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Optimization of Waveforms for Energy-Efficient Communication

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***Abstract*—A new waveform design technique to improve the energy efficiency of a communication system using quadrature amplitude modulation (QAM) for a given throughput is presented. The proposed method optimizes the mutual information using probabilistic shaping for a given power consumption. In certain cases, we show that the optimization problem is convex and can be solved efficiently. Our numerical results show that the proposed waveform design technique outperforms standard QAM signaling regarding spectral efficiency for any power consumption level.**

***Index Terms*—waveform design, energy efficiency, probabilistic shaping, power amplifier, green communications**

I. Introduction

Green communication has attracted much attention in recent years [1], [2]. Communications systems have been studied widely to achieve low energy consumption or high energy efficiency (EE) while upholding the high quality of service and spectral efficiency (SE) requirements.

Power amplifiers are often the most power-hungry components of communication systems, and there has been extensive research to deal with this component in the field of green communication. In [3], authors have surveyed PA-centric technologies to improve EE in wireless communication systems and investigated EE and SE trade-offs while considering nonlinearity. The trade-off between EE and SE has been extensively studied in the literature; see [4] and the references therein.

There are different techniques for improving EE based on the nature of the understudy system. For wireless systems, mainly for Orthogonal Frequency Division Multiplexing systems, peak-to-average power ratio reduction techniques are in use, as these techniques help systems to use less back-off and hence have a better output power to consumed power ratio [5]–[9].

Techniques like probabilistic and geometric shaping are now widely used for shaping the modulations to decrease the gap between modulation SE and capacity curves, i.e., finding the constraint capacity [10]. These techniques, especially probabilistic shaping (PS), are primarily limited to optic systems, but some papers have been published in wireless systems, too. For example, authors in [11] used PS for the WiFi standard.

In this paper, we are searching for the best SE for a specific consumed power. We solve the trade-off rising from linearity issues by including nonlinearity in our problem formulation. We define super symbols by concatenating several consecutive symbols and find the SE and the power consumption as a function of the probabilities of the set of super symbols. Then, we optimize the probabilities to obtain the best achievable SE for a desired power consumption.

The following sections are the system model, problem formulation, simulations, and conclusion. In the system model, we provide a model for power consumption for power amplifiers, which suits the PS framework. Then, we introduce a model for nonlinearity, which can be used in our simulation. In problem formulation, we discuss how we can formulate the problem in a way that makes it tractable in terms of probabilities. Then, we introduce some simplifications that make the problem we formulated. In the numerical solution and simulation section, we introduce our method for solving and provide simulation results.

II. System Model

This section describes the communication system model and how power consumption is modeled. Our system model considers the system using digital linear modulation.

Without considering nonlinearity or other impairments, the selected channel model is simply an additive white Gaussian noise (AWGN) channel

$$Y = X + Z, \tag{1}$$

where Y is the output of the channel, X is the input, and Z is the noise added by the channel, which is complex Gaussian noise with zero mean and variance σ^2.

A. Modeling the amplifier nonlinearity

Linear range in power amplifiers is a limiting factor for system performance, and there are different models for it. In this paper, we use the gain plus complex limiter model, described as

$$V(x) = \begin{cases} x, & |x| \le x_{\max} \\ x_{\max} e^{j\angle x}, & |x| > x_{\max} \end{cases} \tag{2}$$

where $V(x)$ is the output, x is the input, x_{max} is the maximum amplitude of the amplifier and $\angle c$ denotes the phase of the complex number c. The complete amplifier will include gain, here α before the hard limiter, so the output of the amplifier will be $V(\alpha x)$.

B. Consumed power model

As stated in [12], [13], instantaneous consumed power for a relatively wide class of power amplifiers, including class B amplifiers, can be modeled as a function of the absolute value of the waveform in the output of the amplifier,

$$P_{\text{dc}}(x) \propto |V(\alpha x)|^{2\epsilon}, \tag{3}$$

where x is the complex envelope of output and ϵ is a constant for that amplifier. In this this work, we consider that ϵ equals half. For turning (3) into equality, we need a factor that we represent by κ. For the amplifier model that we described in subsection II-A, this factor can be obtained in terms of maximum amplitude and maximum efficiency as

$$\kappa = \frac{x_{\text{max}}}{\eta_{\text{max}}}, \tag{4}$$

where η_{max} is the maximum value of η, power efficiency of the amplifier, or maximum efficiency of the specific power amplifier and x_{max} is the maximum amplitude of amplifier as we have in (2).

By this assumption, we can find the consumed power of the amplifier for a signal in time as

$$P_{\text{dc}} = \kappa \lim_{D\to\infty} \frac{1}{2D} \int_{-D}^{D} |V(\alpha x(t))|\, dt. \tag{5}$$

As a linear digital modulator generates the input signal to the amplifier, we can write $x(t)$ as

$$x(t) = \sum_{k=-\infty}^{\infty} x_k h(t-kT), \tag{6}$$

where x_ks are the digital symbols to be transmitted, $h(t)$ is the impulse response of the pulse shape filter, and T is the symbol length. As (6) suggests, $x(t)$ is a cyclo-stationary process, so by assuming that the source is ergodic, we can rewrite (5) as

$$P_{\text{dc}} = \kappa \mathbb{E}_X \left[\frac{1}{T} \int_T \left| V\left(\alpha \sum_{k=-\infty}^{\infty} x_k h(t-kT) \right) \right| dt \right], \tag{7}$$

where $\mathbb{E}_X$ is the expected value operator over random variable X, which is random variable representing digital symbols generated by the source; And, x_ks, are identically independent distributed (IID) symbols drawn from set $\mathcal{X}$, the set of constellation points, at symbol time k with probability mass function P_X.

As can be seen from (7), the consumed power is a function of symbols' distribution. Besides that, as the pulse shaping filter can be longer than a single symbol, the power consumption is a function of possible sequences of symbols.

Note that (7) is an exact formula for consumed power when symbols are IID. However, assuming IID symbols will result in insufficient degrees of freedom in our feasible set for optimization of mutual information constrained by power consumption. Therefore, we proposed a new concept between one symbol and an infinite sequence of the symbols, namely supersymbols.

1) Introducing super symbol concept and reformulating power consumption formula: A super symbol is defined as a finite sequence of consecutive QAM symbols; in other words, it can be viewed as a multidimensional symbol in which the dimensions are quadrature and in-phase for several consecutive symbols.

We define a super symbol with length N from symbols $x_i \in \mathcal{X}$,

$$\mathbf{s}(n) = [x_{n\cdot N+1}, x_{n\cdot N+2}, ..., x_{(n+1)\cdot N}]^T, \quad x_i \in \mathcal{X} \subset \mathbb{C} \tag{8}$$

where $\mathcal{X}$ is the set of constellation points of QAM (i.e. $\mathbf{s} \in \mathbf{S} = \mathcal{X}^N$).

If we consider that super symbols are IID, then the consumed power can be written as

$$\begin{aligned} P_{\text{dc}} &= \kappa \mathbb{E}\left[\frac{1}{NT} \int_0^{NT} \left| V\left(\alpha \sum_{k=-\infty}^{+\infty} x_k h(t-kT) \right) \right| dt \right] \\ &= \kappa \mathbb{E}\left[\frac{1}{NT} \int_0^{NT} \left| V\left(\alpha \sum_{n=-\infty}^{\infty} \sum_{i=0}^{N-1} x_{n(N-1)+i} \right.\right.\right. \\ &\qquad \left.\left.\left. \times\, h(t-n(N-1)T-iT) \right) \right| dt \right]. \end{aligned} \tag{9}$$

To make (9) completely in terms of super symbols, we define F, time domain super symbol,

$$F(\mathbf{s}(n), n, t) := \sum_{i=0}^{N-1} \mathbf{s}(n)_i h(t-n(N-1)T-iT) \tag{10}$$

where $\mathbf{s}(n)_i$ is the ith element of the vector $\mathbf{s}(n)$. We can rewrite (9) by

$$P_{\text{dc}} = \kappa \mathbb{E}\left[\frac{1}{NT} \int_0^{NT} \left| V\left(\alpha \sum_{n=-\infty}^{\infty} F(\mathbf{s}(n), n, t) \right) \right| dt \right] \tag{11}$$

where $\mathbf{s}(n)$ defined in (8) is the super symbol in nth super symbol's time slot, or as we can define, "super slot."

Since most of the energy of the pulse shaping filter lies in one symbol, we can approximate (11) with

$$P_{\text{dc}} \approx \kappa \mathbb{E}\left[\frac{1}{NT} \int_0^{NT} \left| V\left(\alpha \sum_{n=-K}^{K} F(\mathbf{s}(n), n, t) \right) \right| dt \right] \tag{12}$$

in which K controls the number of super slots we used in the approximation, changing the precision. The expected value operator can be replaced by a summation over super symbol random space

$$\begin{aligned}P_{\text{dc}} \approx \kappa & \sum_{\forall[\mathbf{s}(-K),...,\mathbf{s}(K)]\in\mathbf{S}^{2K+1}} p_{S^{2K+1}}\left(\mathbf{s}(-K),...,\mathbf{s}(K)\right)\\ & \times \frac{1}{NT}\int_0^{NT}\left|V\left(\alpha\sum_{n=-K}^{K}F(\mathbf{s}(n),n,t)\right)\right|dt\\ = \kappa & \sum_{\forall[\mathbf{s}(-K),...,\mathbf{s}(K)]\in\mathbf{S}^{2K+1}} \prod_{k=-K}^{K} p_{\mathbf{S}}(\mathbf{s}(k)))\\ & \times \frac{1}{NT}\int_0^{NT}\left|V\left(\alpha\sum_{n=-K}^{K}F(\mathbf{s}(n),n,t)\right)\right|dt. \quad (13)\end{aligned}$$

The complexity of the computation of consumed power, as the formula in (13) suggests, grows exponentially in terms of the length of the super slot.

The main problem with this formula is that even for $K = 1$, the memory needed can be huge; for example, if we consider our primary constellation as QAM16 and each super symbol consists of 3 consecutive symbols, then the number of the cases we need to sum in (13) will be 16^9 which is more than 64 billion, although the case was almost minimal.

III. Formulation of the optimization problem

Here, we seek to minimize the power consumption for a given system throughput or, equivalently, maximize the throughput under a power consumption constraint. The method consists of adjusting the probabilities of each sequence of input constellation points. We do this optimization under a constraint on consumed power, derived in (13). The multidimensional constellation is formed by concatenating multiple QAM symbols into a vector, as is described in (8). The probabilities are defined over the set $\mathcal{S} = \mathcal{X}^N$, where N is the number of symbols in one super symbol. In the same way, we define $\mathcal{U} = \mathcal{Y}^N$ as the sampler output super symbols' space and $\mathbf{U}$ as the vector formed by concatenating consecutive sampler outputs, Ys,

$$P^*_{\mathbf{S}} = \arg\max_{P_{\mathbf{S}}} \mathbf{I}(\mathbf{S};\mathbf{U}) \qquad \text{s.t.} \quad P_{\text{dc}} < P_{\text{lim}}. \quad (14)$$

According to (13), single-letterization is possible as the constraint is a function of the probability of one super symbol.

Figure 1 shows the block diagram of the system we describe with this optimization problem and the model we described in section II for super symbols consisting of two consecutive QAM16 symbols. The part after the sampler is not shown because here we study mutual information between the super symbol selector output ($\mathbf{S}$) and sampler output ($\mathbf{U}$).

The objective function, the mutual information between source and sink, can be written in terms of probabilities of input and output and their conditionals as

$$\begin{aligned}\mathbf{I}(\mathbf{S};\mathbf{U}) &= h(\mathbf{U}) - h(\mathbf{U}\mid\mathbf{S})\\ &= \sum_{\mathbf{s}\in\mathbf{S}} p_{\mathbf{S}}(\mathbf{s})\int_{\mathbb{C}^N} p_{\mathbf{U}|\mathbf{S}}(\mathbf{u}\mid\mathbf{s})\log_2\left(p_{\mathbf{U}|\mathbf{S}}(\mathbf{u}\mid\mathbf{s})\right)d\mathbf{u}\\ &\quad - \sum_{\mathbf{s}\in\mathbf{S}} p_{\mathbf{S}}(\mathbf{s})\int_{\mathbb{C}^N} p_{\mathbf{U}|\mathbf{S}}(\mathbf{u}\mid\mathbf{s})\log_2\left(p_{\mathbf{U}}(\mathbf{u})\right)d\mathbf{u}. \quad (15)\end{aligned}$$

From (15), computing mutual information, as it should be integrated over a multidimensional volume, is tedious. Therefore, using the well-known Monte Carlo technique can be a good option. In the following subsections, we will investigate two cases of this problem.

A. Without nonlinearity

When $|\alpha x| \leq x_{\text{max}}$ for all input sequences, the conditional pdf in (15) is known and is equal to the entropy of multi-dimensional Gaussian noise because we know $\mathbf{U} = \alpha\mathbf{S} + \mathbf{Z}$ and $h(\mathbf{U} \mid \alpha\mathbf{S}) = h(\mathbf{U} \mid \mathbf{S})$, so it has no impact on our optimization and can be excluded. Therefore, the equivalent objective function is

$$P^*_{\mathbf{S}} = \arg\min_{P_{\mathbf{S}}} \frac{1}{K}\sum_{k=1}^{K}\log_2\left(p_{\mathbf{U}}(\mathbf{u}_k)\right). \quad (16)$$

In Monte Carlo calculations, we need to draw samples from the probability density function at hand. We have opted for importance sampling as it helps mitigate fluctuations caused by varying sample counts for each constellation point, given that probability serves as the optimization argument. For importance sampling, we use N_{G} Gaussian distributed samples per each one of uniformly distributed centroids, amplifier transformed super symbols. So, the sample distribution is

$$q(\mathbf{u}_k) = \frac{1}{M}\sum_{m=1}^{M} p_{\mathbf{U}|\tilde{\mathbf{S}}}(\mathbf{u}_k \mid \tilde{\mathbf{s}}_m), \quad \forall k \quad 1 \leq k \leq K \quad (17)$$

where $\mathbf{s}_m$ is the mth possible $\mathbf{s}$, selected from multidimensional constellation points, M is the size of this constellation, $\tilde{\mathbf{s}}_m = \alpha\mathbf{s}_m$, $K = MN_G$ and $p_{\mathbf{U}|\mathbf{S}}(\mathbf{u}_k \mid \tilde{\mathbf{s}}_m)$ is multivariate normal distribution with center $\tilde{\mathbf{s}}_m$ and covariance matrix equal to $2\sigma^2 I_{\text{N}}$, in which I_{N} is N-dimensional identity matrix.

The difference between the distribution we integrate on it and the sample distribution is that it is not uniform on centroid selection, but it is equivalent to the probabilities of constellation points we are searching to find.

By implementing the idea of importance sampling with this sample distribution, the objective function can be written as

$$\begin{aligned}P^*_{\mathbf{S}} = \arg\min_{P_{\tilde{\mathbf{S}}}} \frac{1}{K}\sum_{k=1}^{K}\log_2&\left(\sum_{m=1}^{M} p_{\mathbf{U}|\tilde{\mathbf{S}}}(\mathbf{u}_k \mid \tilde{\mathbf{s}}_m)\, p_{\tilde{\mathbf{S}}}(\tilde{\mathbf{s}}_m)\right)\\ \times &\left(\frac{\sum_{m=1}^{M} p_{\mathbf{U}|\tilde{\mathbf{S}}}(\mathbf{u}_k \mid \tilde{\mathbf{s}}_m)\, p_{\tilde{\mathbf{S}}}(\tilde{\mathbf{s}}_m)}{\sum_{m=1}^{M} p_{\mathbf{U}|\tilde{\mathbf{S}}}(\mathbf{u}_k \mid \tilde{\mathbf{s}}_m)\,\frac{1}{M}}\right). \quad (18)\end{aligned}$$

The probabilities of $\tilde{\mathbf{S}}$ are equal to the ones of $\mathbf{S}$, so the objective function is actually in terms of probabilities of super symbols. To obtain the final form of the problem, we rewrite all the optimization problems in terms of super symbol probability vector $\mathbf{p} = \left[p_{\mathbf{S}}(\mathbf{s}_1),...,p_{\mathbf{S}}(\mathbf{s}_M)\right]^T$ as

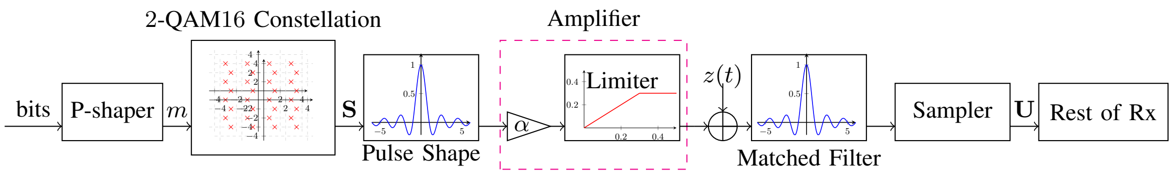

Fig. 1. Block diagram of the whole system for 2-symbol QAM16 super symbol

$$P_{\mathbf{S}}^{*} = \arg\min_{\mathbf{p}} \frac{1}{K}\sum_{k=1}^{K}\log_2\left(\sum_{m=1}^{M} p_{\mathbf{U}|\tilde{\mathbf{S}}}\left(\mathbf{u}_k \mid \tilde{\mathbf{s}}_m\right)\mathbf{p}_m\right)$$
$$\times\left(\frac{\sum_{m=1}^{M} p_{\mathbf{U}|\tilde{\mathbf{S}}}\left(\mathbf{u}_k \mid \tilde{\mathbf{s}}_m\right)\mathbf{p}_m}{\frac{1}{M}\sum_{m=1}^{M} p_{\mathbf{U}|\tilde{\mathbf{S}}}\left(\mathbf{u}_k \mid \tilde{\mathbf{s}}_m\right)}\right)$$
s.t.
$$\kappa \sum_{\substack{\forall k=[k_{-N_s},\ldots,k_{N_s}]\\ k_i\in 1,2,\ldots,M}}\left(\prod_{i=-N_s}^{N_s}\mathbf{p}_{k_i}\right)$$
$$\times\frac{1}{NT}\int_0^{NT}\left|V\left(\alpha\sum_{n=-N_s}^{N_s}F(\mathbf{s}_{k_i},n,t)\right)\right|dt < P_{\text{lim}} \quad (19)$$

where N_s is the number of consecutive super symbols we use to approximate the power consumption, k_i is the index of constellation point selected at i the super slot.

As we discussed earlier, the calculation complexity of constraint is very high. Therefore, we need to simplify the problem to reduce the complexity. Besides that, we should include the cases in which $|\alpha x| > x_{\text{max}}$, i.e., clipping occurs.

B. One super symbol with nonlinearity

To include nonlinearity effects and to avoid the complexity of the ”almost precise” system, we use a simplifying assumption; we assume that all symbols outside of the understudy super symbol are zero. This assumption will lead to a new problem: information content maximization for one super-symbol under power consumption constraint.

There are two ways to map the result of this new problem to the primary one: First if the understudy super symbol's length tends to infinity. Second, if we have all neighboring IID super symbols in both mappings, we can quantify the distance with the primary optimization problem in constraint and objective function.

At the end of the day, we will simulate and see if the system we described with this assumption will outperform uniform QAM in terms of mutual information for all consumed power levels, which is our objective.

In the system with one super symbol, the effect of non-linearity is simply a displacement in the output constellation. We can see that deterministic (even unknown) displacement in $\mathbf{U}$ as it is a function of $\mathbf{S}$, will not change $h(\mathbf{U} \mid \mathbf{S})$ [14]. Hence, to include the effect of the nonlinearity, we only need to add the nonlinearity effect to the constellation points in our calculation of $h(\mathbf{U})$.

Therefore, the optimization problem for one super symbol using importance sampling can be expressed as

$$P_{\mathbf{S}}^{*} = \arg\min_{\mathbf{p}} \frac{1}{K}\sum_{k=1}^{K}\log_2\left(\sum_{m=1}^{M} p_{\mathbf{U}|\tilde{\mathbf{S}}}\left(\mathbf{u}_k \mid \tilde{\mathbf{s}}_m\right)\mathbf{p}_m\right)$$
$$\times\left(\frac{\sum_{m=1}^{M} p_{\mathbf{U}|\tilde{\mathbf{S}}}\left(\mathbf{u}_k \mid \tilde{\mathbf{s}}_m\right)\mathbf{p}_m}{\frac{1}{M}\sum_{m=1}^{M} p_{\mathbf{U}|\tilde{\mathbf{S}}}\left(\mathbf{u}_k \mid \tilde{\mathbf{s}}_m\right)}\right)$$
s.t.
$$\kappa\sum_{k=1}^{M}\mathbf{p}_k\frac{1}{NT}\int_0^{NT}\left|V\left(\alpha F(\mathbf{s}_k,0,t)\right)\right|dt < P_{\text{lim}} \quad (20)$$

where $\tilde{\mathbf{s}}_m$ are the outputs of the sampler at the receiver in the noiseless situation.

It is evident from (20) that the left-hand side of the inequality in the constraint part is a linear combination of the optimization argument, so it represents a convex subspace.

The implicit constraints on the optimization argument that all elements should be between 0 and 1 and its elements should sum up to 1 also represent a convex set. Therefore, the intersection of all sets defined by constraints will result in a convex feasible set for the problem.

To prove the problem's convexity, we should also show that the objective function is convex. For that, we need to compute the Hessian of the objective function,

$$H_{ij} = \frac{\partial^2}{\partial p_j \partial p_i}\frac{1}{K}\sum_{k=1}^{K}\ln\left(\sum_{m=1}^{M} p_{\mathbf{U}|\mathbf{S}}\left(\mathbf{u}_k \mid \mathbf{s}_m\right)\mathbf{p}_m\right)$$
$$\times\left(\frac{\sum_{m=1}^{M} p_{\mathbf{U}|\mathbf{S}}\left(\mathbf{u}_k \mid \mathbf{s}_m\right)\mathbf{p}_m}{\frac{1}{M}\sum_{m=1}^{M} p_{\mathbf{U}|\mathbf{S}}\left(\mathbf{u}_k \mid \mathbf{s}_m\right)}\right)$$
$$= \frac{1}{K}\sum_{k=1}^{K}\left(\frac{p_{\mathbf{U}|\mathbf{S}}\left(\mathbf{u}_k \mid \mathbf{s}_j\right)}{\frac{1}{M}\sum_{m=1}^{M} p_{\mathbf{U}|\mathbf{S}}\left(\mathbf{u}_k \mid \mathbf{s}_m\right)}\right)$$
$$\times\left(\frac{p_{\mathbf{U}|\mathbf{S}}\left(\mathbf{u}_k \mid \mathbf{s}_i\right)}{\sum_{m=1}^{M} p_{\mathbf{U}|\mathbf{S}}\left(\mathbf{u}_k \mid \mathbf{s}_m\right)\mathbf{p}_m}\right). \quad (21)$$

As it is deductible from (21), the Hessian matrix is the summation of K rank one positive semi-definite matrices, which results in a positive semi-definite matrix, so we can conclude that the objective function is convex (not necessarily strictly convex).

We showed that the problem is convex. Therefore, we can employ an Interior-Points method [15] to solve this optimiza-

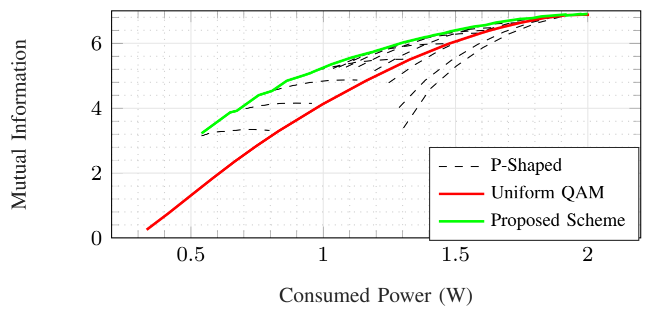

Fig. 2. Mutual information versus consumed power curves for three scenarios at 50 mW noise power : Red: Uniform QAM. Dashed lines: Probabilistic-shaped curves at every chosen gain. Green: combined gain control and PS. Simulations are done for a 2-QAM16 super symbol and amplifier with a maximum amplitude ($x_{\max}$) of 0.3 and maximum efficiency ($\eta_{\max}$) of 0.65

tion problem efficiently, and also, we can be sure that the answer will be the global optimum.

IV. Simulation Results

In this section, we show the simulation results of the system with one super symbol, which we introduced earlier in (20).

For describing the power amplifier with the hard limiter model, we need to specify two amplifier parameters' values $\eta_{\max}$, maximum efficiency of the amplifier, and $x_{\max}$, maximum amplitude of the amplifier. For the pulse shape filter in the simulation, we consider a truncated sinc function.

We can scale the amplifier's power consumption by either changing the amplifier's gain or by using our proposed scheme and introducing a probabilistic back-off. We combine these two techniques in this way: First, we use a back-off, or equivalently a gain, for the amplifier, and then we sweep the P_{lim} from one hundred percent of power consumption at that point, to some percentage of that by using the problem stated in (20). The curves generated by this change in P_{lim} are shown in Figure 2 by dashed lines.

In Figure 2, we also have the uniform QAM curve, the red one, which is obtained by just changing the gain, and the green curve, which shows the maximum mutual information we can get by our combined gain control and PS method. As can be seen, the green curve outperforms the red curve in all consumed power levels in which the green curve is drawn. In the point that the level of consumed power is equal to 2, there is no room for improvement as the system saturates because of $x_{\max}$. Still, in lower power consumption levels, there is room for a probabilistic back-off from a higher gain point, and this probabilistic back-off outperforms a gain back-off.

Based on the results in Figure 2, we can propose this algorithm for finding the best setting of gain and PS probabilities:

- Draw a vertical line to cross the green curve.
- Find the dashed (black) curve that goes through that point and get the gain and probabilities for that curve by following it to the red curve.

It is noteworthy that if both gain and PMF were the arguments of our optimization, the resulting optimization problem would not be convex; but, by using the proposed method, we successfully found the global optimum point for this optimization problem.

V. Conclusion

By probabilistic shaping and metrics based on distortion and energy consumption of power amplifiers, we have proposed a framework for optimizations of waveforms for energy-efficient communication. Using the proposed technique, we have optimized the probabilities and gain for a simple case, and it was shown that the technique can outperform regular uniform QAM systems in terms of mutual information at a specific consumed energy. As a future work, we expect that by extending the study to larger sizes, higher performance gains are within reach.

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