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Augmented geometry assurance digital twin with physics-based incremental learning

Roham Sadeghi Tabar[a,*], Rikard Söderberg (1)[a], Dariusz Ceglarek (1)[b], Pasquale Franciosa[b], Lars Lindkvist[a]

[a] Chalmers University of Technology, Dept. of Industrial and Materials Science, SE 412-96 Gothenburg, Sweden
[b] Warwick Manufacturing Group (WMG), University of Warwick, Coventry, CV4 7AL, UK

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ABSTRACT

This paper presents a novel digital twin framework employing batch incremental learning for geometry assurance. Addressing quality issues caused by part and process variation, the method evaluates three critical tasks: part matching, locator adjustments, and joining sequence. The proposed framework utilizes deep learning architectures, each trained on recursive simulation data. Employing incremental learning, the models adapt to new batch characteristics while maintaining predictive accuracy. A spot welded assembly demonstrated the proposed approach efficiency, achieving prediction accuracies with errors as low as 0.02 mm for part matching and 0.1 mm for locator adjustments.

1. Introduction

Process-induced geometric variation of parts and subassemblies can lead to subsequent geometric variation of assembled products, which can eventually result in rejects and rework. Traditionally, top-down tolerancing activities have been a cumbersome iterative process to control geometric variation [1]. Geometry assurance is defined as a set of activities performed to secure geometric quality during the concept, verification and production phases. Early in the concept phase, part sensitivity and Monte Carlo based variation simulation are performed on immature shell CAD geometries [2,3]. Later, during the verification phase, FEM based variation simulation evaluates fixturing and joining operations. Matching and trimming activities are performed, and the parts and processes are adjusted to adopt the new concepts. In the production phase, the inline and offline measurement data is tracked to assure that final requirements are met. For automatic control and adjustments of the assembly processes, digital twins play a pivotal role [3,4]. The next section introduces the overview of the digital twins in the geometry assurance context.

1.1. Digital twins for geometry assurance

A digital twin (DT), which serves as a digital replica of a physical process or product, integrates real-time data from sensors and simulations to monitor, predict, and optimize manufacturing outcomes, and thereby, controls the manufacturing setup [5]. The development of DT aims to establish a bi-directional information flow between the physical system and the digital replica. Utilizing computational techniques, the parts and processes are monitored and optimized [6].

In assembly processes, DTs are used to model complex interactions between geometric constraints and joining processes. Previous studies have shown applications of DT to predict assembly variation and control spot welding processes to ensure geometric quality across production batches [7,8]. Furthermore, a DT based on skin model shapes has been introduced, representing the product through the manufacturing processes [9]. Deep learning and digital twin have been combined within DTs to establish a closed loop in process quality improvement for remote laser welding showing >96% acceptance rate in a zero defect manufacturing context [3]. In this paper, the geometry assurance DT presented in [4] is augmented by physics-based AI models in order to predict geometric errors in assemblies between production batches.

1.2. Self-compensating assembly line

The DT for a self-compensating assembly is designed for individualized adjustments on each assembly. This DT can use point cloud measurement data of all incoming parts to establish the input variation of incoming parts, i.e., non-ideal parts, as compared to the nominal CAD geometry, i.e., ideal parts. For incoming components prior to the assembly process, matching of the components, also referred to as selective assembly, takes place prior to positioning in the fixture [10]. When the parts are positioned in the fixture, the locators are adjusted to compensate for the existing error [11,12]. After this adjustment, the sequence of the joining is optimized to reduce the geometric deviation for each individual part in the batch [13]. After the assembly is released from the fixture, the assembly is scanned, and the DT model is calibrated to update the model representation [14]. Previous studies have heavily focused on optimization perspectives in the DT context. The optimization loops for large batches are computationally intensive and not suitable for real-time adjustments. Less emphasis has been placed on learning batch information from the optimization loops in the DT. Given that the DT

* Corresponding author.
E-mail address: rohams@chalmers.se (R. Sadeghi Tabar).

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model can also represent the physics of the process, e.g., FEM, and additionally with the introduction of simulations used for training deep networks, there is a possibility to train and update a network from the recursive optimization loops [15]. In this paper, we introduce a generic approach to enhance the geometry assurance digital twin by embedding the physics-based deep learning networks. The details of the model setup, optimization routines, the data generation processing for the AI models, and the architecture of the models are presented. A spot welded assembly is used to showcase the functionality and efficiency of the trained models for prediction geometry improvements for test batches.

1.3. Scope of the paper

To address the current limitations in computational efficiency and real-time response of the DT across batches, this paper proposes a generic approach for augmenting the geometry assurance digital twin with physics-based networks trained on the recursive simulation data. This data is generated during the initial optimization phases, on early production batches. The trained networks are intended to generate a real-time response of the DT for a future batch of components to be assembled.

2. Augmented DT with physics-based incremental learning

The augmented geometry assurance DT with physics-based incremental learning, is composed of three model layers: Process, Part and Deep Learning Models, Fig. 1. The physical assembly cell, including the parts and fixtures during the joining operations, are visualized and marked as 1 and 2 in Fig. 1. Initially, the point cloud data of scanned parts, i.e., non-ideal parts, are fed into the Part Model. Furthermore, the fixturing and joining positions and parameters are fed into the Process Model. The operational data are recursively updated in Part and Process Models to determine required adjustments for the existing errors. The physics of the assembly process, i.e., simulations of the exerted forces during the assembly, are integrated into both Part and Process Models and are used to determine fixture adjustments, update the robot programs for the joining operations and position the matched parts of the batch in the fixture. The control signals are sent to the physical cell for automatic adjustments. The Part and Process Models are connected to the central analysis module, steering the information flow. The focus of this paper is on the following three geometry assurance tasks to adjust the geometric deviation and improve batch quality: part matching, locator adjustments and joining sequence optimization, as discussed in Section 1.3. Incremental deep learning models are integrated into the DT and are trained in parallel. The networks utilize the initial batches of optimization data for the three tasks and incrementally improve by incorporating new observations from the follow up batch optimization data. Next, the details of each optimization task are presented, and the network architectures are introduced.

2.1. Physics-based assembly simulation

The simulation in the DT considers an assembly of multiple parts, joined by a welding process. Here spot welding is utilized, however, the

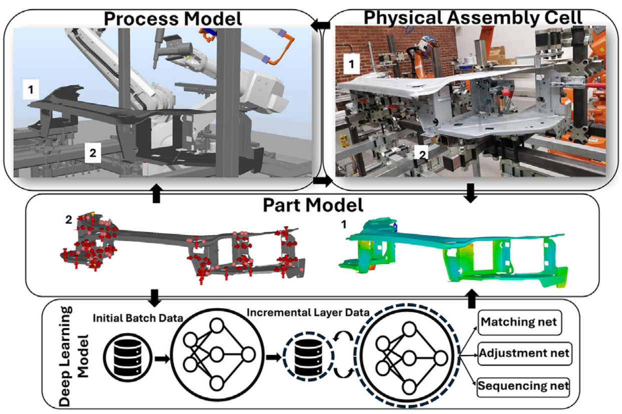

Fig. 1. Overview of the geometry assurance DT augmented with Physics-based incremental learning. Parts and fixtures are marked with 1 and 2.

approach is applicable to other joining processes, such as self-pierced riveting or laser welding [2,3,16]. The method of influence coefficients (MIC) is the foundation of the assembly model [17]. MIC is an exact method of building linear relationships between the part deviation, represented as the scanned geometries, and acting forces on the assembly.

$$\mathbf{K}\boldsymbol{u} = \boldsymbol{f}_{cl} + \boldsymbol{f}_{c} + \boldsymbol{f}_{w} \tag{1}$$

Here, $\mathbf{K}$ is the global stiffness matrix, $\boldsymbol{u}$ is the displacement vector, $\boldsymbol{f}_{cl}$, $\boldsymbol{f}_{c}$ and $\boldsymbol{f}_{w}$ are the forces at the clamping, contact and welding points, respectively. The response of the assembly to these forces is saved in a sensitivity matrix. During the assembly process, contact modelling is utilized to impose no penetration constraints in the adjacent areas. Moreover, considering small displacements, elastic material, and assuming that the local deformations at the position of the weld relative to the total assembly deviation, the spot welding process is introduced by imposing zero contact gap with a stiff beam element [18]. The contact model determines the response of the assembly with respect to the introduced weld points and imposes nonlinearity to the response of the assembly. The contact equilibrium problem is modelled as:

$$\underset{\boldsymbol{f}}{min}\,.\ \frac{1}{2}\boldsymbol{f}_c^T\mathbf{S}\boldsymbol{f}_c + \boldsymbol{f}_c^T\mathbf{D},\ \text{subject to} - \mathbf{S}\boldsymbol{f}_c \le \mathbf{D}, \boldsymbol{f}_c \ge 0, \tag{2}$$

where $\mathbf{S}$ is the compliant matrix, and $\mathbf{D}$ is the deviation vector in the contact nodes. The constraints enforce no penetration condition, and the force is always positive at the contact points. The contact model is efficiently solved through the augmented Lagrangian method [19]. To derive the assembly displacement after welding in a given sequence, the contact and welding forces are calculated at each welding step. Then using updated $\mathbf{S}$ the aggregated deformation after welding in a sequence is derived. Considering this process as the governing physics during the assembly in the DT, the following three optimizations of the assembly parameters are performed.

2.2. Part matching for batch optimization

The optimal matching of geometric deviations between multiple parts within an assembly to be spot welded is considered in the DT. Each part exhibits distinct deformation as the result of part manufacturing processes and the inconsistencies between the batches. The primary objective is to determine an optimal permutation $\boldsymbol{\pi} : \{1, 2, \ldots, N\} \to \{1, 2, \ldots, N\}$, where N is an integer number representing the number of instances of each part in the batch, that matches each instance of parts such that the resulting assembly minimizes the root mean square (RMS) displacement across all nodes after welding. Here, the mean (μ) and standard deviation represented by (6σ) for the batch 1 to N is considered as the objective for minimization. This optimization problem can be formulated as:

$$\underset{\pi}{\min}.\ [\mathrm{w}_1\varphi_\mu(\boldsymbol{\pi}) + \mathrm{w}_2\varphi_{6\sigma}(\boldsymbol{\pi})], \tag{3}$$

where $\varphi_\mu(\pi)$ represents the RMS of the mean displacements and $\varphi_{6\sigma}(\pi)$ denotes the RMS of the six standard deviations of the displacements across all nodes for the given permutation $\boldsymbol{\pi}$. The weights w_1 and w_2 are assigned to balance the contributions of the mean and variation of components in the cost function. To solve this combinatorial optimization problem, we employ a simulated annealing algorithm, which iteratively explores permutations by swapping pairs of instances and evaluates their associated costs using the simulation software RD&T [20]. The simulation software processes each permutation by generating an input file, executing the deformation analysis, and outputting the resulting displacements. The algorithm seeks to identify the permutation that yields the lowest combined RMS values, thereby ensuring the minimal overall displacement of the welded assembly.

2.3. Locator adjustment

As the sheet metal parts can be considered as compliant in one plane, therefore, they are located and securely held by a N-2–1 assembly fixture locating scheme, which locks the main six degrees freedom (3–2–1) and the extra clamps ($N \ge 3$) in the normal direction to the surface geometries, respectively [21,22]. Since the adjustments, also referred to as shims, often occur in the normal direction, realized through adjustable shims, the problem is defined as adjusting the clamps in the normal direction. The optimization problem was formulated to minimize the RMS of mean displacement throughout all the nodes in the assembly for each instance of the batch. This is denoted as $\tau_\mu(\boldsymbol{u})$. Here, $\boldsymbol{u}$ is a vector of scalar adjustments on the position of the clamps. In this study, a Bayesian optimization framework is deployed to identify the optimal configuration for the adjustments. The problem is formulated as:

$$\min_{\boldsymbol{u}} . \ \tau_\mu(\boldsymbol{u}) \text{ for } \boldsymbol{u} = [u_1 \dots u_z], \text{ and } \alpha \le u_i \le \beta \text{ for } i = 1, 2, \dots, z. \quad (4)$$

Here α and β are the allowable adjustments for each fixture point, defined as ± 2 mm in line with common industrial practice.

To solve this optimization problem, the Gaussian Process based Bayesian optimization algorithm is utilized and implemented via the scikit-optimize library in Python. The algorithm iteratively explored the z-dimensional search space, initially performing ten random evaluations to establish a probabilistic model of the objective function. Subsequently, it conducted up to fifty evaluations, balancing exploration and exploitation to efficiently converge towards the optimal locator adjustments. During each iteration, the FEM based variation simulation software RD&T was invoked to simulate the assembly response to the current set of adjustment values, providing the necessary $\tau_\mu(\boldsymbol{u})$ feedback for the optimization process.

2.4. Joining sequence optimization

The sequence of joining operation is to be optimized so that total assembly displacement $\tau_\mu(\boldsymbol{Q})$ is minimized for each instance of parts in the batch. The optimization problem for an assembly with w weld points is formulated as:

$$\min_{\boldsymbol{Q}} . \ \tau_\mu(\boldsymbol{Q}) \text{ for } \boldsymbol{Q} : \{1, \ \dots, \ \mathrm{w}\} \rightarrow \{\mathrm{w}, \ \dots, \ 1\}, \quad (5)$$

where $\boldsymbol{Q}$ is a permutation of joining points from 1 to w. Similar to the locator adjustment problem, the RMS of the displacements in all the nodes is the objective of the optimization. To solve this problem, a stepwise heuristic algorithm with a greedy approach, introduced in [23], is utilized. The algorithm creates all the combinations of the first elements in the sequence $\boldsymbol{Q}$. The simulation software then provides the objective value for the provided solutions, the minimum value is identified, and the corresponding sequence elements are set. This process is then continued until the sequence is complete.

2.5. Incremental learning framework

Utilizing each of the optimization routines, as detailed in Sections 2.2 to 2.4, an incremental learning framework is proposed to estimate the assembly response achieved for a given batch with the specific task. The proposed incremental learning framework is designed to predict the assembly geometric mean (μ) and standard deviation (6σ) of displacements across multiple batches of components. Rather than training a model solely on a fixed, initial dataset, the framework incrementally updates the model parameters as new batches of part data become available.

This approach enables the prediction model to adapt to part distributions, process variation, and changing assembly conditions over time. The incremental training procedure entails two primary phases. During the initial training phase, the model is first trained on a large historical dataset composed of several assembly batches. These initial batches provide sufficient data diversity, allowing the model to learn a robust representation of the assembly process and the influence of specific tasks on resultant geometric quality. The model is optimized using a gradient based optimizer and a mean squared error loss function. During the incremental update phase, new batches of part and assembly data become available. To incorporate these new data without retraining the model from scratch, the previously saved model parameters are reloaded and fine-tuned on the incoming batches. The same data preprocessing steps are applied to ensure consistency. This incremental training phase typically requires fewer training steps due to initialization with tuned parameters. The overview of the network architecture utilized for incremental learning is visualized in the lowest column in Fig. 1, named Deep Learning Model.

The network architecture is adapted to each specific task in the proposed DT. For matching, since the problem is binary and a batch composition is essentially independent of permutations for each part combination in a batch, a set transformer architecture is utilized to address the permutation invariance [24]. Two encoder attention blocks are built to aggregate all elements in the set of part instances in a batch. After encoding, a pooling layer condenses the variable set representation into a fixed embedding. The pooled embedding is passed through two layers of fully connected networks. Each network maps the latent embedding to the predicted RMS mean displacement and RMS 6σ.

For the locator adjustments, a feed forward neural network model is employed. The model is designed as a multi-layer perceptron (MLP) composed of fully connected layers and nonlinear activation functions. The architecture takes as input a feature vector derived from each assembly batch and generates a single continuous value corresponding to the predicted assembly RMS of mean displacements.

For joining sequence, the problem is binary, and each assembly batch of w welds has $w!$ possible permutations of welding sequences. For this task, a decision tree regressor is designed to represent each sequence outcome for the batches. After training the network given the batch RMS mean displacements and the given sequence input, modelled as a binary vector of 6 elements, provides the output displacement.

3. Case study

To further develop the method and evaluate its performance, this study considers an assembly of two parts connected with three weld points. The parts are of steel material with a stiffness of 210 GPa, and both have a 1.5 mm thickness. The assembly has 3 weld points and 173 contact points. Each part is held in the fixture with a 4–2–1 positioning system, including 4 clamping points subject to adjustments. Fig. 2 visualizes the assembly model for this case. The position of the weld points and the locating elements are marked with spheres and filled arrows, respectively.

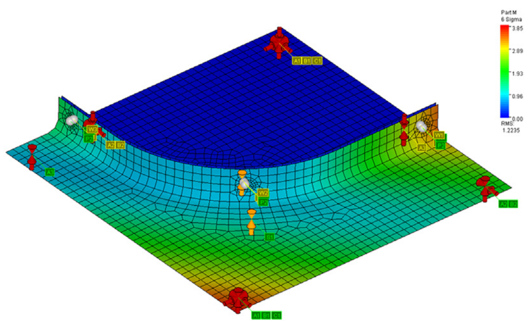

Fig. 2. The reference assembly held in the inspection fixture and welded.

To represent the batch of components, it is considered that each part in each batch is subject to errors, where the source of errors is different across the batches. Inside batches, the errors are in the same area but in different magnitudes. To represent this error, forces are applied to a set of six points. These points remain in the same position in each batch, but the magnitude is scaled to differ to displace the points for a random value within the range ± 1 mm. The position of the force sources changes from batch to batch. For generalization, we consider that the occurrence of the source of forces and their placements are ambiguous and, thereby, randomly defined. For the reference assembly, 105 batches are generated, where, in each batch, there are 100 instances for each part. Deformed shapes of the two parts are created and utilized as the geometry input.

4. Results and discussion

The proposed method is set up for the case study, and the three optimization routines are established and solved for 100 batches of parts. This batch data provides the models with 10,000 data points on each part in the assembly. Dataset and DataLoader classes for efficient batch sampling, memory management, and GPU parallelization with Adam optimizer are utilized within Python pytorch on a Nvidia

GeForce RTX 4090 device. The models are trained end to end with a learning rate of 10^{-3}, and early stopping or validation checks ensure generalization. After an initial training with 75 batches, with 75% train data and 25% validation set, the learned parameters are stored. For incremental updates, the previously trained weights are reloaded, and the same model architecture is fine-tuned on batches 76–100. Batches 101 to 105 are utilized as unobserved data for further testing the predicted models on new batches.

4.1. Evaluation metrics and validation

Performance is evaluated by comparing predicted displacement metrics against ground truth values obtained from DT task optimizations in [4] with simulated data. Root mean square error (RMSE) validation sets and unobserved batches are monitored to ensure the incremental updates improve or maintain prediction accuracy, as shown in Table 1. Additionally, the prediction plots of the unobserved test data are presented in Fig. 3.

Table 1
Prediction results.

Prediction error	Train/Test data Batch 1–100		Unobserved data Batch 101–105	
RMSE	Mean	Variation (6σ)	Mean	Variation (6σ)
Matching	0.9509	0.02572	0.02079	0.0102
Adjustments	0.1011	0.2269	0.4118	0.3653
Joining sequence	0.0719	0.1092	0.2356	0.1121

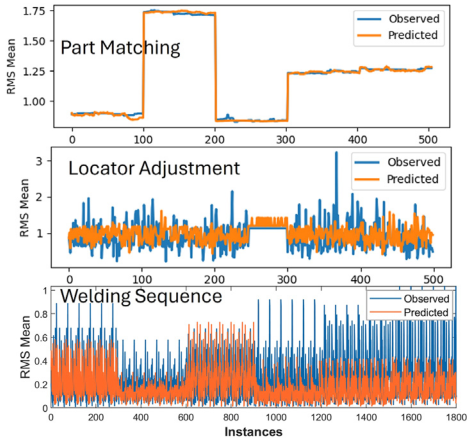

Fig. 3. Predicted and simulated results for batches 101–105.

4.2. Task predictions

For the matching task, the method demonstrates effective performance in capturing the mean value associated with the test batches, achieving an RMSE of 0.02 mm, as shown in Table 1 and Fig. 3. All the reported values are prediction errors compared to the baseline optimization routines in Sections 2.2–2.4. This prediction for the variation measure has been less accurate, with a RMSE of 0.01 mm. However, the main fluctuations between the batches are captured, although the predictions suffer from scaling errors. This is due to the relatively small variation among batches in the observed data, with a difference within 0.01 mm. Given that the prediction error is also within 0.01 mm for the 6σ value, this error is expected to improve incrementally by learning from batches where matching influences total batch variation.

For the locator adjustment task, the augmented DT resulted in a mean RMSE of 0.10 mm on the validation set and a further 0.4 mm on the unobserved data, as presented in Table 1. Fig. 3 further visualizes the generalizability of the model on the unobserved data following the batch differences. However, the accuracy of the prediction within the batch has been less precise. Providing the model with larger inside batch adjustment values, currently 50 iterations per instance, further increases the inside batch predictions.

For the joining sequence task, the augmented DT predicts the unobserved batch data with an RMSE of 0.2 mm. This has been 0.07 mm for the validation set during the training phase. Since the model here is a shallow network, the generalizability of the method to unobserved data has been less precise, as seen in Table 1. However, as visualized in Fig. 3, the model follows the differences across batches. It is to be noted that due to the nature of the sequencing problem, representing a sequential model with a small number of sequences available is challenging. Therefore, shallow separate sequence models are utilized in this case study. Increasing the number of incremental datasets in the case study enhances the model accuracy for sequence output predictions.

5. Conclusions

This study introduced an augmented geometry assurance digital twin, integrated with physics-based incremental learning models to predict geometric responses to part matching, locator adjustments, and joining sequences in self-compensating assembly lines. The augmented DT leverages recursive optimization routines and deep learning architectures to predict assembly geometric outcome, enabling real-time compensation for existing deviations. The results demonstrated the effectiveness of the method in achieving high accuracy for predicting geometric mean and variation metrics, particularly for part matching and locator adjustment tasks. Incremental updates further improved model generalization on unobserved data, supporting the potential of the proposed framework in dynamic, batch level geometry assurance. The developed augmented DT successfully applied to a case study and showcased the adaptability of deep learning models to assembly error compensation tasks with complex constraints.

Despite promising results, challenges such as scalability for joining sequence analysis remain. Also, integrating the online inspection data to model the simulation inaccuracies and adjust accordingly can further enhance augmented DT's accuracy.

Future research directions can focus on enhancing the robustness of the proposed framework to handle diverse joining operations. Furthermore, generalizing the method to detect the impact of batch to batch variation of stamped parts on assembly quality can be studied. Novel architectures, i.e., graph neural networks and physics informed architectures, also hold potential for future work.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Roham Sadeghi Tabar: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Rikard Söderberg:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Dariusz Ceglarek:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Pasquale Franciosa:** Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Lars Lindkvist:** Validation, Software, Methodology, Investigation, Formal analysis, Data curation.

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