G} = \{\mathbf{G}_{k,n}\}_{k=1, n=1}^{K, N_{\text{sc}}}$ ,  $\mathcal{Q} = \{\mathbf{Q}_{k,n}\}_{k=1, n=1}^{K, N_{\text{sc}}}$  and the objective of problem  $\mathcal{Q}_2$  can be transformed into

$$f(\mathcal{G}, \mathbf{U}, \boldsymbol{\Omega}, \bar{\mathbf{W}}_{\text{BB}}, \mathbf{W}_{\text{RF}}, \mathcal{Q}) = \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} L(\mathbf{G}_{k,n}, u_{k,n}, \omega_{k,n}, \mathbf{W}_{\text{BB},n}, \mathbf{W}_{\text{RF}}, \mathbf{Q}_{k,n}), \quad (38)$$

where  $L(\mathbf{G}_{k,n}, u_{k,n}, \omega_{k,n}, \mathbf{W}_{\text{BB},n}, \mathbf{W}_{\text{RF}}, \mathbf{Q}_{k,n}) = \rho \varepsilon_{k,n}^{\text{w}}(\mathbf{G}_{k,n}, u_{k,n}, \omega_{k,n}, \mathbf{Q}_{k,n}) + \frac{1-\rho}{K} d_{k,n}(\mathbf{G}_{k,n}) + \frac{\eta_{k,n}}{2} \|\mathbf{G}_{k,n} - \mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB},n} + \mathbf{Q}_{k,n}\|_F^2$ . The corresponding problem  $\mathcal{Q}_3$  can be then written as

$$\mathcal{Q}_3 : \quad \underset{\mathcal{G}, \mathbf{U}, \boldsymbol{\Omega}, \bar{\mathbf{W}}_{\text{BB}}, \mathbf{W}_{\text{RF}}, \mathcal{Q}}{\text{minimize}} \quad f(\mathcal{G}, \mathbf{U}, \boldsymbol{\Omega}, \bar{\mathbf{W}}_{\text{BB}}, \mathbf{W}_{\text{RF}}, \mathcal{Q}) \quad (39a)$$

$$\text{s.t.} \quad \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \text{Tr} \{ \mathbf{G}_{k,n} \mathbf{G}_{k,n}^H \} \leq PK, \quad (39b)$$

$$\mathbf{W}_{\text{RF}} \in \mathcal{S}, \quad (39c)$$

which can be handled through the ADMM method [59]. In each iteration of the ADMM method, the algorithmic steps mainly depend on the optimization of each variable in problem  $\mathcal{Q}_3$ . It is worth noting that, in each iteration, the architecture of the ADMM allows us to update variables  $\mathbf{G}_{k,n}$ ,  $u_{k,n}$ ,  $\omega_{k,n}$ , and  $\mathbf{W}_{\text{BB},n}$  in parallel [60]. Then, the corresponding steps for handling problem  $\mathcal{Q}_3$  by the ADMM are detailed as follows:

1) *Update of  $\mathbf{G}_{k,n}$* : We formulate the optimization problem to update  $\mathbf{G}_{k,n}$  as

$$\mathcal{Q}_4 : \underset{\mathcal{G}}{\text{minimize}} \quad f(\mathcal{G}) \quad (40a)$$

$$\text{s.t.} \quad \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \text{Tr} \{ \mathbf{G}_{k,n} \mathbf{G}_{k,n}^H \} \leq PK. \quad (40b)$$

To handle problem  $\mathcal{Q}_4$ , we introduce a Lagrange multiplier for constraint in (40b) and the corresponding Lagrange function for  $\mathcal{Q}_4$  is given by [48]

$$f(\mathcal{G}, \mu) = \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} L(\mathbf{G}_{k,n}) + \mu \left( \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \text{Tr} \{ \mathbf{G}_{k,n} \mathbf{G}_{k,n}^H \} - PK \right). \quad (41)$$

Subsequently, by utilizing Karush-Kuhn-Tucker (KKT) conditions,  $\mathbf{G}_{k,n}$  can be updated by  $\mathbf{G}_{k,n} = (\mathbf{A}_{k,n} + \mu \mathbf{I}_{N_t})^{-1} \mathbf{\Psi}_{k,n}$ , where  $\mathbf{A}_{k,n}$  is a Hermitian matrix which can be decomposed as  $\mathbf{A}_{k,n} = \mathbf{D}_{k,n} \mathbf{\Lambda}_{k,n} \mathbf{D}_{k,n}^H$  and the expression for  $\mathbf{A}_{k,n}$ ,  $\mathbf{\Psi}_{k,n}$  is given by

$$\mathbf{A}_{k,n} = \rho \omega_{k,n} |u_{k,n}|^2 \bar{\mathbf{h}}_{k,n}^* \bar{\mathbf{h}}_{k,n}^T + \left( \frac{1-\rho}{K} + \frac{\eta_{k,n}}{2} \right) \mathbf{I}_{N_t}, \quad (42)$$

$$\mathbf{\Psi}_{k,n} = \frac{1-\rho}{K} \mathbf{W}_n^{\text{loc}} + \rho \omega_{k,n} u_{k,n}^* \bar{\mathbf{h}}_{k,n}^* \mathbf{e}_k^T + \frac{\eta_{k,n}}{2} (\mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB},n} - \mathbf{Q}_{k,n}). \quad (43)$$

Then, the left hand side of Eq. (40b) can be written as a function of  $\mu$ , which is given by

$$\delta(\mu) = \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \text{Tr} \{ \mathbf{G}_{k,n}(\mu) \mathbf{G}_{k,n}^H(\mu) \} = \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \sum_{i=1}^{N_t} \frac{[\Phi_{k,n}]_{i,i}}{([\Lambda_{k,n}]_{i,i} + \mu)^2}, \quad (44)$$

where  $\Phi_{k,n} = \mathbf{D}_{k,n}^H \mathbf{\Psi}_{k,n} \mathbf{\Psi}_{k,n}^H \mathbf{D}_{k,n}$ . The Lagrange multiplier  $\mu$  can be found to satisfy the complementarity slackness condition [48]. In particular, if  $\delta(0) \leq PK$ , then  $\mu = 0$ ; otherwise  $\mu$  is selected to satisfy  $\delta(\mu) = PK$  via the bisection search.

2) *Update of  $u_{k,n}$* : The update for  $u_{k,n}$  is  $u_{k,n} = \mathbf{e}_k^T \mathbf{G}_{k,n}^H \bar{\mathbf{h}}_{k,n}^* \left( \sum_{i=1}^K |\bar{\mathbf{h}}_{k,n}^T \mathbf{G}_{k,n} \mathbf{e}_i|^2 + N_0 \right)^{-1}$ .

3) *Update of  $\omega_{k,n}$* : The update for  $\omega_{k,n}$  is given by

$$\omega_{k,n} = \left( |u_{k,n}^* \bar{\mathbf{h}}_{k,n}^T \mathbf{G}_{k,n} \mathbf{e}_k - 1|^2 + \sum_{i \neq k} |u_{k,n}^* \bar{\mathbf{h}}_{k,n}^T \mathbf{G}_{k,n} \mathbf{e}_i|^2 + N_0 |u_{k,n}|^2 \right)^{-1}, \quad \forall k, n. \quad (45)$$

4) *Update of  $\mathbf{W}_{\text{BB},n}$* : The optimization problem to update  $\mathbf{W}_{\text{BB},n}$  is formulated as

$$\mathcal{Q}_5 : \underset{\{\mathbf{W}_{\text{BB},n}\}_{n=1}^{N_{\text{sc}}}}{\text{maximize}} \quad f(\bar{\mathbf{W}}_{\text{BB}}) = \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \frac{\eta_{k,n}}{2} \|\mathbf{G}_{k,n} - \mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB},n} + \mathbf{Q}_{k,n}\|_F^2. \quad (46a)$$

For both fully and partially connected structures, we set the derivative of the objective function  $f(\bar{\mathbf{W}}_{\text{BB}})$  with respect to  $\mathbf{W}_{\text{BB},i}$  to be zero, and then, the expression to update  $\mathbf{W}_{\text{BB},i}$  is given by  $\mathbf{W}_{\text{BB},i} = \frac{1}{\sum_{k=1}^K \eta_{k,i}} (\mathbf{W}_{\text{RF}}^H \mathbf{W}_{\text{RF}})^{-1} \sum_{k=1}^K \eta_{k,i} \mathbf{W}_{\text{RF}}^H (\mathbf{G}_{k,i} + \mathbf{Q}_{k,i})$ . In particular, for partially

connected structure, we have  $\mathbf{W}_{\text{RF}}^H \mathbf{W}_{\text{RF}} = N_g \mathbf{I}_{N_{\text{rf}}}$ .

5) *Update of  $\mathbf{W}_{\text{RF}}$* : The optimization problem to update  $\mathbf{W}_{\text{RF}}$  is formulated as

$$\mathcal{Q}_6 : \underset{\mathbf{W}_{\text{RF}}}{\text{maximize}} \quad \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \frac{\eta_{k,n}}{2} \|\mathbf{G}_{k,n} - \mathbf{W}_{\text{RF}} \mathbf{W}_{\text{BB},n} + \mathbf{Q}_{k,n}\|_F^2 \quad (47a)$$

$$\text{s.t.} \quad \mathbf{W}_{\text{RF}} \in \mathcal{S}. \quad (47b)$$

For the fully connected structure,  $\mathbf{W}_{\text{RF}}$  can be updated as  $\mathbf{W}_{\text{RF}} = \exp \{-j \angle \mathbf{X}^T\}$  [61], where  $\mathbf{X} = \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \frac{\eta_{k,n}}{2} \mathbf{W}_{\text{BB},n} (\mathbf{G}_{k,n} + \mathbf{Q}_{k,n})^H - (\mathbf{T} - \lambda_{\max}(\mathbf{T}) \mathbf{I}_{N_{\text{rf}}}) \mathbf{W}_{\text{RF}}^H$  and  $\lambda_{\max}(\mathbf{T})$  denotes the maximum eigenvalue of  $\mathbf{T} = \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \frac{\eta_{k,n}}{2} \mathbf{W}_{\text{BB},n} \mathbf{W}_{\text{BB},n}^H$  [61]. For the partially connected structure, by expanding the objective in (47a) and utilizing the property that  $\mathbf{W}_{\text{RF}}^H \mathbf{W}_{\text{RF}} = N_g \mathbf{I}_{N_{\text{rf}}}$ , the problem can be rewritten as [62]

$$\mathcal{Q}_7 : \underset{[\mathbf{W}_{\text{RF}}]_{i,j}}{\text{maximize}} \quad \Re \left\{ \text{Tr} \left\{ \mathbf{Y}_{i,j}^H [\mathbf{W}_{\text{RF}}]_{i,j}^* \right\} \right\} \quad \text{s.t.} \quad |[\mathbf{W}_{\text{RF}}]_{i,j}| = 1, \quad \forall i, \quad \forall j = \left\lceil \frac{i}{N_g} \right\rceil, \quad (48)$$

where  $\mathbf{Y}_{i,j} = \sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \frac{\eta_{k,n}}{2} [\mathbf{G}_{k,n} + \mathbf{Q}_{k,n}]_{i,:} [\mathbf{W}_{\text{BB},n}]_{j,:}^H$ . Then, the  $(i, j)$ th element of  $\mathbf{W}_{\text{RF}}$  can be updated by  $[\mathbf{W}_{\text{RF}}]_{i,j} = \exp \{\angle \mathbf{Y}_{i,j}\}$ ,  $\forall i, \quad \forall j = \lceil i/N_g \rceil$ . To further improve the convergence rate, at the end of each iteration, the penalty  $\eta_{k,n}$  is updated according to [59, Eq. (3.13)]

$$\eta_{k,n}^{t+1} = \begin{cases} \varsigma^{\text{mul}} \eta_{k,n}^t, & \frac{\|\mathbf{G}_{k,n}^{t+1} - \mathbf{W}_{\text{RF}}^{t+1} \mathbf{W}_{\text{BB},n}^{t+1}\|_F^2}{\|\mathbf{G}_{k,n}^{t+1} - \mathbf{G}_{k,n}^t\|_F^2} > \varrho, \\ \eta_{k,n}^t / \varsigma^{\text{div}}, & \frac{\|\mathbf{G}_{k,n}^{t+1} - \mathbf{W}_{\text{RF}}^{t+1} \mathbf{W}_{\text{BB},n}^{t+1}\|_F^2}{\|\mathbf{G}_{k,n}^{t+1} - \mathbf{G}_{k,n}^t\|_F^2} < 1/\varrho, \\ \eta_{k,n}^t, & \text{otherwise,} \end{cases} \quad (49)$$

where  $\varrho > 1$  and  $\varsigma^{\text{mul}}, \varsigma^{\text{div}} > 1$ . The whole procedure for computing the hybrid precoders for LEO satellite ICAL is summarized in **Algorithm 1**.

### E. Convergence and Computational Complexity

The convergence of **Algorithm 1** depends on two parts. First, the sum SPEB minimization problem is handled with MM method by transforming the projection matrices, i.e.,  $\text{Tr} \{\mathbf{E}^T \mathbf{J}_{\bar{\eta}_k}^{-1} \mathbf{E}\}$ , with their second-order Taylor expansion, and according to [50], the sequence of feasible points  $\{\mathbf{Z}_{n,t}\}$  can converge to a stationary value. According to [63], when the penalty is properly chosen, the ADMM algorithm is guaranteed to be convergent to a stationary point.

The complexity of **Algorithm 1** is detailed as follows. First, in each iteration of the MM algorithm, the number of the optimized variables for the SDP in (35) is denoted by  $n_{\text{var}} = 9K^2 N_{\text{sc}} +$

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**Algorithm 1** Hybrid Precoding for ICAL
 

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**Input:** Thresholds  $\epsilon_1, \epsilon_2 > 0$ .

**Output:** Hybrid precoders  $\mathbf{W}_{\text{RF}}$  and  $\mathbf{W}_{\text{BB},n}$ ,  $n = 1, \dots, N_{\text{sc}}$ .

- 1: Initialize  $t = 0$ ,  $t_{\text{max}}^{\text{loc}}$  and  $\mathbf{Z}_{n,t}$  such that  $\|\mathbf{L}_n \mathbf{Z}_{n,t} \mathbf{L}_n\|_F^2 = P/N_{\text{sc}}$ .
  - 2: **repeat**
  - 3:   Solve SDP problem  $\mathcal{P}_8^{(t+1)}$  and obtain  $\mathbf{M}_{k,t+1}$ ,  $\mathbf{Z}_{n,t+1}$ .
  - 4:    $t = t + 1$ .
  - 5: **until**  $\left| \sum_{k=1}^K \text{Tr} \{ \mathbf{M}_{k,t} \} - \sum_{k=1}^K \text{Tr} \{ \mathbf{M}_{k,t-1} \} \right| > \epsilon_1$  or  $t \geq t_{\text{max}}^{\text{loc}}$ .
  - 6: Obtain  $\mathbf{C}_n = \mathbf{L}_n \mathbf{Z}_n \mathbf{L}_n^H$  and  $\mathbf{W}_n^{\text{loc}}$  by eigenvalue decomposing  $\mathbf{C}_n$ ,  $n = 1, \dots, N_{\text{sc}}$ .
  - 7: Initialize  $t = 0$ ,  $\mathcal{G}^t$ ,  $\mathbf{U}^t$ ,  $\mathbf{\Omega}^t$ ,  $\bar{\mathbf{W}}_{\text{BB}}^t$ ,  $\mathbf{W}_{\text{RF}}^t$ ,  $\mathbf{Q}^t$ ,  $t_{\text{max}}^{\text{hy}}$ .
  - 8: **while**  $\sum_{k=1}^K \sum_{n=1}^{N_{\text{sc}}} \|\mathbf{G}_{k,n}^t - \mathbf{W}_{\text{RF}}^t \mathbf{W}_{\text{BB},n}^t\|_F^2 > \epsilon_2$  or  $t \leq t_{\text{max}}^{\text{hy}}$  **do**
  - 9:    $t = t + 1$ .
  - 10:    $\mathcal{G}^t \leftarrow \arg \min_{\mathcal{G}} f(\mathcal{G}, \mathbf{U}^{t-1}, \mathbf{\Omega}^{t-1}, \bar{\mathbf{W}}_{\text{BB}}^{t-1}, \mathbf{W}_{\text{RF}}^{t-1}, \mathbf{Q}^{t-1})$  s.t. (40b).
  - 11:    $\mathbf{U}^t \leftarrow \arg \min f(\mathcal{G}^t, \mathbf{U}, \mathbf{\Omega}^{t-1}, \bar{\mathbf{W}}_{\text{BB}}^{t-1}, \mathbf{W}_{\text{RF}}^{t-1}, \mathbf{Q}^{t-1})$ .
  - 12:    $\mathbf{\Omega}^t \leftarrow \arg \min f(\mathcal{G}^t, \mathbf{U}^t, \mathbf{\Omega}, \bar{\mathbf{W}}_{\text{BB}}^{t-1}, \mathbf{W}_{\text{RF}}^{t-1}, \mathbf{Q}^{t-1})$ .
  - 13:    $\bar{\mathbf{W}}_{\text{BB}}^t \leftarrow \arg \min f(\mathcal{G}^t, \mathbf{U}^t, \mathbf{\Omega}^t, \bar{\mathbf{W}}, \mathbf{W}_{\text{RF}}^{t-1}, \mathbf{Q}^{t-1})$ .
  - 14:    $\mathbf{W}_{\text{RF}}^t \leftarrow \arg \min_{\mathbf{W}_{\text{RF}} \in \mathcal{S}} f(\mathcal{G}^t, \mathbf{U}^t, \mathbf{\Omega}^t, \bar{\mathbf{W}}_{\text{BB}}^t, \mathbf{W}, \mathbf{Q}^{t-1})$ .
  - 15:    $\mathbf{Q}_{k,n}^t \leftarrow \mathbf{Q}_{k,n}^{t-1} + \mathbf{G}_{k,n}^t - \mathbf{W}_{\text{RF}}^t \mathbf{W}_{\text{BB},n}^t$ .
  - 16:   Update  $\eta_{k,n}^t$  following Eq. (49).
  - 17: **end while**
- 

$9K$ . The number of linear matrix inequality (LMI) constraints is  $M_{\text{LMI}} = K + N_{\text{sc}}$ , contributed by (35c) and (35d), respectively. Besides, the number of the rows or columns for the matrix of the  $i$ th LMI constraint is given by  $m_i = 9$ ,  $1 \leq i \leq K$  and  $m_i = 3K$ ,  $K + 1 \leq i \leq M_{\text{LMI}}$ . Thus, the complexity of the SDP in (35) is given by  $\mathcal{O}\left(n_{\text{var}}^2 \sum_{i=1}^{M_{\text{LMI}}} m_i^2 + n_{\text{var}} \sum_{i=1}^{M_{\text{LMI}}} m_i^3\right)$  [47]. By assuming the MM algorithm terminates in  $J_{\text{mm}}$  iterations, the overall complexity to handle problem  $\mathcal{P}_5$  can be evaluated as  $\mathcal{O}\left(J_{\text{mm}} \left(n_{\text{var}}^2 \sum_{i=1}^{M_{\text{LMI}}} m_i^2 + n_{\text{var}} \sum_{i=1}^{M_{\text{LMI}}} m_i^3\right)\right)$ . The computational complexity of the eigenvalue decomposition for  $\mathbf{C}_n$ ,  $n = 1, \dots, N_{\text{sc}}$  is given by  $\mathcal{O}(N_{\text{sc}} N_t^3)$ . Subsequently, in each iteration of the ADMM, the corresponding parameters should be updated according to steps 10 – 16, and the major computational complexity lies in the update of  $\mathbf{G}_{k,n}$ . In particular, the update for  $\mathbf{G}_{k,n}$  in step 10 mainly contributes to the pseudo-inverse operation and the eigenvalue decomposition, both of which present the complexity of  $\mathcal{O}(N_t^3)$ . Then, assuming the ADMM algorithm terminates in  $J_{\text{admm}}$  iterations, the complexity for the ADMM can be estimated as  $\mathcal{O}(2J_{\text{admm}} K N_{\text{sc}} N_t^3)$ .

## V. SIMULATIONS

In this section, we evaluate the communication and localization performance for the proposed massive MIMO LEO satellite ICAL system, which is assumed to operate in the S-band. The

performance of the communications and localization is evaluated with the downlink SE and average position error bound (APEB), which is defined as  $\rho_{\text{avg}}^b = \sqrt{\rho_{\text{sum}}^b/K}$ , respectively. Some related typical simulation parameters are given in Table I [64], [65]. The LEO satellite is assumed to be located at  $\mathbf{q} = [0, 0, 0]^T$  and the orientation angle is given by  $\mathbf{o} = [0, 0]^T$ . The maximum nadir angle of the UT is set to be  $\vartheta_{\text{max}} = \pi/6$  and each component of the AoD pair for the UT, i.e.,  $\theta_k^x$  and  $\theta_k^y$ , is assumed to be uniformly distributed in  $[\pi/2 - \vartheta_{\text{max}}, \pi/2 + \vartheta_{\text{max}}]$ . The nadir angle of the  $k$ th UT can be calculated as  $\vartheta_k = \arccos(\sin \theta_k^x \sin \theta_k^y)$  and thus, the elevation angle for UT  $k$  is given by  $\varphi_k = \arccos\left(\frac{R_e + H}{R_e} \sin \vartheta_k\right)$ , where  $R_e$  is the earth radius and  $H$  is the orbit height of the LEO satellite [22], [66]. Then, the distance between the LEO satellite and the  $k$ th UT can be calculated as  $d_k = \sqrt{H^2 + 2HR_e + R_e^2 \sin^2 \varphi_k} - R_e \sin \varphi_k$ . The channel gain  $\gamma_k$  is defined as

$$\gamma_k = G_{\text{sat}} G_{\text{ut}} N_t \left( \frac{c}{4\pi f_c d_k} \right)^2, \quad (50)$$

where  $G_{\text{sat}}$  and  $G_{\text{ut}}$  denote the antenna gain at the satellite and the UTs, respectively. The velocity of each UT at x, y, or z-axes is assumed to be uniformly distributed in  $[-10, 10]$  m/s.

For the LoS scenario, the delay spread depends on the maximal time delay through the UPA, which is given by [32]

$$\tau_{\text{max}} \triangleq \max_{n_x, n_y, \theta_k} \frac{(n_x - 1) \sin \theta_k^y \cos \theta_k^x + (n_y - 1) \cos \theta_k^y}{2f_c}, \quad (51)$$

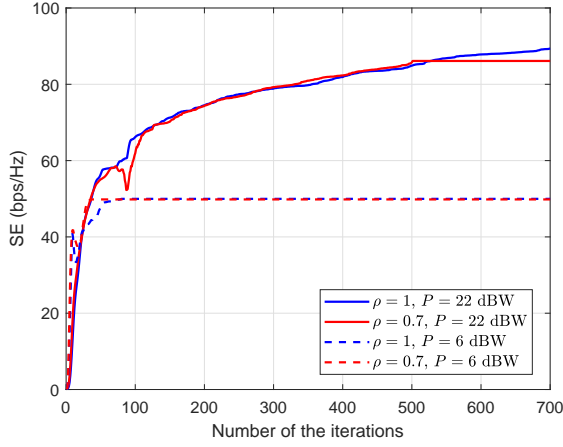
where  $n_x \in \{1, 2, \dots, N_t^x\}$  and  $n_y \in \{1, 2, \dots, N_t^y\}$ . Thus, for a system operating with 15.36 MHz efficient bandwidth with a  $24 \times 24$  UPA, we can divide the valid subcarriers into  $N_{\text{blk}} = 8$  blocks and each block consists of  $N_{\text{bsc}} = N_e/N_{\text{blk}} = 64$  subcarriers. We assign the same hybrid precoders for each block to reduce computational complexity.

Fig. 3 presents the values of the SE and APEB versus the iteration number under different transmit power budgets with different weighting coefficients for **Algorithm 1**. In general, with the increase of the iteration number, the performance of both communications and localization tends to be better. Then, it is observed from Fig. 3 that the proposed method can converge to certain points with satisfactory performance for typical scenarios.

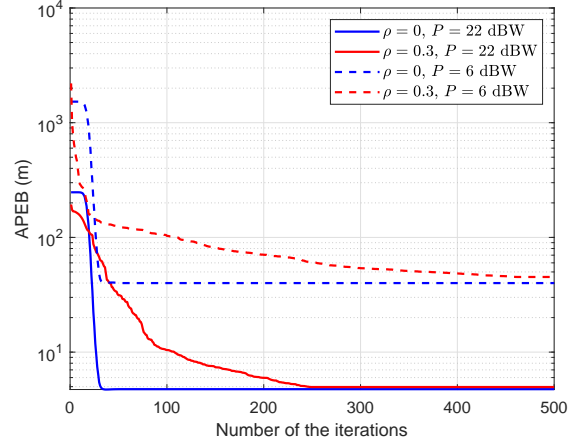
Fig. 4 illustrates the relationship between the transmit power and the SE as well as the APEB. As observed from Fig. 4(a), the tightness of the upper bound in Eq. (13) is verified. It is worth noting that  $\rho = 1$  and  $\rho = 0$  refer to the pure communication and pure localization system, respec-

TABLE I  
SIMULATION PARAMETERS

	Parameter	Value
Channel	System bandwidth $B_w$	$\sim 20$ MHz
	Carrier frequency $f_c$	2 GHz
	Speed of light $c$	$3 \times 10^8$ m/s
	Carrier wavelength $\lambda_c$	0.15 m
	Subcarrier separation $f_s$	30 kHz
	Number of valid subcarriers $N_{sc}$	512
	Sampling rate $1/T_s$	30.72 MHz
	Number of slots per frame $M_s$	20
	Number of CP $N_{cp}$	36
	Rician factor $\kappa_k$	18 dB
	Number of OFDM symbols per slot $M_{sp}, M_{sd}$	2, 12
Satellite	Orbit height $H$	200 km
	Number of antennas $N_t$	$24 \times 24$
	Antenna spacing $r_x, r_y$	$\lambda_c/2$
	Number of RF chains $N_{rf}$	36
	Antenna gain $G_{sat}, G_{ut}$	6 dB, 0 dB
UTs	Number of UTs $K$	9
	Noise spectral density	-174 dBm/Hz



(a) SE.



(b) APEB.

Fig. 3. The SE and APEB versus the number of iterations.

tively. The proposed ICAL system outperforms the pure communication and pure localization ones in APEB and SE, respectively. Besides, with larger transmit power, the SE sees an increase, and the APEB decreases, leading to better performance of both communications and localization.

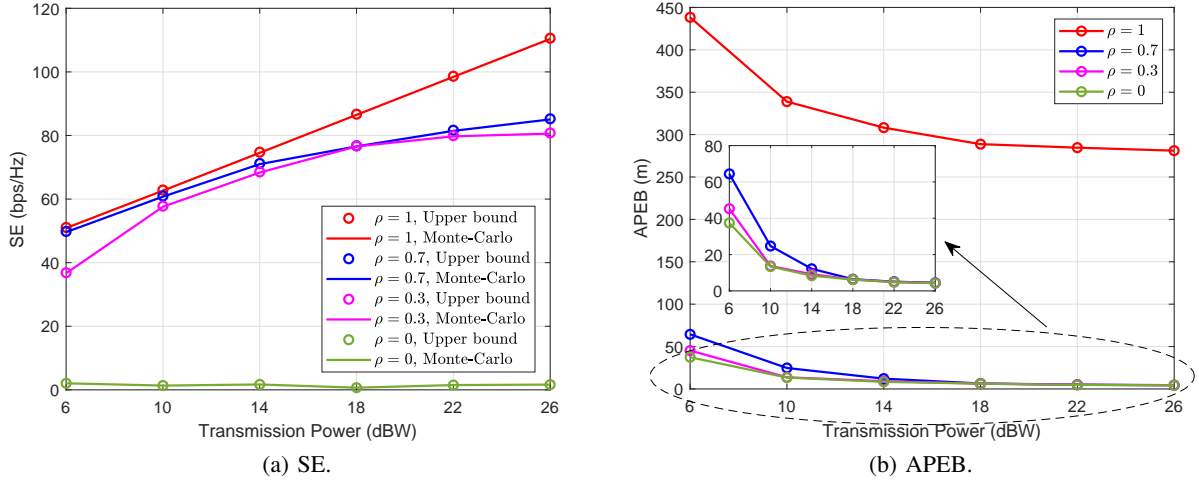


Fig. 4. SE and APEB performance versus transmit power  $P$  with  $N_t = 576$  antennas under the fully connected structure for different weighting coefficients  $\rho$ .

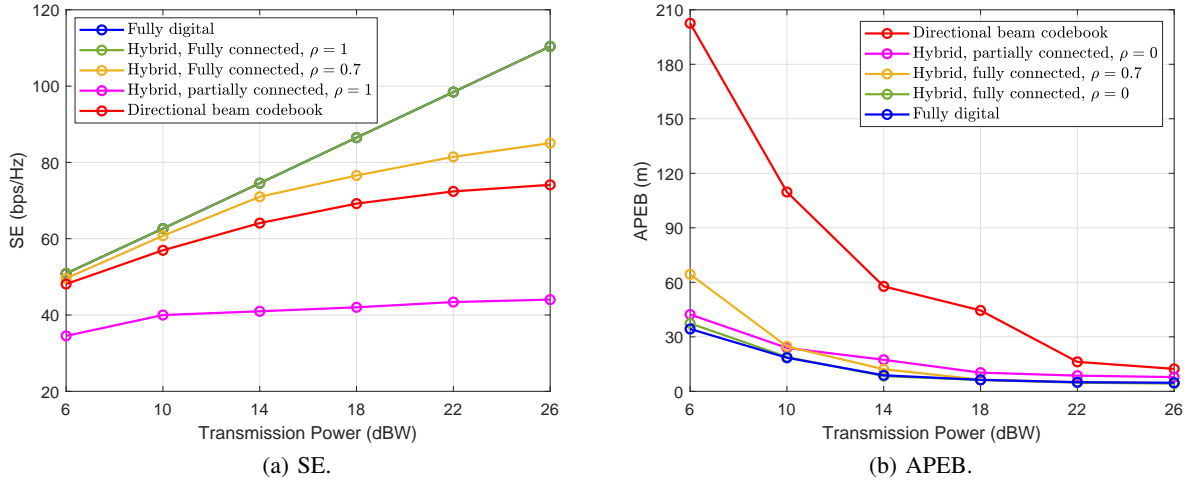


Fig. 5. SE and APEB performance versus transmit power  $P$  with  $N_t = 576$  antennas for different precoding design strategies.

Under the scenario that a weighting coefficient  $\rho = 0.7$  is allocated for communications, both the SE and APEB can be enhanced with satisfactory performance, especially with higher transmit power. With a smaller weighting coefficient, i.e.,  $\rho = 0.3$ , the ICAL system presents better APEB performance at the cost of the SE performance, compared with the case when  $\rho = 0.3$ . Based on the above analysis, it can be concluded that the proposed ICAL system can offer satisfying communication and localization performance, thanks to the high angular resolution brought by massive MIMO.



Fig. 5 compares the SE and APEB performance with different precoding design strategies as follows:

- The hybrid precoders with  $N_{\text{rf}}$  RF chains under both fully and partially connected structures for different weighting coefficients, as discussed in Section IV-A.
- The optimized fully digital precoder with  $N_t$  RF chains, i.e., the hybrid precoder with  $\mathbf{W}_{\text{RF}} = \mathbf{I}_{N_t}$  and  $\mathbf{W}_{\text{BB},n}$  optimized only [7].
- The standard directional beam codebook-based precoder with  $N_t$  RF chains, i.e.,  $\mathbf{W}_n = \sqrt{P/K/N_{\text{sc}}} [\mathbf{v}_{1,n}^*, \dots, \mathbf{v}_{K,n}^*]$  [47].

As observed from Fig. 5, for pure communications and localization, the hybrid scheme with the fully connected structure presents comparable performance with the fully digital scheme with fewer RF chains and thus, less static power consumption. Note that the proposed ICAL scheme with a fully connected structure for a weighting coefficient, e.g.,  $\rho = 0.7$ , is capable for supporting both communications and localization with satisfactory SE and APEB performance, especially with higher transmit power. In particular, with 26 dBW transmit power, the proposed ICAL scheme with a fully connected structure can offer the sum SE of approximately 85 bps/Hz for all the UTs, and guarantee the localization precision  $< 5$  m for each UT, simultaneously. Besides, in the low region of the transmit power, the hybrid scheme with a partially connected structure for  $\rho = 0$  has better APEB performance than with a fully connected one for  $\rho = 0.7$ , at the cost of the SE performance. In general, the directional beam codebook-based precoder performs worst in the medium and high transmit power regions.

## VI. CONCLUSION

In this paper, we proposed to simultaneously perform communications and localization for massive MIMO LEO satellite ICAL systems, by exploiting sCSI. We derived the performance metrics SE and SPEB for communications and localization, respectively, based on which we formulated a multi-objective problem, to design the hybrid transmitter for the ICAL. Then, the SE and SPEB maximization problem was converted into the sum-MSE and Euclidean distance minimization one. We introduced a weighting coefficient to tradeoff between the performance of communications and localization, and formulated an algorithmic framework to handle the corresponding problem. Simulation results confirmed that the proposed scheme could operate communications and localization with satisfactory performance.

## APPENDIX A

### EXPRESSION FOR THE MATRIX $\Xi_k$ IN (19)

We denote the rotated position vectors associated with UT  $k$  as [8],

$$\mathbf{p}_k^r \triangleq \mathbf{R}(\mathbf{o})^{-1} (\mathbf{p}_k - \mathbf{q}) \triangleq [p_k^{r,x}, p_k^{r,y}, p_k^{r,z}]^T, \quad (52)$$

where  $\mathbf{R}(\mathbf{o})$  is the corresponding rotation matrix, given by [27], [67], [68]

$$\mathbf{R}(\mathbf{o}) \triangleq \begin{bmatrix} \cos \varphi_2 & -\sin \varphi_1 \sin \varphi_2 & \cos \varphi_1 \sin \varphi_2 \\ 0 & \cos \varphi_1 & \sin \varphi_1 \\ -\sin \varphi_2 & -\sin \varphi_1 \cos \varphi_2 & \cos \varphi_1 \cos \varphi_2 \end{bmatrix} \triangleq [\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3]. \quad (53)$$

Then, the relationship between the AoD pair and the position of the  $k$ th UT is given by [27], [67], [68]

$$\theta_k^x = \arctan \left( \frac{p_k^{r,z}}{p_k^{r,x}} \right), \quad \theta_k^y = \arccos \left( \frac{p_k^{r,y}}{\|\mathbf{p}_k^r\|_2} \right). \quad (54)$$

Besides, the propagation delay and Doppler shifts of LoS path correspond to the position and the velocity information of the  $k$ th UT as follows [69]

$$\tau_k = \frac{\|\mathbf{p}_k - \mathbf{q}\|_2}{c}, \quad (55)$$

$$\nu_k = -\frac{f_c}{c} \frac{\dot{\mathbf{p}}_k^T (\mathbf{p}_k - \mathbf{q})}{\|\mathbf{p}_k - \mathbf{q}\|_2}. \quad (56)$$

Then, the component  $\Xi_k$  of the transformation matrix in (19) for UT  $k$  can be expressed as

$$\Xi_k = \begin{bmatrix} \frac{\partial \theta_k^x}{\partial \mathbf{p}_k} & \frac{\partial \theta_k^y}{\partial \mathbf{p}_k} & \frac{\partial \tau_k}{\partial \mathbf{p}_k} & \frac{\partial \nu_k}{\partial \mathbf{p}_k} \end{bmatrix} \in \mathbb{R}^{3 \times 4}, \quad (57)$$

where the elements are detailed as

$$\frac{\partial \theta_k^x}{\partial \mathbf{p}_k} = \frac{p_k^{r,x} \mathbf{r}_3 - p_k^{r,z} \mathbf{r}_1}{(p_k^{r,x})^2 + (p_k^{r,z})^2}, \quad \frac{\partial \theta_k^y}{\partial \mathbf{p}_k} = \frac{p_k^{r,y} (\mathbf{p}_k - \mathbf{q}) - \mathbf{r}_2 \|\mathbf{p}_k^r\|_2^2}{\|\mathbf{p}_k^r\|_2^2 \sqrt{(p_k^{r,x})^2 + (p_k^{r,z})^2}}, \quad \frac{\partial \tau_k}{\partial \mathbf{p}_k} = \frac{1}{c} \frac{\mathbf{p}_k - \mathbf{q}}{\|\mathbf{p}_k - \mathbf{q}\|_2}, \quad (58a)$$

$$\frac{\partial \nu_k}{\partial \mathbf{p}_k} = \frac{f_c}{c} \frac{(\dot{\mathbf{p}}_k^T (\mathbf{p}_k - \mathbf{q})) (\mathbf{p}_k - \mathbf{q}) - \|\mathbf{p}_k - \mathbf{q}\|_2^2 \dot{\mathbf{p}}_k}{\|\mathbf{p}_k - \mathbf{q}\|_2^3}. \quad (58b)$$

## APPENDIX B

### EXPRESSION FOR FIM

With the notation  $\mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H = \mathbf{C}_n$ , the elements of the FIM  $\mathbf{J}_{\eta_k} (\mathbf{C}_n)$  is detailed as

$$\mathbf{J}_{\theta_k^d, \theta_k^{d'}} = \sum_{n=1}^{N_{\text{sc}}} \frac{2M}{N_{k,n}^{\text{eq}}} |\alpha_k|^2 \mathbf{v}_{k,n, \theta_k^{d'}}^H \mathbf{C}_n \mathbf{v}_{k,n, \theta_k^d}, \quad d, d' \in \{x, y\}, \quad (59a)$$

$$\mathbf{J}_{\theta_k^d, \tau_k} = \sum_{n=1}^{N_{\text{sc}}} \frac{2M}{N_{k,n}^{\text{eq}}} |\alpha_k|^2 \Re \left\{ -j \frac{2\pi}{T_{\text{sc}}} \left( n \mathbf{v}_{k,n}^H \mathbf{C}_n \mathbf{v}_{k,n, \theta_k^d} \right) \right\}, \quad (59b)$$

$$\mathbf{J}_{\theta_k^d, \nu_k} = \sum_{n=1}^{N_{\text{sc}}} \frac{M(M+1)}{N_{k,n}^{\text{eq}}} |\alpha_k|^2 \Re \left\{ j 2\pi T \left( \mathbf{v}_{k,n}^H \mathbf{C}_n \mathbf{v}_{k,n, \theta_k^x} \right) \right\}, \quad (59c)$$

$$\mathbf{J}_{\theta_k^d, \alpha_k^{\text{R}}} = \sum_{n=1}^{N_{\text{sc}}} \frac{2M}{N_{k,n}^{\text{eq}}} \Re \left\{ \alpha_k^* \left( \mathbf{v}_{k,n}^H \mathbf{C}_n \mathbf{v}_{k,n, \theta_k^x} \right) \right\}, \quad (59d)$$

$$\mathbf{J}_{\theta_k^d, \alpha_k^{\text{I}}} = \sum_{n=1}^{N_{\text{sc}}} \frac{2M}{N_{k,n}^{\text{eq}}} \Re \left\{ j \alpha_k^* \left( \mathbf{v}_{k,n}^H \mathbf{C}_n \mathbf{v}_{k,n, \theta_k^x} \right) \right\}, \quad (59e)$$

$$\mathbf{J}_{\tau_k, \tau_k} = \sum_{n=1}^{N_{\text{sc}}} \frac{2M}{N_{k,n}^{\text{eq}}} |\alpha_k|^2 4\pi^2 f_s^2 \left( n^2 \mathbf{v}_{k,n}^H \mathbf{C}_n \mathbf{v}_{k,n} \right), \quad (59f)$$

$$\mathbf{J}_{\tau_k, \nu_k} = - \sum_{n=1}^{N_{\text{sc}}} \frac{M(M+1)}{N_{k,n}^{\text{eq}}} |\alpha_k|^2 4\pi^2 T f_s \left( n \mathbf{v}_{k,n}^H \mathbf{C}_n \mathbf{v}_{k,n} \right), \quad (59g)$$

$$\mathbf{J}_{\tau_k, \alpha_k^{\text{R}}} = -\mathbf{J}_{\tau_k, \alpha_k^{\text{I}}} = \sum_{n=1}^{N_{\text{sc}}} \frac{2M}{N_{k,n}^{\text{eq}}} \Re \left\{ j 2\pi f_s \alpha_k^* \left( n \mathbf{v}_{k,n}^H \mathbf{C}_n \mathbf{v}_{k,n} \right) \right\}, \quad (59h)$$

$$\mathbf{J}_{\nu_k, \nu_k} = \sum_{n=1}^{N_{\text{sc}}} \frac{2M(M+1)(2M+1)}{6N_{k,n}^{\text{eq}}} |\alpha_k|^2 4\pi^2 T^2 \left( \mathbf{v}_{k,n}^H \mathbf{C}_n \mathbf{v}_{k,n} \right), \quad (59i)$$

$$\mathbf{J}_{\nu_k, \alpha_k^{\text{R}}} = \sum_{n=1}^{N_{\text{sc}}} \frac{M(M+1)}{N_{k,n}^{\text{eq}}} \Re \left\{ j 2\pi T \alpha_k \left( \mathbf{v}_{k,n}^H \mathbf{C}_n \mathbf{v}_{k,n} \right) \right\}, \quad (59j)$$

$$\mathbf{J}_{\nu_k, \alpha_k^{\text{I}}} = \sum_{n=1}^{N_{\text{sc}}} \frac{M(M+1)}{N_{k,n}^{\text{eq}}} \Re \left\{ 2\pi T \alpha_k \left( \mathbf{v}_{k,n}^H \mathbf{C}_n \mathbf{v}_{k,n} \right) \right\}, \quad (59k)$$

$$\mathbf{J}_{\alpha_k^{\text{R}}, \alpha_k^{\text{R}}} = \mathbf{J}_{\alpha_k^{\text{I}}, \alpha_k^{\text{I}}} = \sum_{n=1}^{N_{\text{sc}}} \frac{2M}{N_{k,n}^{\text{eq}}} \left( \mathbf{v}_{k,n}^H \mathbf{C}_n \mathbf{v}_{k,n} \right), \quad (59l)$$

where

$$\mathbf{v}_{k,n, \theta_k^x} \triangleq \frac{\partial \mathbf{v}_{k,n}^x}{\partial \theta_k^x} \otimes \mathbf{v}_{k,n}^y, \quad \mathbf{v}_{k,n, \theta_k^y} \triangleq \frac{\partial \mathbf{v}_{k,n}^x}{\partial \theta_k^y} \otimes \mathbf{v}_{k,n}^y + \mathbf{v}_{k,n}^x \otimes \frac{\partial \mathbf{v}_{k,n}^y}{\partial \theta_k^y}, \quad (60a)$$

$$\frac{\partial \mathbf{v}_{k,n}^x}{\partial \theta_k^x} = \frac{j\varpi_n \sin \theta_k^y \sin \theta_k^x}{\sqrt{N_t^x}} [0 \exp\{-j\varpi_n \sin \theta_k^y \cos \theta_k^x\} \dots (N_t^x - 1) \exp\{-j\varpi_n (N_t^x - 1) \sin \theta_k^y \cos \theta_k^x\}]^T, \quad (60b)$$

$$\frac{\partial \mathbf{v}_{k,n}^x}{\partial \theta_k^y} = \frac{-j\varpi_n \cos \theta_k^y \cos \theta_k^x}{\sqrt{N_t^x}} [0 \exp\{-j\varpi_n \sin \theta_k^y \cos \theta_k^x\} \dots (N_t^x - 1) \exp\{-j\varpi_n (N_t^x - 1) \sin \theta_k^y \cos \theta_k^x\}]^T, \quad (60c)$$

$$\frac{\partial \mathbf{v}_{k,n}^y}{\partial \theta_k^y} = \frac{j\varpi_n \sin \theta_k^y}{\sqrt{N_t^y}} [0 \exp\{-j\varpi_n \cos \theta_k^y\} \dots (N_t^y - 1) \exp\{-j\varpi_n (N_t^y - 1) \cos \theta_k^y\}]^T. \quad (60d)$$

### APPENDIX C

#### PROOF FOR EQ. (31)

The precoder  $\mathbf{W}_n^{\text{loc}}$  can be decomposed as

$$\mathbf{W}_n^{\text{loc}} = \Pi_{\mathbf{L}_n} \mathbf{W}_n^{\text{loc}} + \Pi_{\mathbf{L}_n}^\perp \mathbf{W}_n^{\text{loc}}, \quad (61)$$

where  $\Pi_{\mathbf{L}_n} \triangleq \mathbf{L}_n (\mathbf{L}_n^H \mathbf{L}_n)^{-1} \mathbf{L}_n^H$  denotes the subspace spanned by the columns of  $\mathbf{L}_n$  and  $\Pi_{\mathbf{L}_n}^\perp \triangleq \mathbf{I}_{N_t} - \Pi_{\mathbf{L}_n}$ . Following the equality that  $\mathbf{C}_n = \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H$ , we have  $\mathbf{C}_n = \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H = \Pi_{\mathbf{L}_n} \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H (\Pi_{\mathbf{L}_n})^H + \tilde{\mathbf{C}}_n$ , where  $\tilde{\mathbf{C}}_n = \Pi_{\mathbf{L}_n} \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H \Pi_{\mathbf{L}_n}^\perp + \Pi_{\mathbf{L}_n}^\perp \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H \Pi_{\mathbf{L}_n} + \Pi_{\mathbf{L}_n}^\perp \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H \Pi_{\mathbf{L}_n}^\perp$ . It is worth noting that since  $\mathbf{v}_{k,n}$ ,  $\mathbf{v}_{k,n,\theta_k^x}$  and  $\mathbf{v}_{k,n,\theta_k^y}$  belong to the subspace  $\Pi_{\mathbf{L}_n}$ , it is not difficult to verify that the terms  $\mathbf{v}_{k,n}^H \tilde{\mathbf{C}}_n \mathbf{v}_{k,n}$ ,  $\mathbf{v}_{k,n}^H \tilde{\mathbf{C}}_n \mathbf{v}_{k,n,\theta_k^x}$ ,  $\mathbf{v}_{k,n}^H \tilde{\mathbf{C}}_n \mathbf{v}_{k,n,\theta_k^y}$ ,  $\mathbf{v}_{k,n,\theta_k^x}^H \tilde{\mathbf{C}}_n \mathbf{v}_{k,n,\theta_k^x}$ ,  $\mathbf{v}_{k,n,\theta_k^y}^H \tilde{\mathbf{C}}_n \mathbf{v}_{k,n,\theta_k^y}$ ,  $\mathbf{v}_{k,n,\theta_k^x}^H \tilde{\mathbf{C}}_n \mathbf{v}_{k,n,\theta_k^y}$  are all equal to zeros. Thus, it can be concluded that the FIM does not depend on  $\tilde{\mathbf{C}}_n$ . On the other hand, we have

$$\begin{aligned} \text{Tr}(\tilde{\mathbf{C}}_n) &= \text{Tr}(\Pi_{\mathbf{L}_n} \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H \Pi_{\mathbf{L}_n}^\perp + \Pi_{\mathbf{L}_n}^\perp \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H \Pi_{\mathbf{L}_n} + \Pi_{\mathbf{L}_n}^\perp \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H \Pi_{\mathbf{L}_n}^\perp) \\ &= \left\| (\mathbf{W}_n^{\text{loc}})^H \Pi_{\mathbf{L}_n}^\perp \right\|_F^2 \geq 0. \end{aligned} \quad (62)$$

Furthermore,  $\mathbf{C}_n$  is subject to the constraint in Eq. (30b), and thus, it can be concluded that  $\text{Tr}(\tilde{\mathbf{C}}_n) = 0$ , otherwise the constraint in Eq. (30b) is not satisfied. As observed from Eq. (62),  $\text{Tr}(\tilde{\mathbf{C}}_n) = 0$  is equivalent to  $(\mathbf{W}_n^{\text{loc}})^H \Pi_{\mathbf{L}_n}^\perp = 0$ , i.e.,  $\tilde{\mathbf{C}}_n = 0$ . Therefore,

$$\mathbf{C}_n = \Pi_{\mathbf{L}_n} \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H (\Pi_{\mathbf{L}_n})^H \triangleq \mathbf{L}_n \mathbf{Z}_n \mathbf{L}_n^H, \quad (63)$$

where  $\mathbf{Z}_n = (\mathbf{L}_n^H \mathbf{L}_n)^{-1} \mathbf{L}_n^H \mathbf{W}_n^{\text{loc}} (\mathbf{W}_n^{\text{loc}})^H \mathbf{L}_n (\mathbf{L}_n^H \mathbf{L}_n)^{-1}$ .

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