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ReciprocalShell

A hybrid timber system for robotically-fabricated lightweight shell structures

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Reciprocal timber systems have been widely studied, however they have never been directly applied to the segmented timber shell structures as cross bracing of the polygonal topologies. For the first time, this paper presents an innovative hybrid timber system developed for design and construction of the robotically-fabricated lightweight timber shell structures. The paper integrates two configurations of wood beams: polygonal framing and reciprocal bracing. While, the polygonal topology of facets enables a constant distance offset for the thickness of the shell, the reciprocal configuration allows for cross bracing of polygonal frames where diagonals within the polygons cannot directly connect corners due to geometric constraints resulted by the free-form surface structure of shell shapes. Joining the cross-bracing elements in the center of the polygons with a reciprocal node reduces the complexity of the connection system at nodes while demonstrating the high load-bearing capacity of joints to withstand structural loads throughout the structure, compared to connecting 5, 6 or 7 beams in a single point. The article discusses the application and limitations of the timber system while presenting the design-to-assembly process of a case study of the small-scale shell demonstrator with the maximum span of 7.5 meters made of 144 wood elements for each polygonal and reciprocal configurations. The results show that the timber system has a great capacity for the rapid and precise assembly and disassembly of prefabricated timber structures. Generation of similar but different solid elements, allowed for the development of a custom CAD data interface for the automated production of numerous pieces, where simple joint details are applied for both alignment and attachment of beams, reducing the design complexity and facilitate the construction phase. As the result, the fabrication process was completely carried out with only a saw blade in a multi-axis robotic fabrication set up that enables the rapid, precise, and accurate cuts and grooves. Both timber configurations generate a uniform distribution of beam size, meaning that the production process created only a minimal amount of offcuts that allows for the use of simple and cost-efficient, short solid wood pieces.

Keywords: *Hybrid Timber System, Reciprocal Shell, Robotic Fabrication, Timber Shell, Lightweight Structures.*

INTRODUCTION

Advances in geometry processing and CAM data interfaces have enabled the optimization of historical timber systems for realization of innovative and complex wood structures. The research presented in this paper is a continuation of our work on the segmental timber plate shell structures. Despite the development of efficient timer systems, the construction of plated shell structures holds some disadvantageous. The plated shells which are typically made hexagonal geometry still produces a large amount of waste during the nesting which consequently result in a low material-efficient construction systems with higher production cost. Not only that, disassembly of segments sometimes become much more complicated, time-consuming and expertise-oriented than it looks. In addition, although these shell are truly lightweight with the great level of structural efficiency, lowering the weight of structure is still a hot topic to explore. From the architectural point of view, plate shells produce a darker interior spaces, many of which need artificial lighting systems even under the sharp daylight. These cons led us to rethink the plate systems from a different point of view and come up with an alternative solution which can potentially add to the existing context of lightweight timber structures. Initially, concerning the fact that hexagonal discretization is a highly efficient method for plate generation, the early concept was to keep its polygonal geometric pattern while trying to reconfigure the timber system by adding additional agents to reach a more lightweight, material-efficient, transparent, and structurally optimal with the great possibility for easy and rapid assembly and disassembly. In the case of the research presented in this paper, it includes a dual timber system inspired by reciprocal timber frames. Geometric capacities of reciprocal timber systems for realization of radically complex designs have been widely explored and studied by various scholars (Song et al, 2013; Gherardini, and Leali, 2017; Mesnil et al, 2018; Apolinarska et al, 2021; Wang and Akbarzadeh. 2022). Despite the advances in development of various linear or spatial configurations and improvement of joint systems, reciprocal timber system has never been directly applied as a cross bracing for a polygonal frame. Accordingly, the hypothesis is that by development of a hybrid timber system that uses the reciprocal bracing in a more simple and innovative way, we are able to develop a new lightweight, material-efficient, and cost-effective design solution for the sustainable construction of segmented timber structures. The goal is then to shape a computational and construction system to be applied on the real shell structure.

COMPUTATION

The whole structure of the research is divided into two parts: computation and construction. The computation part includes the timber system, joint details, tagging and marking, as well as CAM data generation.

Timber system

The presented timber system consists of two main configurations: the polygonal framings and reciprocal bracings (Figure 1). The primary topology of the shell are planar polygonal facets, in which always three edges are joined. The reason lies in the valence - geometric properties of the mesh describing the number of edges connected at a node - proving that by the hexagonal segmentation, a constant distance offset for the thickness of the shell is perfectly possible with no technical errors (Robeller and Von Haaren. 2020; Adelzadeh et al, 2022). While the polygon frames shape the main skeleton of the structure, there are no internal forces resisting the displacement of nodes and deformation of the cassette; so, the primitive idea

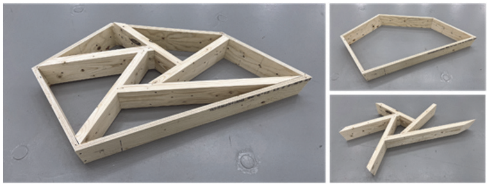

Figure 1
Hybrid timber system consists of polygonal framings and reciprocal bracings

was to develop a cross bracing system connecting the center point of the cassette to the corners, which are the weakest points of the frame where the nodal displacement could occur. The system was failed due to the free-form geometry of structurally optimal shell shape, where diagonals within the polygons cannot directly connect corners. That is, the intersection of two planes at corners rotates the neighboring diagonal along its centroid in a way that the diagonals could not reach a single point without deformation. Although, a similar issue was profoundly studied in the gravitational pavilion 2017, where the lower part of beam elements were cut out at the nodes (Bannwart, 2017), a more simplified node with reduced geometric complexity was targeted. As an alternative, cross bracing is achieved through a reciprocal configuration of wood beams within the polygonal segments. Joining the cross-bracing elements in the center of the polygons with a reciprocal node allows for a more simple joints with better spaces for connectors, compared to connecting beams in a single point. In addition, these diagonals are in the same planar facet with more structural efficiency to counteract the external in-plane forces at corners, without the use of metal plates or brackets. Having two independent timber configuration enabled the use of different material for each set. Since the polygonal topology creates the main skeleton of the shell structure, it requires a stronger material compared to the reciprocal configuration where diagonal bracing are more capable of load circulation and distribution (Karimian et al, 2023); accordingly, the 21 mm building plywood and 45x145mm dimensioned spruce timber were selected for the framing and bracing elements, respectively. In this paper, the system was applied to a structurally optimized three-leg catenary shell shape with maximum span of 7.5 m (Figure 2).

Joint details

In segmented timber structures, joint details simply have a key role to play. After the generation of the timber configurations, joint details for attachment of different elements throughout the structure were studied. Primarily, precise arrangement and alignment of each element is a necessity to the quality of the joint system. Due to the multitude of timber elements, a simple strategy was required to avoid any complexity and constraint in fabrication and assembly. As a solution, wooden spline was selected for several reasons. Spline is a simple and thin strip of wood that attaches two end-to-end connected planks (Figure 3, 4). Using a saw blade enables a much faster fabrication process than

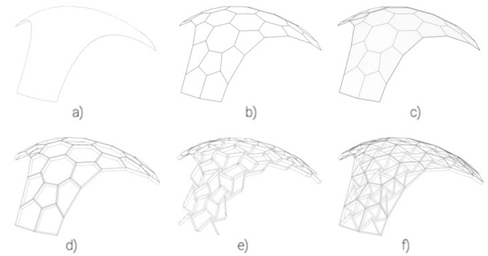

Figure 2
Modeling process
a) input surface
b) discretization
c) planarization
d) polygonals
e) reciprocals
e) hybrid timber system

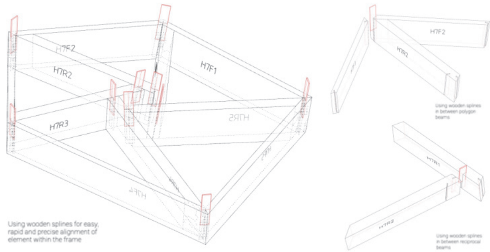

Figure 3
Placement of glued wooden splines in the joints

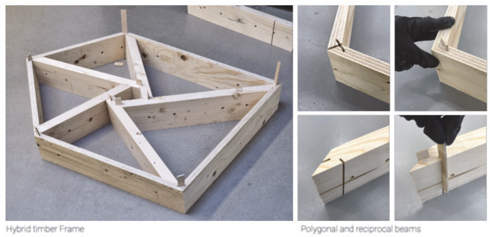

Figure 4
Assembly of beams by placement of glued wooden splines in a frame

milling; accordingly, grooves which can be produced by blade allows for the application of splines made from birch, that are cheap, and easily available for a mass production. To reduce the fabrication time, identical splines were generated at all end-to-end interconnections between polygonal and reciprocal beams in a way that the groove depth in both sides is equal to the half of the spline width. This strategy allowed for a rapid and precise alignment and attachment of neighboring elements in each configuration (Figure 4. For easy placement of splines, small tolerance of gaps between elements was considered, meaning that the 3.2 mm thick splines are slightly thinner than grooves thickness of 3.5 mm. To fasten each beam to its neighboring one, two sets of screws were used for each configuration with respect to the material thickness. To reach the most interlocking capacity of screw with the wood fiber, each joint was screwed perpendicular to the bisectional plane of adjoining elements where wood end-grains meet each other. After production of segments, a simple connection was required for the rapid assembly. As a solution, two neighboring cassettes which are back to back connected were fastened by two bolts (Figure 5). The system enabled us to handle a production constraint. Keeping the exact position and orientation of beams on the jig during the robotic fabrication is a key element for the quality of cuts. In addition, a fluent workflow for placing, fixing, and changing the piece after fabrication was essential for the rapid production of numerous pieces. Having two holes for bolts, allowed for holding and tightening each piece on the worktable precisely, while letting us substitute each piece rapidly. Due to a multitude of geometrically similar but practically different timber element in two sets of configurations, a custom tagging method was required to facilitate the process of assembly. As a solution, a top-down method was developed to first sort all hexagonal cassettes and then represent each one by *Ha* where *a* represents the number of cassette. In each cassette, two classes of polygonal framing *F* and reciprocal bracing *R* were defined in order and with connection to its neighboring beam (Figure 6). The tag ID starts clockwise from the diagonal, which creates the most acute angle with the neighboring frame beam (Figure 7). The reason arises from the lack of enough space for the manual assembly of the last diagonal beam, where there is no possibility to screw it to the neighboring one. To avoid any unnecessary and time-consuming effort to find the position of the reciprocals within the polygonal frame, which can be resulted from unintentional rotation or flipping, the tag ID of polygonal beams follow the order of diagonals.

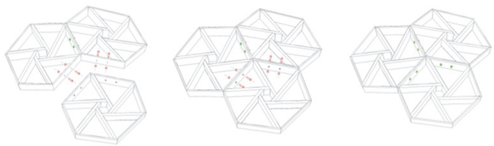

Figure 5
Assembly of polygonal frames: (green) bolted (red) to be bolted

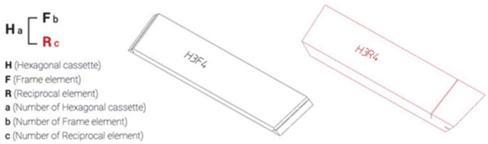

Figure 6
Tagging system for both faming and bracing timber elements

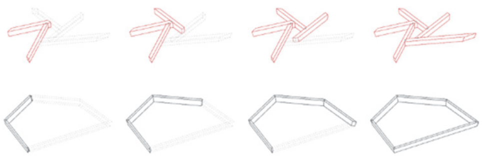

Figure 7
Assembly of the beams starting clockwise from the more acute angle

CAM data interface

Automated production workflow necessitates a CAM data interface by which geometric properties can be translated into the fabrication data. Thanks to the simple cuts and grooves for each beam, we were able to significantly reduce the complexity of the fabrication data generation. As there were two sets of global geometries, a CSV file was generated for each set with the possibility to handle the differences in the cut plane angles and chosen order of operations, which was disseminated into each individual piece (Figure 8). All vectors and origin points are given by the file, enabling the creation of a plane with the correct orientation and location in space (Figure 9). The file includes the complete list of end cuts, side cuts (caused by the cross-section of beams) as well as the dimension of each piece, supplying the distance needed for the start and end positional plane.

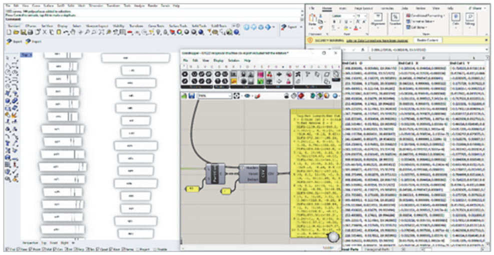

Figure 8
Automated CSV file generation

CONSTRUCTION

The construction phase includes the robotic fabrication, on-site assembly, as well as the experimental load test (Karimian et al, 2023) and disassembly of the project.

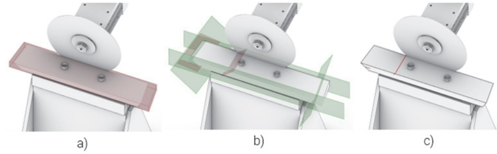

Figure 9
Robotic simulation
a) nested beam
b) generation of cutting planes (green) and grooves (red)
c) final element

Robotic setup

The robotic setup is constituted of a multi-axis KUKA robotic arm, and a worktable, made from various lengths of aluminum extrusions, that was bolted to the ground in front of the robot with three points, enabling an easier calibration of the bed by lifting or lowering sides to change the workpiece plane (Figure 10). The setup was installed at the Robotic Laboratory at Chalmers University of Technology. To run the robot, the fabrication script was split into four distinct parts: data importation, plane creation, plane transformation, and toolpath generation. To generate a movement in the fabrication code, two sets of data were required: a three-dimensional point and a series of three rotations. By defining the tool position and orientation in space, a series of these points and rotations allows for the generation of the toolpath (Figure 11). By keeping each cut separate in the script, we were able to handle problems with the robotic kinematics, such as axis overspeed limits and collisions with the tool. Adding another movement command in between some of the cuts, prolonged the distance of movement for the robot but slowed down the rotational speed of individual axes. This separated way of generating the fabrication data allowed for quick necessary adjustment to the code, enabling an overall increased level of productivity. As the sawblade generates a large amount of centrifugal force while spinning, any movement of the robot divergent from the plane of inertia while the sawblade is powered was removed.

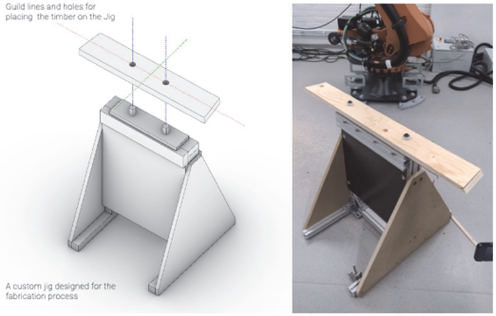

Figure 10
A custom worktable for holding the elements during the fabrication process

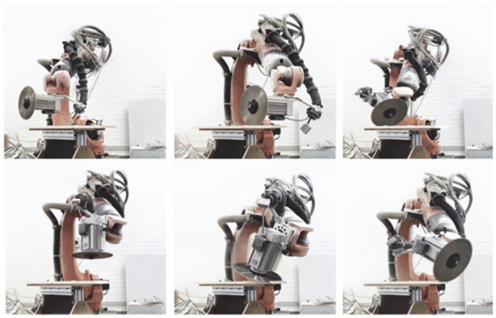

Figure 11
Robotic fabrication process

Robotic fabrication
All timber elements of the structure were completely prefabricated in two days. Thanks to the automated modeling process and data production, all the required data for the fabrication process are extracted directly from the CAD model. The production process of the components is completely carried out with only a saw blade, which allows for very fast and precise cuts and grooves. Thanks to the pre-drilled holes in raw timber elements, placement and substitution of each beam on the jig were very precise and quick by the 10 mm shoulder bolts. To increase the overall material efficiency from the standard parts, a ready-made plugin of OpenNest was used to nest each piece. This allowed for a full picture of the entire piece list and a systematic workflow to create each piece needed. As a result, the fabrication process created only a minimal amount of offcuts which unfold the potential capacity of the construction system for the use of simple and cost-efficient, short solid wood pieces.

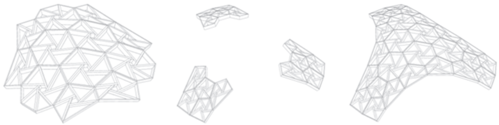

Figure 12
Subdividing the whole structure into a large dome and three legs for on-site assembly

Figure 13
Easy and rapid alignment and assembly of the polygonal and reciprocal beams

Figure 14
Easy and rapid assembly of the polygonal cassettes

On-site assembly
To facilitate the construction phase, the assembly of cassettes were proceeded in parallel to the fabrication process. Thanks to the efficient global and local tagging, the position of each frame or reciprocal element in the cassette. The glued wooden spline allowed for the very straightforward and precise fastening of elements together, acting as a pull-push agent for curation of glued joint. Manual assembly of segments necessitated the use of clamps to tighten the attached beams where they are glued throughout the structure.. The research demonstrator was assembled in only a couple of hours by a team of the workshop leaders and students at the large hall of the Chalmers University of Technology in Gothenburg, Sweden. The assembly of the segments was planned based on the available facilities, including a mega-scale overhead crane system in the hall which allows for lifting the heavy loads; accordingly, the whole structure was divided into four segments, including the main dome (heavier part) to be lifted by the crane and the remaining three shell legs (lighter parts) (Figure 12).

The assembly was started from the bigger segments of the dome, enabling an easier and faster assembly of the cassettes with no need for scaffolding or any sub-structure (Figure 13, 14). After the completion of the segments, the dome was lifted by the crane and flipped, so the three legs could be attached to shape the complete shell (Figure 15). Afterwards, the shell was placed on a temporary foundation made of remaining timber elements, where it could be loaded for verification of the structural FEM analysis. To keep the focus of this paper on the design-to-assembly of the demonstrator, the structural evaluation and experimental load test are presented in another article (Karimian et al, 2023).

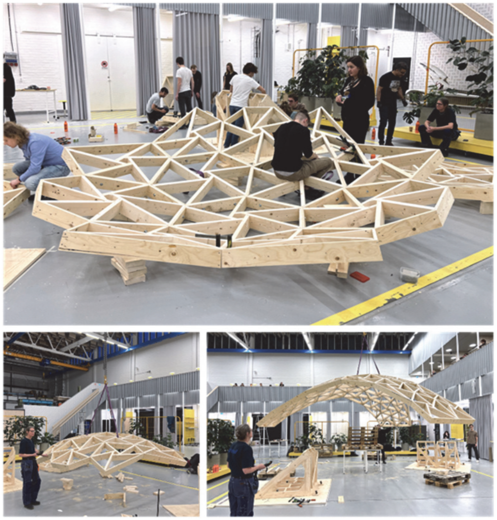

Figure 15
On-site assembly of the cassettes including lifting the dome and attachment of the three legs

DISCUSSIONS

After completion, the project was evaluated from the different points of view. In contrast to the visual complexity of the structure, simple and rapid assembly of elements enabled the participation and physical involvement of students with no prior expertise and experience in wood construction in the on-site assembly, promising a cheap, fast and straightforward construction system with no need for skilled workforce (Figure 13). Compared to the plate shell structures which typically have same polygonal pattern made of CLT plates (Robeller and Von Haaren, 2020; Adelzadeh et al, 2022; Adelzadeh et al, 2023), the material and structural analysis shows that the hybrid timber system is 60% lighter with almost the same structural behavior with maximum calculated deformation difference of 0.1 in respect to the similar CLT shell structure (Figure 16) (Table 1). (Karimian et al, 2023). From the functional and environmental point of view, contrary to the plate shells that typically produce dark interior spaces, the timber system offers a significantly brighter ambient and internal space through receiving the natural daylight while getting benefits from the shadows caused by the complexity of the timber configuration and the height of the beams, leading to a more energy-efficient structure that reduces the need for artificial lighting (Figure 17). Although the roofing surface of high-curvature segmented structure might not be smooth enough for the direct application of overlapping systems and tiling (Karimian et al, 2022), either development of a substructure or panelization of the roof would be a potential solution for the future work. The construction system also has some limitations. Application of the cross bracing is directly dependent on the convexity of frames, meaning that concave segments which usually exist in negative surface curvatures would face modeling problems. The angle of diagonals directly affects the structural capacity and fabrication-to-assembly of the reciprocal bracing elements. The current reciprocal pattern is influenced by several technical parameters. For the more uniform distribution of loads, diagonals are almost generated along the bisector of corners, however to avoid small holes in the center of frames which can cause difficulties for assembly or screwing these angles are slightly manipulated in the bad angles. In addition, the acute angles of diagonals can cause long cuts at the corners, which is technically difficult to produce by saw blade. This issue is also relevant to the size of segments, meaning that for the larger hexagonal frames in the future, the reciprocal pattern can be completely optimized for the structural performance of the shell. In parallel, It is expected by changing the geometric specification of the beams, which would consequently provide both higher structural capacities and more bonding area for joint improvement with better interlocking capacities, significantly larger shell structures could be constructed.

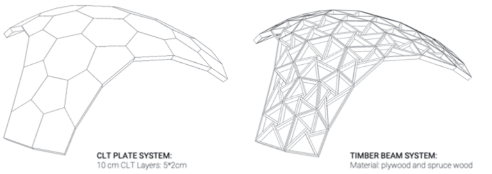

Figure 16
A comparison between the plated and beamed timber shell systems

A COMPARISON BETWEEN THE PLATED AND BEAMED TIMBER SHELL SYSTEMS

CLT PLATE SYSTEM:	TIMBER BEAM SYSTEM:
Thickness: 100mm 5*20mm	Cross section: 40*130mm
Weight: 0.45 KN/M^2	Density: 700 kg/m^3
Total volume: 1.92 m^3	Total volume: 0.75 m^3
Total mass: 865 kg	Total mass: 528 kg
Fabrication time : 6-10 min	Fabrication time : 3-4 min
Waste: Significantly high	Waste: Extremely low
Interior: Dark	Interior: Light

Table 1
A general comparison between the plated and beamed timber shell systems

Figure 17
Interior space of the shell

CONCLUSIONS

Reciprocal configurations have been widely studied for realization of complex timber structures, however they have never been directly applied as cross bracing of planar hexagonal topology of free-form structures. The ReciprocalShell construction system shows a new assembly technique for lightweight segmented shells, made from simple and cost-efficient, short solid wood pieces. The paper adds to the existing research context of reciprocal timber systems by developing a hybrid configuration of wood beams for design to construction of the free-form timber structures, optimized for a rapid and precise assembly and disassembly (Figure 18). While the planar polygonal facets with three-edge connections, enabled a constant distance offset for the thickness of multi-curved surfaces, the reciprocal configurations of wood beams solved geometric constraints caused by free-form structure of shell shapes, where the diagonals within a polygon cannot directly be connected to the corners of a cassette, allowing for the reinforcement of polygonal cassettes without the use of metal plates or brackets. The timber system reduces the complexity of connectors at nodes throughout the structure and allows for more simple joints compared to connecting several beams in a single point with inevitable out cuts (Bannwart, 2017). With the direct generation of machine-ready CSV files, and the simplified robotic toolpaths for cuts and groove, the whole fabrication process was completely done by only a saw blade in two days with maximum fabrication time of 3–4 minutes per each piece. The production process creates only a minimal amount of offcuts, providing the great possibility to use any similar solid wood production waste. The whole assembly process was even faster than the expected time, and was accomplished in two days. Thanks to the prefabrication of the cassettes, no specific scaffolding or substructure was required for the assembly. From the architectural standpoint, the timber system has a great level of visual complexity and esthetic value as well as the porous surface texture for the use of natural daylight. After the experimental load test, the demonstrator was disassembled by the students at Chalmers University of Technology to be exhibited as part of the upcoming festival of the city of Gothenburg in Sweden.

Future work

The research opens up a new opportunity for further development of the joint system and its potential application in the construction of shells with larger spans. Having two completely independent sets of solid wood elements would certainly enable a material-informed timber system where combination of hardwood and softwood with various cross-sections for structural reasons is possible. Due to the great architectural and structural performance of the research demonstrator

(Karimian et al, 2022), the future application of the presented timber system in wood building construction industry is highly expected, especially where a cost-effective, material-efficient and rapid assembly of large-space roofing system is required. Not only that, we also believe the concept can be directly applied to orthogonal systems to modernize the traditional truss. To validate the further structural capacities of the timber system for a real architectural project, construction of the medium-scale research demonstrator with the maximum span of 15m is planned for summer 2023.

Figure 18
(left) visual pattern of the timber system
(right) complete shell structure

ACKNOWLEDGMENT

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