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# TOI-2458 b: A mini-Neptune consistent with in situ hot Jupiter formation<sup>★</sup>

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## ABSTRACT

We report on the discovery and spectroscopic confirmation of TOI-2458 b, a transiting mini-Neptune around an F-type star leaving the main-sequence with a mass of  $M_{\star} = 1.05 \pm 0.03 M_{\odot}$ , a radius of  $R_{\star} = 1.31 \pm 0.03 R_{\odot}$ , an effective temperature of  $T_{\text{eff}} = 6005 \pm 50$  K, and a metallicity of  $-0.10 \pm 0.05$  dex. By combining TESS photometry with high-resolution spectra acquired with the HARPS spectrograph, we found that the transiting planet has an orbital period of  $\sim 3.74$  days, a mass of  $M_p = 13.31 \pm 0.99 M_{\oplus}$  and a radius of  $R_p = 2.83 \pm 0.20 R_{\oplus}$ . The host star TOI-2458 shows a short activity cycle of  $\sim 54$  days revealed in the HARPS S-index and  $H\alpha$  times series. We took the opportunity to investigate other F stars showing activity cycle periods comparable to that of TOI-2458 and found that they have shorter rotation periods than would be expected based on the gyrochronology predictions. In addition, we determined TOI-2458's stellar inclination angle to be  $i_{\star} = 10.6^{+13.3}_{-10.6}$  degrees. We discuss that both phenomena (fast stellar rotation and planet orbit inclination) could be explained by in situ formation of a hot Jupiter interior to TOI-2458 b. It is plausible that this hot Jupiter was recently engulfed by the star. Analysis of HARPS spectra has identified the presence of another planet with a period of  $P = 16.55 \pm 0.06$  days and a minimum mass of  $M_p \sin i = 10.22 \pm 1.90 M_{\oplus}$ . Using dynamical stability analysis, we constrained the mass of this planet to the range  $M_c \simeq (10, 25) M_{\oplus}$ .

**Key words.** techniques: photometric – techniques: radial velocities – techniques: spectroscopic – planetary systems – planets and satellites: dynamical evolution and stability – planets and satellites: formation

## 1. Introduction

The first discovery of an exoplanet around a Sun-like star, 51 Peg b, was an unexpected breakthrough, given the planet's dissimilarity from any known planets within our Solar System (Mayor & Queloz 1995). Subsequent population studies revealed that hot Jupiters exist around roughly 1% of Sun-like stars (Mayor et al. 2011; Wright et al. 2012). Despite over two decades of research, the primary formation channel of hot Jupiters remains elusive. The formation mechanisms for hot Jupiters can be grouped into two categories, the first positing a slow orbital decay as a result of interactions with a protoplanetary disk (e.g., Lin et al. 1996; Ward 1997; Murray et al. 1998), and the second suggesting the tidal circularization of a planet on a highly eccentric orbit, created by gravitational interactions with other massive bodies (e.g., Rasio & Ford 1996; Fabrycky & Tremaine 2007; Naoz et al. 2011). Over the past two decades, migration mechanisms have become an established theoretical paradigm.

The formation of hot Jupiters in situ represents a potential alternative to migration mechanisms (e.g., Pollack et al. 1996; Boss 1997; Batygin et al. 2016; Hasegawa et al. 2019; Poon et al. 2021). Several arguments have been proposed in support of this scenario. Firstly, the rate of disk-driven migration observed in simulations for giant planets appears to be independent of planetary mass (Kley & Nelson 2012), suggesting that the physical properties of hot and cold giant planets should be similar on average. However, recent studies have shown that close-in and distant giant planet populations exhibit intrinsic differences, with hot giant planets generally having smaller masses (Knutson et al. 2014; Bryan et al. 2016). Secondly, in situ formation has previously been discounted due to a lack of sufficiently massive cores at small orbital radii (Rafikov 2006). However, radial velocity (RV) surveys and space-based transit photometry have shown that the inner regions of planetary systems are not typically empty. Instead, hot planets with a period of less than 100 days and a mass of less than  $30 M_{\oplus}$  are highly prevalent, with 30% – 50% of main-sequence stars in the solar neighborhood serving as hosts (e.g., Dressing & Charbonneau 2013; Petigura et al. 2013; Winn & Fabrycky 2015).

Rapid gas accretion onto protoplanetary cores can occur in the innermost regions of the protoplanetary disk if the core's

<sup>★</sup> Based on observations performed with the 3.6 m telescope at the European Southern Observatory (La Silla, Chile) under programs 1102.C-0923, 106.21TJ.001, 110.2438.001, and 111.254R.002.

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mass exceeds  $M_{core} \geq 15 M_{\oplus}$  (Batygin et al. 2016). However, it remains an open question whether there is sufficient gaseous material available within a few astronomical units. Therefore, the initial population of close-in giant planets formed in situ is expected to consist of low-mass planets occupying a similar orbital range. Considering the high occurrence of low-mass planets, it is reasonable to assume that in situ-formed giant planets are often accompanied by low-mass planets on similar orbits. This assumption leads to dynamical evolution that has observable implications (Batygin et al. 2016). Recent observations from space-based observatories and radial velocity follow-up measurements of known transiting hot Jupiters have led to the detection of new low-mass planetary companions to hot Jupiters. Examples of such systems include WASP-47 (Becker et al. 2015), Kepler-730 (Cañas et al. 2019), TOI-1130 (Huang et al. 2020; Korth et al. 2023), WASP-132 (Hord et al. 2022), TOI-2000 (Sha et al. 2023), and WASP-148 (Hébrard et al. 2020). These new discoveries provide evidence that at least some hot Jupiters may have formed in situ. They also suggest that nearby low-mass planetary companions to stars hosting hot Jupiters may be more common than previously thought. Furthermore, such companions could be below the detection limits of many past surveys.

The majority of systems mentioned above exhibit low-mass companions on inner orbits to hot Jupiters, which is expected in the theory (Batygin et al. 2016). In this case, the nodal recession of the inner orbit is always faster, meaning that a secular resonance can never be established. Hence, such planets are expected to remain in unperturbed, coplanar orbits. However, if low-mass companions exhibit on exterior orbits, the initially slower nodal regression of the orbit will accelerate as the inner body gains mass and the star spins down. Once affected by secular resonances, the orbit may become unstable or acquire high mutual inclination, making observations of both interior and exterior planets extremely challenging as, in this case, a non-transiting planet might orbit at a low inclination relative to the line of sight and produce only weak radial-velocity variations. Observation of a system with a transiting low-mass planet orbiting close to the poles of a star with non-transiting hot Jupiter would be an important piece of the puzzle addressing the in situ formation of hot Jupiters.

We have spectroscopically confirmed a transiting planet candidate TOI-2458 b as a mini-Neptune planet orbiting an F-type star. Our paper includes a description of the observations in Section 2, data analysis in Section 3, a discussion in Section 4, and a summary of our results in Section 5.

## 2. Observations

### 2.1. TESS light curves

The Transiting Exoplanet Survey Satellite (TESS; Ricker et al. 2015) conducted a photometric monitoring campaign of TOI-2458 (TIC 449197831) in sector 32 from November 20, 2020, to December 16, 2020, employing a 120-second time sampling. The resulting data for TOI-2458 are publicly available on the Mikulski Archive for Space Telescopes (MAST)<sup>1</sup>. These data were processed by the TESS Science Processing Operations Center (SPOC; Jenkins et al. 2016) located at the NASA Ames Research Center. The SPOC conducted a transit search of the light curve (LC) on January 08, 2021, with an adaptive, noise-compensating matched filter (Jenkins 2002; Jenkins et al. 2010, 2020),

<sup>1</sup> <https://mast.stsci.edu/portal/Mashup/Clients/Mast/Portal.html>

producing a threshold crossing event (TCE) with 3.74 d period for which an initial limb-darkened transit model was fitted (Li et al. 2019) and a suite of diagnostic tests were conducted to help assess the planetary nature of the signal (Twicken et al. 2018). The TESS Science Office reviewed the vetting information and issued an alert for TOI 2458.01.01 on February 03, 2021 (Guerrero et al. 2021).

The LC derived from TESS sector 32 was obtained from the MAST archive using the `lightkurve` software package (Lightkurve Collaboration 2018). We used the Pre-search Data Conditioning Simple Aperture Photometry (PDCSAP) LC (Smith et al. 2012; Stumpe et al. 2012, 2014) processed by the SPOC pipeline, which also removes systematic errors associated with the instrument. The pixel response functions (PRFs) were employed to investigate the crowding, and a correction for crowding was incorporated into the PDCSAP flux time series. Failure to incorporate such a correction may lead to an underestimation of the planet’s radius. In addition, the SPOC Data Validation difference image centroid offset analysis (Twicken et al. 2018) locates the source of the transit signature within  $3.0 \pm 2.7$  arcsec of the target star. This excludes all TIC objects other than TOI-2458 as potential sources of the transit signature.

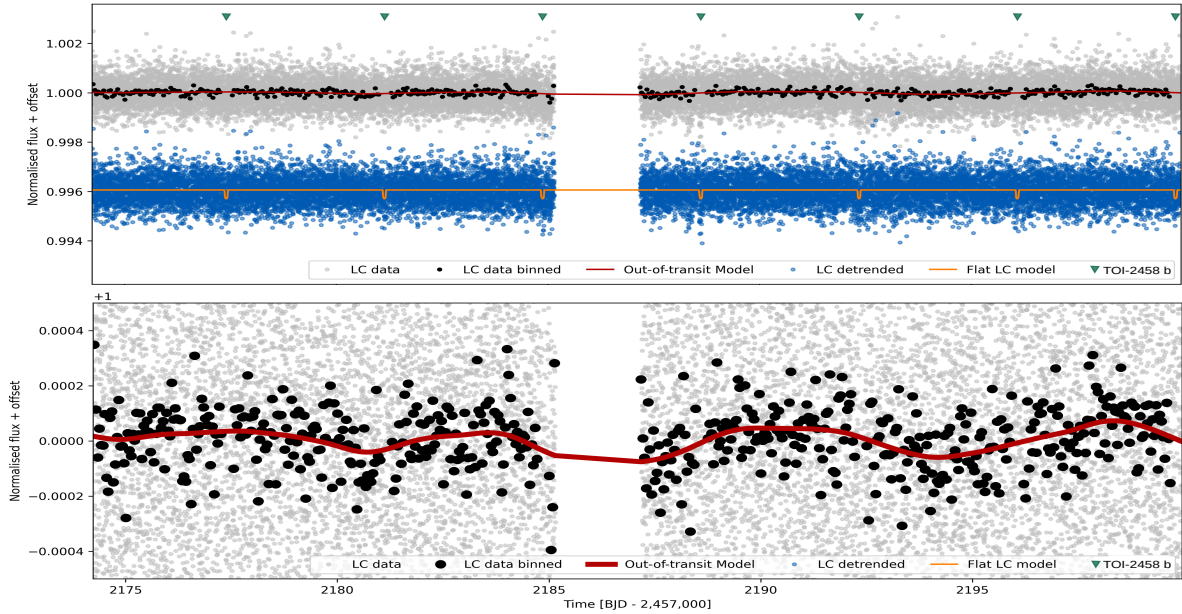
We utilized the Python package, `citlalicue` (Barragán et al. 2022), to remove residual stellar variability from the PDCSAP LCs. Combining a Gaussian process (GP) regression with transit models computed via the `pytransit` code (Parviainen 2015), `citlalicue` generated a model comprising both the light curve variability and the transits. We subtracted the variability part and produced a flattened LC containing only the transit photometric variations. We detected 7 transits. The LCs before and after the process is shown in Fig. 1.

To compare the *Gaia* DR2/DR3 catalogs with the TESS target pixel file (TPF), we utilized the TESS-cont tool (Castro-González et al. 2024). This tool is used to detect any potential stars that could dilute the TESS photometry. Figure 2 presents a heatmap that illustrates the percentage of flux from the target star that falls within each pixel. As shown, within the aperture, TOI-2458 contributes between 63% and 100% of the total flux, depending on the specific pixel analyzed. We found that only 2.6% of the flux in total originates from nearby sources, indicating negligible contamination, which has already been corrected in the PDCSAP. It is worth noting the presence of the dominant contaminating source *Gaia* ID 3233785163260952192, which is relatively close to the aperture used, although it is three magnitudes fainter.

### 2.2. LCOGT photometric follow-up

We observed a full transit window of TOI-2458 continuously for 260 minutes in Pan-STARRS *z*-short band on February 14, 2021, from the Las Cumbres Observatory Global Telescope (LCOGT; Brown et al. 2013) 1-m network node at Cerro Tololo Inter-American Observatory in Chile (CTIO). The  $4096 \times 4096$  LCOGT SINISTRO cameras have an image scale of  $0''.389$  per pixel, resulting in a  $26' \times 26'$  field of view. The images were calibrated by the standard LCOGT BANZAI pipeline (McCully et al. 2018) and differential photometric data were extracted using `AstroImageJ` (Collins et al. 2017).

The TOI-2458 SPOC pipeline transit depth of 310 ppm is generally too shallow to detect with ground-based observations reliably, so we instead checked for possible nearby eclipsing binaries (NEBs) that could be contaminating the TESS photometric aperture and causing the TESS detection. To account for possible contamination from the wings of neighboring star point



**Fig. 1.** SPOC PDCSAP LC from TESS sector 32 for TOI-2458 downloaded with the lightcurve tool. Gray points represent the TESS observations, black points are binned TESS data, while red lines correspond to the out-of-transit GP models created with *citlalicue* to capture the variability in the LC. Datasets were divided by these models, leading to a flattened TESS LC (blue points) with the transit model (orange line). Green triangles indicate the positions of transits. The lower panel is a zoom-in of the upper panel highlighting the out-of-transit GP model.

spread functions (PSFs), we searched for NEBs out to  $2.5'$  from TOI-2458. If fully blended in the SPOC aperture, a neighboring star that is fainter than the target star by 8.8 magnitudes in the TESS band could produce the SPOC-reported flux deficit at mid-transit (assuming a 100% eclipse). To account for possible TESS magnitude uncertainties and possible delta-magnitude differences between TESS-band and Pan-STARRS  $z$ -short band, we included an extra 0.5 magnitudes fainter (down to TESS-band magnitude 18.0). We calculated the RMS of each of the 33 nearby star light curves (binned in 10-minute bins) that meet our search criteria and found that the values are smaller by at least a factor of 5 compared to the required NEB depth in each respective star. We then visually inspected each neighboring star's light curve to ensure no obvious eclipse-like signal. Our analysis ruled out an NEB blend as the cause of the SPOC pipeline TOI-2458 detection in the TESS data. All LCOGT light curve data are available on the EXOFOP-TESS website<sup>2</sup>.

### 2.3. Contamination from nearby sources

To eliminate any potential sources of dilution within the *Gaia* separation limit of  $0.4''$ , we obtained high-resolution images using adaptive optics and speckle imaging of TOI-2458. This was done as part of follow-up observations that were coordinated through the TESS Follow-up Observing Program (TFOP) High-Resolution Imaging Sub-Group 3 (SG3).

On October 16, 2021, TOI-2458 was observed through the Alopeke speckle imager (Scott & Howell 2018), which is mounted on the 8.1 m Gemini-North. The Alopeke system employs high-speed iXon Ultra 888 back-illuminated Electron Multiplying CCDs (EMCCDs) to simultaneously capture data in two bands centered around 562 nm and 832 nm. The data were processed using the procedures outlined in Howell et al. (2011). The final reconstructed image, displayed in Fig. 3, attains

<sup>2</sup> <https://exofop.ipac.caltech.edu/tess/target.php?id=449197831>

a contrast of  $\Delta mag = 6.60$  at a separation of  $0.5''$  in the 832 nm band and  $\Delta mag = 5.63$  at a separation of  $0.5''$  in the 562 nm band. A contrast of  $\Delta mag \sim 5.50$  is achieved in both bands at a separation of  $0.2''$  (22 au). The estimated PSF is  $0.02''$  wide.

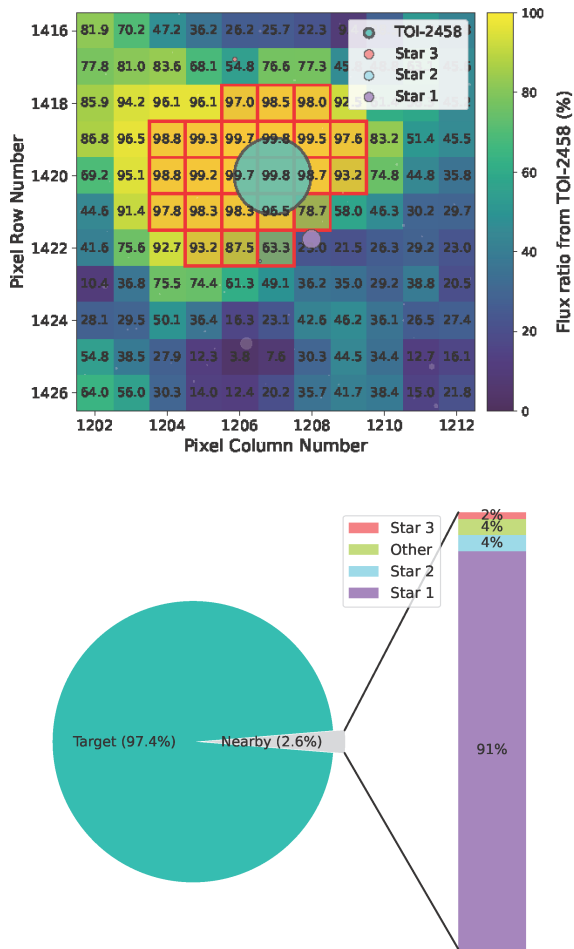
On February 24, 2021, observation of TOI-2458 was conducted using the Palomar High Angular Resolution Observer (PHARO; Hayward et al. 2001) with the JPL Palomar Adaptive Optics System, mounted on the 5.0 m Hale telescope. PHARO uses a  $1024 \times 1024$  HAWAII HgCdTe detector to capture images in the 1 to  $2.5 \mu\text{m}$  range. During the observation, a Bry filter was used. The reconstructed image, as shown in Fig. 3, achieves a contrast of  $\Delta mag = 6.83$  at a separation of  $0.5''$  with an estimated PSF of  $0.09''$  width. We found no companion with a contrast of  $\Delta mag \sim 7$  at projected angular separations ranging from  $0.5$  to  $4.0''$  (55 to 443 au).

On February 27, 2021, the High-Resolution Camera (HRCam; Tokovinin & Cantarutti 2008) speckle interferometry instrument was employed on the Southern Astrophysical Research (SOAR) 4.1m telescope to observe TOI-2458. The observation was made in the Cousins  $I$  filter with a resolution of 36 mas. The observation was carried out according to the observation strategy and data reduction procedures outlined in Tokovinin (2018) or Ziegler et al. (2021). The final reconstructed image achieves a contrast of  $\Delta mag = 5.5$  at a separation of  $0.5''$ , and the estimated PSF is  $0.064$  arcsec wide. Fig. 3 plots a visual representation of the final reconstructed image. We found no companion with a contrast of  $\Delta mag \sim 5.5$  at projected angular separations ranging from  $0.5$  to  $3.0''$  (55 to 332 au).

### 2.4. HARPS spectra

We acquired 87 spectra of TOI-2458 using the High Accuracy Radial velocity Planet Searcher (HARPS; Mayor et al. 2003) spectrograph, which is mounted on the ESO 3.6-m telescope located at La Silla observatory. The observation period spans 590 days, from February 16, 2022, to September 28, 2023. The HARPS instrument has a spectral resolving power of  $R \approx 115000$





**Fig. 2.** Top: heatmap illustrating the percentage of the flux from TOI-2458 that falls within each pixel of the TESS TPF image using *Gaia* DR2/DR3 catalogs. Bottom: pie chart representing the flux from the target and nearby stars inside the photometric aperture. Star 1 (*Gaia* ID 3233785163260952192) is the primary source of contamination.

and covers wavelengths ranging from  $\sim 380$  nm to  $\sim 690$  nm. The observations of TOI-2458 were carried out as part of our follow-up programs of TESS transiting planets (IDs: 1102.C-0923, 106.21TJ.001, 110.2438.001, 111.254R.002; PIs: Gandolfi, Dai, Wilson). We set the exposure time to 1200–1800 s depending on weather conditions, seeing, and nightly schedule constraints. We extracted the spectra using the HARPS data reduction software (DRS; Lovis & Pepe 2007) available at the telescope. The signal-to-noise ratio (S/N) per pixel at 550 nm on the extracted spectra ranges from 57 to 124. We utilized the TERRA algorithm (Anglada-Escudé & Butler 2012) to extract relative radial velocities and spectral activity indicators. The TERRA algorithm achieves this by using a least-squares matching technique of each observed spectrum to a high signal-to-noise ratio template, which was derived from the same observations. The radial velocities were adjusted for barycentric motion, secular perspective acceleration, instrumental drift, and nightly zero points.

### 3. Analysis

#### 3.1. Modeling stellar parameters with *iSpec* and *SpecMatch*

We achieved an S/N of 600 per pixel at 550 nm by co-adding all the high-resolution HARPS spectra that were corrected for

**Table 1.** Basic parameters for TOI-2458.

Parameter	Description	Value	Source
$\alpha_{J2000}$	Right Ascension (RA)	05:18:29.17	1
$\delta_{J2000}$	Declination (Dec)	01:15:13.92	1
$V_T$	Tycho-2 $V_T$ mag	$9.268 \pm 0.020$	2
$B_T$	Tycho-2 $B_T$ mag	$9.900 \pm 0.028$	2
$G$	<i>Gaia</i> $G$ mag	$9.105 \pm 0.003$	1
$B_P$	<i>Gaia</i> $B_P$ mag	$9.384 \pm 0.003$	1
$R_P$	<i>Gaia</i> $R_P$ mag	$8.659 \pm 0.003$	1
$T$	TESS $T$ mag	$8.710 \pm 0.006$	3
$J$	2MASS $J$ mag	$8.196 \pm 0.020$	4
$H$	2MASS $H$ mag	$7.990 \pm 0.044$	4
$K_S$	2MASS $K_S$ mag	$7.891 \pm 0.023$	4
$WIS E1$	WISE1 mag	$7.827 \pm 0.028$	5
$WIS E2$	WISE2 mag	$7.872 \pm 0.019$	5
$\mu_\alpha \cos \delta$	PM in RA (mas/yr)	$-3.023 \pm 0.016$	1
$\mu_\delta$	PM in Dec (mas/yr)	$-99.207 \pm 0.012$	1
$\pi$	Parallax (mas)	$9.027 \pm 0.017$	1
Other identifiers:			
HD 34562			
TIC 449197831			
TYC 0100-00856-1			
2MASS J05182917+0115139			
<i>Gaia</i> DR2 3233785128901216128			

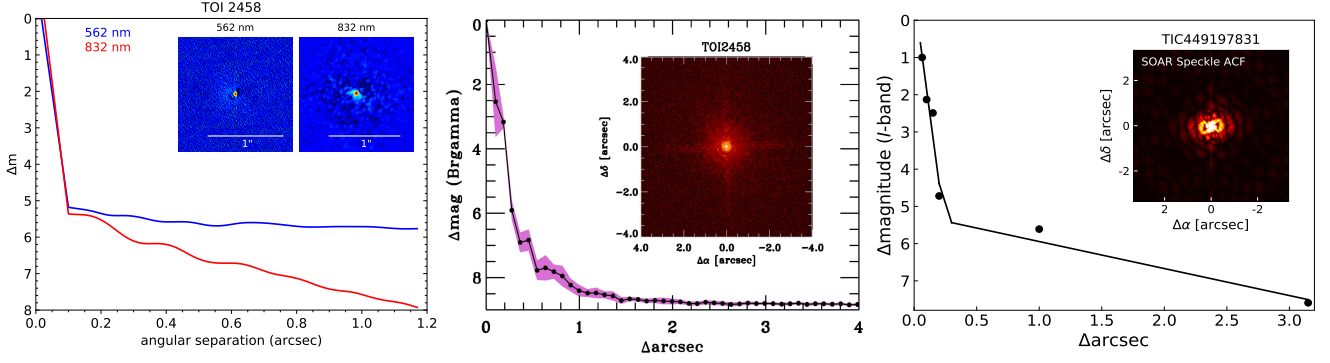
**Notes.** 1 – *Gaia* Collaboration (2021); 2 – Høg et al. (2000); 3 – Stassun et al. (2018); 4 – Skrutskie et al. (2006); 5 – Wright et al. (2010).

RV shifts. The combined spectrum was then used to determine the stellar parameters of TOI-2458 via the Spectroscopy Made Easy radiative transfer code (SME; Valenti & Piskunov 1996; Piskunov & Valenti 2017), which is incorporated into *iSpec* (Blanco-Cuaresma et al. 2014; Blanco-Cuaresma 2019). Complementary to this, we modeled the spectrum with MARCS models of atmospheres (Gustafsson et al. 2008) and utilized version 5 of the GES atomic line list (Heiter et al. 2015). The *iSpec* software calculates synthetic spectra and compares them to the observed spectrum by minimizing the chi-squared value between them using a nonlinear least-squares (Levenberg-Marquardt) fitting algorithm (Markwardt 2009).

In our study, we employed the spectrum region spanning between 490 and 580 nm to determine the effective temperature  $T_{\text{eff}}$ , metallicity [Fe/H], and the projected stellar equatorial velocity  $v \sin i$ . We utilized the Bayesian parameter estimation code PARAM 1.5 (da Silva et al. 2006; Rodrigues et al. 2014, 2017) to compute the surface gravity  $\log g$  from PARSEC isochrones (Bressan et al. 2012) using derived spectral parameters as well as available photometry (refer to Table 1) and *Gaia* DR3 parallax as inputs. The iterative procedure was performed multiple times to achieve convergence to the final parameter values. The final parameters obtained are presented in Table 2. The obtained values agree with those calculated by the *Gaia* DR3 using low-resolution BP/RP spectra<sup>3</sup>.

The final parameters derived from several iterations were used as inputs to the PARAM 1.5 code once again to estimate the mass, radius, and age of the star. The star appears to be

<sup>3</sup> [https://gea.esac.esa.int/archive/documentation/GDR3/Data\\_analysis/chap\\_cu8par/sec\\_cu8par\\_apsis/ssec\\_cu8par\\_apsis\\_gspphot.html](https://gea.esac.esa.int/archive/documentation/GDR3/Data_analysis/chap_cu8par/sec_cu8par_apsis/ssec_cu8par_apsis_gspphot.html)



**Fig. 3.** From left to right: alopeke contrast curve for 562 nm and 832 nm bands with a  $1.2'' \times 1.2''$  reconstructed image of the field. PHARO contrast curve for Brgamma band with a  $8'' \times 8''$  reconstructed image of the field. SOAR contrast curve for Cousins *I* band with a  $6'' \times 6''$  reconstructed image of the field.

**Table 2.** Stellar parameters of TOI-2458.

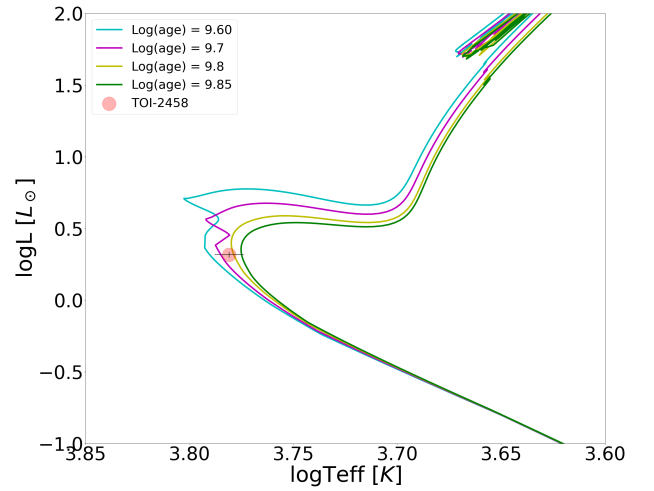
	iSpec & PARAM 1.5 analysis	SpecMatch
$T_{\text{eff}}$ (K)	$6005 \pm 50$	$6002 \pm 110$
[Fe/H] (dex)	$-0.10 \pm 0.05$	$0.05 \pm 0.09$
$\log g$ (cgs)	$4.20 \pm 0.05$	$4.28 \pm 0.12$
$v_{\text{rot}} \sin i_{\star}$ (km/s)	$2.5 \pm 2.5$	
$EW_{\text{Li}}$ (Å)	0.055	
$M_{\star}$ ( $M_{\odot}$ )	$1.05 \pm 0.03$	
$R_{\star}$ ( $R_{\odot}$ )	$1.31 \pm 0.03$	
ARIADNE analysis		
$T_{\text{eff}}$ (K)	$6047 \pm 49$	
[Fe/H]	$-0.11 \pm 0.16$	
$\log g$	$4.22 \pm 0.17$	
$L_{\star}$ ( $L_{\odot}$ )	$2.09 \pm 0.10$	
$R_{\star}$ ( $R_{\odot}$ )	$1.32 \pm 0.03$	
Other parameters		
$P_{\text{rot}}$ (days)	$8.9^{+3.9}_{-2.5}$	
Spectral type	F9–F9.5	
Age (Gyr)	$5.7^{+0.9}_{-0.8}$	

relatively old, with an estimated age of  $5.7 \pm 0.9$  Gyr. The age is also reported in Table 2. In Fig. 4 we overplot the luminosity and effective temperature of TOI-2458 with the MIST stellar evolutionary tracks (Choi et al. 2016). The spectral type of TOI-2458 was estimated via the latest version<sup>4</sup> of the empirical spectral type-color sequence proposed by Pecaut & Mamajek (2013).

### 3.2. Stellar parameters with SpecMatch

As an additional validation of the derived stellar parameters, we used the HARPS co-added spectrum as an input to the SpecMatch package (Yee et al. 2017). The determination of stellar parameters by SpecMatch involves a comparison of the observed spectrum with a library of well-characterized high signal-to-noise ( $S/N > 400$ ) High Resolution Echelle Spectrometer (HIRES) spectra. All the derived parameters are documented in Table 2 and are in agreement with the ones derived from iSpec.

<sup>4</sup> [https://www.pas.rochester.edu/~emamajek/EEM\\_dwarf\\_UBVIJHK\\_colors\\_Teff.txt](https://www.pas.rochester.edu/~emamajek/EEM_dwarf_UBVIJHK_colors_Teff.txt)



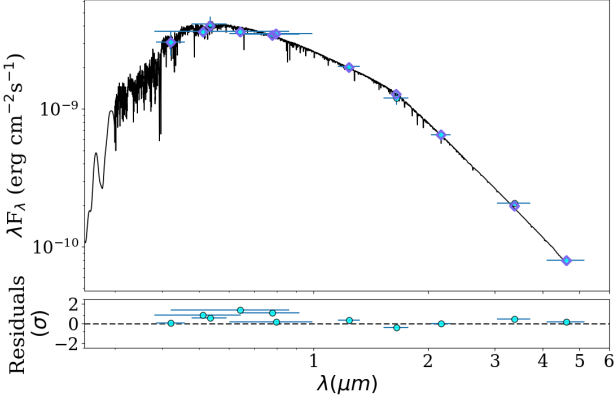
**Fig. 4.** Luminosity vs. effective temperature plot for the two-component model. Curves represent MIST isochrones for ages: 4.0 Gyr (cyan), 5.0 Gyr (purple), 6.3 Gyr (yellow), 7.0 Gyr (green), and for  $[\text{Fe}/\text{H}] = 0.0$ . The red point represents the positions of TOI-2458 with its error bars.

### 3.3. SED analysis with ARIADNE

We conducted an independent verification of the derived stellar parameters by modeling the spectral energy distribution (SED) using the spectrAl eneRgy dIstribution bAyesian moDel averaging fittEr (ARIADNE; Vines & Jenkins 2022). To determine the effective temperature  $T_{\text{eff}}$ , surface gravity  $\log g$ , metallicity  $[\text{Fe}/\text{H}]$ , and stellar luminosity and radius, we employed a combination of 3D dust Bayestar maps for interstellar extinction (Green et al. 2019) and four distinct models: Phoenix (Husser et al. 2013), BT-Settl (Allard et al. 2012), Kurucz (Kurucz 1993), and Ck04 (Castelli & Kurucz 2003). Our priors for these parameters were based on results obtained from iSpec. The derived final parameters, which are listed in Table 2, are in good agreement with previously derived parameters. The SED of TOI-2458 is shown in Fig. 5, along with the best fitting Phoenix atmospheric model (Husser et al. 2013).

### 3.4. Stellar rotation

The SPOC PDC pipeline module often eliminates a portion of the stellar variability. Therefore, we used the lightkurve package to retrieve the TESS target pixel files from the MAST



**Fig. 5.** SED fit for the TOI-2458 system. The blue points with vertical error bars show the observed catalog fluxes and uncertainties from *Gaia*, TESS, Tycho-2, 2MASS, and WISE, while the horizontal error bars illustrate the width of the photometric band. For illustrative purposes, we overplot the best-matching Phoenix atmospheric model (Husser et al. 2013).

archive. Subsequently, a suitable aperture mask was selected to obtain a light curve, which was then normalized and corrected for outliers. To eliminate systematics from the data, we utilized the pixel level decorrelation (PLD) method (Deming et al. 2015). Data points within transits were excluded from the analysis. The SPOC Simple Aperture Photometry (SAP) light curve peak-to-peak range is 1.9% while the peak-to-peak amplitude of the oscillation recovered by PLD appears to be  $\sim 5.6 \times 10^{-4}$ , which is 2.9% of the full range of the SAP light curve. To determine the stellar rotation, we employed the Gaussian Process Regression library, *Celerite2*. In Foreman-Mackey et al. (2017); Foreman-Mackey (2018), a detailed explanation of the library is provided. To account for variations in LCs that arise from heterogeneous surface features such as spots and plages, we used a rotational kernel function in the form of a sum of two stochastically driven damped simple harmonic oscillators (SHOs) with periods of  $P_{rot}$  and  $P_{rot}/2$ . This GP was successfully used in several papers studying stellar rotation (e.g., David et al. 2019; Gillen et al. 2020). The power spectral density of both terms is:

$$S(\omega) = \sqrt{\frac{2}{\pi}} \frac{S_k \omega_k^4}{(\omega^2 - \omega_k^2)^2 + 2\omega_k^2 \omega^2 / Q_k^2}, \quad (1)$$

where,

$$Q_1 = 1/2 + Q_0 + \delta Q, \quad (2)$$

$$\omega_1 = \frac{4\pi Q_1}{P_{rot} \sqrt{4Q_1^2 - 1}}, \quad (3)$$

$$S_1 = \frac{\sigma^2}{(1+f)\omega_1 Q_1}, \quad (4)$$

$$Q_2 = 1/2 + Q_0, \quad (5)$$

$$\omega_2 = \frac{8\pi Q_2}{P_{rot} \sqrt{4Q_2^2 - 1}}, \quad (6)$$

$$S_2 = \frac{f\sigma^2}{(1+f)\omega_2 Q_2}, \quad (7)$$

where  $\omega$  is the angular frequency,  $S_k$  is the amplitude of the oscillation,  $\omega_k$  is the undamped frequency,  $Q_k$  is the quality

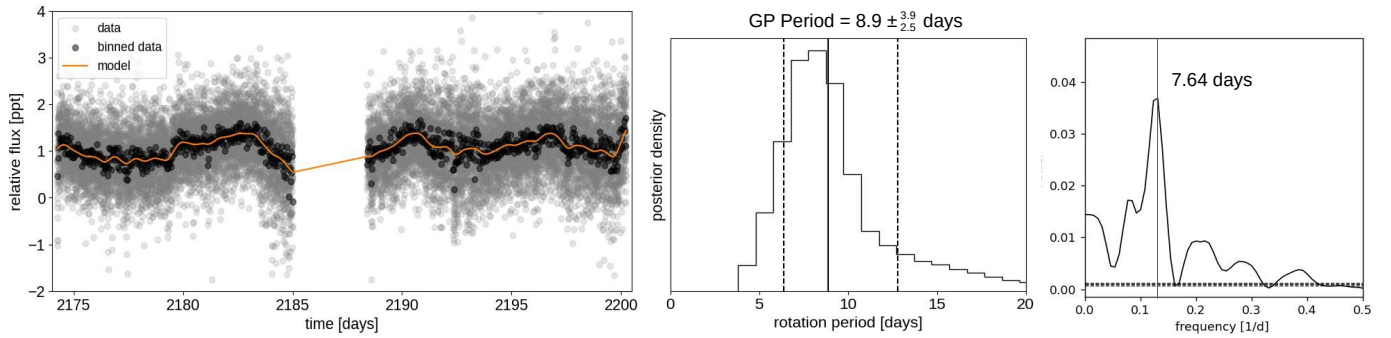
factor, and  $P_{rot}$  is the rotation period. The maximum a posteriori (MAP) parameters are estimated using the L-BFGS-B nonlinear optimization routine (Byrd et al. 1995; Zhu et al. 1997). To obtain the marginalized posterior distributions of free parameters, we ran a Markov chain Monte Carlo (MCMC) analysis using *emcee* (Goodman & Weare 2010; Foreman-Mackey et al. 2013). Our analysis yielded a derived rotational period of  $P_{rot} = 8.9^{+3.9}_{-2.5}$  days, which is shown in Fig. 6 as the probability density of  $P_{rot}$  alongside the MAP model prediction.

We have also applied the generalized Lomb-Scargle (GLS) periodograms (Zechmeister & Kürster 2009) to the light curve obtained from TESS to verify the derived rotational period independently. Our analysis indicates a significant peak around 7.64 days, as illustrated in Fig. 6. The stellar rotation signal observed in the GLS periodogram is consistent within error bars with the results obtained from the GP analysis.

### 3.5. TESS variability validation

The rotation period of TOI-2458 holds significant importance in further discussions regarding the system. To validate the variability in TESS LCs, we extracted cuts of TESS full-frame images around TOI-2458 and analyzed the variations in its neighboring stars. We used the same detrending vectors for each star to remove systematics. While none of the stars exhibit similar variability to TOI-2458, we still hesitate to rule out that we observe residual variations caused by systematic effects, given the suspicious phase that appears to be roughly synced with the spacecraft orbit. The WASP photometry has been able to exclude any variability of TOI-2458 down to a limit of 0.6 mmag, which is insufficient to negate the variability observed in the TESS LC. Hence, the null detection in the WASP photometry is likely due to the small photometric amplitude of the signal.

We investigated the robustness of the isolation of the stellar rotation signal by performing a series of injection and recovery experiments. We used the TESS PRFs to inject sinusoidal signals modeled after the isolated signal, with a period of 7.5 days and an amplitude of 50 electrons/s, into the TPF for TOI-2485 at six phases ( $0^\circ$ ,  $45^\circ$ ,  $90^\circ$ ,  $135^\circ$ ,  $180^\circ$ , and  $270^\circ$ ) with respect to the stellar rotation signal. We also tested an injection with half of the amplitude at  $90^\circ$  with similar results. A customized PLD algorithm with an automatic corrector parameter optimization (Rapetti et al. 2024) (Rapetti et al., in prep.) was then applied to both the original and the modified TPFs. The PLD-corrected light curves from the original TPFs were subtracted from those obtained from the injected TPFs to produce residual light curves that should only contain the injected stellar rotation signal. In general, PLD was able to recover the injected sinusoid at approximately the correct amplitude and phase, with some dependence on the phase angle with respect to long-term trends in the residual light curve and distortions in the recovered waveform near the edges of each orbit that were generally stronger in the first several days of the observations in each orbit. An alternative bespoke corrector based on that of Hedges et al. (2021) was also investigated in this manner and provided comparable results. A third regression corrector utilizing the SPOC cotrending basis vectors (CBVs) tended to overfit the data and not recover the injected signals, except for the case with a phase shift of  $270^\circ$ . These results increase confidence in the interpretation of the 7.5-day signal as being intrinsic to the star. However, the ratio of the peak-to-peak range of the suspected stellar oscillation ( $\sim 56$  ppm) compared to the peak-to-peak range of the systematic effects in the simple aperture light curve ( $\sim 19\,300$  ppm), along with the similarity in the oscillation period with those at which instrumental effects



**Fig. 6.** Left: TESS PLD LC (gray points) with the MAP model prediction. It is important to note that the long trends and transits have been removed prior to analysis. The orange line shows the predictive mean. Middle: probability density of the rotation period. The  $1\sigma$  error bar is indicated with the dashed black lines. Right: GLS periodogram of the TESS PLD LC which was processed in accordance with the methods outlined in the text.

can occur, highlight the need for some caution with respect to this interpretation.

Additional high-precision photometry is required to confirm the rotation period of the star. Nevertheless, we have found a similar signal in the HARPS H $\alpha$  activity indicator, as discussed in Section 5. Furthermore, in Section 4.3, we discuss that TOI-2458’s rapid stellar rotation would not be surprising. It is imperative to confirm the star’s rotation period for a more comprehensive validation of the system architecture. However, corresponding to our current best understanding, we elaborate on this potential scenario and assume that this is the star’s rotation period for the remainder of this work.

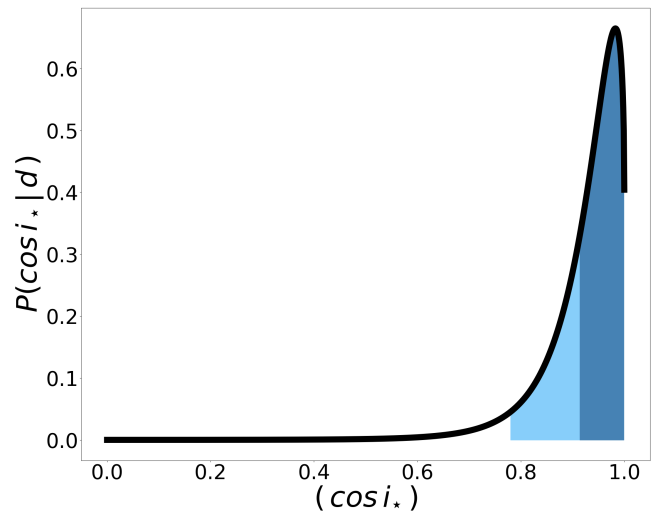
Stellar inclination angle can be determined by combining of  $P_{rot}$ ,  $v \sin i$ , and  $R_*$ , yielding a value for TOI-2458 of  $i_* = 10.6^{+13.3}_{-10.6}$  degrees. The methodology employed in this analysis follows the procedure outlined in Masuda & Winn (2020) and implemented in Bowler et al. (2023). This approach correctly accounts for the correlation between stellar equatorial velocity and projected rotational velocity. The posterior distribution of stellar inclination can be viewed in Fig. 7.

### 3.6. Age analysis

In this section, we have employed various techniques to constrain the age of TOI-2458. Our analysis indicates that stellar isochrones (see Section 3.1) and all age indicators, except gyrochronology, are consistent with a relatively old age of  $5.7 \pm 0.9$  Gyr.

#### 3.6.1. Lithium EW

The 6708 Å lithium line was employed to derive the equivalent width (EW) using the iSpec tool. The EW of 0.055 Å was determined by fitting the line with a Gaussian profile and calculating the area under the resulting curve. Subsequently, in Fig. 8, we compared the EW of lithium to that of well-established members of studied clusters from the literature. These are the Tuc-Hor young moving group (~45 Myr; Mentuch et al. 2008), the Pleiades (~120 Myr; Soderblom et al. 1993b), M34 (~220 Myr; Jones et al. 1997), Ursa Major Group (~400 Myr; Soderblom et al. 1993c), Praesepe (~650 Myr; Soderblom et al. 1993a), Hyades (~650 Myr; Soderblom et al. 1990), and M67 clusters (~4 Gyr; Jones et al. 1999). If necessary, we dereddened clusters using E(B-V) values obtained from Gaia Collaboration (2018). Our findings reveal that TOI-2458 exhibits quite a high abundance of lithium. However, our measurements are still consistent with those obtained for the 4 Gyr-old cluster M67.



**Fig. 7.** Posterior distributions for line-of-sight stellar inclination in the form of  $\cos i_*$ . Blue shadow regions show  $1\text{-}\sigma$  and  $2\text{-}\sigma$  credible intervals.

#### 3.6.2. $R'_{HK}$ index

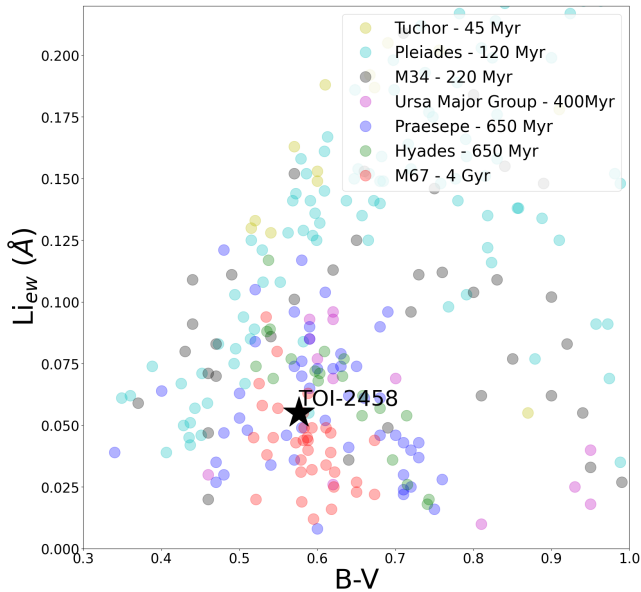
To obtain a time-averaged measurement of the activity index of TOI-2458,  $\log R'_{HK} = -4.98 \pm 0.06$ , we utilized standard relations to convert the time-averaged S-index measurements from our spectroscopy (Noyes et al. 1984; Mittag et al. 2013) using PyAstronomy<sup>5</sup>. The correlation between  $R'_{HK}$  and  $P_{rot}$  observed in numerous stars (e.g., Lovis et al. 2011; Suárez Mascareño et al. 2015). Utilizing the relationship established by Suárez Mascareño et al. (2015), it is possible to convert the value of  $\log R'_{HK} = -4.98 \pm 0.06$  to a rotation period ranging from 23 to 40 days. This conversion indicates that the  $\log R'_{HK}$  index for TOI-2458 is consistent with relatively advanced stellar age, and the expected rotation period is longer than observed. Based on the research conducted by Mamajek & Hillenbrand (2008), the derived value of the  $\log R'_{HK}$  index is consistent with the typical values observed in field stars.

#### 3.6.3. Gyrochronology

To determine the age of TOI-2458, we compared its rotation period with the clusters analyzed in Godoy-Rivera et al. (2021).

<sup>5</sup> <https://pyastronomy.readthedocs.io/en/latest/pyaslDoc/aslDoc/sindex.html>

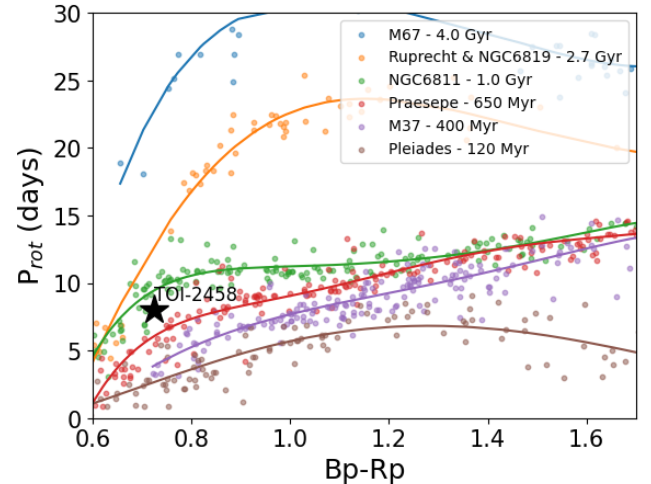




**Fig. 8.** Color vs. EW of lithium line Li 6708 Å of TOI-2458 together with members of well-studied clusters.

These clusters include the Pleiades cluster ( $\sim 120$  Myr), M37 cluster ( $\sim 400$  Myr), the Praesepe cluster ( $\sim 650$  Myr), NGC 6811 ( $\sim 1$  Gyr). Additionally, we used several older clusters: Ruprecht 147 and NGC 6819 clusters ( $\sim 2.7$  Gyr) from [Curtis et al. \(2020\)](#) and M 67 cluster ( $\sim 4$  Gyr) from [Barnes et al. \(2016\)](#). Using the `dustmaps` code ([Green 2018](#)) and three-dimensional Bayestar dust maps ([Green et al. 2019](#)), we estimated the reddening  $E(B-V)$  for each star in these clusters. We then employed the method from [Gaia Collaboration \(2018\)](#) to calculate reddening in the *Gaia* colors. Fig. 9 shows the position of TOI-2458 in the rotation period vs. *Gaia* DR3  $G_{BP}$  minus  $G_{RP}$  magnitudes plane together with all studied clusters. Our findings show that the star TOI-2458, highlighted with a black star symbol, appears much younger than in other age-dating techniques. With an estimated age between 0.65 and 1.00 Gyr, TOI-2458 falls between the ages of the Praesepe cluster and NGC 6811. It is worth noting that although the older clusters have few members with a measured rotation period at TOI-2458’s color, the field stars in this area generally have longer rotation periods ([Dungee et al. 2022](#)).

In contrast to gyrochronology, the techniques previously discussed for determining the age of TOI-2458 are consistent with the older ages derived from stellar isochrones. Furthermore, the predicted rotation period based on the  $\log R'_{HK}$  index ranges from 23 to 40 days (longer than observed). However, the relationship between  $\log R'_{HK}$  and the rotation period stems from a complex relationship that lacks complete theoretical understanding. In particular, the impact of increased stellar rotation resulting from planet engulfment on this correlation remains ambiguous. Similarly, the age derived from gyrochronology cannot be accurately ascertained for stars whose rotation is influenced by either a stellar or planetary companion ([Benbakoura et al. 2019](#)). Therefore, we propose that recent planet engulfment may elucidate the atypical rotation period observed within the context of gyrochronology, as well as the relationship between  $\log R'_{HK}$  and  $P_{rot}$ . Further elaboration on this concept is provided in Section 4.3, where we illustrate that similar unusual rotation periods are also observed for other stars comparable to TOI-2458.

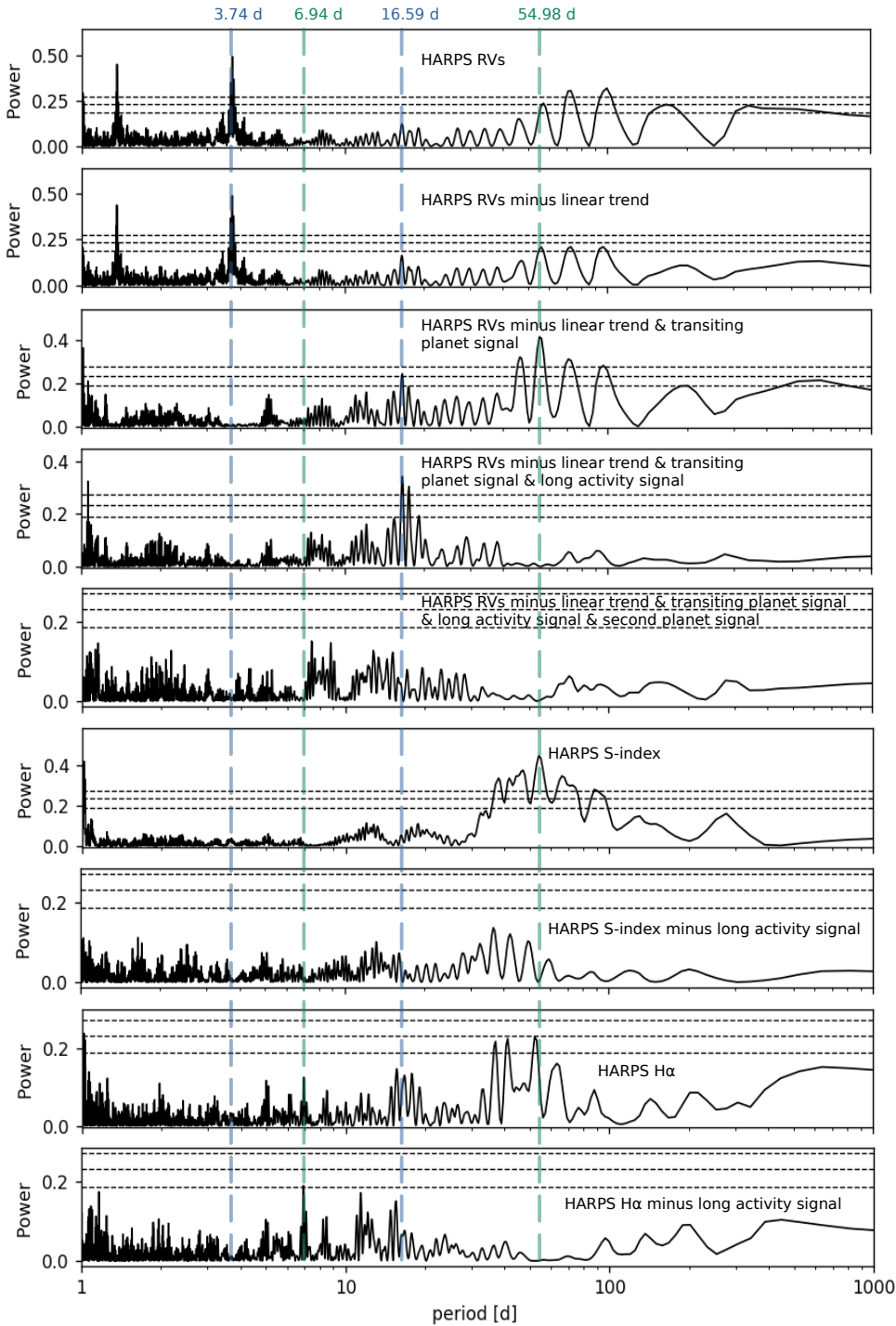


**Fig. 9.** Color-period diagram of TOI-2458 together with members of well-studied clusters. Lines represent the polynomial fit to each cluster sequence.

### 3.7. Frequency analysis

In an effort to distinguish the Doppler reflex motion from planetary candidates and stellar activity and to detect the existence of plausible additional signals, a frequency analysis was performed on the RVs,  $H\alpha$ , and S-index values of the spectra produced by the TERRA pipeline. The GLS periodograms ([Zechmeister & Kürster 2009](#)) of the available time series were computed, and the theoretical false alarm probability (FAP) levels of 10%, 1%, and 0.1% were determined, as depicted in Fig. 10. The observation baseline spans approximately 400 d, resulting in a frequency resolution of roughly  $1/400 = 0.0025 \text{ d}^{-1}$ . Initially, we implemented the pre-whitening technique incorporating the potential linear trend. This method did not alter the signals identified in the periodogram, which are now more clearly delineated. The most significant signal observed in the periodogram of the RVs has a period of 3.74 d, which is consistent with the period of the transiting planet from TESS photometry. After removing this signal, the periodogram of the RV residuals exhibits a signal with a period of  $\sim 54$  d, which is also present in both activity indicators. Although several other peaks are visible due to observation sampling. We attribute this signal to a short-term magnetic cycle observed in a group of other F-type stars. We elaborate further on this matter in Section 4.3. Following the removal of the 54-d signal from RVs, an additional peak at 16.6 d is observed, which we attempted to fit as the second planet in Section 3.8, as it does not have a counterpart in any activity indicator. It is noteworthy that a second peak near the 16.6-d peak of nearly equal amplitude is also observed. These two peaks are 1-year alias to each other, making it challenging to ascertain the real signal. In the joint modeling in Section 3.8, a sufficiently wide prior was adopted to encompass both peaks.

In order to ensure the robustness of results obtained from the TERRA pipeline, we conducted a sanity-check exercise using the SERVAl pipeline ([Zechmeister et al. 2018](#)) to extract RV measurements from the HARPS spectra. Our findings revealed that all signals reported earlier remained significant in both the TERRA and SERVAl datasets. Additionally, the 7-d signal was observed in the  $H\alpha$  activity indicator after the removal of the long activity signal. While the 7-day signal in the  $H\alpha$  activity indicator is at the 10% FAP level, the presence of a peak roughly at the same periodicity detected in the TESS photometry (which



**Fig. 10.** Generalized Lomb-Scargle periodograms of HARPS RVs of TOI-2458 from top to bottom: (a) HARPS RVs, (b) HARPS RVs minus linear trend, (c) HARPS RVs minus linear trend and transiting planet signal, (d) HARPS RVs minus linear trend, transiting planet signal and long activity signal, (e) HARPS RVs minus linear trend, transiting planet signal, long activity signal and second planet signal, (f) HARPS S-index, (g) HARPS S-index minus long activity signal, (h) HARPS  $H\alpha$ , (i) HARPS  $H\alpha$  minus long activity signal. The vertical blue lines highlight the orbital period of the transiting companion and the second companion. The vertical green lines highlight the stellar activity signals. The 6.94-day line indicates the peak observed in the  $H\alpha$  activity indicator (last panel), which is near the rotation period identified in the TESS photometry. The 55-day line highlights the maximum signal from the S-index activity indicator. Horizontal dashed lines show the theoretical FAP levels of 10%, 1%, and 0.1% for each panel.

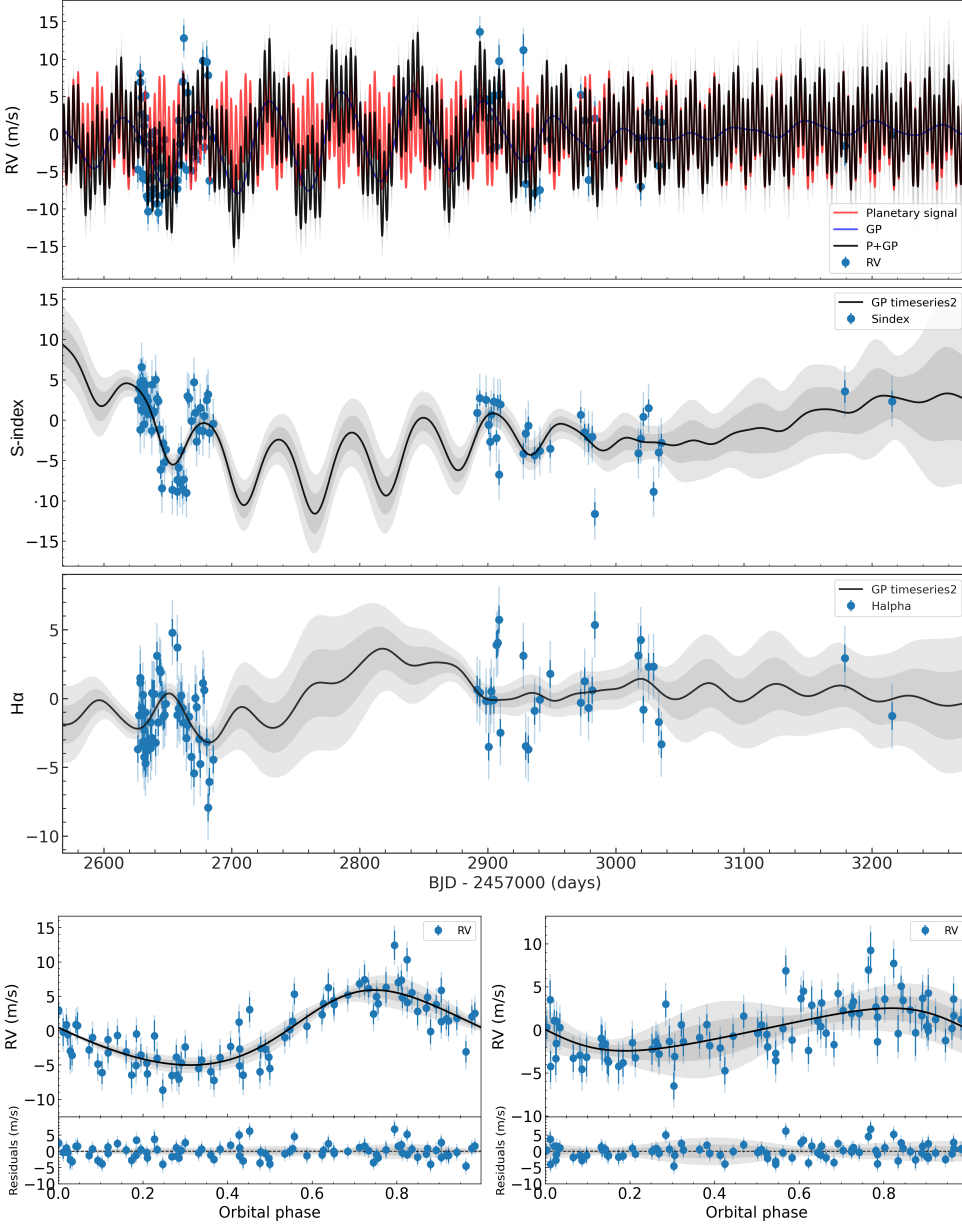
we attribute to the rotation period of the star) supports the short-period scenario. The prominent peak in the S-index periodogram exhibits a period of approximately 55 days, with several additional peaks being visible, which are 1-year aliases of each other. It is important to note that periodograms of several other activity indicators from the TERRA and SERVAL pipelines did not reveal any of the discussed signals, and therefore, they are excluded from this study.

### 3.8. Joint RVs and transits modeling

We utilized the `pyaneti` package (Barragán et al. 2019; Barragán et al. 2022) to conduct a joint analysis of transit

photometry and radial velocity data. `Pyaneti` is a software program written in PYTHON/FORTRAN that utilizes a MCMC sampler based on EMCEE (Foreman-Mackey et al. 2013) to generate marginalized posterior distributions of exoplanet RV and transit parameters. `Pyaneti` is capable of conducting multi-planet fits with RV and transit data, together with multidimensional GP regression to simultaneously model stellar activity signals in Doppler data and activity indicators.

We have performed a comprehensive analysis of the transit photometry and radial velocity data. To account for the variability present in the TESS LC, we employed the activity-corrected LC that was processed using the procedure outlined in Section 2.1. As discussed in Section 5, the RV data contain



**Fig. 11.** The radial velocity curve of TOI-2458, fitted with the `pyaneti` as described in Section 3.8. The top panel shows the HARPS RV time series with all signals included. The middle panels show the HARPS S-index and  $H\alpha$  time series, while the two bottom figures show the phased RV curves for TOI-2458 b (left) and TOI-2458 c (right). The middle panels present the outputs from two separate analyses using RVs with the S-index and RVs with  $H\alpha$ , respectively. This approach was used to save computational time, and we note that both analyses yield consistent results. The phased RV curves have been corrected for stellar activity using the GP model, which has been subtracted from the data.

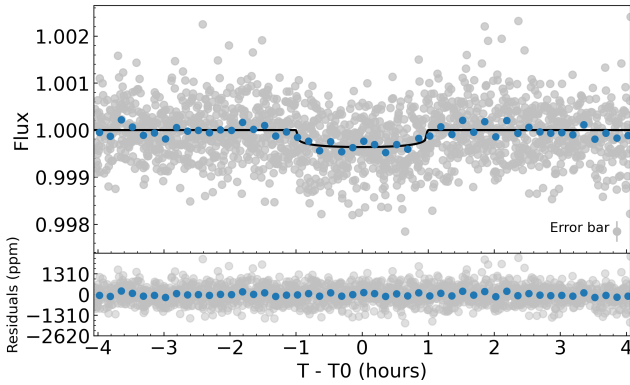
stellar activity. Therefore, we incorporated a multidimensional GP model, which has been previously described by Georgieva et al. (2021) and Barragán et al. (2022), to account for the stellar variability. The RV data and the S-index activity indicator were modeled together, with the assumption that they could be described using the same underlying GP function. To achieve this, we utilized a quasi-periodic covariance function:

$$\gamma_{QP,i,j} = \exp \left[ -\frac{\sin^2 \left[ \pi(t_i - t_j) / P_{GP} \right]}{2\lambda_p^2} - \frac{(t_i - t_j)^2}{2\lambda_e^2} \right], \quad (8)$$

with  $P_{GP}$  representing the period of the activity signal,  $\lambda_p$  indicating the inverse of the harmonic complexity, and  $\lambda_e$  representing the long-term evolution timescale. We have set the prior for  $P_{GP}$  around the 54-day activity signal. The signal is clearly identifiable in RVs and activity indicators, and it corresponds to a short-term magnetic cycle observed in various F-type stars, as discussed in Section 4.3. It is important to note that when we repeated the analysis using the  $H\alpha$  activity indicator

instead of the S-index activity indicator, we obtained consistent results.

In this study, two models were compared, one with a single transiting planet and an activity, and the other additionally accounting for a second planet found in frequency analysis, as explained in Section 5. The models were evaluated by computing the Bayesian model log evidence ( $\ln Z$ ), a widely used metric. If the difference in  $\ln Z$  between the models is less than two, they are considered indistinguishable (Trotta 2008). The analysis produced  $\Delta \ln Z = 16$ , indicating a preference for the two-planet model. The quadratic limb darkening law was employed, with coefficients  $q_1$  and  $q_2$ . The posteriors of the fitted parameters and the derived physical parameters are available as supplementary material on Zenodo (see Section 5). The RVs with the RV model, along with the phased RVs of both planets, are shown in Fig. 11. The phased LC with the transit model is displayed in Fig. 12. Finally, the correlations between the main free parameters and the derived posterior probability distributions are available as supplementary material on Zenodo (see Section 5). We wish to also highlight that we have tried remodeling the data,



**Fig. 12.** The phased LC of TOI-2458 b, fitted with `pyaneti` as described in Section 3.8. The gray points represent TESS data, and the blue points are TESS binned data in the phase curve with a bin width of 10 minutes in the phase. The black line represents the best transit and eclipse model.

permitting a larger prior for the GP long-term evolution timescale of up to 1000 days. Despite this adjustment, the fitted parameters for both planets remained consistent within their respective error bars.

Our study detects a Neptune-mass planet, with an RV semi-amplitude of  $K = 5.3 \pm 0.4 \text{ m s}^{-1}$ , and a transit depth of  $\delta = 363 \pm 51 \text{ ppm}$ . We derived an eccentricity of  $e = 0.085^{+0.072}_{-0.055}$ . To investigate the significance of the noncircular orbit, we computed the Bayesian model log evidence by comparing models with fixed zero eccentricity and eccentricity as a free parameter. The resulting value of  $\Delta \ln Z = 1.5$  indicates that we are unable to differentiate between a circular or noncircular orbit.

The timing of the second planet’s transit remains uncertain, and it could potentially be located anywhere within the TESS LC. Even the gap situated in the middle of the sector cannot be ruled out as a possible location. Despite the lack of any single-transit event indication in the TESS LC, given the planet’s unknown size, it is still possible that the planet transits. Our analysis has revealed a minimum mass of approximately  $10 M_{\oplus}$ , and the planet’s orbital period is approximately 16.6 d. The eccentricity of the planet remains unconstrained.

### 3.9. Dynamical analysis

The TOI-2458 system is fascinating from the point of view of its dynamical architecture and evolution. The orbit of the transiting planet is highly inclined to the stellar spin, which raises questions about how this rare configuration originated. While the semimajor axes of the planets are well determined, the inclination of the outer planet, TOI-2458 c, is unknown. Currently, there is no way to fully determine this inclination and the mass of the outer planet except by analyzing the effects of the mutual gravitational interactions of the companions, which may constrain the orbital architecture of the system. Fortunately, our analysis shows that the long-term stability requirement and the expected orbital evolution within the physically allowed ranges are indeed meaningful in this case. We summarize our findings and provide all details of the analysis with corresponding figures in the Appendix A.

We found that stable regions appear only in a portion of the parameter  $(\Delta\Omega, i_c)$ -plane outside mutual inclinations approximately  $i_{mut} \in (60^\circ, 140^\circ)$ . This implies moderate mutual inclinations. Considering that  $i_b = 84^\circ \pm 0.5^\circ$ , then  $i_c \simeq (24^\circ, 156^\circ)$  and its mass is in the range  $M_c \simeq (10, 25) M_{\oplus}$ . The overall 3D

dynamical structure of the TOI-2458 system is very complex despite apparently well-separated planets with low masses. The results of the dynamical simulations constrain the observed system to relatively small islands of moderate inner, close-in planet eccentricity when the orbital planes are moderately inclined.

## 4. Discussion

### 4.1. Mass-radius diagram

Combining data from the TESS mission and HARPS spectroscopy, we detected and confirmed the planetary nature of the  $P = 3.7 \text{ d}$  transiting candidate around the F-type star TOI-2458. We found the fundamental parameters of TOI-2458 b to be  $M_p = 13.31 \pm 0.99 M_{\oplus}$  and  $R_p = 2.83 \pm 0.20 R_{\oplus}$ . Additionally, HARPS RVs revealed the presence of a second planet with a period of  $P = 16.6 \text{ d}$  and a minimum mass of  $M_p \sin i = 10.22 \pm 1.90 M_{\oplus}$ .

We present a comparative analysis of the mass and radius of TOI-2458 b with other well-characterized planets of comparable mass and size. Fig. 13 showcases the analysis results through three subplots distinguished by their color coding based on different parameters such as orbital period, equilibrium temperature, and transmission spectroscopic metric. The placement of TOI-2458 b on the mass-radius diagram is rather conventional, with several planets positioned within the middle of the radius range observed for planets orbiting F-type stars. These planets align with composition tracks corresponding to water content ranging from 50–100% and exhibit moderate temperatures. Another subset of highly irradiated planets is situated near the composition track for pure silicate rocks. The last subset comprises typically long-period companions with radii of approximately  $3.5 R_{\oplus}$ . Additionally, due to the relatively large size of the primary star, the TOI-2458 b’s transmission spectroscopy metric of 25 is quite low, although comparable with other planets in this region. Nonetheless, it presents a challenging target for atmospheric characterization. When compared with the mass-radius tracks from Zeng et al. (2019), the planet appears to be composed of a large fraction of silicate rocks and water.

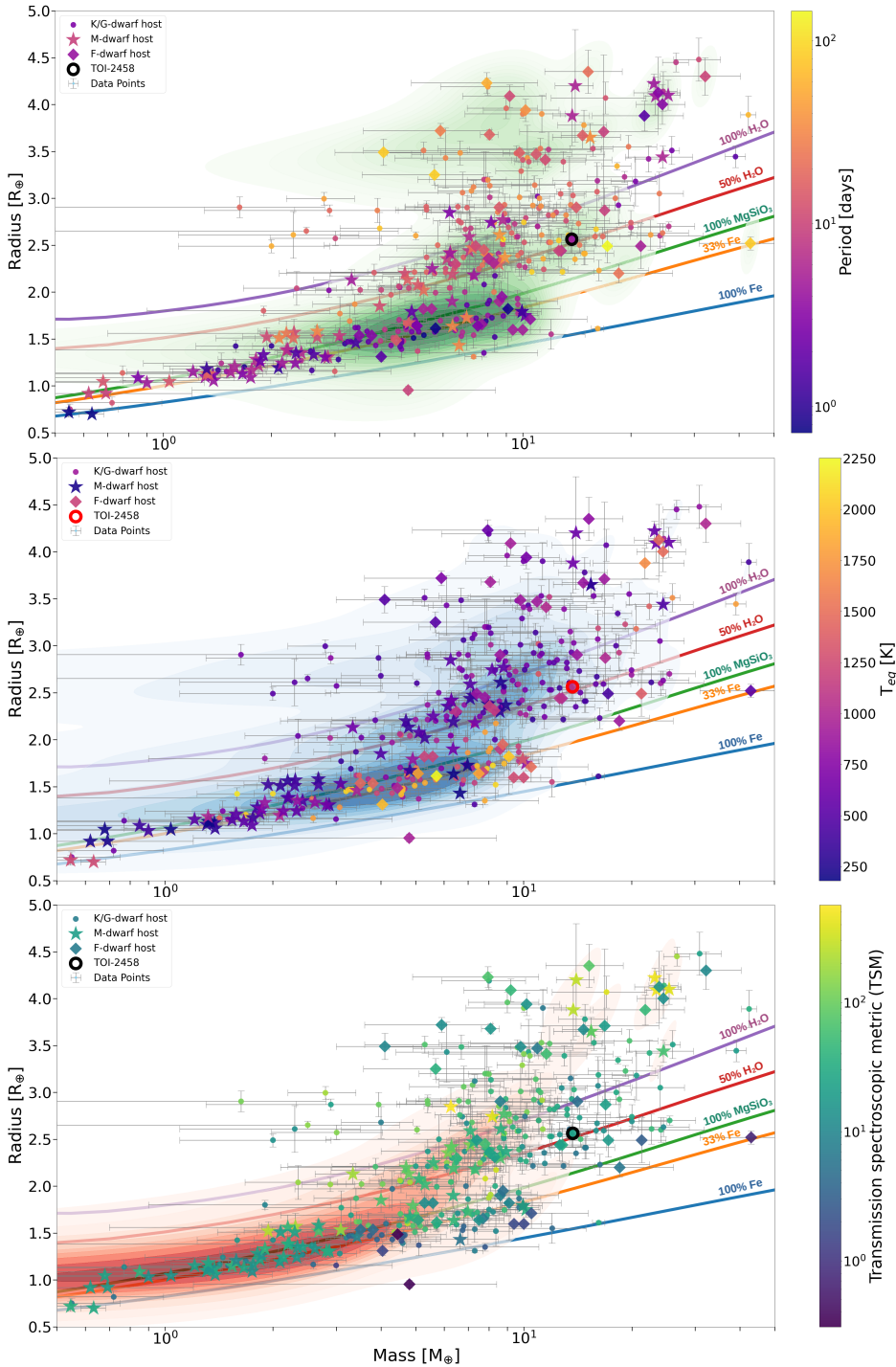
We can compare TOI-2458 b with HD 106315b (Barros et al. 2017), as both exoplanets share comparable mass, radius, and stellar hosts. HD106315b, orbiting an F-type star with a period of 9.55 days, exhibits a mass of  $12.6 \pm 3.2 M_{\oplus}$  and a radius of  $2.44 \pm 0.17 R_{\oplus}$ . Similarly to the TOI-2458 system, the HD 106315 system features a second outer companion with a mass of  $15.2 \pm 3.7 M_{\oplus}$ , a radius of  $4.35 \pm 0.23 R_{\oplus}$ , and an orbital period of approximately 21 days. The authors discuss that the distinct densities of the two planets may stem from disparate formation regions.

TOI-2458 b has a radius that positions it slightly above the known radius valley (Fulton et al. 2017). There are currently several hypotheses behind the origin of these distinct populations of low-mass exoplanets, such as photo-evaporation (Owen & Wu 2017), core-powered mass loss (Ginzburg et al. 2018), or migration after formation beyond the ice-line (Mordasini et al. 2009).

### 4.2. Transiting planet composition

We have utilized a machine learning model called ExoMDN (Baumeister & Tosi 2023), which is based on mixture density networks, to model the interior structure of TOI-2458 b. This model has been trained on a massive dataset of over 5.6 million synthetic planets that are less than 25 Earth masses in order to create a highly accurate representation of their internal

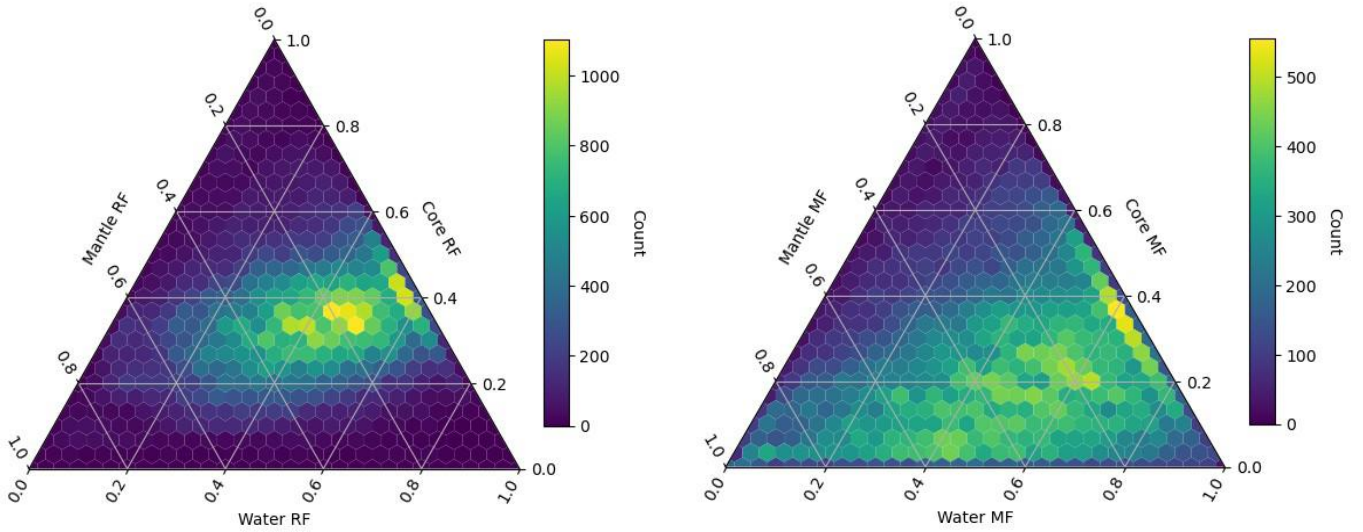




**Fig. 13.** The population of known companions within the mass interval  $0.5\text{--}50 M_{\oplus}$ . The position of TOI-2458 b is highlighted. The top subplot is color-coded based on the orbital period of the planets, the middle one based on the equilibrium temperature, and the bottom one based on the transmission spectroscopic metric of the planets. The background shows the kernel density estimate (KDE) with green, blue, and red colors representing planets around F, K/G, and M stars, respectively. The mass-radius tracks from Zeng et al. (2019) are utilized.

composition. This synthetic dataset, created using the TATOOINE code (Baumeister et al. 2020; MacKenzie et al. 2023), consists of planets with an iron core, a silicate mantle, a water and a high-pressure ice layer, and a H/He atmosphere. ExoMDN consists of two trained models: one trained on planetary mass, radius, and equilibrium temperature, and the second includes the fluid Love number  $k_2$  as an additional input, which is used to overcome degeneracy between layers. The fluid Love number  $k_2$  is a parameter that depends on the density distribution of the planet and can be computed for some planets, as highlighted in previous studies (Csizmadia et al. 2019; Akisanmi et al. 2019). Another potential method to mitigate degeneracy in planetary interior

modeling is to use the elemental abundances of the host star, which may provide insights into the planet’s bulk composition and atmospheric composition (Dorn et al. 2015). However, such an approach requires making additional assumptions regarding the planet’s formation and evolutionary history. In the case of TOI-2458 b, this planet does not conform to the known track of any planet with a  $k_2$  detection. Consequently, we have chosen not to incorporate such a constraint in our analysis, thereby broadening the range of potential interior configurations. To account for the uncertainties associated with planetary parameters, we model normal distributions with a standard deviation equivalent to the uncertainties of each parameter.



**Fig. 14.** Ternary plots of the interior structure of TOI-2458 b. The left panel shows the radius fraction configuration, and the right panel shows the mass fraction configuration.

Analysis of TOI-2458 b suggests a core-mass fraction of  $0.20^{+0.36}_{-0.18}$ , a mantle-mass fraction of  $0.30^{+0.47}_{-0.27}$ , a water-mass fraction of  $0.43 \pm 0.36$ , and a volatile element mass fraction of  $0.001^{+0.020}_{-0.001}$ . The core-radius fraction is estimated to be  $0.29 \pm 0.16$ , while the mantle-radius fraction is  $0.22^{+0.29}_{-0.19}$ , and the water-radius fraction is  $0.34^{+0.23}_{-0.26}$ . The atmosphere-radius fraction constitutes  $0.12^{+0.18}_{-0.09}$  of the total radius. Due to the large uncertainty in these parameters, several different planetary configurations are possible, such as one where the mantle mass fraction is close to zero. To achieve a more precise determination of the radius, additional high-precision photometric observations are necessary. Unfortunately, it is very difficult to establish the actual structure of planets similar to TOI-2458 b using mass and radius alone. These planets lie in a region of the parameter space where several composition tracks are in close proximity to each other. The mass-radius configuration of TOI-2458 b is equally consistent with a water-rich world without an extended volatile atmosphere and a rocky core with an  $H_2$  layer. To fully ascertain the nature of these scenarios, a further characterization of the planet’s atmosphere is required. Ternary plots from the analysis are presented in Fig. 14.

#### 4.3. Stellar activity

We report on the activity cycle of TOI-2458, which exhibits a period of 53 days. This cycle is the second shortest on record for an F-type star. The first star discovered with a short cycle was  $\tau$  Boo (HD 120136), an F7 spectral-type star with a hot Jupiter companion in a close orbit of 3.3 days and an M2-type companion at the separation of 220 au. The rotation period of  $\tau$  Boo is found to be in close proximity to 3.3 days, indicating synchronization. Nonetheless, this star is noteworthy because of additional variability in the Ca II lines, characterized by a surprisingly short period. This finding is supported by Baliunas et al. (1997), who determined a period of 116 days, which was later confirmed by Mengel et al. (2016) and Mittag et al. (2017) investigating the S-index with the NARVAL and TIGRE instruments, respectively. Mittag et al. (2017) further showed that the X-ray data support a periodicity of approximately 120 days, raising an intriguing question about the representative nature of

$\tau$  Boo’s fast activity period for main-sequence F stars. To address this question, Mittag et al. (2019) employed an analysis of the S-index time series of F-type stars taken with the TIGRE telescope. They detected three additional short-term cycles and one candidate cycle between 0.5–1.0 year. Moreover, Šubjak et al. (2023) found that HD 46588 is a twin of  $\tau$  Boo with respect to the activity cycle. The star is of the same F7 spectral type and hosts an L9 type companion at a projected physical separation of 1420 au (Loutrel et al. 2011). Using the SONG spectrograph, the authors found evidence of a chromospheric cycle with a period of 127 days. Recently, Seach et al. (2022) found that stars  $\theta$  Dra and  $\beta$  Vir also exhibit short-term activity cycles both well below 100 days.

We investigate a sample of stars exhibiting short-term activity cycles with the aim of identifying common characteristics among them, focusing on their age, rotation period,  $R'_{HK}$  index, and close companions. The findings are reported in Table 3, which includes the relevant parameters for all stars. Our analysis reveals that TOI-2458 displays the second shortest cycle ever observed and is also the oldest system in the sample. However, we observe no discernible correlation between age and period. Our previous analysis (Section 3.6) highlights that TOI-2458 exhibits a surprisingly short rotation period compared to its age. This may be a feature common to the sample, as, despite their advanced age, many of the stars rotate at a fast speed. Obvious are HD 46588,  $\theta$  Dra, and  $\beta$  Vir, which, similarly to TOI-4987, exhibit low  $R'_{HK}$  indexes and advanced ages yet also possess unexpectedly short rotation periods.  $\beta$  Vir is an analog to TOI-2458. The star has a rotation period of 9 days, an activity cycle of 83 days, an age of 3 Gyr, and a  $\log R'_{HK}$  of field stars ( $-4.91$ ).

The rotational period of stars can be influenced by their nearby companions through tidal interactions (e.g., Fleming et al. 2019; Santos et al. 2019). Tidal friction causes the companion to migrate inward and to spin up the star gradually. Furthermore, the presence of a hot Jupiter in close proximity to a star, as postulated by Fares et al. (2009), may be responsible for accelerating the star’s cycle by synchronizing its outer convective envelope due to tidal interactions. As such, the presence of close companions can offer an explanation for observed phenomena. For instance,  $\tau$  Boo has a near companion with an orbital period of approximately 3.3 days, which appears synchronized

**Table 3.** Sample of F-type stars with a short activity cycle.  $P_{cyc}$  is a period of a short activity cycle.

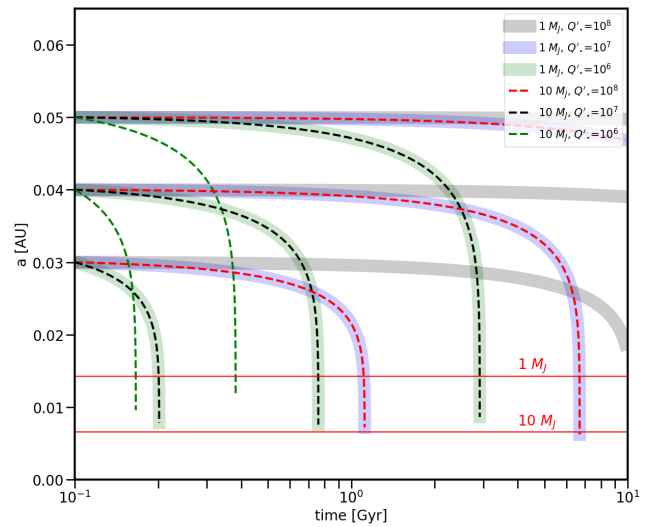
System	TOI-2458	$\tau$ Boo	HD 46588	HD 16673	HD 49933	HD 100563	HD 75332	$\theta$ Dra	$\beta$ Vir
Age (Gyr)	$5.7^{+0.9}_{-0.8}$	$0.85^{+0.81}_{-0.41}$	$2.1^{+1.3}_{-0.8}$	$1.20^{+1.20}_{-0.55}$	$2.2^{+0.3}_{-0.3}$	$1.4^{+0.6}_{-0.4}$	$2.0^{+0.6}_{-0.5}$	$2.1^{+0.2}_{-0.2}$	$3.1^{+0.8}_{-0.8}$
$P_{Rot}$ (days)	$8.9^{+3.9}_{-2.5}$	$3.3 \pm 0.4$	$5.1 \pm 0.6$	$6.0 \pm 0.8$	$3.5 \pm 0.3$	$2.0 \pm 0.3$	$3.75 \pm 0.35$	$2.9 \pm 0.1$	$9.2 \pm 0.3$
$\log R'_{HK}$	$-4.98 \pm 0.06$	$-4.67 \pm 0.06$	$-4.88 \pm 0.06$	$-4.60 \pm 0.06$	$-4.50 \pm 0.06$	$-4.64 \pm 0.06$	$-4.46 \pm 0.06$	$-4.68 \pm 0.06$	$-4.91 \pm 0.06$
$P_{cyc}$ (days)	53	120	127	309	212	223	180	43	83

with the star’s rotation period. However, most stars do not exhibit indications of having short-period giant companions. We cannot completely rule out this scenario, as their orbits’ orientation may possibly hinder detection, or these planets may have been engulfed by their host stars. If the orbital angular momentum is too low to enable spin-orbit synchronization, tidal evolution would ultimately result in planet engulfment (Hut 1980). Massive planets on very tight orbits can tidally migrate inward and spin up their star within the main-sequence lifetime (Carone & Pätzold 2007; Jackson et al. 2009; Tejada Arevalo et al. 2021). In their study, Tejada Arevalo et al. (2021) have uncovered evidence suggesting that hot Jupiters modify the rotation of their host stars during the main sequence. This determination was made by comparing the rotation periods of Sun-like stars hosting hot and cold Jupiters, as well as low-mass planets. Moreover, Jackson et al. (2009) have presented evidence indicating that the observed orbital distribution of exoplanets is consistent with the predictions of tidal theory. This suggests that the tidal destruction of close-in exoplanets is a common phenomenon. For instance, the planetary system WASP-12 is notable for hosting a hot Jupiter with a decaying orbit (Yee et al. 2020).

We employ the same equations used by Jackson et al. (2009) to investigate the tidal evolution of planets with masses of 1 Jupiter mass and 10 Jupiter masses orbiting F-type stars. Specifically, we examine the orbital decay of these planets and the time required for them to cross the Roche limit around the host star, beyond which they would be tidally disrupted and accreted by the star. The equations we use assume that the period of rotation for host stars is always substantially longer than the planet’s orbital period. We also assume zero eccentricity, meaning that only the tides raised on the star by the planet contribute to its tidal evolution. Figure 15 shows a plot of the tidal evolution of the semimajor axis. The results indicate that the time taken by planets to reach the Roche limit is often less than 1 Gyr for several initial conditions and decreases for higher-mass companions at smaller orbital distances. Oetjens et al. (2020); Ahuir et al. (2021) studied the influence of planetary engulfment on the stellar rotation of main-sequence stars. Their results indicate a large increase in stellar rotation velocity while the star is still in the main-sequence evolutionary phase. It is worth noting that such an over-rotation is likely to last for a few billion years (Ahuir et al. 2021). In the subsequent section, we explore the possibility of an unobserved hot Jupiter companion in the TOI-2458 system.

#### 4.4. System architecture

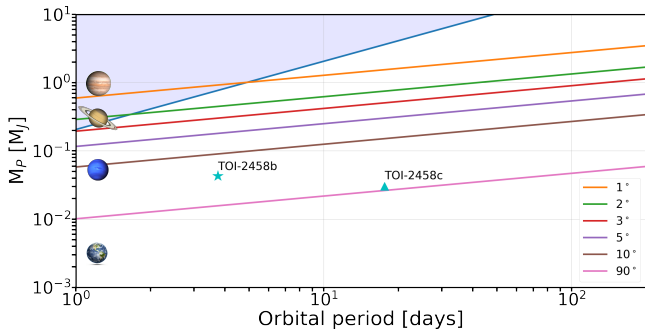
The TOI-2458 system is a promising candidate for testing the hypothesis of the presence of hot Jupiters in systems showing short activity cycles. The system’s transiting small planet that likely orbits close to the star’s rotational poles provides an opportunity to investigate whether such an orientation is dynamically consistent with such an idea. The abundance of hot, low-mass planets with a mass of less than  $30 M_{\oplus}$  on close-in orbits with a


**Fig. 15.** Tidal evolution of the orbit of TOI-2458 b. The horizontal red lines indicate the Roche limit for both planet scenarios.

period of less than 100 days (e.g., Mulders et al. 2015; Winn & Fabrycky 2015), suggests that many in situ formed hot Jupiters conglomerate within systems that also contain low-mass planets. Batygin et al. (2016) have studied the in situ formation and dynamical evolution of such hot Jupiter systems and demonstrated that under conditions appropriate to the inner regions of protoplanetary disks, rapid gas accretion can be initiated by 10–20 Earth-mass planets, which are very common. Their findings suggest that the interaction of in-situ formed hot Jupiters with outer low-mass planets has observable consequences. They determined a critical semimajor axis of low-mass planets beyond which secular resonant excitation of mutual inclination of both planets is guaranteed. Therefore, in the case of an almost circular orbit of a hot Jupiter, dynamical interactions during the early stages of planetary systems’ lifetimes should highly increase the mutual inclinations for the specific range of the semimajor axis. Disk lifetime, stellar rotation rate, and semimajor axis of hot Jupiter are among the critical parameters that influence this specific range. Figure 9 from Batygin et al. (2016) indicates that TOI-2458 b can be located within this range in a large portion of parameter space, which could explain the observed low inclination of its orbit.

Not only is the TOI-2458 system consistent with the scenario of in situ formed hot Jupiter but the orbital orientation of TOI-2458 b would be its natural consequence. If the hot Jupiter has not yet been engulfed, its detection in RVs and photometry would be prevented due to its orbital orientation. Based on our radial velocity observations, we have determined the minimum orbital inclination required to prevent the detection of a hot Jupiter close and massive enough to affect the tidal spin-up of TOI-2458. We used the criterium from Tejada Arevalo et al. (2021) to define the





**Fig. 16.** Detection limits for companions around TOI-2458 based on their orbital inclination. The blue line represents the boundary for planets that would cause a strong tidal spin-up of TOI-2458.

theoretical mass and period boundary for planets causing strong tidal spin-up. [Tejada Arevalo et al. \(2021\)](#) showed that objects within this boundary spin up their hosts, which tend to spin faster than the stars hosting cold Jupiters. Our findings suggest that the orbital inclination would need to be below two degrees to prevent the detection of a hot Jupiter with a period of three days (see Fig. 16), quite an improbable scenario indicating that any hot Jupiter present may have been ingested. To date, no planet system has been observed that convincingly satisfies the implications described in [Batygin et al. \(2016\)](#). WASP-47 may be an undisrupted system with an in situ formed hot Jupiter, where an increase of mutual inclination has been avoided. In this context, TOI-2458 represents evidence supporting the in situ formation of a hot Jupiter in this system, with consequent dynamical evolution matching the observed orbital orientation of TOI-2458 b. Due to the low  $v \sin i$ , it is not possible to measure the misalignment of TOI-2458 b’s orbit using the Rossiter-McLaughlin effect. One more time, we emphasize the importance of confirming the stellar rotation period to thoroughly validate the system architecture.

#### 4.5. Potential debris disk

TOI-2458 shows an infrared excess at 22  $\mu\text{m}$  in the WISE/W4 bandpass, with an excess level of 75% (4.7  $\sigma$ ) compared to its photospheric flux ([Cruz-Saenz de Miera et al. 2014](#)). High-resolution imaging confirms the absence of photometric contaminants around the star, suggesting that the infrared excess may be attributed to a debris disk. While debris disks are typically associated with stars younger than 1 Gyr (e.g., [Siegler et al. 2007](#); [Gáspár et al. 2009](#); [Mizuki et al. 2024](#)), it is important to note that the presence of such a disk does not necessarily indicate the youth of the system. Indeed, some older systems have been found to have debris disks, although their occurrence is less common compared to younger stars (e.g., [Trilling et al. 2008](#)).

Small dust grains present in debris disks are the result of a collisional cascade, where planetesimals are gradually reduced in size ([Wyatt 2008](#)). The timescale for the crossing of orbits within a planetesimal belt due to planet stirring can be calculated using equation (15) from [Mustill & Wyatt \(2009\)](#), taking into consideration the characteristics of both the planet and the disk. The disk temperature of  $166^{+42}_{-30}$  K from [Cruz-Saenz de Miera et al. \(2014\)](#) was employed to determine the location of the debris disk at  $4.0^{+2.2}_{-1.5}$  au using the equation (1) in [Trilling et al. \(2008\)](#). These values were used to illustrate the planetary perturbation for the disk in the TOI-2458 system. The timescale of secular perturbation for planet TOI-2458 b varies

from 0.2 Gyr to 14 Gyr, depending on the precise position of the disk. Planet TOI-2458 c, which orbits slightly beyond the transiting planet, exhibits a larger orbital eccentricity uncertainty and only a lower limit for its mass, indicating its potential role as a perturber for the debris disk. Nevertheless, we note that unseen outer planets in proximity to the debris disk may also exert a more substantial influence on the belt’s dynamics. To comprehensively understand the architecture of the TOI-2458 planetary system, encompassing both planets and a disk, future research endeavors could concentrate on confirming the debris disk and characterizing its location through spectroscopic observations.

## 5. Summary

We have presented a discovery of the transiting mini-Neptune TOI-2458 b around an F-type star. TOI-2458 b has an orbital period of  $P = 3.73659 \pm 0.00047$  days, a mass of  $M_p = 13.31 \pm 0.99 M_\oplus$  and a radius of  $R_p = 2.83 \pm 0.20 R_\oplus$ . The star appears to be relatively old, with an estimated age of approximately 5.7 Gyr. Given this age, we found an unexpectedly fast stellar rotation period of  $8.9^{+3.9}_{-2.5}$  days. Using a combination of  $P_{\text{rot}}$ ,  $v \sin i$ , and  $R_\star$ , we constrained a low stellar inclination angle of  $i_\star = 10.6^{+13.3}_{-10.6}$  degrees. The star also shows a short-term activity cycle of 53 days, which contributes to the increasing sample of F-type stars exhibiting similar behavior. Our investigation of a sample of stars exhibiting short-term activity cycles revealed that a short stellar rotation period may be a common characteristic. Such a rotation period may be explained by tidal interactions with hot Jupiters, which may be engulfed on timescales of a few billion years or less. The TOI-2458 system appears to be consistent with the scenario of an in situ formed hot Jupiter, which may explain the stellar properties (rotation period and magnetic cycle) and orientation of the transiting companion TOI-2458 b. Finally, the RV periodogram revealed the presence of a second planet in the HARPS data with a period of  $P = 16.55 \pm 0.06$  days and a minimum mass of  $M_p \sin i = 10.22 \pm 1.90 M_\oplus$ . Using dynamical stability analysis, we constrained the mass of this planet to the range  $M_c \simeq (10, 25) M_\oplus$ .

Overall, our findings suggest that TOI-2458 is an interesting system for further study, particularly in the context of hot Jupiter formation and its influence on short-term activity cycles of F-type stars.

## Data availability

The supplementary materials, which include a table reporting parameters from the `pyaneti` analysis, a table presenting relative HARPS radial velocities and activity indicators for TOI-2458, and a correlation plot derived from the `pyaneti` analysis, are available on Zenodo at <https://zenodo.org/records/14362145>.

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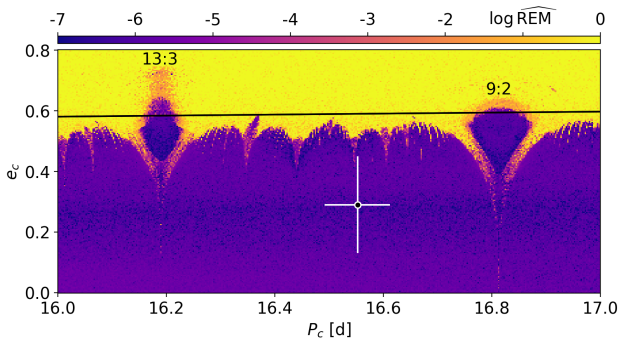
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## Appendix A: Dynamical analysis

First, to investigate the possible proximity of the TOI-2458 system to mean-motion resonances (MMRs) and the dynamical stability of our best-fit solution, we used direct  $N$ -body integrations and a CPU-efficient fast indicator called the reversibility error method (REM; Panichi et al. 2017). REM is closely related to the maximum Lyapunov exponent (MLE). It is based on integrating the equations of motion with a time-reversible (symplectic) scheme forward and backward in time for the same number of time steps. Then, the difference between the initial and final states of the system, normalized by the size of the phase-space trajectory, makes it possible to distinguish between regular and chaotic evolution. REM is scaled so that  $\log \widehat{\text{REM}} = 0$  means that the difference between the initial and final states is on the order of the orbit size.

We integrated REM with the WHFAST integrator with the 17th order corrector, with a fixed time step of 0.05 days, as implemented in the REBOUND package (Rein & Liu 2012; Rein & Spiegel 2015) for  $\approx 2 \times 10^5$  orbital periods of the outer planet. As shown in Fig. A.1,  $\log \widehat{\text{REM}} \leq -5$ , indicating a stable solution. Furthermore, the location of the best-fitting model in phase space indicates its safe, nonresonant character. The orbital period of TOI-2458 c was determined to be  $P_c = 16.553^{+0.059}_{-0.061}$  d, and the dynamical map shows the nominal system shifted by a few sigma from nearby strongest 13:3 MMR and 9:2 MMR, respectively. It is worth noting that the eccentricity error is quite large, which may indicate proximity to the collision curve and increased mutual interactions between the planets.



**Fig. A.1.** REM dynamical map for the found solution in the  $(P_c, e_c)$ -plane of the planet c, the system is coplanar. Small  $\log \widehat{\text{REM}}$  characterize regular (long-term stable) solutions, marked with black/dark blue color. Dynamically chaotic solutions are marked with lighter colors up to yellow. The black curve is for the geometric collision of the orbits, defined by the condition:  $a_b(1 + e_b) = a_c(1 - e_c)$ . The resolution of the map is  $401 \times 201$  points.

In an attempt to constrain the mutual inclination of the orbits by dynamics, we first computed the REM maps in the  $(\Delta\Omega, i_c)$ -plane, where  $\Delta\Omega \in [0^\circ, 360^\circ]$  is the separation of the nodal lines of the orbits and  $i_c \in [1^\circ, 179^\circ]$  is the inclination of the planet c. Then, the mutual inclination of the orbits is:

$$i_{mut} = \cos i_b \cos i_c + \sin i_b \cos i_c \cos \Delta\Omega.$$

The results of the REM simulations are shown in the upper left panel of Fig. A.2. Stable regions appear only in a portion of the parameter plane, outside mutual inclinations approximately  $i_{mut} \in (60^\circ, 140^\circ)$ . This implies moderate mutual inclinations. Considering that  $i_b = 84^\circ \pm 0.5^\circ$ , then  $i_c \approx (24^\circ, 156^\circ)$  and its mass must be in the range  $M_c \approx (10, 25) M_\oplus$ .

The resulting scan of the  $(\Delta\Omega, i_c)$ -plane is reminiscent of the results for the Lidov-Kozai resonance (e.g., Shevchenko 2017; Naoz et al. 2013). However, both the nominal system with an initial coplanar orbit and two initial conditions, labeled “1”, “2” and “3” respectively, in the REM dynamical map, do not exhibit librations of  $\omega_b$ , so none of them belongs to the libration zone of the resonance. This is illustrated in Fig. A.3, where the evolution of the elements has been calculated using the IAS15 integrator of the REBOUND package, which is also suitable for highly eccentric orbits. Since the system is relatively compact, with  $a_b/a_c \approx 0.37$ , the presence of a “classical” LK resonance is uncertain. Its analytical models rely on small factor  $a_b/a_c$ . We found numerically that the global dynamics showed features similar to those of the canonical case, such as the large variability of eccentricity and inclination in the anti-phase.

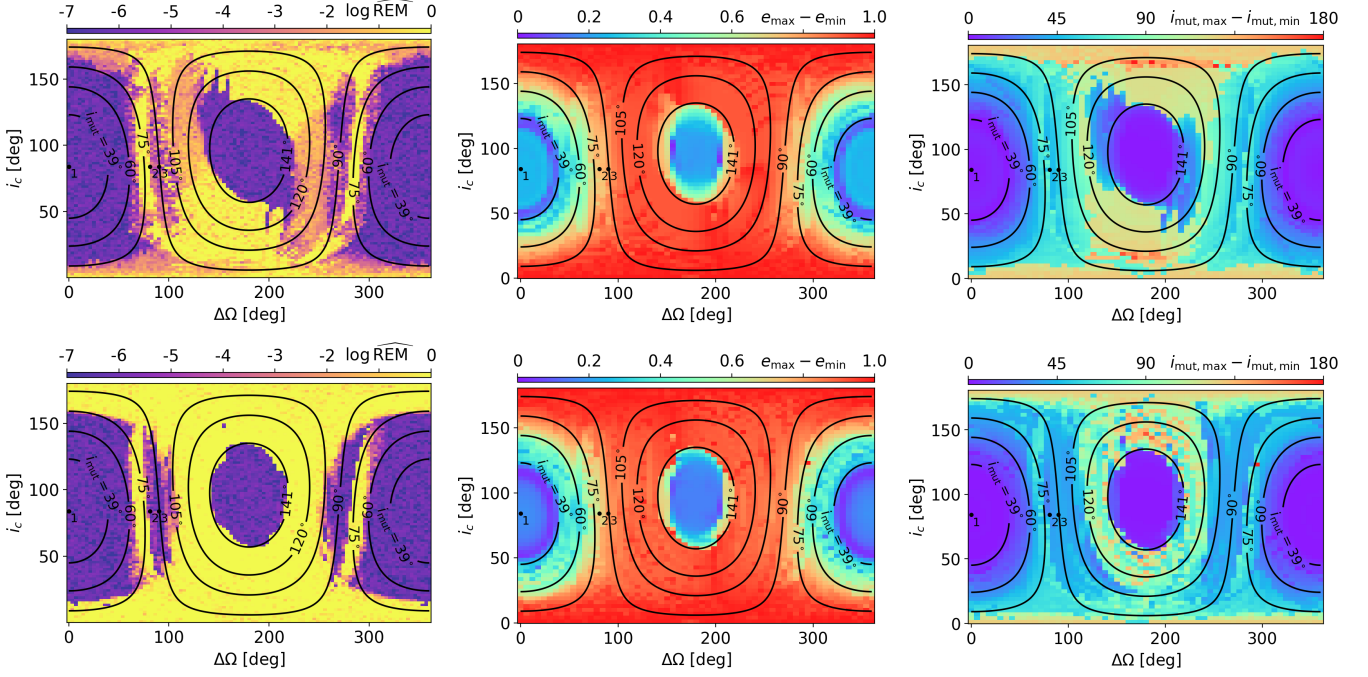
This REM scan is accompanied by panels for the range of eccentricity and mutual inclination of the orbits (the top-middle and top-right panels in Fig. A.2 respectively) for the interval spanning at least a few cycles of the variation of the elements shown in Fig. A.3. Indeed, in a large part of the phase space, the eccentricity can reach extremely large values, exceeding 0.7–0.8 (the upper-middle panel). This would imply the pericenter distance as small as 0.014–0.0096 au, only  $\approx 2$  stellar radii of  $\approx 0.006$  au. At the same time, the mutual inclination in these regions varies up to  $\sim 90^\circ$  (top right panel). Also, such highly eccentric and mutually inclined systems are usually extremely chaotic and strongly unstable, as can be seen for initial condition “2” in Fig. A.3, middle panel.

It is also known (Shevchenko 2017; Naoz et al. 2013) that the LK resonance can be suppressed by General Relativity (GR) perturbations and star-planet tides, which can significantly modify the pericenter precession. We have performed a test of this phenomenon for the GR perturbations in the simplest yet quantitatively correct and CPU-efficient form of the radial and conservative potential  $\sim r^{-2}$  introduced by Nobili & Roxburgh (1986). The dynamical map is then substantially modified, with some structures appearing at the boundary of the stable regions (see the lower right panel in Fig. A.2). The eccentricity and mutual inclination regions remain largely similar to the “pure” Newtonian case, but some structures, such as the central oval, are significantly different. Clearly, the dynamics of the system are globally affected by the GR perturbation of the Newtonian force.

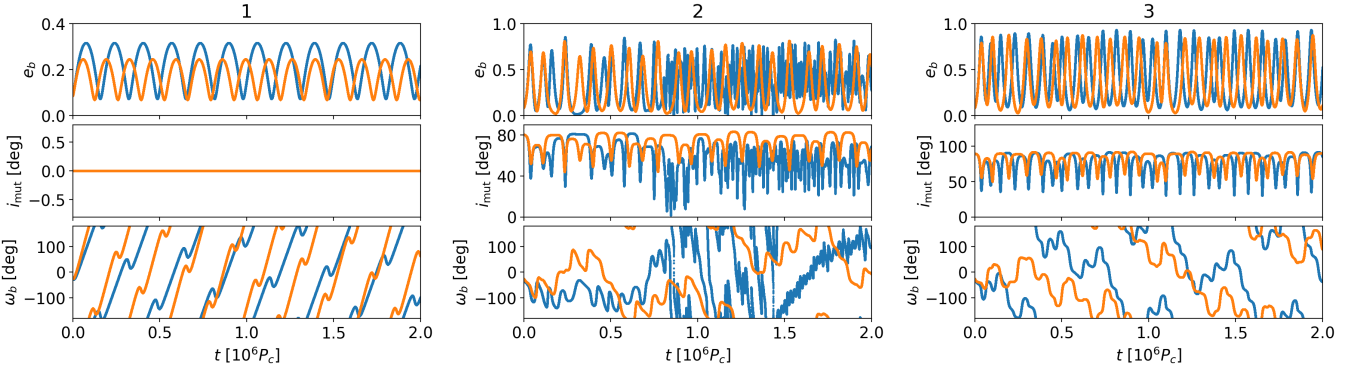
Moreover, the evolution of the elements for the selected initial conditions shows that the GR perturbation quantitatively modifies the dynamics by changing the frequency of the pericenter precession and the range of the elements in Fig. A.3, where orange curves are for the GR model. These simulations also confirm the high sensitivity of REM to chaotic motions and the correct characterization of regular and chaotic solutions in the dynamical maps.

To summarize our results in this section, the overall 3D dynamical structure of the TOI-2458 system is very complex despite apparently well-separated planets with low masses. The results of the dynamical simulations constrain the observed system to relatively small islands of moderate inner, close-in planet eccentricity when the orbital planes are moderately inclined. However, in the case of the hypothetical violent scattering history of the system, we could also observe the orbits just in a phase of increasing or decreasing inner eccentricity. In the case of large variations in this inner eccentricity, the orbital evolution of the inner planet and the whole system would be strongly influenced by the tidal interaction of the star. The properties of the putative LK resonance in the system would then require comprehensive investigation, which is beyond the scope of this paper.





**Fig. A.2.** Dynamic REM maps (left column), variations of the inner eccentricity (middle column), and mutual inclination of the orbits (right column) for several cycles of oscillation of these elements in the  $(\Delta\Omega, i_c)$  plane. The top row is for the Newtonian gravitational interactions; the bottom row is for the GR perturbations included in the equations of motion. Black curves mark some levels of mutual inclination of the orbits. The points marked “1”, “2” and “3” are for the integrated initial conditions shown in Fig. A.3. See text for details.



**Fig. A.3.** The results of the integration of the TOI-2458 system for the best-fitting initial condition with the orbital eccentricity and nodal longitude of the outer orbit marked with “1”, “2” and “3” in Fig. A.2. The time evolution of selected orbital elements for the Newtonian force is shown in blue, and the Newtonian and GR models are shown in orange, respectively. The integrations were performed with the variable-step, high-accuracy IAS15 integrator.