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Spiral Constellations for Nonlinear Channels

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***Abstract*— In this letter, we propose a novel constellation design, specifically tailored to contrast the effect of nonlinear channels. The proposed constellations are based on a spiral expression, which, unlike other unstructured designs, makes them easy to build and optimize. Our design approach allows to optimize the distance between symbols in the angular and radial directions, resulting in a constellation that is robust to the nonlinearities, which affect more severely symbols with higher magnitude. Simulation results show that the proposed constellations outperform the solutions usually adopted for nonlinear channels, especially when the constellation size increases.**

***Index Terms*— Nonlinear channels, spiral constellations, constellation design.**

I. Introduction

POWER amplifiers are the most power-hungry devices of the whole transceiver chain and hence it is vital to use them as efficiently as possible; this means that they must be driven as close to saturation as possible, which, however, is also the region in which the amplifier characteristics exhibit the highest level of nonlinearity. Multiple solutions have been proposed to mitigate the effects of the nonlinearity. Among the most commonly used, it is worth mentioning digital predistortion [1], which consists in the computation, by means of a training phase, of a new constellation to be used in transmission, in place of the real one. Signal predistortion is another option, feasible when it can be applied directly before the nonlinear amplifier, which works directly on the samples of continuous-time signals, rather than on the transmitted symbols [2], [3]. As an alternative to predistortion, transmission schemes insensitive to nonlinearities such as continuous phase modulations can be employed. However, the spectral efficiency can be reduced [4].

Another approach to mitigate nonlinear effects, which is adopted especially when it is necessary to reduce the amount of information sent back from the receiver to the transmitter, is to properly design a constellation which is more robust to nonlinear distortion compared to conventional constellation formats, such as quadrature amplitude modulation (QAM). Satellite communication standards such as the digital video broadcasting, second generation extensions (DVB-S2X) [5], foresee the use of amplitude and phase shift keying (APSK) constellations, which can be designed to achieve satisfactory performance on typical nonlinear satellite channels. However, the constellations described in the standard have been optimized for the additive white Gaussian noise (AWGN) channel [5], while the full optimization for nonlinear channel requires the joint selection of several parameters, such as the number of rings, the number of points on each ring and the radii of the rings. In [6], the authors propose a design, based on a simulated annealing algorithm, to maximize the pragmatic capacity, by jointly optimizing the position of the points and the bits-to-symbols mapping. In [7], constellation design under phase noise impairment and peak to average power ratio constraint is investigated using artificial intelligence methods. An interesting APSK design has been proposed in [8], which relies on the choice of a single parameter to build the whole constellation.

Although effective, most of the design methods proposed in the literature suffer from a large complexity. Even constraining the constellation to be symmetric, the number of parameters is still large. Other optimization algorithms, whose complexity is usually high, often produce unstructured or asymmetric constellations, which are not suitable for a reduced complexity detection or a high speed hardware implementation [9]. These drawbacks become more and more significant when the size of the constellation grows large. The DVB-S2X standard proposes APSK constellations with up to 256 points for high throughput applications. Indeed, high order constellations offer high spectral efficiency over bandlimited channels, but they are also the most sensitive to channel impairments. As such, it is of critical importance to develop efficient high order constellation design methods.

Recently, spiral constellations for channels affected by phase noise have been considered in [10], which depend on a single parameter to be optimized, thus leading to a much simpler optimization procedure with respect to other works available in the literature. The constellation design is based on the observation that the angular noise variance increases with the magnitude of the symbol, while the radial noise is independent of it. Inspired by [10], in this letter we consider the design of spiral constellations for nonlinear channels, but this scenario poses extra challenges with respect to the case of phase noise. In this case, the radial and the angular noise induced by the nonlinearities scale, with different intensity,

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polynomially with the magnitude of the symbol. The proposed constellation design consists in the optimization of a set of parameters which regulate the distance between points in the radial and in the angular direction, and outperforms competing constellations with comparable design complexity.

II. System Model

We consider a typical model of a channel affected by a non-linear amplifier. A sequence of symbols $\{x_k\}_{k=0}^{K-1}$, belonging to an $M-$ary constellation, is linearly modulated by means of a root-raised cosine shaping pulse $p(t)$ to obtain the signal

$$x(t) = \sum_{k=0}^{K-1} x_k p(t - kT). \tag{1}$$

The signal passes through a nonlinear amplifier and is then transmitted over an AWGN channel. Hence, the received signal can be expressed as

$$r(t) = f_{\mathrm{NL}}[x(t)] + w(t), \tag{2}$$

where $f_{\mathrm{NL}}[\cdot]$ represents the effect of the nonlinearity, and $w(t)$ is an AWGN process. At the receiver, a matched filtering with the filter $p^*(-t)$, followed by a sampler at symbol time, is usually performed. The nth received sample in the presence of 3rd-order nonlinearity can be modeled as [11]

$$r_n = x_n K(1 + K'|x_n|^2) + \gamma_n^{(0)} + \gamma_n^{(1)}|x_n| + \gamma_n^{(2)}|x_n|^2 + w_n, \tag{3}$$

where K and K' are deterministic variables which depend on the amplitude-to-amplitude modulation (AM/AM) and amplitude-to-phase modulation (AM/PM) characteristics of the nonlinearity and on the pulse shape, while $\gamma_n^{(i)}$, with $i = 0, 1, 2$, are random variables independent of $|x_n|$, and dependent on the pulse shape, on the characteristics of the nonlinearity, on the transmitted symbols x_j, with $j \neq n$, and on $\angle x_n$ [11]. Constant K is a scaling of the constellation, the term $(1 + K'|x_n|^2)$ is a deformation of the constellation, while the term $\gamma_n^{(0)} + \gamma_n^{(1)}|x_n| + \gamma_n^{(2)}|x_n|^2$ represents the signal-dependent noise induced by the nonlinearity, i.e., the intersymbol interference (ISI). Despite the presence of ISI, symbol-by-symbol detection is usually performed at the receiver. A simple data-aided amplitude estimator can be also employed, in order to take into account of the scaling of the constellation. This can be based on any preamble field, commonly inserted in transmitted frames.

III. Spiral Constellation Design

The design of spiral constellations for a given communication system can be based on the study of the effects of the involved impairments on the nominal constellation points. In the case of systems affected by AWGN only, the best constellation has points placed at the same distance in the radial and in the angular directions. The condition on the radial distance is satisfied by considering an Archimedean spiral, defined by the following equation

$$s(t) = te^{jt}, \tag{4}$$

while the condition on the angular one can be met by placing the points along the spiral according to

$$c_m = t_m e^{jt_m} \quad m = 1, \ldots, M, \tag{5}$$

where M is the cardinality of the desired constellation, and $t_m = \sqrt{4\pi m}$ [10]. When the transmission is affected by phase noise or by nonlinear noise due to the presence of an amplifier, the noise affects more severely symbols with higher magnitude. In the case of phase noise, the angular noise variance increases with the magnitude of the current symbol [10], while the radial noise is independent of it. The spiral constellation can therefore be an Archimedean spiral, since the radial distance between points can be kept constant, while the points along the spiral can be chosen such that the angular distance increases linearly with $|c_m|$.

For the case of nonlinear channels, from (3) it can be observed that the amplifier-induced noise scales as a second-order polynomial of the amplitude of the transmitted symbol and impacts with different intensity in the radial and in the angular directions. For this reason, the design of the proposed spiral constellation is based on the fulfillment of two conditions, represented by the following equations (8) and (11), respectively, i.e., we require that the distance between two constellation symbols increases as a second-order polynomial of the modulus of the symbols both in the radial (*condition 1*) and angular (*condition 2*) directions.

These conditions are accounted for by choosing a proper function to describe the amplitude of the spiral (*condition 1*) and by suitably choosing the points along the new spiral (*condition 2*).

We first consider the radial distance. The Archimedean spiral is not a good choice, since successive laps of the spirals have constant distance. A more general spiral can be described as

$$s(t) = f(t)e^{jt} \tag{6}$$

and constellation points are defined as

$$c_m = f(t_m)e^{jt_m}, \quad m = 1, \ldots, M, \tag{7}$$

where M is the cardinality of the desired constellation and $t_1 < t_2 < \ldots < t_M$. The *condition 1* is satisfied if

$$f(t + 2\pi) - f(t) = \rho_0 + \rho_1 f(t) + \rho_2 f^2(t), \tag{8}$$

where $\boldsymbol{\rho} = (\rho_0, \rho_1, \rho_2)$ is the vector of the parameters that define the proposed spiral. When $\boldsymbol{\rho} = (2\pi, 0, 0)$, equation (8) gives $f(t) = t$ and we obtain the Archimedean spiral. Finding a function that satisfies (8) and has also a proper behaviour to be used as amplitude of constellation symbols is not an easy task. In this work, we build the function $f(t)$ for discrete values of the independent variable t. In particular, we set

$$\begin{aligned} f(\ell 2\pi + j) = \rho_0 + (1 + \rho_1)f((\ell - 1)2\pi + j) \\ + \rho_2 f^2((\ell - 1)2\pi + j) \end{aligned} \tag{9}$$

for $j = 1, \ldots, J$ and $\ell = 1, \ldots L$, and

$$f(j) = 2\pi j/J. \tag{10}$$

Parameters J and L are the number of discrete points in each lap and the number of laps of the discrete spiral, respectively. Their value is not critical, since different values lead to the same result. However, L must be large enough to allow to place the points of the constellation along the spiral, therefore it should increase with M. For example, in the simulation results we set $J = 30$ and $L = 10$ for the case $M = 256$.

The constellation points along the spiral are chosen in order to satisfy *condition 2*. In the following, we describe how to choose the parameters t_m in (7), such that the radial distance increases as a second order polynomial of the modulus of the symbol. We start by assuming the following relation

$$|c_{m+1} - c_m| = \alpha_0 + \alpha_1 |c_m| + \alpha_2 |c_m|^2, \quad m = 1, \ldots, M \tag{11}$$

and replace (7) into (11) to obtain

$$f(t_{m+1}) \left| 1 - \frac{f(t_m)}{f(t_{m+1})} e^{j(t_m - t_{m+1})} \right| = \alpha_0 + \alpha_1 |c_m| + \alpha_2 |c_m|^2. \tag{12}$$

We assume that $|t_m - t_{m+1}| \ll 1$ and use the approximations[1] $e^{j(t_m - t_{m+1})} \simeq 1 + j(t_m - t_{m+1})$ and $f(t_m)/f(t_{m+1}) \simeq 1$

$$f(t_{m+1}) \left| -j \frac{f(t_m)}{f(t_{m+1})} (t_m - t_{m+1}) \right| = \alpha_0 + \alpha_1 |c_m| + \alpha_2 |c_m|^2, \tag{13}$$

that is

$$f(t_m)(t_{m+1} - t_m) = \alpha_0 + \alpha_1 |c_m| + \alpha_2 |c_m|^2, \tag{14}$$

and finally we have the recursive expression for t_{m+1}

$$t_{m+1} = t_m + \frac{\alpha_0}{f(t_m)} + \alpha_1 + \alpha_2 f(t_m) \tag{15}$$

for $m \geq 1$ and $t_1 = 2\pi$. In (15), $f(t_m)$ is obtained though interpolation of the discrete function (9) and $\boldsymbol{\alpha} = (\alpha_0, \alpha_1, \alpha_2)$ is the vector of the parameters that regulate the distance between consecutive constellation points along the spiral.

The parameters $\boldsymbol{\rho}$ and $\boldsymbol{\alpha}$ that define the proposed spiral constellation can be chosen as the ones maximizing a given performance measure. Based on the considered scenario, some of the six parameters can be kept fixed or equal to zero in order to simplify the design procedure. In Section V, we will show the performance of the proposed spiral constellation obtained by optimizing only two parameters.

A low-complexity suboptimal detection algorithm with good complexity-performance trade-off can be derived, which considers only the two closest constellation points to the observable r_k, by following an approach similar to [10].

IV. Performance Metrics

This section describes the figures of merit that will be adopted to evaluate the performance of the proposed constellations. Our aim is to compute both theoretical and more practical results. The first considered performance metric is the achievable information rate (IR), which can be evaluated numerically with the Monte Carlo method proposed in [12]. According to the principle of mismatched detection, this metric is a lower bound to the channel capacity, achievable with joint detection and decoding, and computed without reference to any practical coding scheme. Defining by $\mathbf{x} = \{x_k\}_{k=0}^{K-1}$ a sequence of transmitted symbols, by $\mathbf{r} = \{r_k\}_{k=0}^{K-1}$ the corresponding channel outputs, and by $p(\mathbf{r}|\mathbf{x})$ the selected auxiliary channel law, the adopted lower bound on the channel capacity can be evaluated as

$$I(\mathbf{x}; \mathbf{r}) = \lim_{K \to \infty} \frac{1}{K} \mathrm{E} \left[\log_2 \frac{p(\mathbf{r}|\mathbf{x})}{\sum_{\mathbf{x}'} p(\mathbf{r}|\mathbf{x}') P(\mathbf{x}')} \right], \tag{16}$$

where the summation is computed over the M constellation symbols, and $P(\mathbf{x}')$ is the probability distribution of the transmitted symbols, which is assumed equal to $1/M$, and the expectation is over the channel realizations. In this letter, we considered the AWGN channel as the auxiliary channel model. In this way, we can simplify (16) by replacing the vector PDF with its scalar version $p(r_k|x_k)$, which corresponds to using a symbol-by-symbol detector. In order to maximize the achievable IR with the proposed spiral constellation, we have to select the parameters $\boldsymbol{\rho}$ and $\boldsymbol{\alpha}$ accordingly. We perform the following optimization

$$(\boldsymbol{\rho}^*, \boldsymbol{\alpha}^*) = \arg \max_{\boldsymbol{\rho}, \boldsymbol{\alpha}} I(\boldsymbol{\rho}, \boldsymbol{\alpha}), \tag{17}$$

where I is the bound on the achievable IR, computed as in (16), in which we have explicitly shown the dependence on the adopted constellation, defined by the parameters $\boldsymbol{\rho}$ and $\boldsymbol{\alpha}$. This is the same principle already adopted in [10] for a different spiral design.

An alternative theoretical measure is given by the pragmatic IR, which represents an upper bound to the IR achieved by a practical modulation and coding format (ModCod). As suggested by the name, in this case the receiver follows a *pragmatic* approach, in the sense that it does not perform iterations between detector and decoder, as is usually done in practical systems. The pragmatic IR depends on the adopted bit-to-symbol mapping, which determines how transmitted bits are mapped on the complex constellation symbols.

Commonly used PSK and QAM constellations adopt a Gray mapping, ensuring that adjacent symbols differ by only one bit. APSKs are usually designed to adopt a quasi-Gray mapping, see for example the DVB-S2X standard [5]. When the constellation does not have a regular structure, instead, the mapping needs to be determined through optimization, possibly jointly with the position of the constellation symbols [6]. For the spiral constellations of [10], different mapping schemes were proposed, based on a transformation of the Gray mapping of a QAM constellation (see [10] for the details). For the constellation design proposed in this letter, we adopt a mapping which is Gray along the spiral, meaning that each symbol differs by only one bit with respect to the previous and next symbols along the spiral. This choice is not optimal but ensures good performance and simplifies the constellation design in the considered scenarios. Also in this case, an optimization of the parameters $\boldsymbol{\rho}$ and $\boldsymbol{\alpha}$ is performed as in (17), with the function I replaced by the pragmatic IR. An example of the adopted mapping is shown in Figure 1

[1] The approximation is good for all constellation points except the most internal ones, i.e., when m is very small.

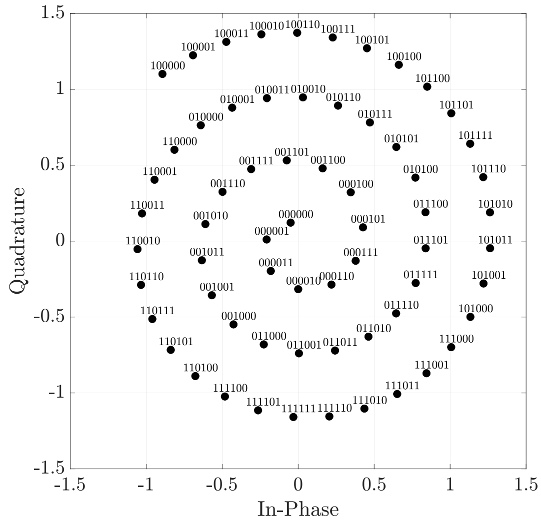

Fig. 1. Example of a spiral with $M = 64$, $\boldsymbol{\rho} = (2\pi, 0, 2.3 \times 10^{-5})$ and $\boldsymbol{\alpha} = (3.5, 0, 0)$ and the corresponding Gray mapping.

for a spiral with $M = 64$, $\boldsymbol{\rho} = (2\pi, 0, 2.3 \times 10^{-5})$ and $\boldsymbol{\alpha} = (3.5, 0, 0)$.

As a final performance measure, we will consider the packet error rate (PER) of coded transmission using the low-density parity-check (LDPC) codes proposed in the DVB-S2X standard [5], which are nearly capacity-achieving on the AWGN channel.

V. NUMERICAL RESULT

In this section, we present some numerical results to evaluate the performance of the proposed spiral constellations. For fair comparisons, we use the definition of signal-to-noise ratio (SNR) as the ratio P/N between the saturation power of the nonlinear amplifier and the noise power in the system bandwidth. The nonlinear AM/AM and AM/PM transfer characteristics of the power amplifier are given in [5, Figure H.1]. We consider a unit power amplifier. This assumption is without loss of generality since we measure the performance in terms of P/N. We consider a shaping pulse with roll-off factor 0.25, constellations with size $M = 64$ and 256, and compare different modulation formats, namely the proposed spirals (*Spiral NL*), the spirals proposed in [10] (*Spiral PN)*, classical QAMs, APSKs foreseen by the DVB-S2X standard [5], and the APSKs with Gray mapping proposed in [8] (*Gray APSK*). All constellations are normalized to have the same average symbol energy. For all cases, we use the same symbol-by-symbol maximum a posteriori probability detection algorithm and, for the error rate simulations, the same log-likelihood-ratio-based belief-propagation decoding algorithm, with a maximum of 50 decoder iterations. The constellations' parameters have been selected to maximize the performance, as described in Section III. For the proposed spirals, we optimized two parameters, ρ_2 and α_0 and fixed the others, i.e., $\boldsymbol{\rho} = (2\pi, 0, \rho_2)$ and $\boldsymbol{\alpha} = (\alpha_0, 0, 0)$. This choice is motivated by the fact that the considered amplifier induces a noise that has larger variance in the radial than in the angular direction. It could be convenient to optimize different parameters for other nonlinearities or different scenarios. For Spirals PN, we optimized the parameter that regulate the

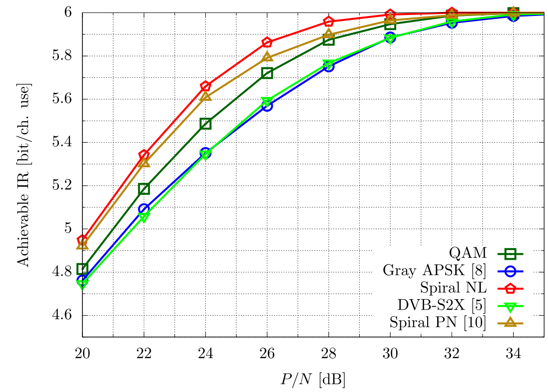

Fig. 2. Achievable information rate for constellations with $M = 64$.

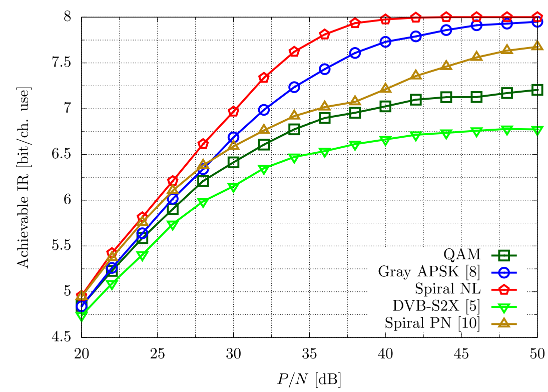

Fig. 3. Achievable information rate for constellations with $M = 256$.

distance between adjacent points on the spiral. For Gray APSKs, we selected the optimal number of circles; for QAMs, DVB–S2X APSKs, and Spiral PN, we optimized the variance of the noise assumed by the detector. For each considered performance metric, we have chosen the input back-off (IBO) that optimizes it, possibly different for different modulation formats, and with a quantization step of 1 dB.

Figures 2 and 3 report the achievable IR for constellations with $M = 64$ and 256 points, respectively. We notice that the proposed spirals ensure performance gains with respect to all the considered alternatives. These gains increase as the ratio P/N increases, and become more significant, in the order of 2-3 dB, for larger constellation size. Figures 4 shows the pragmatic IR for $M = 256$. The conclusions we can draw are similar, with significant gains of the proposed solution, especially for medium-high P/N values.

In Figure 5, we report the PER for $M = 256$ using three LDPC codes selected from the DVB-S2X standard, namely those with rates $r = 3/4$, $5/6$, and $9/10$. The optimal values of the IBO for each curve are reported in Table I, where a sign "–" means that the corresponding curve does not converge. For the code with rate $3/4$ (6 bits/ch. use), Gray APSK and the proposed spiral have the same performance, while Spiral PN, QAM and DVB-S2X suffer significant losses. For the higher code rates, instead, corresponding to an achieved IR of 6.67 and 7.2 bit/ch. use, respectively, spirals ensure increasing gains with respect to Gray APSKs. In particular, for code rate $r = 9/10$, we observe a gain of about 2 dB. When for a given code rate the PER curves are not reported, it means that they are constant and equal to one. This could be expected by

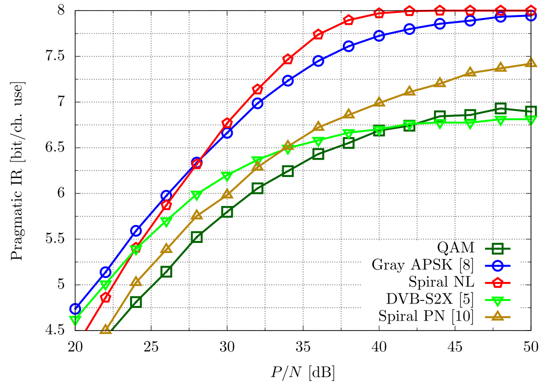

Fig. 4. Pragmatic information rate for constellations with $M = 256$.

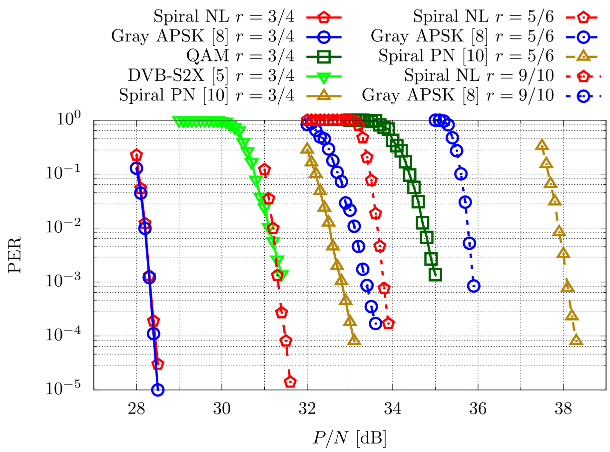

Fig. 5. Packet error rate for constellations with $M = 256$. Optimized spiral parameters: $\rho_2 = 7.8 \times 10^{-3}$, $\alpha_0 = 1.9$ for code rate $3/4$; $\rho_2 = 2.9 \times 10^{-3}$, $\alpha_0 = 2.3$ for code rates $5/6$ and $9/10$.

TABLE I

OPTIMAL IBO VALUES IN DECIBEL FOR THE CURVES IN FIGURE 5

	Spiral NL	Gray APSK	QAM	DVB-S2X	Spiral PN
$r = 3/4$	6	6	7	7	6
$r = 5/6$	6	6	–	–	9
$r = 9/10$	7	8	–	–	–

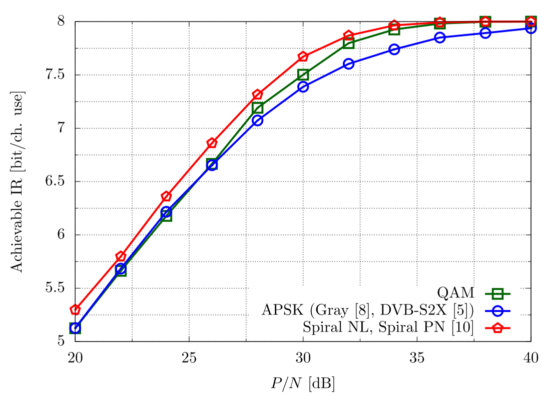

Fig. 6. Information rate for constellations with $M = 256$ with data predistorsion at the transmitter.

looking at Figure 4, where we can see that the corresponding curves are unable to reach the desired rates. This means that coded performance does not converge for any value of P/N.

The performance analysis revealed that the proposed spirals have superior performance at high SNR, where the system is limited by the nonlinear distortion, while the gain is reduced for AWGN-limited scenarios. Furthermore, the gain increases with the cardinality of the constellation, as high-order constellations are more sensitive to nonlinearities and benefit more from the specific design that takes them into account.

Finally, Figure 6 shows the achievable IR when the data predistortion is applied at the transmitter side and reveals that the Spiral NL and Spiral PN have the best performance. This is due to the fact that in this case the best constellation to use at the receiver is close to the optimal one for the AWGN channel, and both Spiral PN and Spiral NL can be made very effective in AWGN through an appropriate choice of the parameters that characterize them.

VI. CONCLUSION

We proposed a novel constellation format, specifically designed to combat the effects of nonlinear channels. The proposed design is based on a geometrical spiral shape, and it relies on a simple optimization to identify the best parameters configuration. This feature makes it particularly attractive when compared to other design methods, especially for high order constellations. We evaluated the performance of the proposed spirals using both theoretical and more practical figures of merit. Simulations show a strong agreement of all the result, demonstrating that significant gains can be achieved over a typical nonlinear channel model.

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