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Research paper

State-of-the-art optimization algorithms in weather routing — ship decision support systems: challenge, taxonomy, and review

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ABSTRACT

Weather routing has been extensively used as a decision support system in merchant ship operations and traffic management. A critical component of such a system is the optimisation method. Over recent years, substantial research efforts have been devoted to developing voyage optimisation algorithms, either to support decision-making of weather routing in merchant shipping or to assist autonomous ships in academic research. The requirements for optimisation methods for merchant shipping differ significantly from those in academic autonomous ship applications. However, many optimisation-related terminologies and algorithms are often used arbitrarily across these two fields, easily leading to confusion. In addition, the emergence of machine learning after 2020 has shown a significant impact on the development of those optimisation algorithms. Still, we see a lack of a systematic review and in-depth summary of recent developments in the optimisation methods focused on weather routing. This paper presents an overview of recent scientific publications to show state-of-the-art research and development status and trends. Focusing on the optimisation methods used in weather routing, we clarify optimisation terminologies. In addition, we propose a general framework to develop voyage optimisation methods to summarise and categorise various developed algorithms. Then, we review scientific papers published in recent years for weather routing developments and applications. Finally, future research and outlooks are discussed for further development of weather routing algorithms.

1. Introduction

1.1. Background and motivations

The International Maritime Organization (IMO) has established an ambitious strategy targeting a 20 % reduction in greenhouse gas (GHG) emissions by 2030, 70 % by 2040, and full decarbonisation by 2050, relative to 2008 emission levels (IMO, 2023; UNCTAD, 2023). However, only a 3.6 % reduction had been achieved by 2023 (DNV, 2024c). The shipping industry is facing the urgent need to develop and implement energy efficiency measures to reduce its carbon footprint and emissions (DNV, 2023, 2024b; Wang et al., 2023; Yan et al., 2024) while avoiding substantial commercial cost increases (DNV, 2024a). Weather routing has a small investment but immediate benefits, such as safety and a reliable estimated time of arrival (ETA). Additionally, it has significant potential to improve the effectiveness and efficiency of current ship operations (DNV, 2024b, c), such as achieving just-in-time (JIT) arrivals considering accurate ETAs (IMO, 2020), being integrated with new technologies such as wind-assisted propulsion ships (Mason et al., 2023; Mason, 2021; Wang et al., 2022b). As a result, relevant research and development have grown significantly during the past years.

Weather routing, as illustrated in Fig. 1, is often used as a decision support tool to help search for optimal routing between two locations based on information provided by weather forecasts and a ship's energy performance (Zis et al., 2020). The decision-making process in a ship's operations often needs to consider many factors in a complex process, as shown in Fig. 2. Large uncertainties involved in the process may bring significant challenges for decisions leading to optimal ship operations

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[1] Coordinate the paper organization and writing

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(Tsai et al., 2021). For example, the open sea poses significant risks due to unpredictable weather (Vettor et al., 2021; Yuan et al., 2022), and various uncertainties arise from the dynamic nature of real-world operations that influence ship performance (Ksciuk et al., 2023; Zhang et al., 2023).

By leveraging superior data processing and computational capabilities offered by modern computers compared to human operators, weather routing systems can predict potential solutions and proactively avoid unfavoured consequences, identifying optimal options in advance. Optimisation algorithms in weather routing are essential for intelligent ship decision support and greatly enhance optimal ship operations. An algorithm needs to handle these uncertainties and dynamic changes in real time, ensuring various ship operation objectives, such as punctuality and energy conservation with emissions reduction. In addition, the sailing process is continuously monitored, and some adjustments in the plan of merchant shipping may also occur because of market fluctuations. Furthermore, factors such as changes to the voyage plan (e.g. destination, ETA) or deviations due to unforeseen incidents (Poulsen and Sampson, 2019; Poulsen et al., 2022) make the development of weather routing algorithms more challenging. All these challenges have driven dedicated research on weather routing algorithms, with many classic algorithms being applied, as well as numerous variations and newly developed ones (Grifoll et al., 2022; Ma et al., 2024).

However, to our knowledge, no reviews have focused on optimisation algorithms in weather routing to summarise the development over the years. Furthermore, in recent years, the challenges and complexity of weather routing have driven researchers to adopt advancements in algorithms from fields other than the maritime sector. These include artificial intelligence (AI) and machine learning (ML) techniques, which have led to significant improvements. The two most recent review papers were published before 2021 (Yu et al., 2021; Zis et al., 2020) and did not cover the latest research developments. Furthermore, we observed that the meanings of commonly used terms are inconsistent and unclear in the weather routing literature. For example, the essence of weather routing is optimising voyages, often referred to as voyage optimisation. However, other maritime studies, such as those on ship scheduling (Gao and Sun, 2023; Luo et al., 2024; Yu et al., 2022) and collision avoidance for maritime autonomous surface ships (MASSs) (Johansen et al., 2016; Tsolakis et al., 2024; Wu et al., 2021; Zhang et al., 2025) also involve voyage optimisation. These are all broadly labelled as 'voyage optimisation' without specifying the scope, or the term is used arbitrarily, resulting in misinterpretations. Furthermore, optimisation algorithms are widely studied and applied beyond the maritime community, drawing on expertise from research fields such as computer science (Liang et al., 2024; Tian et al., 2021), transportation (Lee et al., 2023; Yu et al., 2023), and control engineering (Li et al., 2022; Zhang et al., 2020). This interdisciplinary nature brings difficulties since terms from different fields are frequently adopted, adding to the confusion.

1.2. Research questions, contributions, and outline

To address the above issues, this study aims to illustrate and answer the following research questions.

- What are the general research trends in weather routing, and which aspects have started to receive specific attention in recent years (i.e. after 2020) (Section 2.1)?
- How is weather routing defined and distinguished from other voyage optimisation problems (Section 2.2, 2.3)?
- What are the common optimisation algorithms used in weather routing, along with their strengths and weaknesses (Section 3)?
- How have the optimisation algorithms been developed in recent years incorporating emerging technologies such as AI/ML? What improvements and advancements have been made in their key processes (Section 4)?
- What is the future direction of optimisation in weather routing research? (Section 0)

To investigate the above research questions, we conducted a systematic literature review following the process in Fig. 3. First, the literature retrieval reveals publication trends and focuses, followed by clarifications on work scopes and terminologies. Then optimisation methods used in weather routing are categorised and reviewed using a proposed algorithm framework. Finally, key innovations in recent publications are discussed to pinpoint future research directions. Some

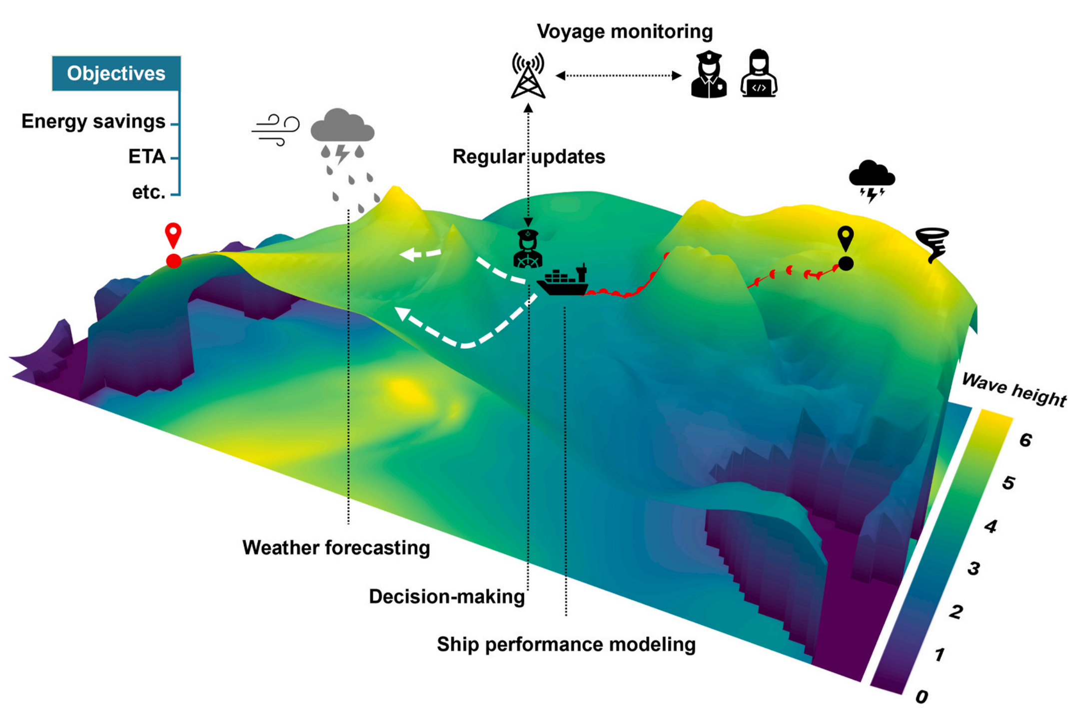


Fig. 1. Graphical illustration for weather routing in open sea shipping.

of the key contributions from this study are listed as follows.

1) Clarifications of common but mixed-used terminologies and algorithms and comparisons of the scopes of similar research issues. This helps researchers and practitioners gain a clear understanding of weather routing and identify the correct problem-solving approaches, avoid misinterpretation, and facilitate efficient development of weather routing services.
2) Discussion of the cons/pros of optimisation algorithms researched in weather routing based on a systematic and in-depth review.
3) Presentation of recent research trends in optimisation algorithms, including the impact of emerging technologies (AI/ML) since 2020, and potential future directions.

2. Overview of publication trends and terminology clarifications

2.1. Publication trends from 2010 to 2024

An automated literature retrieval is used to obtain a general overview of publication trends. The scope of this review was ship weather routing, and the search conditions and results are presented in Table 1. The literature was retrieved from the database Web of Science. The research subject had to be a ship. The search conditions for topics include the following: ‘voyage optimisation’, ‘route optimisation’, ‘voyage planning’, ‘route planning’, or ‘weather routing’ and explicitly exclude ‘collision avoidance’, ‘schedule’, and ‘scheduling’. Open sea conditions were not specifically addressed as they are details typically included in the main body of the paper. Logical operators ‘AND’, ‘OR’, and ‘NOT’ were employed to define search criteria. Considering papers written in English and published after 2010, we filtered the results to include only early-access articles, journals, conference papers, letters, and reviews. The end result was a total of 2151 papers, of which 1588 were published in journals.

First, the retrieved papers were categorised to identify the number of publications in each year, top publishing journals, and contributing fields. Fig. 4 presents the annual trend in publications and citations for this research topic. Since 2010, a continuous increase in the number of publications and citations has occurred, indicating growing interest among researchers. Fig. 5 shows the top ten journals by number of publications, with *Ocean Engineering* leading. When categorised by discipline, as shown in Fig. 6, *Marine Engineering* contributes the most to this research topic. However, the maritime sector does not dominate, as many other fields are also involved, reflecting the interdisciplinary nature of weather routing.

In addition, we employed CiteSpace software to identify key research topics and gain insights into the primary issues that have concerned researchers. Representative terms were extracted by CiteSpace through the analysis of paper titles, abstracts, keywords, and index terms using natural language processing techniques. The trending terms and keywords identified by CiteSpace are presented in Tables 2 and 3, respectively. In each table, the results are divided into two periods, 2010–2019 and 2020–2024, to study changes over time and investigate the shifts in the most recent five years. The counts in 2020–2024 are higher across the board compared to 2010–2019. Along with Fig. 4, these findings demonstrate that research interest and activities in this field have been steadily increasing, especially in the past five years. As indicated in Table 2, the main concerns have consistently been focusing on fuel consumption and energy efficiency. Specifically, Table 3 indicates that the components that have continuously received research attention across both periods are ‘models’, ‘optimisation’, and ‘algorithm’. Of note, the term ‘machine learning’ rises to the fourth place in related research from 2020 (Table 2), indicating significant interest gained over the past five years.

The above observations first demonstrate the need for a systematic review of weather routing, especially over the past five years, as emerging ML technologies have started to significantly impact weather routing research (Table 2). As a keyword, ML first appeared in 2019 twice and then 47 times after 2020 (Table 3). The latest reviews published before 2021 did not sufficiently cover these new advancements in the field. Moreover, optimisation algorithms comprise a key component that continues to receive significant attention and ongoing research (Table 3). Thus, this paper focuses on reviewing optimisation algorithms used in weather routing, including developments in recent years. Furthermore, ‘liner shipping’ in Table 2 should not be at the top of the search results, as it is usually a focus of a ship scheduling problem instead of weather routing. This again demonstrates the mixed and unclear usage of terms, as the search conditions have explicitly excluded ‘ship scheduling’ and ‘schedule’. Thus, this section provides a very general overview of research trends. As the automated literature search cannot effectively distinguish the correct research scope, we further investigate the retrieved papers manually in later sections and, in the next part, clarify the definitions of common terms in voyage optimisation.

2.2. Terminologies and definitions

The interchangeable use of terms, as well as the blurred boundaries among problems, have led to inconsistency within the maritime community. For example, Yu et al. (2021) presented a review on voyage optimisation, where they did not distinguish weather routing from ship routing/scheduling problems. Zis et al. (2020) specifically stated that the scope of ‘ship routing/scheduling’ is completely distinct from that of weather routing. They explicitly excluded ship routing/scheduling from their review and focused on ship weather routing. These two reviews demonstrate different understandings of researchers for the problem’s scope. In this paper, however, the literature search revealed that despite

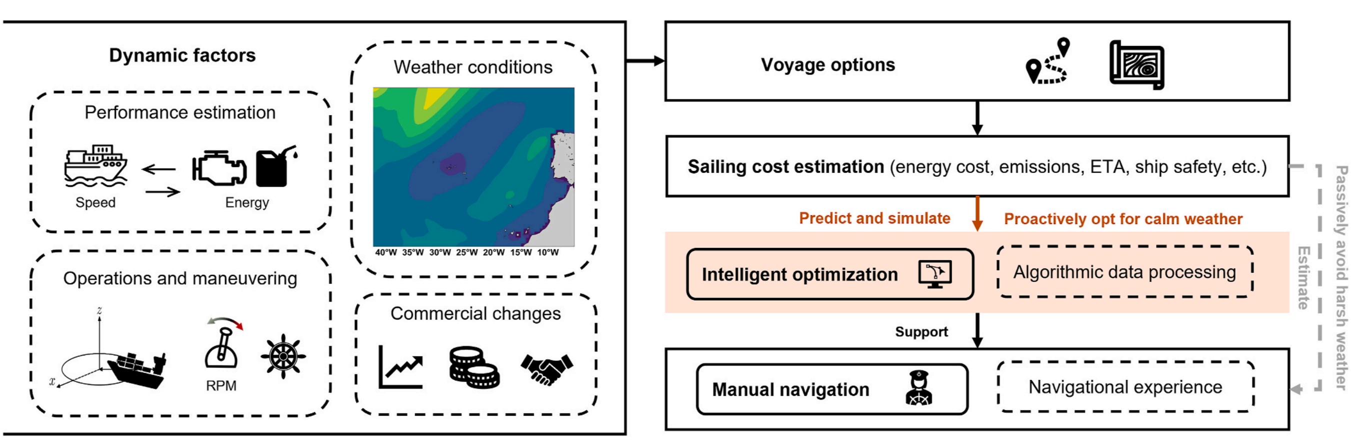


Fig. 2. The general processes with weather routing as the decision support system in shipping.

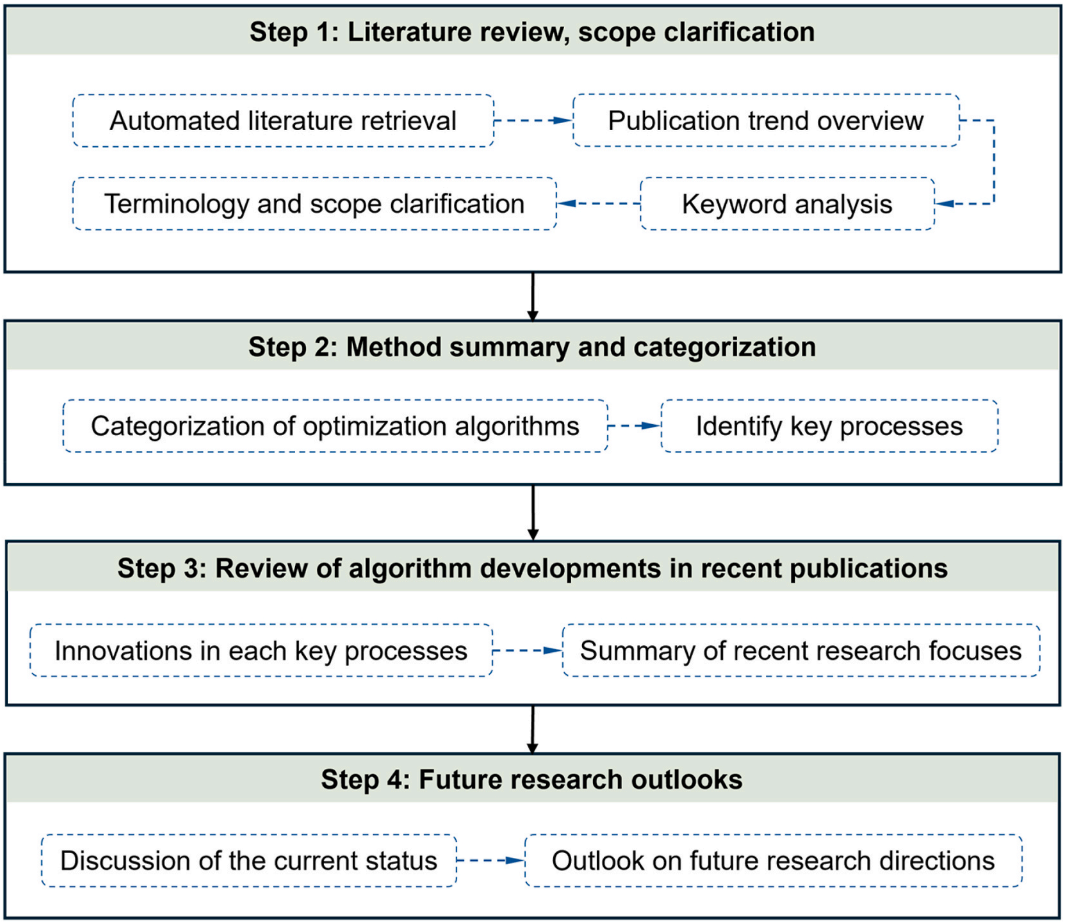


Fig. 3. Steps of systematic literature review on optimisation methods in weather routing.

Table 1
Literature search conditions and results.

Conditions	Results	
Database	Web of Science	
Search keywords	('voyage optimisation' OR 'route optimisation' OR 'voyage planning' OR 'route planning' OR 'weather routing') AND ship NOT ('collision avoidance' OR 'schedule' OR 'scheduling')	
Paper type	Early-access, journal, conferences, letter, review papers	
Language	English	
Time range	January 2010–December 2024	
Number of papers	After 2020	1236
	Total	2151
Number of journal papers	After 2020	1043
	Total	1588

using keywords like 'voyage optimisation' or 'weather routing' and explicitly excluding 'ship scheduling', some studies surfaced that essentially address 'ship scheduling' problems, such as in (Lee et al., 2023). This again demonstrates the ambiguous use of these terms in research. Some common mixed terminologies in voyage optimisation or weather routing are clarified based on authoritative resources as follows.

- **Voyage.** The berth-to-berth concept for voyages is applied according to the European Parliament and of the Council (2015). That is, a voyage starts at the berth of one port of call and ends at the berth of the next port of call.
- **Route.** A route is defined as a way or course taken from a starting point to a destination (Stevenson, 2010).
- **Routing.** The objective of a ship's routing is to improve the safety of navigation in converging areas and in areas where the density of traffic is great or where freedom of movement of shipping is inhibited by restricted sea room, the existence of obstructions to navigation, limited depths, or unfavourable meteorological conditions (IMO, 2003).
- **Weather routing.** Weather routing, by which ships are provided with optimum routes to avoid bad weather, can enhance safety (IMO, 2003). Environmental routing and weather routing are frequently used interchangeably, but the latter is a subset of the former. Both belong to the broader category of voyage optimisation (Christiansen et al., 2007).
- **Voyage planning.** Voyage and passage planning includes four stages: appraisal, planning, execution, and monitoring. At the planning stage, a detailed voyage or passage plan should be prepared, covering the entire voyage or passage from berth to berth. This includes tasks such as plotting the intended route, tracking the voyage or passage, and altering speed, course, and machinery status en route (IMO, 1999).
- **Voyage execution.** The voyage or passage should be executed in accordance with the plan or any changes made thereto. Factors considered include vessel navigation, ETA, meteorological conditions, weather routing information, and traffic conditions (IMO, 1999).
- **Ship scheduling.** The 'ship routing problem', or 'ship routing and scheduling problem', is a distribution problem at the tactical level in which a ship or a fleet of ships has to serve several ports to retrieve

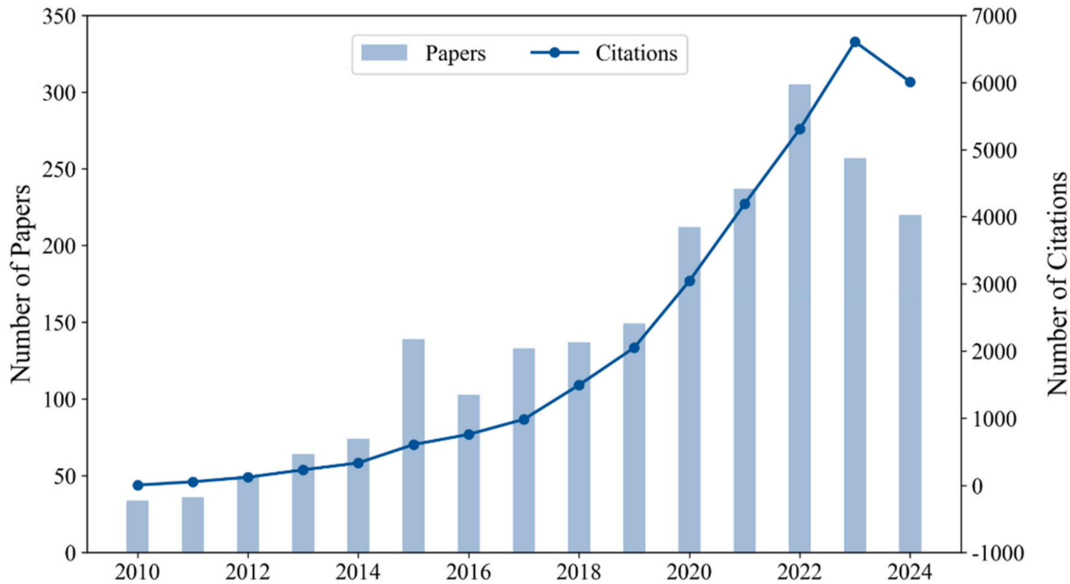


Fig. 4. Numbers of published papers and citations from 2010 to 2024.

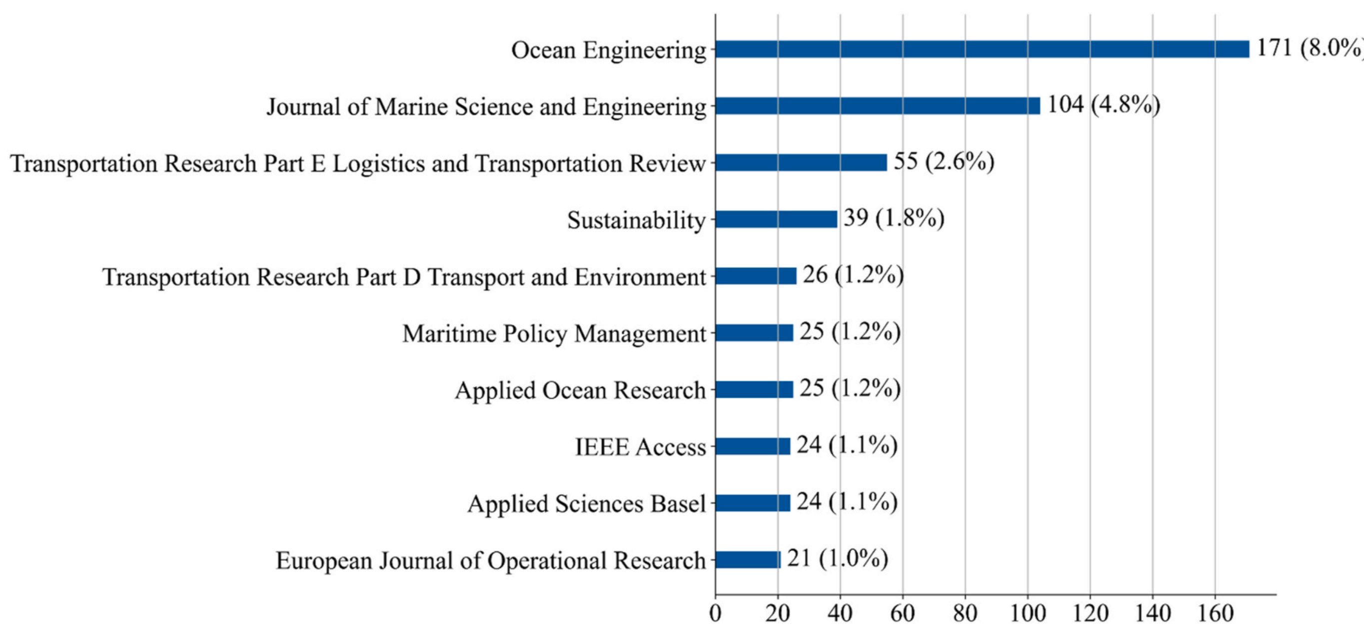


Fig. 5. Top ten publishing journals for ship weather routing literature from 2010 to 2024.

and deliver cargo and is subject to various constraints, such as ship capacity and time windows (Zis et al., 2020).

- **Pathfinding.** Pathfinding is the algorithmic interpretation and implementation of attaining the shortest route(s) from a given source (s) to destination(s). It is a fundamental problem broadly studied in many fields, such as AI, robotics, and computer science (Majumder and Majumder, 2021).
- **Path planning.** Path planning involves finding a collision-free motion between an initial (start) and final configuration (goal) within a specified environment (Gasparetto et al., 2015).

Based on the above definitions, some research topics often appearing alongside voyage optimisation are presented in an overlapping diagram in Fig. 7. Path planning and pathfinding problems, as the broadest and most fundamental topics, have been extensively studied beyond the maritime sector (Majumder and Majumder, 2021). They apply to moving objects in general, including vehicles and robots, while voyage optimisation \ focuses on ships. As optimisation objectives vary, voyage optimisation further results in sub-problems with different implementation scenarios and scopes, e.g. **ship routing**, **weather routing**, **collision avoidance**, and **speed optimisation**.

While **ship routing** often involves multiple voyages between several ports of call, ETAs at each port, or arrival sequences, it is similar to the traditional vehicle routing problem in transportation networks. Although it optimises voyages with speed and ETA, it is a different optimisation problem from weather routing (Zis et al., 2020). Besides energy consumption, these problems emphasise commercial factors, such as freight rate, fuel prices, market fluctuations, and profitability (Lee et al., 2023; Tran and Haasis, 2018). In addition, their voyages are not strictly limited to specific sailing regions, i.e. these routes can involve container ship routing in transoceanic voyages or coastal ships engaged in short sea shipping, as shown in Fig. 8(a).

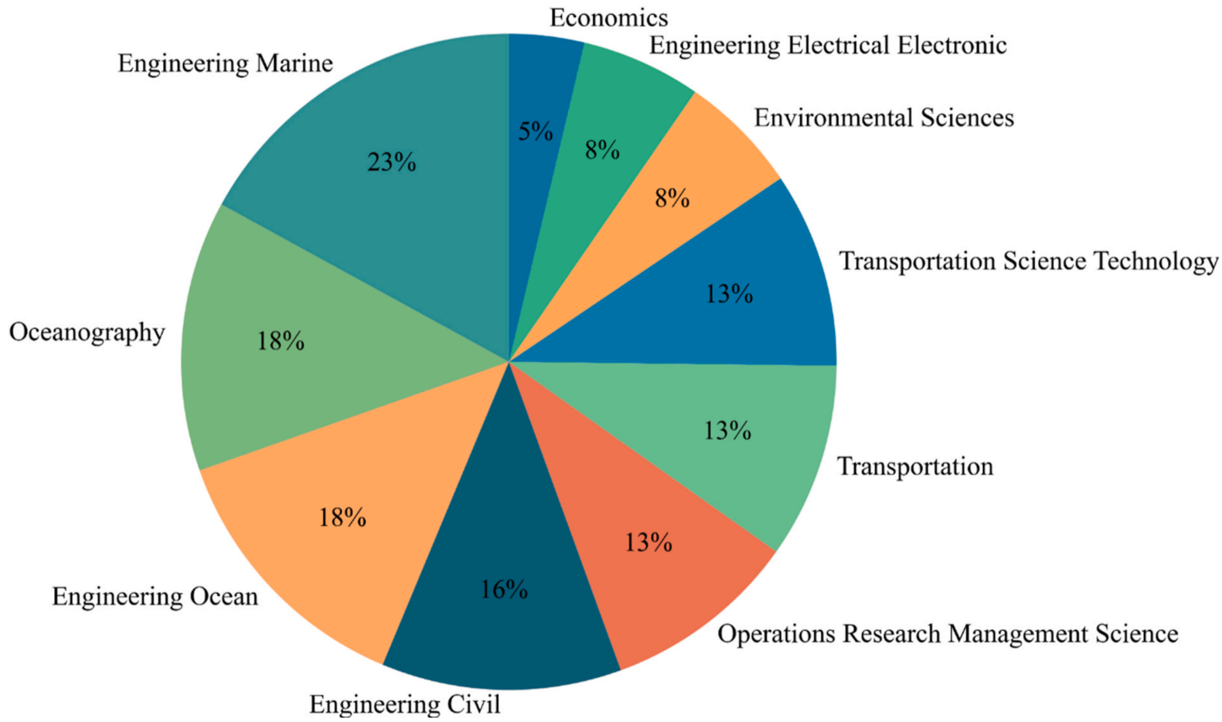


Fig. 6. Fields of ship weather routing literature published from 2010 to 2024, showing the percentage of total number of papers.

Table 2
Trending terms in ship weather routing literature across different periods.

Trending level	2010–2019	Count	2020–2024	Count
1	Fuel consumption	69	Fuel consumption	131
2	Routing problem	39	Route planning	58
3	Liner shipping	39	Energy efficiency	55
4	Genetic algorithm	31	**Machine learning**	55
5	Energy efficiency	30	Routing problem	54

Table 3
Trending keywords in ship weather routing literature across different periods.

Trending levels	2010–2019	Counts	2020–2024	Counts
1	Optimisation	85	Optimisation	204
2	Model	82	Model	155
3	Algorithm	53	Algorithm	93
12	Fuel consumption	26	**Machine learning**	47

Weather routing and collision avoidance are distinct because of differences in sailing areas. **Weather routing** involves optimising a voyage in the open sea from one port of call to another, and weather conditions are the main factor affecting energy consumption. **Collision avoidance** becomes the focus in coastal and inland water areas, where the number of nearby vessels and maritime traffic density increase (Gao et al., 2023; Zhang et al., 2025). Because of more complex traffic and less weather impact, their sailings prioritize compliance with safety and traffic regulations, focusing on collision risk and avoidance strategies (Huang et al., 2020; Tran et al., 2023). Additionally, because of real-time and frequent ship manoeuvres, the algorithm's output is often integrated with control systems (Johansen et al., 2016).

Operation optimisation refers to optimising the operational decisions (e.g. speed (Li et al., 2024a; Sidoti et al., 2023), power (Besikçi et al., 2016; Ma et al., 2023a), or trim (Coraddu et al., 2017; Hu et al., 2022)) along fixed routes to achieve energy savings based on ETA constraints at each waypoint or the destination. These problems include voyage division, separation of the route into segments, and combinatorial optimisation across multiple sub-routes between waypoints. They are similar to ship routing problems with many overlapping solution

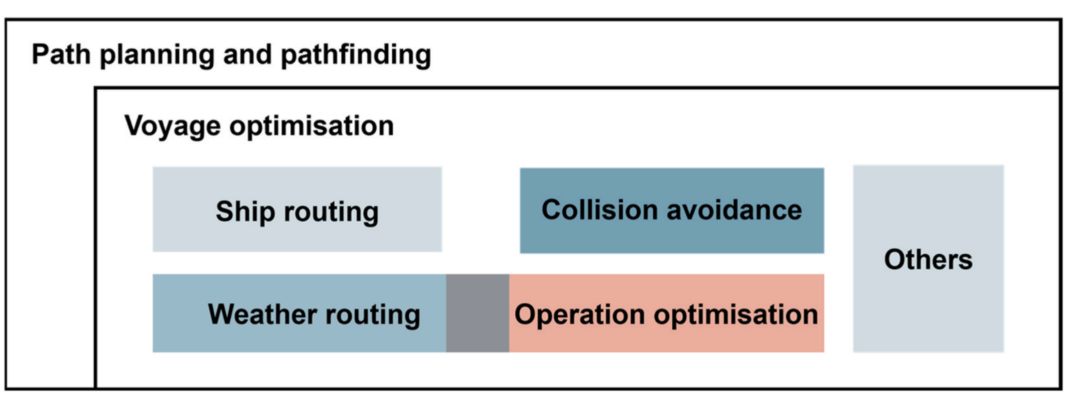


Fig. 7. Overlapping diagram of common research problems related to voyage optimisation.

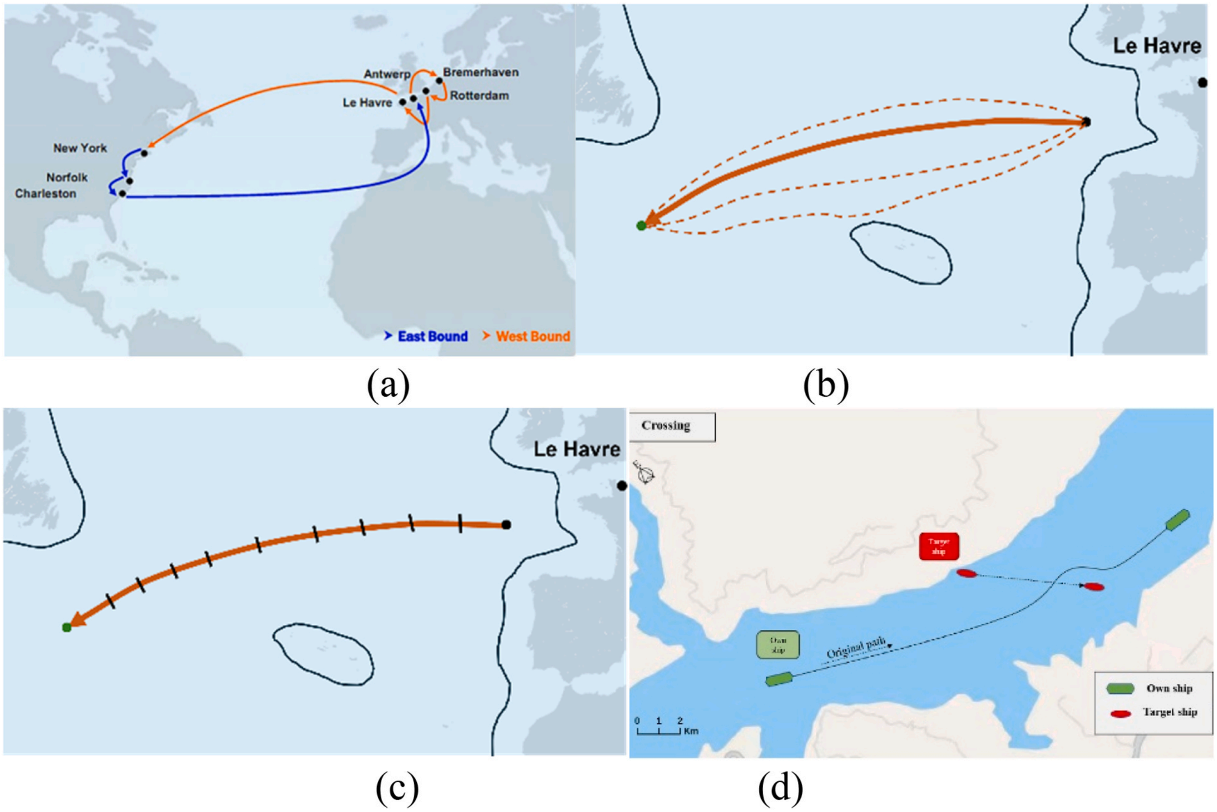


Fig. 8. Examples of (a) an intercontinental routing network for shipping (Tran and Haasis, 2018), (b) a transoceanic voyage between two locations in weather routing, (c) speed optimisation for a transoceanic voyage between two locations, and (d) a collision avoidance scenario (Gao et al., 2023).

approaches.

Clarifying the problem scope is essential to finding effective methods and applying them correctly. These four issues – ship routing, weather routing, collision avoidance, and speed optimisation – share some common aspects, such as concerns related to weather/sea conditions, energy consumption, safety, and emissions. In particular, speed optimisation can also be integrated into weather routing as a sub-task (Ma et al., 2020, 2023a; Wang et al., 2020b), as shown in Fig. 7. As a result, these terms are easily confused with each other, especially when used interchangeably. However, they differ in problem conditions and objectives. These differences first result in some unique solutions, such as integer/mixed-integer programming for ship scheduling (Gürel and Shadmand, 2019; Wang and Meng, 2012), probability random map (PRM) (Guan et al., 2024), rapidly exploring random tree (RRT) (Zaccone and Martelli, 2020) for collision avoidance, and linear/quadratic programming for speed optimisation (Li et al., 2020; Sung et al., 2022).

In this study, weather routing specifically addresses the voyage optimisation problem in open sea conditions as defined above. As an operational-level challenge, it aims to determine the best voyage for a

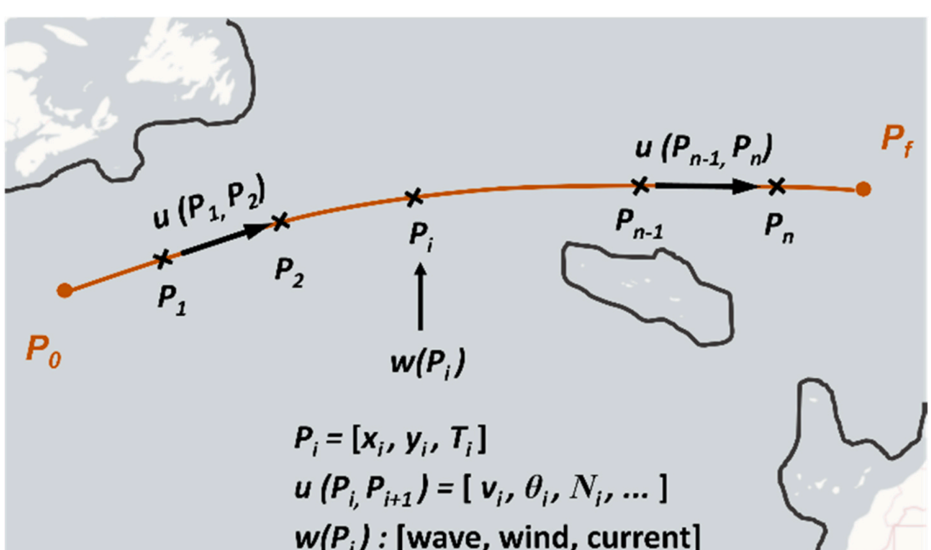


Fig. 9. Illustration of a ship voyage with its waypoints, operational parameters, and sailing conditions.

single seagoing ship to travel from departure port A to destination port B with objectives such as energy efficiency. This process includes optimising the route of a voyage and other operational parameters along the route (e.g. speed, power, heading) by considering metocean conditions. The output from the weather routing is provided to ship operators to assist with their decision-making.

2.3. Energy-efficient weather routing

In a ship's weather routing process, a voyage can be defined as a series of waypoints, along with operational parameters in each sub-route between adjacent waypoints, as illustrated in Fig. 9. Assume $\boldsymbol{P}_0$ is the departure and $\boldsymbol{P}_f$ is the destination. A waypoint between $\boldsymbol{P}_0$ and $\boldsymbol{P}_f$ can be first denoted by Eq. (1) at the ith time stage:

$$\boldsymbol{P_i} = [\, x_i, y_i, T_i \,] \tag{1}$$

where x_i, y_i, and T_i indicate the longitude, latitude, and the time of the ship passing this waypoint, respectively. The encountered metocean conditions at $\boldsymbol{P}_i$ include wave, wind, and current, which can be denoted as

$$\boldsymbol{w} = [\, S(\omega\,|H_s, T_z), V_c, \theta_c, V_w, \theta_w \,] \tag{2}$$

where $S(\omega\,|H_s, T_z)$ is the encountered wave that consists of significant wave height H_s and wave period T_z; $V_c, \theta_c, V_w,$ and θ_w represent ocean current and wind in terms of speed V and direction θ, respectively. Further, the operational parameters at the sub-route from $\boldsymbol{P_i}$ to the waypoint at the next, i.e. $(i+1)$-th, time stage $\boldsymbol{P_{i+1}}$ can be represented by

$$\boldsymbol{u}(\boldsymbol{P}_i, \boldsymbol{P}_{i+1}) = [v_i, \theta_i, N_i, \dots] \tag{3}$$

where $\boldsymbol{u}$ $(\boldsymbol{P_i}, \boldsymbol{P_{i+1}})$ can include different factors, such as the sailing speed v_i, heading θ_i, and engine speed (RPM) N_i. The **optimisation variables** of the weather routing problem are, therefore, the set of waypoints $\boldsymbol{P}$ and associated operational parameters $\boldsymbol{U}$:

$$\boldsymbol{P} = [\, \boldsymbol{P}_0, \boldsymbol{P}_1, \boldsymbol{P}_2, \dots \boldsymbol{P}_{n-1}, \boldsymbol{P}_n \,]$$

$$\boldsymbol{U} = \left[\boldsymbol{u}(\boldsymbol{P}_0, \boldsymbol{P}_1), \dots, \boldsymbol{u}(\boldsymbol{P}_n, \boldsymbol{P}_f)\right] \tag{4}$$

where n denotes the last time stage and $\boldsymbol{P_n}$ denotes the last waypoint before $\boldsymbol{P_f}$.

Referring to a typical optimisation problem, the general structure for weather routing can further be divided into four major components: constraints, objectives, cost function, and optimisation algorithm, as presented in Fig. 10. The **constraints** outline the solution space within which $\boldsymbol{P}$ and $\boldsymbol{U}$ are allowed to take values. Assume $\boldsymbol{S_P}$ is the feasible solution space for $\boldsymbol{P}$, i.e., the allowed sailing area between $\boldsymbol{P_0}$ and $\boldsymbol{P_f}$, and $\boldsymbol{S}_U$ is the feasible solution space for $\boldsymbol{U}$, i.e., the ship's allowed range of operational settings. $\boldsymbol{S_P}$ excludes areas such as land, no-go zones, shallow water or emission control areas, and $\boldsymbol{S}_U$ considers ship's specific maneuverability, etc. The aim of weather routing is to find a set of $\boldsymbol{P}$ and $\boldsymbol{U}$ that can meet a pre-defined **optimisation objective.** Common objectives include energy efficiency, accurate ETAs ensuring JIT arrivals, and ship safety at sea and decarbonisation. Optimisation objectives are achieved by minimising the total **cost function**:

$$J_n = \sum\nolimits_{i=0}^{n} L_i(\boldsymbol{P}_i, \boldsymbol{u}(\boldsymbol{P}_i, \boldsymbol{P}_{i+1})|\boldsymbol{w}) \tag{5}$$

where L_i $(\boldsymbol{P_i}$, $\boldsymbol{u}$ $(\boldsymbol{P_i}$, $\boldsymbol{P_{i+1}})$ $|\boldsymbol{w})$ represents the instantaneous cost function at the ith time stage or sub-route. If the optimisation objective is to achieve minimal fuel consumption, L_i $(\boldsymbol{P_i}$, $\boldsymbol{u}$ $(\boldsymbol{P_i}$, $\boldsymbol{P_{i+1}})$ $|\boldsymbol{w})$ calculates the fuel consumption for sailing at the sub-route following waypoint $\boldsymbol{P_i}$ using operational parameter $\boldsymbol{u}$ $(\boldsymbol{P_i}$, $\boldsymbol{P_{i+1}})$ under the local sea states $\boldsymbol{w(P_i)}$. J_n

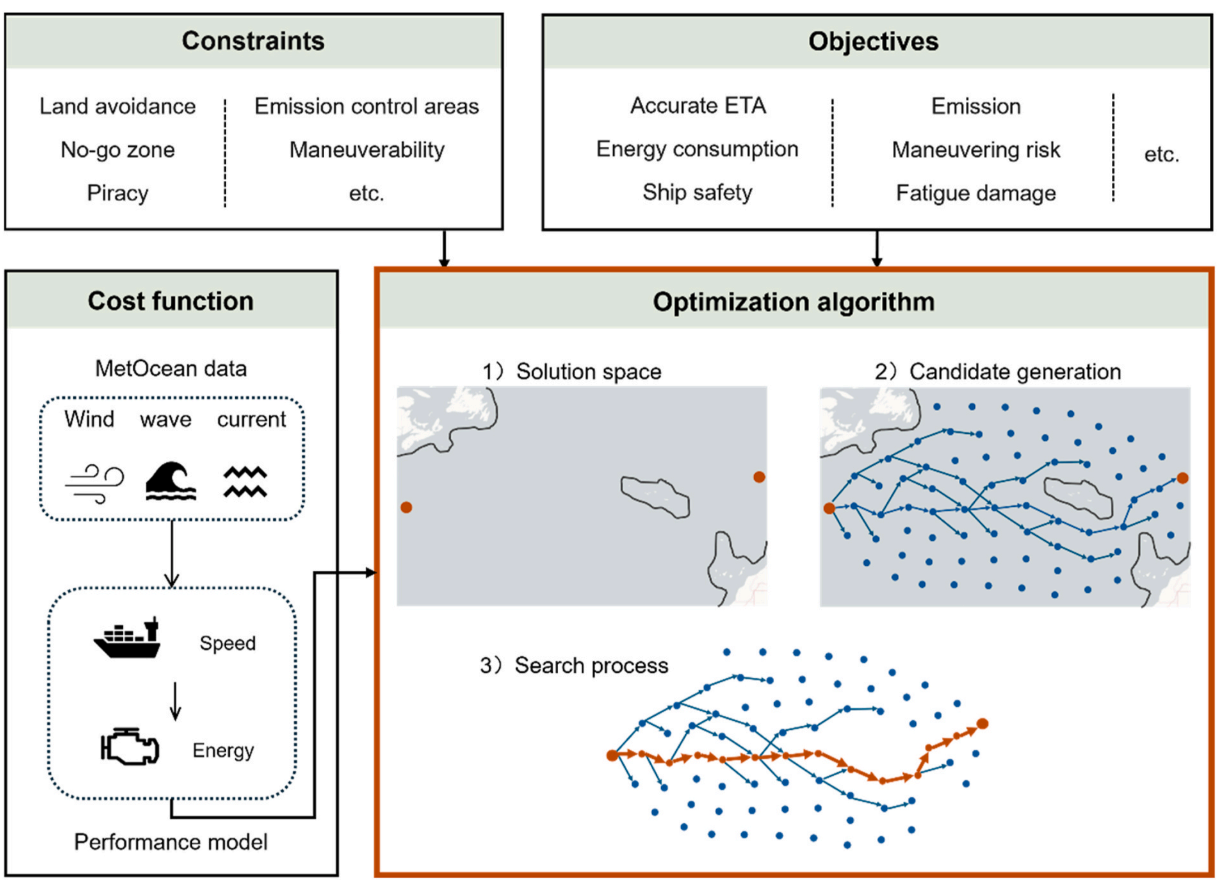


Fig. 10. General structure for a weather routing problem.

represents the accumulated cost function from the start to the final nth time stage, and J_i indicates the current accumulated cost function until the i-th time stage.

In weather routing, the instantaneous cost function L_i in Eq. (5) incorporates a ship model to assess the impact of weather on the ship's performance. Specifically, it evaluates the energy the ship needs to maintain a given speed under the surrounding weather conditions, i.e. presenting a speed–energy relationship. The energy metric can refer to fuel consumption, power usage, or emissions, for example. The cost function uses the ship model to formulate a cost for the algorithm's decision-making, and the formulation of the cost function can be problem specific. Finally, all inputs are provided to the optimisation algorithm, which determines the output of the weather routing, $\boldsymbol{P}^*$ and $\boldsymbol{U}^*$, that can minimise the total cost function:

$$\boldsymbol{P}^*, \boldsymbol{U}^* = \underset{\boldsymbol{P}_i \in \boldsymbol{S}_P, \boldsymbol{u}(\boldsymbol{P}_i, \boldsymbol{P}_{i+1}) \in \boldsymbol{S}_U}{\arg\min} \sum_{i=0}^{n} L_i(\boldsymbol{P}_i, \boldsymbol{u}(\boldsymbol{P}_i, \boldsymbol{P}_{i+1}) | \boldsymbol{w}) \tag{6}$$

The optimisation algorithm is the core component in weather routing. It further includes three processes: identifying the solution space, generating feasible solution candidates, and conducting the search for the optimal solution. Algorithms employ varying strategies for the execution and integration of these three processes, as detailed in Section 3. In general, based on the input constraints, the solution space is first defined. Feasible candidate solutions are generated within the solution space and evaluated with a cost. Then, the search is carried out to find the optimum in all candidates based on their costs, subject to specific objectives. The overall effectiveness of weather routing is highly sensitive to the optimisation algorithm, as well as weather forecasting and ship performance model (Tsai et al., 2021). However, weather forecasting falls more within the domain of meteorological expertise and is therefore beyond the scope of this review paper. Similarly, the ship performance model, which is used to estimate energy cost, is an independent research topic. Interested readers can refer to the following review papers for more information (Fan et al., 2022; Yan et al., 2024). Only the optimisation algorithms are reviewed in the upcoming subsections of this paper.

3. Algorithms for energy-efficient weather routing

This section proposes a general algorithm framework, summarising the algorithms in weather routing research, including state-of-the-art developments. Research on algorithms has been ongoing since weather routing gained attention. The categories of algorithms commonly used in weather routing include dynamic programming (DP), Dijkstra, A*, Isochrone algorithm, evolutionary algorithm (EA), genetic algorithm (GA), ant colony optimisation (ACO), and partial swarm optimisation (PSO). Some algorithms were originally designed specifically for ship weather routing, e.g. Isochrone algorithms, while others were adapted from advancements in other research fields and applied to weather routing, e.g. Dijkstra and GA. Many algorithms have further evolved into numerous variations. However, based on their fundamental characteristics and processes, such as optimality, dependency, search strategy, and efficiency, these algorithms can still be grouped into three categories: exact, heuristic, and learning-based, as described in Fig. 11.

3.1. General categories of algorithms used in weather routing

Exact algorithms can find the globally optimal solution to a problem in the solution space under certain preconditions. Examples of exact algorithms include DP and Dijkstra. Although they can ensure global optimality, exact algorithms may face efficiency challenges when dealing with large-scale, complex problems. Exact algorithms have the following key characteristics.

- **Global optimality.** Exact algorithms can ensure that under certain preconditions, the result is the global optimum among all possible solutions.
- **Mathematical foundations.** Certain preconditions must be met to ensure that exact methods can find the global optimum, and these preconditions are closely connected to the mathematical foundation on which exact methods rely. In addition, the effectiveness of exact algorithms may depend on the mathematical structure of the problem, e.g. linearity, convexity, and the nature of the constraints. When these conditions are met, the algorithm can fully leverage its mathematical principles to explore the solution space efficiently and reliably.
- **Systematic search.** Based on mathematical principles and strategies, the exact methods systematically explore the solution space.
- **Complexity.** Due to these characteristics, exact methods can be time-consuming, especially for large or complex problems. While they avoid brute-force enumeration, their computational complexity remains high and may grow exponentially with input size.

As a result of these complexities, researchers often opt for another type of approach – heuristic methods. **Heuristic algorithms** are those employing empirical or practical knowledge (referred to as heuristics) to

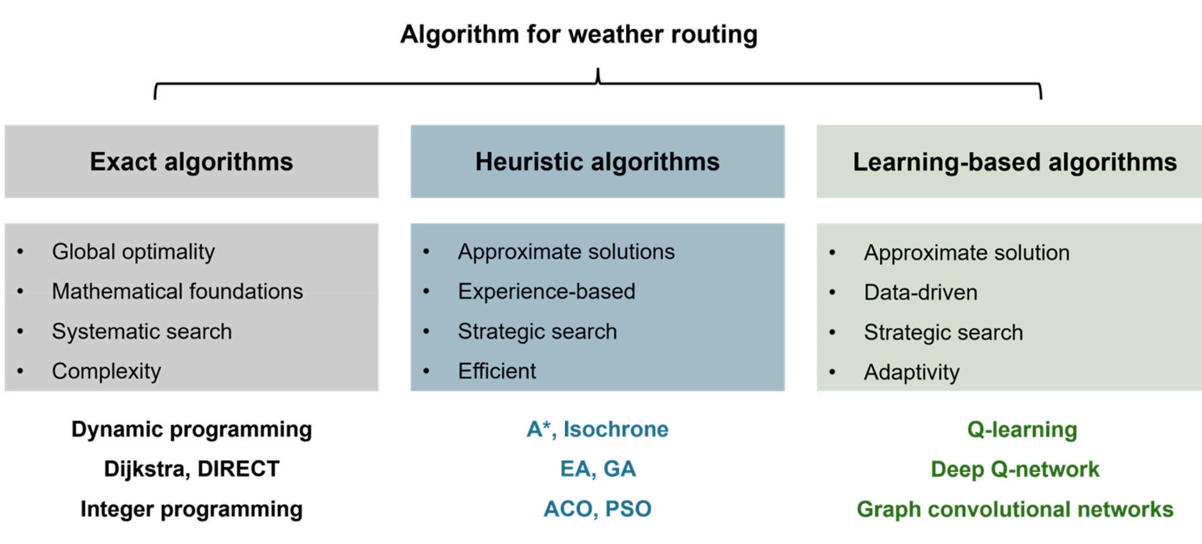


Fig. 11. General categorisation of algorithms for weather routing.

improve the efficiency of solving optimisation problems. Rather than guaranteeing a globally optimal solution, heuristic methods aim to provide satisfactory solutions when the solution space is very large or complex. They may not achieve global optimality, but heuristic algorithms are especially useful and efficient for problems that are computationally intensive and difficult to solve through exact methods. Examples of heuristic algorithms used in weather routing include A*, Isochrone, EA, and GA. As their search is guided by heuristics, the effectiveness and performance of heuristic algorithms heavily rely on the employed heuristic knowledge. However, much of this knowledge may require prior learning and experience. Moreover, the experience may be non-transferable, and changes in problem types may render the algorithm inapplicable, such as transitioning from weather routing to collision avoidance. Heuristic algorithms have the following key characteristics.

- **Approximate solutions.** The aim of heuristic algorithms is often not to find the globally optimal solution but a 'good enough' solution, especially when the solution space is very large or complex.
- **Experience-based.** These algorithms often rely on heuristic knowledge, i.e. empirical and practical knowledge, to more effectively guide the search process. This knowledge may include rules of thumb, known features, or characteristics of the problem.
- **Strategic search.** The search of heuristic algorithms is guided by experience-/heuristic-based strategies. They can prioritize promising areas of the search space based on experience to avoid brute-force enumeration.
- **Efficiency.** Because of their strategic search, heuristic algorithms aim to be efficient and return solutions in a reasonable amount of time.

In more recent years, along with the emergence of AI and ML, another type of algorithm, learning-based algorithms, appeared in weather routing. These can uncover more useful information from the data or performance that may not be clearly recognised through experience while not requiring prior knowledge. Meanwhile, they can leverage the data-driven characteristics of AI and ML to achieve adaptability for different conditions. **Learning-based algorithms** are an emerging field of research for voyage optimisation thanks to the fast advancement in maritime big data and AI. Examples of learning-based algorithms include using reinforcement learning such as Q-learning and deep Q-network. Currently, they are relatively new, with a few applications in weather routing (Moradi et al., 2022). In contrast, they are more widely applied for MASSs as they possess flexible manoeuvrability and involve more dynamic changes and complexity in their environments. They have the following key characteristics.

- **Approximate solutions.** Learning-based methods use the same approximate approach as heuristic algorithms.
- **Data-driven.** Learning-based methods utilize collected data generated by corresponding actions to determine rewarding search directions.
- **Strategic search.** Learning-based methods use the same strategic search approach as heuristic algorithms.
- **Adaptivity.** By using data to adjust their search, learning-based methods can effectively adapt to changing conditions.

3.2. General framework for an optimisation algorithm

Besides their characteristics, algorithms have commonalities in their procedures to achieve optimisation. Therefore, these algorithms can be further summarised and categorised based on their common processes. Fig. 12 provides a general framework for algorithms, presenting three common processes and examples (note that these algorithms are based

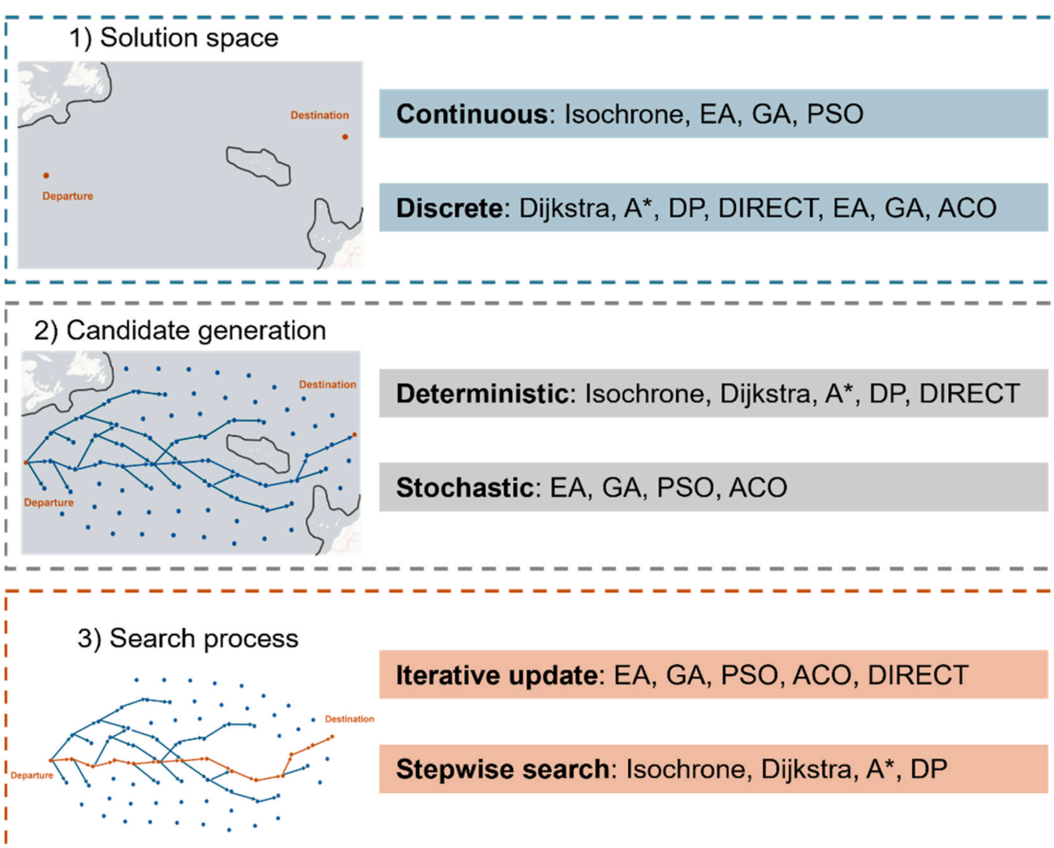


Fig. 12. General framework of a weather routing algorithm.

on their original forms or classic applications). As optimisation algorithms have significantly evolved, various variants or hybrids have emerged. A classic algorithm is difficult to categorise as belonging to one specific type while excluding its applicability to others. This framework does not aim to provide an exact classification for algorithms considering all their existing derivations. Instead, it summarises the potential variations in those common elements for algorithms used in weather routing so that relevant algorithms can find appropriate classifications within this framework. As aforementioned, each optimisation algorithm consists of the following three elements but takes a distinct approach for each element:

Solution space. Solution space refers to the set of all possible values for optimisation variables. Correspondingly, the solution space in weather routing, i.e. $\boldsymbol{P}$ and $\boldsymbol{U}$ in Eq. (4), involves sailing areas that exclude all infeasible regions in space while also considering allowed operational variables, e.g. feasible power and sailing speed. In Fig. 12, infeasible regions are outlined in the blue block by black lines. The solution space employed in algorithms typically has two types, i.e. **continuous** and **discrete.**

In **continuous space**, the optimisation variables can take any real value within the feasible range. Thus, space contains an infinite number of possible candidates and feasible solutions provided to the algorithm for the search. In weather routing, this feasible sailing space can be fully reached by ships; second, any value for operation parameters within allowed ranges can be applied to ships.

Discrete space means that the optimisation variables can only take on discrete values. The solution space is discretised, containing a finite number of distinct and separate feasible solutions. In weather routing, the space contains a set with a limited number of waypoints and sub-routes for ships to select; second, a limited number of values for operational parameters can be applied to ships.

Candidate generation. After defining the solution space, the candidate solutions can be generated, where optimisation variables, i.e. waypoints, headings, powers, or speeds, are assigned with possible values, generating feasible sub-routes or voyages for the search. This assignment can either be **deterministic** or **stochastic**.

Deterministic means that the waypoints, headings, powers, or speeds are assigned following certain rules.

Stochastic means that in addition to rules, random variables are introduced to increase the diversity of candidates.

Search process. Once the candidate solutions set is established, i.e. some feasible sub-routes or voyages are obtained, the search process can be started by either an **iterative update** or a **stepwise search.**

The **iterative update** starts with a feasible initial solution and improves the current solution iteratively through a loop. In weather routing, at the start of the algorithm, a feasible voyage with allowed operational parameters is initially generated. Then, in each step, optimisation variables $\boldsymbol{P}$ and $\boldsymbol{U}$ in Eq. (4) of this initial voyage are gradually refined towards the optimisation objectives. In other words, subsequent candidate voyages are generated and improved based on this initial voyage. An evaluation/fitness function C is used to identify which candidate is improved at each iteration:

$$C = C\,(\boldsymbol{P}, \boldsymbol{U}) \tag{7}$$

This evaluation/fitness function C is usually formulated based on the cost function J_i; however, they are not equivalent. The cost functions J_i and L_i represent the real costs, but the evaluation function C can be flexible. Still, the choice of evaluation function C is also crucial, as this function C guides the search. The initial voyage iteratively approaches towards the optimal voyage until no further significant improvements are found possible, as shown in Fig. 13.

A **stepwise search** begins at the departure and incrementally searches waypoints in each step towards the destination. In weather routing, starting from the departure, the algorithm evaluates the adjacent feasible waypoints and searches for the locally optimal solutions (including $\boldsymbol{P}$ and $\boldsymbol{U}$) to move a step forward. The search for the locally optimal solutions is also based on evaluation function C given in Eq. (7), e.g. choosing a solution with the lowest C. When the destination is reached, the algorithm considers this complete voyage as optimal. This process is shown in Fig. 14.

3.3. Commonly used algorithms in weather routing

These three processes can be reflected in the procedures of each algorithm, but the way they are executed and combined varies. Based on the previous subsection, this subsection introduces some classic algorithms commonly used in weather routing, which are used as examples in Fig. 12.

Isochrone algorithm. The Isochrone algorithm is long-established, first invented in 1957 (James, 1957; Wisniewski, 1991). Different from other methods, it was initially developed for marine navigation with specific consideration for ETA. An isochrone is a contour line or isopleth indicating the farthest distance a ship can reach in an equal sailing time, as shown in Fig. 15 (a). The basic procedures are as follows.

1) Starting from departure, the ship sails in different directions for the same sailing time $\Delta t = T_{i+1} - T_i$ (e.g. 6 h), where T_i is introduced in Eq. (1). All ending waypoints form the first isochrone, as shown in Fig. 15 (a).
2) The next isochrone is extended based on the current one. This extension iterates until the destination is reached, as shown in Fig. 15 (b).
3) Feasible voyages connecting to the destination can be found, and the optimal one has the lowest cost, as shown in Fig. 15 (c).

DP-based algorithms. DP was originally proposed to tackle complex optimal control problems following Bellman's principle of optimality (Bellman, 1952). Gradually, it evolved into a general principle to solve multi-stage problems: the optimal solutions of subproblems can form the optimal solution for the entire problem. DP enables efficient recursive or iterative problem-solving and has been widely applied in many areas, including graph search and voyage optimisation, e.g. by Chen (1978) and Dewit (1990).

In weather routing, DP first initialises the sailing space with a discretised grid as its solution space. The nodes in adjacent stages relate to

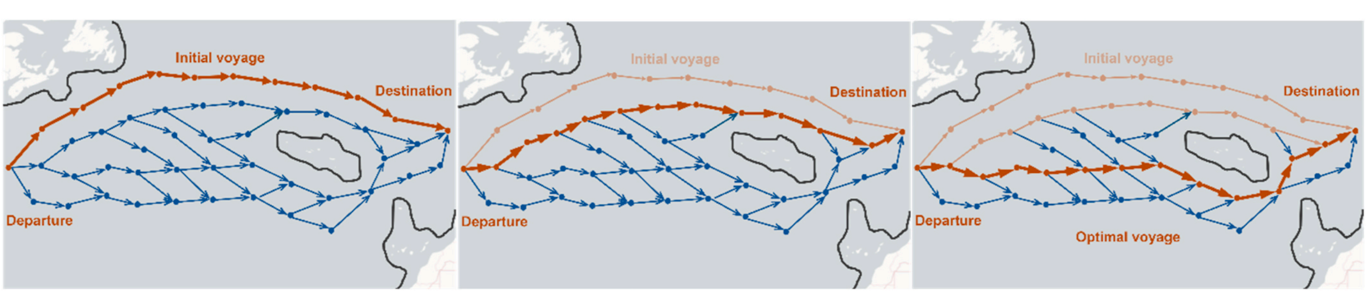


Fig. 13. Graphical illustration of the iterative update process.

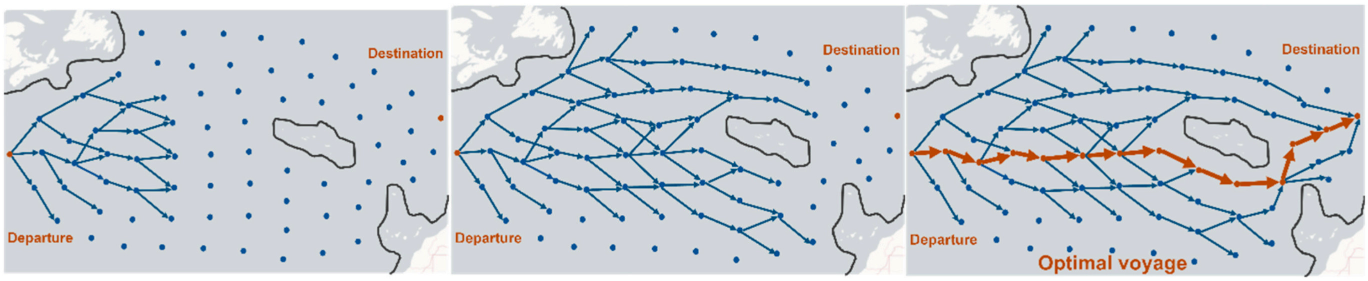
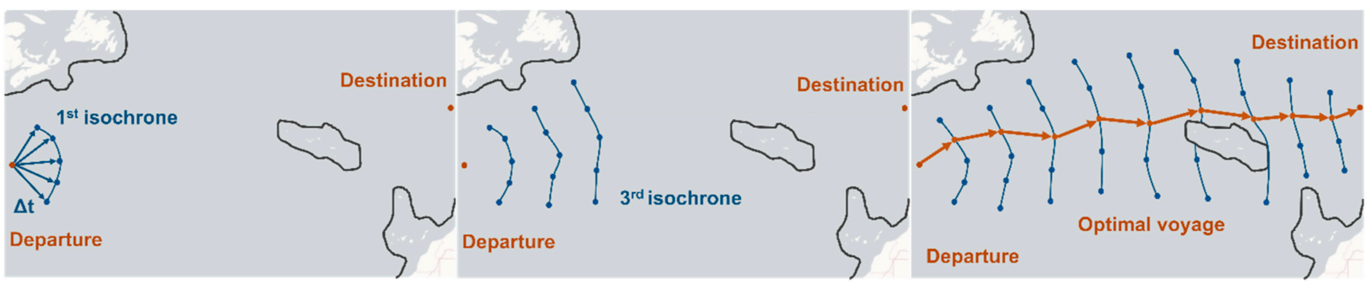


Fig. 14. Graphical illustration of the stepwise search process.



(a) (b) (c)

Fig. 15. Graphical illustrations of the Isochrone algorithm.

edges as sub-routes, as shown in Fig. 14. The search is conducted based on this grid system to solve the DP functional equation following Bellman's principle of optimality. Assuming that $\boldsymbol{P_0}$ is the departure, $\boldsymbol{P_i}$ is a waypoint at the ith stage, and $\boldsymbol{P_{i+1}}$ is a waypoint at the (i+1)th stage. The optimal voyage from $\boldsymbol{P_0}$ to $\boldsymbol{P_{i+1}}$ can be obtained as follows:

Optimal voyage $\boldsymbol{P_0} \rightarrow \boldsymbol{P_{i+1}}$ = Optimal voyage $\boldsymbol{P_0} \rightarrow \boldsymbol{P_i}$ + Optimal voyage $\boldsymbol{P_i} \rightarrow \boldsymbol{P_{i+1}}$ (8).

The entire optimal voyage can be found when $\boldsymbol{P_{i+1}}$ reaches $\boldsymbol{P_f}$. Eq. (8) illustrates a more general form of the DP functional equation, which is further elaborated in Eq. (9):

$$\min J_{i+1} = \min\ (J_i + L_i) \tag{9}$$

where min J_{i+1} is the minimal accumulated cost from $\boldsymbol{P_0}$ to $\boldsymbol{P_{i+1}}$ and L_i is the instantaneous cost from $\boldsymbol{P_i}$ to $\boldsymbol{P_{i+1}}$.

Based on DP, Dijkstra's algorithm (Dijkstra, 1959) employs a greedy strategy to explore the grid. In each step, the algorithm proceeds to the neighbouring waypoint with the minimal cost J_{i+1}. That is, starting at $\boldsymbol{P_i}$, Dijkstra uses J_{i+1} as the evaluation function C in Eq. (7), chooses the point with the lowest J_{i+1} as $\boldsymbol{P_{i+1}}$, and proceeds towards it. At the new point $\boldsymbol{P_{i+1}}$, it updates the cost of reaching adjacent waypoints and repeats the above process.

The **A*** algorithm (Hart et al., 1968) can be further considered an informed variation of Dijkstra's algorithm. It forms a predictive evaluation function C by adding a heuristic function h_{i+1} ($\boldsymbol{P_i}$, $\boldsymbol{P_f}$) to the current cost J_{i+1} to additionally evaluate the consequence of reaching $\boldsymbol{P_{i+1}}$. Compared with the evaluation function of Dijkstra's algorithm given in Eq. (10a), the evaluation function of A* is presented in Eq. (10b):

$$C = J_{i+1} \tag{10a}$$

$$C = J_{i+1} + h_{i+1} \tag{10b}$$

where J_{i+1} is the accumulated cost from $\boldsymbol{P_0}$ to $\boldsymbol{P_{i+1}}$ and h_{i+1} is the heuristic function estimating the cost from $\boldsymbol{P_{i+1}}$ to $\boldsymbol{P_f}$. The heuristic term can guide the search more efficiently. DP and Dijkstra can enumerate the optimal voyage within its grid, making them **exact** methods, while A* can be seen as a **heuristic** version of Dijkstra's algorithm.

EA and GA. EAs, founded by Evolutionary Strategies (Rechenberg, 1989), and GAs (Holland, 1975) were developed based on Darwin's evolutionary theories. They perform optimisation by simulating biological mutation and inheritance. Starting with an initial solution set that includes a population of multiple initial solutions, these algorithms employ various operators to introduce randomness and iteratively refine the solutions, continuously improving the current population. At each step, candidates are evaluated for fitness using evaluation function C in Eq. (7) to retain some of the best, and the evolution continues. The solution is considered converged to the optimum when it cannot be significantly improved. GA is a sub-class of EA, as GA specifically simulates biological inheritance, emphasising crossover and mutation operations. EA includes various flexible operators, incorporating multiple evolution-based algorithms.

In weather routing, candidates (populations) are complete voyages defined by $\boldsymbol{P}$ and $\boldsymbol{U}$ (including variables, e.g. speed) in Eq. (4), as shown in Fig. 16. Operators such as crossover and mutation aim to introduce variations in candidate generations to achieve evolution. Their solution space is not limited to continuous or discrete, depending on how they perform operators such as crossover and mutation. EA and GA are **heuristic** as their evolution is guided by heuristic fitness evaluation (selection) function C.

PSO and ACO. PSO and ACO belong to a broader class of swarm intelligence algorithms. **PSO** initialises a set of particles with updated velocities v, where each particle represents a current solution. At each iteration, every particle records its individual best solution *pBest*, and all particles record the global best solution *gBest*. For each particle, assume the current solution at the ith iteration is x_i, the next solution x_{i+1} at the (i+1)th iteration is calculated based on the following equations (Kennedy and Eberhart, 1995):

$$v_{i+1} = w \cdot v_i + c_1 \cdot r_1 \cdot (pBest_i - x_i) + c_2 \cdot r_2 \cdot (gBest_i - x_i) \tag{11}$$

$$x_{i+1} = x_i + v_{i+1} \tag{12}$$

where w is a weight coefficient, v_i is the last update velocity, c_1 and c_2, are acceleration coefficients, r_2 are two random coefficients r_1, r_2, and v_{i+1} is the new update velocity from x_i to x_{i+1}. In weather routing, a current solution x_i indicates $\boldsymbol{P}$ and $\boldsymbol{U}$ in Eq. (4). At each iteration, elements in $\boldsymbol{P}$ and $\boldsymbol{U}$ in x_i are updated respectively following Eqs. (11) and

Fig. 16. A general workflow of EA and GA in weather routing.

(12), as illustrated in Fig. 17 (left). Then, the particles perform a fitness evaluation using function C in Eq. (7) to identify *pBest* and *gBest* to proceed to the next iteration. The voyage is gradually improved until the optimal voyage is found or the stopping criterion is reached.

ACO mimics the process of ants, guided by pheromones, selecting routes during foraging. In a pre-discretised grid, each route has a pheromone level (Dorigo et al., 1996). The probability of route selection is calculated based on a formula that considers the pheromone concentration and the heuristic value of the route (e.g. time or cost). After a route is selected, the pheromone levels are updated based on their quality, which is evaluated by function C. The concentration is increased on high-quality routes and reduced on lower routes, as illustrated in Fig. 17 (right). This process repeats until the solution quality shows no significant improvement or stopping conditions are met.

4. State-of-the-art development of optimisation algorithms in weather routing

The recent development of weather routing systems gives the decision support system more functionalities in terms of multi-objectives and various control variables for planning. Table 5 lists objectives and variables retrieved from the recent research literature (mainly after 2020). To demonstrate the effectiveness of methods, some studies compared their proposed algorithm with existing methods, while others compared them with actual sailing data. The references used in these studies are inconsistent, making it challenging to compare the effectiveness of these proposed algorithms. The reported optimisation results vary significantly across studies, as shown in Table 4.

Table 4
Average quantitative optimisation results reported from the literature compared with real voyage cases.

Metrics	Average results with references from full-scale measurement
Fuel savings	1–10 % (Chen and Mao, 2024), 9.4 % (Du et al., 2022b), 9 % (Lee et al., 2018)
Time savings	1.65 % (Shin et al., 2020), 5 % (Du et al., 2022a)
Economic profits	2.55 % (Du et al., 2023), 1.5 % (Ma et al., 2024), 7.9 % (Bahrami and Siadatmousavi, 2024)
Emission reductions	19 % (Du et al., 2022a), 6.4 % (Du et al., 2022b), 2–12.5 % (Wang et al., 2021a)
Fatigue	50–90 % (Lang et al., 2021), avg. 50 % (Wang et al., 2019)

The innovation and scientific contribution to the development of the previously four types of categorised algorithms, i.e. Isochrone algorithm, DP-based algorithm, PSO/ACO, and EA/GA, are thoroughly reviewed in the following subsections. Finally, Table A1 in Appendix A summarises innovations of the reviewed papers in terms of the three areas, i.e. solution space, regarding the advancement of grid partition, candidate generation, which involves improvements in generating candidate solutions, and search process, regarding enhancements in search, e.g. refining the cost function.

4.1. Isochrone algorithm

A unique characteristic of the Isochrone algorithm is that it dynamically generates a grid through iteration, i.e. it is a dynamic grid-based algorithm. The key process is the outward expansion of each

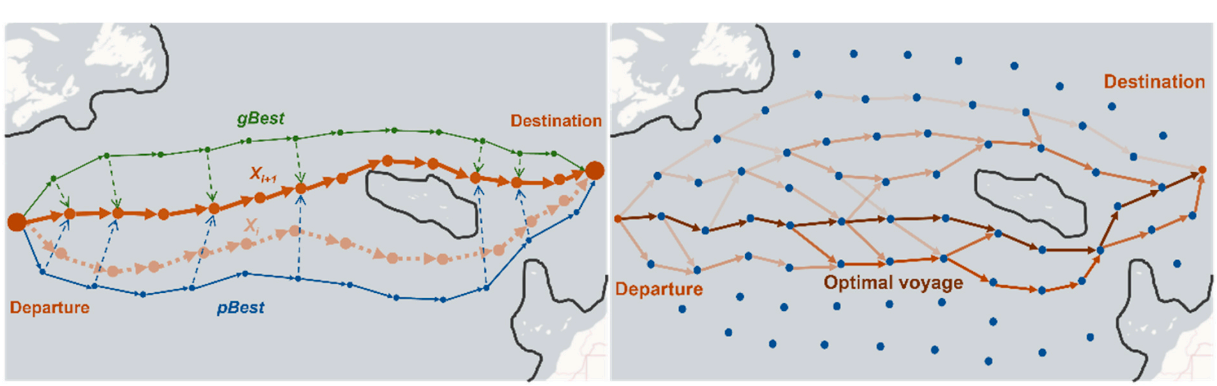


Fig. 17. Graphical illustrations for PSO (left) and ACO (right).

Table 5
Examples of recent literature based on optimization objectives and variables.

		(Lee et al., 2018)	(Wang et al., 2019)	(Gkerekos and Lazakis, 2020)	(Kim et al., 2020)	(Lang et al., 2021)	(Pan et al., 2021a)	(Wang et al., 2021a)	(Du et al., 2022a)	(Du et al., 2022b)	(Grifoll et al., 2022)	(Khan et al., 2022)	(Kytariolou and Themelis, 2022)	(Wang et al., 2022a)	(Zhang et al., 2022)	(Du et al., 2023)	(Li et al., 2023)	(Ma et al., 2023b)	(Mason et al., 2023)	(Qian et al., 2023)	(Szlapczynski et al., 2023)	(Chen and Mao, 2024)	(Guo et al., 2024a)	(Li et al., 2024b)	(Ma et al., 2024)
Objective	Fuel	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■			■	■	■	■	■
	Operational cost															■		■							■
	Emission							■			■			■				■	■					■	■
	Maneuvering risk										■		■		■					■	■				
	Accurate ETA	■	■		■		■	■														■			
	Sailing time								■	■	■	■						■			■		■		■
	Travelling distance	■	■		■		■																		
	Power																			■					
	Fatigue		■			■																			
Variables	Route with heading	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■
	Speed		■		■		■		■	■	■	■		■	■	■	■	■	■	■				■	■
	Power							■					■												
	Trim																							■	
	RPM	■																			■		■		
Algorithm	Isochrone	■																				■			
	DP-based		■	■	■	■		■	■		■						■		■	■					■
	PSO/ACO									■				■	■	■		■							
	EA/GA	■					■	■				■	■								■		■	■	■
Result	Fuel savings	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■	■			■	■	■	■	■
	Time savings								■	■	■	■						■			■		■		■
	Distance savings	■	■		■		■																		
	Economic profits															■		■							■
	Emission reductions		■					■			■			■				■	■					■	■
	Risk/Safety										■		■		■					■	■				
	Fatigue		■			■																			

generation of isochrones, as it involves the three critical elements: identifying the local solution space, generating candidates, and conducting the search process. Initially, the propagation of the next isochrone was based on transforming in the perpendicular direction of the current waypoint, as shown in Fig. 18 (a). For more possible candidates, Hagiwara (1989) used a sector-based expansion for each waypoint in the current isochrone to derive more waypoints for the next isochrone. This approach leads to significantly more waypoints, and he further proposed using 'subsectors' to retain optimal waypoints in each isochrone, as shown in Fig. 18 (b). Based on their work, researchers conducted follow-up studies in greater depth. The most recent study based on Hagiwara (1989) is shown in Fig. 18 (c), where ML prediction is integrated to enhance the heuristic in the cost function to better select waypoints in each isochrone to guide the next generation. Furthermore, based on the innovations in different stages during the optimisation process, the improvements of the type of Isochrone algorithms in the literature are summarised in Table 6. The arrows in Tables 6–9 are used specifically for hybrid algorithms, indicating that the first algorithm is used to initialize or assist in executing the second algorithm. For example, 'Isochrone → GA' indicates that the Isochrone method is used to generate initial populations for GA.

The operation of transoceanic ships is complex, and frequent manoeuvres are difficult to execute, potentially leading to unnecessary energy consumption and operational risks. In practice, keeping the navigation settings as stable as possible is preferable, which makes the constant ship and engine speed settings of the Isochrone method highly practical. The Isochrone method was initially developed specifically for ship navigation, making it both highly practical for real-world voyages and computationally efficient. Further research is also warranted using emerging technologies, such as that by Chen and Mao (2024), to develop effective and efficient weather routing algorithms.

4.2. DP-based algorithms

As classic pathfinding algorithms, DP-based algorithms like Dijkstra and A* demonstrate outstanding performance, having been widely applied to weather routing, particularly in the early stage, owing to their unique advantages. Using an initialised static grid gives them strong adaptability to various application environments. DP and Dijkstra, as exact methods, guarantee an optimal solution within the grid. A* additionally improves efficiency for enhanced applicability. Recent literature adopting these algorithms is presented in Table. The general improvement trends are similar to those outlined in the previous subsection, as shown in Table 7. Because of their versatility, these algorithms have achieved excellent outcomes across many applications.

By discretising the entire solution space into an initial grid, they can search and obtain the best solution within it. However, this approach demands significant computational effort when handling more dimensions of variation, and the quality of results depends heavily on the grid's density. This requires a balance between accuracy and efficiency to achieve optimal performance in practice. For weather routing – an optimisation problem with multiple operational constraints – managing these constraints can be challenging for these algorithms, such as in cases with a fixed ETA.

4.3. PSO and ACO

PSO and ACO processes differ from the graph search methods discussed in the previous two sub-sections. Their applications are not limited by problem dimensions, and increasing dimensions does not impact computational efficiency as significantly. They start with an initial solution and then use the best-recorded solution along with random trials to guide the search, converging when no further

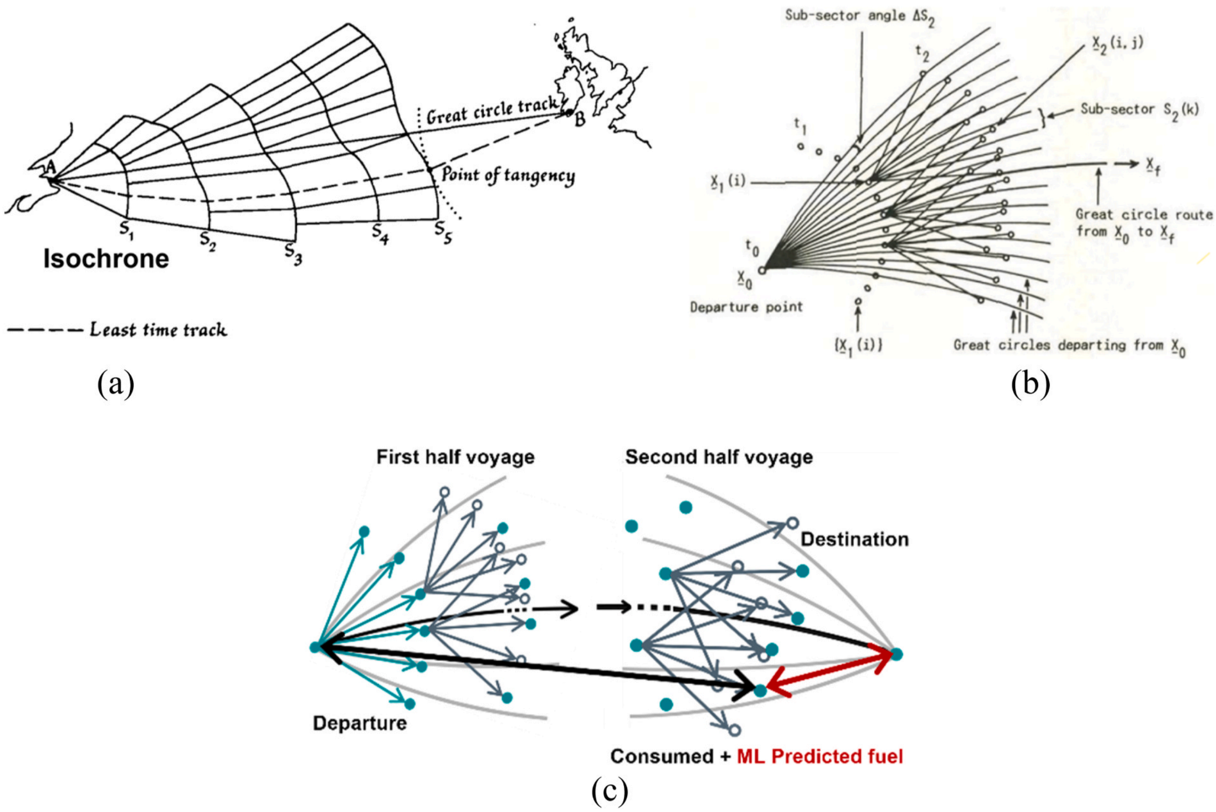


Fig. 18. (a) Least time sailing (Hanssen and James, 1960); (b) minimum time/distance sailing (Hagiwara, 1989); (c) Isochrone-based predictive optimisation (IPO) (Chen and Mao, 2024).

Table 6
Categorisation of literature using Isochrone algorithms.

Feature	Approach	
Enhance the grid	More variables	Include speed in addition to route (Lin et al., 2013)
	Changing the partition to improve	Adaptability of environment (Topaj et al., 2019)
		Route smoothness (Chen and Mao, 2024)
Enhance the search	ML predictive cost in heuristics (Chen and Mao, 2024)	
Practical application	Replace the constant speed to generate isochrones using	Power (Klompstra et al., 1992)
		Engine speed (Lee et al., 2018),
		Self-defined cost (Topaj et al., 2019)
	Integrating ship performance model (Roh, 2013)	
	Considering speed loss and manoeuvrability in ship's movement (Sasa et al., 2021)	
	Dynamically updating weather (Chen and Mao, 2024)	
	Specific optimisation objectives	Considering specific ETA requirements (Chen and Mao, 2024)
Hybrid algorithm	Isochrone → EA (Szlapczynska and Smierzchalski, 2007)	
	Isochrone → GA (Lee et al., 2018)	
	Isochrone → PSO (Lin, 2018)	

improvement can be achieved. By introducing these random variations, they increase solution diversity and help avoid local optimums. Recent improvements using PSO and ACO are summarised in Table 8.

The introduction of stochastic elements is an advantage of these two methods, helping to avoid getting trapped in local optima while incorporating the best-recorded results, which makes improvements more reasonable and effective. Similar to Dijkstra, increasing population size (particles or ants) also impacts solution quality, but PSO and ACO are more suitable for multi-dimensional optimisation problems, while their computational complexity may also be relatively higher. Additionally, their dependence on initial solutions requires special attention, as it significantly affects both performance and outcomes. This reliance is also a key reason they are often used within hybrid algorithms.

4.4. EA and GA

EA and GA processes in weather routing are similar to those of PSO and ACO. They also introduce stochastics in the process, using operators like crossover and mutation, followed by selection, to gradually improve results. Numerous variants of these algorithms have been developed (Wang and Sobey, 2020). In weather routing, the main methods from EA/GA currently applied are as follows: NSGA I-III, Strength Pareto EA (SPEA), multi-objective EA (MOEA), and MOEA based on decomposition (MOEA/D). Recent implementations of EA/GA methods in weather routing are presented in **Table** in Appendix A and summarised in Table 9.

EA and GA are adaptable and well-suited for complex, multi-dimensional, and constrained optimisation problems. Genetic evolution may involve tuning many parameters, which enhances the method's flexibility but introduces the challenge of finding optimal parameters. The choice of population size affects both convergence speed and solution quality, and careful tuning of parameters is often necessary to ensure good performance. While the stochastics introduced by these algorithms promote solution diversity, they may also reduce convergence efficiency, hindering the efficiency in addressing dynamic problems with frequently changing environments. Additionally, some optimisation variables are correlated, and simple operators may not effectively account for these correlations. For instance, special consideration is needed to efficiently handle correlations between voyage legs to achieve smooth operational transitions. As a result, the combination of EA/GA with classic pathfinding algorithms like Dijkstra to form hybrid algorithms may be necessary, where pathfinding algorithms focus on route and EA/GAs on speed optimisation. This aspect is further discussed in the next section.

Table 7
Categorisation of literature using DP-based algorithms.

Feature	Approach	
Enhance the grid	More variables	Include speed with route (Sun et al., 2022; Wang et al., 2019, 2020a; Zaccone et al., 2018)
	Enhanced grid generation	Generate a path search grid efficiently while removing unnecessary vertices (Jeong and Kim, 2023)
	Dynamic gird structure	Enhance computational efficiency (Qian et al., 2023)
Enhance the search	ML predictive time cost (Shin et al., 2020)	
	Iterative and improved search process to optimise under dynamic weather impact (Bahrami and Siadatmousavi, 2024; Gkerekos and Lazakis, 2020)	
Practical applications	Integrating performance model	Wind assisted propulsion ships (WAPS) (Mason et al., 2023; Wang et al., 2024)
		Sailing boats (Sidoti et al., 2023)
		Various performance models, including motor vessels and sailboats (Mannarini et al., 2024)
		Include the uncertainty of ship performance (Dickson et al., 2019)
	Specific optimisation objectives	Manoeuvring risk (Guo et al., 2024b)
		Considering specific ETA requirements (Li et al., 2023)
		Seakeeping performance for container ships by using the Seakeeping Performance Index in the objective function (Pennino et al., 2020)
	Establishing the grid based on the electronic chart (Pan et al., 2021b)	
	Employing parallel computing for efficiency (Qian et al., 2023)	
	Dynamic updating weather for edge weights in the grid (Grifoll et al., 2022; Kurosawa et al., 2020; Mannarini et al., 2020, 2024)	
Hybrid algorithms	A* → GA (Qian et al., 2023)	
	Great circle → Speed optimisation using Dijkstra (Wang et al., 2020a)	
	Dijkstra → Speed optimisation using integer programming (Ma et al., 2020)	
	Dijkstra → Three-dimensional (3D) DP (Choi et al., 2023)	

Table 8
Categorisation of literature using PSO and ACO.

Feature	Approach	
Enhance the grid	More variables	Route with speed (Wang et al., 2020b; Zhang et al., 2022)
		Wing-sail's angle of attack for WAPS (Wang et al., 2022a)
Enhance the search	Refining the stochastic terms to avoid local optimums (Du et al., 2022b, 2023; Zheng et al., 2019)	
Practical applications	Specific optimisation objectives	Manoeuvring risk (Yang et al., 2022)
		Route smoothness (Zhang et al., 2021)
		Risk from sea ice (Zhang et al., 2022)
	Integrating performance model applying to WAPS (Wang et al., 2022a)	
	Including weather dynamics (Du et al., 2022b)	
Hybrid algorithms	NSGA → PSO (Zhao et al., 2022)	
	GA → ACO (Zhang et al., 2021)	
	APF → ACO (Ma et al., 2023b)	
	PSO for route and dynamic collaborative optimisation for speed optimisation (Wang et al., 2021b)	

Table 9
Categorisation of literature using EA and GA.

Feature	Approach	
Enhance the grid	More variables	Route with speed (Yuan et al., 2022), RPM (Guo et al., 2024a; Lee et al., 2018)
		Trim (Li et al., 2024b)
Enhance the search	More efficient guidance of the search direction (Guo et al., 2024a; Szlapczynski et al., 2023)	
	Refined operators to accelerate convergence (Pan et al., 2021a)	
	ML models to predict potential costs (Li et al., 2024b)	
Practical applications	Specific optimisation objectives	Sea ice in Arctic Ocean sailing (Lee et al., 2021)
		Ship stability and ETA constraints (Zhao and Zhao, 2024)
		ETA constraints and piracy risk (Kuhlemann and Tierney, 2020)
		Risk from manoeuvring (Kytariolou and Themelis, 2022)
	Considering the impact of fouling of hull and propeller on ship performance (Kytariolou and Themelis, 2023)	
	Deploy and compare the effectiveness of four state-of-the-art variants of GA in weather routing (Khan et al., 2022)	
	Including weather uncertainties (Yuan et al., 2022)	
Hybrid algorithms	A* → SPEA2 (Szlapczynska and Szlapczynski, 2019)	
	PSO → GA (Zhao et al., 2021)	
	A* → GA (Lee et al., 2021)	
	GA → Simulated annealing algorithm (Zhou et al., 2023)	
	A* → NSGA-III (Ma et al., 2024)	
	Dijkstra + DP → GA (Wang et al., 2021a)	

5. Discussion and future research trends

5.1. Summary of recent improvements

Recent improvements in these algorithms by the research community on weather routing can be categorised into the following three aspects. The first two aspects focus on enhancing a specific algorithm through its search process. Additionally, there is an approach that combines the advantages of different types of algorithms to form new hybrid methods. Specifically, enhancing the search process mainly involves identifying and retaining better candidate solutions at each step, eventually leading to an optimal final solution. This requires effectively addressing the following two key issues:

Improved candidate diversity. First, ensure candidate diversity to avoid premature convergence to local optimums, providing comprehensive information to cost functions to support more informed decision-making rather than relying on localised, partial perspectives. Examples include adding predictive terms to consider future possibilities or accounting for uncertainty and potential dynamic changes due to weather or ship performance (such as biofouling). As a result, selected candidates are more likely to be the global optimum rather than a local optimum. Increasing solution diversity helps explore more comprehensive possibilities at each step, avoiding early convergence to local optimums. One way to achieve this is by enriching the grid with more optimisation variables to formulate a higher-dimensional optimisation problem that includes factors such as ship and engine speeds in addition to the route. Another approach is to introduce stochastic variations during the optimisation process by incorporating stochastic terms into the methods.

Better candidates retained in the process. Second, directing the search can avoid excessive divergence caused by stochastic elements or diversity. Stochastics should be formulated at a moderate level, and the cost function can effectively distinguish valuable candidates from the others. Keeping the candidate pool at a manageable size helps move to the next iteration and prevents divergence issues.

Hybrid structure. In addition to improving a specific algorithm, different algorithms can also be combined to form hybrid methods. The two main types of hybrid forms used in literature are integrated and two-step processes. Regarding an integrated process, for example, the performance of PSO/ACO and EA/GA are highly dependent on the quality of the initial solution set. Therefore, researchers use efficient path-finding algorithms to obtain a high-quality and reasonable initial solution set, outlining areas in the solution space where the global optimal solution most likely lies. Then, more powerful but computationally complex algorithms are applied to further optimise the solution based on this reduced candidate set. Fig. 19 illustrates integrating A* with GA as an example. From the literature presented in Sections 4.3 and 4.4, more examples include Isochrone and A* combined with EA/GA or PSO/ACO.

The two-step process separates the optimisation of the route with other variables (ship and engine speed) as shown in Fig. 19. As introduced in Section 2.2, one type of problem focuses on speed optimisation along fixed routes. Based on these findings, researchers first optimise the route using pathfinding algorithms, then optimise other dimensions (e.g. speed) along the chosen route using methods suitable for speed optimisation, e.g. combinatorial optimisation (Ma et al., 2020, 2023a; Wang et al., 2020b). The advantage of this approach is its simpler execution compared to the first integration approach, as it divides the entire optimisation problem into two sub-tasks. However, a potential challenge is that route and speed are correlated variables. For example, at a specific location, different arrival times can encounter varying weather conditions. As discussed in the literature (Ma et al., 2023b), this separation can make the global optimum difficult to find, potentially resulting in a local optimum.

5.2. Challenges and future trends related to optimisation algorithms

Weather forecast and ship performance models used in weather routing systems contain large uncertainties. A general trend of research on optimisation algorithms is to handle those uncertainties and make the optimisation process more robust and efficient from the following aspects:

Improving performance – enhanced heuristics. Because of the complexity of weather routing problems, achieving high efficiency while providing optimal solutions is the main goal. In the short term, this highlights the importance of constructing effective heuristic terms within heuristic methods, and approaches using AI/ML for predictive optimisation have been developed. With the continuous advancement of ML technology, ML-enhanced heuristics is a highly promising and valuable research direction for future exploration. Alternatively, exploring ways to integrate algorithms in a hybrid form is an option to better leverage the strengths of both efficient and powerful methods, enhancing optimisation performance.

Improving performance –learning-based methods. In the long term, many algorithms have achieved success in other fields. The range of algorithms used in weather routing is still limited compared to other areas. For example, deep learning and reinforcement learning are widely applied in MASSs and vehicle routing, but few studies have explored their use in weather routing. This lack may be due to the numerous constraints on the operation and manoeuvring of large transoceanic vessels, which limit the effectiveness of many advanced algorithms, making their advantages less apparent for now. In the future, with the upgrading of ships and the shift towards digitalisation, developments with these algorithms may see significant improvement.

Applicability in real-world operations – uncertainty handling. Uncertainties, such as weather, ship performance, and operational changes during operation, cannot be avoided; thus, addressing this issue is a key direction. Predicting and simulating these uncertainties is a promising strategy. However, transoceanic voyages are long, often lasting around one month (Notteboom et al., 2022). With current weather forecasting technologies, long-term predictions with high accuracy remain a challenge. Other uncertainties in ship performance or operations also require frequent calibration or identification for updates. In this context, dynamic optimisation with real-time updates becomes the most feasible approach. This involves accounting for dynamic changes during the optimisation process (such as using dynamic grids), following the results, and updating the optimisation in real time. However, this also places high requirements on the efficiency of the algorithm.

Applicability in real-world operations – support **clean fuel–powered ships.** Current methods have considered the impact of different environmental conditions on weather routing, e.g. sailing in the Arctic, and practical operational preferences, e.g. manoeuvring risks and constant engine speed. In the future, weather routing may be applied to more clean fuel–powered ships to better support the energy transition. The current use of weather routing in WAPS is a strong example of how weather routing can greatly benefit these ships. Current research mainly considers traditional fuel oil engine–powered ships and their operational requirements. In the future, weather routing may need to be further upgraded to account for the special navigation requirements of other new-energy ships, optimising specific parameters such as the wind angle of attack in WAPS and sailboats.

Benchmark studies. Benchmark studies are beneficial as they

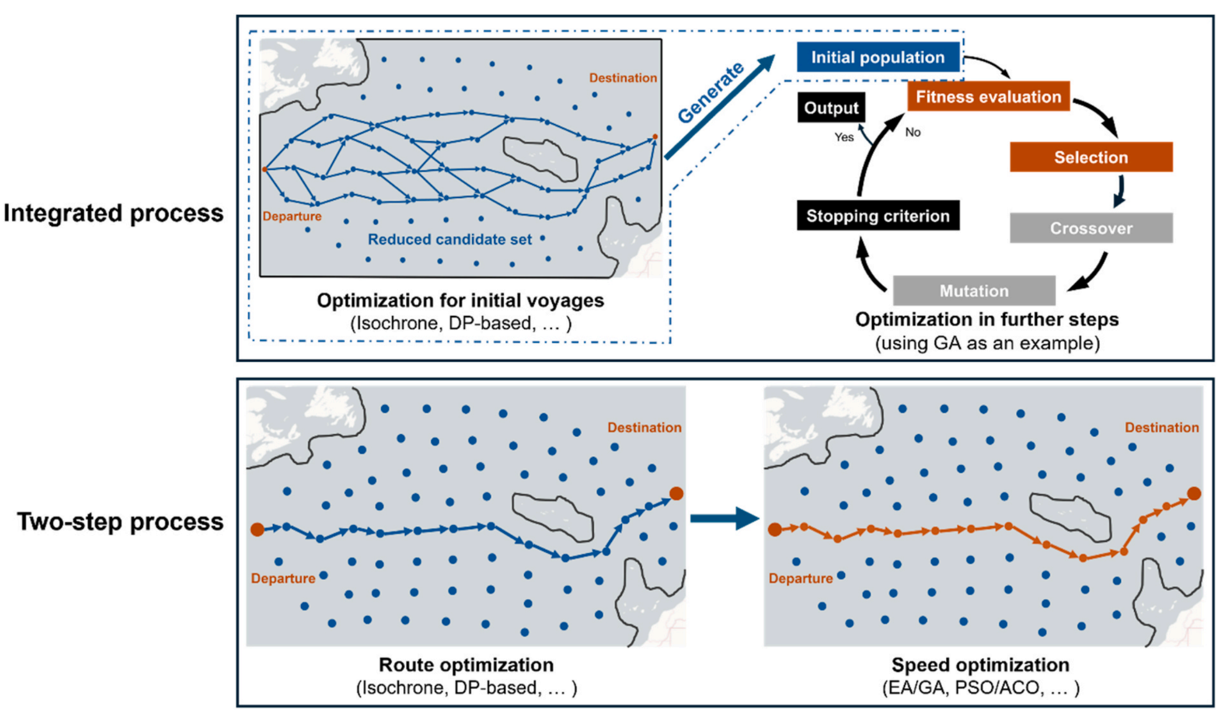


Fig. 19. Integrated process (top) and two-step process with separate optimisations (bottom).

provide a unified framework to evaluate the performance of different methods, identify potential challenges, and drive improvements. They not only offer guidance for specific problems but also contribute to the overall development of the field. However, current observations reveal challenges. When validating the effectiveness of algorithms, some of the literature used existing approaches for comparison, while others incorporated real-world voyage data. Some of the measured voyage data, because of various limitations, may not be publicly available. This inconsistency in references makes the proposed algorithms difficult to compare, and so far, only limited attempts at establishing benchmarks have been found. Furthermore, there are also possibilities to propose performance indicators to evaluate improvements, making it easier to compare the performance of different algorithms. In addition, similar to performance models, optimisation algorithms in weather routing also have their own strengths and limitations, possibly leading to different applicable scenarios. Weather exhibits characteristics of seasonal and regional variation. However, to our knowledge, no study has explored the sensitivity of weather routing algorithms to factors such as variations in typical shipping routes, nor has any investigated whether weather characteristics across sailing times may be better addressed by different algorithms. Research in this direction may offer valuable insights into the industrial application of weather routing.

6. Conclusions

Weather routing is an operational-level optimisation problem that determines the optimal voyage between one port of call to another, accounting for weather impacts. This study begins by presenting an overview of research trends in weather routing starting from 2010. It shows a significant increase in publications in this field after 2020, with algorithms and models being the two research focuses and when applications of ML began to appear in weather routing. Following that, this paper clarifies mixed-used terminologies in voyage optimisation with weather routing to clarify scopes and establish consistency in the research community. From the recent literature, the objectives are typically fuel consumption, emissions, ETA, total operation cost, risk, and fatigue. The optimisation outcomes/variables include a series of waypoints with operational settings for each sub-route, e.g. ship speed, engine speed, and power.

Then, focusing on the optimisation algorithm, the paper proposed a general framework to categorise the commonly used algorithm types in this field, highlighting their characteristics. Based on this framework, we provide a comprehensive literature review of optimisation algorithms with a focus on recent developments. Isochrone, DP-based, EA/GA, and PSO/ACO optimisation algorithms have been frequently used in weather routing. Their development trends in recent years have similarities, and the efforts are devoted to four main aspects.

- Improving the diversity of candidates
- Retaining better candidates in the process
- Forming hybrid algorithms to leverage different algorithms' advantages
- Enhancing practical applications to meet more specific operational requirements

The first two aspects aim to enhance the optimisation performance of an algorithm by avoiding local optimums, preventing excessive divergence, and finding a suitable balance in between. Based on these insights, future directions include improving algorithm performance by enhancing heuristics and incorporating learning-based methods, and enhancing practical applications by effectively handling uncertainties and supporting future clean fuel–powered ships.

Weather routing remains a challenge. Because of its multidisciplinary nature, each involved area of specific knowledge imposes influences on its overall effectiveness. Three main components of weather routing are weather forecasts, ship performance modelling, and optimisation algorithms, with optimisation algorithms playing a key role in decision-making. Beyond the traditional pathfinding problem, weather routing involves complex practical constraints on operations and changes in uncertain factors, which must be comprehensively and correctly considered. Ineffective weather routing can lead to serious consequences, including cargo delays, economic losses, and even risks to navigational safety. Both academia and industry make continuous efforts to improve its effectiveness and efficiency, as it can closely integrate with digitalisation, providing substantial benefits for decarbonisation and future energy transition.

CRediT authorship contribution statement

Yuhan Chen: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Chi Zhang:** Writing – review & editing, Writing – original draft, Visualization, Methodology, Formal analysis, Conceptualization. **Yuhan Guo:** Writing – review & editing, Writing – original draft, Visualization, Investigation, Formal analysis, Conceptualization. **Yiyang Wang:** Writing – review & editing, Supervision, Methodology, Formal analysis, Conceptualization. **Xiao Lang:** Writing – review & editing, Visualization, Investigation, Formal analysis, Conceptualization. **Mingyang Zhang:** Writing – review & editing, Visualization, Methodology, Conceptualization. **Wengang Mao:** Writing – review & editing, Visualization, Supervision, Project administration, Methodology, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix B. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.oceaneng.2025.121198.

Appendix A. Summary of the reviewed papers

Table A1
Summary of representative work reviewed in this paper with innovations in each key process of algorithms.

Literature	Solution space	Candidate generation	Search process
Isochrone algorithm			
Hanssen and James (1960)	Perpendicular transforming to generation isochrones	Constant sailing speed	
Modified Isochrone (Hagiwara, 1989)	Propose subsectors for area partition, to select waypoints	Sector-based expansion	Retain least time/distance waypoints in each subsector to form isochrones
Klompstra et al. (1992)		Constant engine power	
Szlapczynska and Smierzchalski (2007)	Check land-cross for sub-routes between isochrones; hybrid structure, use Isochrone Algorithm to provide initial populations to EA		
Roh (2013)		Incorporate ship performance modeling to calculate fuel	
Lin et al. (2013)	Three-dimensional (3D) dynamic grid including speed variation		Retain only one optimal waypoint at each stage
Lee et al. (2018)		Constant RPM	Hybrid structure, Isochrone Algorithm for route + GA for speed optimisation
Lin (2018)	Hybrid structure, use Isochrone Algorithm to provide particles to PSO		
Topaj et al. (2019)	Employ parallel sub-channels for area partition, to select waypoints	Constant cost, where cost is defined by multi-objectives cost function	Retain the least time-consuming waypoints in each sub-channel to form isocost lines
Sasa et al. (2021)		Consider involuntary speed loss and maneuverability	
Chen and Mao (2024)	Symmetric subsectors to select waypoints for smoother routes, derived from (Hagiwara, 1989); dynamically updated weather in the process		Integrate ML predictive cost to select waypoints in each subsector to form isochrones
Dynamic programming (DP)			
Zaccone et al. (2018)	3D DP including speed variation		
Dickson et al. (2019)			Consider numerical error and uncertainty in ship performance
Du et al. (2022a)	Refined grid for 3D DP to increase efficiency		
Mason et al. (2023)		Incorporate WAPS performance models	Characterize the stochastic weather uncertainty to assist decision-making
Sidoti et al. (2023)		Incorporate with models of sailing boat	Iterative search process to deal with dynamic environmental changes
Debski and Drezewski (2024)			Employ an adaptive Markov chain-based route generator to direct the search process in DP
Choi et al. (2023)	Employ the Dijkstra algorithm to generate an initial route before applying DP		
Jeong and Kim (2023)	Use Delaunay triangulation to generate the graph, and the quadtree method to reduce the number of vertices in the graph to enhance search efficiency		
Dijkstra			
Wang et al. (2019)	3D grid including speed variation		
Mannarini et al. (2020)	Dynamically update the weather information in the grid of Dijkstra		Model dynamic wind-wave changes
Gkerekos and Lazakis (2020)			Iterative search to optimise with bi-objectives: the shortest distance and least fuel consumption
Ma et al. (2020)			Consider time constraints in Dijkstra graph
Wang et al. (2020a)	3D grid including speed variation		
Pennino et al. (2020)			Consider seakeeping performance of container ships
Pan et al. (2021b)	Incorporate electronic chart		
Bahrami and Siadatmousavi (2024)	Dynamically update the weather information in grid		Iterative search based on the dynamic grid
Mannarini et al. (2024)	Implementing dynamic graph edge weights considering weather changes	Incorporate performance models of various ships, e.g., sailboats and motor vessels	
A*			
Kurosawa et al. (2020)	Dynamically update weather changes		Integrate regional oceanic and atmospheric models
Shin et al. (2020)	Adaptive grid		Use predicted time cost based on ML
Sun et al. (2022)	3D grid in A* including speed variation		
Grifoll et al. (2022)	Dynamically update environmental changes		Employ wave prediction
Li et al. (2023)			Incorporate specific ETA constraints of voyages in optimisation
Qian et al. (2023)	Dynamically update and improve grid for efficiency; adopt operators in GA to include speed variations;	A* using parallel computing to increase efficiency	Use adaptive heuristic function for removing useless grid for efficiency
Guo et al. (2024b)			Consider sailing maneuvering risks and requirements for speed adjustments in specific sea areas
Wang et al. (2024)		Incorporates WAPS performance model	

(continued on next page)

Table A1 (*continued*)

Literature	Solution space	Candidate generation	Search process
PSO			
Zheng et al. (2019)			Propose and compare several strategies to improve the search process of PSO
Wang et al. (2020b)	3D PSO including speed variation		
Wang et al. (2021b)			Combine PSO for route optimisation and dynamic collaborative optimisation for speed optimisation
Du et al. (2022b)	Consider the real-time update of weather	FO-PSO, use fractional order in updating the velocity of particles to avoid local optimums	
Wang et al. (2022a)	Consider wing-sail's angle of attack to utilize wind energy		
Zhao et al. (2022)	Use Non-domain Sorting GA (NSGA) to generate initial partials for PSO		
Du et al. (2023)		Improve second order oscillating in particle velocity and position update formula in PSO, to avoid local optimums	
ACO			
Zhang et al. (2021)	Adopt crossover and mutation operators in GA to provide candidates to ACO		Consider route smoothness in probability function in choosing routes
Zhang et al. (2022)	3D ACO including speed variation		Consider sea ice risk
Yang et al. (2022)			Use Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) for evaluation of maneuvering risk and fuel
Ma et al. (2023b)	Bi-layer mapping, lower layer artificial potential fields (APF) provide pheromone for ACO		
EA			
Szlapczynska and Szlapczynski (2019)	A* is used to provide initial populations for Strength Pareto Evolutionary Algorithm (SPEA2)		Consider decision makers' preference as weight intervals to limit objectives' space, for faster convergence and better solutions
Yuan et al. (2022)			MOEA, integrate a probabilistic model considering uncertainty to predict weather
Szlapczynski et al. (2023)	Dynamic update weather to consider uncertainties in predictions		Incorporate decision maker's preferences in MOEA/D
Guo et al. (2024a)			Learning-based MOEA, learn from selected optimums in each generation to guide the next evolution direction; incorporate weather forecasts uncertainties
GA			
Lee et al. (2018)	Replacing waypoints with heading and RPM		
Kuhlemann and Tierney (2020)			Consider ETA constraints and piracy risk
Pan et al. (2021a)			Refined selection operators for candidates to accelerate convergence
Lee et al. (2021)	A* provides initial populations for GA		Considering sea ice risk
Zhao et al. (2021)	Use PSO to provide improved initial population for GA		
Wang et al. (2021a)	Use Dijkstra and DP to provide improved initial populations for GA		
Khan et al. (2022)			Compare the effectiveness of four different variants of GA in weather routing
Kytariolou and Themelis (2022)			Include weather forecast in cost function and maneuvering safety in decision-making
Kytariolou and Themelis (2023)		Considering the impact of fouling of hull and propeller on ship performance	
Zhou et al. (2023)	Use GA to provide initial populations for simulated annealing algorithm		
Ma et al. (2024)	A* provides initial populations for improved NSGA-III		
Zhao and Zhao (2024)			NSGA-II considering ETA constraints and ship stability
Li et al. (2024b)	Includes trim as an optimisation parameter		NSGA-III using ML ship models for performance prediction
Other algorithms			
Kim et al. (2020)	Propose the real number grid method for grid refinement		Monte Carlo method tests the cost variation to choose the best voyage
Moradi et al. (2022)			Employ and compare three reinforcement learning approaches to investigate their effectiveness in weather routing
Vettor et al. (2021)	Investigate and account for the impact of weather uncertainty on weather routing		
Sang et al. (2023)		Investigate the impact of operational strategies on fuel consumption, finally influencing the result of weather routing	
Charalambopoulos et al. (2023)	Dynamically update weather		Investigate the effectiveness of probabilistic roadmaps (PRM) method in ship weather routing

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