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REVIEW ARTICLE

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An overview of solutions to the bus bunching problem in urban bus systems

Abstract Bus bunching has been a persistent issue in urban bus system since it first appeared, and it remains a challenge not fully resolved. This phenomenon may reduce the operational efficiency of the urban bus system, which is detrimental to the operation of fast-paced public transport in cities. Fortunately, extensive research has been undertaken in the long development and optimization of the urban bus system, and many solutions have emerged so far. The purpose of this paper is to summarize the existing solutions and serve as a guide for subsequent research in this area. Upon careful examination of current findings, it is found that, based on the different optimization objects, existing solutions to the bus bunching problem can be divided into five directions, i.e., operational strategy improvement, traffic control improvement, driver driving rules improvement, passenger habit improvement, and others. While numerous solutions to bus bunching are available, there remains a gap in research exploring the integrated application of methods from diverse directions. Furthermore, with the development of autonomous driving, it is expected that the use of modular autonomous vehicles could be the most potential solution to the issue of bus bunching in the future.

Keywords bus bunching, operation strategy, traffic control, driver driving rules, passenger habits

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1 Introduction

In urban bus operation, headway refers to the time interval between two consecutive buses traveling along the same route. For bus service to be deemed reliable and efficient, the headway between each bus at a given stop should remain consistent, without experiencing abrupt changes during operation (El-Geneidy et al., 2011; Durán-Hormazábal and Tirachini, 2016; Muñoz et al., 2020; Yu et al., 2022). However, the nature of public transport operations on a typical urban network is highly stochastic because of interactions with automotive traffic, passenger demand, and signal timing, all of which are stochastic processes (Loder et al., 2017; Dakic et al., 2020; Hu et al., 2022a; Ji et al., 2022; Qu et al., 2022a; Li et al., 2022a; 2022c; Liu et al., 2022a; 2023a; Lin et al., 2023a; 2023b; Zeng and Qu, 2023). As a result, delays can occur at any time when buses are running, as shown in Fig. 1. When a bus is delayed, its headway to the preceding vehicle is likely to increase, causing it to arrive at the station later than planned. The delayed bus is then more likely to encounter a higher volume of passengers at downstream stops, which increases its dwell time at those stops, leading to further delays and creating a positive feedback loop (Liu et al., 2022c). Conversely, the bus following the delayed one is likely to face fewer passengers and shorter dwell time at the next stop, thus increasing the likelihood of many buses “clustering” together with short headways. This phenomenon is described as bus bunching, which was initially analyzed and described by Newell and Potts (1964) and empirically demonstrated in other several studies (e.g., Strathman et al., 1999; Hammerle et al., 2005; El-Geneidy et al., 2011; Cats, 2014; Feng and Figliozzi, 2015; Byon et al., 2018).

Bus bunching can have many adverse effects on urban public transport. It can lead to not only an increase in expected waiting times for passengers but also less predictable waiting times (Li et al., 2010). Meanwhile, Verbich et al. (2016) found that when bunching occurs, both bus dwell time and running time will increase,

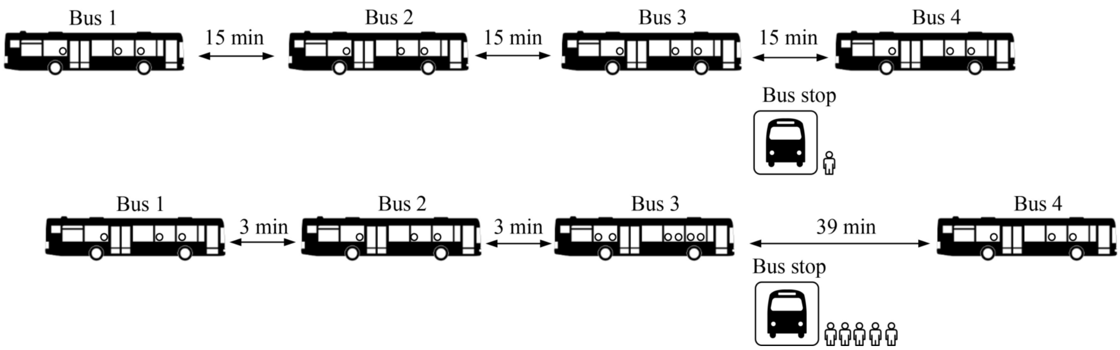

Fig. 1 Schematic diagram of bus bunching.

thereby reducing the overall traveling experience of passengers. As for society as a whole, from a macro perspective in terms of operations, bus bunching also has a negative impact on urban transport. It contributes to route-wide congestion, which in turn prolongs the dwell times for buses, lowering the system's overall operational efficiency. In urban public transport systems, to mitigate the inefficiencies induced by bus bunching and meet the demands of public transport, it is often necessary to allocate additional manpower and vehicles, which puts a burden on the urban public transport system and also causes financial losses (Furth and Muller, 2000).

There are many triggering reasons for bus bunching, including but not limited to the following ones. Inefficient scheduling: One of the primary issues lies in bus scheduling, for example, often designed without sufficient consideration of the number of people waiting at each stop and the rate at which waiting passengers increase. Such oversights can result in buses not having enough time to complete their routes, leading to erratic driving speeds and, consequently, bus bunching (Gkiotsalitis and Cats, 2018). Irregular dwell time and variability in passenger demand: Bus stops often experience varying levels of passenger demand, sometimes even with a sudden influx of passengers (Zhong et al., 2023). The surge in boarding and alighting passenger numbers typically increases the boarding and alighting time per individual, which can increase the variability of the dwell time, contributing to the increased headways between consecutive buses (Milkovits, 2008; Sun et al., 2014). Poor traffic conditions: Traffic congestion caused by accidents on the roads can induce delays in bus travel times, resulting in certain buses falling behind their scheduled timetable and clustering together at subsequent stops (Comi et al., 2017; Byon et al., 2018). Driving behavior: Unreasonable driving behaviors of certain vehicles also cause vehicles to brake or yield multiple times (Liu et al., 2021). Some bus drivers, particularly those who have less experience, may struggle to accurately determine the optimal running speed of the bus and identify imminent bunching scenarios. This inability to properly adjust the speed or trajectory of the vehicle in time when the bunching occurs may exacerbate the occurrence of the bunching (El-Geneidy et al., 2011; Li et al., 2022d; Lyu et al., 2022). Furthermore, bus bunching often results from a combination of these factors. Tirachini et al. (2022) highlighted contributors to headway variations, including initial headway differences, passenger volume, and the frequency of stops. These factors cumulatively contribute to bus bunching.

In summary, while advancements have been made, the challenge of bus bunching persists. This paper aims to shed light on existing mitigation techniques, categorizing them for clarity. Solutions typically align with problem causes. Thus, Section 2 outlines and categorizes the underlying causes of bus bunching. Based on these, we will explore targeted solutions, also touching upon other potential remedies not directly linked to the aforementioned categories. Our review identifies five broad solution categories: Operational strategy, traffic control, driver guidelines, passenger habits, and others. Subsequent sections detail strategies under each category: Section 3 on operational strategies, Section 4 on traffic control, Section 5 on driver guidelines, Section 6 on passenger habits, and Section 7 on miscellaneous solutions. Section 8 wraps up our review and suggests future directions for addressing bus bunching.

2 Analysis of the causes of bus bunching

In a comprehensive review on headway variability, by Tirachini et al. (2022), they categorized the causes of bus bunching into seven categories, each of which is supported by numerous examples. To better analyze the reasons and search the solutions for bus bunching, we base our classification of the cause on Tirachini's paper and make a new classification, as shown in Table 1. The primary motivation for this is that Tirachini's categories are highly specific, while a coarse classification may work better for summarization. By using our new categories to investigate solutions for bus bunching, it is easier to expand the scope of research and achieve a more comprehensive understanding, since it receives fewer

Table 1 Classification of causes of bus bunching and related literature

Tirachini et al. (2022)'s classification	Paper	Contribution	New classification
Headway variability at the beginning of the route	Hammerle et al. (2005); El-Geneidy et al. (2011); Diab et al. (2016); Arriagada et al. (2019); Soza-Parra et al. (2021)	Find that the headway variability at the beginning of the route is one of the most important variables that influence the headway regularity of buses along a route	Operational strategy
	Godachevich and Tirachini (2021)	Identify the bus frequency, number of bus services per terminal, distance from the bus depot to the first stop, route length, bus demand, and operating speed as important variables that are significant in influencing headway variability at vehicle dispatching	
	Cham (2006)	Argues that unreasonable assignment of drivers of different ages or experiences on the roads contributes to the headway variability	
Scheduled frequency	Figliozzi et al. (2012); Diab et al. (2016); Arriagada et al. (2019)	Proof of positive correlation between scheduled service frequency and bus bunching probability	
	Tirachini et al. (2014); Gkiotsalitis and Cats (2018)	Show that scheduled service frequency has a negative correlation with passenger waiting time, which should be taken into account when searching for the optimal service frequency	
Distance traveled from the beginning of the route	Chen et al. (2009); Figliozzi et al. (2012); Sáez et al. (2012); Soza-Parra et al. (2021)	Demonstrate that the probability of bus bunching will increase as the total bus travel distance increased	
Traffic conditions and right-of-way	Comi et al. (2017); Byon et al. (2018)	Point out that accidents and congestion are the sources of bunching and travel time variability	Traffic control
	Durán-Hormazábal and Tirachini (2016)	Indicate that the right-of-way for buses reduces not only the average travel time but also the travel time variability	
	Arriagada et al. (2019)	Prove that segregated busways can reduce headway variability, but bus lanes that can also be used by right-turning cars do not necessarily improve headway regularity	
Number of traffic signals downstream of a bus stop	Abkowitz and Engelstein (1983); Cats (2019)	Show that the number of traffic signals is positively correlated with the mean and variance of traveling time	
	Arriagada et al. (2019); Soza-Parra et al. (2021)	Find that traffic signals increase the headway variability	
	Furth and Muller (2000); Albright and Figliozzi (2012); Danaher (2020b)	Show that traffic signals can also reduce headway variability if transit signal priority is applied considering headway variations from the schedule	
Driver behavior	Danaher (2020a)	Indicates that the headway reliability can be improved if the driver is in good shape and working conditions	Driver behavior
	Strathman et al. (2002); Martinez et al. (2018); Cats (2019)	Find that different travel time can occur due to personal reasons or habits of the drivers	
	El-Geneidy et al. (2011)	Prove that driver experience is negatively related to headway variability and travel time variability	
Passenger demand and dwell time variability	El-Geneidy et al. (2011); Albright and Figliozzi (2012); Arriagada et al. (2019)	Indicate that the number of boarding and alighting passengers is an important variable affecting bus bunching	Passenger demand and dwell time variability
	Milkovits (2008)	Indicates that large demand due to the crowding effect would increase the amount of time it takes each passenger to board and alight from the bus	
	Sun et al. (2014)	Prove that the amount of time each passenger spends boarding and alighting the bus will increase the headway variability	
	Danaher (2020b); Soza-Parra et al. (2021)	Point out that upgrading the fare collection system, increasing the speed of bus fare collection, and changing the rules for boarding buses could reduce headway variability	
	El-Geneidy et al. (2011); Albright and Figliozzi (2012)	Indicate that frequent stop activity will increase headway variability	

restrictions under the coarse categories when exploring the solutions. According to Table 1, it can be seen that bus bunching is caused by many factors. The first three categories "Headway variability at the beginning of the route", "Scheduled frequency", and "Distance traveled from the beginning of the route" in Tirachini's classification essentially correspond to operational strategy. Hence, to simplify the summarization of subsequent solutions, we have combined these three causes into one category, termed "Operational strategy". The next two causes "Traffic conditions and right-of-way" and "Number of traffic signals downstream of a bus stop" in Tirachini's classification are essentially due to traffic control. Similarly, for easier summarization, we have combined them into the category "Traffic control". For "Driver behavior" and "Passenger demand and dwell time variability", the original classification is still maintained. In this way, the main causes of bus bunching can be divided into four main categories as shown in the column "New classification". Subsequent summaries of solutions to bus bunching problems are organized based on each category of the causes. Additionally, it is worth noting that some causes of bunching may not be captured by either Tirachini's or our new classification framework. As such, these causes are excluded from Table 1. Moreover, some of the solutions may not correspond to any of the categories of causes mentioned above. However, to provide a better and more comprehensive summary of the solutions to bus bunching, we still identify and summarize them in the end. In the subsequent sections, the solutions to the problem of bus bunching are summarized across the following five directions, i.e., operational strategy improvement, traffic control improvement, driver driving rules improvement, passenger habits improvement, and other directions.

3 Solutions for operational strategy improvement

In the direction of operational strategy improvement, the problem of bunching is mitigated by redesigning bus operational strategies to include more dynamic designs that can adjust to actual demand. The main solutions include the establishment of a dynamic timetable, self-organizing control, dynamic waiting times, and bus insertion/substitution. Detailed descriptions of each solution are provided below, and a summary of all references can be found in Table 2 for convenience.

Dynamic timetable: The implementation of a dynamic timetable entails updating the schedule in real-time based on current road conditions, so as to guide passengers waiting for the bus and bus operation in a more rational way. Dynamic timetable brings advantages in terms of saving both the waiting time of passengers and the total transport costs of passengers and companies. Koppisetti et al. (2018) investigated the inherent trade-offs of the relative speeds between the leading and trailing buses to derive a cost-optimal bus schedule. Using Monte Carlo simulations, the study validated the efficacy of its proposed Poisson arrival system and identified the optimal frequency for bus operations. The findings suggest that as the stochasticity of traffic (variance of travel time) increases, it becomes optimal to decrease the bus frequency. This holds true even when the load (passenger arrivals) increases; a low bus frequency can still save more waiting time for passengers. Accordingly, a dynamic timetable is designed so that bus routes vary in departure times throughout the day to alleviate the bus bunching. Additionally, Bueno-Cadena and Muñoz (2017) examined the cost implications of bus bunching from the aspect of frequency and found that sensible control operations contribute to lower costs for both passengers and operations. However, their study also points to the problem of dynamic schedules, i.e., schedules are changing irregularly, which can cause disruption to passengers' travel plans.

Self-organizing control: Self-organizing control involves setting up specific control points — different from bus stops — along the bus routes. When there is a tendency of bunching, certain buses are instructed to wait at the control points according to actual conditions, so as to interrupt the positive feedback loop that leads to bus bunching. Research on self-organizing control primarily focuses on the number and distribution of control points and the waiting times of the buses at these points. Berrebi et al. (2018) assessed various methods of allowing passengers to board at control points, measuring their impact on headway instability and average waiting time. The sensitivity of these methods to parameterization and the number of control points was also examined. The result suggested that the methods can keep the bus traveling stable for a long time at high control point density.

Table 2 Summary of the solutions for operational strategy improvement direction

Classification	Paper	Contribution
Dynamic timetable	Koppisetti et al. (2018)	Derive joint-cost optimal bus schedule and find the relationship between bus service frequency and variance of travel time
	Bueno-Cadena and Muñoz (2017)	Derive the relationship between the cost and the bus frequency and find the corresponding solutions
Self-organizing control	Berrebi et al. (2018)	A predictive approach to managing control points was found to be more effective than a progress-based approach
	He et al. (2020)	Drive the relationship between the number of control points and the alleviation of the bus stop
Dynamic waiting time	He (2015)	Investigate a strategy for adaptively determining the waiting time at the platform and the speed of inbound regulation
	Andres and Nair (2017)	Combine data-driven headway prediction with a dynamic holding strategy that allows for adjusting the dwell time of a bus at a stop
	Moreira-Matias et al. (2016)	Optimize bus dwell time using a mixed machine-learning framework
	Wang and Sun (2020)	Propose a multi-agent deep reinforcement learning to manage static and dynamic holding control
Bus insertion/ substitution	Morales et al. (2019)	Develop a stochastic model based on the second moments of headway distribution to determine whether one or more buses are worth retaining for insertion into a public transport service
	Petit et al. (2018)	Establish a discrete infinite horizon approximate dynamic programming method to find the optimal strategy that minimizes the total agent and passenger costs
	Petit et al. (2019)	Model the insertion decision and the repositioning decision of the alternate buses, as stochastic dynamic programming to obtain the optimal strategy that minimizes the system-wide cost

Prediction-based methods achieve an optimal balance between maintaining headway regularity and minimizing holding time over a wide range. The effectiveness of the method was confirmed through a case study involving the route of TriMet 72 in Portland, Oregon, USA. However, there are some negative consequences of this approach, as some residents of Portland do not understand the intent of this practice, so often there are complaints from passengers while waiting at control points. He et al. (2020) smartly utilized the average instantaneous headway as the dynamic target headway to determine waiting time at various control points. Their research indicated that service efficiency initially improves but later decreases as the number of control points increases. Also, a new strategy was proposed to cope with off-peak, single-peak, and double-peak situations, based on the study of control waiting time above. Both Berrebi et al. (2018) and He et al. (2020) studied the number of control points, but He et al. (2020) made a more detailed study. The difference of the number of the control points for off-peak, single-peak, and double-peak situations of traffic flow are explained in their research.

Dynamic waiting time: The dynamic waiting time strategy adjusts the length of time a bus stops at a bus stop in real-time to prevent buses from bunching. This strategy involves holding buses at stops as needed, rather than departing immediately after all passengers have boarded, aiming to ensure consistent headways. While this strategy typically does not shorten the waiting time — the bus will not leave before all passengers are on board — it does differ from self-organized control. The key distinction is that dynamic waiting time is applied at bus stops, whereas self-organized control operates at control points. He (2015) explored an adaptive strategy for determining the waiting time at the stops and regulating the speed of incoming buses to minimize the risk of bunching. With the ability to mitigate bunching, buses are able to adhere closely to schedules while maintaining high operating speeds. However, this will also bring about some moral problems, such as buses not departing immediately after passengers have finished boarding, and some of the passengers who are on the buses will complain. Andres and Nair (2017) addressed the problem of bunching by combining data-driven headway prediction with a dynamic hold strategy, which allows for adjusting the dwell time at bus stops to minimize headway deviation. Several prediction methods were compared on the data collected from a busy bus route in Dublin, Ireland. Both He (2015) and Andres and Nair (2017) analyzed the residence time of vehicles at the station in their studies. But because of moral problems, such as buses not departing immediately after passengers have finished boarding, some of the passengers who are on the buses will complain. He (2015) proposed another method in his study, which deliberately made the bus run at a lower speed after it leaves from the bus stop, so as to achieve the same delaying effect as waiting at the bus stop. From the perspective of experience, this approach can reduce passenger complaints to a certain extent. Interestingly, machine learning techniques have also been increasingly employed to optimize waiting times in recent years. Deep learning, supported by bus operational data, has also been used to identify optimal waiting time (Moreira-Matias et al., 2016; Liu et al., 2019; 2022b; 2023b; Wang and Sun, 2020; Ding et al., 2022; Li et al., 2022b; Xu et al., 2022b; Qu et al., 2022b; 2023; Cheng et al., 2023).

Bus insertion/substitution: Bus insertion and substitution involves directing spare buses at bus terminals or specific points along the route. When bus bunching occurs, these buses will be inserted and fill in long gaps between buses in service. While there is no strict distinction between bus insertion and substitution, their primary aim is to alleviate bus bunching by introducing spare buses to bus routes. Extensive research has been conducted on determining the criteria for when to insert these additional vehicles, as well as on the optimal number of spare buses to have on standby. Morales et al. (2019) proposed a bus insertion strategy that keeps spare buses at specific locations along a route, ready to be dispatched in cases of severe bus bunching. They developed a stochastic model based on the second moments of headway distribution to determine whether retaining one or more buses for potential insertion is beneficial. The model employs a single-stop approach to derive an expression for the optimal headway threshold that would trigger insertion. A comprehensive service model is then developed to determine when an empty bus should be inserted within the headway interval. According to their calculations, a bus should be inserted when the headway exceeds 57% of the normal value. Simulations based on real-world data were conducted to verify the performance of the model in measuring the impact on waiting times. Results showed that reserving one bus for insertion is more effective than running the entire fleet continuously. Petit et al. (2018) investigated an alternative strategy for public transit currently implemented by a number of transit agencies in an ad-hoc manner. This strategy involves deploying a fleet of standby buses to take over any early or late bus service in order to alleviate deviations from the schedule. They established a discrete infinite horizon approximate dynamic programming method to find the optimal strategy that minimizes the total agent and passenger costs. Numerical examples showed that scheduling deviations can be controlled by periodically inserting spare buses as substitutes. The proposed strategy exhibits strong performance and outperforms conventional schedule-based control schemes in some scenarios. Furthermore, in comparison to other studies, they mentioned that if we considering the emerging opportunities associated with autonomous driving, the performance of this strategy may become even stronger in the future thanks to the reduced cost of retaining a fleet of spare buses. Petit et al.

(2019) further investigated strategies for dynamic substitution of multi-bus routes in time-varying and non-time-varying setups. An alternate fleet of buses can be utilized dynamically to save costs. They used stochastic dynamic programming to model both the insertion decision and the repositioning decision of the alternate fleet, aiming to find the optimal strategy that minimizes system-wide cost. Numerical analysis demonstrated that this dynamic substitution strategy can achieve economies of scale by concentrating the spare fleet on individual routes. The dynamic substitution strategy also proves beneficial when faced with fluctuating transport demand. Numerical analysis confirmed the feasibility and superiority of this strategy; it not only is more cost-effective than traditional holding strategies but also can be used as a supplement to other methods for better control of highly volatile bus systems. However, in the conclusion section of their study, they still raised the problem of the bus insertion/substitution, that is, the situation is different in different regions, and if the spare vehicles are often left idle, it will still result in a certain amount of wastage of vehicle resources.

4 Solutions for traffic control improvement

Traffic control improvement mainly involves utilizing traditional traffic control methods to adjust particular vehicle actions, thereby enhancing overall traffic flow. Rather than solely improving public transport itself, the aim is to optimize the whole traffic system to minimize headway variability, thus alleviating bus bunching issues. Key strategies include vehicle trajectory adjustment, signal light control, and right-of-way management. Further details on these solutions follow, and all references are summarized in Table 3 for ease of reading.

Vehicle trajectory adjustment: The main purpose of vehicle trajectory adjustment is to refine driving behaviors like overtaking, converging, and turning, ultimately aiming to streamline overall traffic flow (Aramrattana et al., 2022; Fang et al., 2022; Hu et al., 2022b; Mo et al., 2022; Zhao et al., 2022; Andreotti et al., 2023; He et al., 2023; Yanumula et al., 2023). It is expected to reduce both the severity and frequency of congestion, bringing about a better traffic environment. This, in turn, assists in reducing the headway variability and lowering the likelihood of bus bunching. Research specifically focusing on vehicles turning at intersections is popular in this area. Xu et al. (2022a) found that most vehicles tend to deviate to the left from the center of the lane. Moreover, vehicles are inclined to move towards the road center on the bridge section. The study also observed that female drivers pay more attention to controlling vehicle trajectories compared to male. Such behavioral variances can contribute to congestion, subsequently affecting the stability of the headway of buses. It was suggested that drivers should actively strive to adjust their vehicles' trajectories to reduce road congestion. Xu et al. (2022c) applied mixed integer linear programming to analyze various trajectories when approaching an intersection. The problem was simplified and optimized through a discretization approach. Results showed that their model significantly improves the efficiency of traffic flow at intersections in scenarios with both high and medium traffic volume, and both road congestion and bus bunching can be alleviated. However, it was also pointed out that the model is more demanding in terms of driver psychology and behavioral norms, which means that it is less effective if the drivers do not follow their planned routes. Furthermore, Yang et al. (2022) devised an optimal control model for left-turning vehicle platoons at intersections. Their model simultaneously optimizes lateral lane-changing and longitudinal acceleration, aiming to boost traffic efficiency and smoothness. By transforming their model into a mixed integer linear programming challenge and employing branch-and-bound techniques, they achieved impressive control performance under varying control conditions. Moreover, their solution was found to be versatile, performing effectively under different safety time headways and primary signal green durations. This

Table 3 Summary of the solutions for traffic control improvement direction

Classification	Paper	Contribution
Vehicle trajectory adjustment	Xu et al. (2022a)	Find that gender, as well as the driving section, can have impacts on the driver's trajectory, which is detrimental to the stability of traffic and should be consciously avoided
	Xu et al. (2022c)	Use mixed integer linear programming to analyze various trajectory scenarios of a car arriving at an intersection; the efficiency of passing the cross is improved by the optimization scheme
	Yang et al. (2022)	Propose an optimal control model for a left-turn platoon at a vehicular left-turn lane intersection which can reduce the vehicle passing time at the intersection and reduce the number of brakes per vehicle, thus facilitating the passage of buses
Signal light control	Furth and Muller (2000)	Propose a signal control strategy whereby the conditional priority of buses at signal light intersections is that late-arriving buses are given priority and early-arriving buses are not
	Han et al. (2022)	Propose an adjustment strategy of traffic signal cycle duration and obtain the trend of traffic load based on the theory of flexible neural tree
Right-of-way management	Koehler et al. (2019)	Present a real-time integrated holding and priority control strategy for BRT and high-frequency segregated bus systems
	Ramírez et al. (2015)	Introduce the variation of average speed, online information of bus position, and system operation scheduling table to its scheme to prevent bunching in BRT

method notably reduced the time vehicles take to pass through intersections and minimized braking events, facilitating smoother bus movements.

Signal light control: Signal light control primarily focuses on designing a dynamic signal-switching scheme to facilitate smooth vehicle flow at intersections and accommodate the needs of public transportation (Cai et al., 2023). Furth and Muller (2000) proposed a signal control strategy that prioritizes late-arriving buses over those arriving early at intersections. This strategy can improve the quality of service by keeping buses on time. Their method was validated through a case study in Eindhoven, the Netherlands. Results show significant improvements in schedule adherence and headway variability reduction with the help of bus prioritization. However, the study noted that as buses are not the only vehicles using the intersection, other types of traffic may also be affected. Han et al. (2022) proposed a traffic signal scheduling framework that adapts to varying traffic fluctuations by integrating computational intelligence technology with urban transport systems. Based on the theory of a flexible neural tree, a predictive algorithm was proposed to analyze the traffic flow and obtain the traffic load trend. They then developed an adjustment strategy for the traffic signal cycle to address the problem of overloaded or light traffic flow in upcoming time periods. Experimental results showed that their framework can effectively reduce vehicle delays and stopping rates, thus increasing the adaptability to bus traffic flows.

Right-of-way management: Optimizing right-of-way is pivotal for bolstering the consistency of bus systems. By offering priority to specific buses, it is possible to diminish fluctuations in bus headways. Unlike signal light control, right-of-way management is done not just at intersections, and it is for buses only, unlike signal light control which is for all vehicles. Several investigations have honed in on Bus Rapid Transit (BRT). For instance, Koehler et al. (2019) introduced a real-time strategy that seamlessly merges holding and priority controls, tailored for BRT and similar high-frequency isolated bus systems. The integrated control strategy adjusts bus arrival intervals and grants priority at traffic lights, aiming to reduce delays for passengers both on the bus and waiting at stations. The problem was formulated as a mathematical optimization problem that describes the behavior of the bus system, including BRT. An iterative algorithm was designed to solve the optimization problem in a simple and efficient way. The effectiveness of the proposed strategy and the potential for practical application were evaluated through simulation on Arterial 10 in Blumenau, Brazil. But the study also pointed out that BRT construction often requires more government investment and better urban infrastructure. In some small- and medium-sized cities, the establishment of such BRTs may not be practical due to financial and urban infrastructure constraints. Ramírez et al. (2015) pointed out that although only buses are allowed in some BRT systems, the lack of appropriate management can still lead to bus bunching. Therefore, they proposed an optimization model aimed at keeping the BRT system organized, which incorporates the variation of average speed, online information on bus position, and system operation scheduling table. Results demonstrated that the proposed control system helps to maintain bus operation regularity.

5 Solutions for driver-driving rules improvement

The focus of enhancing driving rules is to guide drivers in changing their driving behavior to alleviate the problem of bus bunching. It typically includes methods such as stop-skipping and overtaking in order. A detailed description of these solutions is provided below, and all references are summarized in Table 4 for ease of reading.

Stop-skipping: The stop-skipping strategy involves a bus selectively skipping some stops when it falls behind its schedule or slows down below a particular speed. This helps the bus catch up with the preceding one and alleviates bus bunching. Research on stop-skipping strategies is

Table 4 Summary of the solutions for driver-driving rules improvement

Classification	Paper	Contribution
Stop-skipping	Liu et al. (2013)	Minimize the waiting time, total in-vehicle travel time, and total operating cost to solve the optimization stop-skipping method
	Tang et al. (2023)	Propose a genetic algorithm to solve the model and generate quasi-optimal results for stopping plans
	Liu et al. (2013)	Look at the impact of stop-skipping strategies at a more macro level at the tactical level and considering side effects
	Gkiotsalitis et al. (2019)	Derive the strategy that aborts an ongoing trip and performs a short-turning or an interlining plays the same part as stop-skipping
Overtaking in order	Schmöcker et al. (2016)	Use a set of discrete state equations giving the degree of mitigation of bunching for different overtaking scenarios
	Wu et al. (2017)	Propose an ad-hoc bus propagation model considering vehicle overtaking and distributed passenger boarding behavior and use it for dynamic queuing exchanges
	Wu et al. (2019)	Indicate that overtaking and demand dynamics have significant impacts on the performance of limited-stop service and that the stop patterns are distinct when overtaking and demand dynamics are considered

usually concentrated on determining the optimal conditions or thresholds for stop-skipping. Liu et al. (2013) studied the optimal stop-skipping strategy to minimize the waiting time, total in-vehicle travel time, and total operating cost. A genetic algorithm with Monte Carlo simulation was combined to solve the optimization problem. The effectiveness of the algorithm was validated through numerical examples. Tang et al. (2023) proposed a planning model that considers stop-skipping in optimizing schedules for single-line electric buses, with the objective of minimizing operator costs and passenger expenditures, instead of focusing on minimizing the waiting time and total in-vehicle travel time, as mentioned earlier in the research of Liu et al. (2013). They also proposed a genetic algorithm to solve the model, yielding near-optimal results in terms of fleet size, vehicle trip chain, charging decisions, and parking plans. A case study was also conducted on a real bus route in Dandong, China. Experimental results showed that the introduction of the stop-skipping strategy reduces the total system cost by 15.09% and improves the average energy utilization by 9.02% compared to the no-stop strategy. Liu et al. (2013), on the other hand, examined the impact of stop-skipping strategies at a more macro level at the tactical level and operational level, respectively. It is worth noting that while stop-skipping has been shown to be effective, it has significant side effects, which should be aware of by both the public and operators. For instance, it prevents passengers at skipped stops from boarding and passengers on the bus from alighting at those stops, resulting in negative outcomes like passenger dissatisfaction. On top of stop-skipping, alternative strategies that work on similar principles but in different forms are also under investigation, such as aborting ongoing trips and implementing short-turnings or interlinings. Gkiotsalitis et al. (2019) evaluated these strategies on the bus network of Hague, the Netherlands, for 24 weekdays. Sensitivity analysis results demonstrated a significant reduction in passenger waiting time and operational costs by introducing only a few short turns and interlinings.

Overtaking in order: Different from previous strategies, overtaking in order does not prevent the occurrence of bus bunching from the beginning, but acts as a remedial measure once bunching has occurred to break the positive feedback loop. In other words, when bus bunching happens, the trailing buses can overtake the leading buses based on specific rules, forming a new bus sequence. Most existing studies on bus bunching have assumed that buses neither overtake nor run in parallel. This does not align with reality, as it has been shown that overtaking in order can help reduce bus bunching. Schmöcker et al. (2016) investigated the impact of overtaking and parallel driving on bus bunching. They used a set of discrete state equations to evaluate how much different overtaking schemes can mitigate bus bunching. They also provided insights into achieving balanced queuing among passengers. One case study illustrated that forbidding overtaking leads to a decline in service regularity along the bus route. On the contrary, allowing overtaking enables regular bus routes to self-adjust more effectively. Wu et al. (2017) proposed an ad-hoc bus propagation model that accounts for vehicle overtaking and distributed passenger boarding behavior. When congestion occurs at bus stops and bus capacity constraints are explicitly considered, dynamic queuing exchanges between buses are performed. A simulation environment was built using the enhanced propagation model and different control strategies were tested. The new strategy was applied to the progress and headway-based hold control design to allow overtaking and minimize deviation from the target headway. The effectiveness of the control strategy was tested on both simulated bus routes in different operating environments and real-world bus routes. Results showed that the control strategy with consideration of overtaking and queue-swapping behaviors is more effective in mitigating bus bunching, reducing waiting time, and minimizing total vehicle travel time, compared to strategies without consideration of these behaviors. Wu et al. (2019) proposed a robust optimization model for limited-stop bus service with vehicle overtaking and demand dynamics. Results from a real-world case study showed that both vehicle overtaking and demand dynamics have significant impacts on the performance of limited-stop service.

6 Solutions for passenger habit improvement

Passenger habit improvement aims to alleviate the issue of bus bunching by influencing passengers' behaviors. This approach focuses on controlling the boarding and alighting process and the number of passengers to reduce bunching at bus stops. Strategies for improving passenger habits include on- and off-board management, raising public awareness, and introducing intelligent billing systems. Details of each solution are provided below, and all references are summarized in Table 5 for ease of reading.

On- and off-board management: The main objective of on- and off-board management is to improve the efficiency of boarding and alighting through strategies like allowing rear door boarding, thereby reducing the dwell time of buses at the station. This approach can mitigate bus bunching without any significant hardware modifications to the buses themselves. Notably, research by Jara-Díaz and Tirachini (2013) and West and Cats (2017) revealed that enabling multi-door boarding can significantly cut down the per-passenger boarding time. Furthermore, Delgado et al. (2016) and Ishaq and Cats (2020) suggested that ticketing procedures, including card swiping, can significantly affect passenger boarding time, but these actions do not necessarily have to be done on the bus. Instead, tickets can be purchased beforehand,

Table 5 Summary of the solutions for passenger habits improvement

Classification	Paper	Contribution
On- and off-board management	Jara-Díaz and Tirachini (2013)	Show that diminishing boarding time reduces optimal bus frequency and increases optimal bus capacity
	West and Cats (2017)	Use a simulation model BusMezzo to analyze the impact of isolation and combinations of measures
	Delgado et al. (2016)	Test the operation of buses on the BRT if the new ticketing boarding method is used
	Ishaq and Cats (2020)	Study the operations of BRT related to service reliability and service utilization and derive lessons for planning and operations
Raising public awareness	Archanaa et al. (2017)	Build a mobile phone program, so as to guide the passengers to board the bus
	Zhou et al. (2022)	Show that providing in-vehicle congestion information is as effective as scheduling-based and headway-based control methods; passengers traveling long distances are more inclined to wait for a few more trips
	Yu et al. (2016)	The least squares support vector machine method is established to detect bus bunching using the predicted headway and provide information notification for bus stops
	Drabicki et al. (2021)	Provide a recommendation index on whether the next bus is worth traveling on, which can guide people's judgment
Intelligent billing system	Wang (2014)	Propose a FlexiFare Bus system to change the price of the bus dynamically according to the number of passengers on the bus

e.g., through mobile phone applications, and have them validated after the bus is running. This strategy was proven effective in reducing the bus stopping time at stops by a case study on the BRT system. However, it should be noted that some fare evaders may take advantage of the opportunity to get onto the bus, making it challenging to remove them once the bus is running. Therefore, refining these management strategies remains a research priority.

Raising public awareness: Raising public awareness consists of two strategies: Firstly, guiding passengers on efficient boarding and alighting practices and discouraging them from entering already overcrowded buses; and secondly, proactively communicating to waiting passengers — either through announcements or mobile notifications — about the seating capacity of approaching buses, which empowers them to make informed decisions about boarding. Such initiatives can diminish waiting times and ease congestion at bus stops, consequently alleviating the issue of bus bunching. Archanaa et al. (2017) developed a mobile phone application to guide passengers during boarding, aiming at reducing the overcrowding issue. This application was tested on the New Delhi bus system in India and showed that adherence to the app's guidance can substantially reduce bus bunching. Zhou et al. (2022) provided passengers with real-time information about waiting times and the level of congestion on the bus, allowing passengers to assess the overall travel cost and decide whether to board the incoming bus or wait for another. This study introduced a public transport propagation model to simulate bus movements, through which system performance metrics such as the distribution of the shortest vehicle spacing and the average number of passengers per bus vehicle per stop can be evaluated under different bus control measures. Numerical results showed that providing in-vehicle congestion information is as effective as scheduling-based and headway-based control methods in reducing bus congestion. Passengers traveling long distances are more inclined to wait for a few more trips to get on a not crowded bus. Yu et al. (2016) proposed a headway irregularity prediction framework based on bus smart card data, which incorporates historical headway, passenger demand, and travel time. The least squares support vector machine model was devised to detect bus bunching using the predicted headway and provide information notification at bus stops. Empirical experiments on two bus routes in Beijing, China revealed that, compared with other well-established prediction algorithms, their proposed model can successfully identify more than 95% of bus bunching events. The system provides timely and accurate information to waiting passengers to prevent possible bus bunching and informs passengers when the next bus will arrive. Drabicki et al. (2021) informed passengers at the current stop whether there is a need to wait for a bus based on real-time bunching information (RTBI) about the departure of the next bus. In addition to provide the crowd information similar to other research, the system also provides a recommendation index on whether the next bus is worth boarding, aiding the passengers in decision-making. This approach has been shown to prevent the development of bunching effects. Particularly, in moderately saturated bus networks, the use of RTBI has shown substantial benefits in reducing on-board crowding and headway variations.

Intelligent billing system: The intelligent billing system can dynamically adjust fares based on the number of passengers already on the bus. Generally, the fare increases as more passengers board the bus. This approach discourages buses from carrying too many passengers, thereby reducing boarding and alighting time, and reducing the likelihood of bus bunching. Since most people are loss-averse, they prefer to spend as little as possible for the same service. This psychological inclination can be leveraged to mitigate the problem of bus bunching, which can be done by dynamically adjusting bus fares, thus preventing a large group of passengers from flocking to a single bus. Wang (2014) proposed a new system based on the Automated Fare Collection system, which includes a FlexiFare algorithm and various vehicle sensors. If the vehicle identifies bus bunching, the system will raise the fare to decrease the number of passengers, subsequently reducing the boarding and alighting time.

7 Other solutions

Recent research has increasingly leveraged cutting-edge technologies such as autonomous driving and artificial intelligence to address the bus bunching dilemma. Concurrently, there is a burgeoning interest in hardware-centric innovations that revamp both the bus structure and its supporting infrastructure, primarily encompassing fleet coordination and modular buses. A comprehensive breakdown of these solutions follows, with a consolidated reference list provided in Table 6.

Fleet coordination: Fleet coordination functions on the basis of the navigation system and real-time monitoring system (Guo et al., 2023). When evidence of bus bunching is identified, the speed of the bus is controlled through prompt information exchanges via the vehicle-to-vehicle communication system to prevent bus bunching (Chen et al., 2023). Recent studies have focused more on location tracking and its extensions to machine learning. Wang and Sun (2020) proposed a multi-agent deep reinforcement learning framework to obtain a dynamic and flexible bus waiting control strategy. Specifically, each bus is modeled as an agent that interacts with not only the vehicles ahead/behind but also all other vehicles in the fleet. To better explore potential strategies, an effective progress-based reward function is designed. Moreover, the fleet coordination scheme is modeled by using an action tracker, which can better characterize the complex interactions between agents during training, and a proximal algorithm was used to improve learning performance. Comi et al. (2022) proposed a machine learning-based approach to solve the bus bunching problem, leveraging information and communications technology (ICT) innovations. This model can measure the distance between the leading and trailing vehicles, regulating the upper limit of speed accordingly. A real-world case study verified the effectiveness of the proposed model in helping operators make better decisions throughout the day to improve bus service quality. Daganzo and Pilachowski (2011) developed an adaptive control scheme that adjusts the cruising speed of buses in real-time based on the front and rear spacing, as if consecutive buses are connected by springs. Informing the leading vehicle to decelerate can harmonize the speed with trailing vehicles, thus preventing the occurrence of bus bunching. Compared with conventional methods, this solution offers more responsive speed control and faster overall operating speed. Its straightforward, decentralized logic automatically counteracts traffic disruptions and erratic driving behaviors, with minimal demand for hardware and data. Iliopoulou et al. (2018) employed spatial analysis to gain insight into the mechanism of bus bunching. They conducted local and global spatial autocorrelation tests on real-world automatic vehicle location data to analyze the spatial dynamics of bus operation. The spatial and temporal variations of bus bunching patterns throughout the day are further modeled using the spatio–temporal Density-Based Spatial Clustering of Applications with Noise (DBSCAN) algorithm. Results revealed that the further the stations are from the bus departure point, the more severe and frequent bunching is observed. It was also observed that bus bunching lasts longer on route segments located in the central business district of the city.

Modular bus: The conceptual design of modular buses, which differs from classical buses, is shown in Fig. 2. The volume and capacity of each bus in the modular context are reduced, and multiple bus modules can operate either docked together or separately. During the docked mode, passengers can traverse between modules seamlessly. Such unique operation mode of modular buses allows for the development of new operation schemes, where the relative speed between buses can be controlled, and the boarding and alighting time can be minimized, thus solving the issue of bus bunching. Modular buses serve as an emerging solution for improving the efficiency of the public transport system, which requires significant modifications to the bus structure and integration with autonomous driving technology and has garnered extensive

Table 6 Summary of other solutions

Classification	Paper	Contribution
Fleet coordination	Wang and Sun (2020)	Propose a machine learning framework to manage all bus positions on an entire road as a whole
	Comi et al. (2022)	Take advantage of innovations in ICT to detect the distance between the leading and trailing vehicles and control the upper limit of the speed in time
	Daganzo and Pilachowski (2011)	Propose an adaptive control scheme that adjusts the cruising speed of a bus in real-time based on the front and rear spacing
	Iliopoulou et al. (2018)	Study the spatial structure in the data by performing local and global spatial autocorrelation tests on real-world automatic vehicle location data
Modular bus	Khan et al. (2023)	Based on the skip-stop strategy, a kind of modular bus is proposed that can be divided into two parts according to the situation
	Dai et al. (2020)	Formulate the bus bunching problem as an integer nonlinear programming model for transit systems operating with mixed human-driven and autonomous buses

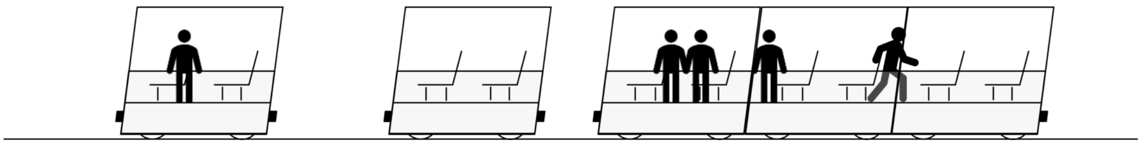

Fig. 2 Modular autonomous bus.

attention recently (Wang et al., 2023). Khan et al. (2023) discussed the application of autonomous modular vehicle (AMV) technology in bus scheduling and operations. Taking advantage of AMV's capabilities, specifically en-route coupling and decoupling of modules, a new alternative stop-skipping was introduced. Their strategy employs bus segregation, guiding modular buses to split into separate entities once headway surpasses a specified threshold. This ensures that boarding and non-boarding passengers are transported distinctively. Macroscopic simulations were then used for evaluation. Compared with conventional uncontrolled and stop-skipping strategy, the proposed strategy was found superior in reducing three travel time components: Waiting time, in-vehicle time, and transferring time (which is completely eliminated). Furthermore, various control thresholds for the proposed strategy were assessed. The study recommends using a small threshold, i.e., small headway deviations can trigger control. However, this strategy works on a simplified scenario of only two modules. A more complex scenario was illustrated by Dai et al. (2020), where modular buses can be decoupled into more than two modules. It was found that a larger number of modules provides more flexible vehicle capability.

8 Conclusions

Since Newell and Potts (1964) first formally articulated the problem of bus bunching, a multitude of operations research analyses have delved into its root causes and potential solutions. Although substantial strides have been made in tackling the bus bunching dilemma, it persists as an unresolved challenge. The academic value of this study is to summarize and classify the solutions to the bus bunching problem, so as to find out the rules and provide guidance ideas for the subsequent solutions. It can be seen that future ideas for solving the problem mainly tend to be towards more dynamic design, more combination of multiple methods, more use of self-driving technology, and more consideration of moral and ethical issues. Numerous promising avenues await further investigation in upcoming research, as detailed below.

• **Increase dynamic design in the management**

Many current solutions for bus bunching, including dynamic timetables, dynamic waiting times, and dynamic pricing, manage to transform the original static operation into an adaptive one, as mentioned in Sections 3, 4, and 5. In the future, this idea can be further used to promote more dynamic designs, enabling bus operation to better align with real-world conditions and encompass higher flexibility. For example, a dynamic bus routing system could be implemented to adjust routes in real-time to mitigate bus bunching. Alternatively, implementing a dynamic queuing system could help distribute passengers more evenly across multiple buses.

• **Investigate the effects of combining multiple methods**

Many existing studies typically highlight a single solution and compare it with cases where no change is made, while some only make a comparison between similar approaches and lack a mixture of approaches. Usually, the combination of different solutions may produce different effects. For example, one can apply different strategies for buses depending on whether they are running or halted at stops. It should also be noted that some combinations of methods are not simply superimposed, but may produce an unexpected effect. It is also crucial to recognize that contradictions may arise when multiple strategies are implemented at the same time.

• **Use autonomous vehicle technologies**

With the development of autonomous vehicles, one can anticipate the widespread adoption of autonomous buses in the future. Based on the Cooperative Vehicle Infrastructure System, the entire bus system will be controlled by computers at a macro level, which can dramatically increase its stability and reduce headway variability in operation. Emerging solutions such as modular autonomous vehicles (as mentioned in Section 7) also possess the potential to completely eliminate the impact of dwell time at stops on bus operations. All these innovations lay the groundwork for addressing the bus bunching issue with autonomous driving technology.

• **Consider ethical issues from multiple perspectives**

In addition to technical concerns, urban public transport system also has to address ethical issues, given its interdependence with and constant interaction with the external environment. While strategies like stop-skipping and dynamic waiting times have potential benefits, as discussed in Sections 3 and 5, they carry a high risk of passenger dissatisfaction, which has received limited attention in previous research. Therefore, future studies should consider more environmental variables to allow for more comprehensive solutions to address the bus bunching issue.

Competing Interests The authors declare that they have no competing interests.

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