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# Impact of E-plane Misalignment on THz Diagonal Horn Antennas

Divya Jayasankar, *Graduate student member, IEEE*, Andre Koj, Jeffrey Hesler, *Fellow, IEEE*, and Jan Stake, *Senior Member, IEEE*

**Abstract**—A key challenge in developing terahertz front-ends is achieving high coupling efficiency between the waveguide feed horn and the optical beam. In this paper, we have quantified the alignment requirements for the widely used E-plane split diagonal horn antenna through theoretical analysis, electromagnetic simulation, and experimental validation within the 325-500 GHz frequency range. The results from our analytical models, simulations, and measurements are consistent and shows good agreement. They reveal that even minor geometric asymmetries can cause significant increases in fractional power radiated to the cross-polar component due to amplitude and phase imbalances in the TE<sub>10</sub> and TE<sub>01</sub> modes. Furthermore, a misalignment of approximately 8% of the wavelength was observed to result in a 3-dB degradation in the optical coupling to a Gaussian beam (Gaussicity) in middle of the waveguide band. These findings highlight the critical importance of precise alignment and feed horn machining for the successful implementation of terahertz front-end systems.

**Index Terms**—Antenna measurements, Aperture antennas, Horn antennas, Near-field measurements, Optical coupling, Sensitivity, Submillimeter wave propagation, Tolerance analysis.

## I. INTRODUCTION

FEED horns [1] play a pivotal role in various terahertz quasi-optical applications such as astronomy [2], remote sensing [3], [4], imaging [5], material characterization [6], and spectroscopy [7]. These applications often use horns integrated

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with detectors to maximize the signal coupling to non-linear elements like diodes, [8], [9] or to radiate to free-space [10]. For astronomical receivers, corrugated feed horns an ideal candidate [11]. Electroforming has effectively produced corrugated horns with high Gaussicity (98%), but achieving uniform  $\lambda/4$  corrugations and coating at sub-millimeter wavelengths is challenging and expensive [12]. Alternative horn profiles, such as Pickett-Potter horns [13], conical horns [14], and smooth-walled spline horns [15], offer easier fabrication compared to electroforming. However, in general these horns are fed by circular waveguides, which require circular-to-rectangular waveguide transitions for circuit integration. Therefore, the classical diagonal horn is widely utilized among feed horns due to its simplified manufacturing process and seamless integration with rectangular waveguides. Notably, it has symmetrical E- and H-planes, suitable for both circular and linear polarization.

First introduced by Li in 1952 [16], diagonal horns feature a square aperture fed by a rectangular waveguide, with two transition sections converting the transverse-electric TE<sub>10</sub> mode to TE<sub>11</sub> in a circular waveguide and then to a diagonally polarized wave in the square waveguide - a superposition of two TE<sub>10</sub> and TE<sub>01</sub> waveguide modes. Subsequently, Love experimentally verified its radiation pattern and coined the term 'Diagonal horn' [17]. Later, Johansson and Whyborn showed that the intermediate circular section could be omitted and proposed E-plane split-block realization for sub-millimeter wavelengths [18]. However, fabricating these components at high frequencies (>1 THz) with micron-level precision presents significant challenges. The performance of diagonal horns depends on the phase and amplitude balance between the TE<sub>10</sub> and TE<sub>01</sub> modes. Any asymmetry, such as E-plane misalignment, can lead to mode imbalance, increasing cross polarization levels, and result in an elliptically polarised beam. The influence of mode amplitude imbalance was discussed by Withington and Murphy in [19], but the role of a phase between the modes has yet to be reported in the literature. Slight differences in propagation constants will cause a significant phase difference [20] at the antenna aperture. Previous studies have documented similar issues; Kerr *et al.* noticed degradation in isolation for ortho-mode transducers (OMT) [21] and Ellison *et al.* observed discrepancies in the radiation pattern of a 3.5-THz quantum-cascade laser (QCL) integrated with dual diagonal feed horns due to fabrication imperfections and misalignment [10]. Montofre *et al.*, [22] noticed unbalanced cross polarization levels in the D-plane diagonal-spline horn and concluded that this is the effect of

asymmetry between the blocks. Jayasankar *et al.* also reported higher conversion losses in a 3.5-THz harmonic mixer with a diagonal horn in a WM64 waveguide and attributed it to high loss in the RF chain [23].

In this paper, we study the performance degradation of THz diagonal horn antennas due to E-plane misalignment since it is the most probable error due to alignment tolerances of split-block components. The diagonal horn is a scaled version of the 4.7-THz antenna presented in [24] and the experiment is conducted at the WM570 frequency band to precisely control misalignment and to have a reliable antenna characterization. The rest of the paper is organized as follows: in section II, we have proposed an analytical model to compute the phase and amplitude imbalance caused by E-plane asymmetry in diagonal horns. We present the analytical results on the impact of misalignment on cross polarization loss at three frequencies,  $k/k_c = 1.3, 1.5,$  and  $1.8$ , along with electromagnetic (EM) simulations. Following this, we present the mechanical design that allows controlled misalignment and machining of the WM570 diagonal horn. In section III, we present the near-field measurement setup and experimental characterization of the horn. We compare three alignment scenarios, a) perfectly aligned, b) misaligned by  $22 \mu\text{m}$  and c) misaligned by  $40 \mu\text{m}$ . Finally, we conclude by summarizing our findings and proposing an alternative solution to address this issue.

## II. METHOD

A standard WM570 diagonal horn (without the intermediate circular transition) was designed as described by Johansson and Whyborn [18], see Fig. 1. A square aperture with the side,  $d = 2.86 \text{ mm}$ , length  $L = 15 \text{ mm}$ , and corresponding flare angle  $\theta = \tan^{-1}(d/\sqrt{2}L) = 7.7^\circ$  were used, which results in a nominal antenna gain of circa 23 dBi. The feed is a standard rectangular waveguide ( $a \times b$ ) of dimension  $570 \times 285 \mu\text{m}^2$ . To experimentally study the effect of E-plane misalignment  $\delta$ , guide structures were machined in a split block to allow for alignment and controlled misalignments up to  $40 \mu\text{m}$  corresponding to about 6% of free-space wavelength. This section describes the theory, EM simulations, mechanical design, machining and near-field characterization for analyzing the impact of E-plane misalignment.

### A. Theoretical analysis

The aperture field distribution of a diagonal horn antenna can be approximated as an in-phase superposition of two orthogonal TE modes [18],  $\text{TE}_{10}$  and  $\text{TE}_{01}$ ,

$$\mathbf{E}_{\text{ap}} = \left[ E_{01} \cos\left(\frac{\pi y}{d}\right) \hat{x} + E_{10} \cos\left(\frac{\pi x}{d}\right) \hat{y} \right] \cdot e^{jk_\rho z} \quad (1a)$$

$$k_\rho = \frac{2\pi}{\lambda_o} \left[ \frac{d^2 - 2x^2 - 2y^2}{4L} \right] \quad (1b)$$

where  $\lambda_o$  is the free space wavelength and  $k_\rho$  is the spherical phase front [25]. When the split blocks are aligned as shown in Fig. 1a, the modes have phase and amplitude balance and exhibit the same propagation phase constant,  $\beta$ , which is not valid for the misaligned and distorted cross-section shown in

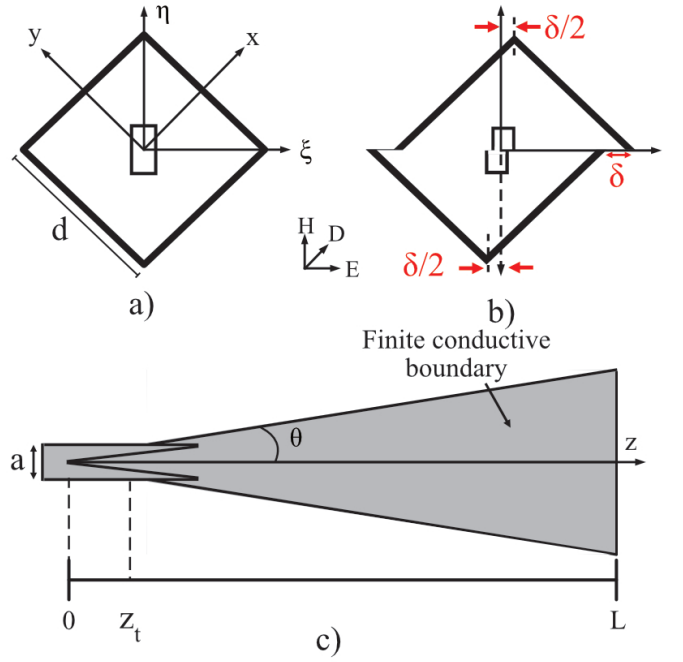


Fig. 1: a-b) Schematic of aligned and misaligned diagonal horn aperture. c) Diagonal horn with rectangular waveguide of width  $a$ , length  $L$  and flare angle  $\theta$ .

Fig. 1b. However, when the split blocks are misaligned, the original square cross-section of the horn, see Fig. 1a, can be replaced with a rectangular shape, Fig. 1b, where one side of the cross-section becomes effectively narrower  $d - \delta/\sqrt{2}$  and the other broader  $d + \delta/\sqrt{2}$ . As a result, the two corresponding TE modes will have different propagation phase constants ( $\beta^+$ ,  $\beta^-$ ) and wave impedances, creating a phase and amplitude imbalance along the length of the diagonal horn. In general, the imbalance between the two modes at the aperture can be expressed as

$$\frac{E_{10}}{E_{01}} = \sqrt{\Omega} \cdot e^{j\varphi} \quad (2)$$

where  $\Omega$  is power balance factor proposed by Withington and Murphy [19], and  $\varphi$  is the total phase imbalance at the aperture. For a modest and typical flare angle ( $< 10^\circ$ ), neglecting reflections, the total phase imbalance can be obtained as the sum of propagating infinitesimal waveguide sections  $dz$ .

$$\begin{aligned} \varphi &= \int_{z_t}^L \left( \beta^+(z) - \beta^-(z) \right) dz \\ &= \int_{z_t}^L \left[ \sqrt{k^2 - \left( \frac{\pi}{z \frac{d}{L} + \frac{\delta}{\sqrt{2}}} \right)^2} - \sqrt{k^2 - \left( \frac{\pi}{z \frac{d}{L} - \frac{\delta}{\sqrt{2}}} \right)^2} \right] dz \end{aligned} \quad (3)$$

where  $L$  is the length of the horn,  $z_t = c \cdot L/d$  is the assumed starting point of the horn (throat), and  $c$  is the corresponding cross-section width at the throat. Note that the wavenumber of each mode,  $k_c$ , changes with the position along the diagonal horn  $z$ -axis, and is corrected with the

misalignment factor  $\pm \frac{\delta}{\sqrt{2}}$ . The integral can be solved using analytical methods, leading to a lengthy expression. However, a first-order approximation can be found by introducing the parameter,  $h = L\delta/(\sqrt{2}d)$ , and reorganizing Eq. (3) as:

$$\begin{aligned}\varphi &\cong 2h \int_{z_t}^L \left( \frac{\beta(z+h) - \beta(z-h)}{2h} \right) dz \\ &\cong 2h \int_{z_t}^L \left( \frac{\beta(z)}{dz} \right) dz \\ &= \frac{\sqrt{2}L\delta}{d} \left[ \beta(z) \right]_{z_t}^L \\ &= \frac{\sqrt{2}L\delta}{d} \left[ \sqrt{k^2 - \left( \frac{\pi}{d} \right)^2} - \sqrt{k^2 - \left( \frac{\pi}{c} \right)^2} \right]\end{aligned}\quad (4)$$

Hence, the phase imbalance depends mainly on the flare angle, frequency, and the cross-section at the throat. For a large aperture  $d \gg c$ , the phase imbalance can be further approximated as  $\frac{\delta}{c} \frac{\pi}{\tan\theta}$ . Note that  $c \in (0, 3a/\sqrt{8})$  is a model fitting parameter representing the effective width of the throat cross-section.

Next, the amplitude or power imbalance results from uneven excitation of the two modes in the transition between the rectangular waveguide and the diagonal horn. For simplicity, we neglect losses and assume that the power imbalance only depends on the ratio of mode impedances at the throat. Using the power-voltage characteristic impedance definition (wave

impedance scaled with waveguide height and width) [26], [27], the power balance,  $\Omega$ , can be estimated as:

$$\begin{aligned}\Omega &= \frac{Z^-(z_t)}{Z^+(z_t)} = \left( \frac{c^-}{c^+} \right)^2 \cdot \frac{\beta^-(z_t)}{\beta^+(z_t)} \\ &= \left( \frac{c - \delta/\sqrt{2}}{c + \delta/\sqrt{2}} \right)^2 \cdot \frac{\beta^-(z_t)}{\beta^+(z_t)}\end{aligned}\quad (5)$$

Together with the aperture field, given in Eq.1a), the analytical model (4-5) provides insights and can be used to predict imbalance in phase and amplitude between the two modes, thereby allowing the calculation of the distorted aperture field for small misalignments ( $\delta \ll \lambda$ ). The analytical model was validated with electromagnetic simulations of cross polarization coupling loss  $\eta^-$ :

$$\eta^- = \frac{P_{cr}}{P_{cr} + P_{co}},\quad (6)$$

where  $P_{co}$  and  $P_{cr}$  are the radiated copolar and cross polar power, respectively. For a perfectly aligned antenna, 9.5% of the total power is radiated into the cross polarized component as shown in Fig. 3 and it agrees with the theoretical prediction in [18], [19]. For the analytical model, a throat cross-section ( $c/a$ ) of 0.82, 0.77, and 0.74 was used for the lower ( $1.3f_c$ ), mid ( $1.5f_c$ ) and upper ( $1.8f_c$ ) frequency bands, respectively, which resulted in a good agreement with the EM simulations. Note:  $f_c$  is the cutoff frequency of the waveguide.

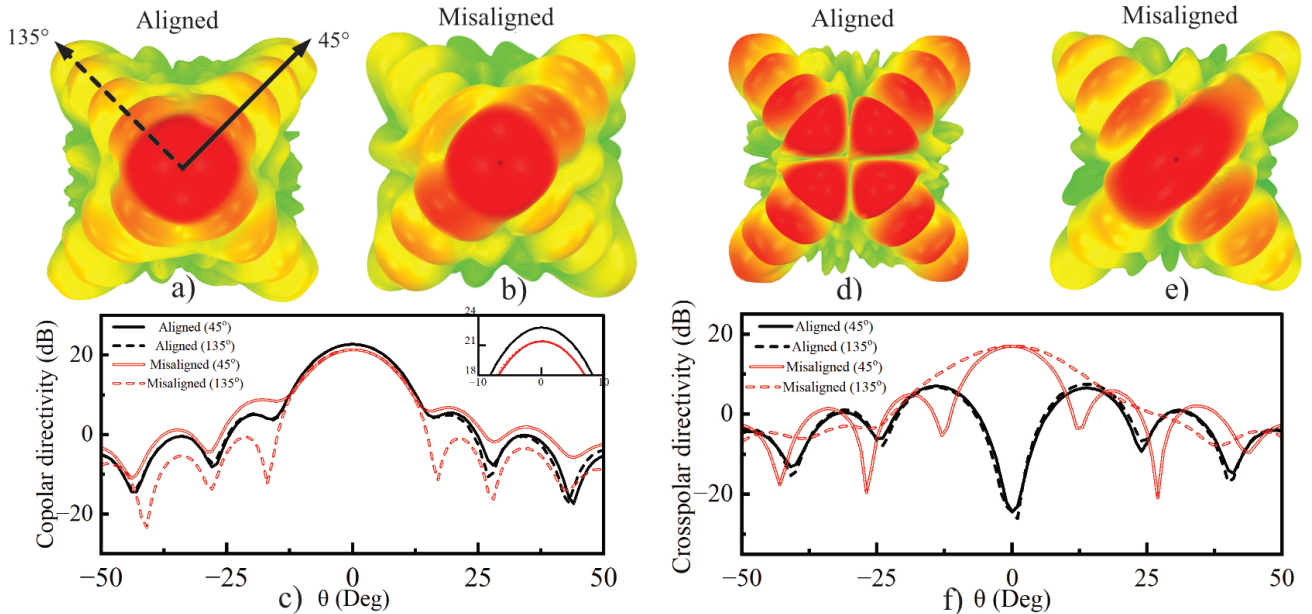


Fig. 2: Simulation of the diagonal horn's copolar and cross polar far-field radiation pattern at 470 GHz. (a-b) Copolar directivity of aligned and misaligned antenna ( $40 \mu\text{m}$ ) (c-f) D-plane cuts of the copolar far-field pattern, solid line for ( $\phi = 45^\circ$ ) and dashed line for ( $\phi = 135^\circ$ ) cut for both aligned (black) and misaligned (red) cases. The inset highlights the reduction in copolar directivity of about 1.5 dB due to misalignment. (d-e) Cross polar directivity of aligned horn antenna with a null in boresight and misaligned antenna shows a central cross polarization component.

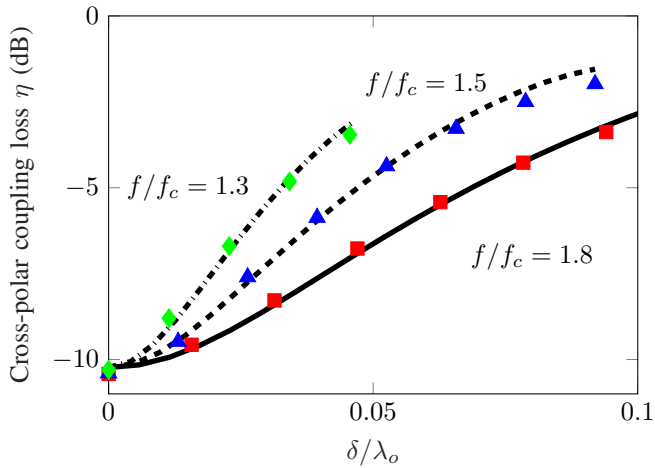


Fig. 3: Cross-polar coupling loss. Increase in the fraction of power radiated into the cross polarized component due to E-plane misalignment for a diagonal horn antenna with a flare angle of  $7.7^\circ$  and an aperture size  $d/a = 5$ . The black lines represent the analytical model and the markers correspond to EM simulations.

### B. EM simulation

The EM simulation of the diagonal integrated with rectangular waveguide as shown in Fig. 1 was carried out using a commercial finite-element method based solver. A finite conductive boundary (aluminum) with electrical conductivity  $\sigma = 3.8 \times 10^7$  S/m was assigned to the antenna and waveguide walls. A radiation boundary box was placed at the end of the horn to obtain the far-field responses. The side with the horn aperture was assigned a finite conductive boundary, and the rest of the faces were assigned radiation boundaries as mentioned in [28]. Two-fold symmetry can significantly reduce the computational time for the perfectly aligned case. However, symmetrical boundary conditions are inapplicable for misaligned configurations, necessitating the simulation of the entire horn and thus increasing the computational time. Simulated directivity is about 23 dB and E-plane half-power beam width (HPBW) is 13 degrees at 470 GHz. Fig. 2a-b,d-e shows the simulated far-field radiation pattern of co and cross polar component of the diagonal horn at 470 GHz, respectively, for both aligned and misaligned antenna. Fig. 2c,f

shows the D-plane cuts at  $\phi = 45^\circ$  and  $135^\circ$ . In Fig. 2f, we see a null in the cross polar component at boresight as expected for the aligned case [18], [29] and when misaligned, shows an increased cross polar component at the boresight. Likewise, the copolar directivity also reduces by 1.5 dB when misaligned, as highlighted in the inset of Fig. 2c.

### C. Mechanical design

Fig. 4a shows the CAD model of the 470-GHz diagonal horn antenna integrated with a WM570 rectangular waveguide in an E-plane split-block housing. It has a waveguide flange based on UG-387 specification [30]. The length of the access waveguide to the horn was designed as short as possible, about one wavelength, to minimize attenuation. A  $45^\circ$  chamfer was implemented on the aperture plane to redirect the reflected signal away from the optical axis.

The top split block has trenches along the edge of the waveguide and horn to ensure a tight fit between the blocks while assembling. To facilitate accurate alignment, two rectangular guide structures were incorporated onto the top surface of the split blocks. The bottom block features guide holes measuring  $1045 \times 510 \mu\text{m}^2$ , while the top block employs pins sized at  $1000 \times 500 \mu\text{m}^2$ . The intentional allowance in tolerance permits controlled sliding of the blocks, thus enabling effortless alignment and misalignment of the split blocks. Micrographs of two extreme cases are shown in Fig. 4c-d. The misalignment was measured using a metrology microscope with  $\pm 1 \mu\text{m}$  accuracy. Using guide structures offered advantages over traditional alignment methods, such as dowel pins, by minimizing rotational misalignments.

### D. Machining

The diagonal horn was machined from a single block of aluminum (Al 6082) alloy using a CNC machine (KERN Micro HD) with micrometer precision. First, planarization of the top surface was carried out using an end mill of  $250 \mu\text{m}$ -diameter. After machining the UG387 flange, the diagonal horn's aperture was machined using a  $45^\circ$  chamfer mill with a top radius of  $5 \mu\text{m}$ , which resulted in a fillet with the same radius as the tool. Following that, a rectangular waveguide was machined using a  $200\text{-}\mu\text{m}$  tungsten end mill with sharp corners.

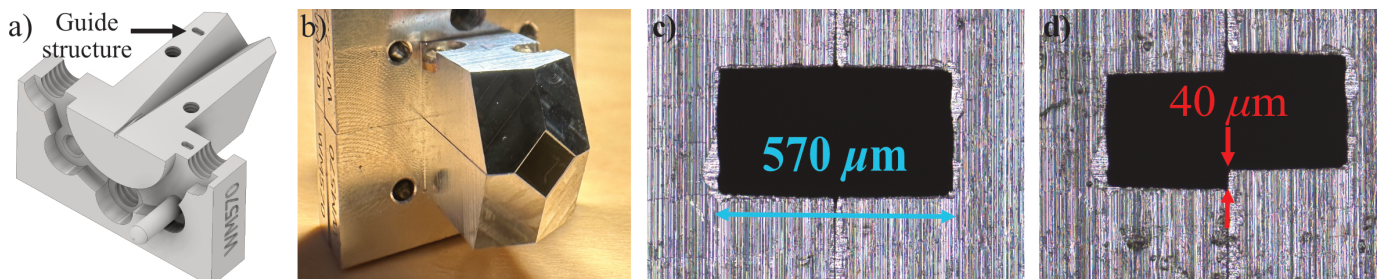


Fig. 4: WM570 diagonal horn antenna. a) CAD model of the antenna. b) Photograph of the machined horn antenna in Aluminum E-plane split-blocks. c) Micrograph of WM570 waveguide aperture when aligned. d) Micrograph of WM570 waveguide aperture with  $40\text{-}\mu\text{m}$  misalignment.

Burrs along the edge of the waveguide were removed since it could get folded into the waveguide cavity when the split-block halves are assembled. In addition, minor machining artifacts, especially at the horn's throat, were removed to avoid any reflections affecting the antenna's performance. After deburring, blocks were cleaned in an ultrasonic bath with acetone and isopropanol. The diagonal horn antenna machined in the E-plane split-block is shown in Fig. 4b. The cross-section of the WM570 waveguide aperture for the aligned and misaligned cases is shown in Fig. 4c-d.

### E. Measurement setup

Near-field pattern of the antenna was measured using a Keysight Vector Network Analyzer (VNA) N5242B and WM570 VDI frequency extenders as shown in the Fig. 5. The antenna under test (AUT) was connected to extender 1 on the left. For near-field scanning, a WM570 open-ended rectangular waveguide was connected to the right, which was mounted on a motorized linear XY stage with a step resolution of  $2.5 \mu\text{m}$ . Input signals from direct digital synthesizers (DDS) in the VNA were fed to the cascaded Schottky multiplier chain in the frequency extenders to generate output signals in the range of 325 GHz to 500 GHz.

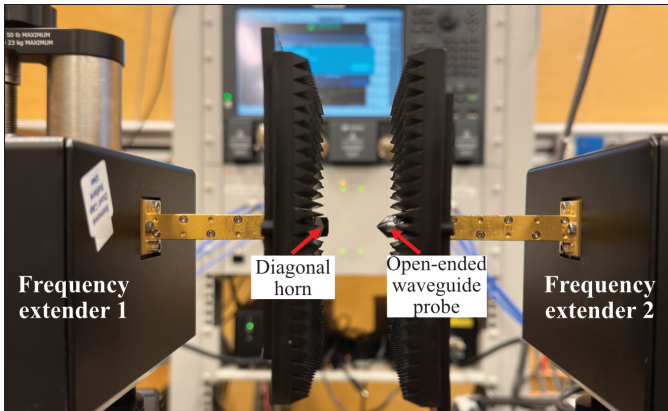


Fig. 5: Photograph showing the near-field measurement setup with VDI's WM570 frequency extenders, diagonal horn antenna and an open-ended waveguide probe.

The amplitude and phase distribution of the diagonal horn antenna was measured at a distance of approximately  $1.5 \text{ cm}$  between horn's aperture and the probe. The separation distance is a trade-off; while reducing the distance gives a smaller required scan area, care has to be taken to limit the standing waves and evanescent fields [31]. A raster pattern was implemented to scan the rectangular XY grid of size  $22 \times 22 \text{ mm}^2$  with uniform spacing set to  $\Delta x = \Delta y \leq \lambda/2$  ( $300 \mu\text{m}$ ). The setup was mounted on a vibration-free optical test bench from Thor Labs; absorbers were used to reduce the free-space standing waves in the optical axis, and the bench was layered with Eccosorb AN-73.

## III. RESULTS

First, the input return loss of the aligned horn antenna and WM570 open-ended waveguide probe was measured.

Two-port short-offset short-load-thru (SOLT) calibration was carried out, and the IF bandwidth was set to 300 Hz. Measured input return loss of the horn is better than 22 dB throughout the whole band from 325 GHz to 500 GHz as shown in Fig. 6. For comparison, simulation results of the diagonal horn are also included which match closely to the measurement results.

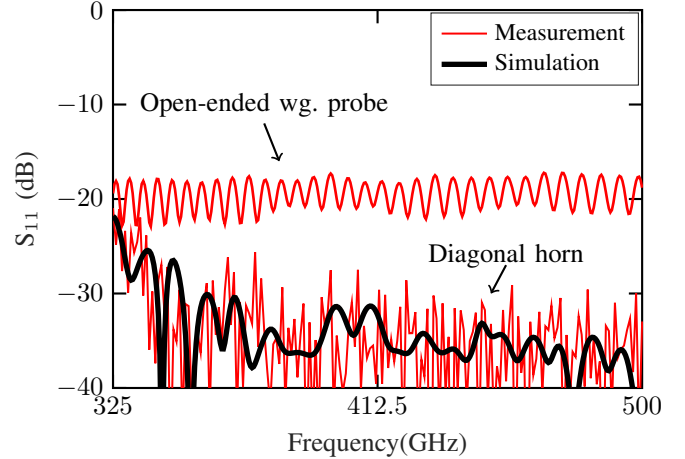


Fig. 6: Return loss. Comparison of measured and simulated  $S_{11}$  versus frequency of the aligned diagonal horn antenna and measured return loss of the WM570 open-ended rectangular waveguide probe.

The probe was moved at a low-speed for near-field scanning to avoid smearing effects. The port cables for extender 1 with AUT are stationary. However, as the cable movement in extender 2 is unavoidable, a reference measurement was recorded at the boresight of the AUT after scanning each row, which was later used to make linear compensation of phase drift. An average magnitude drift of about  $\pm 0.01 \text{ dB}$  and phase drift of  $\pm 10$  degrees were recorded during the complete scan. The probe was aligned to the boresight of the AUT by employing a phase-detection scheme that finds the local phase minima of  $S_{21}$ . The magnitude and phase of the tangential electric field components of the antenna were measured, and standard near-field to far-field transformation technique (*NF/FF*) [32], [33] was employed to compute the far-field radiation pattern of the antenna. For cross polar measurements, a  $90^\circ$  WM570 VDI waveguide twist was used.

Three cases were studied in detail: i) perfectly aligned, ii) misaligned by  $\delta = 0.035\lambda_o$  ( $22 \mu\text{m}$ ), and iii) misaligned by  $\delta = 0.064\lambda_o$  ( $40 \mu\text{m}$ ) where  $\lambda_o$  is the free space wavelength at 470 GHz. The far-field cuts of an aligned diagonal horn antenna in E ( $\phi = 0^\circ$ ) and H ( $\phi = 90^\circ$ ) are shown in Fig. 7. The directivity of the perfectly aligned diagonal antenna is measured at 22.8 dB which is in good agreement with the simulation. As expected, the E- and H-planes are symmetrical, with equal beam widths and high sidelobe levels about 15 dB in the D-plane, refer Fig. 8a.

The far-field radiation pattern of aligned and misaligned antennas is shown in Fig. 8. When misaligned by  $40 \mu\text{m}$ , the copolar directivity reduces by 1.5 dB at 470 GHz as shown in Fig. 8c. The mode imbalance due to the E-plane misalignment

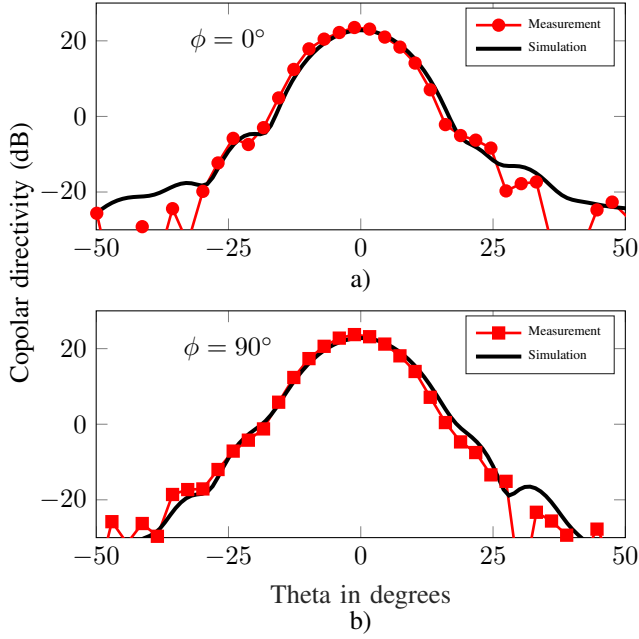


Fig. 7: E- and H-plane far-field radiation patterns. Comparison of simulation and measured copolar radiation pattern of aligned diagonal horn antenna at 470 GHz in two principal planes: a) E ( $\phi = 0$ ), and b) H ( $\phi = 90$ ).

results in a central cross-polarized lobe [19]. A 2D-contour of co- and cross polar radiation patterns of both aligned and misaligned antenna ( $40 \mu\text{m}$ ) is shown in Fig. 9. A short planar scanning range was chosen to reduce the measurement time, which restricts the validity to a limited angular span [34].

#### IV. DISCUSSION

The optical coupling to a Gaussian beam (Gaussicity) is an important parameter for quasi-optical system design. It is defined as the maximum power coupling of the beam produced by the horn to a linearly polarized Gaussian beam and it can be

calculated from aperture E-fields as shown below [35], [36],

$$\eta_{gauss} = \frac{|\langle \hat{E}_{ap} | \hat{g} \rangle|^2}{\langle \hat{E}_{ap} | \hat{E}_{ap} \rangle \langle \hat{g} | \hat{g} \rangle} \quad (7)$$

where  $\hat{g}$  is the first-order Gaussian beam function. Fig. 10 shows the estimated performance degradation of the diagonal horn antenna due to E-plane misalignment along with the analytical model described earlier in Section IIA. The beam waist was kept constant as  $w = 0.86 \cdot d/2$  [18] and note that the analytical model is valid only for small misalignment ( $\delta$ ). Evidently, Gaussicity follows the same trend as the cross polarization loss in Fig. 3 and is more sensitive when operating close to the cut-off frequency of the waveguide feed. We can observe 3-dB degradation in Gaussicity at 8% misalignment relative to the wavelength when operating in the middle of the waveguide band. The degradation is even more severe in the low-frequency part of the waveguide band due to a larger dispersion ( $\beta - k$ ), resulting in a larger imbalance between the two modes. Hence, operating in the upper part of the waveguide band as shown in Fig. 10 and using a larger flare angle,  $\theta$ , will reduce the influence of E-plane misalignment. Hence, there will be a trade-off between flare angle for optimum gain and robustness against fabrication and assembly tolerances. A possible alternative at very short wavelengths is to use a standard pyramidal horn, which is more or less immune to E-plane misalignment, thanks to the 'single' mode ( $\text{TE}_{10}$ ) operation. Alternatively, use a horn with a throat that expands fast, such as the smooth-walled spline-profile diagonal horn [37], or similar conical spline-horns [15], [38].

#### V. CONCLUSION

We have conducted a comprehensive study on the performance degradation of THz diagonal horn antennas due to E-plane misalignment. The consequences of misalignment have been quantified through a combination of theoretical analysis, EM simulations, and near-field antenna pattern measurements in the 325-500 GHz frequency range. The finite-element

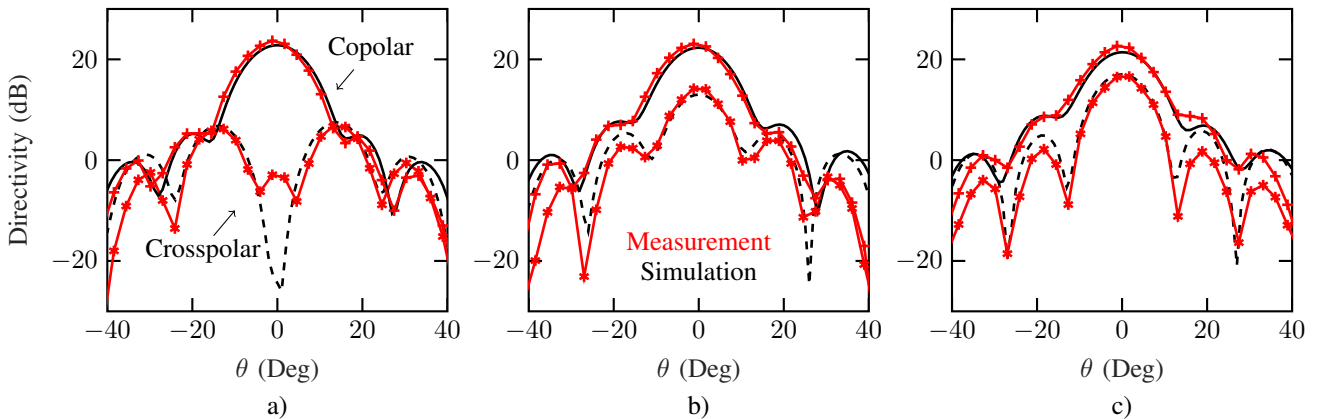


Fig. 8: D-plane far-field radiation patterns. Comparison of simulation and measured copolar and cross polar radiation pattern of diagonal horn at 470 GHz in the D-plane ( $\phi = 135^\circ$ ). a) perfectly aligned horn, b) misaligned by  $22 \mu\text{m}$  and c) misaligned by  $40 \mu\text{m}$ .

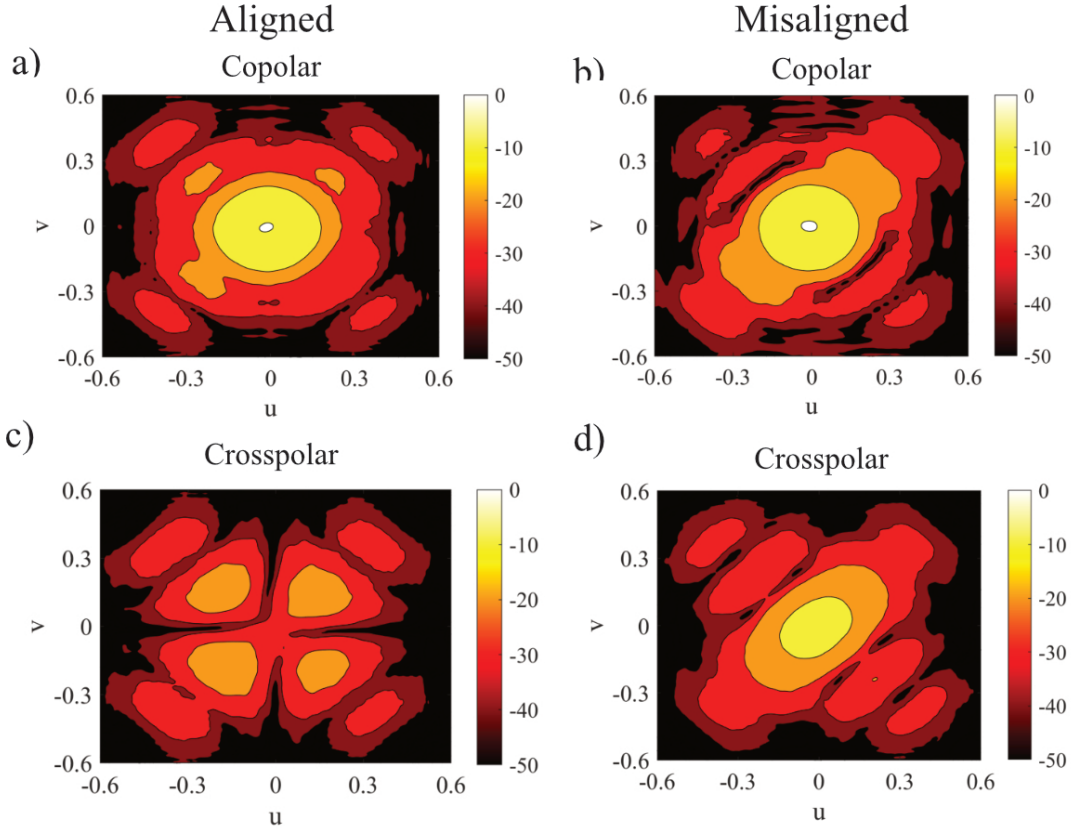


Fig. 9: Measured radiation pattern of aligned (left) and misaligned antenna (right) at 470 GHz. a-b) co-polarization, c-d) cross-polarization. Normalized to peak copolar directivity of 23 dB.

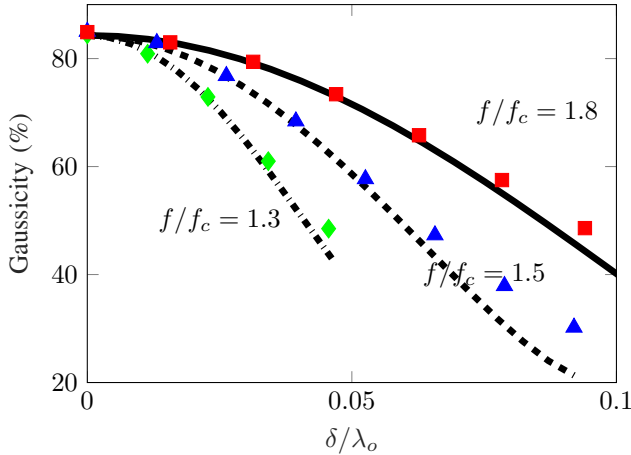


Fig. 10: Coupling to a Gaussian beam. Degradation of Gaussiaicity due to E-plane misalignment for a diagonal horn antenna. For the analytical model, the same throat cross-section ( $c/a$ ) in the range of 0.7-0.8 was used similar to Fig. 3 for three frequency bands. The black lines represent the analytical model and the markers are data points from the corresponding EM simulation.

modeling matched very well with the proposed analytical model and experimental results, indicating that misalignment

causes amplitude and phase imbalance in the  $TE_{01}$  and  $TE_{10}$  modes leading to high cross polarization loss. Particularly, when operating close to the cut-off frequency,  $k_c$ , of the waveguide feed, the diagonal horn is severely impacted by misalignment. The  $\delta/\lambda_o$  normalization in Fig.3 and 10 allows our findings to be easily scalable and applicable across various frequency bands. These results offer valuable practical design guidelines that are critical, especially beyond 3 THz. For future work, tolerance analysis work can be extended by addressing other critical horn parameters and comparing with other type of common terahertz horns.

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