Modelling traffic scenarios via Markovian Opinion Dynamics

Citation for the original published paper (version of record):
Gaetan, E., Giarré, L., Sacone, S. et al (2023). Modelling traffic scenarios via Markovian Opinion Dynamics. IEEE Conference on Intelligent Transportation Systems, Proceedings, ITSC: 1072-1077.
http://dx.doi.org/10.1109/ITSC57777.2023.10422274

(article starts on next page)

Modelling traffic scenarios via Markovian Opinion Dynamics*

Elisa Gaetan[1], Laura Giarré[1], Simona Sacone[2], Paolo Falcone[1,3], Carl-Johan Heiker[3]

Abstract— **We address the question of whether opinion dynamics models can be exploited in novel scenarios, such as traffic flow on highway lanes. In this paper, we design a Markovian model and compare its predictions with those obtained from the widely recognized Cell Transmission Model (CTM) for the same traffic scenario. We identify potential challenges that may arise and propose strategies to address them. Furthermore, we present a concise demonstration showcasing the predictive capabilities of our proposed model through a small-scale example.**

I. Introduction

Road transportation is one of the main actors in the world-wide economy, while also being one of the major sources of environmental pollution [1]. The massive and still increasing use of road transportation induces huge annual costs due to negative externalities (environmental pollution, noise, and safety issues), besides the social impact of road congestion. In Europe, this impact has been estimated at about 100 billion euros only in 2020 (equivalent to about 1% of the European Union's gross domestic product) [2]. A smarter and more efficient use of road infrastructures still is a fundamental objective, necessary to serve an always increasing mobility demand in a sustainable way. Besides a suitable design of new infrastructures, a crucial role in improving road networks is played by effective traffic management strategies, including traffic control methods.

Traffic control strategies have been studied for several decades and a wide scientific literature is available on the topic (see [3] and [4]). Among the existing traffic control strategies, *mainstream control* refers to the direct regulation of vehicular flows traveling on the mainstream. More specifically, mainstream control actions include variable speed limitations and/or lane changing indications (as in [5], [6]), displayed to users through variable message signs placed at significant locations on the road or by taking advantage of intelligent vehicles. The design of mainstream control schemes has received considerable attention but applications are still limited. This is also due to the fact that the actuation of these policies takes place through variable message panels which can merely provide "collective" recommendations to all drivers. Moreover, the speed or lane advice shown on variable message signs can only be displayed at the panel locations. Indeed, advice to individual vehicle categories would certainly be more effective and, as regards the vehicle positioning in the different lanes, it is even more necessary that these indications are individual and are possible at every point of the network and not only where panels are located. Mainstream control actions tailored to individual vehicles or small clusters of vehicles would be instead a very promising approach that is becoming available thanks to the development of Connected and Automated Vehicles (CAVs) having vehicle-to-vehicle (V2V) vehicle-to-infrastructure (V2I) communication capabilities. These vehicles become an enabling factor also for actuating mainstream control actions to be implemented "continuously" along the road.

An important contribution to the effectiveness of traffic control schemes is the possibility of including predictive capabilities either obtained with model-based approaches or data-driven techniques. The modeling of mixed-traffic (including CAVs) or traffic only consisting of CAVs is a topic that has not yet been fully explored and understood from a scientific point of view. A vast set of models have been studied in the last decades to describe the traffic dynamics on high-density arterials and some of these models have proven their effectiveness in predictive control schemes.

However, the presence of connected vehicles, their dynamics, and their interactions with other vehicles are aspects that have not yet found reliable modeling or adequate identification techniques. The complexity arising from the presence of vehicles with very different features and the interactions between vehicles also makes the use of traditional modeling techniques complex. In this respect, an important contribution may come from the adoption of learning-based approaches that, together with the availability of large traffic data sets, could support the development and the deployment of powerful prediction tools specifically oriented to control [7].

The idea at the basis of this work is the development of a modeling framework based on opinion dynamics oriented to traffic control that aims to overcome the limits of model-based methods and those related to the application of purely data-driven methodologies. In a mixed traffic environment, with CAVs and human-driven Vehicles, it is possible to control the traffic flows through the CAVs moves. For this reason, we may see such "moves" as choices/decisions and model them by exploiting models used in opinion dynamics. Specifically, in this work, we design an opinion dynamics

*This work is partially supported by the EU Next-Generation (PNRR) within the Italian National Ph.D. Program in Autonomous Systems (DAuSy), the Mobility National Center, SPOKE 6 and by the Swedish Innovation Agency (VINNOVA) under the grant 2018-05005,

[1]Elisa Gaetan, Laura Giarré and Paolo Falcone are with the Engineering Department 'Enzo Ferrari', University of Modena and Reggio Emilia, Modena, Italy. {elisa.gaetan, laura.giarre, paolo.falcone}@unimore.it

[2]Simona Sacone is with the Department of Informatics, Bioengineering, Robotics and Systems Engineering 'DIBRIS', University of Genova, Italy. simona.sacone@unige.it

[3]Paolo Falcone and Carl-Johan Heiker are with the Mechatronics group, Department of Electrical Engineering, Chalmers University of Technology, Gothenburg, Sweden. heikerc@chalmers.se

framework for traffic scenarios. The performance of the model is tested on an illustrative small-scale case study demonstrating the consistency of our framework's predictions with an established traffic model. To the best of our knowledge, just a few papers have tried to find a connection between models of opinion dynamics and traffic control models and predictions. A previous attempt to use opinion dynamics models for modeling traffic intersections in urban environments has been reported in [8].

This paper is organized as follows. Section II describes the Cell Transmission Model (CTM) framework considering a one-way road, while Section III first introduces a generic Discrete Time Markov Chain (DTMC) model and then adapts it to the traffic scenario. Section IV explains how to use the data to fit DTMC, which results are shown in Section V, for different scenarios. Lastly, conclusions are drawn in Section VI.

II. Highway Traffic model

In this section, we recall the Cell Transmission Model (CTM), a widely used modeling framework that describes highway traffic [9]-[10], which our model will be compared against. Additionally, we discuss how the vehicle arrivals into the model can be described in this setting.

A. Cell Transmission Model

The CTM builds upon the decomposition of the considered road stretch into a series of short sections, referred to as "cells" and denoted as $\mathcal{C} = c_1, ..., c_L$. The CTM is based on the following set of two equations:

$$n_c(t+\Delta t) = n_c(t) + q_c(t)\Delta t - q_{c+1}(t)\Delta t, \tag{1a}$$

$$q_c(t) = \min\left\{\frac{n_{c-1}(t)}{\Delta t}, Q_c(t), \frac{N_c(t)-n_c(t)}{\Delta t}\right\} \tag{1b}$$

where, $n_c(t)$ represents the number of vehicles in cell c, $q_c(t)$ is the inflow to cell c and $q_{c+1}(t)$ is the outflow from the same cell, and Δk represents the time step size. Additionally, $N_c(t)$ represents the maximum number of vehicles that cell c can contain, and $Q_c(t)$ denotes the capacity flow into cell c, which corresponds to the maximum inflow of vehicles allowed in that cell. The term $N_c(t) - n_c(t)$ represents the available space within cell c. Moreover, the parameters $N_c(t)$ and $Q_c(t)$ can also be constant over time.
Regardless of the number of cells, a CTM always includes *source*, *gate*, and *sink* cells. Specific properties hold for each cell type, recalled next. The source cell is assumed to contain a potentially infinite initial number of vehicles (i.e., $n_{\text{source}}(0) = \infty$), which enter the gate cell, thus resulting in an infinitely large cell ($N_{\text{gate}}(t) = \infty$). After traveling through the series of cells, the vehicles eventually arrive at the sink cell, which has an infinite capacity ($N_{\text{sink}}(t) = \infty$) and can accommodate all entering vehicles. The source-gate and the sink cells are the input and output cells respectively, which in turn are the unknown road network structure outside the scenario's boundaries, namely the segments of road before and after the modeled road stretch, that we are not interested in investigating.

B. Modelling the input to the traffic scenario

In the previous subsection, we mentioned the infinite capacity of the gate cell in the CTM. However, the literature does not clearly explain the transition of vehicles from the gate to the first cell of the road. That is, the function $n_{\text{gate}}(t)$, which influences the inflow $q_1(t)$ of vehicles into the first cell:

$$q_1(t) = \min\left\{\frac{n_{gate}(t)}{\Delta t}, Q_1(t), \frac{N_1(t) - n_1(t)}{\Delta t}\right\}, \tag{2}$$

is not explicitly defined. Nevertheless, in traffic scenario modeling, the process of arrivals in the gate cell is often assumed to follow a Poisson Process [11]. This approach is inspired by queuing theory from the telecommunications field [12], where the one-way road can be seen as a queue. The Poisson Process assumes infinite capacity, with the arrivals inflow as a stochastic variable. The probability mass function of a Poisson distributed variable n_{gate}, defined as

$$\mathbb{P}\Big(n_{gate}(t) = a\Big) = \frac{(\lambda t)^a}{a!}e^{-\lambda t},$$

describes the probability of having a arrivals at time t, where λt is the average number of arrivals. Therefore, the outflow from the gate cell to the first cell is a random variable.

III. An opinion dynamics Markovian model for traffic scenarios

In this section, we present our opinion dynamics framework for modeling highway line traffic, focusing on capturing and predicting the dynamics of traffic. We start by introducing the general formulation of a Markovian model in its discrete-time form. Next, we adapt and customize it to fit the characteristics of the considered scenario, resulting in our specific opinion dynamics model.

A. Discrete-Time Markovian Model

The decision process of a single agent r can be described as a Discrete-Time Markov Chain (DTMC) over a set $\mathcal{S} = \{s_1, ..., s_M\}$ of M decision states. State transition probabilities $\Pr(s^r(t+1) = s_j | s^r(t) = s_i) = p^r_{ij}(t)$ at the discrete time step $t \in \mathbb{Z}$ are defined in the non-negative, row-stochastic matrix $P^r(t) \in \mathbb{R}^{M\times M}$, such that $P^r_{i,j}(t) = p^r_{ij}(t)$ for all $i, j \in \{1, \dots, M\}$. The probability distribution $\Pi^r(t) = [\pi^r_1(t)...\pi^r_M(t)]^T$ over $\mathcal{S}$ describes the decision probabilities of r, and is the solution of

$$\Pi^r(t+1) = (P^r(t))^T\Pi^r(t) \tag{3}$$

from some initial decision probability $\Pi^r(0)$. The DTMC is said to be *time-homogeneous* if the transition matrix is independent of t, such that $P^r(t) = P^r\ \forall t$. We define a network $\mathcal{X}$ of Z discrete-time Markovian agents, with more than one agent undergoing a state transition at step t. Similar to [13], we denote a network state as $X = \langle s^1, \dots, s^Z\rangle$, and the state set of the entire network as $\mathcal{S}_\mathcal{X} = \mathcal{S}^1 \times \cdots \times \mathcal{S}^Z$. As $\mathcal{S}^r$ represents the decisions in $\mathcal{S}$ for each r, the network has M^Z states. The transition matrix of $\mathcal{X}$ is

$$P_0(t) = P^1(t) \otimes \cdots \otimes P^r(t) \otimes \cdots \otimes P^Z(t), \tag{4}$$

where $\otimes$ is the Kronecker product. The decision probabilities of the entire network are thus the solution of

$$\Pi_{\mathcal{X}}(t+1) = (P_0(t))^T \Pi_{\mathcal{X}}(t), \tag{5}$$

given the initial network decision probability distribution $\Pi_{\mathcal{X}}(0)$.

B. Fitting the Markovian model for traffic scenarios

In this section, we adapt the generic formulation of a Discrete-Time Markov Chain (DTMC) from the previous section to a one-way road traffic scenario. In particular, we are interested in describing the same scenario as the CTM, namely a road stretch divided into cells crossed by vehicles. We start by defining the sets of agents $\mathcal{R}$ and states $\mathcal{S}$ for the network of Markovian agents. Considering each vehicle as an individual agent may seem intuitive. However, due to the property of Markov Chains where dynamics can potentially be observed for an infinite time, defining vehicles as agents would restrict observations to a finite time interval, undermining this property. To preserve such characteristic of the Markov Chains, we propose considering the cells in the CTM as agents of the DTMC:

$$\mathcal{C} \equiv \mathcal{R} \iff \{c_1, ..., c_L\} \equiv \{r_1, ..., r_Z\}.$$

Henceforth, we replace the index r for a generic Markovian agent with c. Next, we discuss the possible states that a single agent can assume. We define three distinct states:

- $s_1 =$ *"The number of vehicles increases"*;
- $s_2 =$ *"The number of vehicles decreases"*;
- $s_3 =$ *"The number of vehicles remains constant"*.

Such states describe *modes* that can be useful also when dealing with lane change or lane merging decisions. Thus, for our traffic application, the set of decision states is $\mathcal{S} = \{s_1, s_2, s_3\}$. A relationship between such decision states and the states of the CTM can be easily established by rewriting (1a) as

$$n_c(t+\Delta t) - n_c(t) = q_c(t)\Delta t - q_{c+1}(t)\Delta t, \tag{6}$$

where the lhs represents the variation of vehicles in cell c over Δt. Hence, the decision states in $\mathcal{S}$ correspond to the sign of such variation as follows

- $n_c(t+\Delta t) - n_c(t) > 0 \iff s_1^c(t)$,
- $n_c(t+\Delta t) - n_c(t) < 0 \iff s_2^c(t)$ and
- $n_c(t+\Delta t) - n_c(t) = 0 \iff s_3^c(t)$.

Without loss of generality, we choose a unitary step size $\Delta t = 1$. Thus, the above discrete derivative of the number of vehicles in cell r corresponds to the state s_i. Furthermore, it is important to note that the fundamental property of a Markov chain, known as the *memoryless property* [14], holds for the proposed model. This property states that given the current state s_i, the next state depends only on s_i and not on the past history. Indeed, according to (6), the next state can be also expressed as $s_i^c(t) = q_c(t)\Delta t - q_{c+1}(t)\Delta t$. In turn, the inflow $q_c(t)$ and outflow $q_{c+1}(t)$ of the cell c are evaluated following (1b). Thus, the number of vehicles $n_{c-1}(t)$ and $n_c(t)$ depend on the variation expressed by the previous state $s_i^c(t-1)$ and not rely on the past history. Therefore, our model respects memoryless property.

IV. Learning the transition matrix of DTMC

Next, we address the problem of inferring from data the transition matrices $P^c(t)$ for each agent c. This task involves finding the Maximum Likelihood Estimator (MLE) $\hat{P}^c(t)$ [15]. In the literature, the frequentist and Bayesian approaches are commonly considered for this purpose [16]. In our case, since we lack any prior information about the probability distribution of the transition matrix, we adopt the frequentist approach. As described in [17], the MLE $\hat{p}_{ij}(t) = [\hat{P}^c(t)]_{(i,j)}$ for the transition probability $p_{ij}(t) = [P^c(t)]_{(i,j)}$ in a non-homogeneous system is given by:

$$\hat{p}_{ij}(t) = \frac{V_{ij}(t,t+1)}{V_{i+}(t,t+1)}, \tag{7}$$

where $V_{ij}(t,t+1)$ represents the number of transitions from state s_i to s_j in the time interval $[t,t+1]$, and $V_{i+}(t,t+1)$ is the number of chains that experience a transition starting from state s_i in the same time interval.
The result in (7) can be adapted to a homogeneous system, as explained in [18]-[19]. In this case, the MLE for p_{ij} becomes:

$$\hat{p}_{ij} = \frac{v_{ij}}{v_{i+}}, \tag{8}$$

Here, v_{ij} is the number of observations from state s_i to s_j, and $v_{i+} = \sum_{j=1}^{M} v_{ij}$ represents the total number of observations starting from state s_i. Although it is of practical relevance to building either the (7) or the (8) upon experimental data, in this paper we use state trajectories from the CTM. Using the state trajectories of the CTM instead of experimental data is meant to gain an understanding on the DTMC capability of reproducing the behavior emerging from a well-known and widely accepted model like the CTM.
While the MLE in the homogeneous case can be easily estimated by counting the occurrences of state transitions in a single state trajectory of CTM, the estimator for the non-homogeneous case requires multiple trajectories to gather information about the time variability of the state transition rates.

A. Issue in the transition matrix learning process and its possible solution

The issue of learning non-row-stochastic matrices $P^c(t)$ is a potential challenge in both homogeneous and non-homogeneous systems. This can arise because the predictions from the deterministic CTM model may not cover all possible network configurations at each time instant, leading to missing transitions in the inferred matrices. In the CTM framework, while the state of the first cell can vary from the initial instants due to the stochastic input $n_{\text{gate}}(t)$, the states of the remaining $Z-1$ cells are deterministic and remain constant over the initial executions. Consequently, it is not guaranteed that all possible network configurations are reached at least once in the CTM executions, resulting in zero elements in certain rows of the matrix $P_0(t)$. Inspired by [20], where the authors developed a stochastic version of CTM, we add a stochastic term, namely a noise in (1) which becomes

$$n_c(t+\Delta t) = n_c(t) + q_c(t)\Delta t - q_{c+1}(t)\Delta t + \eta(t), \tag{9a}$$

$$q_c(t) = \min\left\{\frac{n_{c-1}(t)}{\Delta t}, Q_c(t), \frac{N_c(t)-n_c(t)}{\Delta t}\right\}. \tag{9b}$$

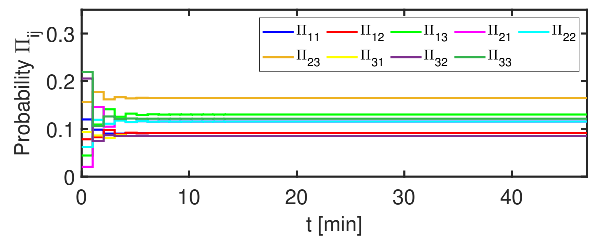
The noise describes additional vehicles entering the system and is constrained to be an integer to preserve the feasibility of the model. To ensure that the vehicle count remains non-negative ($n_c(t+\Delta t) \geq 0$), the noise term $\eta(t)$ must be non-negative as well. The addition of a disturbance term can help in resolving the issue, but it may not be sufficient on its own. Based on our simulations, we observed that a few time instants must be discarded to accurately infer the transition matrices. This means that we may need to exclude certain initial time steps to ensure the row-stochasticity of the inferred matrices. This initial transient phase should be discarded and corresponds to the *burn-in* (or *warm-up*) phase of Markov chains ([21]). Moreover, owing to the memoryless property, this does not negatively affect the Markov chain. In conclusion, incorporating a disturbance term in the CTM equations can partially address the issue of non-row-stochastic matrices. However, further considerations and adjustments, such as discarding initial time steps, may be necessary.

V. Results

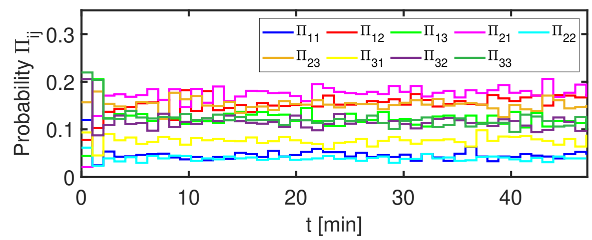
In this section, we compare the CTM and DTMC models and highlight the obtained results. To begin, we claim that the non-homogeneous DTMC model is to be preferred over the homogeneous. Such claim builds on the superior accuracy of the non-homogeneous model in capturing the dynamics of the CTM, as shown next by the simulation results of a small-scale example.

A. Comparing homogeneous and non-homogeneous DTMCs

In this example, we consider a CTM with 2 cells to derive the transition matrices of DTMCs with $Z = 2$ and $M = 3$. Fig.1a illustrates the probability vector evolution $\Pi_{\mathcal{X}}(t)$ of the homogeneous DTMC. Due to the reduced dimension of the Markov chain, we assume that all the network configurations are visited at least once during the simulation, and therefore the corresponding Markov chain is ergodic ([14]). Consequently, $\Pi_{\mathcal{X}}(t)$ converges to a unique, stationary state $\bar{\Pi}_{\mathcal{X}}$. This plot is compared with Fig. 1b, which shows the probability evolution of a non-homogeneous DTMC in order to establish which of the two models must be preferred. If we assume to consider the one with the highest probability as the occurred configuration, then we verify that while the homogeneous DTMC can only predict which network configuration is most likely to occur in a single state trajectory of CTM, the non-homogeneous DTMC can predict the configurations at each instant t and the transitions between them. Thus, a non-homogeneous DTMC should be employed, as far as predicting the configurations' evolution over time is concerned.



(a) Homogeneous system.



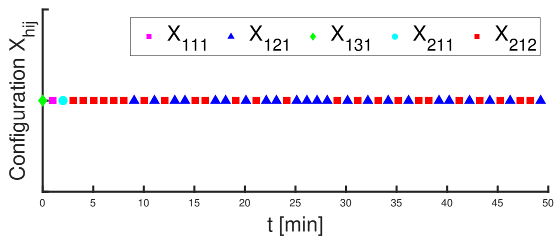
(b) Non-homogeneous system.

Fig. 1: Comparison between probabilities evolution of the homogeneous and non-homogeneous DTMCs with $Z = 2$ and $M = 3$. If we consider the configuration with the highest probability at each instant t as the actual configuration, we can observe that the homogeneous DTMC can only predict which of the M^Z configurations is most probable to occur in a single execution of CTM. In contrast, the non-homogeneous DTMC can not only predict the most probable configurations at each instant t, but it also allows us to foresee the transitions between configurations.

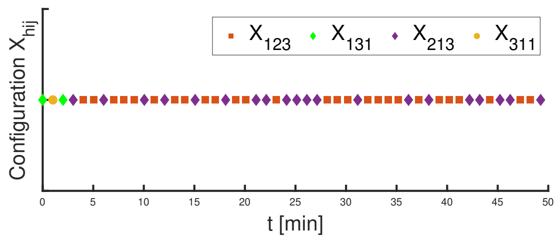
However, choosing a non-homogeneous DTMC introduces a partial limitation. Multiple state trajectories of the CTM are necessary to infer the transition matrices of a single non-homogeneous DTMC. Consequently, the configurations predicted by a non-homogeneous DTMC should be compared against multiple CTM's state trajectories. For future comparisons, we decide to evaluate the DTMC results against the *modal* configurations of CTMs. However, it is worth noting that being able to predict the average behavior of one-way roads is still of considerable interest. For instance, by integrating the DTMC framework into a predictive control algorithm, one could leverage the model's capabilities to anticipate the average traffic behavior of a specific road stretch. This predictive capability allows us to gain valuable insights into future traffic conditions and plan control actions accordingly. More generally, the DTMC framework enables a macroscopic description of the traffic scenario, focusing on the collective behavior of traffic flow rather than individual vehicles. This macroscopic perspective allows us to capture the overall trends and patterns in traffic dynamics, such as variations in cell density, flow rates, and congestion levels.

B. Exploiting a small-scale network example to assess the agreement between CTM and DTMC predictions

We aim to investigate the consistency between CTM and DTMC in terms of their predictions for various traffic scenarios. Furthermore, we emphasize the impact of parameter

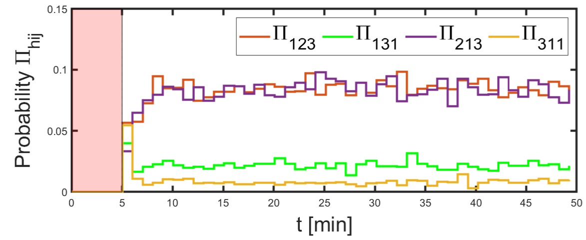


(a) Low arrivals rate λ_{low}.

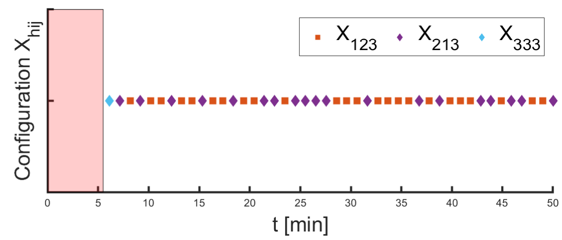


(b) High arrivals rate λ_{high}.

Fig. 2: Comparing CTM modal configurations under low and high arrivals rate of vehicles. In both figures, the distinct symbols (varying in shape and color) represent the different configurations denoted as X_{hij} that CTM can exhibit over time.



(a) Probability evolution of DTMC for λ_{high}. To enhance clarity, we present a graphical representation of the most probable states, which have probability Π_{123} and Π_{213}, along with the probabilities of the additional two configurations observed in Fig. 2b, namely Π_{131} and Π_{311}.



(b) DTMC configurations for λ_{high}. This graph is derived from the probability evolution plot shown in Fig. 3a. Specifically, the configuration denoted as X_{hij} corresponds to the highest probability Π_{hij} observed.

Fig. 3: Evolution of probability estimated configuration chain of DTMC with λ_{high}. The highlighted red zone indicates the inability to make predictions due to non-row-stochastic inferred transition matrices.

values on the resulting configurations. The evolution of the vehicle count, as described by (1), primarily depends on parameters $Q_c(t)$ and $N_c(t)$. Additionally, as elaborated in Section II-B, the arrival rate λ also influences the system. By varying the values of $(Q_c(t), N_c(t), \lambda)$, we can design diverse traffic scenarios. We assume that the parameters are time-invariant, assuming a consistent road structure over time. Our analysis is limited to two types of one-way roads:

1) Uniform roads, where $N_c = N$ and $Q_c = Q$ for all $c \in \mathcal{C}$ (see Case 1).
2) Roads with a narrowing in the last cell, where $N_1 = \ldots = N_c = \ldots = N_{c_{L-1}} > N_{c_L}$ (see Case 2).

To conduct the tests, we employ a small-scale network with $Z = 3$ agents and $M = 3$ states. Consequently, the DTMC has a total of $M^Z = 27$ possible configurations. Utilizing (9), we derive the data needed to compute the state trajectories of CTM using the procedure outlined in Section III-B. This allows us to infer the transition matrices $P^c(t)$ for each agent $c \in \mathcal{C}$. We then construct the transition matrices for the entire network as described in (4) and simulate the network with random initial conditions. Finally, we compare the results obtained from the DTMC with the modal predictions of the CTM. Furthermore, for Case 1, we examine the effect of λ, while for Case 2, we consider both Q_c and λ as variables of interest.

Case 1: Assuming $N_c = N = 50\,\text{veh}$ and $Q_c = Q = 25\,\text{veh/min}$, with an outflow from the network of $Q_{out} = Q$. Additionally, considering the potential issues outlined in Section IV-A, we set $\eta(t) = \mathcal{U}(0\,\text{veh}, 2\,\text{veh})$. To investigate the impact of λ, we conduct two tests:

- A road with a low arrival rate $\lambda_{low} < Q_{out}$,
- A road with a high arrival rate $\lambda_{high} \geq Q_{out}$.

Without loss of generality, we choose $\lambda_{low} = 5\,\text{veh/min}$ (Fig. 2a) and $\lambda_{high} = 50\,\text{veh/min}$ (Fig. 2b). The modal plots demonstrate that a low arrival rate causes the system to alternate between configurations X_{121} and X_{212}, while a high arrival rate induces the system to switch between X_{123} and X_{213}. Consequently, we conclude that the arrival rate influences the number of vehicles in the last cell, specifically in the case of uniform one-way roads. Notably, a high arrival rate results in a consistent density of vehicles in the last cell. By utilizing the data obtained from the CTMs, we derive the transition matrices and derive the probability evolution of the DTMC. As an example, we present and discuss the simulations for λ_{high} (Fig. 3). In Fig. 3a, we plot the evolution of $\Pi_{\mathcal{X}}$ according to (5). To enhance clarity owing to the high dimension of the state vector ($\Pi_{\mathcal{X}} \in \mathbb{R}^{27\times 1}$), we present just some of the state probabilities Π_{hij}. Thus, we deduce the corresponding configurations (Fig. 3b), by stating that the configuration $X_{hij}(t)$ that occurs must have the highest probability

$$\max_{\substack{\Pi_{hij}(t)\in\Pi(t),\\ h\in\mathcal{S}^1,\ i\in\mathcal{S}^2,\ j\in\mathcal{S}^3}} \Pi_{hij}(t) = X_{hij}(t). \tag{10}$$

It should be noted that the configurations at the initial moments cannot be reliably inferred due to non-row-stochastic transition matrices. These instances are highlighted by the red zone in Fig. 3. Finally, we compare the predictions of the CTM and DTMC. Except for a brief transient period in the beginning, the predictions of the two models align. Consequently, we can assert that our framework accurately predicts the average behavior of a uniform one-way road.

Case 2: Considering $N_1 = N_2 = 50\,\text{veh}$, $N_3 = 30\,\text{veh}$, and $Q_1 = Q_2 = 25\,\text{veh/min}$, we continue to employ $\eta(t) = \mathcal{U}(0\,\text{veh}, 2\,\text{veh})$ as in Case 1. In this case, we conduct tests for four different scenarios by varying the parameters $Q_3 = \{5, 25\}\text{veh/min}$ and $Q_{out} = \{5, 25\}\text{veh/min}$, while keeping $\lambda_{low} = 5\,\text{veh/min}$ and $\lambda_{high} = 50\,\text{veh/min}$ fixed. Physically, the values of the maximal inflow Q_3 and outflow Q_{out} can be interpreted as the width of the entrance and exit of the last cell, respectively. Therefore, altering their values corresponds to either widening or narrowing the access and exit points. The results are summarized in Tab. I, which presents the configurations X_{hij} assumed by the CTM for each combination of parameters (Q_3, Q_{out}, λ). Unlike Case 1, we now observe that the network is primarily influenced by Q_3 and Q_{out} rather than λ. The configurations assumed by the system remain consistent for both λ_{low} and λ_{high}. Consequently, our analysis focuses on the effects of Q_3 and Q_{out}.

In the case of parameter combination *A*, where both the inflow and outflow of the last cell are minimal and only a few vehicles enter at each time interval (maximum 5 units), it is reasonable to expect a constant density in the last cell ($s^3(t) = s_3$ for all t).

Increasing the outflow (combination *B*) results in a constant density in all cells. This is a direct consequence of $Q_{out} > Q_3$, as only a limited number of vehicles can be transferred from the second to the third cell. In essence, the small value of Q_3 acts as a restriction on traffic flow toward the last cell, consequently affecting the densities of the preceding cells.

On the other hand, when the inflow is increased, a constant density is observed in the third cell, as seen in combination *C*. In this scenario, the limited capacity of the outflow acts as a constraint on vehicles leaving the network. As a result, only a few vehicles are able to exit the last cell, leading to a tendency for the density to remain constant.

Finally, combination *D* yields the same results as in Case 1. The inflow and outflow values are insufficient to fill the last cell ($Q_3 = Q_{out} < N_3$), and thus no traffic congestion is observed.

Comparisons between the modal CTM predictions and the DTMC predictions have been conducted for all the presented combinations, considering both low and high arrival rates. Satisfactory agreement between the two frameworks has been obtained.

TABLE I: Results from the conducted tests for Case 2.

Network Parameters	**Network Configurations**	
(Q_3, Q_{out}, λ)	$\lambda_{low} = 5\,\text{veh/min}$	$\lambda_{high} = 50\,\text{veh/min}$
A. $(5, 5)$ veh/min	X_{123}, X_{213}	X_{123}, X_{213}
B. $(5, 25)$ veh/min	X_{333}	X_{333}
C. $(25, 5)$ veh/min	X_{123}, X_{213}	X_{123}, X_{213}
D. $(25, 25)$ veh/min	X_{121}, X_{212}	X_{121}, X_{212}

VI. Conclusion

In this paper, we have introduced an opinion dynamics Markovian framework for traffic control. Our framework models and analyzes traffic dynamics in a one-way road scenario. To capture the variations of density in different cells along the road, we have employed a Discrete-Time Markov Chain as the underlying mathematical model. By utilizing DTMC, we are able to represent the system dynamics as a sequence of discrete configurations and transitions between these configurations. Each cell in the one-way road is treated as an individual agent in the DTMC, and the density of vehicles in each cell is considered as a key parameter of interest. Although our study has shown a satisfactory level of agreement between the predictions of the CTM and DTMC, it is important to acknowledge that the DTMC model has been tested in a relatively simple scenario. To further validate and assess the robustness of our DTMC framework, it is essential to test it in a variety of real-world and more demanding traffic scenarios. In future work, we intend to expand the application scenarios of our model and explore various highway layouts, including situations such as lane merging and lane changing. Tests using real data are also ongoing. Additionally, we believe that our framework could be integrated into a control scheme as a predictive tool in place of other widely recognized traffic models.

References

[1] "Transport: Increasing oil consumption and greenhouse gas emissions hamper eu progress towards environment and climate objective," European Environment Agency, Tech. Rep., 2020.

[2] R. Filippova and N. Buchoud, "A handbook on sustainable urban mobility and spatial planning," *Promoting Active Mobility*, vol. No. ECE/TRANS/298, 2020.

[3] S. Siri, C. Pasquale, S. Sacone, and A. Ferrara, "Freeway traffic control: a survey," *Automatica*, vol. 130, 2021.

[4] C. Pasquale, S. Sacone, S. Siri, and A. Ferrara, "Traffic control for freeway networks with sustainability-related objectives: Review and future challenges," *Annual Reviews in Control*, vol. 48, 2019.

[5] R. Carlson, I. Papamichail, and M. Papageorgiou, "Local feedback-based mainstream traffic flow control on motorways using variable speed limits," *IEEE Transactions on Intelligent Transportation Systems*, vol. 12, 2011.

[6] Y. Zhang and P. Ioannou, "Combined variable speed limit and lane change control for highway traffic," *IEEE Transactions on Intelligent Transportation Systems*, vol. 18, 2016.

[7] H. Yu, S. Park, A. Bayen, and M. Krstic, "Reinforcement learning versus pde backstepping and pi control for congested freeway traffic," *IEEE Transactions on Control Systems Technology*, vol. 30, 2021.

[8] C.-J. Heiker and P. Falcone, "Decision modeling in markovian multi-agent systems," in *2022 IEEE 61st Conference on Decision and Control (CDC)*, 2022, pp. 7235–7240.

[9] C. Daganzo, "The cell transmission model: A dynamic representation of highway traffic consistent with the hydrodynamic theory," *Transportation Research Part B: Methodological*, vol. 28, no. 4, pp. 269–287, 1994.

[10] C. F. Daganzo, "The cell transmission model, part ii: Network traffic," *Transportation Research Part B: Methodological*, vol. 29, no. 2, pp. 79–93, 1995.

[11] B. Chandrasekaran, "Survey of network traffic models," *Washington University in St. Louis CSE*, vol. 567, 2009.

[12] R. B. Cooper, *Introduction to queueing theory [by] Robert B. Cooper*. Macmillan New York, 1972.

[13] P. Bolzern, P. Colaneri, and G. De Nicolao, "Opinion influence and evolution in social networks: A markovian agents model," *Automatica*, vol. 100, pp. 219–230, 2019.

[14] C. Cassandras and S. Lafortune, *Introduction to Discrete Event Systems*, 01 2010, p. 800.

[15] P. Singer, D. Helic, B. Taraghi, and M. Strohmaier, "Detecting memory and structure in human navigation patterns using markov chain models of varying order," *PLOS ONE*, vol. 9, no. 7, pp. 1–21, 07 2014.

[16] D. R. Cox, *Principles of Statistical Inference*. Cambridge University Press, 2006.

[17] T. R. Fleming and D. P. Harrington, "Estimation for discrete time nonhomogeneous Markov chains," *Stochastic Processes and their Applications*, vol. 7, no. 2, pp. 131–139, June 1978.

[18] B. A. Craig and P. P. Sendi, "Estimation of the transition matrix of a discrete-time markov chain." *Health economics*, vol. 11 1, pp. 33–42, 2002.

[19] T. W. Anderson and L. A. Goodman, "Statistical inference about markov chains," *The Annals of Mathematical Statistics*, vol. 28, no. 1, pp. 89–110, 1957.

[20] R. Boel and L. Mihaylova, "A compositional stochastic model for real time freeway traffic simulation," *Transportation Research Part B: Methodological*, vol. 40, no. 4, pp. 319–334, 2006.

[21] M. K. Cowles and B. P. Carlin, "Markov chain monte carlo convergence diagnostics: A comparative review," *Journal of the American Statistical Association*, vol. 91, no. 434, pp. 883–904, 1996.