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The electrification of road transport is a cornerstone in the global effort to reduce greenhouse gas emissions and fossil fuel dependence. However, modern electric vehicles (EVs)—particularly plug-in hybrids (PHEVs) and battery electric vehicles (BEVs) with modular and multi-actuator powertrains—introduce new control challenges due to their high degree of freedom in power delivery and drivetrain configuration. Effectively managing the energy flow in such systems is essential to maximize their efficiency and performance in real-world driving.

This thesis investigates the development of computationally efficient, model-based supervisory energy management strategies for over-actuated EVs. These strategies coordinate both continuous and discrete control decisions—such as power-split between propulsion sources, gear selection, and clutch engagement—while explicitly accounting for the transient and hybrid dynamics of powertrain components. The overarching objective is to enhance energy efficiency without compromising key vehicle performance attributes.

To achieve this, customized control-oriented models of key powertrain components are developed and integrated into mixed-integer model predictive control (MI-MPC) strategies, with specialized solution algorithms proposed to enable online implementation within the computational constraints of embedded automotive systems. The effectiveness of the proposed methods is demonstrated across multiple EV architectures through high-fidelity simulations. Additionally, the thesis explores torque vectoring mechanisms in dual-motor BEVs to improve handling and energy efficiency during dynamic driving maneuvers.

Altogether, this work presents a unified, online-capable energy management framework for over-actuated electric vehicles, laying the foundation for intelligent, energy-efficient, and performance-aware vehicle control.

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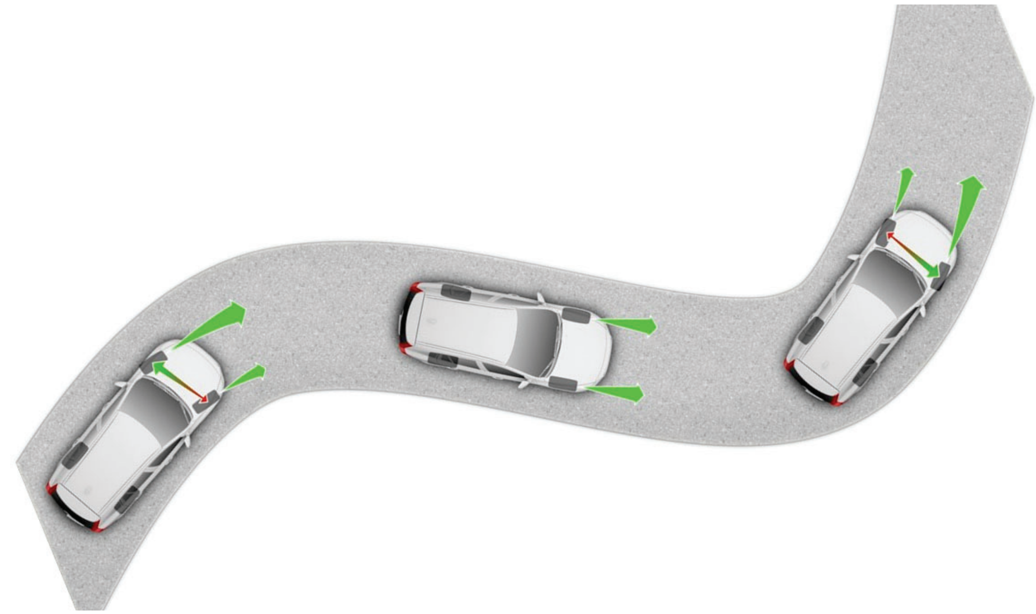

Computationally Efficient Energy Management of Modern Electric Vehicles

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THESIS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

Computationally Efficient Energy Management of Modern Electric Vehicles

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Gothenburg, Sweden, 2025

Computationally Efficient Energy Management of Modern Electric Vehicles

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ISBN 978-91-8103-235-2

Doktorsavhandlingar vid Chalmers tekniska högskola
Ny serie nr 5693
ISSN 0346-718X

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Cover:
Torque distribution, applied to front axle for illustration, in action performing energy-efficient traction control in corners while negotiating winding roads.

Printed by Chalmers Digital Printing
Gothenburg, Sweden, August 2025

Computationally Efficient Energy Management of Modern Electric Vehicles

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Abstract

Modern electric vehicles, particularly plug-in hybrid electric vehicles (PHEVs) and battery electric vehicles (BEVs), often feature over-actuated powertrains with modular architectures that offer high degree of control freedom. Efficient energy management is essential to maximize the operational efficiency (driving range) of these EVs, without compromising performance.

This thesis presents an efficient model-based supervisory energy management framework that co-optimizes torque allocation and discrete decisions online, in over-actuated EVs. Control models capturing powertrain hybrid dynamics are explicitly incorporated into the optimization problem to minimize energy consumption and reduce frequent discrete transitions that degrade performance. Time-scale separation in the supervisory control structure is leveraged to ensure model tractability. To solve the resulting mixed-integer nonlinear problems, customized solution strategies are proposed that exploit their problem structures: relaxation-based methods for PHEVs and bilevel programming approach for BEVs. The framework is implemented using model predictive control and validated with high-fidelity simulations.

The results demonstrate that explicit inclusion of engine dynamics in power-split optimization yields up to 10 % energy savings over a rule-based baseline in PHEVs. At least an additional 3.6 % energy savings is achieved by co-optimizing torque allocation and discrete decisions in both EVs with only a marginal increase in discrete transitions.

Finally, this work also investigates the integration of torque vectoring mechanisms in dual-motor BEVs through a comprehensive torque distribution strategy. This proposed approach enhances energy efficiency, steering performance and dynamic handling, illustrating the potential in advancing the performance envelope of multi-motor EVs.

Keywords: Numerical optimization, nonlinear programming, mixed-integer programming, dynamic programming, model predictive control.

To my family, friends, mentors and guiding spirits.

List of Publications

This thesis is based on the following publications:

[A] **Anand Ganesan**, Sebastien Gros, Nikolce Murgovski, Chih Feng Lee, Martin Sivertsson, "Effect of Engine Dynamics on Optimal Power-Split Control Strategies in Hybrid Electric Vehicles". Published in 2020 IEEE Vehicle Power and Propulsion Conference, Nov. 2020.

[B] **Anand Ganesan**, Sebastien Gros, Nikolce Murgovski, "Numerical Strategies for Mixed-Integer Optimization of Power-Split and Gear Selection in Hybrid Electric Vehicles". Published in IEEE Transactions on Intelligent Transportation Systems, Mar. 2023.

[C] **Anand Ganesan**, Nikolce Murgovski, Derong Yang, Sebastien Gros, "Mixed-Integer Energy Management for Multi-Motor Electric Vehicles with Clutch On-Off: Finding Global Optimum Efficiently". Published in IEEE Transactions on Vehicular Technology, Jul. 2025.

[D] **Anand Ganesan**, Nikolce Murgovski, Derong Yang, "Optimal Torque Vectoring for Performance Enhancement of Multi-Motor Electric Vehicles". Submitted to a peer-reviewed scientific journal.

Other publications by the author, not included in this thesis, are:

[E] **Anand Ganesan**, Nikolce Murgovski, Derong Yang, Sebastien Gros, "Real-Time Mixed-Integer Energy Management Strategy for Multi-Motor Electric Vehicles". *Proc. 2023 IEEE Transportation Electrification Conference & Expo (ITEC)*, Detroit, USA, Jun. 2023.

[F] Alexandre Rocha, **Anand Ganesan**, Derong Yang, Nikolce Murgovski, "Energy-Optimal Trajectory Planning for Electric Vehicles using Model Predictive Control". *Proc. 2024 European Control Conference (ECC)*, Stockholm, Sweden, Jun. 2024.

Contents

Acknowledgments

Every PhD is a long journey of ideas, setbacks, progress, and personal growth, made possible only by the guidance, collaboration, and kindness of extraordinary people. I'm deeply grateful to all who supported me along the way.

First and foremost, I like to express my heartfelt gratitude to Professor Nikolce Murgovski, my main academic supervisor, for his guidance, belief in my work, and unwavering support through setbacks and breakthroughs—it has meant a great deal. Your mentorship and warmth were both inspiring and empowering, which shaped this journey profoundly.

I am equally grateful to Professor Sébastien Gros, first as my main supervisor and later co-supervisor. Your deep scientific insights, hands-on approach and thoughtful feedback were central in shaping the technical foundation of this thesis. I am also grateful to my co-supervisors from Volvo Cars—Dr. Derong Yang, Dr. Martin Sivertsson, and Dr. Chih Feng Lee—for constantly aligning the academic goals with practical relevance to navigate both technical developments and evolving strategic goals.

Special thanks to Professor Torsten Wik, my examiner, for his consistent support and feedback. I also thank Associate Professor Christofer Sundström from Linköping University, whose valuable comments during my mid-term review broadened my perspective at a critical juncture.

This work was funded by Sweden's Innovation Agency (Vinnova) under Grant 2017-05506 and Volvo Cars—for which I am truly thankful. I also wish to thank the VIPP (Volvo Industrial PhD Program) committee for fostering a vibrant platform for industrial research and innovation. Special thanks to Professor Tomas McKelvey, external reviewer at VIPP conferences, for his thoughtful feedback and guidance over the years.

At Chalmers, I thank Professor Anders Grauers, Professor Torbjörn Thiringer, and Professor Balázs Kulcsár for insightful technical discussions. I also thank Senior Researcher Mats Jonasson and Professor Bengt Jacobson, whose input through steering group meetings and thesis projects enriched my work.

At Volvo Cars, I want to thank my line managers: Ole-Fredrik Dunderberg, for being a mentor and consistent support across the years, and Hampus Tengfjord, for helping me navigate the final stretch. Thanks also to Anna Söderlund for her thoughtful guidance during her time as my manager. I gratefully acknowledge the valuable contributions of Mattias Kroon, Daniel Claesson, Anders Brandt, Mats Bohman, Viktor Larsson, Mitra Pourabdol-

lah, and many others across the LVC, Vehicle State, and Energy Manager teams—including Per, Federik, Henrik, Lakshmi, Patrik, Andreas, Roger, Johan, Marcus, Darshan, Babak, Peter, Anette, Forsman, Dhananjay, Avanish and Maryam—for creating a collaborative and supportive environment.

To my academic and industrial PhD colleagues who made workdays brighter and more memorable—thank you. Special shout-out to my academic peers: Godwin, Albert, Ahmet, Faris, Amal, Remi, Ahad, Constantin, Masoud, Yixiao, Sondre, Maxi, Qian, Yao, Fangting, Meng, Qingbo, Xiaolei, Mattia, Betlehem, Alvin, Attila, Francesco, Lorenzo, Hasith, Rita, and Elena; and my industrial office-mates: Huang, Yizhou, Sten Elling, Mudassar, Ivo, Stefan, Erik, Prashant, and Mikael—thank you for your camaraderie and energy.

Thanks also to Shemshedin, Andreas, Filip, Chirag, and Christofer, the co-supervisers of my student projects, your collaboration and trust meant a lot. To my fellow VIPP pals—Niklas, Ektor, Juliette, Russ, Asuka, Ali, Angel, Shuangshuang, Ilhan, and Rickard—thank you for making conferences and social gatherings so lively and enjoyable.

To my long-time friends who were always there—Kanagavel, Prakash, Bharath, Yuvaraj, and Vinil—thank you for your strength and support through it all. To those who made Sweden feel like home—Abirami, Kiruthiga, Leo, Ancy, Benny, Meenu, Axel, and Muhammad—thank you for being my second family and filling my days with warmth and joy.

To my dearest ones—Amma, Appa, Ram, and Sindhu—your unwavering belief in me and unconditional love were the foundation of this journey. Even from afar, you were always with me.

Finally, to all those I may not have named but who stood by me—thank you. This journey had its share of pandemic, detours, curveballs, pleasant surprises and quiet triumphs, and I'm grateful for every part of it.

To all of you—thank you for being part of this adventure.
This thesis is as much yours as it is mine.

Acronyms

AMT: Automated Manual Transmission

BEV: Battery Electric Vehicle

CA: Control Allocation

CO_2: Carbon Dioxide

CTD: Comprehensive Torque Distribution

DoF: Degree-of-Freedom

DP: Dynamic Programming

ECM: Engine Control Module

ED: Electric Drive

EM: Electric Machine

EMS: Energy Management System

eTV: electric Torque Vectoring differential

EV: Electric Vehicle

GHG: Greenhouse Gas

HEV: Hybrid Electric Vehicle

HVAC: Heating, Ventilation and Air Conditioning

i-DP: implicit Dynamic Programming

ICE: Internal Combustion Engine

IP: Interior Point

IPOPT: Interior Point OPTimizer

ISG:	Integrated Starter Generator
IWD:	Individual Wheel Drive
MI:	Mixed-Integer
MIMPC:	Mixed-Integer Model Predictive Control
MINLP:	Mixed-Integer Nonlinear Program
MIOCP:	Mixed-Integer Optimal Control Problem
MIQP:	Mixed-Integer Quadratic Problem
MMEV:	Multi-Motor Electric Vehicle
MPC:	Model Predictive Control
NLP:	Nonlinear Programming
OCP:	Optimal Control Problem
PHEV:	Plug-in Hybrid Electric Vehicle
PMSM:	Permanent Magnet Synchronous Machine
RQ:	Research Question
RSA:	Round-n-Search Approach
RTI:	Real-Time Iteration
SRA:	Selective Relaxation Approach
TTW:	Tank-to-Wheel
TV:	Torque Vectoring
TVDC:	Torque Vectoring Dual Clutch
WTM:	Wheel-to-Miles

Part I

Overview

CHAPTER 1

Introduction

1.1 Motivation and Background

Transportation has long been a catalyst for economic and social development, facilitating global trade, mobility, and access to opportunities. However, its heavy reliance on fossil fuels makes it a major contributor to greenhouse gas (GHG) emissions, accounting for approximately 23 % of global CO_2 emissions [1], [2]. With transport demand expected to more than double by 2050, emissions could increase by over 65 % unless significant mitigation strategies are implemented [2], [3]. To address this challenge, there has been a strong societal push toward sustainable mobility solutions over the past two decades, particularly through electrification of transportation systems [4], [5].

Electric vehicles (EVs) are at the forefront of this transition, offering considerable reductions in GHG emissions throughout the energy production and consumption cycle by employing highly efficient powertrains and enabling integration of renewable energy sources [2], [6], [7]. Advances in electric propulsion technologies have bolstered the acceptance of EVs as a viable alternative to conventional internal combustion engine (ICE) vehicles, leading to mass adaptation over the last decade [5], [7]. However, achieving "Net Zero by 2050" emission levels requires global EV penetration to nearly triple by 2030, reaching 60 % of total car sales [7], [8]. To meet these targets, persistent barriers to EV adoption must be addressed with greater urgency.

Among EVs, battery electric vehicles (BEVs) are recognized as a crucial component of the transition due to their superior net energy efficiency, zero tailpipe emissions, and lower carbon footprint [1], [7], [9]. However, compared to fossil fuel vehicles, the lower energy density and high cost of current battery technologies have led to limited driving ranges and longer charging (fueling) times for BEVs [5], [7]. Consequently, range anxiety still remains a significant barrier to widespread adoption of BEVs, along with concerns about high initial cost and charging infrastructure maturity [5]. (*Range anxiety* refers to the concern that a BEV may not have sufficient charge to complete a journey, considering the available driving range per charge and the accessibility of chargers along a route.) Addressing these challenges is essential to further accelerate BEV adoption. To mitigate range anxiety, improving the driving range per charge of a BEV through strategies that enhance its operational efficiency is crucial. For example, integrating energy management into vehicle motion control algorithms allows for maximizing operational efficiency without compromising performance [10]–[12].

Until a decade ago, hybridization was touted as a transitional solution towards full electrification [2], [8]. However, this perception is evolving, as hybrids are less affected by resource constraints—such as the availability of rare and critical earth elements, silicon shortages, and technological challenges in battery development—compared to BEVs [8], [9]. Additionally, shifts in political stances and policies, including the withdrawal of electrification incentives in major economies and pricing-related trade wars, have further contributed to this change [3], [8], [9]. This is evident from global EV sales in the last decade, where plug-in hybrid electric vehicles (PHEVs) have consistently garnered 30 % market share against BEVs [9]. PHEVs offer the best of both worlds, using gasoline for longer trips to mitigate concerns related to charging infrastructure and range anxiety, while using electricity for shorter trips to reduce emissions. They also feature shorter charging times than BEVs due to smaller battery capacities and boosts cost structures comparable to conventional ICE vehicles. Hence, hybrids are here to stay and are expected to maintain a significant market share of the EV landscape in the coming decades, until a major breakthrough in alternative energy vehicles occur [8].

Beyond ecological benefits, the evolution of PHEVs and BEVs presents new opportunities for innovation and optimization, to revolutionize vehicle performance and efficiency. Electrification is not merely about replacing an ICE and fuel tank with an electric machine and battery. Instead, it necessitates a comprehensive rethinking of vehicle architecture, from power generation and storage to delivery. This transition requires a complete redesign of powertrain and energy management systems to effectively optimize energy storage, distribution, and utilization. For instance, in addition to meeting complex legislation and stringent emissions requirements, modern EV powertrains are increasingly tailored to diverse customer segments to balance cost competitiveness with performance and efficiency, resulting in many variants.

A crucial aspect of this evolution is the modularization of electric powertrains, which increases inherent complexity and introduces significant control challenges. These powertrains integrate various components such as advanced batteries, actuators such as electric motors and ICEs, and sophisticated control units, each with unique operational characteristics and efficiency considerations. For example, an electric machine (EM) and a battery are added to enable regenerative braking and efficient operation of ICE in hybrids, while modern BEVs employ multiple EMs to enhance performance and efficiency [12]–[15]. Like BEVs, PHEVs also allow battery recharging from the grid, expanding their operational flexibility. This modularization quite often leads to over-actuated systems, where the driving demand can be met by a single propulsive or braking actuator or through different combinations of actuators [15]–[18]. It also allows for strategic placement of motors and their independent control to improve vehicle dynamics and energy management. Furthermore, such configurations are particularly advantageous when the advanced strategies employed can exploit these control freedoms to adapt to varying driving conditions and energy usage requirements, thereby enhancing both the drivability and environmental footprint of the vehicle.

Another advancement in modern electric powertrain architectures is the use of decoupling devices like clutches to reduce drag and idle losses of actuators. Typically, three-phase synchronous machines (e.g., permanent magnet synchronous motors, synchronous reluctance motors, and brushless DC motors) or asynchronous machines are used in such applications, and controlled via three-phase inverters. These machines exhibit differences in their rotational dynamics based on factors such as the relative difference in their inertia (electrical and mechanical) and the presence of clutches in the torque delivery path, which can disconnect the electric machine(s) during free rolling scenarios to reduce drag losses [11], [19], [20]. Furthermore, like conventional ICE vehicles, multispeed transmissions are employed in PHEVs to operate ICE efficiently across a wide speed range. An effective way to achieve higher energy efficiency and extend the driving range of such powertrains is to optimize the torque distribution between the actuators and the choice of gear selection and decoupling decisions, allowing the total energy demand and the system losses to be minimized. Also, to fully realize the benefits of such modularized powertrains, it is essential that their dynamic behaviors are adequately considered in the respective control strategies used.

Furthermore, as vehicles become more connected and capable of processing complex information, energy management and vehicle motion control strategies at the route-level planning layer can increasingly leverage both real-time and predictive data from cloud services and connected infrastructure (e.g., traffic conditions, route information, etc.). Meanwhile, modern EVs are often equipped with advanced electronics, sensors, and software—largely driven by advancements in autonomous driving—offering enhanced sensing, localization, and traffic behavior prediction capabilities for trajectory planning and control layers. These developments enable vehicles to interact more intelligently with their environment, creating significant opportunities for better-informed decisions that balance energy efficiency, performance, safety, and driving range. Leveraging this information effectively in real time is essential for adapting to varying driving conditions and user demands in practical scenarios. In addition, ensuring low computational demand is critical for online implementation of these control strategies, as the balance between modeling complexity and computational efficiency directly impacts the cost and feasibility of deploying such advanced systems in mass-produced vehicles.

These opportunities necessitates sophisticated supervisory control frameworks and strategies capable of leveraging the higher control freedom of modern EVs, integrating predictive data, and adapting to driving conditions, user demands, dynamics of systems and components, and operational requirements and limitations, while balancing energy utilization against other vehicle attributes. In addition, the ability to handle powertrain and vehicle variants through generic and flexible control structures is vital for real-world applications.

1.2 Challenges and Prospects for Efficient Control of Over-Actuated EVs

Research in EV control strategies has been intensive, driven by the urgent need for sustainable transportation and the increasing complexity of EV architectures. This work focuses on addressing a key question: How can the high degree of control freedom offered by modularized EV propulsion systems, coupled with advances in sensing and computational capabilities, be leveraged to enhance or balance energy efficiency and performance in modern over-actuated EVs in a computationally efficient manner?

As discussed earlier, modern EV powertrain architectures are becoming increasingly decentralized, incorporating multiple actuators—such as internal combustion engines (ICEs), electric machines (EMs), decoupling devices, and friction brakes—which offer a high degree of control flexibility and modularity [12], [13]. Key control decisions to optimize the operational efficiency of such over-actuated or multi-motor electric vehicles (MMEVs) are typically grouped into control allocation (CA) [16], [21], [22], and discrete decisions [18], [19], [23]. Among these, CA is a continuous decision that refers to the strategic distribution of driving demand between the propulsive and braking actuators such as ICEs, electric drives and friction brakes [22]. Depending on the powertrain layout, CA is further categorized into front-rear [11], [24] and left-right distributions [15], [19]. Based on the chosen control variable, it can be described in terms of torque, power, or force allocation (or split) [21], [25], [26]. Similarly, discrete decisions include gear selection that involves choosing the optimal gear for efficient operation [23] and decoupling decisions that use mechanisms such as clutches to disconnect drivetrain components and reduce idle losses [18]–[20]. In addition to these decisions, vehicle speed can also be optimized to improve energy efficiency, especially in scenarios like autonomous driving and cruise control [14], [27].

Despite significant advancements, several key challenges remain in controlling these decisions, particularly in PHEVs and BEVs. From the perspective of vehicle motion, most studies in the literature have focused primarily on longitudinal motion, optimizing the front-rear torque distribution and decoupling in both PHEVs [28]–[31] and BEVs [11], [19], reporting notable energy savings [19], [24] and braking improvements [32], [33]. Some studies have also investigated torque vectoring (TV), which manages the left-right torque distribution, showing improvements in safety, cornering agility, and energy efficiency [15], [34]–[36]. TV influences multiple degrees of vehicle motion including lateral, yaw, pitch, and roll. In recent studies, combined control of front-rear and left-right torque distributions has been shown to improve traction, stability, maneuverability, and at-the-limit driving [37]–[40]. However, further research is needed to understand the potential of this holistic strategy, referred to in this work as the comprehensive torque distribution (CTD), in improving the operation efficiency of over-actuated EVs.

CTD strategies have been predominantly studied in individual wheel drive (IWD) architectures [38], [41], owing to their high control flexibility. However, IWD systems are generally more expensive and complex than dual-motor architectures, where each motor powers an axle [26], [42]. The latter architecture is widely used in modern EVs as it provides a favorable balance between cost and performance, but lacks TV capabilities inherent in the former. Therefore, establishing the energy efficiency and performance enhancement potential of integrating torque vectoring mechanisms in dual-motor EVs remains an active area of research.

From the perspective of control decision type, most studies on EV energy management have focused solely on CA, typically handling it using offline optimization approaches [12], [24] or heuristic methods such as rule-based or fuzzy logic controllers based on empirical insights [43], [44]. These approaches suffer from suboptimal solutions, as discrete decisions are neglected. Moreover, even advanced techniques like equivalent consumption minimization strategy and Pontryagin minimum principle struggle to handle system dynamics and ignore discrete decisions [16], while dynamic programming (DP), although effective, is often impractical in real time due to computational burden [45].

Studies that jointly optimized CA and decoupling decisions have shown improved operational efficiency and reduced consumption [11], [17], [19], [31]. However, frequent changes in discrete decisions, resulting from the static nature of these purely heuristic or offline strategies, can degrade the drivability, comfort, and useful life of components [17], [46], [47]. Hence, such frequent changes should be addressed when handling discrete decisions in MMEVs. Also, the effect of reducing these frequent changes on energy consumption remains to be established. Furthermore, co-optimizing these continuous and discrete decisions leads to mixed integer nonlinear programming (MINLP) problems [17], [48], [49], which are computationally intensive and often NPhard [50]. Algorithms such as DP, branch-and-bound, and cutting planes can find global solutions to these problems [51]–[54], but exponential worst-case time complexity and substantial run-time variations limit real-time use [55], [56]. Therefore, developing computationally efficient algorithms that provide near-optimal results is critical for the practical deployment of advanced control strategies in EVs.

Furthermore, a common modeling simplification in these energy management studies is the use of steady-state efficiency maps and static optimization or control approaches. Although computationally efficient, these methods can lead to mismatches between expected output and actual vehicle behavior under dynamic conditions [57], [58]. This mismatch is particularly pronounced during high transient loads on ICEs, and during engagement and disengagement of mechanisms with discrete states. Specifically, compared to conventional vehicles, PHEVs experience more frequent ICE transients due to discrete operations such as start-stop and mode switching, and the use of potentially downsized engines. Thus, incorporating dynamic models into control strategies is essential to accurately capture the real-time behavior and performance of actuators.

Based on the research gaps discussed above, key research questions (RQs) that frame the scope of this work are as follows:

RQ1 Can the modeling of transient dynamics of powertrain components and the co-optimization of torque allocation and discrete decisions in modular powertrains be effectively leveraged by model-based supervisory energy management strategies to enhance the operational efficiency of over-actuated electric vehicles without compromising performance attributes?

RQ2 How can the unique properties of mixed-integer energy management problems in over-actuated electric vehicle variants be effectively leveraged to customize advanced solution methods, reducing their computational demands and enabling online implementation?

RQ3 Are there benefits in employing torque vectoring systems along with a comprehensive torque distribution strategy in axle-driven dual-motor electric vehicles?

1.3 Contributions

This work addresses the complex control challenges associated with leveraging modular powertrain architecture and their component dynamics, as well as efficiently solving mixed-integer problems, to establish a comprehensive online-capable energy management framework for over-actuated EVs. Accordingly, it presents a model-based supervisory control framework that co-optimizes torque allocation and discrete decisions (gear selection and clutch on-off) online, enabling a balance among the competing control objectives: energy consumption against other key performance attributes. To realize this, state-of-the-art control concepts, including model predictive control (MPC) and mixed-integer programming, are used alongside conventional vehicle dynamics and control approaches. Furthermore, computationally efficient solution algorithms are developed to solve the resulting mixed-integer problems, enabling online deployment feasibility of the proposed framework in real-world over-actuated EVs.

To address **RQ1** in PHEVs, control-oriented dynamic models of gasoline ICEs have been developed that capture both slow and fast dynamics, enhancing the energy savings realized by power-split controllers. These models account for air mass flow dynamics, fuel flow dynamics, and kinetic energy in engine components, and have been directly integrated into the MPC framework. This integration allows for more precise prediction of actuator behavior, enhancing the controller's ability to manage transient loads effectively.

Addressing **RQ1** and **RQ2**, to minimize energy consumption in PHEVs, a centralized mixed-integer optimal control strategy has been implemented to co-optimize power split and gear choice. This strategy, novel in its explicit consideration of the engine dynamics model stated above and a gear dynamics model (which captures its discrete changes and associated energy loss), provides considerable benefits to fuel economy without frequent gear changes. This work uses relaxation and reformulation techniques that reduce computational demands, enabling online solution of complex mixed-integer optimal control problems (MIOCP). Furthermore, two new numerical strategies, the Selective Relaxation Approach (SRA) and the Round-n-Search Approach (RSA), have been proposed to solve discretized MIOCP efficiently and feasibly across various driving missions. Based on a virtual evaluation of the proposed concept, detailed analysis of performance and computational efficiency against conventional strategies like rule-based gear selection and DP has been provided.

Addressing **RQ1** and **RQ2** in the context of BEVs, a dynamic clutch model (which captures clutch engagement changes and its losses) and a bi-level solution approach have been developed. The proposed control strategy optimizes CA and decoupling decisions to achieve higher energy savings without frequent changes in clutch engagement, thus avoiding negative impacts on vehicle comfort and component wear. The computationally efficient bi-level solution strategy ensures global optimality for complex mixed-integer problems encountered online in energy management of over-actuated BEVs. These strategies have been conceptually validated in high-fidelity closed-loop virtual test environments to indicate robustness and real-world applicability.

Furthermore, addressing **RQ3** in the context of BEVs, this work investigates the performance and energy efficiency of various powertrain layouts with TV capabilities, particularly focusing on differential mechanisms suitable for different MMEV configurations. A systematic approach is adopted to optimize vehicle path, trajectory, and control decisions—-including torque distribution and steering angles—-to maximize potential benefits across various performance objectives and dynamic driving maneuvers. The relative benefits of proposed MMEVs with TV capabilities are established against a conventional open differential layout using high-fidelity vehicle models in a virtual test environment.

The main contributions of the thesis are summarized as follows:

- Customized control-oriented models that capture the dynamic torque response and fuel consumption of gasoline ICEs, and the discrete dynamics and associated energy losses of gear selection and clutch engagement-disengagement are developed to leverage the capability of model-based control frameworks in the mixed-integer energy management of over-actuated EVs. (**RQ1** addressed in **Papers A**, **B**, and **C**.)

- An MPC-based mixed-integer (MI) energy management framework, which explicitly considers powertrain dynamics and employs a supervisory control structure, is proposed to enhance the operational efficiency of over-actuated PHEVs and BEVs, while minimizing the negative consequences of frequent changes of discrete decisions. (**RQ1** addressed in **Papers B**, and **C**.)

- To solve the centralized MI problem of power-split and gear selection in

PHEVs to near-optimality, two computationally efficient numerical solution strategies are presented, in which relaxation strategies were customized by leveraging the iterative nature of the proposed MPC based energy management strategy. (**RQ2** addressed in **Paper B**.)

- A bi-level decomposition approach is proposed to optimally and efficiently solve the MI problem of torque allocation and clutch on-off decisions in multi-motor BEVs, by leveraging the convexity of the allocation subproblem and employing analytical solution and implicit-DP to solve the decomposed subproblems. (**RQ2** addressed in **Paper C**.)

- The potential benefits of integrating TV systems and the CTD strategy in an axle driven dual-motor BEV are established using high-fidelity vehicle models and a combined path and trajectory planning problem with steering angle, torque distribution, and clutch on-off as control variables. (**RQ3** addressed in **Paper D**.)

1.4 Thesis Outline

This compendium is structured as two parts. Part-I consists of five chapters that provides an introduction and overview of the research articles appended in Part-II. Within Part-I, Chapter 1 motivates and introduces the research topic, Chapter 2 describes the modeling approaches considered for control and plant models, Chapter 3 discusses the energy management problem in EVs, control architecture and control synthesis approach adopted in the work, Chapter 4 summarizes the research articles, and Chapter 5 presents some concluding remarks and directions for future research in the field.

CHAPTER 2

Vehicle Modeling

This chapter outlines the modeling approaches adopted in this research. First, the considered EV powertrain architectures and configurations are introduced. Next, the control-oriented modeling approach is described, which leverages time-scale separation in supervisory control to simplify actuator, clutch, transmission, and driveline models, balancing accuracy and computational efficiency. Finally, vehicle dynamics models of varying fidelity, tailored for both control design and high-fidelity simulations, are briefly discussed.

2.1 Powertrain Architectures

Modern EV powertrains are characterized by a high degree of modularity and diversity in design, offering varying levels of performance, efficiency, and complexity. These architectures encompass a wide range of design aspects, including the number, type and configuration of electric machines, coupling and switching devices, transmission systems, and the topological arrangement of components. This complexity is further compounded by the number of operational modes supported by an architecture, enabled by the interaction of coupling devices, actuators, and transmissions, as well as by the strategies used to control these components.

The powertrain architectures of these modern EVs, classified based on the degree of electrification, range from mild-hybrid EVs to pure EVs [14], [59]. Depending on the number and rated capacity ratio of energy sources and power delivery mechanisms in their propulsion systems, hybrid EVs are further sub-classified as mild hybrids to plug-in hybrids (PHEVs) [14], [59]. For example, PHEVs use two or more energy sources, like a battery in combination with gasoline, where one or more electric machines (EMs) contribute partially or entirely to propulsion alongside an ICE. Correspondingly, these vehicles exhibit increasing voltage levels, with most commercially available PHEVs operating above 300 V and typically offering electric ranges under 100 km [60], sufficient enough to cover average daily-commute distance for most drivers [61], [62]. Furthermore, based on operating modes and power flow between the powertrain components, these vehicles are also categorized as series, parallel, and power-split hybrid vehicles [14], [59].

Similarly, pure EVs are classified on the basis of the mechanism used to store or convert electrical energy, including battery electric vehicles (BEVs) and fuel cell electric vehicles. Compared to hybrids, pure EVs exhibit more diversity in their configuration. powertrain configurations and architectures, each designed to optimize chosen performance characteristics [12], [13], [63], [64]. While hybrids use both electricity and gasoline, pure EVs use only electrical energy from one or more sources such as fuel cells, ultra-capacitors, and batteries, each with their own advantages and challenges [65], [66]. On the actuator side, distributed architectures featuring multiple motors and drivetrains have gained prominence due to their modularity, control flexibility, and superior performance over centralized architectures limited to a single motor driving an axle [12], [13], [15]. These multi-drivetrain configurations range from simple dual-motor setups [11] to more complex designs with independent motors for each wheel [19], [37]. As discussed in Section 1.2, individual wheel drive (IWD) configurations offer the highest degree of control freedom and performance capabilities, but suffers from a relatively higher cost, weight, and control complexity than simpler single-motor setups, where one EM drives a single axle. Comparatively, dual motor setups (where each motor drives an axle) offers the best of both worlds, providing higher control freedom than single motor setups and lower cost, weight, and complexity than IWDs [26], [42]. Consequently, IWDs are preferred in performance-oriented EVs, while

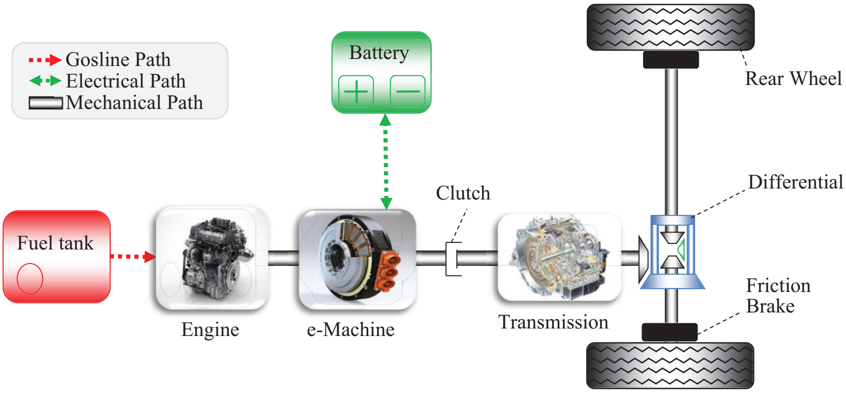

Figure 2.1: Parallel PHEV powertrain configuration.

dual-motor configurations are common in modern EVs that aim to balance cost and performance. Single-motor setups remain the predominant and economical choice for entry-level models due to their simplicity and affordability.

Plug-in Hybrid Electric Vehicle Powertrain

In this work, the PHEV powertrain configuration shown in Fig. 2.1 is used in **Papers A** and **B**. It consists of an ICE and an EM connected to the wheels via a clutch, a multispeed transmission, and associated driveline, with gasoline and a battery being the energy sources. From a power flow perspective, this represents a parallel-hybrid, where both electric and gasoline based propulsion systems can operate separately or combined to deliver the traction demand. Although ICE and EM are mechanically connected to the same drive shaft via a pre-transmission clutch, the system is designed and rated to support the seven distinct operational modes of a typical parallel-hybrid [14], [59]:

(1) Pure electric mode in which EM propels the vehicle while ICE is off.

(2) Conventional ICE mode, where ICE drives the vehicle, with EM off.

(3) Hybrid drive (power assist) mode, where both ICE and EM share traction demand.

(4) Generation mode, where ICE recharges the battery in addition to delivering traction demand.

(5) Regenerative braking mode in which the EM recuperates the kinetic energy during braking into the battery, with the ICE off.

(6) Recharge mode, where both actuators are mechanically disconnected from the driveline, but the ICE still charges the battery with the EM.

(7) Coasting mode, in which ICE and EM are disconnected (both mechanically and electrically) from their sources and transmission.

Mode 1 is typically preferred in city driving with frequent start stops, as ICE has poor efficiency in low-speed high-torque operations, while mode 2 is preferred in high-speed and long-distance driving to address range anxiety.

Compared to traditional PHEVs, the powertrain layout considered in this work has some unique features and limitations. Due to the absence of a clutch between the actuators, the ICE remains connected to the driveline even when off, incurring additional frictional losses in modes 1 and 5. Similarly, EM remains connected and rotates passively in mode 2, introducing magnetic drag losses–particularly for permanent magnet synchronous machines (PMSMs) widely used in EVs [11]–[15]. Also, notice that this system does not support series hybrid operation, which requires two EMs to be connected separately to the ICE and wheels. In contrast, a unique feature of this architecture is the ability to isolate both actuators with a single clutch in modes 6 and 7. This improves powertrain efficiency by reducing drag losses and allowing free rolling (coasting) in flat-road or downhill driving, with or without battery charging, respectively.

In addition to these modes, the multi-speed transmission used effectively emulates the presence of multiple powertrains by scaling the speed-torque characteristics of the actuators according to the selected gear ratio, thereby increasing the control complexity.

Battery Electric Vehicle Powertrain

In this work, to investigate the potential benefits offered by the multi-motor BEVs, different powertrain configurations are considered, wherein the strategic distribution of driver demand among electric drives and friction brakes is exploited by advanced control algorithms. For instance, the dual motor powertrain configuration shown in Fig. 2.2 is commonly used in production BEVs and, therefore, it is considered a baseline configuration in **Papers C** and **D**.

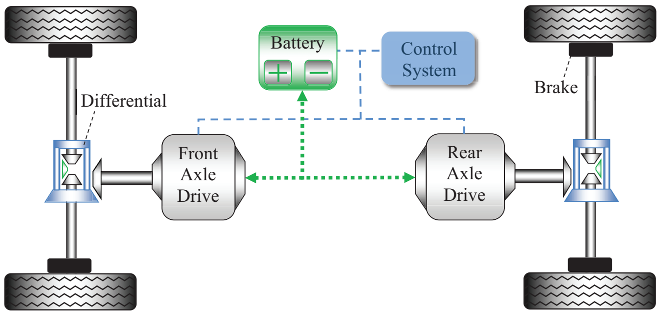

Figure 2.2: Baseline dual motor powertrain configuration.

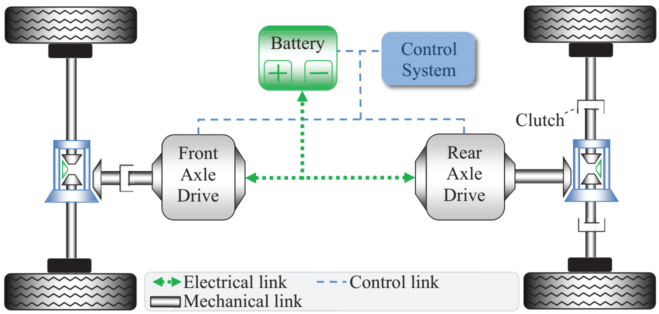

Figure 2.3: Dual motor powertrain with torque vectoring dual clutches in the rear.

This configuration allows only the front-rear distribution, affecting longitudinal and pitch motions. The powertrains of such EVs widely use permanent magnet machines whose magnetic drag losses are notably higher than induction machines. Consequently, a variant of this powertrain with decoupling clutches, similar to that shown in Fig. 2.3, is used in **Papers C** and **D** to understand the impact of minimizing idle losses by isolating an electric drive (ED) under zero load, on the overall energy consumption. This drag loss reduction potential exists in scenarios such as coasting, where all EDs are idling, and in two-wheel-drive mode, where only one ED delivers load demand. In contrast, there is no drag reduction potential in an all-wheel drive scenario, where both EDs are active.

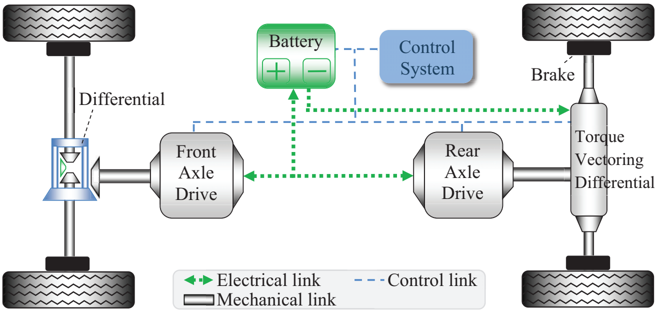

Figure 2.4: Dual motor powertrain with electric torque vectoring differential in the rear.

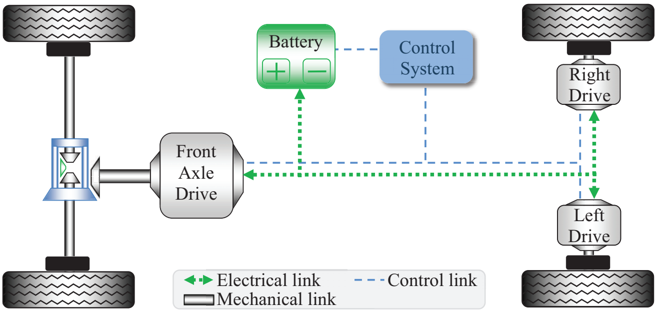

Figure 2.5: BEV powertrain with individual wheel drive in the rear.

Furthermore, the baseline configuration lacks the left-right distribution capability, commonly referred to as torque vectoring (TV). TV allows control of both the magnitude and direction of the wheel torques, creating a differential torque between the left and right wheels. This torque differential generates a yaw moment that turns the vehicle in the direction of the wheels with the lower torque, enhancing cornering performance while reducing steering effort. To impart this capability in dual motor EVs, different TV mechanisms have been explored in **Paper D**. For instance, Fig. 2.4 shows a conceptual powertrain configuration in which a TV mechanism, referred to as electric TV differential (eTV), is integrated in the rear axle. Similarly, Fig. 2.3 presents a configuration equipped with a TV dual clutch (TVDC) mechanism in the rear axle.

Finally, Fig. 2.5 portrays a variant of the IWD configuration, often preferred in high-performance MMEVs. This configuration supports both front-rear and left-right distribution, offering enhanced vehicle control capabilities.

2.2 Propulsive Actuators (Prime Movers)

To improve the energy efficiency of EVs using advanced model-based control techniques and to perform effective simulations, the dynamics of the powertrain actuators must be modeled, along with their efficiency or losses, total energy consumption and limitations. Among the actuators used in EVs, ICE is unique to hybrids, while EM is common to all EVs. Both actuators pose different challenges in modeling their dynamic behaviors and consumption.

Challenges in Modeling Actuator Dynamics

Accurately modeling the dynamics of ICE is quite challenging because of the highly complex nonlinear interactions of its subsystems. Among the main factors that affect the dynamics of a gasoline ICE, the air-mass flow dynamics in the intake and exhaust manifolds, the fuel flow dynamics in the intake, and the kinetic energy in the crankshaft and flywheel of the engine generally exhibit a slower response to control, while the combustion efficiency dynamics exhibits a faster torque response [57], [67], [68]. Modeling approaches that capture these dynamics with sufficiently high accuracy [67] have been used as plant models in the dynamic simulation setups used in this work (**Papers A** abd **B**). However, their computational demand is often prohibitive for online energy management systems.

Similarly, in the case of an EM, it is important to capture the dynamics in three domains–magnetic, electrical, and mechanical–by modeling the interaction and effects of quantities such as direct and quadrature axis currents, voltages, stator and rotor flux linkages, inductance, frequencies, etc., and their relationship to the speed and torque of the machine, to implement effective motor controllers and plant models [69]–[71]. Subsequently, detailed models capturing multi-domain dynamics (including thermal, electrical, chemical, and mechanical) of the actuators and its components have been used as plant models for simulation, while their control-oriented models are used in the proposed strategies in this study.

A holistic description of these plant models is omitted for practical reasons.

Leveraging Time-Scale Separation in Hierarchical Control for Modeling Actuator Dynamics

From the perspective of a supervisory controller that sets references or targets for a lower-level actuator controller (such as engine or motor controllers), simpler models–significantly less complex than those discussed above–have been widely used [14], [68], [72]–[75]. Such a model simplification is adopted in this work as follows.

In **Papers A-C**, hierarchical control architectures in which supervisory controllers at higher levels (strategic layers) set references that reduce energy consumption over a longer time horizon. The lower-level controllers use these references to control their respective actuators along with the task of handling rapid dynamic events. Compared to lower-level controllers, supervisory controllers typically operate at a time scale several orders higher and preview future conditions over a longer horizon, enabling energy-efficient operation while adapting to system limitations. (This hierarchical structure aligns with established approaches in EV energy management [17], [18], [49], [74], whose details are further discussed in Section 3.2.) For such supervisory controllers with preview capability, control-oriented models that capture both transient and steady-state behaviors of an actuator-controller closed-loop system have been used to reduce complexity and be computationally tractable [14], [57], [68], [72]–[74]. Here, closed-loop dynamics refers to the overall dynamic behavior of the actuator under feedback control, encompassing both the intrinsic response of the actuator and the ability of its internal controller to track a reference signal.

In line with these principles, the dynamics of a supercharged gasoline ICE with its integrated engine control module (ECM) is modeled as a closed-loop actuator-controller system, and used as control models in **Papers A** and **B**. This approach enables us to capture the closed-loop dynamics of the ICE corresponding to its fuel consumption rate (proportional to the fuel power, P_{f}) and torque delivery (M_{ic}), as a function of ICE speed and torque (or power) request, at time scales relevant to the supervisory controllers. In contrast, the torque response dynamics of an EM is significantly faster than gasoline engines, reaching steady-state values within the time scales relevant to supervisory control. This enables further simplified treatment and greater flexibility

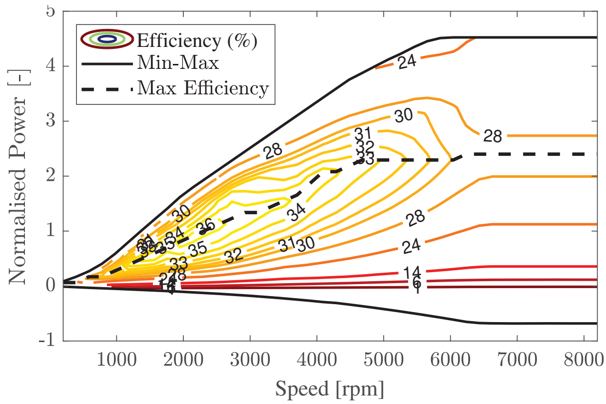

Figure 2.6: ICE Efficiency map with power limitation and maximum efficiency lines.

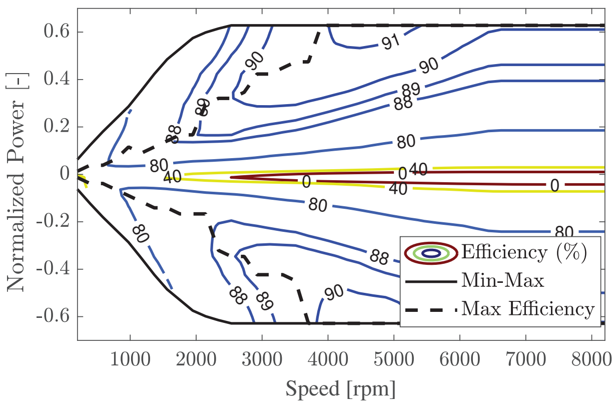

Figure 2.7: Efficiency map of an electrical machine with power limitation and maximum efficiency lines, for propulsive and regenerative operational regions.

in control. Therefore, for the controllers in **Papers A-D**, the torque delivery of an EM (M_{em}) is modeled as a static relation to its torque request ($M_{\text{em,req}}$), i.e. $M_{\text{em}} = M_{\text{em,req}}$. Similarly, system losses, power and torque limits, efficiency and energy consumption rate of the actuators are modeled as static relationships including steady-state measurements.

Fig. 2.6 shows the steady-state efficiency map (measured) of ICE used in **Papers A** and **B**, as a function of normalized power and speed. It also presents the speed-dependent operational limits and maximum efficiency values. The negative power region indicated on the map corresponds to engine braking due to retard function or speed-dependent internal friction. Fig. 2.7 shows a similar efficiency map of an EM used in those papers, which also includes inverter losses in both the propulsive and regeneration regions.

2.3 Decoupling and Power Transfer Components

In EV powertrains, a clutch plays a critical role in enhancing energy efficiency by disconnecting idling actuators from their power source or wheels, thus minimizing drag losses during periods such as coasting or in two-wheel drive operations in the case of an MMEV. As discussed in the previous chapter, the magnitude of these losses varies with the type of actuator employed. For instance, ICE and permanent magnet EMs typically exhibit higher drag losses (friction losses in the former while magnetic drag in the latter) compared to an induction machine. Moreover, the overall impact of a clutch operation on energy consumption depends not only on the duration of the idling periods and the inherent drag losses of the EDs but also on the energy consumed during each clutch transition. In addition, frequent clutch engagements can lead to increased overall losses, negatively affecting operational efficiency, drivability, comfort, and clutch durability of an EV [46]. Similarly, power transfer components vital for EV energy management include transmissions, drive and half shafts, power links–between energy sources, actuators, and inverters–and other driveline parts. Among them, depending on the transmission type, frequent changes in the gear chosen also negatively affect energy consumption and vehicle characteristics, similar to clutches.

In this work, similar to actuators, detailed dynamic models of clutches and power transfer components are used as plant models for simulation, while their simplified representations based on time-scale separations are used as control models. Specifically, the plant model used in **Paper C** captures both the transitional and steady-state phases of clutch dynamics, the key aspects affecting the operation and consumption of an EV powertrain. However, to improve the computational tractability, only the discrete (steady-state) phases of the clutches are captured in the control models. Here, the transitional dynamics is simplified by approximating the losses incurred during each state change as the clutch transition cost. This modeling approach is acceptable for the strategic scope of supervisory control, as it allows control of the frequency of clutch operations.

Similarly, in **Papers A-C**, detailed plant models of power transfer components are used that capture their dynamics due to inertia and shaft torsion, and the discrete effects including the backlashes, shaft shuffle, and shunting [14], [76]. However, similar to actuators and clutches, only the steady-state behavior of these components is captured in the control models. Particularly, in **Paper B**, a discrete dynamic model that captures the integer nature of gear changes in multispeed transmission, along with a transition cost similar to that of clutch models, is used as control model. In these models, inertia of the rotational components is considered for equivalent mass estimations, whereas steady-state maps of transmission losses approximate the total losses in the driveline.

In contrast to the above approach, control models alone are used in **Paper D** for the offline investigation of MMEV performance due to the strategic nature of the decisions involved, ensuring computational tractability for parametric simulations. In **Paper D**, the discrete states of the clutches are uniquely modeled as continuous decisions, drastically reducing the computational demand compared to the model in **Paper C**. Importantly, this approach does not introduce relaxation in the optimal solution (i.e., it guarantees a binary solution), while enabling the use of smooth nonlinear programming solvers. However, a limitation of this model is that losses during the clutch transition are not captured, unlike the clutch model in **Paper C**, which is acceptable for the driving maneuvers investigated in **Paper D**.

Furthermore, in addition to powertrain components, auxiliary electrical loads, such as heating, ventilation and air conditioning (HVAC) systems, lighting, fans, pumps, etc., also contribute to the overall energy consumption of the vehicle. These auxiliary loads can be categorized into two types: those directly related to the powertrain operation (like electric coolant pumps and valves) and those operating at the vehicle level (like infotainment and climate systems). Changes in energy consumption due to the dynamics of all these loads are considered in the plant models to obtain an accurate estimate of energy consumption. However, these auxiliary losses are approximated as a constant to penalize inefficient driving in **Paper D** that focuses on speed and trajectory optimization. In contrast, since the auxiliary loads are not influenced by the control decisions in the EV energy management problems considered, they are neglected in **Papers A–C**.

2.4 Modeling Vehicle Dynamics and Kinematics

Accurately capturing nonlinear vehicle dynamics encompassing multiple degrees of motion, weight transfer, kinematics, resistive (drag) forces, and tire-road interaction effects is essential for optimizing and analyzing vehicle performance, especially under dynamic maneuvers including acceleration, braking, and cornering [77]–[80]. Accordingly, in **Paper D**, a high-fidelity double-track vehicle model with six-degree-of-freedom (DoF) is used to assess the performance of MMEVs under such maneuvers. This model includes two translational motions (longitudinal and lateral), three rotational motions (roll, pitch, and yaw), and load transfer dynamics.

Furthermore, tire forces and moments are modeled using a detailed dynamic tire model based on weighted Magic formula [80], [81], which accounts for the effects of both pure and combined slip on normal forces, accurately capturing tire–road interactions. In addition, total drag forces are modeled to act at the center of gravity of the vehicle. Such model fidelity is crucial for **Paper D** as it focuses on optimal and limit-driving maneuvers, where changes in traction force due to roll- and pitch-induced load transfer significantly impact performance and consumption. However, such complex models poses significant computational challenges for use in online-capable controllers.

In contrast, supervisory controllers in energy management applications often use simpler models, which predominantly focus on longitudinal motion and may include steady-state approximation of pitch-induced load transfer effects [14], [30], [45], [49], [72]. These models typically ignores tire-slip, lateral and rotational dynamics, and tire-road interactions to ensure computational tractability. Subsequently, for the energy management controllers in **Papers A-C**, the traction force demand is calculated using a point mass vehicle model that excludes load transfer effects but incorporates gravitational forces, road slope, air drag, and rolling resistance. This simplified model is deemed sufficient, as the focus in these papers is on optimizing powertrain efficiency under typical driving conditions.

CHAPTER 3

Controller Design and Synthesis

This chapter introduces the broader scope of energy management in modern electric vehicles and outlines the specific problems addressed in this work. The control architecture adopted in the research is then presented, followed by generic formulations of the optimal control problems concerning motion planning and powertrain energy management. Finally, the solution approaches employed—namely, model predictive control, dynamic programming, bi-level programming, and interior-point methods—are briefly described with respect to their suitability for the specific problem structures considered in this thesis.

3.1 Energy Management Problem

In electric vehicles (EVs), energy-efficient operation is crucial to optimize battery usage, improve vehicle performance, reduce cost of ownership, and address range anxiety by extending the driving range. In a broader sense, this involves various strategies and systems that have been proposed to efficiently manage energy consumption while moving from one location to another. Among these, energy management strategies can be categorized into two groups, (1) strategies that address energy flow within the vehicle such as the management of powertrain operation [82], [83] and thermal demand [84], [85], and (2) strategies that focus on minimizing energy losses or consumption during vehicle-terrain interaction including route selection [86], [87], charging point selection [88] and vehicle path-trajectory planning [27], [89]–[91]. Among these strategies, the former enables the improvement of tank-to-wheel (TTW) efficiency, defined as the relationship between the energy output at wheels and the energy content of the fuel sources stored in the vehicle [92], [93], while the latter focuses on improving the wheel-to-miles (WTM) efficiency, which refers to how effectively the energy from the battery is converted into driving distance. In addition, battery charging [94] and recent interest in the industry to integrate EVs into the power grid and home energy systems [95], [96] form the extension of EV operations under stationary conditions, in which the management of the energy flow between the vehicle and the infrastructure is proposed to reduce the cost of ownership and incentivize EV customers. Among these, strategies that address TTW efficiency (energy flow within the vehicle) are of primary interest in this work. Specifically, in **Papers A, B and C**, the focus is on powertrain energy management strategies that utilize the unique opportunity of high degree of control freedom offered by EV powertrain architectures to energy efficiently realize the potential of these systems, despite their inherent complexity. Additionally, to understand the performance potential and operational efficiency of different powertrain configurations, vehicle path and trajectory planning strategies (which focus on improving WTM efficiency) are considered in **Paper D**.

From the perspective of problem definition, in the literature, EV energy management commonly refers to the problem of distributing torque or power demand between multiple propulsive and braking actuators and the energy sources present in EVs [42], [97]–[99]. The key objectives of such energy management strategies is to meet power or energy demands efficiently, while reducing emissions and minimizing the overall operational cost of the propulsion system, as well as ensuring system limitations and satisfactory performance in terms of acceleration, range, and handling. However, extensions have been suggested to include additional discrete control decisions in EV powertrains such as clutch on-off, gear choice and engine on-off, further improving the powertrain efficiency [19], [28], [29], [31], [100]. A notable update to the objective of this extended problem is the inclusion of losses that occur during the transition and steady-state operation of the components or sub-systems

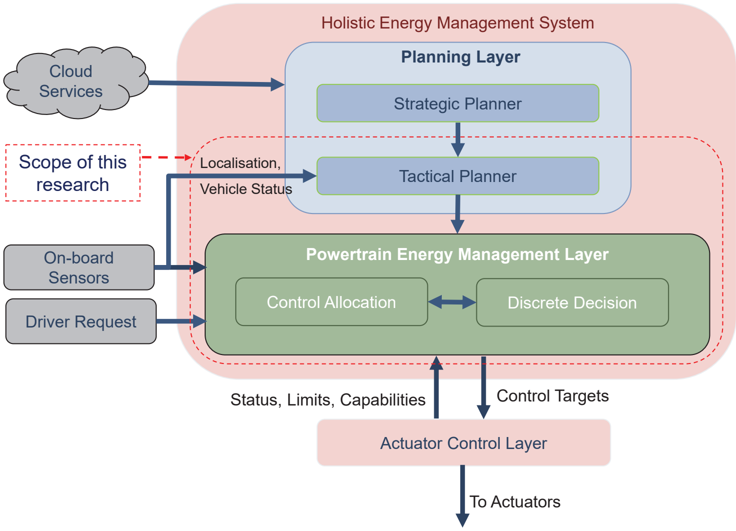

Figure 3.1: Hierarchical control structure of an EV energy management system. As indicated by the red dashed line, the tactical planner and the powertrain energy management layers are the focus in this work.

corresponding to these additional decisions. These decisions and objectives collectively improve the overall operational efficiency of the powertrain and, therefore, are the scope of the powertrain energy management problem addressed in the **Papers A, B and C**. In addition, as discussed earlier, this powertrain energy management strategy is one among the many strategies adopted to reduce overall vehicle energy consumption. From the control architecture perspective these strategies are interlinked and therefore, it is necessary to understand the relationship between these strategies to get an overview of a holistic energy management system for EVs, enabling effective design and synthesis of controllers.

3.2 Control Architecture

The control architecture of a widely adopted holistic energy management system is presented in Fig. 3.1. In particular, a hierarchically distributed control structure is used, owing to its low computational burden while enabling close enough approximation of the solution of centralized approaches [17], [48], [74], [101]. It consists of three main layers: the planning layer, the powertrain management layer, and an actuator control layer. Among these, the first stage within the planning layer is the strategic motion planner, which includes both route planning and route-level planning. Route planning is the process of finding a feasible route between two points on a road network, while minimizing travel time or distance, energy usage, and maximizing battery life [87], [102]. Similarly, route-level planning involves determining energy-efficient targets for vehicle thermal needs (like cabin climatization, battery heating, and propulsion system cooling), vehicle path and speeds, battery state of charge or energy, charge-point selection, etc. [84], [88], [102]. These planners use road network and route-level information such as traffic flow, road topography, speed limits, signals, intersections, and energy consumption from cloud-based navigation service providers. To balance computational efficiency and local uncertainty handling, energy-efficient route-level trajectories are typically averaged over road segments and passed as references to tactical planners.

In this second stage of motion planning, near-future vehicle paths and trajectories—covering a subset of the complete mission—are determined to align with strategic references. This process leverages both current and predicted near-future road segment data, including static road conditions (e.g., slope and speed limits) and real-time traffic information from cloud-based navigation services, following approaches similar to [27], [91], [103]. Among tactical planners, path planning predicts an obstacle-free local path that executes the global path without collision, while trajectory planning creates a set of possible vehicle trajectories targets such as vehicle speed, position, acceleration, etc. that enables navigation of the planned path safely and efficiently. In addition to cloud and map data, these planners often use an array of on-board sensors like Lidars, radars and cameras, etc. for localization and to enhance perception of the local environment [104], while current vehicle status information from other sensors and estimators allows closed-loop control [105]. These information are used to detect roads, lanes, obstacles or objects, etc., enabling the vehicle to perform local maneuvers such as lane changes, merges, forks, and obstacle avoidance in real-time, while optimizing certain performance objectives. To realize such local maneuvers, the planned trajectories are set as targets for the next layer.

After motion planners, next is the powertrain energy management layer, whose objective is to determine control allocation (CA) and discrete decisions, while energy-efficiently meeting driver request, targets set by trajectory planners, and system and component limitations. Here, CA refers to the distribution of power or torque demand between the actuators and the energy sources, while discrete decisions refer to choices such as gear selection, and engine and clutch on-off. Similarly, driver request and trajectory targets are prioritized based on the level of autonomy chosen for a mission. The bottom layer of the control structure is the actuator control layer, where the primary objective is to control the actuators so that the energy-efficient operational targets are met unless safety critical or other operational limitations necessitate a local change. Refer to [17], [18], [49], [74], [106] for a more detailed yet generic understanding of these two layers.

Furthermore, from the above discussions it is noticeable that the layers except the actuator control layer are supervisory control layers. The controllers in these supervisory layers can preview future conditions for a longer horizon than the lower-layer controllers, enabling efficient energy management. For example, the strategic planner addresses the complete mission, while the actuator controller often works on an instantaneous problem [102], [106]. Consequently, to balance computational efficiency and local uncertainty handling, distinct time scales (discretization) and model abstractions are adopted at each level. Specifically, the upper-level layers function at a time scale several orders higher than the lower-level layers [18], [49], [74], [102]. Subsequently, the targets sent from the upper layers are typically averaged over several time samples of the lower layers, as discussed earlier in the case of planners. Due to this, the upper layers commonly handle slow dynamics, while the lower layers handle fast dynamics. For example, in planning layers slow dynamics such as battery state of charge and temperature are addressed, while the actuator control layer handles sudden variations in motor current and voltage, and driving conditions like road friction that require fast adjustments, typically within milliseconds, to maintain vehicle stability and performance. Similarly, the bottom layer uses high-fidelity models of powertrain components whose abstraction increases as we move up the hierarchy with planning layers even approximating the energy consumption and travel time [17], [87], [102], [106].

In line with this framework, we have adopted a similar division of responsibility in our work. However, the focus has been on tactical planning and powertrain energy management layers in the control structure (highlighted in Fig. 3.1 with red-dashed line). As stated in previous section, **Paper D** focuses on the tactical layers while **Papers A, B and C** address only the latter layer. These papers describe the relevant controllers and their interactions with other layers, without detailing the other layers. For example, it is commonly assumed in the papers that a route is given, a decision of strategic planner. However, since the papers have different focus on the rest of the planning layers, the details of their descriptions vary accordingly. Similarly, depending on the chosen mission or route, powertrain configuration, and system dynamics in the control problem considered, different horizon sizes, time scales, control decisions, and model abstractions are employed in the papers. For example, the problem-specific choices of horizon, time scales, etc., are different in **Papers B and C** as they employ different powertrain configurations with unique actuator dynamics. In addition, localization and perceptive sensors are ignored in these papers due to offline nature of the planning problem and the traffic-free scenario considered. Refer to the appended papers for further details on these aspects.

3.3 Problem Formulation

In this section, we first formulate the segment-level problem addressed in **Paper D**, focusing on path and trajectory planning. Then, we formulate a generic mixed-integer powertrain energy management of EVs, addressed in **Papers B and C**, which focuses on CA and discrete decisions.

Path-Trajectory Planning Problem

In **Paper D**, the path-trajectory planning problem aims to optimize the motion of an EV over a predefined route while satisfying physical and operational constraints that includes powertrain components and dynamics. The objective is to determine an optimal trajectory that minimizes a given cost function, such as travel time, energy consumption, or steering effort, while ensuring compliance with vehicle dynamics, control limitations, and road boundaries.

The problem is formulated as an optimal control problem in the spatial domain, where the system dynamics and constraints are expressed as a differential-algebraic system:

$$0 \leq \hat{h}(\hat{x}(s), u(s), \zeta(s)), \tag{3.1}$$

$$0 = \hat{g}(\hat{x}(s), u(s), \zeta(s)), \tag{3.2}$$

$$\hat{x}' = \hat{f}_s(\hat{x}(s), u(s), \zeta(s)), \quad s \in [s_0, s_t], \tag{3.3}$$

where:

- $\hat{x}(s)$ represents the state vector, including vehicle kinematics, powertrain states, and control-related variables.
- $u(s)$ is the control input vector, which includes powertrain decisions such as steering angle, torque distributions and clutch on-off.
- $\zeta(s)$ represents exogenous inputs such as road curvature or environmental factors.
- $\hat{h}(\cdot)$ defines inequality constraints, including control limits, state constraints, road boundaries, and travel time bounds.
- $\hat{g}(\cdot)$ represents algebraic equality constraints, such as power balance conditions specific to the EV powertrain configuration.
- $\hat{f}_s(\cdot)$ describes the spatial nonlinear dynamics of the EV.

The optimization problem seeks to minimize a performance metric, stated as:

$$\min_{\hat{x},u} \quad J_t(\hat{x}(s_t), \zeta(s_t)) + \int_{s_0}^{s_t} J_s(\hat{x}(s), u(s), \zeta(s))\, ds \tag{3.4}$$

subject to:

- The system dynamics and constraints given above.
- Boundary conditions: $\hat{x}(s_0) \in \mathcal{X}_0, \quad \hat{x}(s_t) \in \mathcal{X}_t$.
- If applicable, constraints that ensure repeated motion for periodic maneuvers.

Depending on the scenario under evaluation, different cost functions are considered, such as minimum-time or energy-efficient trajectory planning. To balance travel time and energy consumption, a travel time constraint $t(s_t) \leq \bar{t}(s_t)$ is enforced within the inequality constraints $\hat{h}(\cdot)$. For more details on specific implementations, refer to the main paper.

Mixed-Integer Powertrain Energy Management Problem

The objective of the powertrain energy management problem in **Papers B** and **C** is to minimize the energy consumption of an EV, optimizing both continuous and discrete control decisions. The optimization problem is formulated as a Mixed-Integer Optimal Control Problem (MIOCP), where continuous decisions include power or torque distribution, and discrete decisions involve gear selection or clutch engagement states.

Defined over a specific time horizon $t \in [t_s, t_f]$ with the dynamics and constraints of powertrain components and systems, a generalized MIOCP is given by:

$$\min_{x,u} \int_{t_s}^{t_f} \Phi(x, u, \theta, t) \mathrm{d}t \tag{3.5}$$

$$\text{s.t.} \quad \dot{x}_{\mathrm{c}}(t) = f_{\mathrm{c}}(x, u, \theta, t), \quad \text{(Continuous state dynamics)} \tag{3.6}$$

$$x_{\mathrm{d}}(t^{+}) = f_{\mathrm{d}}(x_{\mathrm{d}}, u_{\mathrm{d}}), \quad \text{(Discrete state transitions)} \tag{3.7}$$

$$h(x, u, \theta) \leq 0, \quad \text{(Path and feasibility constraints)} \tag{3.8}$$

$$u(t) \in \mathcal{U}(t) \subseteq \mathbb{R}^n \times \mathbb{Z}^m, \quad \text{(Admissible controls)} \tag{3.9}$$

$$x(t) \in \mathcal{X}(t) \subseteq \mathbb{R}^n \times \mathbb{Z}^m. \quad \text{(Admissible states)} \tag{3.10}$$

where:

- The objective function Φ in (3.5) may consists of:
  - $P_{\mathrm{f}}(x, u, \theta, t)$ - Gasoline fuel power consumption.
  - $P_{\mathrm{b}}(x, u, \theta, t)$ - Battery power consumption.
  - $\lambda_{\mathrm{b}}^{*}(t)$ - Costate decision used to weight battery consumption in hybrids.
  - $W(u_{\mathrm{d}})$ - Penalty for frequent changes in discrete decisions like gear shifts or clutch engagements.

  For instance, a representative form may include:

$$\Phi = \int_{t_s}^{t_f} \left[P_{\mathrm{f}}(\cdot) + \lambda_{\mathrm{b}}^{*}(t) P_{\mathrm{b}}(\cdot) + W(u_{\mathrm{d}})\right] \mathrm{d}t.$$

- States $x = [x_{\mathrm{c}}^T \;\; x_{\mathrm{d}}^T]^T$ are defined as:
  - $x_{\mathrm{c}}(t)$ - Continuous states (e.g., actuator dynamics).

  - $x_{\mathrm{d}}(t)$ - Discrete states (e.g., gear position, clutch status).
- Control inputs $u = [u_{\mathrm{c}}^T \;\; u_{\mathrm{d}}^T]^T$ are given as:
  - $u_{\mathrm{c}}(t)$ - Continuous control inputs (e.g., requested engine power, torque distribution).
  - $u_{\mathrm{d}}(t)$ - Discrete control inputs (e.g., gear selection, clutch engagement).
- The term θ represents a vector of time-varying system parameters. It may include predicted or optimized quantities from higher-level strategic layers—such as vehicle speed, road slope, driver demand, battery states, costates (e.g.,λ_b^*) and penalty factors—as well as state estimates from lower-level controllers.
- Constraints are defined as:
  - $h(x, u, \theta) \leq 0$ - Includes demand-supply constraints, system and component limitations.
  - $\mathcal{X}(t)$,$\mathcal{U}(t)$ - Limits on states and control inputs, respectively.
  - $(\cdot)^n, (\cdot)^m$ - Superscripts representing the dimension of continuous and discrete states, respectively.

The approach ensures efficient usage of stored energy in any EV configuration, while maintaining smooth-enough transitions of discrete actuators. For further details on implementation, readers are referred to **Papers A** and **B**.

3.4 Solution Approaches

The first step in solving the MIOCPs described in Section 3.3 is to discretize them into finite-dimensional MINLPs—a process known as transcription [107]. Since the original OCPs are infinite-dimensional due to their continuous-time formulation, they are generally intractable except for special cases. Transcription makes these problems amenable to numerical optimization by applying typical approximations such as time discretization. In this work, direct numerical optimization methods are employed for this purpose [107]. Specifically, direct multiple shooting is used for time discretization in all the papers except **Paper D**, where direct collocation with the Radau scheme (an implicit

Runge-Kutta variant) is adopted for spatial discretization to ensure numerical stability during integration of stiff nonlinear dynamics. This transcription step also facilitates the exploitation of the problem structure during the solution process [107].

In the literature, several numerical approaches exist to solve non-linear problems effectively and efficiently, including interior point methods and sequential quadratic programming [108], [109]. However, solving mixed-integer (MI) problems is challenging due to their combinatorial nature. Algorithms such as DP, branch-and-bound, cutting planes, etc. are proficient in finding global solutions to MI problems [51]–[54]. But exponential worst-case time complexity and runtime variations that are typically exhibited by such algorithms [55], [56] are manageable only in offline scenarios, often rendering them impractical for real-time usage. Therefore, the development of computationally efficient solution strategies is also a focus in **Papers B and C**.

Furthermore, depending on the complexity and property of the problem being addressed, different solution approaches have been adopted in the papers. Particularly, in **Papers A and D**, nonlinear programming (NLP) problems are solved with the interior point method using IPOPT [109], while mixed-integer NLP (MINLP) is solved using a combination of relaxation techniques and IPOPT in **Paper B**. Similarly, in **Paper C**, DP is used to solve the integer decision, while continuous decisions are solved analytically after a bi-level decomposition of a mixed integer quadratic problem (MIQP).

Model Predictive Control Framework

Model Predictive Control (MPC) is an advanced optimization-based control strategy designed for real-time implementation in dynamic systems. Unlike conventional control approaches, MPC predicts future system behavior and optimizes control actions over a finite prediction horizon while enforcing system constraints. By repeatedly solving a discretized optimal control problem (OCP) at each time step, the MPC ensures an adaptive and constraint-satisfactory closed-loop control strategy. A key feature of MPC is its inherent ability to handle multi-input multi-output (MIMO) systems by design, allowing for the coordinated and simultaneous control of multiple actuators and regulation of multiple system outputs. Another feature of MPC is its multi-objective capability that allows simultaneously optimization of performance criterias (e.g., energy efficiency, trajectory tracking). MPC is widely applied in electric vehicles including energy management, path-following, obstacle avoidance, and platooning [18], [110]–[114]. MPC operates in a receding horizon framework which can be described using four steps:

1. Prediction - A model of the system predicts the evolution of states over a finite time horizon.
2. Optimization - An objective function is minimized subject to system dynamics and constraints.
3. Control Execution - Only the first optimal control input is applied.
4. Repetition - The horizon shifts forward, and the optimization problem is solved again at each step.

To implement MPC, an OCP must be discretized using direct methods, such as multiple-shooting and orthogonal collocation [107], to transform the OCP into a finite-dimensional problem, as discussed earlier, making it computationally tractable for online implementation. At each update time step k and prediction time step i, given the current state estimate $\hat{x}_k$, the finite-horizon MPC problem is formulated as:

$$\min_{\mathbf{u}_k,\mathbf{x}_k,\mathbf{z}_k} \quad \Phi_N(\mathbf{x}_{N|k}) + \sum_{i=0}^{N-1} \Phi_D(\mathbf{x}_{i|k}, \mathbf{u}_{i|k}, \mathbf{z}_{i|k}, \theta_{i|k}) \tag{3.11a}$$

$$\text{s.t.} \quad \mathbf{x}_{i+1|k} = f_D(\mathbf{x}_{i|k}, \mathbf{u}_{i|k}, \mathbf{z}_{i|k}, \theta_{i|k}), \qquad i = 0, \ldots, N-1 \tag{3.11b}$$

$$h(\mathbf{x}_{i|k}, \mathbf{u}_{i|k}, \mathbf{z}_{i|k}, \theta_{i|k}) \leq 0, \quad i = 0, \ldots, N-1 \tag{3.11c}$$

$$\mathbf{x}_{0|k} = \hat{x}_k \tag{3.11d}$$

$$\mathbf{x}_{N|k} \in \mathcal{X}_{f|k} \tag{3.11e}$$

$$\underline{\mathbf{x}}_k \leq \mathbf{x}_{i|k} \leq \overline{\mathbf{x}}_k, \quad \underline{\mathbf{u}}_k \leq \mathbf{u}_{i|k} \leq \overline{\mathbf{u}}_k, \quad \underline{\mathbf{z}}_k \leq \mathbf{z}_{i|k} \leq \overline{\mathbf{z}}_k, \tag{3.11f}$$

where:

- The decision variable $\mathbf{x}_k = \{\mathbf{x}_{0|k}, \ldots, \mathbf{x}_{N|k}\}$ refers to the state trajectory over the prediction horizon N, $\mathbf{u}_k = \{\mathbf{u}_{0|k}, \ldots, \mathbf{u}_{N-1|k}\}$ denotes the control input sequence, and $\mathbf{z}_k = \{\mathbf{z}_{0|k}, \ldots, \mathbf{z}_{N-1|k}\}$ represents the algebraic states (e.g., slack variables in soft constraints).
- The objective or cost terms in (3.11a):

  - Terminal cost $\Phi_N(\mathbf{x}_{N|k})$: Defines the cost of the terminal state, often used to stabilize constraints or penalize the final deviation.
  - Stage cost $\Phi_D(\mathbf{x}_{i|k}, \mathbf{u}_{i|k}, \mathbf{z}_{i|k}, \theta_{i|k})$: Penalizes states and controls at each stage, typically enforcing objectives such as energy efficiency, tracking accuracy, and control effort minimization.

- The system dynamics in (3.11b), i.e., the discretized system evolution equation $\mathbf{x}_{i+1|k} = f_D(\mathbf{x}_{i|k}, \mathbf{u}_{i|k}, \mathbf{z}_{i|k})$, is obtained through methods such as orthogonal collocation or Euler discretization.
- The path constraints in (3.11c), $h(\mathbf{x}_{i|k}, \mathbf{u}_{i|k}, \mathbf{z}_{i|k}, \theta_{i|k}) \leq 0$, $\forall i$ enforce physical limits, such as road boundaries or actuator saturation.
- The terminal constraints in (3.11e), $\mathbf{x}_{N|k} \in \mathcal{X}_{f|k}$, ensure stability and constraint satisfaction at the end of the horizon.
- The state, control and algebraic bounds in (3.11f) enforce upper and lower limits on states, controls, and algebraic states.

This problem is solved iteratively at each MPC instance $k \in \{1, 2, ..., (t_\mathrm{f}/\Delta t) - N\}$, where Δt denotes the fixed sampling time, and only the first control input is applied to the system. The horizon then shifts forward and the process repeats in the next control cycle.

For further details on the implementation and numerical techniques, the reader is referred to [114]. However, to understand the adaptation of the framework to the problems addressed in the work, refer to the **Papers A, B and C**.

Dynamic Programming

Dynamic programming (DP) is a powerful optimization method solving optimal control problems, particularly in the non-linear, non-convex and mixed integer cases. Developed by Richard Bellman in the 1950s, DP is based on Bellman's principle of optimality [115], which states that the optimal solution to a problem can be found by recursively combining the optimal solutions to its sub-problems.

DP has been widely used in automotive applications such as eco-driving, hybrid energy management, gear shifting, and optimization of driving strategy [51]–[53], [97], [116], [117]. A key advantage of DP is its ability to handle complex constraints on both states and control inputs, while ensuring a global optimum. However, a major drawback is the curse of dimensionality, where computational time increases exponentially with the number of state variables and control inputs. DP solves an optimal control problem over a discrete time horizon by recursively computing the cost-to-go function. The general discrete-time system dynamics are given by:

$$x_{k+1} = f_k(x_k, u_k, \theta_k), \quad k = 0, 1, ..., N-1, \tag{3.12}$$

where:

- $x_k \in X_k$ is the system state at time step k,
- $u_k \in U_k$ is the control input at time step k,
- $\theta_k \in \Theta_k$ is the parameter at time step k.
- f_k represents the system evolution function.

The objective is to find the control policy $\pi(x_0)$ that minimizes a given cost function for an initial state x_0:

$$J_\pi(x_0) = \tilde{g}_N(x_N) + \sum_{k=0}^{N-1} \tilde{g}_k(x_k, u_k(x_k)), \tag{3.13}$$

where:

- $\tilde{g}_k(x_k, u_k)$ represents the stage cost (running cost) at time step k,
- $\tilde{g}_N(x_N)$ is the terminal cost at the final state x_N.

The optimal cost function is obtained by minimizing over a set of all feasible policies, Π:

$$J^*(x_0) = \min_{\pi \in \Pi} J_\pi(x_0). \tag{3.14}$$

Now, using Bellman's optimality principle, the problem is solved using backward recursion as:

$$J^*(x_N) = \tilde{g}_N(x_N), \qquad \text{(Terminal cost)} \tag{3.15a}$$

$$J^*(x_k) = \min_{u_k \in U_k} \left[\tilde{g}_k(x_k, u_k) + J^*(x_{k+1}) \right], \qquad \forall k = N-1, ..., 0. \tag{3.15b}$$

where the optimal control policy is determined as,

$$u_k^*(x_k) = \underset{u_k \in U_k}{\arg\min} \left[\tilde{g}_k(x_k, u_k) + J^*(x_{k+1}) \right]. \tag{3.16}$$

For further theoretical background and implementation details, refer [14], [115]. Furthermore, DP is used as a benchmark strategy to solve an MI problem in **Paper B**, enabling quantification of the optimality gap of the solution of proposed strategies. Additionally, as stated earlier, DP is used to efficiently solve the integer decision of a subproblem after the bilevel decomposition of a mixed integer quadratic problem (MIQP) in **Paper C**.

Bi-Level Programming

Bi-Level Programming is a hierarchical optimization framework where one optimization problem is embedded within another. The structure consists of an upper-level (or outer) problem, which defines the main optimization task, while a lower-level (or inner) problem acts as a constraint for the upper-level optimization. Consequently, the decision variables are split into upper-level decisions (x) and lower-level decisions (y), depending on the subproblem where it is solved. A generic bi-level optimization problem is formulated as:

$$\min_{x \in X, y \in Y} \quad F(x, y) \qquad \text{(Upper-level objective)} \tag{3.17a}$$

$$\text{s.t.} \quad H_i(x, y) \leq 0, \quad i = 1, ..., I, \qquad \text{(Upper-level constraints)} \tag{3.17b}$$

$$y \in \arg\min_{z \in Y} f(x, z) \qquad \text{(Lower-level problem)} \tag{3.17c}$$

$$\text{s.t.} \quad h_j(x, z) \leq 0, \quad j = 1, ..., J. \qquad \text{(Lower-level constraints)} \tag{3.17d}$$

where $F(x, y)$ is the upper-level objective function, $f(x, y)$ is the lower-level objective function, $H_i(x, y)$ and $h_j(x, y)$ define the inequality constraints at the upper and lower levels, and X and Y denote the feasible regions for the upper and lower variables, respectively.

This bi-level approach provides an efficient framework for solving hierarchical decision problems, particularly in automotive control applications where computational efficiency is critical. By separating high-dimensional tasks into upper and lower subproblems, it enables scalable and real-time implementable solutions. An advantage of such a decomposition is that lower-level solutions could be precomputed, enabling real-time feasibility. However, a challenge is that it also requires specialized solution techniques, such as reformulating the lower-level problem as constraints for the upper level. These are the exact aspects exploited in **Paper C** to reduce the computational complexity of solving an MI problem while finding its optimal solution.

Interior Point Methods

Interior Point Methods (IPMs) are widely used to solve a broad spectrum of large-scale optimization problems, ranging from linear to nonlinear programming [118]–[120]. For convex problems, IPMs converge to a global optimum, whereas for nonconvex problems, they typically find local optima. Unlike the Simplex method, which optimizes linear problems by moving along the boundary of the feasible region, IPM iteratively finds optimal solutions by traversing the interior of the feasible region [109], [118]. Modern solvers often use primal-dual IPM, which solves for both primal (decision variables) and dual (Lagrange multipliers) simultaneously, offering enhanced numerical stability and convergence compared to explicit barrier function-based methods.

In this work, the IPOPT software [109], [118], which uses IPM, has been commonly used to locally solve nonlinear programming problems that arise in the energy management of EVs.

CHAPTER 4

Summary of included papers

This chapter provides a summary of the included papers.

4.1 Paper A

Anand Ganesan, Sebastien Gros, Nikolce Murgovski, Chih Feng Lee, Martin Sivertsson
Effect of Engine Dynamics on Optimal Power-Split Control Strategies in Hybrid Electric Vehicles
Published in 2020 IEEE Vehicle Power and Propulsion Conference, Gijon, Spain, pp. 1–8, Nov. 2020.

In this study, a model predictive control (MPC) based supervisory power-split control strategy is proposed to optimize fuel and energy consumption in Hybrid Electric Vehicles (HEVs) by incorporating powertrain actuator dynamic models. Traditional methods often use steady-state maps to approximate actuator energy conversion dynamics, which can lead to sub-optimal control policies and increased fuel and energy consumption, especially under high transient load demands. To address this issue, simpler dynamic models that capture the torque response and fuel consumption of a gasoline internal combustion engine are proposed. Experimental validation of these models shows a mean absolute percentage error of around 3 % in predictions, within the specified operating speed range. These models are then integrated into an MPC-based power-split controller, enhancing the ability of the controller to predict the trajectory of dynamics accurately. A detailed analysis of the sensitivity of HEV energy consumption concerning its actuator dynamics and the transients in its load demands. The results show that including actuator dynamic models in the power-split controller can enable the realization of significant energy savings of atleast 4.25 % compared to a baseline controller without dynamic models, depending on the severity of transient load demands in driving cycles. This validation underscores the effectiveness of dynamic models in capturing both transient and steady-state behaviors relevant to fuel consumption and torque production.

Anand Ganesan contributed with ideas, planing, implementation, results, analysis and writing of the paper. All other co-authors contributed to the ideas, planning and review of the work.

4.2 Paper B

Anand Ganesan, Sebastien Gros, Nikolce Murgovski
Numerical Strategies for Mixed-Integer Optimization of Power-Split and Gear Selection in Hybrid Electric Vehicles
Published in IEEE Transactions on Intelligent Transportation Systems, vol. 24, no. 3, pp. 3194–3210, Mar. 2023.

In the study, a computational strategy for mixed-integer energy management in hybrid electric vehicles (HEVs) is proposed through the co-optimization of power-split and gear selection. The methodology involves formulating a mixed-integer optimal control problem (MIOCP), which is transcribed into a mixed-integer nonlinear program (MINLP) and then tackled using nonlinear model predictive control (MPC). Two primary numerical solution strategies are proposed in the work: the Selective Relaxation Approach (SRA) and the Round-n-Search Approach (RSA), both aimed at solving the MINLP efficiently. These approaches are evaluated against typical rule-based strategies and demonstrate a potential energy savings of approximately 3.6 %, which constitutes a near-optimal solution as it remains within 1 % of the global solution found via DP. Furthermore, both SRA and RSA are approximately 99 times faster than DP, highlighting their computational efficiency and feasibility for real-time implementation in HEVs. The strategies can be extended to address similar mixed-integer problems in future intelligent transportation systems, improving energy efficiency and reducing operational costs in HEVs.

Anand Ganesan contributed with ideas, planing, implementation, results, analysis and writing of the paper. All other co-authors contributed to the ideas, planning and review of the work.

4.3 Paper C

Anand Ganesan, Nikolce Murgovski, Derong Yang, Sebastien Gros
Mixed-Integer Energy Management for Multi-Motor Electric Vehicles with Clutch On-Off: Finding Global Optimum Efficiently
Published in IEEE Transactions on Vehicular Technology, Jul. 2025.

This study introduces a novel energy management strategy for multi-motor electric vehicles (MMEVs) using mixed-integer model predictive control (MI-MPC). It aims to co-optimize torque allocation and clutch on-off decisions to minimize energy consumption and the frequency of clutch engagement changes. A bi-level programming approach is proposed to handle the computational challenges inherent in solving mixed-integer (MI) problems. The inner level deals efficiently with the torque allocation subproblem using an explicit closed-form analytical solution, while the outer level optimizes clutch decisions through implicit dynamic programming (i-DP). An evaluation in a high-fidelity virtual environment demonstrates energy savings exceeding 4 % compared to heuristic controllers used in modern electric vehicles. This strategy also exhibits real-time implementation capability with average solution time of 1 ms on a standard laptop (Intel Core i9-9880H 2.3 GHz processor, 48 GB RAM, 0.294 trillion operations per second capacity), suggesting potential for practical use in MMEVs.

Anand Ganesan contributed with ideas, planing, implementation, results, analysis and writing of the paper. All other co-authors contributed to the ideas, planning and review of the work.

4.4 Paper D

Anand Ganesan, Nikolce Murgovski, Derong Yang
Optimal Torque Vectoring for Performance Enhancement of Multi-Motor Electric Vehicles
Submitted to a peer-reviewed scientific journal.

This study explores the advantages of employing torque vectoring systems, specifically comparing two torque vectoring mechanisms–an electric torque vectoring differential (eTV) and a torque vectoring dual clutch (TVDC)–against an open differential-based conventional and an individual wheel drive-based performance powertrain setup in multi-motor electric vehicles. The objective is to analyze the potential to improve vehicle performance through optimized dynamic maneuvers using steering angle and torque distribution controls. The findings indicate that the proposed torque vectoring variants and control strategies significantly improve performance and energy efficiency, especially at higher speeds, compared to traditional setups. For example, compared to open differentials, the torque vectoring systems studied demonstrate greater lateral force capabilities with reduced steering effort. The eTV variant shows superior performance and handling at high speeds, while the TVDC variant excels at lower speeds. The study concludes that a comprehensive torque distribution in multi-motor electric vehicles can considerably advance vehicle dynamics, realizing up to 11 % energy savings when employing these advanced torque vectoring systems.

Anand Ganesan contributed with ideas, planing, implementation, results, analysis and writing of the paper. Nikolce Murgovski contributed to the ideas, planning, implementation and review of the work. Derong Yang contributed to the ideas, planning, and review of the work.

CHAPTER 5

Concluding Remarks and Future Work

Incorporating the insights from extensive research in the appended **Papers**, this chapter presents conclusive remarks regarding the progress and innovations in energy management strategies for over-actuated EVs, with a focus on optimizing their modular powertrain efficiency across various configurations and driving scenarios.

5.1 Conclusion

This thesis addressed several key challenges in the energy management of over-actuated electric vehicles (EVs), particularly in the context of modular powertrain architectures. A unified energy management framework capable of handling both continuous and discrete decisions online was proposed to improve the operational efficiency of EVs without compromising vehicle performance. This framework leveraged powertrain component dynamics, customized mixed-integer (MI) solution approaches, and advanced control strategies. Addressing the research questions (RQ1–RQ3) through four interconnected studies, the results show that explicit inclusion of engine dynamics in power-split optimization yields up to 10 % energy savings compared to a rule-based baseline in PHEVs. An additional energy saving of at least 3.6 % is achieved by co-optimizing torque allocation and discrete decisions in both EVs with only a marginal increase in the number of discrete transitions. Similarly, the proposed torque vectoring solution in dual-motor BEVs enhances energy efficiency, steering performance and dynamic handling, illustrating their potential to expand the performance envelope of multi-motor EVs. More specifically, the research questions are addressed as follows:

RQ1 Can the modeling of transient dynamics of powertrain components and the co-optimization of torque allocation and discrete decisions in modular powertrains be effectively leveraged by model-based supervisory energy management strategies to enhance the operational efficiency of over-actuated electric vehicles without compromising performance attributes?

To answer **RQ1**, this work established the importance of modeling and integrating transient dynamics of powertrain components–namely, ICEs, multi-speed transmissions, and clutch mechanisms–in enhancing the effective performance of energy management strategies. These models leveraged the time-scale separation in the hierarchical control architecture to ensure low computational complexity. For instance, in **Paper A**, control-oriented models of a gasoline ICE were developed to capture both fuel and torque dynamics, which were explicitly integrated into an MPC-based supervisory energy management controller. Simulations on a PHEV platform showed that actuator dynamics inclusion significantly improves energy savings, up to 10 %, depending on the severity of transient load demands.

Similarly, in **Paper B**, the discrete dynamics and associated energy losses of a multi-gear transmission were modeled and explicitly integrated into a PHEV energy management controller, while in **Paper C**, a similar approach was applied to capture clutch dynamics in multi-motor BEVs. An MPC-based MI energy management framework proposed in both studies effectively utilized these dynamic models within a supervisory control structure, co-optimizing torque allocation and discrete decisions online to enhance the operational efficiency of the EVs. Consequently, the proposed energy management controllers achieved at least 3.5 % additional energy savings compared to the instantaneous baseline controllers, where these discrete decisions were based on heuristic rules while torque allocation was optimized. Another common outcome in these studies is that the potential energy savings in a driving mission is influenced by the intensity of its traction demand, with urban driving being a key benefactor.

Furthermore, by incorporating dynamic models, the proposed controllers also minimized the frequency of discrete decision changes–a common drawback in instantaneous controllers, particularly those using offline-optimized MI policies, as demonstrated in **Paper C**. However, it is essential to note that the energy savings against suboptimal rule-based controllers are accompanied by at least a 5 times increase in the changes of discrete decisions. These frequent changes adversely affect vehicle attributes such as comfort, drivability, and warranty, which the automobile manufacturers prefer to address further. Consequently, the ability of the proposed framework to balance the energy savings and frequency of discrete decision changes (using a calibratable discrete transition cost) was also demonstrated in **Paper C** using Pareto analysis.

These findings answer **RQ1** by underscoring the importance of both online co-optimization of MI decisions and the explicit inclusion of powertrain component dynamics in supervisory energy management strategies, to improve the energy efficiency of over-actuated EVs without sacrificing performance.

RQ2 How can the unique properties of mixed-integer energy management problems in over-actuated electric vehicle variants be effectively leveraged to customize advanced solution methods, reducing their computational demands and enabling online implementation?

To answer **RQ2**—i.e., to ensure online implementation feasibility of the proposed strategies—the unique properties of MI energy management problems in the over-actuated EV variants were leveraged to customize advanced solution methods, reducing their computational demands. Specifically, the iterative nature of the MPC framework was exploited and two customized relaxation strategies, the Selective Relaxation Approach and the Round-n-Search Approach, were used to numerically solve the mixed integer nonlinear problem (MINLP) resulting in PHEVs to near-optimality in **Paper B**. Whereas, in **Paper C**, a bilevel decomposition-based approach is used to optimally solve the resulting mixed-integer quadratic problem (MIQP) in BEVs, where the convexity of the torque allocation subproblem is leveraged to solve it analytically, while discrete clutch decisions are handled separately using implicit

DP. In addition, reformulation and constraint relaxations were performed in these studies to reduce the computational demand of the MI problems. With mean solution times of less than 140 ms[1] for relaxation strategies in PHEVs and 6 ms[1] for i-DP based bi-level approach in BEVs, these proposed strategies demonstrate favorable computational demand, suggesting their conceptual potential for real-time application in over-actuated EVs.

RQ3 Are there benefits in employing torque vectoring systems along with a comprehensive torque distribution strategy in axle-driven dual-motor electric vehicles?

To answer **RQ3**, the use of torque vectoring (TV) systems along with a comprehensive torque distribution (CTD) strategy was demonstrated to enhance the performance of axle-driven dual-motor EVs. Specifically, among the proposed TV mechanisms in **Paper D**, the electric torque vectoring differential (eTV) achieved up to 2 % reduction in travel time and at least 20 % reductions in steering effort, while the torque vectoring dual clutch (TVDC) improved energy efficiency up to 10 %. An open differential configuration preferred in entry-level MMEVs and an individual wheel drive configuration deployed in performance MMEVs were utilized to establish the relative benefits. An unbiased comparison of the configurations was ensured by employing a combined path and trajectory planning framework that incorporates steering angle and CTD decisions as control variables to optimize performance of MMEVs for specific objectives and dynamic driving maneuvers.

5.2 Future Work

Building upon the foundation laid in this work for computationally efficient advanced energy management strategies for over-actuated electric vehicles (EVs), several promising directions to enhance their theoretical robustness and practical applicability under real-world conditions are as follows.

[1] The reported solution times were achieved on a laptop-based platform (powered by an Intel Core i9-9880H 2.3 GHz octa-core processor with atleast 32 GB RAM, providing a peak computational capacity of 0.294 trillion operations per second (TOPS) [121]), using a standalone prototype implementation of the proposed MI strategies within a high-fidelity simulation environment.

First, while the impact of design choices such as unique shift penalties, prediction horizon length, and prediction inaccuracies on net energy savings has been partially evaluated, their influence on the optimality and computational demand of the proposed strategies remains to be systematically investigated. Furthermore, it is essential to analyze the effect of real-world parameter variations–such as road slope, wind speed, and road friction–on both energy consumption and computational complexity. In addition, ensuring the adaptability of dynamic variations in actuator capabilities due to their thermal dynamics and tire-road interaction, including wheel slip, is critical to robustness. These analyses would help quantify robustness more realistically and support the development of strategies that are resistant to a wide range of operating conditions.

To address these challenges more comprehensively, a natural extension is the development of robust or stochastic energy management frameworks. A stochastic model predictive control (MPC) framework could incorporate uncertainty by ensuring that constraints are satisfied on average or with a specified probability, while robust MPC would guarantee constraint satisfaction under worst-case (bounded) uncertainty. Comparing these two approaches, particularly in terms of their conservatism, optimality, and real-time feasibility, would provide valuable insights into the balance of robustness and efficiency. This also motivates further study of advanced mathematical techniques to solve these problems more efficiently. In addition, investigating their implications on robustness, optimality, and real-time tractability will help establish clearer design trade-offs.

Finally, an important step toward real-world adoption is validation of the proposed strategies through implementation in concept or prototype vehicles. Given the increasing computational power onboard and advancements in real-time optimization solvers, deploying these algorithms in embedded automotive systems is becoming increasingly feasible. Tailored software packages for real-time MPC and production-grade code, such as real-time iteration (RTI) schemes, could be employed to further accelerate execution, enhancing the practical viability of the strategies. A successful real-time implementation would pave the way for evaluating these algorithms under realistic driving conditions, and ultimately enabling their deployment into production-level systems for intelligent and energy-efficient transportation.

References

[1] IEA (2023), "CO2 Emissions in 2022 – Analysis," IEA, Paris, Tech. Rep., Mar. 2023, www.iea.org/reports/co2-emissions-in-2022, License: CC BY 4.0.

[2] ITF (2017), "ITF Transport Outlook 2017," OECD Publlishing, Paris, Tech. Rep., Jan. 2017.

[3] ITF (2023), "ITF Transport Outlook 2023," OECD Publlishing, Paris, Tech. Rep., May 2023.

[4] M. F. Felgenhauer, M. A. Pellow, S. M. Benson, and T. Hamacher, "Evaluating co-benefits of battery and fuel cell vehicles in a community in California," *Energy*, vol. 114, pp. 360–368, Nov. 2016.

[5] IEA (2021), "Global EV Outlook 2021," IEA, Paris, Tech. Rep., Apr. 2021, www.iea.org/reports/global-ev-outlook-2021, License: CC BY 4.0.

[6] F. Knobloch, S. Hanssen, A. Lam, *et al.*, "Net emission reductions from electric cars and heat pumps in 59 world regions over time," *Nature Sustainability*, vol. 3, no. 6, pp. 437–447, Jun. 2020.

[7] IEA (2023), "Tracking Clean Energy Progress 2023," IEA, Paris, Tech. Rep., Jul. 2023, www.iea.org/reports/tracking-clean-energy-progress-2023, License: CC BY 4.0.

[8] IEA (2021), "Net Zero by 2050 - A Roadmap for the Global Energy Sector," International Energy Agency, Paris, Tech. Rep., May 2021, www.iea.org/reports/net-zero-by-2050, Licence: CC BY 4.0.

[9] IEA (2022), "Global EV Outlook 2022," IEA, Paris, Tech. Rep., Apr. 2022, www.iea.org/reports/global-ev-outlook-2022, Licence: CC BY 4.0.

[10] M. V. Castro, S. Mukundan, C. L. Filho, *et al.*, "Non–Dominated Sorting Genetic Algorithm Based Determination of Optimal Torque–Split Ratio for a Dual–Motor Electric Vehicle," in *IECON 2021 – 47th Annual Conference of the IEEE Industrial Electronics Society*, Toronto, ON, Canada, Oct. 2021, pp. 1–6.

[11] X. Yuan and J. Wang, "Torque Distribution Strategy for a Front- and Rear-Wheel-Driven Electric Vehicle," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 8, pp. 3365–3374, Oct. 2012.

[12] S. De Pinto, P. Camocardi, C. Chatzikomis, *et al.*, "On the Comparison of 2- and 4-Wheel-Drive Electric Vehicle Layouts with Central Motors and Single- and 2-Speed Transmission Systems," *Energies*, vol. 13, no. 13, p. 3328, Jun. 2020.

[13] O. Nezamuddin, R. Bagwe, and E. Dos Santos, "A Multi-Motor Architecture for Electric Vehicles," in *2019 IEEE Transportation Electrification Conference and Expo (ITEC)*, Detroit, MI, USA, Jun. 2019, pp. 1–6.

[14] L. Guzzella and A. Sciarretta, *Vehicle Propulsion Systems*, 3rd ed. Springer Berlin, Heidelberg, 2013.

[15] G. De Filippis, B. Lenzo, A. Sorniotti, P. Gruber, and W. De Nijs, "Energy-Efficient Torque-Vectoring Control of Electric Vehicles With Multiple Drivetrains," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 6, pp. 4702–4715, Jun. 2018.

[16] A. Sciarretta and L. Guzzella, "Control of hybrid electric vehicles," *IEEE Control Systems Magazine*, vol. 27, no. 2, pp. 60–70, Apr. 2007.

[17] N. Murgovski, B. Egardt, and M. Nilsson, "Cooperative energy management of automated vehicles," *Control Engineering Practice*, vol. 57, pp. 84–98, Dec. 2016.

[18] L. Schmitt, M. Keller, T. Albin, and D. Abel, "Real-Time Nonlinear Model Predictive Control for the Energy Management of Hybrid Electric Vehicles in a Hierarchical Framework," in *2020 American Control Conference (ACC)*, Denver, CO, USA, Jul. 2020, pp. 1961–1967.

[19] J. Torinsson, “Power loss minimization in electric cars by wheel force allocation,” Licentiate Thesis, Chalmers University of Technology, Göteorg, Nov. 2020.

[20] R. Camilleri, P. Armstrong, N. Ewin, R. Richardson, D. A. Howey, and M. D. McCulloch, “The value of a clutch mechanism in electric vehicles,” in *2013 World Electric Vehicle Symposium and Exhibition (EVS27)*, Nov. 2013, pp. 1–11.

[21] P. Pisu and G. Rizzoni, “A Comparative Study Of Supervisory Control Strategies for Hybrid Electric Vehicles,” *IEEE Transactions on Control Systems Technology*, vol. 15, no. 3, pp. 506–518, May 2007.

[22] T. A. Johansen and T. I. Fossen, “Control allocation—A survey,” *Automatica*, vol. 49, no. 5, pp. 1087–1103, May 2013.

[23] M. Joševski and D. Abel, “Gear shifting and engine on/off optimal control in hybrid electric vehicles using partial outer convexification,” in *2016 IEEE Conference on Control Applications (CCA)*, Sep. 2016, pp. 562–568.

[24] H. Fujimoto and S. Harada, “Model-Based Range Extension Control System for Electric Vehicles With Front and Rear Driving–Braking Force Distributions,” *IEEE Transactions on Industrial Electronics*, vol. 62, no. 5, pp. 3245–3254, May 2015.

[25] M. Montazeri-Gh, Z. Pourbafarani, and M. Mahmoodi-k, “Comparative study of different types of PHEV optimal control strategies in real-world conditions,” *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 232, no. 12, pp. 1597–1610, Oct. 2018.

[26] C. Xu, X. Guo, and Q. Xun, “Loss Minimization Based Energy Management for a Dual Motor Electric Vehicle,” in *2022 IEEE Transportation Electrification Conference and Expo, Asia-Pacific (ITEC Asia-Pacific)*, Oct. 2022, pp. 1–6.

[27] A. Sciarretta, G. De Nunzio, and L. L. Ojeda, “Optimal Ecodriving Control: Energy-Efficient Driving of Road Vehicles as an Optimal Control Problem,” *IEEE Control Systems Magazine*, vol. 35, no. 5, pp. 71–90, Oct. 2015.

[28] S. East and M. Cannon, "Fast Optimal Energy Management With Engine On/Off Decisions for Plug-in Hybrid Electric Vehicles," *IEEE Control Systems Letters*, vol. 3, no. 4, pp. 1074–1079, Oct. 2019.

[29] N. Robuschi, M. Salazar, P. Duhr, F. Braghin, and C. H. Onder, "Minimum-Fuel Engine On/Off Control for the Energy Management of a Hybrid Electric Vehicle via Iterative Linear Programming," *IFAC-PapersOnLine*, 9th IFAC Symposium on Advances in Automotive Control AAC 2019, vol. 52, no. 5, pp. 134–140, Jan. 2019.

[30] R. Takapoui, N. Moehle, S. Boyd, and A. Bemporad, "A simple effective heuristic for embedded mixed-integer quadratic programming," *International Journal of Control*, vol. 93, no. 1, pp. 2–12, Jan. 2020.

[31] C. Kirches, *Fast Numerical Methods for Mixed-Integer Nonlinear Model-Predictive Control*, 1st ed. Wiesbaden: Vieweg+Teubner Verlag, Springer, 2011, https://doi.org/10.1007/978-3-8348-8202-8.

[32] T. Wu, F. Wang, and P. Ye, "Regenerative Braking Strategy of Dual-Motor EV Considering Energy Recovery and Brake Stability," *World Electric Vehicle Journal*, vol. 14, no. 1, p. 19, Jan. 2023.

[33] Q. He, Y. Yang, C. Luo, J. Zhai, R. Luo, and C. Fu, "Energy recovery strategy optimization of dual-motor drive electric vehicle based on braking safety and efficient recovery," *Energy*, vol. 248, p. 123 543, Jun. 2022.

[34] J. Ghosh, A. Tonoli, and N. Amati, "A Torque Vectoring Strategy for Improving the Performance of a Rear Wheel Drive Electric Vehicle," in *2015 IEEE Vehicle Power and Propulsion Conference (VPPC)*, Oct. 2015, pp. 1–6.

[35] L. De Novellis, A. Sorniotti, P. Gruber, L. Shead, V. Ivanov, and K. Hoepping, "Torque Vectoring for Electric Vehicles with Individually Controlled Motors: State-of-the-Art and Future Developments," *World Electric Vehicle Journal*, vol. 5, no. 2, pp. 617–628, Jun. 2012.

[36] W. Grzegożek and K. Weigel-Milleret, "Torque vectoring for improving stability of small electric vehicles," *IOP Conference Series: Materials Science and Engineering*, vol. 148, no. 1, p. 012 008, Sep. 2016.

[37] L. Cai, Z. Liao, S. Wei, and J. Li, "Improvement of Maneuverability and Stability for Eight wheel Independently Driven Electric Vehicles by Direct Yaw Moment Control," in *2021 24th International Conference on Electrical Machines and Systems (ICEMS)*, Gyeongju, Republic of Korea, Oct. 2021, pp. 2482–2487.

[38] A. Pennycott, L. D. Novellis, P. Gruber, A. Sorniotti, and T. Goggia, "Enhancing the Energy Efficiency of Fully Electric Vehicles via the Minimization of Motor Power Losses," in *2013 IEEE International Conference on Systems, Man, and Cybernetics*, Oct. 2013, pp. 4167–4172.

[39] A. M. Dizqah, B. Lenzo, A. Sorniotti, P. Gruber, S. Fallah, and J. De Smet, "A Fast and Parametric Torque Distribution Strategy for Four-Wheel-Drive Energy-Efficient Electric Vehicles," *IEEE Transactions on Industrial Electronics*, vol. 63, no. 7, pp. 4367–4376, Jul. 2016.

[40] M. Vignati, M. Belloni, D. Tarsitano, and E. Sabbioni, "Optimal Cooperative Brake Distribution Strategy for IWM Vehicle Accounting for Electric and Friction Braking Torques," *Mathematical Problems in Engineering*, vol. 2021, e1088805, Jul. 2021.

[41] B. Lenzo, G. De Filippis, A. M. Dizqah, *et al.*, "Torque Distribution Strategies for Energy-Efficient Electric Vehicles With Multiple Drivetrains," *Journal of Dynamic Systems, Measurement, and Control*, vol. 139, no. 12, Aug. 2017.

[42] E. Scolaro, D. Troncon, and L. Alberti, "Energy Management of a Dual-Motor Electric Vehicle Based on Particle Swarm Optimization," in *2021 IEEE Vehicle Power and Propulsion Conference (VPPC)*, Gijon, Spain, Oct. 2021, pp. 1–6.

[43] T. Hofman, M. Steinbuch, R. M. van Druten, and A. F. A. Serrarens, "Rule-Based Energy Management Strategies for Hybrid Vehicle Drivetrains: A Fundamental Approach in Reducing Computation Time," *IFAC Proceedings Volumes*, 4th IFAC Symposium on Mechatronic Systems, vol. 39, no. 16, pp. 740–745, Jan. 2006.

[44] F. Khoucha, M. Benbouzid, and A. Kheloui, "An optimal fuzzy logic power sharing strategy for Parallel Hybrid Electric Vehicles," in *2010 IEEE Vehicle Power and Propulsion Conference*, Lille, France, Sep. 2010, pp. 1–5.

[45] N. Murgovski, L. Johannesson, J. Hellgren, B. Egardt, and J. Sjöberg, "Convex Optimization of Charging Infrastructure Design and Component Sizing of a Plug-in Series HEV Powertrain," *IFAC Proceedings Volumes*, 18th IFAC World Congress, vol. 44, no. 1, pp. 13 052–13 057, Jan. 2011.

[46] P. Dolcini, "Contribution to the clutch comfort," [Online] Available: https://theses.hal.science/tel-00172303, PhD Thesis, Departement Automatique / Robotique, Institut National Polytechnique de Grenoble - INPG, France, May 2007.

[47] N. Li, K. Han, I. Kolmanovsky, and A. Girard, "Coordinated Receding-Horizon Control of Battery Electric Vehicle Speed and Gearshift Using Relaxed Mixed-Integer Nonlinear Programming," *IEEE Transactions on Control Systems Technology*, vol. 30, no. 4, pp. 1473–1483, Sep. 2021, https://doi.org/10.1109/TCST.2021.3111538.

[48] M. Joševski and D. Abel, "Distributed predictive control approach for fuel efficient gear shifting in hybrid electric vehicles," in *2016 European Control Conference (ECC)*, Jun. 2016, pp. 2366–2373.

[49] V. Ngo, T. Hofman, M. Steinbuch, and A. Serrarens, "Optimal Control of the Gearshift Command for Hybrid Electric Vehicles," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 8, pp. 3531–3543, Oct. 2012.

[50] L. Liberti, "Undecidability and hardness in mixed-integer nonlinear programming," *RAIRO - Operations Research, Publisher: EDP Sciences*, vol. 53, no. 1, pp. 81–109, Jan. 2019.

[51] D. Putz, D. Schwabeneder, H. Auer, and B. Fina, "A comparison between mixed-integer linear programming and dynamic programming with state prediction as novelty for solving unit commitment," *International Journal of Electrical Power & Energy Systems*, vol. 125, p. 106 426, Feb. 2021.

[52] A. Lieder, D. Briskorn, and R. Stolletz, "A dynamic programming approach for the aircraft landing problem with aircraft classes," *European Journal of Operational Research*, vol. 243, no. 1, pp. 61–69, May 2015.

[53] N. Murgovski, L. Johannesson, J. Sjöberg, and B. Egardt, "Component sizing of a plug-in hybrid electric powertrain via convex optimization," *Mechatronics*, vol. 22, no. 1, pp. 106–120, Feb. 2012.

[54] P. Belotti, C. Kirches, S. Leyffer, J. Linderoth, J. Luedtke, and A. Mahajan, "Mixed-integer nonlinear optimization*†," *Acta Numerica*, vol. 22, pp. 1–131, May 2013.

[55] L. Fortnow and S. Homer, "A Short History of Computational Complexity," Boston University Computer Science Department, Technical Report, Oct. 2003.

[56] A. Basu, M. Conforti, M. Di Summa, and H. Jiang, "Complexity of branch-and-bound and cutting planes in mixed-integer optimization," *Mathematical Programming*, vol. 198, no. 1, pp. 787–810, Mar. 2023.

[57] F. Yan, J. Wang, and K. Huang, "Hybrid Electric Vehicle Model Predictive Control Torque-Split Strategy Incorporating Engine Transient Characteristics," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 6, pp. 2458–2467, Jul. 2012.

[58] M. Bidarvatan and M. Shahbakhti, "Analysis and Control of Torque Split in Hybrid Electric Vehicles by Incorporating Powertrain Dynamics," *Journal of Dynamic Systems, Measurement, and Control*, vol. 140, no. 11, Nov. 2018.

[59] G. Wu, X. Zhang, and Z. Dong, "Powertrain architectures of electrified vehicles: Review, classification and comparison," *Journal of the Franklin Institute*, Special Issue on Control and Estimation of Electrified vehicles, vol. 352, no. 2, pp. 425–448, Feb. 2015.

[60] IRENA (2017), *Electric Vehicles: Technology Brief*, Abu Dhabi.

[61] N. S. Pearre, W. Kempton, R. L. Guensler, and V. V. Elango, "Electric vehicles: How much range is required for a day's driving?" *Transportation Research Part C: Emerging Technologies*, vol. 19, no. 6, pp. 1171–1184, Dec. 2011.

[62] X. Yang, C. Zhuge, C. Shao, *et al.*, "Characterizing mobility patterns of private electric vehicle users with trajectory data," *Applied Energy*, vol. 321, p. 119 417, Sep. 2022.

[63] S. Rinderknecht and T. Meier, "Electric power train configurations and their transmission systems," in *SPEEDAM 2010*, Pisa, Italy, Jun. 2010, pp. 1564–1568.

[64] L. De Novellis, A. Sorniotti, and P. Gruber, "Design and comparison of the handling performance of different electric vehicle layouts," *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 228, no. 2, pp. 218–232, Feb. 2014.

[65] K. T. Chau, Y. S. Wong, and C. C. Chan, "An overview of energy sources for electric vehicles," *Energy Conversion and Management*, vol. 40, no. 10, pp. 1021–1039, Jul. 1999.

[66] M. A. Hannan, M. M. Hoque, A. Mohamed, and A. Ayob, "Review of energy storage systems for electric vehicle applications: Issues and challenges," *Renewable and Sustainable Energy Reviews*, vol. 69, pp. 771–789, Mar. 2017.

[67] L. Guzzella and C. H. Onder, *Introduction to Modeling and Control of Internal Combustion Engine Systems*, 2nd ed. Springer Berlin, Heidelberg, 2010.

[68] M. Bidarvatan and M. Shahbakhti, "Impact of Engine Dynamics on Torque Split Management of a Hybrid Electric Vehicle," in *ASME 2014 Dynamic Systems and Control Conference*, San Antonio, Texas, USA, Dec. 2014, p. 10.

[69] S. Morimoto, Y. Tong, Y. Takeda, and T. Hirasa, "Loss minimization control of permanent magnet synchronous motor drives," *IEEE Transactions on Industrial Electronics*, vol. 41, no. 5, pp. 511–517, Oct. 1994.

[70] C. Mademlis and N. Margaris, "Loss minimization in vector-controlled interior permanent-magnet synchronous motor drives," *IEEE Transactions on Industrial Electronics*, vol. 49, no. 6, pp. 1344–1347, Dec. 2002.

[71] T. Windisch and W. Hofmann, "Loss minimizing and saturation dependent control of induction machines in vehicle applications," in *IECON 2015 - 41st Annual Conference of the IEEE Industrial Electronics Society*, Nov. 2015, pp. 001 530–001 535.

[72] H. Fujimoto, S. Egami, J. Saito, and K. Handa, "Range extension control system for electric vehicle based on searching algorithm of optimal front and rear driving force distribution," in *IECON 2012 - 38th Annual Conference on IEEE Industrial Electronics Society*, Montreal, QC, Canada, Oct. 2012, pp. 4264–4269.

[73] Y. Chen and J. Wang, "Fast and Global Optimal Energy-Efficient Control Allocation With Applications to Over-Actuated Electric Ground Vehicles," *IEEE Transactions on Control Systems Technology*, vol. 20, no. 5, pp. 1202–1211, Sep. 2012.

[74] L. Johannesson, N. Murgovski, E. Jonasson, J. Hellgren, and B. Egardt, "Predictive energy management of hybrid long-haul trucks," *Control Engineering Practice*, vol. 41, pp. 83–97, Aug. 2015.

[75] R. Isermann, *Engine Modeling and Control: Modeling and Electronic Management of Internal Combustion Engines*, 1st ed. Springer Berlin, Heidelberg, 2014.

[76] A. Balluchi, L. Benvenuti, C. Lemma, P. Murrieri, and A. L. Sangiovanni, *Hybrid Models of an Automotive Driveline.*

[77] K. Berntorp, B. Olofsson, K. Lundahl, and L. Nielsen, "Models and methodology for optimal trajectory generation in safety-critical road–vehicle manoeuvres," *Vehicle System Dynamics*, vol. 52, no. 10, pp. 1304–1332, Oct. 2014.

[78] K. Berntorp, *Derivation of a Six Degrees-of-Freedom Ground-Vehicle Model for Automotive Applications* (Technical Reports TFRT-7627). Lund, Sweden: Department of Automatic Control, Lund Institute of Technology, Lund University, Feb. 2013.

[79] P. Anistratov, B. Olofsson, and L. Nielsen, "Analysis and design of recovery behaviour of autonomous-vehicle avoidance manoeuvres," *Vehicle System Dynamics*, vol. 60, no. 7, pp. 2231–2254, Jul. 2022.

[80] V. Fors, "Autonomous Vehicle Maneuvering at the Limit of Friction," PhD dissertation, Linköping University Electronic Press, Linköping, 2020.

[81] H. B. Pacejka, *Tire and Vehicle Dynamics*, 2nd ed. Oxford: Butterworth-Heinemann, 2006.

[82] A. Sciarretta, M. Back, and L. Guzzella, "Optimal control of parallel hybrid electric vehicles," *IEEE Transactions on Control Systems Technology*, vol. 12, no. 3, pp. 352–363, May 2004.

[83] B. Yang, L. Guo, J. Ye, and J. M. Velni, "Energy Management Strategy for Dual-Motor-Based Electric Vehicle Powertrain Using Nonlinear Model Predictive Control," in *2021 IEEE Transportation Electrification Conference & Expo (ITEC)*, Chicago, IL, USA, Jun. 2021, pp. 206–211.

[84] D. Dan, Y. Zhao, M. Wei, and X. Wang, "Review of Thermal Management Technology for Electric Vehicles," *Energies*, vol. 16, no. 12, p. 4693, Jan. 2023.

[85] J. Jaguemont and J. Van Mierlo, "A comprehensive review of future thermal management systems for battery-electrified vehicles," *Journal of Energy Storage*, vol. 31, p. 101 551, Oct. 2020.

[86] W. Zhou and L. Wang, "The Energy-Efficient Dynamic Route Planning for Electric Vehicles," *Journal of Advanced Transportation*, vol. 2019, no. 1, p. 2 607 402, 2019.

[87] T. Perger and H. Auer, "Energy efficient route planning for electric vehicles with special consideration of the topography and battery lifetime," *Energy Efficiency*, vol. 13, no. 8, pp. 1705–1726, Dec. 2020.

[88] A. Hamednia, N. Murgovski, J. Fredriksson, J. Forsman, M. Pourabdollah, and V. Larsson, "Optimal Thermal Management, Charging, and Eco-Driving of Battery Electric Vehicles," *IEEE Transactions on Vehicular Technology*, vol. 72, no. 6, pp. 7265–7278, Jun. 2023.

[89] E. Mejri, S. Kelouwani, Y. Dube, O. Trigui, and K. Agbossou, "Energy Efficient Path Planning for Low Speed Autonomous Electric Vehicle," in *2017 IEEE Vehicle Power and Propulsion Conference (VPPC)*, Dec. 2017, pp. 1–6.

[90] H. Zhang, Y. Zhang, C. Liu, and Z. Zhang, "Energy efficient path planning for autonomous ground vehicles with ackermann steering," *Robotics and Autonomous Systems*, vol. 162, p. 104 366, Apr. 2023.

[91] A. Hamednia, N. K. Sharma, N. Murgovski, and J. Fredriksson, "Computationally Efficient Algorithm for Eco-Driving Over Long Look-Ahead Horizons," *IEEE Transactions on Intelligent Transportation Systems*, vol. 23, no. 7, pp. 6556–6570, Jul. 2022.

[92] O. A. Hjelkrem, P. Arnesen, T. Aarseth Bø, and R. S. Sondell, "Estimation of tank-to-wheel efficiency functions based on type approval data," *Applied Energy*, vol. 276, p. 115 463, Oct. 2020.

[93] M. Weiss, K. C. Cloos, and E. Helmers, "Energy efficiency trade-offs in small to large electric vehicles," *Environmental Sciences Europe*, vol. 32, no. 1, p. 46, Mar. 2020.

[94] J. Oladigbolu, A. Mujeeb, and L. Li, "Optimization and energy management strategies, challenges, advances, and prospects in electric vehicles and their charging infrastructures: A comprehensive review," *Computers and Electrical Engineering*, vol. 120, p. 109 842, Dec. 2024.

[95] P. Kumar, H. K. Channi, R. Kumar, *et al.*, "A comprehensive review of vehicle-to-grid integration in electric vehicles: Powering the future," *Energy Conversion and Management: X*, vol. 25, p. 100 864, Jan. 2025.

[96] M. Blonsky, P. Munankarmi, and S. P. Balamurugan, "Incorporating Residential Smart Electric Vehicle Charging in Home Energy Management Systems," in *2021 IEEE Green Technologies Conference (GreenTech)*, Apr. 2021, pp. 187–194.

[97] D. Ambühl, "Energy management strategies for hybrid electric vehicles," https://doi.org/10.3929/ethz-a-005902053, Doctoral Thesis, ETH Zurich, 2009.

[98] G. Li and D. Görges, "Energy management strategy for parallel hybrid electric vehicles based on approximate dynamic programming and velocity forecast," *Journal of the Franklin Institute*, vol. 356, no. 16, pp. 9502–9523, Nov. 2019.

[99] R. Schmid, J. Buerger, and N. Bajcinca, "Energy Management Strategy for Plug-in-Hybrid Electric Vehicles Based on Predictive PMP," *IEEE Transactions on Control Systems Technology*, pp. 1–13, 2021.

[100] J. Ritzmann, A. Christon, M. Salazar, and C. Onder, "Fuel-Optimal Power Split and Gear Selection Strategies for a Hybrid Electric Vehicle," in *14th International Conference on Engines & Vehicles*, SAE Technical Paper 2019-24-0205, 2019.

[101] N. Robuschi, M. Salazar, N. Viscera, F. Braghin, and C. H. Onder, "Minimum-Fuel Energy Management of a Hybrid Electric Vehicle via Iterative Linear Programming," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 12, pp. 14 575–14 587, Dec. 2020.

[102] L. Claussmann, M. Revilloud, D. Gruyer, and S. Glaser, "A Review of Motion Planning for Highway Autonomous Driving," *IEEE Transactions on Intelligent Transportation Systems*, vol. 21, no. 5, pp. 1826–1848, May 2020.

[103] C. Ngo, E. Solano-Araque, M. Aguado-Rojas, A. Sciarretta, B. Chen, and M. E. Baghdadi, "Real-time eco-driving for connected electric vehicles," *IFAC-PapersOnLine*, 6th IFAC Conference on Engine Powertrain Control, Simulation and Modeling E-COSM 2021, vol. 54, no. 10, pp. 126–131, Jan. 2021.

[104] M. Reda, A. Onsy, A. Y. Haikal, and A. Ghanbari, "Path planning algorithms in the autonomous driving system: A comprehensive review," *Robotics and Autonomous Systems*, vol. 174, p. 104 630, Apr. 2024.

[105] A. K. Basu, S. Tatiya, and S. Bhattacharya, "Overview of Electric Vehicles (EVs) and EV Sensors," in *Sensors for Automotive and Aerospace Applications*, S. Bhattacharya, A. K. Agarwal, O. Prakash, and S. Singh, Eds., Singapore: Springer, 2019, pp. 107–122.

[106] B.-M. Nguyen, J. P. F. Trovão, and M. C. Ta, "Double-Layer Energy Management for Multi-Motor Electric Vehicles," *IEEE Transactions on Vehicular Technology*, vol. 72, no. 7, pp. 8623–8635, Jul. 2023.

[107] M. Diehl and S. Gros, *Numerical Optimal Control*, 2017-draft ed. Optimization in Engineering Center (OPTEC), 2011.

[108] J. Nocedal and S. Wright, *Numerical Optimization* (Springer Series in Operations Research and Financial Engineering), 2nd ed. New York: Springer-Verlag, 2006.

[109] A. Wächter and L. T. Biegler, "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming," *Mathematical Programming*, vol. 106, no. 1, pp. 25–57, Mar. 2006.

[110] L. Stella, A. Themelis, P. Sopasakis, and P. Patrinos, "A simple and efficient algorithm for nonlinear model predictive control," in *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, Melbourne, VIC, Australia, Dec. 2017, pp. 1939–1944.

[111] A. Sathya, P. Sopasakis, R. Van Parys, A. Themelis, G. Pipeleers, and P. Patrinos, "Embedded nonlinear model predictive control for obstacle avoidance using PANOC," in *2018 European Control Conference (ECC)*, Jun. 2018, pp. 1523–1528.

[112] J. Wang, S. Gong, S. Peeta, and L. Lu, "A real-time deployable model predictive control-based cooperative platooning approach for connected and autonomous vehicles," *Transportation Research Part B: Methodological*, vol. 128, pp. 271–301, Oct. 2019.

[113] W. Zhao, H. Wei, Q. Ai, N. Zheng, C. Lin, and Y. Zhang, "Real-time model predictive control of path-following for autonomous vehicles towards model mismatch and uncertainty," *Control Engineering Practice*, vol. 153, p. 106 126, Dec. 2024.

[114] J. B. Rawlings, D. Q. Mayne, and M. Diehl, *Model predictive control: theory, computation, and design*, 2nd ed. Madison, Wisconsin: Nob Hill Publishing, 2017.

[115] R. Bellman, "Dynamic Programming," *Science*, vol. 153, no. 3731, pp. 34–37, 1966.

[116] O. Sundström, "Optimal control and design of hybrid-electric vehicles," https://doi.org/10.3929/ethz-a-005902040, PhD Thesis, ETH Zurich, 2009.

[117] C.-C. Lin, J.-M. Kang, J. Grizzle, and H. Peng, "Energy management strategy for a parallel hybrid electric truck," in *Proceedings of the 2001 American Control Conference. (Cat. No.01CH37148)*, vol. 4, Arlington, VA, USA, Jun. 2001, pp. 2878–2883.

[118] F. E. Curtis, O. Schenk, and A. Wächter, "An Interior-Point Algorithm for Large-Scale Nonlinear Optimization with Inexact Step Computations," *SIAM Journal on Scientific Computing*, vol. 32, no. 6, pp. 3447–3475, Jan. 2010.

[119] J. E. Mitchell, K. Farwell, and D. Ramsden, "Interior Point Methods for Large-Scale Linear Programming," in *Handbook of Optimization in Telecommunications*, M. G. C. Resende and P. M. Pardalos, Eds., 1st ed., Boston, MA: Springer US, Mar. 2006, pp. 3–25.

[120] J. Gondzio, "Interior point methods in the year 2025," *EURO Journal on Computational Optimization*, vol. 13, p. 100 105, Jan. 2025.

[121] Intel Corporation, *APP Metrics for Intel Microprocessors - Intel Core Processors (Rev. 06)*, Export Compliance Metrics for Intel Microprocessors, Product Information & Documentation, Article ID: 000005755. [online] https://www.intel.com/content/dam/support/us/en/documents/processors/APP-for-Intel-Core-Processors.pdf, Jan. 2024.