
# Progressive and efficient verification for digital signatures: extensions and experimental results



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RESEARCH ARTICLE

# Progressive and efficient verification for digital signatures: extensions and experimental results

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## Abstract

Digital signatures are widely deployed to authenticate the source of incoming information, or to certify data integrity. Common signature verification procedures return a decision (accept/reject) only at the very end of the execution. If interrupted prematurely, however, the verification process cannot infer any meaningful information about the validity of the given signature. This limitation is due to the algorithm design solely, and it is not inherent to signature verification. In this work, we provide a formal framework to extract information from prematurely interrupted signature verification, independently of why the process halts: we propose a generic verification procedure that progressively builds confidence on the final decision. Our transformation builds on a simple but powerful intuition and applies to a wide range of existing schemes considered to be post-quantum secure, including some lattice-based and multivariate equations based constructions. We demonstrate the feasibility of our approach through an implementation on off-the-shelf resource-constrained devices. In particular, an intensive testing activity has been conducted measuring the increase of performance on three IoT boards—i.e., Arduino, Raspberry, and Espressif—and a consumer-grade laptop. While the primary motivation of progressive verification is to mitigate unexpected interruptions, we show that verifiers can leverage it in two innovative ways. First, progressive verification can be used to intentionally adjust the soundness of the verification process. Second, our transformation splits verification into a computationally intensive offline set-up (run once), and an efficient online verification that is faster than the original algorithm. We conclude showing how to tweak our compiler for progressive verification to work on a wide range of signatures with properties, on three real-life use cases, and in combination with efficient verification.

**Keywords** Digital signatures · Efficient verification · IoT · Provable security

## 1 Introduction

Digital signatures allow one party (the signer) to use her secret key to authenticate a message in such a way that, at any later point in time, anyone holding the corresponding public key (the verifiers) can check its validity. The typical nature of signature verification procedures is monolithic: the validity of a signature is determined only *after* a sequence of tests is completed. In particular, if the execution is interrupted *in media res* (Latin for "in the midst of things"), no conclusive answer can be drawn from the outcomes of the partial tests. Although this monolithic nature is not a burden in many application scenarios, e.g., validating financial transactions (Bitcoin protocol), installing certified software

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updates (Android OS), or delivering e-services (e-Health, electronic tax systems), it is a major limitation to the adoption of digital signatures in cyber-physical systems [39] and in secure eager or speculative executions [30], where the speed at which verification is performed plays a crucial role.

Le et al. [29] proposed to address unexpected interruptions using a new cryptographic primitive called *signatures with flexible verification*. In a nutshell, such schemes admit a verification algorithm that increasingly builds confidence on the validity of the signature while it performs more steps. In this way, at the moment of an interrupt, the verifier is left with a value $\alpha \in [0, 1] \cup \perp$ that probabilistically quantifies the validity of the signature, or rejects it. While the primary motivation of flexible verifications is to mitigate unexpected interruptions, we observe that the overarching idea of *progressive verification* has further impacts. In particular, progressive verification can be used to customize the soundness of the verification process. For example, a smart device may decide to verify at a 30-bit security level, if the signatures come from specific sources or the battery is below 30%. From a theoretical perspective, progressive verification (as introduced in this work later on) draws interesting connections between classical, information-theoretic and post-quantum security notions.

### 1.1 Our contribution

This work sets out to dismantle the monolithic nature of signature verification by designing *new* verification methods for *existing* signature schemes. Concretely, we investigate two approaches. The first one is to speed-up the verification process for polynomially many signatures by the same signer leveraging a one-time computation on the public key (efficient verification). The second approach is to re-design the verification process so that it allows one to extract sensible information even when the algorithm is executed only partially (progressive verification). In this setting it is of particular interest to investigate the security implications of this new model and what additional features it may bring.

In detail, we introduce formal definitions and security models for both efficient (Sect. 2) and progressive (Sect. 3) verification. In terms of realizations, we focus on a specific family of schemes that we call with **Mv**-style verification (in brief, the verification includes matrix–vector multiplications). For schemes in this class, we propose two compilers, i.e., two information-theoretic transformations that turn monolithic **Mv**-style verifications into provably-secure efficient or progressive ones (Sect. 4). We emphasize that our compilers leave the signing algorithm and the signatures as they are; they only provide an alternative, probabilistic, verification method for them. Our compilers apply to multivariate polynomials based schemes including MAYO [7], Rainbow [18, 19] and LUOV [8]; and lattice-based schemes including GPV [24] (hash & sign), MP [32] (Boyen/BonsaiTree), and GVW [25] (homomorphic) (details in Sect. 5). A large part of the security proof is devoted to a detailed analysis of the leakage due to verification queries (that now involve secret randomness). We consider this leakage analysis a result of independent interest as it can be used to estimate leakage in similar information-theoretic approaches to provably secure algorithmic speed-ups or eager executions.

This extended version of [10] includes additional details on the aforementioned contributions, as well as new material containing the following contributions. We extend our models for efficient and progressive verification to include signatures with advanced properties: ring, threshold, homomorphic multi-key, attribute-based and constrained (Sect. 7.1). We also show how to combine efficient and progressive verification in a secure manner (Sect. 8). To analyze the impact of our technique on constrained devices, we developed a library for **Mv**-style efficient and progressive verification, and tested it against the Rainbow signature scheme. The testing activities were performed on three IoT boards and a consumer-grade laptop, and we show the gathered results in our experimental evaluations (Sect. 6).

Rainbow was chosen because, among the NIST finalists, it is the only proposal on which our technique applies directly with interesting efficiency boosts, and has an available, ready-for-use implementation. In spite of the recent attacks against Rainbow [6], we believe that the results of our work can still be useful to showcase the performance boost obtainable by applying efficient verification to existing constructions. In particular, we emphasize that our technique is rather general and applies to any scheme with **Mv**-style verification. Therefore we believe that understanding the benefit of our approach, even from an experimental perspective, can be useful for future work, including future attempts to counter the recent attacks, e.g., [13].

### 1.2 Related work

The problem of trading security for less computation during a verification has been considered first by Fischlin [21] and Armknecht et al. [2] in the context of message authentication codes (MACs). Le et al. [29] and by Taleb and Vergnaud [36] consider the same question for digital signatures.

Le et al. [29] introduce the notion of flexible signatures and a construction based on the Lamport–Diffie one-time signature [28] with Merkle trees. Taleb and Vergnaud [36] put forth realizations of progressive verification for three specific signature schemes (RSA, ECDSA and GPV). Differently from us, both works demand a modification of the signing or key generation algorithm of the original signature scheme and also a time variable be input to the progressive or flexible verification.

One main difference between our model and those of [21, 29, 36] is that we aim to capture progressive verification as an independent feature that can enhance existing schemes, rather than a standalone primitive that requires one to change some of the core algorithms of a signature scheme. This is in a way more challenging as it leaves less design freedom when crafting these algorithms. In addition, we define progressive verification as a *stateful* algorithm in contrast to stateless [21, 29, 36]: although this makes our model slightly more involved, it is comparably more general and can capture more (existing) schemes.

Our model for efficient verification is close the offline-online paradigm used in homomorphic authentication [3, 15] and verifiable computation [23]; where a preprocessing is done with respect to a function $f$, and its result can be used to verify computation results involving the same $f$. An early instantiation of this technique for speeding up the verification of Rabin-Williams signatures appears in [5]. More recently, Sipasseuth et al. [35] investigate how to speed up lattice-based signature verification while reducing the memory (storage) requirements. The overall idea in [35] is similar to ours (and inspired to Freivalds' Algorithm): to replace the inefficient matrix multiplication in the verification with a probabilistic check via an inner product computation. However, [35] focuses on the DRS signature [34], and investigates the trade-off between pre-computation time for verification and memory storage for this scheme only. Moreover, the work lacks a formal, abstract analysis of the security impact of such a shift in the verification procedure. In contrast, we devise a general framework to model 'more efficient' and 'partial' signature verification. Albeit we developed our approach independently of [35], our techniques can be seen as a generalization of what presented in [35].

### 1.3 Notation

We denote the set of real values by $\mathbb{R}$, integers by $\mathbb{Z}$, natural numbers by $\mathbb{N}$, and finite fields of integers by $\mathbb{Z}_q$, where $q$ is a (power of a) prime number. Throughout the paper, $\lambda \in \mathbb{Z}_{\geq 0}$ denotes the parameter for computational security of a cryptographic scheme. A function $\varepsilon : \mathbb{Z}_{\geq 0} \rightarrow [0, 1]$ is negligible if $\varepsilon(\lambda) < 1/\mathsf{poly}(\lambda)$ for every univariate polynomial $\mathsf{poly} \in \mathbb{R}[\mathsf{X}]$ and a large enough integer $\lambda \in \mathbb{Z}_{\geq 0}$. The abbreviation PPT refers to algorithms that are probabilistic and run in polynomial time. We denote vectors by bold, lower-case letters, and matrices by bold, upper-case letters. We use $\mathbf{v}[i]$ to identify the $i$-th entry of a vector $\mathbf{v}$, and $\mathbf{A}[i, j]$ to identify the entry in the $i$-th row and $j$-th column of a matrix $\mathbf{A}$. The norm of a vector is denoted as $\|\mathbf{v}\|$ and unless otherwise specified, it is assumed to be the infinity norm, i.e., $\|\mathbf{v}\| = \max_i\{|\mathbf{v}[i]|\}$. $\mathbf{A}^T$ denotes the transposed of a matrix. We use $rows(\mathbf{A})$, $cols(\mathbf{A})$, and $rk(\mathbf{A})$ to respectively refer to the number of rows, the number of columns, and the rank of a matrix $\mathbf{A}$; $\mathbf{1}_{1\times n}$ (resp. $\mathbf{0}_{1\times n}$) denotes the row vector of length $n$ that has all entries equal to 1 (resp. 0), while $\mathbf{I}_n$ denotes the $n$ by $n$ identity matrix of dimension $n$. We omit the explicit dimensions when they are clear from the context. We denote the span (linear space) generated by a set of vectors $\mathbf{z}_1, \ldots, \mathbf{z}_i$ in the discrete vector space $\mathbb{Z}_q^m$ as $\langle \mathbf{z}_1, \ldots, \mathbf{z}_i \rangle_q = \{\mathbf{z} \in \mathbb{Z}_q^m \ : \ \mathbf{z} = \sum_{j=1}^{i} a_j \mathbf{z}_j \mod q, \exists a_1, \ldots, a_i \in \mathbb{Z}_q\}$. We denote by $L_1 \| L_2$ the result of appending a list of elements $L_2$ to $L_1$. Given two values $a < b$, we denote a continuous interval as $[a, b] \subseteq \mathbb{R}$, and a discrete interval as $\{a, \ldots, b\} \subseteq \mathbb{Z}$. A signature scheme $\Sigma = (\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Ver})$ with message space $\mathcal{M}$ includes a key generation algorithm $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$, a sign algorithm $\boldsymbol{\sigma} \leftarrow \mathsf{Sign}(\mathsf{sk}, \mu)$ that outputs a signature $\boldsymbol{\sigma}$ on a message $\mu$, and a verification algorithm $\mathsf{Ver}(\mathsf{pk}, \mu, \boldsymbol{\sigma})$ that outputs 1 (accept) or 0 (reject). Throughout the paper, $\Sigma$ satisfies the properties of correctness and existential unforgeability as defined in [36].

## 2 Efficient verification for digital signatures

The core idea of efficient signature verification is to split the verification process into two steps. The first step is a one-time and signature-independent setup called 'offline verification'. Its purpose is to produce randomness to derive a (short, secret) verification key $\mathsf{svk}$ from the signer's public key $\mathsf{pk}$. Note that the offline verification does not change the signature, which remains publicly verifiable; instead it 'randomizes' $\mathsf{pk}$ to obtain a concise verification key $\mathsf{svk}$ that essentially enables one to verify signatures with (almost) the same precision as the standard verification, but in a more efficient way. We remark that for secure efficient verification $\mathsf{svk}$ should be hidden to the adversary, yet, the knowledge of $\mathsf{svk}$ gives no advantage in forging signatures verified in the standard way using just $\mathsf{pk}$. The second verification step consists of an 'online verification' procedure. It takes as input $\mathsf{svk}$ and can verify an unbounded number of message-signature pairs performing significantly less computation than the standard verification algorithm. For security, it is fundamental $\mathsf{svk}$ remains unknown to the adversary. We remark that generating $\mathsf{svk}$ during the offline phase achieves efficient online verification with no impact on the original signing or key generation algorithms, which was a drawback of previous work [29, 36].

### 2.1 Syntax for efficient verification

Our definition of efficient verification lets the verifier set the confidence level $k$ at which she wishes to carry out the signature verification. Notably $k$ determines the amount of computation to be performed and thus plays a central role in the security and the efficiency of the new verification.

**Definition 1** (*Efficient verification*) A signature scheme $\Sigma$ admits efficient verification if there exist two PPT algorithms (offVer, onVer) with the following syntax[1]:

- offVer(pk, $k$): this is a randomized algorithm that on input a public verification key pk, and a positive integer $k \in \{1, \ldots, \lambda\}$ (where $\lambda$ is the security parameter of $\Sigma$), returns a secret verification key svk.
- onVer(svk, $\mu, \sigma$): on input a secret verification key svk, a message $\mu$, and a signature $\sigma$, the efficient online verification algorithm outputs 0 (reject) or 1 (accept).

For convenience we will refer to the signature scheme augmented with the efficient verification algorithms as $\Sigma^E = (\Sigma, \text{offVer}, \text{onVer})$, and to the integer value $k$ as confidence level.

To be meaningful, a realization of efficient verification needs to satisfy the properties of correctness, concrete atomized efficiency and security.

**Definition 2** (*Correctness of efficient verification*) A scheme $\Sigma^E = (\Sigma, \text{offVer}, \text{onVer})$ realizes efficient verification correctly if the following conditions hold. For a given security parameter $\lambda$, for any honestly generated key pair $(\text{sk}, \text{pk}) \leftarrow \text{KeyGen}(\lambda)$, for any message $\mu \in \mathcal{M}$, for any signature $\sigma$ such that $\text{Ver}(\text{pk}, \mu, \sigma) = 1$, and for any confidence level $k \in \{1, \ldots, \lambda\}$; it holds that $\Pr\left[\text{onVer}(\text{svk}, \mu, \sigma) = 1 \mid \text{svk} \leftarrow \text{offVer}(\text{pk}, k)\right] = 1$ for any choice of randomness used in offVer.

Amortized efficiency relies on the fact that running offVer once and reusing its output r times to run onVer is computationally less demanding than running the standard verification Ver r times. To formalize this, we will use the function cost$(\cdot)$ that given as input an algorithm returns its computational cost (in some desired computational model). In addition, we parameterize concrete amortized efficiency with two intertwined variables: $\text{r}_0$ (number of instances of verification), and $\text{e}_0$ (ratio between the cost of $\text{r}_0$ efficient verifications over $\text{r}_0$ standard verifications). The lower the value of $\text{r}_0$ the sooner $\Sigma^E$ amortizes the computational cost of offVer. The lower the value of $\text{e}_0$ the more efficient $\Sigma^E$ is with respect to the standard verification.

**Definition 3** (Concrete Amortized Efficiency) Let $\text{r}_0$ be a non-negative integer and $0 < \text{e}_0 < 1$ be a real constant. A scheme $\Sigma^E$ realizes $(\text{r}_0, \text{e}_0)$-concrete amortized efficient verification for $\Sigma$ if given a security parameter $\lambda$ and a confidence level $k$; for any key pair $(\text{sk}, \text{pk}) \leftarrow \text{KeyGen}(\lambda)$, for any pair $(\mu, \sigma)$ with $\mu \in \mathcal{M}$ and $\sigma$ such that $\text{Ver}(\text{pk}, \mu, \sigma) = 1$; we have that $\forall\, \text{r} \geq \text{r}_0$:

$$\frac{\text{cost}\big(\text{offVer}(\text{pk}, k)\big) + \text{r} \cdot \text{cost}\big(\text{onVer}(\text{svk}, \mu, \sigma)\big)}{\text{r} \cdot \text{cost}\big(\text{Ver}(\text{pk}, \mu, \sigma)\big)} < \text{e}_0 \quad (1)$$

### 2.2 Security model for efficient verification

Intuitively, $\Sigma^E$ realizes efficient verification in a secure way if onVer accepts a signature that would be rejected by Ver only with negligible probability. In the security game (see Fig. 1), the adversary $\mathcal{A}$ has access to the signing oracle $O\text{Sign}$ as well as the efficient verification oracle $O\text{onVer}$. The goal of the adversary is to produce a signature $\sigma^*$ for a message $\mu^*$ that was never queried to $O\text{Sign}$ and for which Ver returns 0 (reject) and onVer returns 1 (accept).

**Definition 4** (*Security of efficient verification*) A scheme $\Sigma^E$ realizes a secure efficient verification for $\Sigma$ if for a given security parameter $\lambda$ and for any confidence level $k \in \{1, \ldots, \lambda\}$, for all PPT adversaries $\mathcal{A}$ the success probability in the cmvEUF experiment reported in Fig. 1 is negligible, i.e.: $Adv^{\text{cmvEUF}}_{\mathcal{A},\Sigma}(\lambda, k) = \Pr\left[\text{Exp}^{\text{cmvEUF}}_{\mathcal{A},\Sigma}(\lambda, k) = 1\right] \leq \varepsilon(\lambda, k)$.

Line 7 of the cmvEUF experiment excludes forgeries against the original signature scheme. This is justified by the correctness of efficient verification and by the fact that $\Sigma$ is existentially unforgeable. Notably, both the security game and the advantage depend on the confidence level $k$ and assume all algorithms are entirely executed.

## 3 Progressive verification for digital signatures

The goal of progressive verification is to incrementally increase the confidence on the validity of a signature, for a given message against a public key. Intuitively, the "confidence" should be proportional to the amount of computation invested: the further in the execution we go, the higher the accuracy of the decision, and thus the confidence of the final outcome (accept/reject).

### 3.1 Existing approaches to progressive verification of signatures

Taleb and Vergnaud give a very intuitive definition of progressive verification for digital signatures [36]. They model digital signatures with progressive verification as a 4-tuple of PPT algorithms (KeyGen, Sign, Ver, ProgVer) such that: $\Sigma = (\text{KeyGen}, \text{Sign}, \text{Ver})$ is a correct digital signature scheme; and ProgVer takes in input a public verification key pk, a message $\mu$, a signature $\sigma$, and some timing parameter $t$,

[1] Here pk denotes a public verification key output by KeyGen.

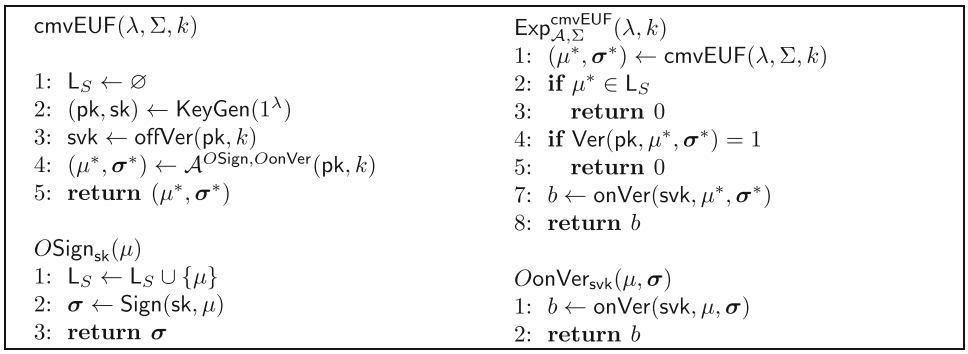



**Fig. 1** Security model for efficient verification of signatures: existential unforgeability under adaptive chosen message and verification attack (security game, experiment and oracles). $\mathcal{A}$ is a PPT algorithm that can query the oracles in an adaptive and parallel way. $\mathsf{L}_S$ is the list of messages queried to the signing oracle

and outputs $\alpha \in \{[0,1] \cap \mathbb{R}\} \cup \{\perp\}$, interpreted as an estimate on the accuracy of its decision whether the signature be valid. Moreover, the scheme satisfies the following properties:

*Correctness* If for some tuple of inputs $\mathsf{ProgVer}(\mathsf{pk}, \mu, \boldsymbol{\sigma}, t)$ outputs $\perp$, then $\mathsf{Ver}(\mathsf{pk}, \mu, \boldsymbol{\sigma}) = 0$.

*Security* If for some tuple of inputs $\mathsf{ProgVer}(\mathsf{pk}, \mu, \boldsymbol{\sigma}, t)$ outputs $\alpha \in [0,1]$, then this implies $\Pr\left[\mathsf{Ver}(\mathsf{pk}, \mu, \boldsymbol{\sigma}) = 0\right] \leq 1 - \alpha$ (where the probability is taken over the random coins of $\mathsf{ProgVer}$).

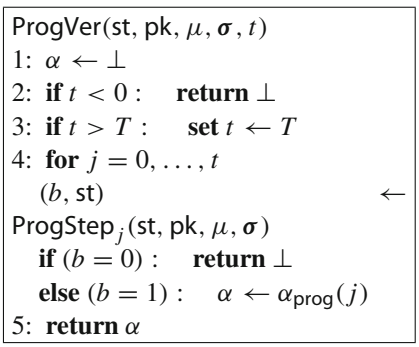
In a nutshell, if $\alpha = \perp$, the progressive verification deems the signature to be invalid (with 100% accuracy). If $\alpha \in [0,1]$, the algorithm considers the signature valid, and $\alpha$ tells how accurate this statement is. Since progressive verification may be interrupted at any arbitrary point $t$ during its execution, in practice $\alpha$ is (the output of) a function $\alpha_{\mathsf{prog}}(t)$ that "converts" the progress in the verification process into a value representing the accuracy of a positive outcome.

*Shortcomings* First, similarly to [29], also [36] sees signatures with progressive verification as a stand-alone primitive. In contrast we view progressive verification as a feature that can augment existing schemes without requiring change to the core algorithms. Second, the definition lacks a precise notion of time complexity and does not model how unexpected interrupts are handled. The model we introduce in the remainder of this section takes care of these aspects. In addition, we generalize progressive verification to be (possibly) stateful, which can capture more signature schemes as well as reuse the same syntax to model both efficient and progressive verification (cf. Sect. 8).

### 3.2 Syntax for progressive verification

In order to model progressive verification as an add-on algorithm we need to derive from $\mathsf{Ver}$ an alternative algorithm $\mathsf{ProgVer}$ (as introduced in Sect. 3.1), that builds confidence on the final verification outcome in an increasing way. Without loss of generality, this task boils down to identifying a sequence of $T+1$ atomic instructions that we call $\mathsf{ProgStep}$ with the following properties. Each $\mathsf{ProgStep}$ performs a check of some sort on the input it receives. If one step fails, the progressive verification returns $\alpha = \perp$. If none of the initial $t$ steps fails, the progressive verification returns the output of a function $\alpha_{\mathsf{prog}}(t) \in [0,1]$ that measures the probability the input will be accepted by $\mathsf{Ver}$. The fact of increasingly building confidence is reflected by functions $\alpha_{\mathsf{prog}}$ that are non-decreasing in $t$, the number of instructions checked before returning the answer. Figure 1 in [36] provides an intuitive and graphical representation of this statement.

**Definition 5** (Stateful Progressive Verification) Let $T \in \mathbb{Z}_{>0}$ and $\alpha_{\mathsf{prog}} : \{0, \ldots, T\} \to [0,1]$ be an efficiently computable function. A signature scheme $\Sigma$ admits $(T, \alpha_{\mathsf{prog}})$-progressive verification if there exists a stateful PPT algorithm $\mathsf{ProgVer}$ that takes in input $\mathsf{pk}$, $\mu$, $\boldsymbol{\sigma}$ and some interruption parameter $t \in \mathbb{Z}_{>0}$, outputs $\alpha \in \{[0,1] \cap \mathbb{R}\} \cup \{\perp\}$, and satisfies the following syntax:




For convenience we will refer to the signature scheme augmented with progressive verification as $\Sigma^P = (\Sigma, \mathsf{ProgVer}, T, \alpha_{\mathsf{prog}})$.

Concretely, $\mathsf{ProgVer}$ is made of $T+1$ algorithms $\mathsf{ProgStep}_j$, for $j = 0$ to $T$, that progressively update the state $\mathsf{st}$. We remark that the formalization into steps is without loss of generality: $\mathsf{Ver}$ realizes a trivial progressive verification for $T = 0$ where the only step is $\mathsf{Ver}$ itself. Finally, the interruption value $t$ is input to $\mathsf{ProgVer}$ only, and it is *not* given to each $\mathsf{ProgStep}_j$. Thus our syntax models the fact that the steps are agnostic of the interruption value and must

work without knowing when to stop, which is essential to capture arbitrary interruptions.

Correctness essentially states that signatures accepted by the standard verification should also be accepted by the progressive one, with the highest confidence allowed by the number of steps performed.

**Definition 6** (Progressive Verification Correctness) Let $\Sigma^P$ be a signature scheme with progressive verification; ProgVer satisfies progressive verification correctness if, for any value $t \in \{0, \ldots, T\}$, for any given security parameter $\lambda$, for any key pair $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(\lambda)$, for any admissible state st generated by ProgVer, for any admissible message, given a signature $\boldsymbol{\sigma}$ such that $\mathsf{Ver}(\mathsf{pk}, \mu, \boldsymbol{\sigma}) = 1$ it holds that: $\Pr\left[\mathsf{ProgVer}(\mathsf{st}, \mathsf{pk}, \mu, \boldsymbol{\sigma}, t) = \alpha_{\mathsf{prog}}(t)\right] = 1$.

We follow the approach of [29] and let the progressive verification algorithm output a value $\alpha$ that either rejects the signature ($\alpha = \perp$), or accepts it with certainty $\alpha$ in the real interval [0, 1]. We use the same interruption variable $t$ as in [29] to model runtime interruptions of the algorithm execution.[2]

*Efficient versus progressive verification* At a first glance, efficient verification and progressive verification seem to have the common goal of reducing the computational cost of a signature verification. However the way this objective is achieved in the two models is quite different.

In progressive verification, the verifier (and thus each $\mathsf{ProgVer}_i$) is unaware of when the computation will be interrupted, and *its execution is independent of* $t$. In contrast, in efficient verification the verifier (running offVer) determines the confidence level $k$ prior to any actual verification (running onVer).

In the latter, the (online) verification is aware of the confidence level $k$ (seen as interruption value), and *adapts its execution to* $k$.

*Stateful versus stateless verification* We define progressive verification as stateful. This allows us to keep the framework as general as possible. Stateless progressive verification, á la [29, 36], can be obtained setting st to $\varnothing$, this also removes the need for analyzing any cross-query leakage due to state reuse.

### 3.3 Security model for progressive verification

Our notion of unforgeability states that signatures rejected by the standard verification should also be rejected by the progressive one, except for an inaccuracy factor due to interruptions. More formally, Ver and ProgVer should have the same behavior (accept/reject) with discrepancies happening with probability negligibly close to $\alpha_{\mathsf{prog}}(t)$.

Our security game has three main differences compared to [29]:

*State:* in order to take into account that ProgVer maintains a possibly non-trivial state we allow the adversary $\mathcal{A}$ to interact with the progressive verification oracle $O\mathsf{ProgVer}$ during the query phase, as well as the signing oracle $O\mathsf{Sign}$, in a concurrent manner.

*Interruption:* queries to $O\mathsf{ProgVer}$ have the form $(\mu, \boldsymbol{\sigma}, t')$, where $t'$ is the desired interruption value submitted by $\mathcal{A}$ (and chosen adaptively).

*Output:* instead of a single bit, our experiment returns a pair $(b, t^*)$. The bit $b \in \{0, 1\}$ flags the absence or the potential presence of a forgery, while $t^* \in \{0, \ldots, T\}$ reports the interruption position used in the final progressive verification. Including $t^*$ in the output of the experiment allows us to measure security in terms of *how close* the probability of $\mathcal{A}$ wining the experiment is from the expected accuracy value $1 - \alpha_{\mathsf{prog}}(t^*)$.

**Definition 7** (*Security of progressive verification* (progEUF)) Let $\Sigma$ be a signature scheme that admits a progressive verification realization $\Sigma^P$. $\Sigma^P$ realizes a secure progressive verification for $\Sigma$ if for any given security parameter $\lambda$, for all PPT adversaries $\mathcal{A}$ the success probability in the progEUF experiment in Fig. 2 is negligible, i.e.,:

$$Adv^{\mathsf{progEUF}}_{\mathcal{A},\Sigma^P}(\lambda) = \Pr\left[\mathsf{Exp}^{\mathsf{progEUF}}_{\mathcal{A},\Sigma^P}(\lambda) = (1, t^*)\right] - (1 - \alpha_{\mathsf{prog}}(t^*)) \leq \varepsilon(\lambda).$$

Intuitively, Definition 7 states that an adversary has only negligible probability to make ProgVer output a confidence value $\alpha^*$ higher than the expected one. Let $\mathsf{bad}(t)$ denote the probability of accepting a forgery after $t$ verification steps. Then by setting $\alpha_{\mathsf{prog}}(t) = 1 - \mathsf{bad}(t)$, we get $Adv^{\mathsf{progEUF}}_{\mathcal{A},\Sigma^P}(\lambda) = \Pr\left[\mathsf{Exp}^{\mathsf{progEUF}}_{\mathcal{A},\Sigma^P}(\lambda) = (1, t^*)\right] - \mathsf{bad}(t^*) \leq \varepsilon(\lambda)$.

In this work, we prove security in the strongest model where $t' = t$, i.e., $\mathcal{A}$ has the power to choose when to stop the verification. Since we put no restriction on the values $t$ queried by $\mathcal{A}$ to $O\mathsf{ProgVer}$ during the game, we will see that by running $O\mathsf{ProgVer}$ on 'too few' steps, $\mathcal{A}$ may learn information about the internal state st.

*Modelling interruptions*

In [29], unexpected interruptions are modeled via an interruption oracle $\mathsf{iOracle}(\lambda)$ that returns a value $t \in \{0, \ldots, T\}$ used by the progressive verification.

However, it is not clear whether $\mathcal{A}$ may control iOracle or not.

We overcome these ambiguities by letting $\mathcal{A}$ output $t'$ with every progressive verification query. For the purpose

[2] Our $\alpha_{\mathsf{prog}}(\cdot)$ is essentially the inverse of the function $\mathsf{iExtract}_{\Sigma}(\cdot)$ in [29].

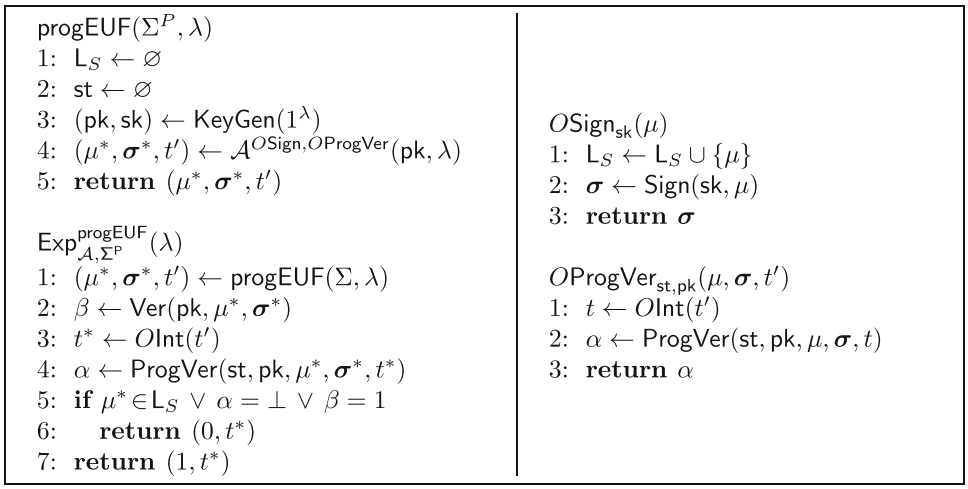



**Fig. 2** Security model for progressive verification of signatures: existential unforgeability under adaptive chosen message and progressive verification attack (security game, experiment and oracles). $\mathcal{A}$ can query the oracles adaptively, in parallel and polynomially many times in $\lambda$. $\mathsf{L}_S$ is the list of messages queried to the signing oracle

of this work, we consider the strongest security model in which the interruption oracle returns the adversary's value, i.e., $t \leftarrow O\mathsf{Int}(t')$ with $t = t'$. This resembles side-channel attack settings, where $\mathcal{A}$ may try to freeze the execution of the verification.

It is possible to relax and generalize our model by setting a different interruption oracle $O\mathsf{Int}$, programmed at the beginning of the game. At each verification query, $O\mathsf{Int}$ takes as input the adversary's suggestion for an interruption position $t'$ and outputs the value $t$ to be used by the progressive verification. In case $t = t'$, we are modelling side channel attacks, but we can also let $t$ be independent of $t'$. A realistic definition of $O\mathsf{Int}$ is outside the scope of this work.

## 4 Generic compilers

In this section, we present generic transformations (compilers) that augment a signature scheme $\Sigma$ with either efficient (Sect. 4.1) or progressive verification (Sect. 4.2).

Our technique works for a specific class of signature schemes that we call *with* **Mv** *-style verification*. In such schemes, $\mathsf{Ver}$ can be seen as the combination of two types of verification checks: a matrix–vector multiplication (referred to as $\mathbf{Mv} = \mathbf{0}$, for appropriate matrix $\mathbf{M}$ and vector $\mathbf{v}$) and other generic checks (collected in the $\mathsf{Check}$ subroutine), see Fig. 3 for details and an explanatory example. Among the schemes with **Mv**-style verification we highlight some of the seminal lattice-based signatures [11, 14, 24, 32], homomorphic signatures [9, 20, 25], and multivariate signatures [8, 19].

### 4.1 A compiler for efficient Mv-style verifications

We present a generic way to realize efficient verification for signatures with **Mv**-style verification, whenever the computational complexity of $\mathsf{Ver}$ is dominated by the matrix–vector multiplication, i.e., $\mathsf{cost}(\mathsf{Check}) \ll \mathsf{cost}(\mathbf{Mv}) \sim mn$ field multiplications (for $\mathbf{M} \in \mathbb{Z}_q^{n \times m}$).

Our compiler for efficient verification is detailed in Fig. 4 with a sketch of instantiation for the lattice-based scheme GPV08 [24] as a running example. Further details on this scheme as well as instantiations and details on the concrete efficiency estimates for MP12 [32], Rainbow [19] and LUOV [8], are deferred to Sect. 5. Table 1 summarizes the efficiency results. We obtain secure efficient online verification using as little as 0.4% (resp. 50%) of the computational cost of the standard verification for lattice-based signatures on exponentially large fields (resp. for Rainbow).

*Overview of our technique* Our transformation takes as input $\Sigma$, a signature scheme with **Mv**-style verification; and it returns $\Sigma^E = (\Sigma, \mathsf{offVer}, \mathsf{onVer})$ that securely instantiates efficient verification for $\Sigma$. The heart of our compiler leverages the fact that for any pair of vectors $\boldsymbol{\sigma}$ and $\mathbf{u}$ (often derived from the message $\mu$), and for any matrix $\mathbf{A}$ (of opportune dimensions) if $\mathbf{A} \cdot \boldsymbol{\sigma} = \mathbf{u}$ then for any random vector $\mathbf{c}$ (of opportune dimension) it holds that $\mathbf{c} \cdot (\mathbf{A} \cdot \boldsymbol{\sigma}) = \mathbf{c} \cdot \mathbf{u}$. Collecting variables on the left hand yields $(\mathbf{c} \cdot [\mathbf{A}| - \mathbf{I}_n]) \cdot \begin{bmatrix} \boldsymbol{\sigma} \\ \mathbf{u} \end{bmatrix} = 0$. Thus one can precompute the vector $\mathbf{z} \leftarrow \mathbf{c} \cdot [\mathbf{A}| - \mathbf{I}_n]$ and run the efficient online verification check $\mathbf{z} \cdot \mathbf{v} \stackrel{?}{=} 0$, where $\mathbf{v} \leftarrow (\boldsymbol{\sigma}, \mathbf{u})$. In a nutshell the idea is to replace the matrix–vector multiplication with a vector-vector multiplication in a sound way. Correctness and efficiency are immediate. Soundness essentially comes from the fact that if $\mathbf{z} \cdot \mathbf{v} = 0$, then with all but negligible probability the original system of linear equations $\mathbf{A} \cdot \boldsymbol{\sigma} = \mathbf{u}$ is satisfied too, as proven in Theorem 1.

We emphasize that our compiler leaves the signing algorithm and the signatures as in the original scheme. It only provides an alternative, probabilistic, verification algorithm.

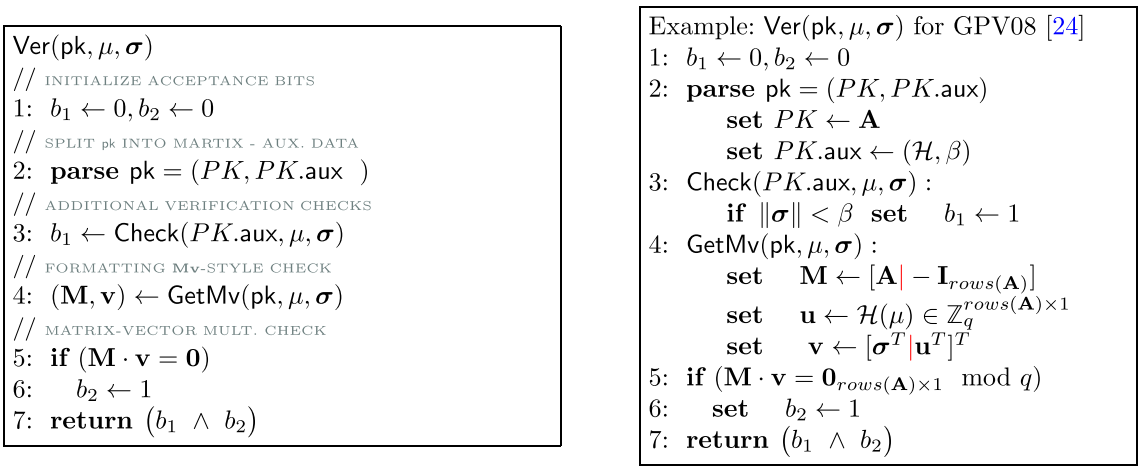



**Fig. 3** General structure of a signature with **Mv**-style verification (on the left); an instructive example: the GPV08 [24] signature verification (on the right)

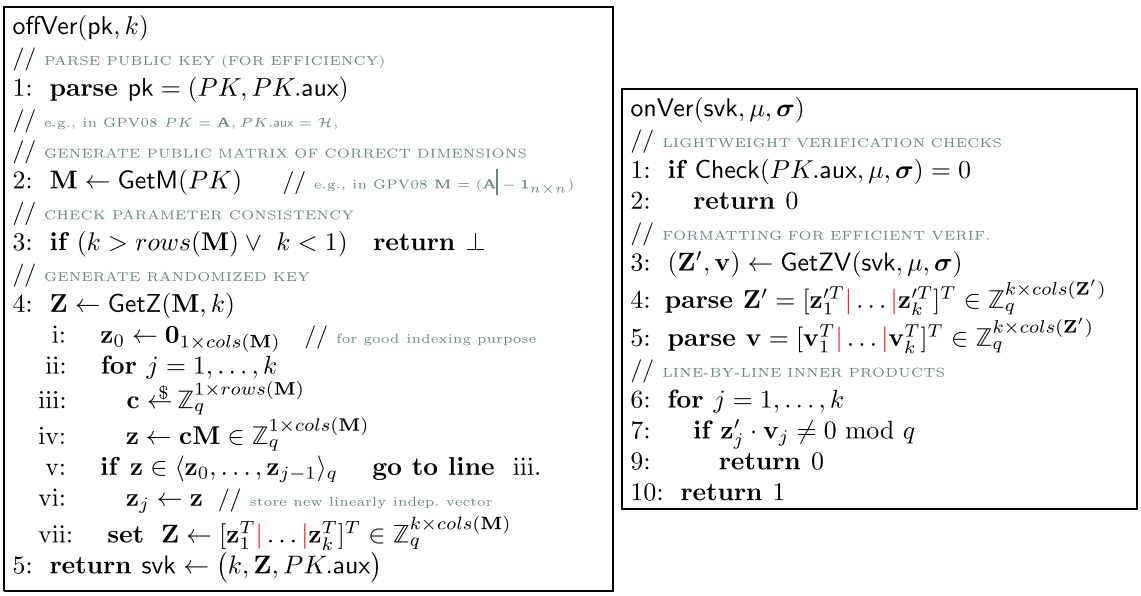



**Fig. 4** Our compiler for efficient verification of signatures with **Mv**-style verification. The four scheme-dependent subroutines are: **parse** pk and GetZ (in onVer)[ and Check and GetZV (in offVer). The computational complexity of onVer is linear in $k$, the chosen confidence level

#### 4.1.1 Security analysis

Despite the construction being intuitive, analysing the leakage due to verification queries that reuse the same svk is not trivial and is one main technical contribution of this result.

**Theorem 1** *Let* $\Sigma$ *be an existentially unforgeable signature scheme with* **Mv***-style verification (as in Fig. 3). The scheme* $\Sigma^E = (\Sigma, \mathsf{offVer}, \mathsf{onVer})$ *obtained via our compiler depicted in Fig. 4 is existentially unforgeable under adaptive chosen message and efficient verification attacks. Concretely, the advantage is* $Adv^{CMVA}_{\mathcal{A},\Sigma}(\lambda, k) \leq \frac{q_V+1}{q^k-q_V}$ *where* $k \in \{1, \ldots, rk(\mathbf{M})\}$ *denotes the chosen confidence level that grows up to the rank of the matrix* $\mathbf{M}$*,* $q_V = \mathsf{poly}(\lambda) << q^k$ *is a bound on the total number of verification queries and* $q$ *is the modulo of the algebraic structure on which* $\Sigma$ *is built.*

***Remark 1*** For simplicity, Theorem 1 considers only existential unforgeability. The statement and the proof adapt with ease to other security models such as strong and selective unforgeability.

***Proof of Theorem 1*** In the cmvEUF security experiment (Fig. 1), the winning conditions require $\mathcal{A}$ to produce a message-signature pair $(\mu^*, \sigma^*)$ such that $\mu^*$ has not been queried to the signing oracle during the game (existential unforgeability); $\sigma^*$ is invalid under the standard verification, i.e., $\mathsf{Ver}(\mathsf{pk}, \mu^*, \sigma^*) = 0$; and the pair is accepted by the

online verification, i.e., $\mathsf{onVer}(\mathsf{svk}, \mu^*, \boldsymbol{\sigma}^*) = 1$. The goal of the proof is to bound the probability this event occurs.

Let $\mathsf{Win}$ be the event $\{\mathsf{Exp}^{\mathsf{cmvEUF}}_{\mathcal{A},\Sigma}(\lambda, k) = 1\}$. Let $i = 1$ to $q_V$ be the index of the queries $(\mu_i, \boldsymbol{\sigma}_i)$ submitted by $\mathcal{A}$ to the $O\mathsf{onVer}$ oracle. Define the family of events $\mathsf{bad}_i$ (for $i = 1$ to $q_V + 1$) as:

$$\mathsf{bad}_i := \{\mathsf{Ver}(\mathsf{pk}, \mu_i, \boldsymbol{\sigma}_i) = 0 \wedge \mathsf{onVer}(\mathsf{svk}, \mu_i, \boldsymbol{\sigma}_i) = 1\}$$

where $\mathsf{bad}_{q_V+1}$ corresponds to $\mathcal{A}$ returning a valid forgery $(\mu^*, \boldsymbol{\sigma}^*) := (\mu_{q_V+1}, \boldsymbol{\sigma}_{q_V+1})$ at the end of the experiment. We can rewrite the winning condition of the security experiment as $\mathsf{Win} = \{\mathsf{bad}_{q_V+1} \wedge \mu^* \notin \mathsf{L}_S\}$ (recall that $\mathsf{L}_S$ is the list of messages queried to the signing oracle in the game execution). Consider the event $\mathsf{Bad}$ defined as "there exists at least one query index $i$ in the game execution for which $\mathsf{bad}_i$ occurs". Formally:

$$\mathsf{Bad} := \left\{ \exists\, i \in \{1, \ldots, q_V\} : \begin{array}{c} \mathsf{Ver}(\mathsf{pk}, \mu_i, \boldsymbol{\sigma}_i) = 0 \\ \wedge \\ \mathsf{onVer}(\mathsf{svk}, \mu_i, \boldsymbol{\sigma}_i) = 1 \end{array} \right\}.$$

Hence,

$$\begin{aligned} Adv^{CMVA}_{\mathcal{A},\Sigma}(\lambda, k) &= \Pr\left[\mathsf{Win} \wedge \mathsf{Bad}\right] + \Pr\left[\mathsf{Win} \wedge \neg\mathsf{Bad}\right] \\ &\leq \Pr\left[\mathsf{Bad}\right] + \Pr\left[\mathsf{Win} \mid \neg\mathsf{Bad}\right] \end{aligned}$$

where the inequality comes from applying the definition of conditional probability and upperbounding $\Pr\left[\mathsf{Win} \mid \mathsf{Bad}\right]$ and $\Pr\left[\neg\mathsf{Bad}\right]$ by 1.

We notice that $\Pr\left[\mathsf{Win} \mid \neg\mathsf{Bad}\right]$ is essentially the probability that the event $\mathsf{bad}_i$ occurs only for $i = q_V + 1$ and never before, i.e.,

$$\Pr\left[\mathsf{Win} \mid \neg\mathsf{Bad}\right] \leq \Pr\left[\mathsf{bad}_{q_V+1} \mid \bigwedge_{i=1}^{q_V} \neg\mathsf{bad}_i\right]$$

In order to bound $\Pr\left[\mathsf{Bad}\right]$, we define events $\mathsf{Bad}^*_i$ (for $i = 1$ to $q_V$) as "$\mathsf{bad}_i$ occurs *for the first time* at query $i$", namely $\mathsf{Bad}^*_i = \mathsf{bad}_i \wedge \left(\bigwedge_{j=1}^{i-1} \neg\mathsf{bad}_j\right)$. Then we have

$$\begin{aligned} \Pr\left[\mathsf{Bad}\right] = \Pr\left[\bigvee_{i=1}^{q_V} \mathsf{Bad}^*_i\right] &= \sum_{i=1}^{q_V} \Pr\left[\mathsf{Bad}^*_i\right] \\ &\leq \sum_{i=1}^{q_V} \Pr\left[\mathsf{bad}_i \;\mid\; \bigwedge_{j=1}^{i-1} \neg\mathsf{bad}_j\right] \end{aligned}$$

where the second equality holds because the events $\mathsf{Bad}^*_i$ are all disjoint, and the inequality follows from applying the definition of conditional probability and upperbounding $\Pr\left[\bigwedge_{j=1}^{i-1} \neg\mathsf{bad}_j\right]$ by 1, for all $i$. Thus:

$$Adv^{CMVA}_{\mathcal{A},\Sigma}(\lambda, k) \leq \sum_{i=1}^{q_V+1} \Pr\left[\mathsf{bad}_i \mid \bigwedge_{j=1}^{i-1} \neg\mathsf{bad}_j\right]. \tag{2}$$

The final steps of the proof rely on the following lemma.

**Lemma 1** *For every $i = 1$ to $q_V + 1$, it holds that*

$$\Pr\left[\mathsf{bad}_i = 1 \mid \bigwedge_{j=1}^{i-1} \neg\mathsf{bad}_j\right] \leq \frac{1}{q^{k-(i-1)}}.$$

The proof of Lemma 1 is deferred momentarily to let us complete the reasoning that proves the theorem. Using the inequality provided by Lemma 1, it is easy to see that $\sum_{i=1}^{q_V+1} \Pr\left[\mathsf{bad}_i = 1 \mid \bigwedge_{j=1}^{i-1} \neg\mathsf{bad}_j\right] \leq \sum_{i=1}^{q_V+1} \frac{1}{q^{k-(i-1)}}$. Indeed, $\frac{1}{q^{k-(i-1)}} \leq \frac{1}{q^k - q_V}$ for all integers $i$ in $[1, q_V + 1]$ and for all $q_V, q, k \in \mathbb{N}$ satisfying $q_V < q^k$. Thus $\sum_{i=1}^{q_V+1} \frac{1}{q^{k-(i-1)}} \leq \frac{q_V+1}{q^k - q_V}$, which proves the bound on the advantage. □

***Proof of Lemma 1*** To upperbound $\Pr\left[\mathsf{bad}_i = 1 \mid \bigwedge_{j=1}^{i-1} \neg\mathsf{bad}_j\right]$ we need to analyze the information leakage due to verification queries. First of all, by correctness $\mathsf{onVer}(\mathsf{svk}, \mu_i, \boldsymbol{\sigma}_i) = 0 \Rightarrow \mathsf{Ver}(\mathsf{pk}, \mu_i, \boldsymbol{\sigma}_i) = 0$ and $\mathsf{Ver}(\mathsf{pk}, \mu_i, \boldsymbol{\sigma}_i) = 1 \Rightarrow \mathsf{onVer}(\mathsf{svk}, \mu_i, \boldsymbol{\sigma}_i) = 1$ for every possible $\mathsf{svk}$ generated by $\mathsf{offVer}$ from $\mathsf{pk}$. Leakage about $\mathsf{svk}$ happens in two cases: when an event $\mathsf{bad}_i$ occurs ($O\mathsf{onVer}$ accepts where the standard verification would reject); and when $O\mathsf{onVer}$ rejects a query (here $\mathcal{A}$ may learn that some combination of rows of $\mathsf{pk}$ must appear in $\mathsf{svk}$). Equation (2) gives us a way to bound the adversary's advantage (and thus, the magnitude of this leakage) in terms of the events $\mathsf{bad}_i$ and $\neg\mathsf{bad}_i$.

Consider the $i$-th query $(\mu_i, \boldsymbol{\sigma}_i)$ to $O\mathsf{onVer}$. If the oracle returns 0, the adversary learns that $\mathbf{C} \cdot (\mathbf{M}_i \cdot \mathbf{v}_i) \neq \mathbf{0} \mod q$. In other words, there is *at least one* row of $\mathbf{C} \in \mathcal{C} := \{\mathbf{C} \in \mathbb{Z}_q^{k \times n} : rk(\mathbf{C}) = k\}$, say $\mathbf{c}_j$, that is *not* in the hyperplane orthogonal to $\mathbf{w}_i := \mathbf{M}_i \cdot \mathbf{v}_i$, i.e., $\mathbf{c}_j \cdot \mathbf{w}_i \neq 0 \mod q$. Note that $\mathcal{A}$ knows $\mathbf{w}_i$ since $(\mathbf{M}_i, \mathbf{v}_i)$ can be computed from the $\mathsf{pk}$, $\mu_i$ and $\boldsymbol{\sigma}_i$.

Let us introduce the sets $\mathcal{H}_i \subseteq \mathcal{C}$ of full-rank matrices $\mathbf{C} \in \mathcal{C}$ whose rows are *all orthogonal* to $\mathbf{w}_i$, formally:

$$\mathcal{H}_i := \left\{ \mathbf{C} \in \mathcal{C} \;:\; \mathbf{C} = \begin{bmatrix} \mathbf{c}_1 \\ \ldots \\ \mathbf{c}_k \end{bmatrix} \wedge \begin{array}{c} \mathbf{c}_j \cdot \mathbf{w}_i = 0 \mod q \\ \forall\, j = 1, \ldots, k \end{array} \right\}.$$

We assume $\mathcal{A}$ be able to pick the vectors $\mathbf{w}_i \in \mathbb{Z}_q^n \setminus \{0\}$ of her choosing (e.g., by generating suitable pairs $(\mu_i, \boldsymbol{\sigma}_i)$). This assumption is generous as it gives the adversary a large amount of power and freedom in the game. The restriction $\mathbf{w}_1 \neq \mathbf{0}$ is technical, as otherwise $\mathsf{Ver}(\mathsf{pk}, \mu_1, \boldsymbol{\sigma}_1) = 0$, which is a necessary condition for $O\mathsf{onVer}$ leaking information about $\mathsf{svk}$.

At the first verification query $(\mu_1, \boldsymbol{\sigma}_1)$, $\mathcal{A}$ has no information about $\mathbf{C}$ beyond the fact that it was uniformly sampled from the set $\mathcal{C} := \{\mathbf{C} \in \mathbb{Z}_q^{k \times n} : rk(\mathbf{C}) = k\}$. Therefore, for any choice of $\mathbf{w}_1 \neq \mathbf{0}$, if the event $\mathsf{bad}_1$ occurs,

then $\mathsf{bad}_1 = \{\mathbf{C}\cdot\mathbf{w}_1 = \mathbf{0} \mod q \ \wedge\ \mathbf{C} \xleftarrow{\$} \mathcal{C}\}$, thus $\Pr[\mathsf{bad}_1] = \Pr\left[\mathbf{C}\cdot\mathbf{w}_1 = \mathbf{0} \mod q \ \wedge\ \mathbf{C} \xleftarrow{\$} \mathcal{C}\right] = \frac{|\mathcal{H}_1|}{|\mathcal{C}|}$. The first (rejected) verification query leaks the fact that $\mathbf{C} \in \mathcal{C} \setminus \mathcal{H}_1$.

For the second verification query, without loss of generality let $\mathbf{w}_2$ be linearly independent from $\mathbf{w}_1$, i.e., $\mathbf{w}_2 \notin \langle \mathbf{w}_1 \rangle_q$. In this case, we have

$$
\begin{aligned}
&\Pr[\mathsf{bad}_2 \mid \neg\mathsf{bad}_1] = \\
&= \Pr[\mathbf{C}\cdot\mathbf{w}_2 = \mathbf{0} \mod q \mid \mathbf{C} \xleftarrow{\$} \mathcal{C} \ \wedge\ \mathbf{C} \in (\mathcal{C}\setminus\mathcal{H}_1)] \\
&= \frac{\Pr\left[\mathbf{C}\cdot\mathbf{w}_2 = \mathbf{0} \mod q \ \wedge\ \mathbf{C} \xleftarrow{\$} \mathcal{C} \ \wedge\ \mathbf{C} \in (\mathcal{C}\setminus\mathcal{H}_1)\right]}{\Pr\left[\mathbf{C} \xleftarrow{\$} \mathcal{C} \ \wedge\ \mathbf{C} \in (\mathcal{C}\setminus\mathcal{H}_1)\right]} \\
&\leq \frac{\Pr\left[\mathbf{C}\cdot\mathbf{w}_2 = \mathbf{0} \mod q \ \wedge\ \mathbf{C} \xleftarrow{\$} \mathcal{C}\right]}{\Pr\left[\mathbf{C} \xleftarrow{\$} \mathcal{C} \ \wedge\ \mathbf{C} \in (\mathcal{C}\setminus\mathcal{H}_1)\right]} \\
&= \frac{\frac{|\mathcal{H}_2|}{|\mathcal{C}|}}{\frac{|\mathcal{C}\setminus\mathcal{H}_1|}{|\mathcal{C}|}} = \frac{|\mathcal{H}_1|}{|\mathcal{C}\setminus\mathcal{H}_1|}
\end{aligned}
$$

where the inequality follows from the fact that, given three events $E_1, E_2, E_3$, it always holds that $\Pr[E_1 \wedge E_2 \wedge E_3] \leq \min\{\Pr[E_1 \wedge E_2],\ \Pr[E_1 \wedge E_3],\ \Pr[E_2 \wedge E_3]\}$; and the last equality follows since the hyperplanes $\mathcal{H}_1$ and $\mathcal{H}_2$ have the same dimension.

The same reasoning applies to the generic $i$-th verification query, where, w.l.o.g., $\mathcal{A}$ chooses $\mathbf{w}_i$ outside the space generated by the previous $\mathbf{w}_j$'s, i.e., $\mathbf{w}_i \notin \langle \mathbf{w}_1, \ldots, \mathbf{w}_{i-1} \rangle_q$. At such query, $\mathcal{A}$ knows that $\mathbf{C} \in \mathcal{C} \setminus \left(\bigcup_{j=1}^{i-1} \mathcal{H}_j\right)$. Analogously as before we get that

$$
\begin{aligned}
&\Pr\left[\mathsf{bad}_i = 1 \mid \bigwedge_{j=1}^{i-1} \neg\mathsf{bad}_j\right] \\
&\leq \frac{\Pr\left[\mathbf{C}\cdot\mathbf{w}_i = \mathbf{0} \mod q \ \wedge\ \mathbf{C} \xleftarrow{\$} \mathcal{C}\right]}{\Pr\left[\mathbf{C} \in \mathcal{C} \setminus \left(\bigcup_{j=1}^{i-1} \mathcal{H}_j\right) \ \wedge\ \mathbf{C} \xleftarrow{\$} \mathcal{C}\right]} \\
&= \frac{|\,\mathcal{H}_1|}{\left|\mathcal{C} \setminus \left(\bigcup_{j=1}^{i-1} \mathcal{H}_j\right)\right|}\,. \qquad (3)
\end{aligned}
$$

Lemma 2 (stated and proven next) concludes the proof showing that $\forall\, i = 2, \ldots, q_V + 1$:

$$
\left|\mathcal{C} \setminus \left(\bigcup_{j=1}^{i-1} \mathcal{H}_j\right)\right| \geq |\mathcal{H}_1| \cdot \left(\frac{q^n - q}{q^{n-k} - 1} - (i-1)\right).
$$

Substituting this value into Eq. (3) returns:

$$
\Pr[\mathsf{bad}_i = 1 \mid \bigwedge_{j=1}^{i-1} \neg\mathsf{bad}_j] \leq \frac{1}{q^k \cdot \frac{1-q^{1-n}}{1-q^{k-n}} - (i-1)} \leq \frac{1}{q^k - (i-1)}
$$

where the last bound follows from the chain:

$$
q^{n-1} > q^{n-k} \ \Leftrightarrow\ \frac{1}{q^{n-1}} < \frac{1}{q^{n-k}}
$$

$$
\Leftrightarrow\ 1 - \frac{1}{q^{n-1}} > 1 - \frac{1}{q^{n-k}} \ \Leftrightarrow\ \frac{1 - \frac{1}{q^{n-1}}}{1 - \frac{1}{q^{n-k}}} > 1\,,
$$

as $1 < k < n$ and $q > 1$. □

**Lemma 2** *Let $\mathcal{C} := \{\mathbf{C} \in \mathbb{Z}_q^{k\times n} \ :\ rk(\mathbf{C}) = k\}$ be the set of $k \times n$ matrices that are full rank and have entries in $\mathbb{Z}_q$ (as introduced in the proof of Lemma 1).*

*For any given set of vectors $\{\mathbf{w}_1, \ldots, \mathbf{w}_{q_V+1}\} \subseteq \mathbb{Z}_q^n$ such that $\mathbf{w}_i \notin \langle \mathbf{w}_j \rangle_q$ for every $i \neq j$, define the collection of sets $\{\mathcal{H}_i := \{\mathbf{C} \in \mathcal{C} \ :\ \mathbf{C}\cdot\mathbf{w}_i = 0 \mod q\}\}_{i=0}^{q_V+1} \subseteq \mathcal{C}$ containing the matrices having the corresponding $\mathbf{w}_i$ in their right kernel and $\mathcal{H}_0 = \{\varnothing\}$. It holds that $\forall\, i = 1, \ldots, q_V$:*

$$
\left|\mathcal{C} \setminus \left(\bigcup_{j=0}^{i-1} \mathcal{H}_j\right)\right| \geq |\mathcal{H}_1| \cdot \left(\frac{q^n - 1}{q^{n-k} - 1} - (i-1)\right). \qquad (4)
$$

***Proof of Lemma 2*** It is easy to see that the cardinality of $\mathcal{C}$ is:

$$
\begin{aligned}
|\mathcal{C}| &= (q^n - 1)\cdot(q^n - q)\cdot\ldots\cdot(q^n - q^{k-1}) \\
&= q^{\frac{k(k-1)}{2}} \cdot \prod_{j=0}^{k-1} (q^{n-j} - 1).
\end{aligned}
$$

as we have full freedom for how to pick the first row of $\mathbf{C}$ (except for $\mathbf{c}_1 \neq 0$); for the second row, we can pick any vector $\mathbf{c}_2$ that is in $\mathbb{Z}_q^n$ but not in the span of the previous row of $\mathbf{C}$ (to keep the matrix full rank), and so on. The formula after the final equality follows from decomposing each $(q^n - q^j)$ factor as $q^j(q^{n-j} - 1)$ and observing that $\prod_{j=1}^{k-1} q^j = q^{\frac{k(k-1)}{2}}$. The same reasoning applies to computing the cardinality of a generic $\mathcal{H}_i$ (for $i > 0$), after noticing that the rows of the matrices $\mathbf{C} \in \mathcal{H}_i$ must be picked (as linearly independent vectors) from the $(n-1)$-dimensional hyperplane orthogonal to $\mathbf{w}_i$; thus

$$
\begin{aligned}
|\mathcal{H}_i| &= \prod_{j=0}^{k-1} (q^{n-1} - q^j) = q^{\frac{k(k-1)}{2}} \cdot \prod_{j=0}^{k-1} (q^{(n-1)-j} - 1) \\
&= q^{\frac{k(k-1)}{2}} \cdot \prod_{j=1}^{k} (q^{n-j} - 1)
\end{aligned}
$$

This proves the base case ($i = 1$) of the bound in (4): $|\mathcal{C} \setminus \varnothing| = |\mathcal{H}_1| \cdot \frac{q^n - 1}{q^{n-k} - 1}$, since, compared to $|\mathcal{H}|$, $|\mathcal{C}|$ has the additional factor $j = 0$ and the missing factor $j = k$.

Concretely this means that: $\Pr[\mathsf{bad}_1] \leq \frac{q^{n-k}-1}{q^n-1}$. For $i = 2$, again we get $|\mathcal{C}\backslash\mathcal{H}_1| = |\mathcal{C}| - |\mathcal{H}_1| = |\mathcal{H}_1|\left(\frac{q^n-1}{q^{n-k}-1} - 1\right)$. For $i = 3$, we remove from the pool of eligible $\mathbf{C}$ all those matrices in $\mathcal{H}_1 \cup \mathcal{H}_2$, i.e., that have either $\mathbf{w}_1$ or $\mathbf{w}_2$ in their right kernel.[3] In other words, matrices composed by only rows orthogonal to $\mathbf{w}_1$ or to $\mathbf{w}_2$. The hyperplanes $\mathbf{w}_1^{\perp}$ and $\mathbf{w}_2^{\perp}$ both have dimension $n-1$, and since we are in a space of dimension $n$, they must intersect in a subspace of dimension $n-2$. For a tighter bound we use: $|\mathcal{H}_2 \setminus \mathcal{H}_1| = |\mathcal{H}_2| - |\mathcal{H}_1 \cap \mathcal{H}_2|$ and recall that $0 \leq |\mathcal{H}_2| - |\mathcal{H}_1 \cap \mathcal{H}_2| \leq |\mathcal{H}_1|$. Hence after the second rejected query the number of possible $\mathbf{C}$ becomes:

$$
\begin{aligned}
&|\mathcal{C} \setminus (\mathcal{H}_1 \cup \mathcal{H}_2)| = |\mathcal{C}| - |\mathcal{H}_1| - |\mathcal{H}_2| + |\mathcal{H}_1 \cap \mathcal{H}_2| \\
&= |\mathcal{H}_1| \cdot \left(\frac{q^n-1}{q^{n-k}-1} - 2\right) + |\mathcal{H}^{(n-2)}| \\
&= |\mathcal{H}_1| \cdot \left(\frac{q^n-1}{q^{n-k}-1} - 2\right) + q^{k(k-1)/2} \prod_{j=2}^{k-1} (q^{n-j} - 1) \\
&= |\mathcal{H}_1| \cdot \left(\frac{q^n-1}{q^{n-k}-1} - 2 + \frac{1}{(q^{n-1}-1)(q^{n-k}-1)}\right) \\
&\geq |\mathcal{H}_1| \cdot \left(\frac{q^n-1}{q^{n-k}-1} - 2\right).
\end{aligned}
$$

Remark that $\mathbf{C}$ could be still composed by some elements of $\mathcal{H}_1$ *and* some of $\mathcal{H}_2 \setminus \mathcal{H}_1$; this would be consistent with $\mathcal{A}$'s view at this point.

We can now proceed by induction, assuming (4) holds for the query index $i$, prove it for $i+1$.

$$
\begin{aligned}
&\left|\mathcal{C} \setminus \bigcup_{j=0}^{i} \mathcal{H}_j\right| = \left|\mathcal{C} \setminus \left(\left(\bigcup_{j=0}^{i-1} \mathcal{H}_j\right) \cup \mathcal{H}_i\right)\right| \\
&= |\mathcal{C}| - \left|\bigcup_{j=0}^{i-1} \mathcal{H}_j\right| - |\mathcal{H}_i| + \left|\left(\bigcup_{j=0}^{i-1} \mathcal{H}_j\right) \cap \mathcal{H}_j\right| \\
&= \left|\mathcal{C} \setminus \bigcup_{j=0}^{i-1} \mathcal{H}_j\right| - |\mathcal{H}_1| + \left|\left(\bigcup_{j=0}^{i-1} \mathcal{H}_j\right) \cap \mathcal{H}_j\right| \\
&\geq \left|\mathcal{C} \setminus \bigcup_{j=0}^{i-1} \mathcal{H}_j\right| - |\mathcal{H}_1| \\
&\geq |\mathcal{H}_1| \cdot \left(\frac{q^n-1}{q^{n-k}-1} - i + 1\right) - |\mathcal{H}_1| \\
&= |\mathcal{H}_1| \cdot \left(\frac{q^n-1}{q^{n-k}-1} - i\right).
\end{aligned}
$$

□

[3] The vectors $\mathbf{w}_i$ are assumed not to be multiples of one another. Otherwise, $\mathcal{A}$ does not extract new information from a rejection, i.e., there is no additional leakage.

### 4.2 A compiler for progressive Mv-style verification

Our compiler for progressive verification builds on the result presented in Sect. 4.1. Given a signature scheme $\Sigma$ with **Mv**-style verification, we define the $T$ steps of a progressive verification $\Sigma^P$ for $\Sigma$ as shown in Fig. 5.

The value $T$ sets the upper bound on the number of linear constraints the verifier wants to check, hence $T = rows(\mathbf{M})$, where $\mathbf{M}$ is the matrix employed in the original signature verification of $\Sigma$. The set of admissible states $\mathcal{S}$ includes $\emptyset$ and any possible state output by some $\mathsf{ProgVer}_i$, specifically $\mathcal{S} = \{0,1\} \times \mathbb{Z}_q^{rows(\mathbf{Z}') \times cols(\mathbf{Z}')} \times \mathbb{Z}_q^{rows(\mathbf{v}) \times cols(\mathbf{v})} \times \{0,1\}^{\lambda} \cup \varnothing$. We extract the confidence level from the probability of a progressive forgery (as motivated by the proof of security given in Theorem 1). It is easy to see that the probability that an adversary creates a progressive forgery for an interruption step $t$ is at most $\frac{q^{n-t}-1}{q^n-1}$, this follows from the same reasoning as in the proof of Theorem 1 for efficient verification. Concretely, the bound is derived from the proof of Lemma 2, where we only consider $\Pr[\mathsf{bad}_1]$ as $\mathsf{svk}$ is refreshed with every new efficient verification query, and so there is no useful cross-query leakage, and we replace the confidence level $k$ of the efficient verification with the interruption parameter $t$. If the size of the underlying algebraic structure is $q = 2^{\mathsf{poly}(\lambda)}$ this probability is negligible already for $t = 1$. In other words, for signatures with **Mv**-style verification defined on *exponentially large* algebraic structures *efficient verification and progressive verification coincide*, trivially. The interesting case is $q = \mathsf{poly}(\lambda)$, as the adversary could create a progressive forgery with non-negligible probability. We remark that in this section we are not targeting efficiency, and our instantiations of progressive verification refresh the $\mathsf{svk}$ produced by $\mathsf{offVer}$ at every verification query. This way, $\mathcal{A}$ cannot exploit the information possibly leaked by a progressive forgery in future forgery attempts.

Finally, we note that, as in the case of efficient verification, our compiler leaves the signing algorithm and the signatures as in the original scheme, and only provides an alternative verification algorithm which achieves the progressive verification property.

#### 4.2.1 Security analysis

**Theorem 2** *Let $\Sigma$ be an existentially unforgeable signature scheme with* **Mv***-style verification (as of Fig. 3). Then the scheme $\Sigma^P$ obtained via our compiler (in Fig. 5) is a secure realization of progressive verification for $\Sigma$.*

***Proof*** Recall that an adversary $\mathcal{A}$ wins the security experiment in Definition 7 if it outputs a message-signature pair $(\mu^*, \sigma^*)$ and an interruption $t'$ such that:

(1) $(\mu^*, \sigma^*)$ is rejected by $\mathsf{Ver}$, but accepted $\mathsf{ProgVer}$ when it is interrupted at step $t^* \leftarrow \mathit{OInt}(t')$; and

(2) the progressive verification algorithm outputs a too high confidence level $\alpha_{\mathsf{prog}}(t^*)$.

Following Definition 7, we can realize secure progressive verification by setting $\alpha_{\mathsf{prog}}(t) = 1 - \Pr\left[\mathsf{Exp}^{\mathsf{progEUF}}_{\mathcal{A},\Sigma}(\lambda) = (1,t)\right] + \varepsilon(\lambda)$ for all $t = 0,\ldots,T$. The core part of the proof is to estimate this probability.

Recall that our compiler for efficient **Mv**-style verification (in Fig. 5) runs offVer at every verification query (line 1 in $\mathsf{ProgVer}_0$). This means that every verification query is answered using a freshly generated svk. In particular, the final verification (line 4 in the $\mathsf{Exp}^{\mathsf{progEUF}}_{\mathcal{A},\Sigma^P}(\lambda)$ in Fig. 2) checks $\mathcal{A}$'s output using independent randomness from the previous queries. So, whatever information the adversary may have collected from previous queries is useless to win the experiment. As a consequence, the probability that the adversary wins the game equals the probability that the adversary outputs a valid forgery *without querying* $O\mathsf{ProgVer}$. The latter is precisely the probability of the event $\mathsf{bad}_1$ defined in the proof of Theorem 1, where now we consider the matrix $\mathbf{C}$ to have $t^*$ rows instead of $k$. Hence from Lemma 1 it follows that $\Pr\left[\mathsf{Exp}^{\mathsf{progEUF}}_{\mathcal{A},\Sigma}(\lambda) \le (1,t^*)\right] = \frac{1}{q^{t^*}}$ and:

$$
\begin{aligned}
&Adv^{\mathsf{progEUF}}_{\mathcal{A},\Sigma}(\lambda) \\
&\quad = \Pr\left[\mathsf{Exp}^{\mathsf{progEUF}}_{\mathcal{A},\Sigma}(\lambda) = (1,t^*)\right] - (1 - \alpha_{\mathsf{prog}}(t^*)) \\
&\quad \le \frac{1}{q^{t^*}} - \left(1 - \left(1 - \frac{1}{q^{t^*}}\right)\right) = 0\,.
\end{aligned}
$$

□

## 5 Examples of efficient verification

Because any instantiation of our compiler is completely determined by the four subroutines **parse** pk, GetM, Check, and GetZV, in what follows we explain only how these four algorithms work. The complete descriptions of offVer and onVer are derived using the general structure given in Fig. 4.

### 5.1 From lattices

We present concrete instantiations of our compiler for two categories of LBS: 'hash & sign' with representative the GPV08 signature [24], and 'Boyen/BonsaiTree' style with representative MP12 [32].

*Efficient verification for GPV08* [24]. The **parse** pk procedure splits the public key into $PK = \mathbf{A} \in \mathbb{Z}_q^{n\times m}$ (the matrix identifying the signer's public key), and the auxiliary public information $PK.\mathsf{aux} = (\mathcal{H}, \beta)$, i.e., a description of a full-domain hash function $\mathcal{H} : \{0,1\}^* \to \mathbb{Z}_q^n$ and the norm bound $\beta \in \mathbb{R}$. The Check procedure is exactly as in the original verification (enforcing the norm bound $\beta$ on the signature). The GetM algorithm takes in input the public matrix $PK = \mathbf{A}$, and tails to it the identity matrix: $\mathbf{M} = [\mathbf{A}|-\mathbf{I}_n]$. The GetZV routine returns the matrix $\mathbf{Z}'$ (explained momentarily) and the vector $\mathbf{v} = [\boldsymbol{\sigma}|\mathcal{H}(\mu)\cdot\mathbf{1}_{1\times n}]$. The matrix $\mathbf{Z}'$ is made up of the same 'randomized key' vectors produced by GetZ during the offline verification, i.e., $\mathbf{z}'_j = \mathbf{z}_j \leftarrow \mathbf{c}_j\mathbf{M} = [\mathbf{c}_j\mathbf{A}|-\mathbf{c}_j]$. Thus the core verification check (line 7 in onVer) is actually ensuring that $\mathbf{z}'_j\mathbf{v}_j = 0$, i.e., $\mathbf{c}_j\cdot\mathbf{A}\cdot\boldsymbol{\sigma} = \mathbf{c}_j\mathcal{H}(\mu)$ which is the probabilistic check of the original verification equality.

*Efficient Verification for MP12* [32].

The **parse** pk procedure assigns $PK \leftarrow \mathbf{A} = [\tilde{\mathbf{A}}|\mathbf{A}_0 \mid \ldots \mid \mathbf{A}_\ell] \in \mathbb{Z}_q^{n\times(\bar{m}+n\lceil\log q\rceil\ell)}$ (the matrix identifying the signer's public key), where $\bar{m} = O(n\lceil\log q\rceil)$, and $\ell$ denotes the number of bits in the message, i.e., $\mu \in \{0,1\}^\ell$. The auxiliary public information is $PK.\mathsf{aux} = (\mathbf{u}, \beta)$. The Check procedure is exactly as in the original verification (enforcing the norm bound $\beta$ on the signature). The GetM algorithm takes in input the public matrix $PK = \mathbf{A}$, and appends to it the identity matrix to obtain $\mathbf{M} = [\mathbf{A}|-\mathbf{I}_n]$. The GetZV routine returns the matrix $\mathbf{Z}'$ and the vector $\mathbf{v}$. The matrix $\mathbf{Z}'$ is made up of vectors of the form $\mathbf{z}'_j = [\tilde{\mathbf{z}}_j \mid \mathbf{z}_j^0 + \sum_{i=1}^{\ell}\mu[i]\mathbf{z}_j^i|\mathbf{c}_j]$ that identify a message-dependent lattice (called $\mathbf{A}_\mu$ in [32]). The vector $\mathbf{v}$ is the concatenation of the signature with the auxiliary vector, i.e., $\mathbf{v} = [\boldsymbol{\sigma}|\mathbf{u}]$. Note that $\mathbf{u}$ is the same for all messages; thus, one could further optimize the online verification by computing (once and for all) the $k$ inner products $\mathbf{z}_j[\bar{m}+n\lceil\log q\rceil+1] = \mathbf{c}_j\cdot\mathbf{u}$ during the offline phase. To conclude we notice that the online verification ensures that $\mathbf{z}'_j\mathbf{v}_j = 0$, i.e., $\mathbf{c}_j\cdot\mathbf{A}_\mu\cdot\boldsymbol{\sigma} = \mathbf{c}_j\cdot\mathbf{u}$ which is the probabilistic check of the original verification equality.

### 5.2 From multivariate equations

For signatures based on multivariate equations we take Rainbow, MAYO, and LUOV as examples. We consider Rainbow, despite it being broken, as it allows us to anayze the impact of our technique in practice, thanks to its implementation made available for the NIST standardization process.

*Efficient Verification for Rainbow* [18]. In the description below we consider the standard Rainbow verification. A similar approach can be used to speed up the verification also in the "cyclic" and the "compressed" Rainbow variants as in those cases the verification includes an additional initial phase to reconstruct the full public key. We recall that in this scheme the public key contains a system of $m$ multivariate quadratic polynomials in $n$ variables. For convenience, let $N = n(n+1)/2$ and consider the field $\mathbb{F} = \mathbb{F}_{2^r}$. Using a Macaulay matrix representation we can visualize this system as a wide matrix composed of a quadratic term $\mathbf{Q}$ (actually a $m \times N$ submatrix), a linear term $\mathbf{L}$ ($m \times n$

**Table 1** A summary of the concrete efficiency achieved by various instatiations of our compiler for efficient verification

| Ring or Field Size (representative schemes) | Min. Accuracy Level for 128-bit security | Concrete Amortized Efficiency (see Definition 3) | Online Efficiency $\frac{\text{cost(onVer)}}{\text{cost(Ver)}} = \frac{k_0}{n}$ |
|---|---|---|---|
| exponential: $q = 2^{128}$ (FMNP [20]; GVW [25]) | $k_0 = 1$ | $(\mathsf{r}_0 = 2, \mathsf{e}_0 = 0.51)$ | $\frac{1}{256} < 0.4\%$ |
| large poly.: $q = 2^{30}$ (Boyen [11]; GPV [24]; MP [32]) | $k_0 = 5$ | $(\mathsf{r}_0 = 6, \mathsf{e}_0 = 0.86)$ | $\frac{5}{256} < 2\%$ |
| small poly.: $q = 16$ (Rainbow [19] $\mathbb{F}_{2^4}$-(36, 32, 32)) | $k_0 = 32$ | $(\mathsf{r}_0 = 65, \mathsf{e}_0 = 0.99)$ | $\frac{32}{64} = 50\%$ |

In the table, $k_0$ denotes the minimum accuracy level that ralizes efficient verification with 128 bits of security, i.e., for which $\Pr[\mathsf{Bad}] \leq 2^{-128}$ is negligible (cf. proof of Theorem 1, with $q_V = 2^{30}$); $\mathsf{r}_0$ is the smallest positive integer for which $\frac{\text{cost(offVer(pk},k_0))+\mathsf{r}\cdot\text{cost(onVer)}}{\mathsf{r}\cdot\text{cost(Ver)}} < 1$, and $\mathsf{e}_0$ is a (tight) upperbound on this ratio

$\mathsf{ProgStep}_0(\mathsf{st}, \mathsf{pk}, \mu, \boldsymbol{\sigma})$
1: $\mathsf{svk} \leftarrow \mathsf{offVer}(\mathsf{pk}, T)$
2: **parse** $\mathsf{svk} = (T, \mathbf{Z}, PK.\mathsf{aux})$
3: $b \leftarrow \mathsf{Check}(PK.\mathsf{aux}, \mu, \boldsymbol{\sigma})$
4: $\mathsf{st} \leftarrow \mathsf{GetZV}(\mathsf{svk}, \mu, \boldsymbol{\sigma})$
5: **return** $(b, \mathsf{st})$

$\mathsf{ProgStep}_i(\mathsf{st}, \mathsf{pk}, \mu, \boldsymbol{\sigma})$
1: $b \leftarrow 0$
2: **parse** $\mathsf{st} = (\mathbf{Z}', \mathbf{v})$
3: **if** $\mathbf{Z}'[i, *] \cdot \mathbf{v}[*, i] = 0 \bmod q$
4: **return** $(b \leftarrow 1, \mathsf{st})$
5: **return** $(b \leftarrow 0, \mathsf{st})$

$\alpha_{\mathsf{prog}} : \{0, \ldots, T\} \to [0, 1], \qquad \alpha_{\mathsf{prog}}(t) = (1 - \frac{1}{q^t})$

**Fig. 5** Our compiler for progressive verification of signatures with **Mv**-style verification. The algorithms offVer, Check and GetZV are precisely as defined in Sect. 4.1, Fig. 4, and $T = rows(\mathbf{M})$. The notation $\mathbf{Z}'[i, *]$ describes the $i$-th row of the matrix $\mathbf{Z}'$, similarly $\mathbf{v}[*, i]$ describes the $i$-th column of $\mathbf{v}$ (which is usually a vector $\mathbf{v}$, but may be a matrix in some constructions)

submatrix) and a constant term $\mathbf{C}$ (a $m \times 1$ vector). The **parse** pk procedure extracts from the public key $PK$ this matrix $\mathsf{pk} = [\mathbf{Q}|\mathbf{L}|\mathbf{C}] \in \mathbb{F}^{m\times(N+n+1)}$ and a description of a full-domain hash function $\mathcal{H} : \{0, 1\}^* \to \mathbb{F}^m$ as the auxiliary public information $PK.\mathsf{aux} = \mathcal{H}$. The Check procedure is trivial and always returns 1. This is because the whole verification can be written as a matrix–vector multiplication. The GetM algorithm extracts from $PK$ the matrix representing the system of quadratic multivariate equations $[\mathbf{Q}|\mathbf{L}|\mathbf{C}]$. Finally, it appends to this the identity matrix, so $\mathbf{M} \leftarrow [\mathbf{Q}|\mathbf{L}|\mathbf{C}| - \mathbf{I}_m]$. We remark that $\mathbf{M}$ can be seen as a matrix of blocks, where any block has the same height ($m$ = number of rows), but different length (number of columns). The GetZV routine reads the matrix $\mathbf{Z}' = \mathbf{Z}$ made up of the rows $\mathbf{z}'_j = \mathbf{z}_j \leftarrow \mathbf{c}_j\mathbf{M} = [\mathbf{c}_j \cdot \mathbf{Q}|\mathbf{c}_j \cdot \mathbf{L}|\mathbf{c}_j \cdot \mathbf{C}| - \mathbf{c}_j] \in \mathbb{F}^{1\times N+n+1}$. In addition, this algorithm parses the signature as $\boldsymbol{\sigma} = (\mathbf{s}, \mathsf{salt})$, computes the (salted) hash of the message $\mathbf{d}$ as $\mathbf{h} \leftarrow \mathcal{H}(\mathcal{H}(\mathbf{d})\|\mathsf{salt})$ and outputs the vector $\mathbf{v} = [\tilde{\mathbf{s}}|\mathbf{s}|1|\mathbf{h}]$, where $\mathbf{s}$ is part of the signature and $\tilde{\mathbf{s}}$ is the 'quadratic vector' obtained by computing all products of pairs of elements in $\mathbf{s}$ (with monomials ordered lexicographically ), i.e., $\tilde{\mathbf{s}} \leftarrow [\mathbf{s}[1]^2, \mathbf{s}[1]\mathbf{s}[2], \ldots, \mathbf{s}[n-1]\mathbf{s}[n], \mathbf{s}[n]^2]$. Clearly $\mathbf{z}'_j \cdot \mathbf{v} = 0$ if and only if $\mathbf{c}_j \cdot (\mathbf{Q}\tilde{\mathbf{s}} + \mathbf{L}\mathbf{s} + \mathbf{C}) = \mathbf{c}_j \cdot \mathbf{h}$, which is a probabilistic check of the original system of verification equations in Rainbow.

*Efficient Verification for MAYO* [7]. MAYO signatures are a "whipped up" version of unbalanced oil and vinegar (UOV) signatures. The public key is a trapdoored multivariate map $\mathcal{P} : \mathbb{F}_q^n \to \mathbb{F}_q^m$, which consists of a system of $m$ multivariate quadratic polynomials in $n$ variables, where all polynomials share a common vanishing linear space $O \subseteq \mathbb{F}_q^n$ (the oil space). Beullens optimizes the public key generation so that each polynomial $p_i$ can be represented by a matrix of the form $\mathbf{P}_i = \begin{pmatrix} \mathbf{P}_i^{(1)} & \mathbf{P}_i^{(2)} \\ \mathbf{0} & \mathbf{P}_i^{(3)} \end{pmatrix} \in \mathbb{F}_q^{n\times n}$ and the evaluation of an input $\mathbf{x} \in \mathbb{F}_q^n$ is done in the natural way: $p_i(\mathbf{x}) = \mathbf{x}^T\mathbf{P}_i\mathbf{x}$. MAYO signatures also employ a full-domain hash function $\mathcal{H}$, and $\ell^2$ "emulsifier" matrices $\mathbf{E}_{i,j} \in \mathbb{F}_q^{m\times m}$ that are part of the public parameters, that are the same for all signers.

The **parse** pk procedure extracts from the public key the matrices $PK = (\mathbf{P}_1, \ldots, \mathbf{P}_m)$ and other public parameters $PK.\mathsf{aux} = (\mathcal{H}, \{\mathbf{E}_{i,j}\}_{i,j=1}^{\ell})$.

The GetM algorithm extracts the emulsifier matrices $\mathbf{E}_{i,j}$ from $PK.\mathsf{aux}$ and combines them in the matrix $\mathbf{M} = (\mathbf{E}_{1,1}, \mathbf{E}_{2,2}, \ldots, \mathbf{E}_{\ell,\ell}, \mathbf{E}_{1,2}, \ldots, \mathbf{E}_{\ell-1,\ell}, -\mathbf{I}_m) \in \mathbb{F}_q^{k\times m(L+1)}$, where $L = \ell(\ell+1)/2$ is the total number of emulsifier matrices. In this case svk consists of: $k$ the confidence level; $\mathbf{Z}$ the compressed version of the train of emulsifier matrices with the final block being the randomness used for compression; $\mathcal{H}$ the hash function; and $PK$ the polynomials of the signers' public key. Notably, in MAYO we do not apply our transformation to $PK$, but rather to the emulsifier contained in $PK.\mathsf{aux}$. Thus $\mathbf{Z}$ is general and can be used to efficiently verify signatures by any signer using the same MAYO setting (the public parameters).

The Check algorithm is trivial and always returns 1.

The GetZV algorithm parses $\boldsymbol{\sigma}$ as $(\mathsf{salt}, \mathbf{s}_1, \ldots \mathbf{s}_\ell)$. It uses $\mathbf{Z}' = \mathbf{Z} \in \mathbb{F}_q^{k \times m(L+1)}$ and $\mathbf{v} = (\tilde{\mathbf{s}}, \tilde{\mathbf{v}})$, where $\tilde{\mathbf{v}} = \mathcal{H}(\mu \| \mathsf{salt})$ and $\tilde{\mathbf{s}} = (\mathcal{P}(\mathbf{s}_1), \ldots, \mathcal{P}(\mathbf{s}_\ell), P'(\mathbf{s}_1, \mathbf{s}_2), \ldots \mathcal{P}'(\mathbf{s}_{\ell-1}, \mathbf{s}_\ell)) \in (\mathbb{F}_q^m)^{\frac{\ell(\ell+1)}{2}}$, here $\mathcal{P}'$ is the polar form of $\mathcal{P}$.

*Efficient Verification for LUOV* [8]. The **parse** pk procedure splits the public key into $PK = (\mathsf{public.seed}, \mathbf{Q}_2)$ (the concise information needed to retrieve the full signer's public key), and the auxiliary public information $PK.\mathsf{aux} = \mathcal{H}$, i.e., a description of a full-domain hash function $\mathcal{H} : \{0,1\}^* \to \mathbb{F}^m$, where $m = rows(\mathbf{Q}_2)$ and $\mathbb{F} = \mathbb{F}_{2^r}$. The Check procedure is trivial and always returns 1. This is because the whole LUOV verification can be written as a matrix–vector multiplication The GetM algorithm takes in input $PK = (\mathsf{public.seed}, \mathbf{Q}_2)$ and derives the full public key as done in the original verification: it runs $[\mathbf{C}|\mathbf{L}|\mathbf{Q}_1] \leftarrow \mathcal{G}(\mathsf{public.seed})$ to get the constant constant (vector), the linear (matrix) and the first quadratic (matrix) parts of the verification equation; and then it reconstructs the full quadratic term as $\mathbf{Q} \leftarrow [\mathbf{Q}_1|\mathbf{Q}_2]$. Finally it appends to the public key the identity matrix $\mathbf{M} \leftarrow (\mathbf{C}, \mathbf{L}, \mathbf{Q}, -\mathbf{I}_{rows(\mathbf{Q})})$, we remark that $\mathbf{M}$ can be seen as a matrix of blocks, where any block has the same height (number of rows), but different length (number of columns). The GetZV routine reads the matrix $\mathbf{Z}' = \mathbf{Z}$ made up of the rows $\mathbf{z}'_j = \mathbf{z}_j \leftarrow \mathbf{c}_j\mathbf{M} = (\mathbf{c}_j \cdot \mathbf{C}, \mathbf{c}_j \cdot \mathbf{L}, \mathbf{c}_j \cdot \mathbf{Q}, -\mathbf{c}_j)$. It also outputs the vector $\mathbf{v} = [1|\mathbf{s}|\tilde{\mathbf{s}}|\mathbf{h}]$, where $\mathbf{s}$ is part of the signature $\boldsymbol{\sigma} = (\mathbf{s}, \mathsf{salt})$, $\tilde{\mathbf{s}}$ is the 'quadratic vector' obtained by computing all products of pairs of elements in $\mathbf{s}$, i.e., $\tilde{\mathbf{s}} \leftarrow [\mathbf{s}[1]^2, \mathbf{s}[1]\mathbf{s}[2], \ldots, \mathbf{s}[n-1]\mathbf{s}[n], \mathbf{s}[n]^2]$, finally $\mathbf{h}$ is the hash of the message and the salt, i.e., $\mathbf{h} \leftarrow \mathcal{H}(\mu \| 0\text{x}0 \| \mathsf{salt})$. Clearly $\mathbf{z}'_j \cdot \mathbf{v} = 0$ if and only if $\mathbf{c}_j \cdot (\mathbf{C} + \mathbf{L}\mathbf{s} + \mathbf{Q}\tilde{\mathbf{s}}) = \mathbf{c}_j \cdot \mathbf{h}$, which is a probabilistic check of the original verification equation in LUOV.

### 5.3 Efficiency estimates

In what follows, we evaluate the efficiency gains provided by our compiler using the $(\mathsf{r}_0, \mathsf{e}_0)$-concrete efficiency notion of Eq. (1). In brief, a $\Sigma^E$ achieves $(\mathsf{r}_0, \mathsf{e}_0)$-concrete amortized efficiency if $\mathsf{r}_0$ is the smallest, non-negative integer for which it holds that $\mathsf{e}_0 < 1$, where $\mathsf{e}_0$ is an upperbound on the ratio between the cost of running the offline verification once and using its outcome in $\mathsf{r}_0$ online verifications, over the cost of running $\mathsf{r}_0$ standard signature verifications. For convenience, we estimate only the cost of the most expensive 'steps' in the verification, namely the ones involving several field element *multiplications* (e.g., matrix–vector products), and disregard the cost of adding elements, generating random values, reading algorithm inputs or evaluating hash functions. Moreover, we do not consider ad-hoc optimizations of matrix multiplication due to probabilistic checks using, e.g., Freivalds' Algorithm or its variant [35]. Table 2 collects the common notation, while Table 1 displays a summary of our findings, that we motivate below. Observe that we only compute concrete parameters for Rainbow, so that they can be compared with the results of our implementation. The analysis of MAYO and LUOV can be conducted analogously when optimized implementations will be available.

The computational complexity of Ver for signature with **Mv**-style verification, e.g., [8, 18, 19, 24, 25, 32], is dominated by a matrix–vector multiplication. Let $n = rows(\mathbf{M})$ and $m = cols(\mathbf{M})$, with $m \geq n$. The cost of computing $\mathbf{M} \cdot \mathbf{v}$ is, in the worst case, $nm$ filed multiplications. Our offline verification algorithm executes $k$ vector–matrix multiplications (one for each $\mathbf{z}'_j$ in $\mathbf{Z}'$), resulting in $knm$ multiplications in the worst case. The computational complexity of our online verification is dominated by the $k$ vector-vector (inner) products $\mathbf{z}_i \cdot \mathbf{v}$, resulting in $km$ multiplications in the worst case. Thus, the compiler presented in Sect. 4.1 outputs an efficient verification for signature with **Mv**-style verification that has the following concrete amortized efficiency:

$$\frac{\mathsf{cost(offVer)} + \mathsf{r} \cdot \mathsf{cost(onVer)}}{\mathsf{r} \cdot \mathsf{cost(Ver)}} = \frac{knm + \mathsf{r}km}{\mathsf{r}nm} = \frac{k}{\mathsf{r}} + \frac{k}{n}. \tag{5}$$

Clearly the first addend in Eq. (5) comes from amortizing the cost of offVer (over verifying r signatures), while the second term is the fix trade-off between the computational costs of onVer and Ver (at each and every verification). Table 1 collects the figures for three representative classes of signature schemes, if we apply our compiler for efficient verification at 128 bit of security. The values are extrapolated as explained in the reminder of the section. In detail, $k_0$ depends on the signature $\Sigma$ as it is the minimal value of the confidence level $k$ for which $\Sigma^E$ is existentially unforgeable; $k_0$ determines the length of the svk. The value $\mathsf{r}_0$ is the minimum number of verifications to run in order to achieve a concrete efficiency gain of $\mathsf{e}_0$. Thus, lower values of $\mathsf{e}_0$ and $\mathsf{r}_0$ correspond to better efficiency gains. The last column in Table 1 displays the ratio $k_0/n$ that essentially tells how much *cheaper* onVer is compared to the original verification Ver (ignoring the one-time cost of running offVer). Again, lower values in this column correspond to better efficiency; for instance, a ratio of 0.4% means that the computational cost of Ver is 99.6× higher than the one of onVer (i.e., onVer is expected to be about 99× faster).

For convenience we categorize signatures according to the size of their underlying algebraic structure.

*The modulo $q$ is exponential in $\lambda$:* To the best of our knowledge, the only LBS constructions that fall in this category are the homomorphic signatures by Gorbunov et al. [25] and by Fiore et al. [20]. In this case, using our compiler (with some caveats, as we show in the next section) yields

**Table 2** Parameters involved in the performance analysis of our compiler for efficient verification

| | |
|---|---|
| $q$ | Modulus of the lattice or size of the field |
| $n$ | Number of rows in the public key |
| $m \in \Omega(n \log q)$ | Number of columns in the public key |
| $\beta$ | Bound on the noise / size of signatures |
| $\boldsymbol{\sigma}$ or $\mathbf{U}$ | Vector or matrix signatures |
| $k$ | Number of steps in the online verification (confidence level) |
| $\mathsf{r}$ | Number of signatures verified (repetitions of onVer) |
| $\mathsf{cost}(alg)$ | Number of field multiplications needed to compute $alg$ |

**Table 3** Details about IoT boards adopted

| Board | CPU | SRAM | Flash |
|---|---|---|---|
| Arduino Due | ARM Cortex-M3 (single-core @84MHz) | 96KB | 512KB |
| Espressif ESP32 | Tensilica Xtensa LX6 (dual-core @240MHz) | 520KB | 4MB |
| Raspberry Pi Pico | Arm Cortex-M0+ (dual-core @133MHz) | 264KB | 2MB |

that the advantage in the cmvEUF experiment (as per Definition 4) is negligible in the security parameter $\lambda$ for any confidence level $k \geq k_0 = 1$. However, in [20, 25] the complexity of Ver is dominated by the matrix-matrix multiplication $\mathbf{AU}$ where $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$ is the fixed public key, and $\mathbf{U} \in \mathbb{Z}_q^{m \times m}$ is the signature.[4] We computed parameters for this family of schemes according to Albrecht et al.'s methodology [1]. Setting $\lambda = 128$, $q = 2^{\lambda}$ and $n = 256$ yields that reduction algorithms (in particular, the optimized BKZ algorithm) would have runtime $2^{128}$ and would solve at most $\mathrm{SIS}_{256,2^{128},65536,2^{80}}$, while the security of the scheme relies on a SIS instance with norm bound $\beta = 2^{49d}$, where $d$ is the depth of the circuit. We can now use this set of parameters to determine the concrete amortized efficiency reached by our compiler for [20, 25]. Setting $k = k_0 = 1$ and $n = 256$ in Eq. (5), we want to extract the minimum $\mathsf{r}_0$ for which $1/\mathsf{r}_0 + 1/256$ is smaller than 1, formally $\mathsf{r}_0 = \min\{\mathsf{r} \in \mathbb{Z}_{>0} \mid 1/\mathsf{r} + 1/256 < 1\}$. It is easy to see that $\mathsf{r}_0 = 2$ suffices and we get $1 > \mathsf{e}_0 = 0.504 > 1/2 + 1/256$. In other words, the cost of setting up the online verification (running offVer) *plus* performing $\mathsf{r} = 2$ online verifications is about half of the cost of running 2 standard verifications, while preserving the security level. Moreover, for this set of parameters $\frac{\mathsf{cost(onVer)}}{\mathsf{cost(Ver)}} = \frac{k_0}{n} = \frac{1}{256} < 0.004$, i.e., our online verification requires about 0.4% of the computational cost of running the standard verification algorithm; alternatively, we can read this results as our onVer is 99× faster than Ver.

*The modulo $q$ is a large polynomial in $\lambda$ :* This is the most common setting given the 'small' size of $q$. In this category fall the standard signature schemes by Gentry et al. [24], Boyen [11], and its improved version by Micciancio and Peikert [32]; as well as the linearly homomorphic scheme by Boneh and Franklin [9]. For the lattice-based constructions, in order to guarantee a negligible advantage in the cmvEUF experiment (see Definition 4) we need to set an appropriate value of $k \geq k_0 > 1$. We argue that 'appropriate' values of $k$ are still 'small' in comparison to $n$ and lead to a 'good' amortized efficiency even for 'few' verifications. We recall that for these constructions Ver computes a product $\mathbf{A}\boldsymbol{\sigma}$ where $\mathbf{A} \in \mathbb{Z}_q^{n \times m}$ and the signature is just a vector $\boldsymbol{\sigma} \in \mathbb{Z}_q^m$. To guarantee the security of our efficient verification, the value $k$ should be set so that $q^{-k}$ be negligible. In other words, for the cmvEUF advantage to be negligible it must hold that $q^{-k} \leq 2^{-\lambda}$. Hence, to estimate $k$, one needs to first fix the value of $\lambda$, compute the corresponding $q$ that can guarantee such level of security, and then extract the minimum value $k_0$ for which the above relation holds.

Computing parameters for lattice-based schemes is not straightforward, as so far there is no unique way to derive the parameters from a given $\lambda$. However, a good measure of the security of a set of parameters can be extracted computing a the root Hermite factor $\delta$ introduced in [22]. Concretely, $\delta$ provides an indication of how reduction algorithms would perform against the hardness assumption underlying the lattice-based construction. Generally, the 'smaller' the $\delta$, the 'more secure' the scheme.

For Boyen's signature [11] and its variant by Micciancio and Peikert [32], we use the parameters provided in [32, Fig. 2]. Since in [32] they set $\delta = 1.007$, to ensure a fair comparison, we compute the parameters Gentry et al.'s signature [24] for the same value of $\delta$. As a result, we observed that for this $\delta$ all of the schemes require about the same modulo $q = 2^{30}$ (for $n = 256$). For this set of parameters, our efficient verification provides 80 (resp. 128; 250) bits

[4] In [20] the dimension $m$ additionally depends on the number $t \geq 1$ of distinct identities (users) involved in labeled program. For simplicity, in what follows we consider $t = 1$.

of security with just $k_0 = 3$ (resp. $k_0 = 5$; $k_0 = 9$). Thus our compiler achieves a (4, 0.77)-concrete amortized efficiency (resp.(6, 0.86); (10, 0.94)), and a concrete tradeoff between onVer and Ver of $k_0/n < 0.02$ (resp. 0.02; 0.04). In particular, for the lower security settings this means that onVer is about 98× faster than Ver.

*The modulo q is a sub-polynomial in n:* The only signature schemes in this category are the ones based on multivariate quadratic polynomial equations and stem from the (unbalanced) oil and vinegar approach. As a case of study we consider Rainbow with the parameters given for its last NIST submission and available on the the official website.[5] For Level I we have $\mathbb{F} = \mathbb{F}_{2^4}$ and $(v_1, o_1, o_2) = (36, 32, 32)$, which lead to $m = 100$ and $n = 64$ (for consistency in this paper we set $n$ to be the number of rows of a matrix and $m$ to denote the number of columns, classically the variables are swapped for multivariate signatures). Setting $k_0 = 32$ is suitable for good security since $q^{-k_0} = 2^{-4 \cdot 32} = 2^{-128}$. Thus, the minimum number of repetitions $\mathsf{r}_0$ to achieve amortized efficiency (i.e., for which we have $k_0/\mathsf{r} + k_0/n < 1$) is $\mathsf{r}_0 = 65$, the corresponding amortization factor is $\mathsf{e}_0 = 0.9923 = 32/65 + 32/64$. For this set of parameters we have $\frac{\mathsf{cost(onVer)}}{\mathsf{cost(Ver)}} = \frac{k_0}{n} = \frac{32}{64} = 0.5$, in other words, our compiler produces an online verification that is 2× *faster* than the standard verification. For Level III, $\mathbb{F} = \mathbb{F}_{2^8}$ and $(v_1, o_1, o_2) = (68, 32, 48)$, we have $m = 148$ and a suitable $k$ in this case would be $k_0 = 16$, since $q^{-k_0} = 2^{-8 \cdot 16} = 2^{-128}$. For Level V, $\mathbb{F} = \mathbb{F}_{2^8}$ and $(v_1, o_1, o_2) = (96, 36, 64)$. As a result we have $m = 196$, $n = 100$ and again $k_0 = 16$ but a better amortize efficiency factor $\mathsf{e}_0 = 0,96$ already for $\mathsf{r}_0 = 20$. We remark that for this set of parameters $\frac{\mathsf{cost(onVer)}}{\mathsf{cost(Ver)}} < 0.16$, i.e., our compiler produces an online verification procedure that is 6× *faster* than the standard verification.

## 6 Experimental evaluations with rainbow

In order to test the feasibility of our verification frameworks, we developed a library called BLEP [37] that offers a rather straightforward API in C and C++ to perform standard, efficient, and progressive verification for signature schemes with **Mv**-style verification. BLEP has been developed with a strong orientation towards performance and portability.

We benchmarked our library using three IoT boards: Arduino Due, Espressif ESP32, and Raspberry Pi Pico. Central Processing Unit (CPU), Random Access Memory (RAM), flash memory and other details about these devices can be found in Table 3. The Zephyr Real-Time Operating System (RTOS) version 3.2.0 has been selected to build our test bed since it supports a variety of boards and several C/C++ language features.

**Table 4** **Mv**-style verification details regarding Rainbow classic versions

| Version | Field used | PK matrix size |
|---|---|---|
| Rainbow I | GF(16) | (64 × 5050)/2 bytes |
| Rainbow III | GF(256) | 80 × 11026 bytes |
| Rainbow V | GF(256) | 100 × 19306 bytes |

**Table 5** Look-up configurations offered by BLEP

| Config. | Type of Optimization |
|---|---|
| 0 | No look-up table |
| 1 | compressed look-up tables (for roughly half possible inputs) |
| 2 | Complete look-up tables (for all possible inputs) |

We used a consumer-grade laptop as a reference platform, more precisely, a MSI Prestige 14 A10SC equipped with an Intel i5-10210U CPU@1.60GHz and 16GB LPDDR4 RAM. On this laptop we installed Linux Mint 20.3.

For our implementation we focus on Rainbow since, among all the **Mv**-style signatures, this is the only one designed with practical constrains in mind and has an available, ready-for-use implementation. Albeit Rainbow's security has been recently challenged [6], we consider our implementation insightful and a useful blueprint for the analysis of other schemes based on multivariate polynomial equations. We believe that the results of our testing activity can still be useful to showcase the performance boost obtainable by applying efficient verification to existing constructions. Table 4 shows the parameters used for testing.

In order to shorten verification times, our library BLEP allows the use of look-up tables to speed up addition and multiplication over finite fields. More precisely, BLEP offers three possible configurations—i.e., complete, compressed, or no look-up tables—as shown in Table 5. Users can select the appropriate type of optimization based on the desired performance criteria and the availability of storage space. Since we are dealing with IoT devices, we have to deal with tight storage constraints, thus the usage of look-up tables and the choice of the storage medium has to be considered carefully. Usually, RAM allows for faster access times, while flash memory has way more space to work with. Certainly better performance is desirable, but the random access memory can easily fill up, drastically reducing the space available for the program stack. On the other hand, relying entirely on flash memory can drastically limit the space available for storing the "short" verification key (svk). This would inevitably affect the maximum security level achievable when implementing efficient, and progressive verification on specific constrained devices.

[5] https://www.pqcrainbow.org/ (accessed on 20/02/2023).

**Table 6** Reference device: Avg efficient verification times (in microsecs) for Rainbow I, III and V

| svk size / pk size (%) | Language | Configuration | Rainbow I (μs) | Rainbow III (μs) | Rainbow V (μs) |
|---|---|---|---|---|---|
| 10% | C | Conf. 0 | 364 | 2984 | 6390 |
| | | Conf. 1 | 320 | 1152 | 2290 |
| | | Conf. 2 | 83 | 352 | 780 |
| | C++ | Conf. 0 | 390 | 2152 | 4550 |
| | | Conf. 1 | 262 | 1192 | 2570 |
| | | Conf. 2 | 83 | 351 | 760 |
| 20% | C | Conf. 0 | 729 | 5698 | 12780 |
| | | Conf. 1 | 640 | 2304 | 4580 |
| | | Conf. 2 | 166 | 704 | 1560 |
| | C++ | Conf. 0 | 780 | 4304 | 9100 |
| | | Conf. 1 | 524 | 2384 | 5140 |
| | | Conf. 2 | 166 | 703 | 1520 |
| 30% | C | Conf. 0 | 1094 | 8952 | 19170 |
| | | Conf. 1 | 960 | 3456 | 6870 |
| | | Conf. 2 | 249 | 1056 | 2340 |
| | C++ | Conf. 0 | 1171 | 6456 | 13650 |
| | | Conf. 1 | 787 | 3576 | 7710 |
| | | Conf. 2 | 249 | 1055 | 2280 |
| 40% | C | Conf. 0 | 1459 | 11936 | 25560 |
| | | Conf. 1 | 1280 | 4608 | 9160 |
| | | Conf. 2 | 332 | 1408 | 3120 |
| | C++ | Conf. 0 | 1561 | 8608 | 18200 |
| | | Conf. 1 | 1049 | 4768 | 10280 |
| | | Conf. 2 | 332 | 1407 | 3040 |
| 50% | C | Conf. 0 | 1824 | 14920 | 31950 |
| | | Conf. 1 | 1600 | 5760 | 11450 |
| | | Conf. 2 | 416 | 1760 | 3900 |
| | C++ | Conf. 0 | 1952 | 10760 | 22750 |
| | | Conf. 1 | 1312 | 5960 | 12850 |
| | | Conf. 2 | 412 | 1759 | 3800 |
| 100% | C | Conf. 0 | 3648 | 29840 | 63900 |
| | | Conf. 1 | 3200 | 11520 | 22900 |
| | | Conf. 2 | 832 | 3520 | 7800 |
| | C++ | Conf. 0 | 3904 | 21520 | 45500 |
| | | Conf. 1 | 2624 | 11921 | 25700 |
| | | Conf. 2 | 830 | 3519 | 7600 |

With the aim of evaluating the verification times on different devices, and since the **Mv**-style verification time scales linearly in the number of rows present in the svk, we simply collected the cost for a single row-vector product. To do so, we wrote two benchmark programs in C and C++ using the API offered by BLEP. These programs have been compiled with gcc and g++, both version 11.3.0, and with the -O3 optimization flag enabled. Results of our testing activity are shown in Table 6 and in Figs. 6, 7 and 8.

In Table 6, we present the average efficient verification times collected on our reference platform with the different versions of Rainbow, and with various svk sizes (here shown as percentages with respect to the number of rows of the corresponding public key). This data shows the remarkable impact of lookup tables on the experimental results (configuraration 1 and 2) even on a standard laptop. This increase in performance is even more evident when analyzing verification times of Rainbow III and V.

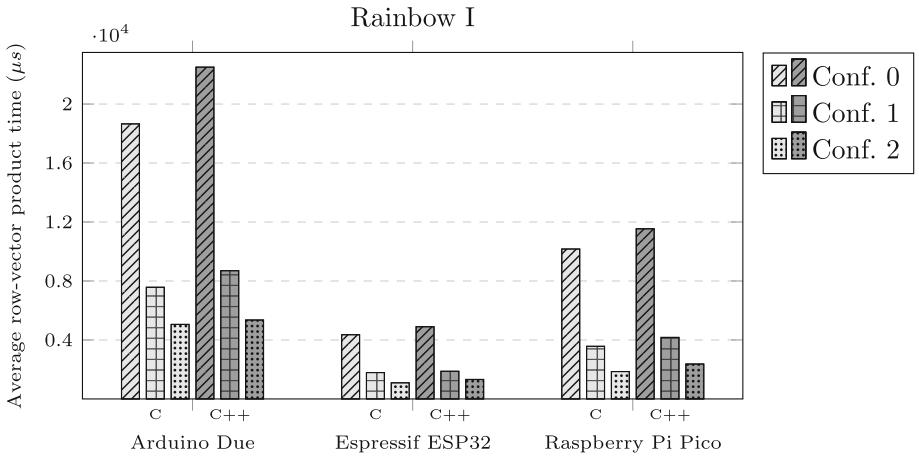



**Fig. 6** Rainbow I: Avg row-vector product times (in microsecs) obtained on our IoT devices

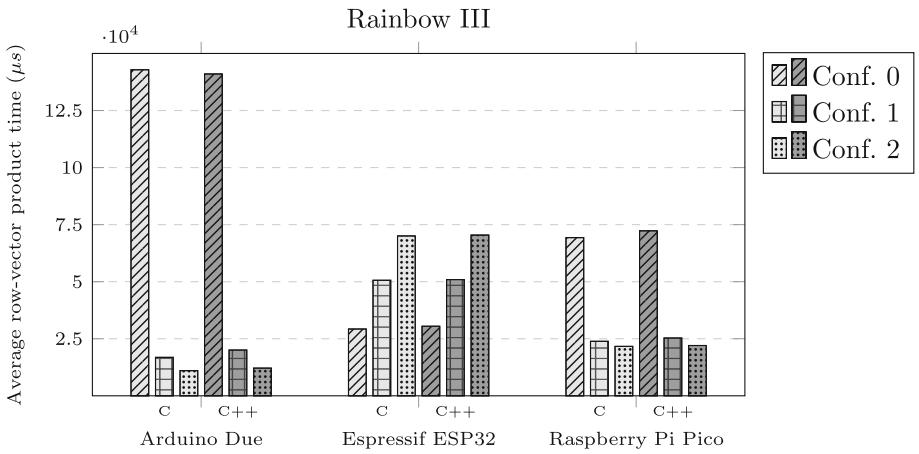



**Fig. 7** Rainbow III: Avg row-vector product times (in microsecs) obtained on our IoT devices

In Figs. 6, 7 and 8, we summarize data related to Rainbow I, III and V, respectively. More precisely, we store the look-up tables in flash memory (unless otherwise indicated), run our benchmark on the three versions of Rainbow, and then collect computation times on our IoT boards (see Table 3). Testing activity executed with Rainbow I (see Fig. 6) shows that using look-up tables can reduce the verification times by more than 50% on all IoT boards. On the other hand, by storing the look-up tables in RAM we gain an additional performance improvement of about 8% and 18% on the Arduino Due board, for the Configuration 1 and 2 respectively. No significant improvement has been observed on the Espressif ESP32 and the Raspberry Pi Pico.

The behaviour of Rainbow III and V is similar to that of Rainbow I, except for the ESP32 (see Figs. 7 and 8). In fact, if we use this board with large look-up tables stored in flash memory, performance deteriorates considerably. This abnormal behaviour does not happen with the Arduino Due, nor with the Raspberry Pi Pico. We suppose that the hardware constraints of ESP32 negatively affect the performance of our testing activities. On the other hand, if we store look-up tables in RAM only the Raspberry Pi Pico is capable of holding them completely. The finite field used by Rainbow III and V is $GF(256)$ and the look-up tables are larger than those used by Rainbow I over $GF(16)$. Therefore, Raspberry is able to get additional timing improvements of about 62% and 76% for Configuration 1 and 2, respectively.

## 7 Extensions

### 7.1 Signatures with properties

Efficient verification can be easily generalized to the case of signatures with different security notions, such as *strong* or *selective* unforgeability, or with advanced properties. This is of particular interest for LBS, where the versatility of well-established hardness assumptions has already given life to a variety of constructions under different security models and realizing advanced properties, including *homomorphic* [25], *threshold* [4], *constrained* [38] and *indexed attribute based* signatures [27]; and yet relying on an **Mv**-style verification (as introduced in the beginning of the section, and displayed in Fig. 3).

Signatures with properties require more complex security definitions than plain existential unforgeability. Figure 9

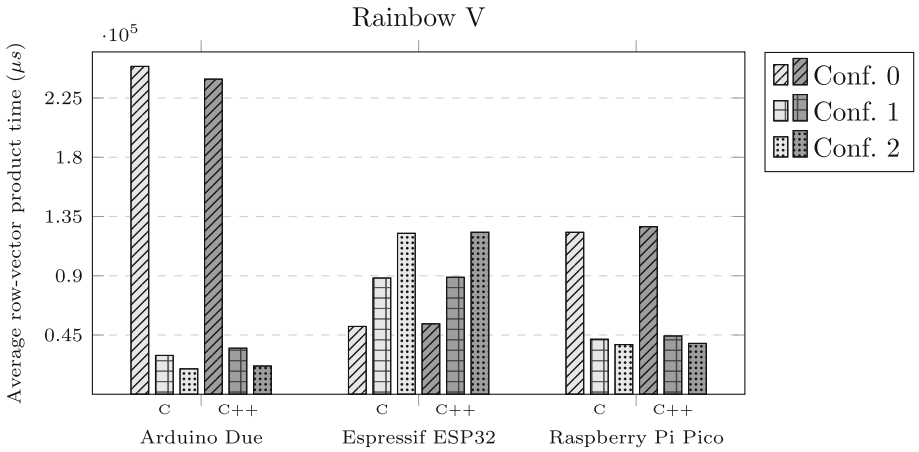



**Fig. 8** Rainbow V: Avg row-vector product times (in microsecs) obtained on our IoT devices

$\mathsf{Exp}^{\mathsf{generic-unf}}_{\mathcal{A},\Sigma}(\lambda)$
1: $val \leftarrow \mathcal{A}(1^\lambda)$
2: $(\mathsf{pval}, \mathsf{sval}) \leftarrow \mathsf{Setup}(1^\lambda)$
3: $\mathsf{st}_K \leftarrow \emptyset,\ \mathsf{st} \leftarrow \emptyset$
4: $\mathcal{O} := \{OK(\ \cdot\ ;\ \mathsf{sval}, \mathsf{st}_K),\ O(\ \cdot\ ;\ \mathsf{st})\}$
5: $(\mu^*, \boldsymbol{\sigma}^*, \mathsf{aux}^*) \leftarrow \mathcal{A}^{\mathcal{O}}(\mathsf{pval}, val)$
6: $b \leftarrow \mathsf{WinCond}(\mu^*, \boldsymbol{\sigma}^*, \mathsf{aux}^*, \mathsf{pval}, \mathsf{st}_k, \mathsf{st}, val)$

7: **if** $\mathsf{Ver}(\mathsf{pk}, \mu^*, \boldsymbol{\sigma}^*, \mathsf{aux}^*) = 1\ \wedge\ b$
8: **return** 1
9: **else return** 0.

**Fig. 9** Generic description of the unforgeability under adaptive chosen message attacks experiment for signatures with properties

provides a generic formalism to unify the description of the unforgeability experiments for signatures with properties. In a nutshell the common requirements are:

1. If the signature guarantees *selective* unforgeability, the first step in the experiment is for $\mathcal{A}$ to declare the target messages for the forgery; in Fig. 9 this is handled via the *val* variable. If unforgeability is adaptive, *val* is set to $\perp$.
2. A setup phase, where a probabilistic routine (denoted Setup in Fig. 9) generates a set of secret values sval – handed over to the oracles– and other public auxiliary values pval, that include verification keys, delivered to the adversary.
3. A challenge phase, where the adversary is given access to some, possibly stateful, oracles (usually, at least an oracle that returns signatures by honest users), and has to output a message and a forged signature on it. We model this by defining two oracles:
   - $OK(\cdot; \mathsf{sval}, \mathsf{st}_K)$: Returns signing/secret keys (of users or other entities that $\mathcal{A}$ may corrupt).
   - $O(\cdot; \mathsf{st})$: Encompasses all the other possible oracles (signing, opening for group signatures, etc.).
4. A check phase, where the experiment checks whether the signature output by $\mathcal{A}$ is valid and if $\mathcal{A}$ won the experiment. The former requires an execution of the verification algorithm; the latter includes a variety of additional checks to ensure the signature is actually a forgery (and is not trivially derivable from the adversary's view, e.g., because it was output by the signing oracle). We model this second check with the WinCond predicate. Clearly, the specification of WinCond depends on each primitive, and on the type of unforgeability: If selective, it checks that the queries and the forgery are consistent with the values *val* declared at the beginning of the game. If existential, it checks that $\mu^*$ was not queried to the signing oracle. If strong, it checks that the queries to the signing oracle are all distinct.

Adapting the syntax and security experiment of efficient verification to signatures with properties is rather straightforward. Similarly, our compiler of Sect. 4.1 can be easily adjusted to work on signatures with properties and with **Mv**-style verification, as we discuss momentarily. Regarding security, the core part of the proof of Theorem 1 is information-theoretical, and therefore it does not significantly change when considering signatures that are only selectively unforgeable, or strongly unforgeable. In the following we analyze the impact of our compiler on the efficiency of some schemes whose verification is structured as in Fig. 3: the constrained LBS in [38], the (indexed) attribute-based LBS in [27], the homomorphic LBS in [20], the threshold LBS in [4], and the multivariate-based ring signature RingRainbow [33]. This list is by no means an exhaustive list. Indeed, in this work we decided to ignore lattice-based signatures with properties that are obtained using the Fiat-Shamir with abort technique from [31], despite the fact that wherever the result of Chen et al. [16] is applicable, our compiler is too. The reason is that signatures with properties that rely on such technique are many, and the effi-

ciency gain computation is similar to the one performed in Sect. 5

*Constrained Signatures (CS).* CS allow a signer to sign a message only if either the message or the key satisfies certain preset constraints. The verification algorithm of the lattice-based instantiation of CS by Tsabary [38] includes an **Mv**-style check (where the matrix has $n$ rows) and a norm check. Hence, our compiler applies directly to this scheme. Unforgeability requires that $n \geq \lambda$ and $q \leq 2^{\lambda}$, so for an average value $q \sim 2^{32}$, we can set $k = 9 \ll \lambda$ so that the advantage of $\mathcal{A}$ in Theorem 1 is $\frac{q_V+1}{q^k-q_V} < 1/2^{256}$. Remark that larger values of $q$ (that could be required to have higher security guarantees) imply smaller values of $k$. Therefore, for this less conservative choice of parameters the efficiency gain is $\frac{\mathsf{cost(onVer)}}{\mathsf{cost(Ver)}} = \frac{k}{n} = \frac{9}{256} < 0.036$, i.e., the online verification requires about 3.6% of the computational cost of running the standard verification algorithm.

*Indexed Attribute-based Signatures (iABS), and Homomorphic Signatures (HS).* iABS allow a signer to generate a valid signature on a message only if the signer holds a set of attributes that satisfy some policy (represented by a circuit $\mathsf{C}$). HS allow a signer to sign messages $\mu_i$ so that it is possible to publicly derive a valid signature for a message $\mu$ that corresponds to the output of a computation on the original messages, i.e., $\mu = \mathsf{C}(\mu_1, \ldots, \mu_r)$. According to the type of homomorphism supported by the scheme, the circuit $\mathsf{C}$ can encode only linear functions, polynomial functions, or any function of bounded multiplicative degree. In both iABS in [27] and HS in [20] the signature verification is composed by three steps:

1. Computation of the public matrix $\mathbf{M}$ from the circuit $\mathsf{C}$ (either the policy, or the homomorphic computation specified by the labelled program);
2. An '**Mv**'-style check;
3. A norm check on the signature.

The first step is critical because the public matrix $\mathbf{M}$ can be generated through a non-linear transformation, i.e., it might include multiplications of the public matrix by itself (or by a gadget matrix). This would not allow to compute the first step online from the $\mathbf{z}_i$'s, but the verifier would have to use $\mathbf{M}$ and the $\mathbf{c}_i$'s instead, defying the purpose of our compiler. Hence, our compiler can be applied to these signatures in an efficient way only if either (1) $\mathsf{C}$ involves solely linear operations on the public matrix, or (2) $\mathsf{C}$ is fixed, or (3) $\mathsf{C}$ is known before running verification.[6] In these cases, we achieve efficient verification by letting $\mathsf{offVer}$ take as (additional) input $\mathsf{C}$ and compute $\mathbf{M}$ using the algorithm $\mathsf{PubEval}$ from [26]. The vectors $(\mathbf{Z}', \mathbf{v})$ used in the verification might (as in [20]) or might not (as in [27]) depend on the message. In the latter case the subroutine $\mathsf{GetZV}$ in $\mathsf{onVer}$ simply returns the input. The impact of the compiler on the efficiency of HS was already analyzed in Sect. 5. Regarding the iABS, the suggested value of the modulo $q$ is such that $q \geq n^8$. The standard requirement $n \geq 2$ already implies that $1/q^k \leq 1/(2^8)^k = 1/256^k$. However, to guarantee the hardness of lattice-based problems usually $n$ needs to be at least $n = 128$. In this case $q \geq 2^{56}$, hence already $k = 6$ guarantees that $\frac{q_V+1}{q^k-q_V} < 1/2^{305}$, thus the unforgeability of this iABS. As $n = O(d \log d)$ (where $d$ is the depth of $\mathsf{C}$) and the efficiency gain can be bounded as follows: $\frac{\mathsf{cost(onVer)}}{\mathsf{cost(Ver)}} \leq \frac{k}{O(d \log d)} = \frac{6}{O(d \log d)}$. From this inequality is clear that already for a circuit of depth 4 the online verification only requires 75% of the computation required by standard verification; the impact of our compiler increases for larger size of the circuit.

*Threshold Signatures (TS).* TS allow $h$ out of $\ell$ parties to produce a signature on a message. Unforgeability is guaranteed for up to $t$ colluding parties. Bendlin, Krehbiel, and Peikert [4] introduced a compiler that allows to distribute the signature generation step of the GPV08 signature, and convert it into a TS. The idea is to share the signing trapdoor among the parties using a $h$-out-of-$\ell$ secret sharing scheme. Signing requires at least $h$ parties to come together to generate a signature satisfying a **Mv**-type equation (where $\mathbf{M}$ is the public verification key). Verification is composed by the standard **Mv** equation and norm checks. Therefore, the thresholdizing compiler is composable with our compiler for efficient verification. As neither of them change the parameters of the underlying GPV08 scheme, the efficiency gain is the same (Sect. 5).

*RingRainbow* [33]. RingRainbow is a ring signature scheme —i.e., a signature that allows a user to sign a message anonymously on behalf of a group – based on multivariate equations. This scheme is a hash-and-sign type of signature built as a modification of Rainbow. As thus, it is affected by the recent attacks, but we believe its analysis is still important, as it can serve as a blueprint for the analysis of future variants avoiding such attacks. Verification requires to check whether the signature satisfies a multivariate quadratic system, and can be converted in a **Mv**-style verification with the same technique used for Rainbow (cf. Sect. 5). Therefore, our compiler can be applied to RingRainbow as well. To evaluate the efficiency gain due to our compiler, we consider the efficient version of RingRainbow, (whose parameters can be found in Table 2 in [33]). For $\lambda = 128$ and a group of 5 users the authors set $\mathbb{F} = \mathbb{F}_{2^8}$ and $(v_1, o_1, o_2) = (36, 21, 22)$,

[6] The construction of group signature in [27] has this iABS as building block, but it does not satisfy any of these conditions, as the verification circuit depends strongly on the signature. The authors did not find a straightforward way to modify this construction to have efficient verification without significantly impacting the signature length.

which yield $m = 5 * (v_1 + o_1 + o_2) = 395$ and $n = 43$. Theorem 1 requires at least $\frac{q_V+1}{q^k-q_V} = 1/2^{256}$ for 128 bits of post-quantum security, which is ensured by $k \geq k_0 = 36$. Plugging these values in our amortized efficiency formula $\frac{k_0}{\mathsf{r}} + \frac{k_0}{n}$ (that is the formula derived from Definition 3 for signatures with **Mv**-style verification) yields that the minimum number of repetitions $\mathsf{r}_0$ to achieve meaningful amortized efficiency is $\mathsf{r}_0 = 580$, and the corresponding amortized efficiency factor is $\mathsf{e}_0 = 0.8992 > 36/580 + 36/43$. In this case, our compiler produces an online verification such that $\frac{\mathsf{cost(onVer)}}{\mathsf{cost(Ver)}} = \frac{k_0}{n} = \frac{36}{43} < 0.86$, in other words, our compiler produces an online verification that requires only 86% of the computation required by the standard verification.

### 7.2 Applications

*Optimistic Verification.* Speculative execution is an optimization technique that relies on pre-computing some tasks so that the information be ready when the user prompts for it. This technique is increasingly used to in a wide range of commodity devices to boost performance. In these settings, progressive verification is a valuable candidates for early rejection of incorrect inputs. In particular, with 'little' computations the CPU can identify the most 'promising' branch and further optimize its performance. Another concrete use case is smart vehicle updates, where multiple software need to be installed 'at the same time' to have compatible versions. Thus, a successful upgrade of a number of components entails checking, say, $N$ signatures (to verify the authenticity of each new piece of software). There are two standard approaches to face this task: the conservative one, where the user verifies *all* of the $N$ signatures before launching the software update; and the lazy one, where the user verifies *each* signature before launching the corresponding update. In the first case, one may incur into long waiting time (since the signatures have high security level, given the threats that rise from installing malware on smart vehicles). In the second case, one may have to interrupt the update and revert some components to previous versions. In contrast, using progressive verification with an optimistic approach lets the verifier start the $N$ verifications at the same time and interrupt them after a number $t$ of steps to check the 'partial' results. If one signature is invalid, the user should not even bother to start with the installation. If all signature appear valid (until step $t$), the user can proceed with the lazy approach, checking the remaining constraints while updating its vehicle, knowing there is a low chance the process fails 'at the very end'.

*Sidestepping Boundaries* The offline/online verification approach allows bypassing known algorithmic barriers by changing the task performed by the signature verification. Instead of performing a multiplication between a (public) matrix and a (public) vector—in the worst case, a multiplication has complexity $O(n^2)$ but it can be easily improved by dividing the polynomials of size $n$ into smaller pieces and performing the multiplication on such pieces [12, 17],—our efficient verification employs a (secret) vector and a (public) vector, which obviously reduces the complexity to $O(n)$, and does not necessarily sacrifice security (verification soundness).

## 8 Combining progressive and efficient verification

Progressive verifications obtained with our transformation (Sect. 4.2) can be split into two parts: a one-time, computationally intensive, setup ($\mathsf{ProgStep}_0$); and an efficient online verification ($\mathsf{ProgStep}_1$ to $\mathsf{ProgStep}_T$, for some opportune integer $T \geq 1$). This gives rise to custom, i.e., intentionally adjustable, verification soundness. This property makes post-quantum-secure verification accessible to a larger range of devices, and at the same time draws interesting connections between classical, information-theoretic, and post-quantum security notions.

The security guarantees discussed thus far hold as long as the employed $\mathsf{svk}$ is fresh. In what follows, we investigate sound ways to amortize the setup cost by reusing (and refreshing) the $\mathsf{svk}$ produced by $\mathsf{ProgStep}_0$ for several online efficient verifications. We remark that naïvely reusing $\mathsf{svk}$ makes the confidence function degrade with every new verification, since progressive verification allows for premature verification outcomes that may leak a substantial amount of information about $\mathsf{svk}$, unless the modulo $q$ is exponential, as shown in the following theorem.

**Theorem 3** *For signatures with* **Mv***-style verification relying on algebraic structures of size* $q = 2^{\mathsf{poly}(\lambda)}$ *our compiler for efficient and progressive verification outlined above is unforgeable according to Definition 7 and achieves* $(2, 1/2 + 1/rows(\mathbf{M}))$*-concrete amortized efficiency as per Definition 3.*

***Proof*** The proof relies on the same argument used in the proof of Theorem 1. The security experiment is essentially the experiment $\mathsf{Exp}^{\mathsf{cmvEUF}}_{\mathcal{A},\Sigma}(\lambda, k)$ with a few changes: (1) any appearance of $\mathsf{onVer}$ is replaced by $\mathsf{ProgVer}$, (2) $\mathsf{svk}$ is *not* refreshed at every verification query, and (3) the number $k$ of rows of $\mathbf{C}$ is replaced by the interruption parameters $t$ output by $O\mathsf{Int}$ at each verification query. Since $q$ is exponential in the security parameter, $\mathbf{C}$ can be a single row vector. In other words $T = 1$, thus we can set all interruption values $t$ to 1. The advantage of an attacker is bounded through Lemma 1 as in the proof of Theorem 1.

In this setting, a concrete efficiency gain is achieved already with $\mathsf{r}_0 = 2$ repetitions. The corresponding amor-

tized efficiency value is

$$\begin{aligned} \mathsf{e}_0 &= \frac{\mathsf{cost}(\mathsf{offVer}(\mathsf{pk},1))}{\mathsf{r}_0 \cdot \mathsf{cost}(\mathsf{Ver}(\mathsf{pk},\mu,\boldsymbol{\sigma}))} + \frac{\mathsf{r}_0 \cdot \mathsf{cost}(\mathsf{onVer}(\mathsf{svk},1,\mu,\boldsymbol{\sigma}))}{\mathsf{r}_0 \cdot \mathsf{cost}(\mathsf{Ver}(\mathsf{pk},\mu,\boldsymbol{\sigma}))} \\ &= \frac{1}{2} + \frac{1}{rows(\mathbf{M})}\,. \end{aligned}$$

□

Next we focus on the trickier and more interesting case where $q$ is polynomial.

### 8.1 Efficient & progressive signature verification with r-bounded randomness reuse

We introduce the concept of progressive and efficient (pref) verification with r-bounded randomness reuse. Similarly to Definition 5 (progressive signatures), this *sustainable* variant is defined for a given value $k$, that determines the maximum desired confidence level achievable by the verification. In addition to $k$, we need a second parameter, r, that determines the maximum number of times svk can be reused while guaranteeing verification soundness. For correctness and security, both $k$ and r are input to the confidence function, which now is named $\alpha_{\mathsf{pref}}$.

**Definition 8** (*Progressive and Efficient Verification*) A signature scheme $\Sigma = (\mathsf{KeyGen}, \mathsf{Sign}, \mathsf{Ver})$ admits a $(\mathsf{r}, k)$-efficient and $(T, \alpha_{\mathsf{pref}})$-progressive verification realization $\Sigma^{F+E} = (\Sigma, \mathsf{prefVer})$ if there exist

- two positive integers: r (number of reuses of the secret randomness) and $k$ (interruption step);
- an efficiently computable confidence function $\alpha_{\mathsf{pref}} : \{0,\ldots,k\} \times \{0,\ldots,\mathsf{r}\} \to [0,1]$;
- a set of admissible sequences of states $\mathcal{S} = \{st^{(1)}, st^{(2)}, \ldots\}$ (each sequence $st^{(j)}$ contains $\mathsf{r}+1$ states $\mathsf{st}_i$, i.e., $st^{(j)} = (\mathsf{st}_i)_{i=0}^{\mathsf{r}}$, $\mathsf{st}_0 = \varnothing$);

and

- a progressive verification algorithm prefVer consisting of $k+1$ steps $\mathsf{prefVer}_0, \ldots, \mathsf{prefVer}_k$ with the same syntax as in Definition 5.

**Definition 9** (r-Reuse $k$-Progressive Correctness)Let $\Sigma$ be a signature scheme that admits progressive and efficient verification realized by the tuple $(\mathsf{r}, k, \alpha_{\mathsf{pref}}, \mathsf{prefVer}, \mathcal{S})$. Then $\Sigma^{F+E} = (\Sigma, \mathsf{prefVer})$ satisfies $(\mathsf{r}, k)$-correctness if, for a given security parameter $\lambda$, for any key pair $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(\lambda)$, for any one sequence of admissible states $st \leftarrow \mathcal{S}$, $st = (\mathsf{st}_i)_{i=0}^{\mathsf{r}}$, for any choice of r message-signature pairs $(\mu_i, \boldsymbol{\sigma}_i)_{i=1}^{\mathsf{r}}$ with $\mu_i \in \mathcal{M}$ and $\boldsymbol{\sigma}_i$ such that $\mathsf{Ver}(\mathsf{pk}, \mu_i, \boldsymbol{\sigma}_i) = 1$ and for any sequence of interruption values $(t_i)_{i=1}^{\mathsf{r}} \subseteq \{1,\ldots,k\}$, it holds that, for all $i = 1,\ldots,\mathsf{r}$:

$$\Pr\left[\mathsf{prefVer}(\mathsf{st}_i, \mathsf{pk}, \mu_i, \boldsymbol{\sigma}_i, t_i) = \alpha_{\mathsf{pref}}(t_i, i)\right] = 1$$

**Definition 10** (Concrete Amortized Efficiency) Let $\mathsf{r}_0 > 0$ be an integer, and $0 < \mathsf{e}_0 < 1$ a small, real constant. A scheme $\Sigma^{F+E} = (\Sigma, \mathsf{prefVer})$ realizes $(\mathsf{r}_0, \mathsf{e}_0)$-concrete amortized efficiency if, for a given security parameter $\lambda$, for any key pair $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KeyGen}(\lambda)$, for any tuple of pairs $(\mu_i, \boldsymbol{\sigma}_i)$ with $\mu_i \in \mathcal{M}$ and $\boldsymbol{\sigma}_i$ such that $\mathsf{Ver}(\mathsf{pk}, \mu_i, \boldsymbol{\sigma}_i) = 1$, for any sequence of admissible states $(st_i)_{i=0}^{\mathsf{r}} \subseteq \mathcal{S}$, we have that for every $\mathsf{r} \geq \mathsf{r}_0$ the following holds true:

$$\frac{\sum_{i=0}^{\mathsf{r}-1} \mathsf{cost}\big(\mathsf{prefVer}(\mathsf{st}_i, \mathsf{pk}, \mu_i, \boldsymbol{\sigma}_i, k)\big)}{\mathsf{r} \cdot \mathsf{cost}\big(\mathsf{Ver}(\mathsf{pk}, \mu, \boldsymbol{\sigma})\big)} < \mathsf{e}_0 \tag{6}$$

### 8.2 Security model for pref verification

Figure 10 collects a description of our security game and experiment for existential unforgeability under adaptive chosen message attack for signatures with *progressive and efficient verification* (r-prefEUF).

**Definition 11** (r-*Bounded Progressive Security* (r-prefEUF)) Let $\Sigma$ be a signature scheme that admits a non-trivial realization of (r,$k$)-efficient and progressive verification $\Sigma^{F+E}$. Then, for a given security parameter $\lambda$, $\Sigma^{F+E}$ is existentially unforgeable under adaptive chosen message attack with progressive and efficient verification (r-prefEUF) if for all efficient PPT adversaries $\mathcal{A}$ the success probability in the r-prefEUF experiment is:

$$\Pr\left[\begin{array}{c} \mathsf{Exp}^{\text{r-prefEUF}}_{\mathcal{A},\Sigma,\mathsf{r}}(\lambda,k,\mathsf{r}) = (\mathsf{ctr}^*, t^*) \\ \wedge\ (\mathsf{ctr}^*, t^*) \neq (0,0) \end{array}\right] \leq (1 - \alpha_{\mathsf{pref}}(t^*, \mathsf{ctr}^*)) + \varepsilon(\lambda).$$

### 8.3 A compiler for pref Mv-style verification with polynomial $q$

We now present a compiler for signatures with **Mv**-style verification and $q = \mathsf{poly}(\lambda)$ that realizes efficient bounded progressive verification. This compiler builds on top of the two compilers presented in Sect. 4.1. Intuitively, the problem with progressive verification is that if interrupted after $t < k$ steps the process may erroneously accept an invalid signature with a non-negligible probability $\approx 1/q^t$. In Sect. 4.2 we mitigate this leakage of information between queries by refreshing the vectors in svk after every verification. This conservative approach clearly impacts efficiency. Here we want to prioritize efficiency at the cost of accuracy, and investigate how the confidence function degrades when the same set of vectors $\mathbf{z}_i$ is used to perform r progressive verifications (Fig. 11).

Our compiler works essentially as the efficient verification compiler in Fig. 5, except that the offVer algorithm (that generates a fresh svk) is run only *once every* r *verifications*.

r-prefEUF$(\Sigma, \lambda, k)$
1: $\mathsf{ctr} \leftarrow 0, \mathsf{st}_0 \leftarrow \varnothing, \mathsf{L}_S \leftarrow \varnothing$
2: $(\mathsf{pk}, \mathsf{sk}) \leftarrow \mathsf{KeyGen}(1^\lambda)$
3: $(\mu^*, \boldsymbol{\sigma}^*, t^*) \leftarrow \mathcal{A}^{O\mathsf{Sign}, \mathsf{prefVer}}(\mathsf{pk}, \lambda)$
4: **return** $(\mathsf{ctr}, \mu^*, \boldsymbol{\sigma}^*, t^*)$

$O\mathsf{prefVer}_k(\mathsf{st}_{\mathsf{ctr}}, \mathsf{pk}, \mu, \boldsymbol{\sigma}, t')$
1: **if** $(\mathsf{ctr} \geq \mathsf{r})$ **return** $\perp$
2: $t \leftarrow O\mathsf{Int}(t')$
3: $\alpha \leftarrow \mathsf{prefVer}(\mathsf{st}_{\mathsf{ctr}}, \mu, \boldsymbol{\sigma}, t)$
4: $\mathsf{ctr} \leftarrow \mathsf{ctr} + 1$
5: **return** $\alpha$

$\mathsf{Exp}^{\mathsf{r\text{-}prefEUF}}_{\mathcal{A},\Sigma,\mathsf{r}}(\lambda, \mathsf{k})$
1: $(\mathsf{ctr}, \mu^*, \boldsymbol{\sigma}^*, t') \leftarrow \mathsf{r\text{-}prefEUF}(\Sigma, \lambda, k\ \ )$
2: $\beta \leftarrow \mathsf{Ver}(\mathsf{pk}, \mu^*, \boldsymbol{\sigma}^*)$
3: $t^* \leftarrow O\mathsf{Int}(t')$
4: $\alpha \leftarrow \mathsf{prefVer}_k(\mathsf{st}_{\mathsf{ctr}}, \mathsf{pk}, \mu^*, \boldsymbol{\sigma}^*, t^*)$
5: **if** $(\mu^* \in \mathsf{L}_S \vee \alpha = \perp \vee \beta = 1)$
6: **return** $(0, 0)$
7: **return** $(\mathsf{ctr}, t^*)$

$O\mathsf{Sign}_{\mathsf{sk}}(\mu)$
1: $\mathsf{L}_S \leftarrow \mathsf{L}_S \cup \{\mu\}$
2: $\boldsymbol{\sigma} \leftarrow \mathsf{Sign}(\mathsf{sk}, \mu)$
3: **return** $\boldsymbol{\sigma}$

**Fig. 10** Security model for existential unforgability under chosen message and progressive verification for signatures with stateful, $(k, \mathsf{r})$-efficient and progressive verification: queries security game, experiment and oracles

To further optimize the scheme, we replace the $\mathsf{GetZV}$ algorithm by $k$ algorithms $\mathsf{GetZV}_i$ each of which is run by the corresponding $\mathsf{prefVer}_i$. The behavior of $\mathsf{GetZV}_i$ depends on the signature scheme and in what follows we define it for each of the three major classes we identified in this paper. Each algorithm takes as input the corresponding $i$-th vectors $((\mathbf{c}_i, \mathbf{z}_i), PK.\mathsf{aux}, \mu)$ and returns $(\mathbf{z}'_i, \mathbf{v}_i)$ that are defined according to the scheme considered:

GPV08 [24]: the $\mathsf{GetZV}_i$ routine returns $\mathbf{z}'_i = \mathbf{z}_i = \mathbf{c}_i\mathbf{M}$, and $\mathbf{v}_i = [\boldsymbol{\sigma}|\mathcal{H}(\mu)]$.
MP12 [32]: the $\mathsf{GetZV}_i$ routine outputs $\mathbf{z}'_i = [\tilde{\mathbf{z}}_i \mid \mathbf{z}_i^0 + \sum_{j=1}^{\ell} \mu[j]\mathbf{z}_i^j | \mathbf{c}_i]$ and $\mathbf{v}_i = [\boldsymbol{\sigma}|\mathbf{u}]$.
Rainbow [18]: the $\mathsf{GetZV}_i$ routine outputs $\mathbf{z}'_i = \mathbf{z}_i = \mathbf{c}_i\mathbf{M}$, and $\mathbf{v}_i = [\tilde{\mathbf{s}}|\mathbf{s}|1|\mathbf{h}]$.

Finally, the confidence function $\alpha_{\mathsf{pref}}(\cdot, \cdot)$ is defined as:

$$\alpha_{\mathsf{pref}}(t, \mathsf{ctr}) = \begin{cases} \left(1 - \frac{1}{q^t - \mathsf{ctr}} - \frac{\mathsf{ctr}}{q - (\mathsf{ctr}-1)}\right) & \text{if } t > 0 \\ 0 & \text{if } t = 0 \end{cases} \tag{7}$$

#### 8.3.1 $\mathsf{r}_0$-concrete amortized efficiency estimates

The cost of $\mathsf{prefVer}_i$ varies depending on whether $i = 0$ or $i > 0$. When $\mathsf{prefVer}$ is run the first time (or with an empty state), the step $\mathsf{prefVer}_0$ generates the state. This includes computing $(knm)$ multiplications, in the worst case. After that, every step $\mathsf{prefVer}_i$ computes at most $(n+m)$ multiplications (the first term represents the cost of running $\mathsf{GetZV}_i$). Therefore,

$$\mathsf{cost}\big(\mathsf{prefVer}(\mathsf{st}_0, \mu_0, \boldsymbol{\sigma}_0, k)\big) = knm + k(n+m)\ .$$

However, this is true only for the first execution of $\mathsf{prefVer}$, as when executing the verification $1 < \mathsf{r}_0 < \mathsf{r}$ times, the algorithm $\mathsf{prefVer}_0$ does not refresh the multipliers. Hence, for $i > 0$

$$\mathsf{cost}\big(\mathsf{prefVer}(\mathsf{st}_i, \mu_i, \boldsymbol{\sigma}_i, k)\big) = k(n+m)\ .$$

This yields $\sum_{i=0}^{\mathsf{r}_0-1} \mathsf{cost}\big(\mathsf{prefVer}(\mathsf{st}_i, \mathsf{pk}, \mu_i, \boldsymbol{\sigma}_i, k)\big) = knm + \mathsf{r}_0 k(n+m)$. The cost of a verification is dominated by $\mathsf{cost}\big(\mathsf{Ver}(\mathsf{pk}, \mu, \boldsymbol{\sigma})\big) = nm$ multiplications, in the worst case. Therefore, Eq. (6) yields $knm + \mathsf{r}_0 k(n+m) < \mathsf{r}_0 nm \Rightarrow \mathsf{r}_0 > \frac{knm}{nm - k(n+m)}$. From the above formula we can derive a lower bound on values of $\mathsf{r}$ that yield efficiency (recall that by definition $\mathsf{r}_0 \leq \mathsf{r}$). A concrete security approach should lead to a meaningful upper bound on the value $\mathsf{r}$ that can be safely used in realistic applications. Intuitively, lower values of $\mathsf{r}$ yield higher accuracy (and unfogeability), higher ones guarantee better amortized efficiency.

## 9 Conclusions and future work

We presented a study on how to achieve efficient and progressive verification for digital signatures. In addition to putting forth these notions and formal models for them, we presented two compilers that allow one to realize efficient (resp. progressive) verification for a wide class of existing constructions including lattice-based and multivariate-based. We demonstrated the feasibility of our approach through an implementation on off-the-shelf resource-constrained devices. Finally, we showed how to extend our compiler to work with digital signatures with advanced properties, such as ring, threshold, homomorphic multi-key, attribute-based and constrained signatures. While our constructions show the feasibility of the desired properties, they also raise some natural follow up questions. For instance, is it possible to realize a compiler for LBS with $q \sim \mathsf{poly}(\lambda)$ that simultaneously pro-

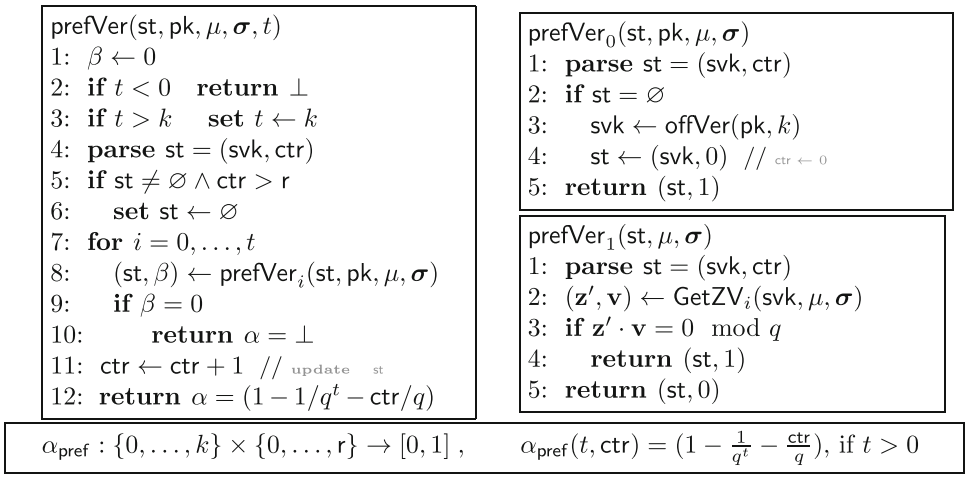



**Fig. 11** Generic compiler to obtain efficient and progressive verification of signature schemes with **Mv**-style verification and $q$ polynomial in the security parameter

vides efficient *and* progressive verification? We address this question in a positive way in Sect. 8, albeit in weaker security model. A solution with full fledged progressive security remains an interesting open problem. Another question is, is it possible to generalize our approach to other classes of digital signatures, e.g., code-based or LBS obtained through the Fiat-Shamir heuristic or from ideal lattices?

Finally, it would be worth to explore more applications of progressive and efficient verification. On top of the already mentioned applications to real-time systems, another possible avenue is parallel and distributed verification of digital signatures. Consider a public bulletin board that stores authenticated (signed) data. For security reasons, one may be tempted to use post quantum signature schemes such as LBS. However, the large sizes of the public keys and signatures and the slow speed of the verification are notorious bottlenecks to deploy them in such scenarios. Using our approach, a pool of parties –acting as verifiers– can be made in charge of running each a single verification check (i.e., ProgVer includes only $\mathsf{ProgVer}_0$ and $\mathsf{ProgVer}_1$). In terms of security, although a single verifier may be wrong with non-negligible probability $1/q$, the probability that $k$ honest verifiers are all wrong becomes negligible already for $k = 5$. Finally, we think that it would be interesting to explore the study of efficient and progressive verification also for more cryptographic primitives, such as commitments and zero-knowledge proofs.

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## Declarations

**Conflict of interest** The authors declare no conflict of interest.



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