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Squashed 7-spheres, octonions and the swampland

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Abstract. We give a brief account of how to derive the entire eigenvalue spectrum of the operators on the squashed S^7 that appear in the compactification of eleven-dimensional supergravity. These spectra determine the mass spectrum of the fields in AdS_4 and are important for the corresponding $\mathcal{N} = 1$ supermultiplet structure. By an orientation-flip on the squashed S^7 we can also determine the spectrum of the corresponding non-supersymmetric theory, and, e.g., its spectrum of marginal operators on the boundary of AdS_4 which has some relevance for the AdS stability conjecture in the swampland program. Here we review recent work in [1, 2, 3] which is a continuation of the work in [4] where the complete spectrum of irreducible isometry representations of the fields in AdS_4 was derived for this compactification. Details are here given primarily for 2-forms while comments are also made on the key role of G_2 and octonions for the structure of the operator equations and mode functions on the squashed S^7. Key features of these improved methods were obtained in Joel Karlsson's 2021 MSc thesis [5]. This is a write-up of a talk given at ISQS28, Prague, Czech Republic, July 4, 2024.

1 Introduction

Compactification of D=11 supergravity on the squashed S^7 dates back to the first half of the 1980s [6, 7] (see also [8]) but has regained some interest recently. This is partly due to the swampland program and in particular to one of the conjectures proposed in this context, namely the *AdS stability conjecture* [9]: Any non-supersymmetric compactification leading to an AdS spacetime will be unstable. Our universe is strongly believed, from observations, to be of de Sitter type for which another set of swampland conjectures indicate that such compactifications can never be stable, something that again may be due to the lack of supersymmetry in de Sitter space. For these reasons it is of utmost important to prove these conjectures which so far have met with huge challenges. A second best approach is to find examples supporting their correctness or to search for counterexamples showing that the conjectures fail to be true (at least in their current formulation).

In the case of the AdS conjecture mentioned above a huge number of non-supersymmetric examples exist that are BF (see below) stable but which ultimately have been demonstrated to be unstable due to various kinds of decay channels. We will not discuss these any further here but instead concentrate on the few cases that so far have not been proven unstable. Without claiming to be exhaustive we mentioned here two cases in this category:

1. S-folds [10] and
2. Right-squashed S^7 [7]

Note that without supersymmetry it seems very hard to prove stability directly but there are interesting attempts involving fake supersymmetry, see [11]. The rest of this talk is devoted to explaining recent results [4, 1, 3] concerning the second example, the right-squashed S^7 compactification of D=11 supergravity. Of course, it is not possible to claim stability just by testing known decay modes since there may be new ones found in the future. This is why supersymmetry, or possibly fake supersymmetry, is so utterly important for proving stability.

When compactifying D=11 supergravity on $AdS_4 \times S^7$ there are three cases to consider, the round case with eight supersymmetries, the left-squashed with one and the right-squashed case with no supersymmetry. The two squashed cases are the skew-whiffed (orientation flipped) versions of each other. Being supersymmetric the left-squashed case is absolutely stable from which one can show that also the right-squashed case is BF stable [12] since all unitarity bounds are respected also in the non-supersymmetric case [13].

Going beyond BF stability one may ask if introducing interactions may ruin stability in the non-supersymmetric case. There are many aspects of this issue. One such is addressed in [14, 15], the up-shot of which is that marginal operators in the 1/N expansion of beta-functions in the boundary field theory are dangerous objects that might lead to the removal of fix-points and hence instabilities in the bulk theory. It should be mentioned that the relevance of these claims are put in question by some authors, see, e.g., the Introduction in [3] and work cited there. Here we will only discuss the presence or not of marginal operators.

The author is very grateful to M.J. Duff, C.N. Pope, A. Padellaro, S. Ekhammar, and J. Karlsson for the many collaborations that have led to the results presented here.

2 D=11 supergravity and its compactification on S^7

To find the background solutions we need the bosonic part of D=11 supergravity:

$$\mathcal{L} = \frac{1}{\kappa^2}\big(R - \frac{1}{12}F_{MNPQ}F^{MNPQ} + \frac{8}{12^4}\epsilon_{M_1\ldots M_{11}}A^{M_1M_2M_3}F^{M_4\ldots M_7}F^{M_8\ldots M_{11}}\big). \tag{1}$$

The ansatz for the D=11 background is given in terms of an 11= 4+7 split $X^M = (x^\mu,\, y^m)$, a diagonal product metric and a spacetime volume form for the non-zero components of the 4-form field strength:

$$G_{MN} = \mathrm{diag}(g_{\mu\nu}(x), g_{mn}(y)),\;\; F_{\mu\nu\rho\sigma}(x) = 3m\epsilon_{\mu\nu\rho\sigma}(x), \tag{2}$$

where m is a positive constant parameter of dimension $1/L$.

Inserting this ansatz into the field equations one finds the background conditions

$$\bar{R}_{\mu\nu} = -12m^2\bar{g}_{\mu\nu},\;\; \bar{R}_{mn}(y) = 6m^2\bar{g}_{mn}. \tag{3}$$

Thus the background is AdS_4 times a seven-dimensional compact manifold K^7, examples of which are plentiful, see ,e.g., [8]. As mentioned above we will here only discuss $K^7 = S^7$. However, as explained in full detail in, e.g., [8] there exist one round and one squashed Einstein metric on this manifold. While the orientation in the round case is irrelevant for the theory in AdS_4 this is not the case for the squashed metric as will be explained below.

One aim of this presentation is to summarize the recent results on the squashed S^7 spectrum analysis, in particular the one for 2-forms, which finally has lead to an almost complete understanding of all the fine details. We start below by a short review of the irreps content relevant in AdS_4, i.e., the unitary irrepses of its isometry group $SO(2,3)$.

3 SO(2,3) irrep diagrams and singletons

The unitary irreps of $SO(2,3)$, which is the isometry group of AdS_4 and also the conformal group on the boundary of AdS_4, are denoted $D(E_0, s)$ where E_0 is the energy of the lowest state (see the state diagram below copied from Nicolai [16]) and s its spin. In the case of a massless (pseudo)scalar ($s = 0$) field E_0 is equal to 1 or 2 depending on the boundary conditions chosen for the field. Note that the diagram extends indefinitely upwards.

An interesting point for this presentation is the fact that the irreps sitting at the unitarity bound values, $D(E_0 = \frac{1}{2}, s = 0)$ and $D(E_0 = 1, s = \frac{1}{2})$ are known as singletons. These have the special property that they fluctuate (or "live") only on the boundary of the AdS_4 space, being in some sense topological as bulk fields. These irreps have state diagrams with states only along the main trajectory (the first lower right one in the diagram below). Arguments for why various kinds of singletons are needed in order to make sense of the different S^7 spectra and their relations are presented for the first time in [4], see also [2, 3].

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In some cases it is possible to follow the change in E_0, or rather the operator eigenvalues [17], see also [2, 3], when squashing which indicates that the singletons on the round S^7 must become ordinary bulk scalars and fermions in the squashed vacua [4]. For this to be possible it seems that one must add to the singleton state diagrams states corresponding to bulk scalar or fermion fields which, if true, would be a new kind of Higgs effect. We will return to this issue again below. Unfortunately, so far there is no Lagrangian realization of this in the literature as far as we know.

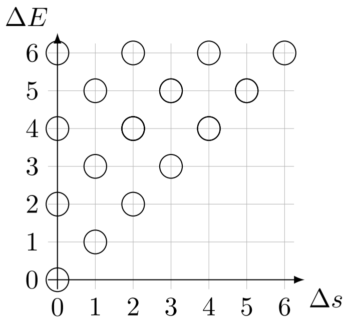

Figure 1: Example of a state diagram for unitary irreps of $SO(2,3)$ denoted $D(E_0, s)$: For massless (pseudo)scalars the states have energy and spin (E, j) with $E = E_0 + \Delta E$ where $E_0 = 1$ or 2 (depending on boundary conditions) and $j = s + \Delta s$ for $s = 0$. The scalar singleton has $E_0 = \frac{1}{2}$ and a state diagram consisting of only the main trajectory (the first lower right one above).

4 $D(E_0, s)$ and the relation of E_0 to the S^7 operator eigenvalues via the mass matrix

In a Lagrangian formulation of a field theory on AdS_4 there are relations between the E_0 of the irreps $D(E_0, s)$ and the masses of the fields. These relations are given in the table below where also the unitarity bound is given for each spin.

$s = 2$	$E_0 = \frac{3}{2} + \frac{1}{2}\sqrt{(M/m)^2 + 9} \geq 3$
$s = \frac{3}{2}$	$E_0 = \frac{3}{2} + \frac{1}{2}\lvert M/m - 2\rvert \geq \frac{5}{2}$
$s = 1$	$E_0 = \frac{3}{2} + \frac{1}{2}\sqrt{(M/m)^2 + 1} \geq 2$
$s = \frac{1}{2}$	$E_0 = \frac{3}{2} \pm \frac{1}{2}\lvert M/m\rvert \geq 1$
$s = 0$	$E_0 = \frac{3}{2} \pm \frac{1}{2}\sqrt{(M/m)^2 + 1} \geq \frac{1}{2}$

Table 1: E_0 for AdS_4 fields of given mass M and spin s (in $Spin(2,3)$-irreps $D(E_0, s)$) and the corresponding unitarity bounds, see, e.g., [8].

From the full $D = 11$ supergravity Lagrangian linearized around any Freund-Rubin solution $AdS_4 \times K^7$ one can derive relations between the mass matrices and operators on the compact internal manifold K^7. The idea is simply a generalization of the fact that in a flat spacetime a D=11 box defines a mass matrix M^2 in D=4:

$$\Box_{11} = \Box_4 + \Box_7 \equiv \Box_4 - M^2. \quad (4)$$

In the table below the graviton tower is given by masses $M^2 = \Delta_0 = -\Box$, acting on scalars on the compact manifold. From the table above we then get the corresponding values of E_0. A final step is thus to express the eigenvalues of the operators in the table below in terms Casimirs for the isometry group in question. This will be made clear below for the squashed S^7.

spin$^{\text{parity}}$	Mass operator M^2 or M depending on the spin
2^+	Δ_0
$\frac{3}{2}_{(\pm)}$	$-i\not{D}_{1/2} + \frac{7m}{2}$
$1^-_{(\pm)}$	$\Delta_1 + 12m^2 \pm 6m\sqrt{\Delta_1 + 4m^2} = (\sqrt{\Delta_1 + 4m^2} \pm 3m)^2 - m^2$
1^+	Δ_2
$\frac{1}{2}_{(\pm)}$	$-i\not{D}_{1/2} - \frac{9m}{2}$
$\frac{1}{2}_{(\pm)}$	$i\not{D}_{3/2} + \frac{3m}{2}$
$0^+_{(\pm)}$	$\Delta_0 + 44m^2 \pm 12m\sqrt{\Delta_0 + 9m^2} = (\sqrt{\Delta_0 + 9m^2} \pm 6m)^2 - m^2$
0^+	$\Delta_L - 4m^2 = (\Delta_L - 3m^2) - m^2$
$0^-_{(\pm)}$	$Q^2 + 6mQ + 8m^2 = (Q + 3m)^2 - m^2$

Table 2: Mass operators in Freund–Rubin compactifications, see, e.g., [8]. For spins with two tower assignments, the subscripts $(\pm)$, the plus and minus signs refer to branches of the M^2 formulas or to the positive and negative parts of the spectrum for linear operators (includes $Q = \star d$ on 3-forms).

By using the definition $\Delta_p = \delta d + d\delta$ an interesting new form of the Laplacian was found in [5]

$$\Delta = -\Box - R_{abcd}\Sigma^{ab}\Sigma^{cd}, \tag{5}$$

called the *universal Laplacian* since it turns out to be valid for any tensor field and thus unifies, e.g., Δ_p and Δ_L. It was then heavily used in [3]. Below we will provide examples of how this is done.

5 Strategy to get the spectra of operators on the squashed S^7

When we now turn to the squashed seven-sphere we need a realization of it and a method to compute the spectra of the various operators appearing in the mass matrix table above. These are Hodge-de Rham operators $\Delta_p \equiv d\delta + \delta d$ for p-forms with $p = 0, 1, 2, 3$, the Lichnerowicz operator Δ_L and the Dirac operators $\not{D}_s$ for spin $s = \frac{1}{2}$ and $s = \frac{3}{2}$. There is also the linear operator $Q = \star d$ acting on transverse 3-forms related to Δ_3 by $\Delta_3 = Q^2$. By squaring the linear operators they can all be represented by the above universal Laplacian.

There are several ways to define the geometry of the squashed Einstein metric. The metric can for instance be obtained as the distance sphere in the quaternionic projective space $\mathbf{H}P^2$ as explained in detail in [8], but perhaps a more intuitive picture is provided by the Hopf fibration, or Kaluza-Klein, form

$$ds^2_{Hopf}(S^7) = d\mu^2 + \frac{1}{4}\sin^2\mu\ \Sigma_i^2 + \lambda^2\,(\sigma_i - A_i)^2, \tag{6}$$

where Σ_i and σ_i are two sets of left-invariant 1-forms satisfying the $SU(2)$ Lie algebra, while A_i is the $SU(2)$ $k = 1$ instanton gauge field on S^4. From this description of the squashed S^7 we see that squashing refers to a change in the relative size of the fibre S^3 relative the base S^4. The fact is that there are only two Einstein metrics which arise for $\lambda = 1$, the round case, and for $\lambda = 1/\sqrt{5}$ which is the squashed case. Although these two representations are frequently used we will here instead utilize a structure constant based approach for $Sp_2 \times Sp_1^C/(Sp_1^A \times Sp_1^{B+C})$ developed in [18]. The strategy is then as follows:

1. Derive the full spectrum of isometry irreps on the squashed S^7: Done in [4].
2. Derive all possible operator eigenvalues: Done in [1, 5, 3].
3. Tie the irreps in 1. to the eigenvalues in 2.: Done in [3], see also [2].
4. Derive the possible values of E_0 for all fields in AdS_4 and form Heidenreich $\mathcal{N} = 1$ supermultiplets [19]: Done in [3], see also [2].
5. Skew-whiff to the right-squashed non-supersymmetric squashed S^7 and study the possible single- and multi-trace operators on the AdS_4 boundary, and identify the marginal operators: Done in [3], see also [2].

A different approach to finding the squashed spectrum in AdS_4 was used in [20] giving overlapping results.

6 Explicit 2-form mode results and their implications

We start by explaining the method used to obtain the isometry irrep content of the squashed S^7 compactification. To be concrete and simple we start by discussing how to get the spectrum of spin two modes in AdS_4. Recall that the $SO(2,3)$ irreps are denoted $D(E_0, s)$ where for $s = 2$ we have $E_0 = \frac{3}{2} + \frac{1}{6}\sqrt{M^2/m^2 + 9}$ where the mass matrix $M^2 = \Delta_0$. Thus to find the spin 2 spectrum in AdS_4 we need to derive the eigenvalue spectrum of $\Delta_0 = -\Box$ on scalar modes on the squashed S^7 and the spectrum of isometry irreps that can occur. To obtain the latter the general procedure for any tensor field on a coset manifold G/H is as follows:

1. Split the tangent space irrep of the field into H irreps,
2. The isometry spectrum contains any G irrep that when decomposed under H contains any of the H irreps found in the previous step.

The result for the squashed S^7 is presented in terms of *cross diagrams* in [4] where each cross corresponds to an $G = Sp_2 \times Sp_1^C$ irrep $(p, q; r)$. For scalars $\Delta_0 = \frac{20m^2}{9} C_G$, where $C_G(p, q; r)$ is the Casimir [17], and the spectrum is given by the diagram (note that $r = p$),

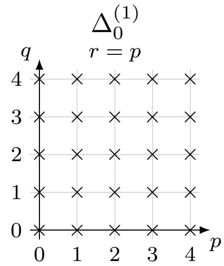

Figure 2: Scalar cross diagram as given in [4]. Each cross corresponds to an isometry irrep $(p, q; r)$, for $p \geq 0$, $q \geq 0$, and $r = p$ of eigenmodes ϕ of Δ_0 with eigenvalues $\Delta_0^{(1)} = \frac{m^2}{9} 20C_g$ [17].

In the case of 2-forms [4] the isometry irrep spectrum contains 21 cross diagrams 15 of which contain the transverse modes that are relevant for the spectrum of fields in AdS_4. These 15 diagrams are divided into sets that make up Sp_1^C irreps, in fact we have one **1**, three **3** and one **5** [3] corresponding to the vertical sets in the 2-form cross diagram below. By inspection we see that some diagrams lack crosses along some lines parallell to the p-axes which is due to these irreps having zero norm. This phenomenon was mentioned for spin 1/2 modes already in [17] but first fully analyzed for 2-forms and several other simpler cases in [3]. The eigenvalues for these transverse 2-form modes were originally calculated in [1] using a method relying heavily on G_2 and octonions. This method did not, however, connect the eigenvalues to the cross diagrams.

An improved version of this method was later developed in [5, 3] giving rise to the universal Laplacian mentioned above and the following novel formulas: The algebraic approach to a coset G/H gives a relation between a differential operator and a Lie algebra element T_a in the coset, namely acting on Fourier modes $\check{D}_a = -T_a$ where $\check{D}_a = D_a + \frac{1}{2} f_{abc}\Sigma^{bc}$ is an H-covariant derivative. Using that for the squashed S^7 $f_{abc} = -\frac{1}{\sqrt{5}} a_{abc}$ [18] this derivative becomes G_2 covariant in the sense that $\check{D}_a a_{abc} = 0$ [1] where a_{abc} are the octonionic structure constants.

From these facts we get the very simple formula

$$\check{\Box} = T_a T^a = -(C_g - C_h), \tag{7}$$

which can easily be related to $\Box$ [5] and hence to the eigenvalue equations we seek to solve.

In fact, on the squashed S^7 one finds the following group theoretic version of the operator equation valid for any tangent space tensor [5]:

$$\Delta = C_g + \frac{6}{7} C_{so(7)} - \frac{3}{2} C_{g_2} - \frac{1}{\sqrt{5}} a_{abc} \Sigma^{ab} \check{D}^c, \tag{8}$$

where the Cs are Casimir operators and Σ^{ab} are the $so(7)$ generators. Note that projectors onto G_2 irreps are needed when solving the eigenvalue equations involving Δ above and that they are all written in terms of octonionic structure constants [5, 3].

For 2-form modes Y_{ab} the expression above for the universal Laplacian leads to the eigenvalue equation

$$\Delta_2 Y_{ab} = C_g Y_{ab} + 3P_7 Y_{ab} - \frac{2}{\sqrt{5}} a_{[a_1}{}^{bc} \check{\mathcal{D}}_c Y_{|b|a_2]} = \kappa_2^2 Y_{ab}, \tag{9}$$

which can be solved giving four sets of different eigenvalues. Thus one set must correspond to two sets of cross diagrams and hence mode functions. This will verified below.

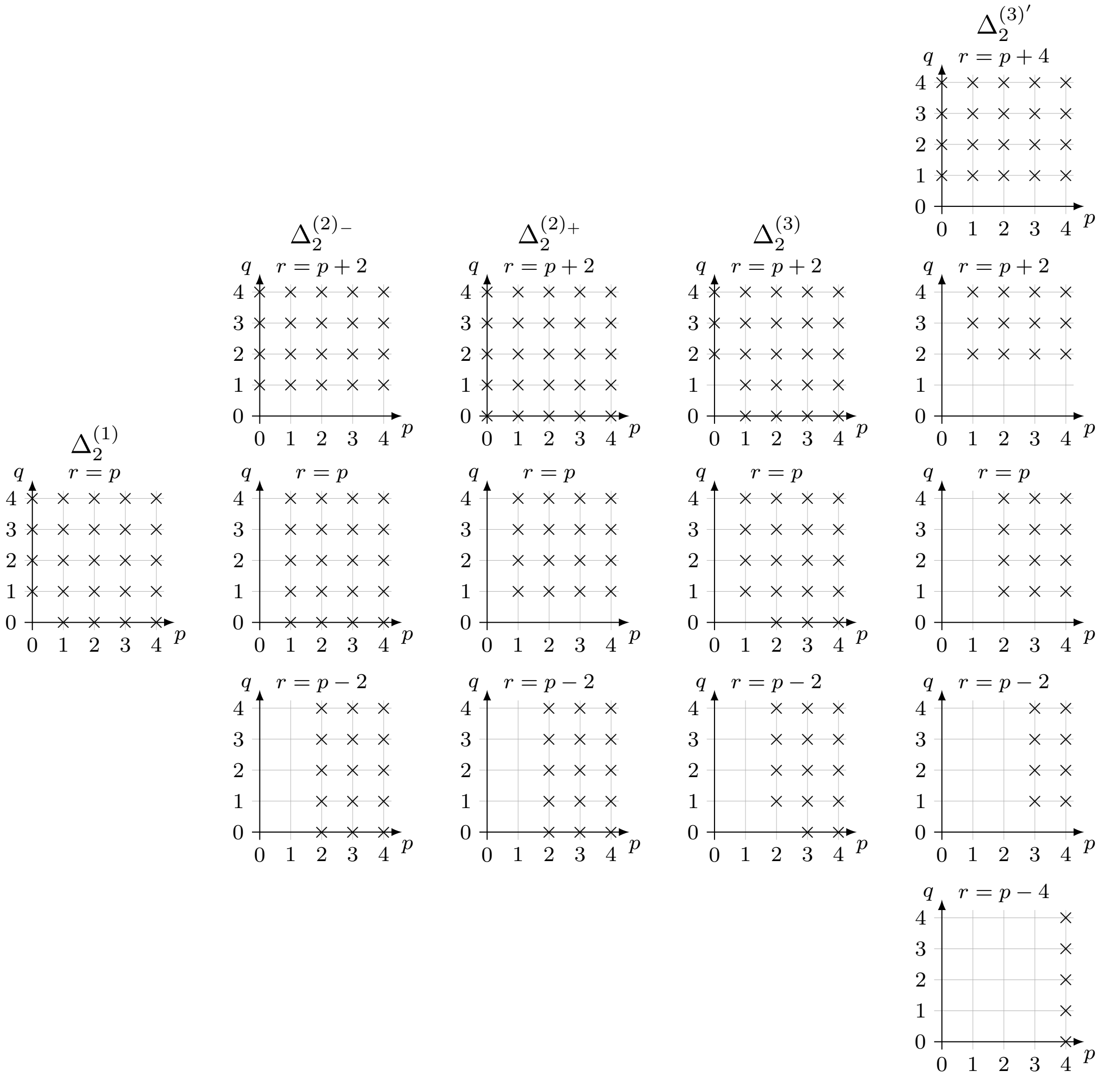

Figure 3: Transverse two-form cross diagrams.

To understand the mode-eigenvalue issue we need explicit expressions: The operators below acting on scalar modes give all 21 2-form modes. Note that they contain the octonionic structure constants a_{abc} and its dual c_{abcd} as well as the Sp_1^C Killing vectors s^i.

$$\mathcal{Y}^{(1)}_{ab} = a_{ab}{}^c \check{D}_c, \quad \mathcal{Y}^{(2)i}_{ab} = a_{ab}{}^c s_c{}^i, \quad \mathcal{Y}^{(3)i}_{ab} = \epsilon^i{}_{jk} s_a{}^j s_b{}^k, \quad \mathcal{Y}^{(4)i}_{ab} = s_{[a}{}^i \check{D}_{b]}, \tag{10}$$

$$\mathcal{Y}^{(5)i}_{ab} = c_{ab}{}^{cd} s_c{}^i \check{D}_d, \quad \mathcal{Y}^{(6)i}_{ab} = a_{[a|}{}^{cd} s_c{}^i \check{D}_{d|b]}, \quad \mathcal{Y}^{(7)ij}_{ab} = s_{[a}{}^{\{i|} a_{b]}{}^{cd} s_c{}^{|j\}} \check{D}_d. \tag{11}$$

The novel aspect of these mode functions is present in $\mathcal{Y}^{(6)i}_{ab}$: It contains a two derivative operator $\check{D}_{ab} = \check{D}_{(a}\check{D}_{b)}$. This was found in [2] and will be explained in [21]. Deriving the eigenvalues for these modes shows that there is indeed a degeneracy in the eigenvalues [3] as is seen in the table below.

s^p	E_0	$E_0 - \frac{1}{2}$	$E_0 + \frac{1}{2}$	E_0	Sp_1^C	E_0 values
1_1^+	$1^+(\Delta_2^{(3)})$	$\frac{1}{2}(i\not{D}_{3/2}^{(2)}{}_-)$	$\frac{1}{2}(i\not{D}_{3/2}^{(2)}{}_+)$	$0^+(\Delta_L^{(1)})$	**3**	$\frac{3}{2} + \frac{1}{6}\sqrt{20C_g + 9}$
1_2^+	$1^+(\Delta_2^{(3)'})$	$\frac{1}{2}(i\not{D}_{3/2}^{(2)'}{}_-)$	$\frac{1}{2}(i\not{D}_{3/2}^{(2)'}{}_+)$	$0^+(\Delta_L^{(1)'})$	**5**	$\frac{3}{2} + \frac{1}{6}\sqrt{20C_g + 9}$

Table 3: Supermultiplets with spin and parity $s = 1^+$. The two tables of all spins up to $s = 2$ are given in [3]. Here, each entry, represented by $s(\text{operator}^{\text{eigenvalue}}_{\text{mode type}})$, corresponds to a specific spin component of a Heidenreich $\mathcal{N} = 1$ supermultiplet, but with the highest spin first. The notation indicates also the relevant cross diagrams of Δ_p, for $p = 2, L$, and $i\not{D}_{3/2}$ as given in [3]. The Sp_1^C irrep entries specify the number of cross diagrams belonging to the supermultiplet.

When the entire squashed spectrum was derived in [4] it was also found that all states fit into the following $\mathcal{N} = 1$ supermultiplets (referred to by their maximum spin component): 1 spin 2, 6 spin 3/2, 6 spin 1^-, 8 spin 1^+ and 14 Wess-Zumino multiplets. However, when comparing this squashed spectrum to the round one some states do not fit into a Higgs picture relating the two spectra. These states were of two types [4]:

1. States occurring in the round SSB spectrum but not in the squashed:
Lichnerowicz modes of Δ_L : $(\mathbf{4}, \mathbf{2}), (\mathbf{5}, \mathbf{3})$
Rarita-Schwinger modes of $i\not{D}_{3/2}$: $(\mathbf{4}, \mathbf{2}), (\mathbf{5}, \mathbf{1}), (\mathbf{1}, \mathbf{3})$
2. A state occurring in the squashed spectrum but not in the round: $i\not{D}_{3/2}$ mode $(\mathbf{1}, \mathbf{1})$ [2].

A way to make these facts fit with a Higgs/deHiggs kind of picture was suggested in [4] (see also [2, 3]) involving adding singletons to the spectra: $\mathcal{N} = 8$ supersingleton to the round spectrum and a fermionic singlet singleton to right-squashed spectrum (while nothing is added to the left-squashed spectrum).

7 Conclusions

We end by listing a number of conclusions related to the results presented above:
1. *Singletons*: There are modes in the round case that do not appear in either of the squashed cases although the latter are a kind of Higgsed version of the former. However, by adding the $\mathcal{N} = 8$ singleton supermultiplet to the round spectrum, and a fermionic singlet singleton to the right-squashed spectrum, one can argue that the three spectra are consistent with each other. There is also a spin 3/2 mode in the squashed spectrum that does not exist in the round spectrum. This fact can also be seen to be consistent by invoking a deHiggsing in the spin 3/2 sector of the two squashed cases [4, 2].
2. *The left-squashed spectrum 1*: The $\mathcal{N} = 1$ supersymmetric spectrum in the left-squashed case is completely understood apart from the degeneracy encountered in all supermultiplets containing fields whose masses are related to the Lichnerowicz operators on S^7 [3].
3. *The left-squashed spectrum 2*: We find a rather surprising feature namely that supersymmetry does not fix the boundary conditions of scalar and spin 1/2 fields in the left-squashed case. In other words, the values of E_0 for these fields fit in two different ways into Wess-Zumino supermultiplets using different assignments of boundary conditions (that is different choices of $\pm$ in the formulas for E_0). In fact, there are irreps whose boundary conditions in the round case cannot be retained in the squashed case [3].
4. *The right-squashed spectrum 1*: By skew-whiffing (orientation-flipping) one obtains the spectrum of the right-squashed non-supersymmetric compactification which again is BF stable [13].
5. *The right-squashed spectrum 2*: Having established in the left-squashed case that boundary conditions can be chosen in different ways, the right-squashed spectrum becomes very dependent on boundary conditions and how they are chosen. In particular, it is possible to choose boundary conditions so that there are no marginal operators on the boundary at all [2, 3], something that might be relevant for the *AdS* swampland stability issue [9] as mentioned in the Introduction.

References

[1] Ekhammar S and Nilsson B E W 2021, "On the squashed seven-sphere operator spectrum", *JHEP* **12**, 057 [arXiv:2105.05229 [hep-th]].

[2] Nilsson B E W and Pope C N 2024, "De-Higgsing in eleven-dimensional supergravity on the squashed S^7," *J. Phys. A* **57** no.4, 045402, [arXiv:2302.03842 [hep-th]].

[3] Karlsson J and Nilsson B E W 2024, "The complete Kaluza-Klein spectra of $\mathcal{N} = 1$ and $\mathcal{N} = 0$ M-theory on $AdS_4 \times$ squashed S^7)," *JHEP* **02**, 144, [arXiv:2305.00916 [hep-th]].

[4] Nilsson B E W, Padellaro A and Pope C N 2019, "The role of singletons in S^7 compactifications," *JHEP* **07**, 124, [arXiv:1811.06228 [hep-th]].

[5] Karlsson J 2020, "Compactifications of String/M-Theory and the Swampland: A Study of the AdS_4 Mass Spectrum of Eleven-Dimensional Supergravity on the Squashed Seven-Sphere," [arXiv:2110.09885 [hep-th]]. Chalmers Univ. of Tech., Göteborg, MSc thesis.

[6] Awada M A, Duff M J and Pope C N 1983, "N = 8 Supergravity Breaks Down to N = 1," *Phys. Rev. Lett.* **50**, 294.

[7] Duff M J, Nilsson B E W and Pope C N 1983, "Spontaneous Supersymmetry Breaking by the Squashed Seven Sphere," *Phys. Rev. Lett.* **50** no.26, 2043.

[8] Duff M J, Nilsson B E W and Pope C N 1986, "Kaluza-Klein Supergravity," *Phys. Rept.* **130**, 1 - 142.

[9] Ooguri H and Vafa C 2016, "Non-supersymmetric AdS and the Swampland," [arXiv:1610.01533 [hep-th]].

[10] Giambrone A, Guarino A, Malek E, Samtleben H, Sterckx C and Trigiante M 2022, "Holographic evidence for nonsupersymmetric conformal manifolds," *Phys. Rev. D* **105** no.6, 066018, [arXiv:2112.11966 [hep-th]].

[11] Giri S, Martucci L and Tomasiello A 2022, "On the stability of string theory vacua," *JHEP* **04**, 054, [arXiv:2112.10795 [hep-th]].

[12] Breitenlohner P and Freedman D Z 1982, "Stability in Gauged Extended Supergravity," *Annals Phys.* **144**, 249.

[13] Duff M J, Nilsson B E W and Pope C N 1984, "The Criterion for Vacuum Stability in Kaluza-Klein Supergravity," *Phys. Lett.* **139B** 154.

[14] Ahn C h and Rey S J 2000, "Three-dimensional CFTs and RG flow from squashing M2-brane horizon," *Nucl. Phys. B* **565**, 210, [arXiv:hep-th/9908110 [hep-th]].

[15] Murugan A 2016, "Renormalization group flows in gauge-gravity duality," Princeton University PhD thesis (2009), [arXiv:1610.03166 [hep-th]].

[16] Nicolai H 1984, "REPRESENTATIONS OF SUPERSYMMETRY IN ANTI-DE SITTER SPACE," CERN-TH-3882, April 1984.

[17] Nilsson B E W and Pope C N 1983, *Phys. Lett.* **133B** 67.

[18] Bais F A, Nicolai H and van Nieuwenhuizen P 1983, "Geometry of Coset Spaces and Massless Modes of the Squashed Seven Sphere in Supergravity," *Nucl. Phys. B* **228**, 333-350.

[19] Heidenreich W 1982, "ALL LINEAR UNITARY IRREDUCIBLE REPRESENTATIONS OF DE SITTER SUPERSYMMETRY WITH POSITIVE ENERGY," *Phys. Lett. B* **110**, 461-464.

[20] Duboeuf B, Malek E and Samtleben H 2023, "Kaluza-Klein spectrometry beyond consistent truncations: the squashed S^7," *JHEP* **04**, 062 [arXiv:2212.01135 [hep-th]].

[21] Karlsson J and Nilsson B E W 2024, work in progress.