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Optimized Beamforming for Joint Bistatic Positioning and Monostatic Sensing

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Abstract—We investigate the performance tradeoff between *bistatic positioning (BP)* and *monostatic sensing (MS)* in a multi-input multi-output orthogonal frequency division multiplexing scenario. We derive the Cramér-Rao bounds (CRBs) for BP at the user equipment and MS at the base station. To balance these objectives, we propose a multi-objective optimization framework that optimizes beamformers using a weighted-sum CRB approach, ensuring the weak Pareto boundary. We also introduce two mismatch-minimizing approaches, targeting beamformer mismatch and variance matrix mismatch, and solve them distinctly. Numerical results demonstrate the performance tradeoff between BP and MS, revealing significant gains with the proposed methods and highlighting the advantages of minimizing the weighted-sum mismatch of variance matrices.

Index Terms—Radio positioning, ISAC, Cramér-Rao bound, beamforming, multi-objective optimization.

I. INTRODUCTION

Thanks to its inherent dual functionality, integrated sensing and communication (ISAC) is expected to be a cornerstone in 6G development, enabling modern wireless networks to incorporate sensing capabilities [1]. Sensing refers to the ability of a network to detect and interpret its surroundings, while positioning is a critical component that involves determining the location of objects or users. By sharing the same infrastructure, accurate radio positioning can not only support communications but also support various location-information-driven applications, particularly in scenarios with poor satellite visibility [2], which is anticipated to drive promising applications such as massive twinning, autonomous driving, immersive telepresence, and more.

Radio positioning can be implemented in either bistatic or monostatic modes. In bistatic positioning (BP), the transmitter and receiver are located at different positions, creating additional signal processing challenges, particularly due to the need for synchronization (or orientation estimation if the user equipment (UE) has an antenna array). A common application of BP in cellular networks involves UEs estimating their positions from pilots sent by base stations (BSs) [3]. In contrast, radar-like monostatic sensing (MS) uses co-located

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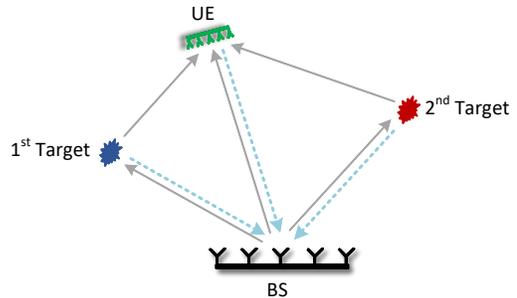


Fig. 1. Illustration of joint BP and MS, where the BS transmits pilot signals, functioning as a monostatic radar to sense passive targets and the UE. Meanwhile, the UE uses the received pilot signals to position itself.

transmitters and receivers, simplifying synchronization and signal processing, allowing cellular networks (e.g., BSs) to position environmental objects such as vehicles.

As ISAC evolves toward 6G, the increased frequency and path loss in communication systems necessitate high beamforming gain, which requires the use of large antenna arrays [2]. Beamforming optimization must not only consider communication performance but also account for positioning and sensing objectives. For example, traditional directional beamforming techniques are not optimal for positioning purposes [4]. Given that ISAC systems share resources between sensing and communication, significant research has been dedicated to exploring the tradeoff between these two functions [5]–[7] and multi-objective optimization (MOO) problems can be formulated. Another way of dealing with the ISAC tradeoff is to maximize sensing performance while guaranteeing the minimum required SINR of communication users [8]. In [9], the existing ISAC beamforming design is extended to a general case by considering the full-duplex capability. Usually, the beam design involves the formulation of a convex (or convexified) problem, which may not work well in reality with impairment, making learning-based approaches favorable [10].

In the above-mentioned works, bistatic positioning (BP) and monostatic sensing (MS) are usually discussed individually, and the complementary advantages of their co-existence are largely ignored. The authors in [11] initiated the exploration by integrating BP and MS from a simultaneous localization and mapping (SLAM) perspective. However, no existing works have been identified to design beamformers on the tradeoff between these two paradigms. This paper considers

a multi-input-multi-output (MIMO) orthogonal frequency division multiplexing (OFDM) scenario and characterizes the performance tradeoff between BP and MS by designing beamforming judiciously. The key contributions are summarized as follows.

- 1) We systematically derive the Cramér-Rao bounds (CRBs) for BP and MS as functions of beamformers, addressing distinct objectives: the UE estimating its own position (considering clock bias and orientation mismatch) and the BS sensing and positioning passive targets (including the UE);
- 2) To strike a tradeoff between BP and MS, we formulate a MOO problem for beamforming design. A weighted-sum CRB approach is proposed to ensure a weak Pareto boundary. Additionally, weighted-sum mismatch-minimizing approaches, based on beamformer mismatch and variance matrix mismatch, are introduced and solved using distinct techniques;
- 3) We provide comprehensive numerical results that reveal the tradeoff between BP and MS. Specifically, compared to other baselines, the proposed approaches demonstrate significant performance gains. Furthermore, we highlight the superiority of minimizing the weighted-sum mismatch of variance matrices over the beamformer.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. Signal Model

As illustrated in Fig. 1, we consider a MIMO OFDM-based joint BP and MS system with M subcarriers, where a BS equipped with M_T transmit antennas transmits positioning pilot signals across L slots to a UE equipped with M_U antennas, who uses the received signals to positioning itself, referred to as BP. Meanwhile, the BS acts as a monostatic radar with M_R colocated receive antennas, sensing the environments by receiving echoes from passive targets¹ and the UE, then estimating their positions, referred to as MS.

Let N denote the number of OFDM pilot symbols in each slot. The transmit signal associated with the n -th symbol in the l -th slot over the m -th subcarrier is given by

$$\mathbf{x}_{l,n,m} = \mathbf{w}_l s_{n,m}, \quad (1)$$

where $\mathbf{w}_l \in \mathbb{C}^{M_T}$ is the beamformer for the l -th slot, and $s_{n,m}$ is the unit-modulus pilot symbol over the m -th subcarrier of the n -th symbol.

1) *Receive Signal at BP:* The signal received at the UE is

$$\bar{\mathbf{y}}_{l,n,m} = \bar{\mathbf{H}}_m \mathbf{x}_{l,n,m} + \bar{\mathbf{z}}_{l,n,m}, \quad (2)$$

where $\bar{\mathbf{H}}_m \in \mathbb{C}^{M_U \times M_T}$ is the channel between the BS and the UE over the m -th subcarrier, given by

$$\bar{\mathbf{H}}_m = \sum_{k=0}^K \alpha_k e^{-j2\pi m \Delta f \tau_k} \mathbf{a}_U(\theta_{U,k}) \mathbf{a}_T^H(\theta_{B,k}), \quad (3)$$

¹To be noted, a passive target in MS creates a multipath in BP.

and $\bar{\mathbf{z}}_{l,n,m} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{M_U})$ is the additive white Gaussian noise (AWGN) at the UE receiver. Here, $\sigma^2 = FN_0 \Delta f$ is the noise power with F , N_0 , and Δf being the noise figure, single-side power spectral density (PSD), and subcarrier spacing, respectively, K denotes the number of targets, and α_k , τ_k , $\theta_{U,k}$, and $\theta_{B,k}$ are the complex channel gain, delay, angle-of-arrival (AOA), and angle-of-departure (AOD), respectively, associated with the k -th path². Finally, $\mathbf{a}_T(\theta) \in \mathbb{C}^{M_T}$ and $\mathbf{a}_U(\theta) \in \mathbb{C}^{M_U}$ are the steering vectors at the BS (transmitter side) and the UE, respectively.

2) *Receive Signal at MS:* Similarly, the signal received at the BS receiver is

$$\mathbf{y}_{l,n,m} = \mathbf{H}_m \mathbf{x}_{l,n,m} + \mathbf{z}_{l,n,m}, \quad (4)$$

where $\mathbf{H}_m \in \mathbb{C}^{M_R \times M_T}$ is the round-trip channel between the BS and the passive targets (including the UE) over the m -th subcarrier, given by

$$\mathbf{H}_m = \sum_{k=0}^K \beta_k e^{-j2\pi m \Delta f \epsilon_k} \mathbf{a}_R(\theta_{B,k}) \mathbf{a}_T^H(\theta_{B,k}), \quad (5)$$

and $\mathbf{z}_{l,n,m} \sim \mathcal{CN}(\mathbf{0}, \sigma^2 \mathbf{I}_{M_R})$ is the AWGN at the BS receiver. Here, β_k and ϵ_k represent the complex channel gain and delay, respectively, associated with the k -th target³, while $\mathbf{a}_R(\theta) \in \mathbb{C}^{M_R}$ is the receiver-side steering vector at the BS.

B. CRB-Based Performance Metric

For both BP and MS, we consider a two-stage positioning process, where the channel domain parameters are estimated in the first stage, and the position domain parameters are inferred from the channel domain parameters in the second stage.

1) *Performance Metric of BP:* In BP, the channel domain parameters are collected by $\bar{\boldsymbol{\xi}} = [\theta_B^T, \theta_U^T, \boldsymbol{\tau}^T, \boldsymbol{\alpha}_R^T, \boldsymbol{\alpha}_I^T]^T \in \mathbb{R}^{(5K+5)}$, where $\theta_B = [\theta_{B,0}, \dots, \theta_{B,K}]^T \in \mathbb{R}^{(K+1)}$ is the collection of AODs, $\theta_U = [\theta_{U,0}, \dots, \theta_{U,K}]^T \in \mathbb{R}^{(K+1)}$ is the collection of AOAs, $\boldsymbol{\tau} = [\tau_0, \dots, \tau_K]^T \in \mathbb{R}^{(K+1)}$ represents the delays, and $\boldsymbol{\alpha}_R = [\Re\{\alpha_0\}, \dots, \Re\{\alpha_K\}]^T \in \mathbb{R}^{(K+1)}$ and $\boldsymbol{\alpha}_I = [\Im\{\alpha_0\}, \dots, \Im\{\alpha_K\}]^T \in \mathbb{R}^{(K+1)}$ are the collections of the real and imaginary parts of the complex channel gains, respectively. Using the Slepian-Bangs formula [4], the element at the i -th row and j -th column of the channel-domain Fisher information matrix (FIM) $\mathbf{I}_c(\bar{\boldsymbol{\xi}})$ is derived as

$$\begin{aligned} [\mathbf{I}_c(\bar{\boldsymbol{\xi}})]_{i,j} &= \frac{2}{\sigma^2} \sum_{l=1}^L \sum_{n=1}^N \sum_{m=1}^M \Re \left\{ \frac{\partial \bar{\boldsymbol{\mu}}_{l,n,m}^H}{\partial [\bar{\boldsymbol{\xi}}]_i} \frac{\partial \bar{\boldsymbol{\mu}}_{l,n,m}}{\partial [\bar{\boldsymbol{\xi}}]_j} \right\} \\ &= \frac{2N}{\sigma^2} \sum_{m=1}^M \Re \left\{ \text{tr} \left(\frac{\partial \bar{\mathbf{H}}_m}{\partial [\bar{\boldsymbol{\xi}}]_j} \mathbf{W} \mathbf{W}^H \frac{\partial \bar{\mathbf{H}}_m^H}{\partial [\bar{\boldsymbol{\xi}}]_i} \right) \right\}, \end{aligned} \quad (6)$$

where $\bar{\boldsymbol{\mu}}_{l,n,m} = \bar{\mathbf{H}}_m \mathbf{x}_{l,n,m}$ denotes the noise-free observation from (2) and $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_L] \in \mathbb{C}^{M_B \times L}$ collects L beamformers.

²For notational convenience, the line-of-sight (LOS) path of the channel is indexed by $k = 0$. Specifically, $\theta_{U,0}$ and $\theta_{B,0}$ denote the AOA and AOD with respect to the BS and the UE, respectively.

³Note that the UE is also a target (indexed by $k = 0$) in the MS scenario.

The position-domain parameters are collected in $\bar{\boldsymbol{\eta}} = [\mathbf{p}_U^T, \phi, \mathbf{p}_1^T, \dots, \mathbf{p}_K^T, \Delta t, \boldsymbol{\alpha}_R^T, \boldsymbol{\alpha}_I^T]^T \in \mathbb{R}^{(4K+6)}$, where $\mathbf{p}_U \in \mathbb{R}^2$ represents the position of the UE, and $\mathbf{p}_k \in \mathbb{R}^2$ represents the position of the k -th target. The variable ϕ denotes the relative orientation of the BS (in the UE's local coordinate system), while Δt characterizes the clock bias that reflects the asynchronism between the BS and UE in the bistatic setting. Note that the nuisance parameters $\boldsymbol{\alpha}_R$ and $\boldsymbol{\alpha}_I$ from the channel-domain parameter $\bar{\boldsymbol{\xi}}$ remain part of the position-domain parameter $\bar{\boldsymbol{\eta}}$, as they do not contribute useful information for position estimation. Using the channel-domain FIM, the position-domain FIM $\mathbf{I}_p(\bar{\boldsymbol{\eta}})$ is computed as follows

$$\mathbf{I}_p(\bar{\boldsymbol{\eta}}) = \bar{\mathbf{J}}^T \mathbf{I}_c(\bar{\boldsymbol{\xi}}) \bar{\mathbf{J}}, \quad (7)$$

where $\bar{\mathbf{J}} \in \mathbb{R}^{(5K+5) \times (4K+6)}$ is the Jacobian matrix, with the element in the i -th row and j -th column given by $[\bar{\mathbf{J}}]_{i,j} = \partial[\bar{\boldsymbol{\xi}}]_i / \partial[\bar{\boldsymbol{\eta}}]_j$. The CRB is used to quantify the BP accuracy concerning \mathbf{p}_U , providing a lower bound on the sum of the variances for estimating \mathbf{p}_U , and is expressed as

$$\overline{\text{CRB}}(\mathbf{p}_U) = \text{tr} \left(\left[\mathbf{I}_p(\bar{\boldsymbol{\eta}})^{-1} \right]_{1:2,1:2} \right). \quad (8)$$

2) *Performance Metric of MS*: Following similar steps, the position-domain FIM for MS is given by

$$\mathbf{I}_p(\boldsymbol{\eta}) = \underline{\mathbf{J}}^T \mathbf{I}_c(\boldsymbol{\xi}) \underline{\mathbf{J}}, \quad (9)$$

where $\boldsymbol{\xi} = [\boldsymbol{\theta}_B^T, \boldsymbol{\kappa}^T, \boldsymbol{\beta}_R^T, \boldsymbol{\beta}_I^T]^T \in \mathbb{R}^{(4K+4)}$ and $\boldsymbol{\eta} = [\mathbf{p}_U^T, \mathbf{p}_1^T, \dots, \mathbf{p}_K^T, \boldsymbol{\beta}_R^T, \boldsymbol{\beta}_I^T]^T \in \mathbb{R}^{(4K+4)}$ are the channel-domain and position-domain parameters, respectively. Here, $\boldsymbol{\kappa} = [\kappa_0, \dots, \kappa_K]^T \in \mathbb{R}^{(K+1)}$ represents the delay measurements, while $\boldsymbol{\beta}_R = [\Re\{\beta_0\}, \dots, \Re\{\beta_K\}]^T \in \mathbb{R}^{(K+1)}$ and $\boldsymbol{\beta}_I = [\Im\{\beta_0\}, \dots, \Im\{\beta_K\}]^T \in \mathbb{R}^{(K+1)}$ represent the real and imaginary parts of the complex channel gains, respectively. The CRB for MS, concerning the passive targets (as well as the UE), provides a lower bound on the sum variance for estimating $\mathbf{p} = [\mathbf{p}_U^T, \mathbf{p}_1^T, \dots, \mathbf{p}_K^T]^T \in \mathbb{R}^{(2K+2)}$ at the BS, and is given by

$$\underline{\text{CRB}}(\mathbf{p}) = \text{tr} \left(\left[\mathbf{I}_p(\boldsymbol{\eta})^{-1} \right]_{1:2K+2,1:2K+2} \right). \quad (10)$$

C. Problem Formulation

We observe that both $\overline{\text{CRB}}(\mathbf{p}_U)$ and $\underline{\text{CRB}}(\mathbf{p})$ are functions of \mathbf{W} , which can be optimized by designing the beamformers \mathbf{W} [4], [12]. However, due to the different objectives, a performance tradeoff between BP and MS emerges. Specifically, this bistatic-monostatic performance tradeoff is characterized by a MOO problem [13], expressed as

$$\min_{\mathbf{W}} \left[\overline{\text{CRB}}(\mathbf{p}_U), \underline{\text{CRB}}(\mathbf{p}) \right] \quad (11a)$$

$$\text{s.t.} \quad \text{tr}(\mathbf{W}\mathbf{W}^H) \leq P/M, \quad (11b)$$

where P is the power budget⁴. Note that the optimal solution to (11) represents the Pareto boundary of $[\overline{\text{CRB}}(\mathbf{p}_U), \underline{\text{CRB}}(\mathbf{p})]$,

⁴Without loss of generality, the right-hand side of (11b) is set as P/M such that the total transmit power over M subcarriers is P .

which is challenging to find due to the MOO nature. Additionally, neither $\overline{\text{CRB}}(\mathbf{p}_U)$ nor $\underline{\text{CRB}}(\mathbf{p})$ is convex with respect to \mathbf{W} , further complicating the problem.

III. TRADEOFF BETWEEN BP AND MS

A. Weighted-Sum CRB Optimization

To solve (11) and characterize the performance tradeoff, we first employ the weighted-sum approach, a classical method capable of obtaining the weak Pareto boundary of MOO problems [13]. Specifically, (11) is reformulated as

$$\min_{\mathbf{W}} \quad \rho \overline{\text{CRB}}(\mathbf{p}_U) + (1 - \rho) \underline{\text{CRB}}(\mathbf{p}) \quad (12a)$$

$$\text{s.t.} \quad \text{tr}(\mathbf{W}\mathbf{W}^H) \leq P/M, \quad (12b)$$

where $\rho \in [0, 1]$ is a constant that adjusts the priority between BP and MS, determined by the specific application scenario and quality of service (QoS) requirements.

Problem (12) remains challenging to solve due to its non-convexity. By defining $\mathbf{V} = \mathbf{W}\mathbf{W}^H$, we lift (12) into a relaxed form (by omitting the constraint $\text{rank}(\mathbf{V}) = L$) as

$$\min_{\mathbf{V}} \quad \rho \overline{\text{CRB}}(\mathbf{p}_U) + (1 - \rho) \underline{\text{CRB}}(\mathbf{p}) \quad (13a)$$

$$\text{s.t.} \quad \text{tr}(\mathbf{V}) \leq P/M, \quad \mathbf{V} \succeq \mathbf{0}. \quad (13b)$$

Next, note that the matrices on the right-hand sides of (8) and (10) can be reformulated as [3]

$$\left[\mathbf{I}_p(\bar{\boldsymbol{\eta}})^{-1} \right]_{1:2,1:2} = \left[\bar{\mathbf{F}} - \bar{\mathbf{G}}\bar{\mathbf{Z}}^{-1}\bar{\mathbf{G}}^T \right]^{-1}, \quad (14a)$$

$$\left[\mathbf{I}_p(\boldsymbol{\eta})^{-1} \right]_{1:2K+2,1:2K+2} = \left[\underline{\mathbf{F}} - \underline{\mathbf{G}}\underline{\mathbf{Z}}^{-1}\underline{\mathbf{G}}^T \right]^{-1}, \quad (14b)$$

where $\bar{\mathbf{F}} = [\mathbf{I}_p(\bar{\boldsymbol{\eta}})]_{1:2,1:2}$, $\bar{\mathbf{G}} = [\mathbf{I}_p(\bar{\boldsymbol{\eta}})]_{1:2,3:4K+6}$, $\bar{\mathbf{Z}} = [\mathbf{I}_p(\bar{\boldsymbol{\eta}})]_{3:4K+6,3:4K+6}$, $\underline{\mathbf{F}} = [\mathbf{I}_p(\boldsymbol{\eta})]_{1:2K+2,1:2K+2}$, $\underline{\mathbf{G}} = [\mathbf{I}_p(\boldsymbol{\eta})]_{1:2K+2,2K+3:4K+4}$, and $\underline{\mathbf{Z}} = [\mathbf{I}_p(\boldsymbol{\eta})]_{2K+3:4K+4,2K+3:4K+4}$. By introducing auxiliary variables $\bar{\mathbf{U}} \in \mathbb{R}^{2 \times 2}$ and $\underline{\mathbf{U}} \in \mathbb{R}^{(2K+2) \times (2K+2)}$, (13) can be reformulated into an equivalent form as

$$\min_{\mathbf{V}, \bar{\mathbf{U}}, \underline{\mathbf{U}}} \quad \rho \text{tr}(\bar{\mathbf{U}}^{-1}) + (1 - \rho) \text{tr}(\underline{\mathbf{U}}^{-1}) \quad (15a)$$

$$\text{s.t.} \quad \begin{bmatrix} \bar{\mathbf{F}} - \bar{\mathbf{U}} & \bar{\mathbf{G}} \\ \bar{\mathbf{G}}^T & \bar{\mathbf{Z}} \end{bmatrix} \succeq \mathbf{0}, \quad \begin{bmatrix} \underline{\mathbf{F}} - \underline{\mathbf{U}} & \underline{\mathbf{G}} \\ \underline{\mathbf{G}}^T & \underline{\mathbf{Z}} \end{bmatrix} \succeq \mathbf{0}, \quad (15b)$$

$$\bar{\mathbf{U}} \succeq \mathbf{0}, \quad \underline{\mathbf{U}} \succeq \mathbf{0}, \quad (15c)$$

$$\text{tr}(\mathbf{V}) \leq P/M, \quad \mathbf{V} \succeq \mathbf{0}. \quad (15d)$$

The above problem is a convex semi-definite programming (SDP) problem that can be efficiently solved using off-the-shelf optimization tools such as CVX. Once solved, the beamformers \mathbf{W} can be recovered from \mathbf{V} via matrix decomposition or a randomization procedure [14].

B. Weighted-Sum Mismatch Approaches

Inspired by the weighted waveform mismatch minimization approach commonly used in the ISAC literature to balance sensing and communication performance [5], [15], we propose two alternative methods to effectively balance BP and MS by minimizing the weighted-sum mismatch of various metrics. Specifically, the optimal beamformers $\overline{\mathbf{W}}$ for BP and $\underline{\mathbf{W}}$ for MS are obtained by solving (15) for $\rho = 1$ and $\rho = 0$, respectively. The *balanced* beamformers are then derived from these extremes using different strategies to weigh the mismatch: one approach minimizes the weighted-sum mismatch of the beamformers, while the other minimizes the weighted-sum mismatch of the variance matrices.

1) *Weighted-Sum Mismatch of Beamformers*: Upon obtaining $\overline{\mathbf{W}}$ and $\underline{\mathbf{W}}$, we formulate the following optimization problem to minimize the weighted-sum mismatch of beamformers

$$\min_{\mathbf{W}} \rho \|\mathbf{W} - \overline{\mathbf{W}}\|_{\text{F}}^2 + (1 - \rho) \|\mathbf{W} - \underline{\mathbf{W}}\|_{\text{F}}^2 \quad (16a)$$

$$\text{s.t. } \text{tr}(\mathbf{W}\mathbf{W}^{\text{H}}) = P/M, \quad (16b)$$

where (16b) represents the full power transmission constraint. Problem (16) can be equivalently reformulated as

$$\min_{\mathbf{W}} \|\mathbf{A}\mathbf{W} - \mathbf{B}\|_{\text{F}}^2 \quad (17a)$$

$$\text{s.t. } \text{tr}(\mathbf{W}\mathbf{W}^{\text{H}}) = P/M, \quad (17b)$$

where $\mathbf{A} = [\sqrt{\rho}\mathbf{E}_{M_{\text{T}}}; \sqrt{1-\rho}\mathbf{E}_{M_{\text{T}}}]$ and $\mathbf{B} = [\sqrt{\rho}\overline{\mathbf{W}}; \sqrt{1-\rho}\underline{\mathbf{W}}]$.

This problem is non-convex due to the equality constraint in (17b). Notably, $\mathbf{A}^{\text{H}}\mathbf{A} = \mathbf{E}_{M_{\text{T}}}$. By defining $\Xi = \mathbf{A}^{\text{H}}\mathbf{B}$ and $\tilde{\mathbf{W}}_l = \tilde{w}_l \mathbf{w}_l^{\text{H}}$, where $\tilde{w}_l = [1, \mathbf{w}^{\text{T}}]^{\text{T}}$, we can relax the constraint $\text{rank}(\tilde{\mathbf{W}}_l) = 1$. This enables us to lift and reformulate (17) into

$$\min_{\tilde{\mathbf{W}}_l} \sum_{l=1}^L \text{tr} \left(\begin{bmatrix} 0 & -\Xi(:, l)^{\text{H}} \\ -\Xi(:, l) & \mathbf{E}_{M_{\text{T}}} \end{bmatrix} \tilde{\mathbf{W}}_l \right) \quad (18a)$$

$$\text{s.t. } \sum_{l=1}^L \text{tr}(\tilde{\mathbf{W}}_l) = P/M + L, \quad (18b)$$

$$\left[\tilde{\mathbf{W}}_l \right]_{1,1} = 1, \quad \tilde{\mathbf{W}}_l \succeq \mathbf{0}, \quad l = 1, \dots, L. \quad (18c)$$

Problem (18) is a convex SDP problem. Although relaxed, it falls under the category of trust-region subproblems, which exhibit strong duality and are guaranteed to yield rank-one solutions [16]. This implies that the optimal \tilde{w}_l , and thus the optimal w_l for (16), can be recovered from the obtained $\tilde{\mathbf{W}}_l$.

2) *Weighted-Sum Mismatch of Variance Matrices*: We observe that the position-domain FIM, and consequently the CRBs in (8) and (10), are directly determined by $\mathbf{W}\mathbf{W}^{\text{H}}$. This can be viewed as the *variance matrices* of the transmit signal (ignoring scaling) and reflects the accumulated effect of different beamformers. Inspired by this observation, we propose minimizing the weighted sum mismatch of the variance matrices. The beamformers can then be recovered from

the obtained variance matrix. Specifically, the optimization problem can be formulated as

$$\min_{\mathbf{V}} \rho \|\mathbf{V} - \overline{\mathbf{V}}\|_{\text{F}}^2 + (1 - \rho) \|\mathbf{V} - \underline{\mathbf{V}}\|_{\text{F}}^2 \quad (19a)$$

$$\text{s.t. } \text{tr}(\mathbf{V}) = P/M, \quad (19b)$$

$$\mathbf{V} \succeq \mathbf{0}, \quad \text{rank}(\mathbf{V}) = L, \quad (19c)$$

where $\overline{\mathbf{V}} = \overline{\mathbf{W}}\overline{\mathbf{W}}^{\text{H}}$ and $\underline{\mathbf{V}} = \underline{\mathbf{W}}\underline{\mathbf{W}}^{\text{H}}$.

Problem (19) can be efficiently solved using the semi-definite relaxation (SDR) technique. Specifically, by temporarily dropping the rank constraint, (19) becomes an SDP problem, from which the optimal variance matrix \mathbf{V} can be obtained. Subsequently, the beamformers \mathbf{W} are retrieved using a randomization procedure [14].

C. Complexity Analysis

According to [4], the computational complexity of an SDP problem is given by $\mathcal{O}(I^2 \sum_{j=1}^J d_j^2 + I \sum_{j=1}^J d_j^3)$, where I and J represent the numbers of optimization variables and linear matrix inequality (LMI) constraints, respectively, and d_j denotes the row/column size of the matrix associated with the j -th LMI.

- For (15): $I = M_{\text{T}}^2 + (2K + 2)^2 + 4$, $J = 5$, $d_1 = 4K + 6$, $d_2 = 4K + 4$, $d_3 = 2$, $d_4 = 2K + 2$, and $d_5 = M_{\text{T}}$.
- For (18): $I = M_{\text{T}}^2 L$, $J = L$, and $d_j = M_{\text{T}}$.
- For (19): $I = M_{\text{T}}^2$, $J = 1$, and $d_j = M_{\text{T}}$.

When $K \ll M_{\text{T}}$ (e.g., in scenarios with channel sparsity), the computational complexity of solving each problem can be approximated as $\mathcal{O}(M_{\text{T}}^6)$. Note that, although (18) and (19) are subject to roughly the same order of complexity, their actual running times in simulations are usually significantly lower than that of (15).

IV. NUMERICAL RESULTS

A. Scenarios

Unless stated otherwise, the simulation parameters are as follows: The BS is equipped with $M_{\text{T}} = 16$ transmit antennas and $M_{\text{R}} = 16$ colocated receive antennas, positioned at $\mathbf{p}_{\text{B}} = [0 \text{ m}, 0 \text{ m}]^{\text{T}}$. The UE, equipped with $M_{\text{U}} = 16$ antennas, is located at $\mathbf{p}_{\text{U}} = [-5 \text{ m}, 20 \text{ m}]^{\text{T}}$. Additionally, there are $K = 3$ targets, positioned at $\mathbf{p}_1 = [-10 \text{ m}, 15 \text{ m}]^{\text{T}}$, $\mathbf{p}_2 = [5 \text{ m}, 15 \text{ m}]^{\text{T}}$, and $\mathbf{p}_3 = [0 \text{ m}, 17 \text{ m}]^{\text{T}}$, respectively. The transmit power is set to $P = -20$ dBm, with a carrier frequency of $f_c = 28$ GHz and a bandwidth of $W = 120$ MHz. The number of subcarriers is $M = 1024$, the noise figure is $F = 10$ dB, and the noise PSD is $N_0 = -173.855$ dBm/Hz. The system simulates $L = 16$ slots, each with $N = 100$ pilots, and the clock bias is $\Delta t = 1 \mu\text{s}$. The UE's relative orientation is $\phi = (110/180)\pi$. The channel gains are generated using a standard free-space path loss model. For the k -th path, the phases $\bar{\zeta}_k$ (for BP) and $\underline{\zeta}_k$ (for MS) are uniformly distributed over $[-\pi, \pi]$. In BP, the LOS channel gain is $\alpha_0 = e^{j\bar{\zeta}_0} \lambda / (4\pi \|\mathbf{p}_{\text{B}} - \mathbf{p}_{\text{U}}\|)$, while the non-line-of-sight (NLOS) channel gain is

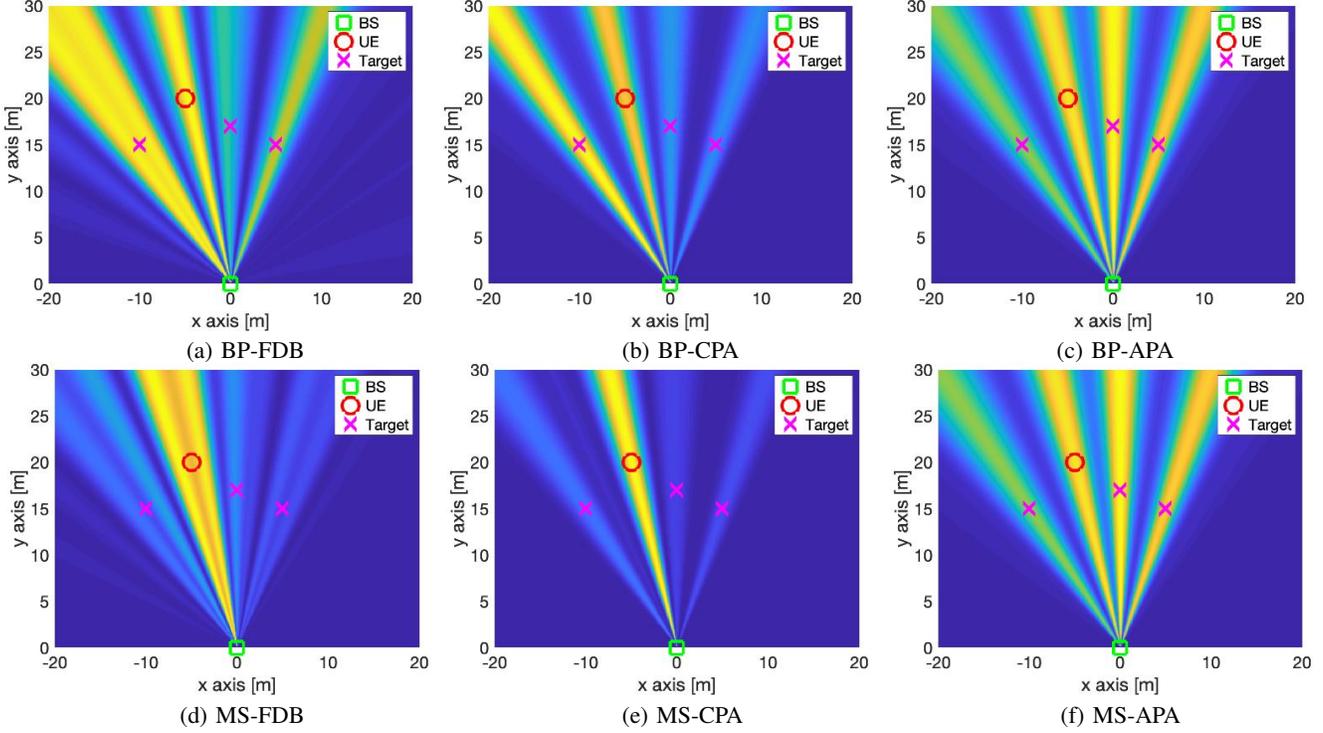


Fig. 2. Beampatterns: (a) BP-FDB; (b) BP-CPA; (c) BP-APA; (d) MS-FDB; (e) MS-CPA; (f) MS-APA.

$\alpha_k = \bar{\sigma}_{\text{RCS}} e^{j\bar{\zeta}_k} \lambda / ((4\pi)^{3/2} \|\mathbf{p}_U - \mathbf{p}_k\| \|\mathbf{p}_k - \mathbf{p}_B\|)$. Here, $\bar{\sigma}_{\text{RCS},k}$ (for BP) and $\underline{\sigma}_{\text{RCS},k}$ (for MS) represent the radar cross section (RCS) of the k -th target while λ is the wavelength. Specifically, $\underline{\sigma}_{\text{RCS},0} = 10 \text{ m}^2$, while $\bar{\sigma}_{\text{RCS},k} = \underline{\sigma}_{\text{RCS},k} = 100 \text{ m}^2$ ($k = 1, \dots, K$).

B. Compared Schemes

We characterize the performance tradeoff boundary between BP and MS through three categories of approaches, which are detailed in the following.

1) *FDB*: The first category focuses on solving full-dimensional beamforming (FDB) optimization problems, represented by (12), (16), and (19). These are further categorized as FDB-weighted CRB (FDB-WCRB), FDB-weighted beamformer (FDB-WBF), and FDB-weighted variance matrix (FDB-WVM), respectively.

2) *CPA*: As a low-complexity alternative, we introduce a codebook-based power allocation (CPA) approach. The core idea is that the optimal variance matrix minimizing the CRB can be expressed as $\mathbf{V} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, where $\mathbf{\Lambda} \in \mathbb{C}^{(2K+2) \times (2K+2)}$ and $\mathbf{U} = [\mathbf{a}_T(\theta_{B,0}), \dots, \mathbf{a}_T(\theta_{B,K}), \dot{\mathbf{a}}_T(\theta_{B,0}), \dots, \dot{\mathbf{a}}_T(\theta_{B,K})] \in \mathbb{C}^{M_T \times (2K+2)}$, with $\partial \mathbf{a}_T(\theta_{B,k}) / \partial \theta$ [4], [12]. Moreover, by restricting $\mathbf{\Lambda}$ to be diagonal, this approach simplifies to a lower-dimensional, low-complexity power allocation problem over the predetermined codebook \mathbf{U} . In other words, the optimization problem in (12) can be reformulated as a low-complexity power allocation task, and the solutions for (16) and (19) can be derived using the variance matrix

obtained from solving (12). These methods are further categorized as CPA weighted CRB (CPA-WCRB), CPA weighted beamformer (CPA-WBF), and CPA weighted variance matrix (CPA-WVM), respectively.

3) *APA*: Lastly, a simple baseline is the average power allocation (APA) across the given codebook \mathbf{U} , where no distinction or tradeoff is made between BP and MS.

C. Results and Discussion

1) *Beampatterns*: In Fig. 2, we show the optimized beam patterns. The top row illustrates the case of BP ($\rho = 1$), while the bottom row corresponds to MS ($\rho = 0$). For each row, the results for the FDB, CPA, and APA schemes are displayed from left to right, respectively. We observe that under FDB and CPA, the beam patterns exhibit distinct characteristics between BP and MS, whereas under APA, there is no difference. This is because adaptive design (tailored for different ρ) is applied in the FDB and CPA schemes but not the APA case.

Focusing on the cases under the FDB and CPA schemes, we observe that for BP, strong beams are directed toward the UE in both schemes, while the power allocated to beams directed at the targets varies. This is because the BS acts as an essential anchor in BP, whereas the targets serve as auxiliary anchors, with their positions being estimated simultaneously. Depending on the relative locations, different amounts of information are provided regarding the UE's position. Therefore, beams with varying power levels are allocated to illuminate the targets, maximizing the UE's positioning accuracy. In MS, all targets and the UE need to be accurately positioned for optimal

performance. Since the UE has the smallest RCS, stronger beams are directed toward it under both the FDB and CPA schemes to ensure balanced positioning performance across all targets.

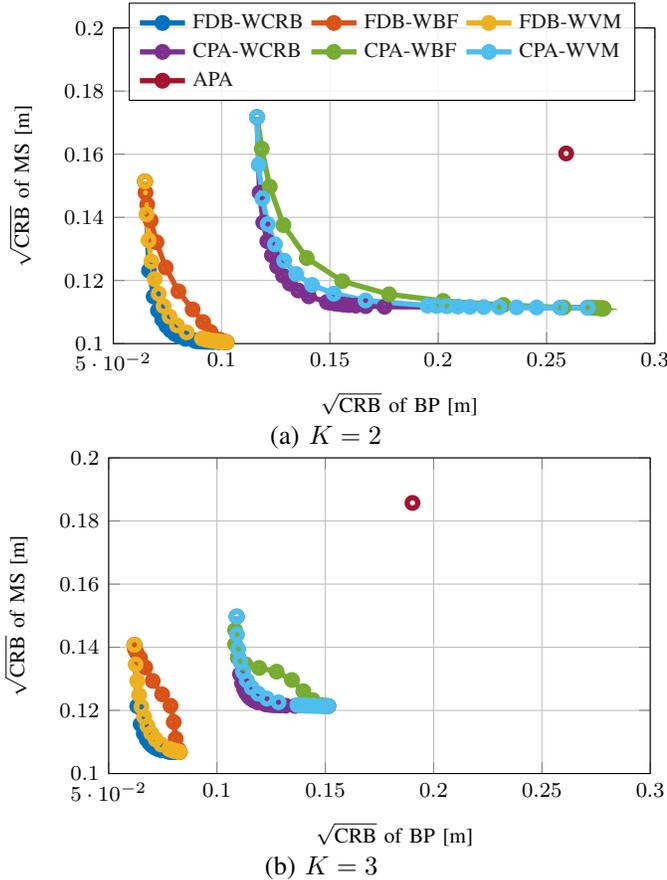


Fig. 3. Tradeoff (in terms of square root of CRB) between BP and MS: (a) $K = 2$ (Targets 1 and 2); (b) $K = 3$ (Targets 1, 2, and 3).

2) *Tradeoff between BP and MS*: In Fig. 3, the performance tradeoff between BP and MS is evaluated and compared across different schemes. Specifically, the results are examined for $K = 2$ and $K = 3$ targets. We observe that, except for the non-adaptive APA scheme, all curves exhibit the fundamental tradeoff between BP and MS. Additionally, compared to CPA, the FDB schemes deliver significantly better performance in terms of the bistatic-monostatic performance tradeoff, owing to a higher degree of freedom in optimization. Furthermore, compared to the weighted-sum mismatch approaches (FDB-WBF, FDB-WVM, CPA-WBF, and CPA-WVM), the weighted-sum CRB approaches (FDB-WCRB and CPA-WCRB) achieve the best bistatic-monostatic performance tradeoff, as they yield weak Pareto boundaries. It is also worth noting that, compared to schemes relying on WBF, those relying on WVM consistently achieve a better bistatic-monostatic performance tradeoff, with performance very close to the weak Pareto boundary. Moreover, the performance gap between WBF and WVM becomes more pronounced in scenarios with more targets. The underlying insight is that approximating the variance matrix is more direct and effective

in preserving the desired spatial characteristics of the transmit signal compared to approximating the beamformers, as the elements in the FIM are directly determined by the variance matrix.

V. CONCLUSION

In this paper, we characterized the tradeoff between BP and MS in a MIMO-OFDM system. We derived the CRBs for both paradigms and formulated a MOO problem. A weighted-sum CRB approach was proposed, ensuring the weak Pareto boundary. Additionally, we introduced and solved two mismatch-minimizing criteria based on beamformer mismatch and variance matrix mismatch. Numerical results demonstrated the performance tradeoff between the two paradigms and highlighted the superiority of the weighted-sum variance matrix mismatch approach, emphasizing the importance of balancing BP and MS in future ISAC systems.

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