

Comments and Corrections

Corrections to “Energy-Efficient Wheel Torque Distribution for Heavy Electric Vehicles With Adaptive Model Predictive Control and Control Allocation”

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Following the publication of our paper [1], several typographical and minor transcription errors were identified. While these errors do not alter the validity of the reported results or the conclusions drawn, they may lead to ambiguity in the interpretation and reproduction of the proposed methods. The corrections presented below are intended to improve clarity and maintain the accuracy of the published work.

CORRECTIONS TO SECTION I

- 1) In the last paragraph describing the organisation of the paper there is a missing description of system overview and vehicle configuration presented in Section II.

CORRECTIONS TO SECTION II

- 1) The steering wheel angle request from driver model, as specified in driver interpreter description, should be denoted using δ_{SWA} instead of δ_f which represents the steering angle at the wheel and is consistent with (2) and Fig. 1 of [1].

CORRECTIONS TO SECTION III

A. GLOBAL FORCE REFERENCE GENERATOR

1. VEHICLE MODEL USED BY THE MODEL PREDICTIVE CONTROLLER

- 1) The notation used in the continuous-time state equation (5) may lead to ambiguity by evaluating the state dynamics at the sampling instant t_s . The correct equation is

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}_{mpc}(t)). \quad (5)$$

The sampling instant t_s should only be used to denote the current operating point for model linearization and initialization of the MPC prediction, e.g., $\mathbf{x}(t_s) = \mathbf{x}_{init}$, while the continuous-time vehicle dynamics are defined with respect to the continuous-time variable t .

- 2) *Clarification of longitudinal state definition:* In the originally published paper [1], the state vector was defined as

$$\mathbf{x} = [\dot{v}_x \ v_x \ \omega_z \ \beta]^T$$

where $\dot{v}_x$ is introduced as an augmented state to enable acceleration-based requests within the discrete-time MPC formulation. This should not be interpreted as introducing an additional independent physical state of the vehicle, as the underlying longitudinal dynamics remain governed by the force balance in (9). Equation (10) is used solely within the discretized prediction model to propagate the augmented acceleration state via a finite-difference approximation over the MPC sampling time, and does not modify the continuous-time vehicle model. This clarification concerns only the interpretation of the control-oriented formulation and does not affect the implementation, optimization problem, or results of the paper.

- 3) *Clarification of tire model linearization:* The tire model used in the MPC prediction model is linear in formulation to preserve convexity of the optimization problem. However, the corresponding cornering stiffness is updated online at each time step t_s based on the nonlinear tire characteristic in (6). Specifically, the nonlinear tire force model

$$F_{y,i} = \mu F_{z,i} \tanh\left(\frac{C_i \alpha_i}{\mu F_{z,i}}\right)$$

is linearized around the current operating point $\alpha_i(t_s)$. The effective cornering stiffness used in the MPC model is obtained from the first-order derivative of (6) as

$$\tilde{C}_i = \left. \frac{\partial F_{y,i}}{\partial \alpha_i} \right|_{\alpha_i(t_s)}$$

This results in a linear time-varying tire representation within the MPC formulation, where the stiffness parameters are updated at each sampling instant while maintaining a linear prediction model for optimization.

- 4) *Clarification of vehicle state dynamics formulation:* The state dynamics formulation of the vehicle model is obtained from the longitudinal force balance, lateral force balance, and yaw moment balance equations. Specifically, the longitudinal acceleration is derived from the longitudinal force equilibrium, while the body side-slip and yaw-rate dynamics are obtained from the lateral force equilibrium and yaw moment equilibrium, respectively and these are detailed in [2]. The resulting equations are expressed in (7)–(10) using the localized cornering stiffness parameters obtained from the linearization of the non-linear tire model. These equations form the basis of the state-space representation used in the MPC prediction model.
- 5) In (9), the term v_y represents the lateral velocity estimated from the vehicle plant model and is not an independent state included in the MPC prediction model. The lateral velocity is related to the body side-slip angle through $v_y = v_x \beta$; however, the estimated plant value is used in the longitudinal force balance to account for the coupling effect between lateral and longitudinal vehicle motion. Therefore, v_y is treated as an available parameter updated at each MPC iteration and held constant over the prediction horizon, similar to other operating-point quantities.
- 6) In (9), the rolling resistance term was denoted as $C_r F_z/m$, which may lead to ambiguity due to the use of C_r for tire cornering stiffness in the vehicle model. The rolling resistance coefficient should be denoted by c_r as presented in Table 2, consistent with the definition of rolling resistance force:

$$F_{RR} = c_r F_z$$

This correction is only due to a typographical error and does not affect the vehicle model or the results presented.

- 7) A typographical error is present in the description of the model simplifications. The sentence “The control-oriented model is further simplified **not** by two assumptions” should be corrected to “The control-oriented model is further simplified by two assumptions.” The two assumptions are: (i) longitudinal and lateral tire forces are decoupled, and (ii) wheel dynamics are neglected.
- 8) The lateral tire force expression in (11) should be clarified as an estimated quantity obtained from vehicle feedback under the assumption of steady-state lateral conditions. The corrected equation (11) is:

$$F_{y,i,est} = F_{z,i}(t_s) \frac{a_y(t_s)}{g}$$

The estimated lateral tire forces are calculated using the estimated axle loads and measured lateral acceleration at the current sampling instant and are assumed constant over the MPC prediction horizon based on the steady-state assumption.

- 9) Consequently, the combined tire force constraint in (13) should use the corrected version of the estimated lateral tire force:

$$\mu F_{z,i} = \sqrt{F_{x,i}^2 + F_{y,i,est}^2}$$

- 10) The longitudinal tire force limit in (14) should therefore be expressed as:

$$F_{x,lim,tyre,i} = n_{conf} \sqrt{(\mu F_{z,i}(t_s))^2 - F_{y,i,est}^2}$$

2. ACTUATOR AND TYRE FORCE OPERATING LIMITS

1) *Correction of actuator torque formulation and constraint domains:* In Section III-A.2 of the originally published paper, variable definition inconsistencies were identified in the actuator torque constraint formulation in (16) and (17) contained a notation error. The expressions for $T_{lim,max,EMi}$ and $T_{lim,min,EMi}$ incorrectly combined the physical actuator capability with the dynamically achievable torque, which could imply that the actuator limits vary with the requested torque.

The actuator torque limits should instead represent the physical capability of the actuator and must be separated from the dynamically achievable torque response. The electric machine dynamics are represented using a first-order actuator lag:

$$\dot{T}_{EMi} = \frac{T_{req,EMi} - T_{EMi}}{\tau_{EMi}} \quad (1)$$

The dynamically achievable electric machine torque is approximated as:

$$T_{EMi}^{dyn} = T_{req,EMi}(t_s) - \tau_{EMi} \frac{dT_{req,EMi}(t_s)}{dt}$$

The achievable electric machine torque is obtained by applying saturation based on the physical actuator torque limit:

$$T_{EMi} = \max \left(-T_{lim,EMi}, \min \left(T_{lim,EMi}, T_{EMi}^{dyn} \right) \right)$$

Similarly, the friction brake actuator dynamics are described by:

$$\dot{T}_{Brki} = \frac{T_{req,Brki} - T_{Brki}}{\tau_{Brki}} \quad (2)$$

and the achievable braking torque is obtained as:

$$T_{Brki} = \max \left(-T_{lim,Brki}, \min \left(0, T_{Brki}^{dyn} \right) \right)$$

where T_{Brki}^{dyn} is calculated using the same first-order lag approximation.

This correction replaces the previously defined actuator torque limit expressions in (16) and (17). The correction only affects the representation of the actuator constraints and does not affect the optimization formulation, controller implementation, or reported results.

Additionally, the original paper did not explicitly state the domains in which the available actuator torque limits are defined. The torque limits of the actuators are defined in different domains depending on the purpose of the constraint. The electric machine torque limits are initially defined at the actuator (motor shaft) side, whereas the brake limits, tyre force constraints and the MPC input constraints are formulated in the wheel torque domain. Therefore, the electric machine torque limits are transformed into the equivalent wheel torque domain using the drivetrain gear ratio g_i :

$$T_{max,EMi} = g_i T_{lim,EMi}$$

$$T_{min,EMi} = -g_i T_{lim,EMi}$$

The friction brake torque limits are directly defined at the wheel side. Since friction brakes can only generate braking torque, the

combined actuator torque limits are:

$$\begin{aligned} T_{\min,acti} &= T_{\min,EMi} - T_{\lim,Brki} \\ T_{\max,acti} &= T_{\max,EMi} \end{aligned}$$

The admissible wheel torque range used in the MPC formulation is obtained by combining actuator and tyre force constraints:

$$\begin{aligned} T_{\max,i} &= \min(T_{\lim,tyrei}, T_{\max,acti}) \\ T_{\min,i} &= \max(-T_{\lim,tyrei}, T_{\min,acti}) \end{aligned}$$

The MPC input constraint vector is therefore defined in the wheel torque domain as:

$$\mathbf{u}_{l,mpc} \leq \mathbf{u}_{mpc} \leq \mathbf{u}_{u,mpc} \quad (3)$$

where:

$$\begin{aligned} \mathbf{u}_{u,mpc} &= [T_{\max,fl} \cdots T_{\max,rr}]^T \\ \mathbf{u}_{l,mpc} &= [T_{\min,fl} \cdots T_{\min,rr}]^T \end{aligned}$$

2) *Clarification of control allocation actuator constraints:* Unlike the MPC formulation, the control allocation problem considers the instantaneous achievable actuator torque limits, including actuator dynamic limitations [2]. The actuator rate constraints are calculated from the first-order actuator models in (1) and (2) defined in this correction paper.

Using forward Euler discretization with controller sampling time T_s , the achievable torque variation within one sampling interval is given by:

$$\begin{aligned} \Delta T_{\text{req},EMi}^+ &= \frac{T_s}{\tau_{EMi}} (T_{\lim,EMi} - T_{\text{req},EMi}) \\ \Delta T_{\text{req},EMi}^- &= \frac{T_s}{\tau_{EMi}} (-T_{\lim,EMi} - T_{\text{req},EMi}) \\ \Delta T_{\text{req},Brki}^+ &= \frac{T_s}{\tau_{Brki}} (-T_{\text{req},Brki}) \\ \Delta T_{\text{req},Brki}^- &= \frac{T_s}{\tau_{Brki}} (-T_{\lim,Brki} - T_{\text{req},Brki}) \end{aligned}$$

The final allocation constraints combine the static actuator limits and actuator dynamic response. The tyre force constraints are enforced at the MPC level through the wheel torque constraints. During allocation, the demanded wheel torque is redistributed between the electric machines and friction brakes while satisfying actuator limitations and the torque demand generated by the MPC. The individual actuator limits are defined as:

$$\begin{aligned} T_{\max,Brki}^{CA} &= \min(0, T_{\text{req},Brki} + \Delta T_{\text{req},Brki}^+) \\ T_{\min,Brki}^{CA} &= \max(-T_{\lim,Brki}, T_{\text{req},Brki} + \Delta T_{\text{req},Brki}^-) \\ T_{\max,EMi}^{CA} &= \min\left(T_{\lim,EMi}, T_{\text{req},EMi} + \Delta T_{\text{req},EMi}^+, \frac{T_{\max,tyrei}}{g_i}\right) \\ T_{\min,EMi}^{CA} &= \max\left(-T_{\lim,EMi}, T_{\text{req},EMi} + \Delta T_{\text{req},EMi}^-, \frac{T_{\min,tyrei}}{g_i}\right) \end{aligned}$$

$$\mathbf{u}_u = \begin{bmatrix} T_{\max,EMfl}^{CA} \\ \vdots \\ T_{\max,EMrr}^{CA} \\ T_{\max,Brkfl}^{CA} \\ \vdots \\ T_{\max,Brkrr}^{CA} \end{bmatrix}, \quad \mathbf{u}_l = \begin{bmatrix} T_{\min,EMfl}^{CA} \\ \vdots \\ T_{\min,EMrr}^{CA} \\ T_{\min,Brkfl}^{CA} \\ \vdots \\ T_{\min,Brkrr}^{CA} \end{bmatrix}$$

The resulting allocation constraints are:

$$\mathbf{u}_l \leq \mathbf{u} \leq \mathbf{u}_u \quad (4)$$

This correction distinguishes between actuator-side torque limits used for actuator dynamics and control allocation, and wheel-side torque limits used in MPC input constraints. The correction only modifies the presentation of the constraint formulation and does not alter the implemented controller or the reported results.

3. SAFE OPERATING LIMITS OF VEHICLE MOTION

1) *Clarification of Yaw-Rate State Limits:* The yaw-rate constraint presented in (18) is derived from the lateral acceleration limit $|a_y| \leq \mu g$, using the relationship $a_y = v_x \omega_z$. Therefore, the resulting yaw-rate bound is valid for vehicle operation at non-zero longitudinal velocities. The formulation does not include an explicit low-speed regularization of v_x ; hence, the constraint should be interpreted within the intended operating region where v_x remains above zero. This clarification concerns the applicability of the state constraint formulation and does not affect the reported results, which are obtained within the intended operating range.

2) *Clarification of Side-Slip State Limits:* The maximum tyre slip angles used in the state constraint formulation of (19)-(20) were derived using the linear approximation of the tyre force relationship. Although the MPC prediction model employs a nonlinear tyre force representation with local linearization of the cornering stiffness, the side-slip limits were retained from the linear tyre model formulation for computational simplicity. These limits are used only for defining the safe operating region and do not modify the nonlinear tyre force model used within the MPC prediction. This clarification concerns the interpretation of the state constraint formulation and does not affect the reported results.

4. MPC OPTIMIZATION PROBLEM FORMULATION

1) *Clarification of reference prediction notation:* The notation used for the future yaw-rate reference prediction in (22) was ambiguous. The expression $\omega_{z,\text{ref}}(kT_{s,\text{mpc}})$ should be interpreted as the predicted reference at a future MPC prediction step relative to the current simulation time t_s . The corrected expression is:

$$\omega_{z,\text{ref}}(t_s + kT_{s,\text{mpc}}) = \omega_{z,\text{ref}}(t_s) + \dot{\omega}_{z,\text{ref}}(t_s)kT_{s,\text{mpc}}$$

Here, t_s denotes the current simulation sampling instant, $T_{s,\text{mpc}}$ denotes the MPC update interval, and k denotes the prediction step within the MPC horizon. This correction only clarifies the time reference used for the Taylor expansion and does not affect the implementation, optimization formulation, or results presented in the paper.

2) *Correction of MPC optimization formulation:* Several notation inconsistencies were identified in (23).

- First, the quadratic cost terms were incorrectly written as $\mathbf{e}_x(k)Q(k)\mathbf{e}_x^T(k)$ and $\mathbf{u}_{mpc}(k)R(k)\mathbf{u}_{mpc}^T(k)$. These terms should

be expressed as the dimensionally consistent quadratic forms $\mathbf{e}_x^T(k)Q(k)\mathbf{e}_x(k)$ and $\mathbf{u}_{mpc}^T(k)R(k)\mathbf{u}_{mpc}(k)$, respectively.

- Second, the state constraint was incorrectly written as $\mathbf{r}_{\min}(k) \leq \mathbf{r}_{\text{ref}}(k) \leq \mathbf{r}_{\max}(k)$. The constraint should instead be applied to the predicted vehicle states as $\mathbf{r}_{\min}(k) \leq \mathbf{x}(k) \leq \mathbf{r}_{\max}(k)$.
- Third, N denotes the prediction horizon in seconds; therefore, the number of discrete prediction steps is given by $N/T_{s,\text{mpc}}$. Accordingly, the summation limits and prediction constraints are expressed using $N/T_{s,\text{mpc}}$.
- Fourth, the MPC input constraint vector representing actuator and tyre force limits were incorrectly represented. The constraint instead should be defined as in (3), of this paper, in the wheel torque domain.

With these corrections, the MPC optimization problem in (23) is formulated as:

$$\mathbf{u}_{mpc}^* = \arg \min_{\mathbf{u}_{mpc}} \left(\sum_{k=1}^{\frac{N}{T_{s,\text{mpc}}}} \mathbf{e}_x^T(k)Q(k)\mathbf{e}_x(k) + \sum_{k=1}^{\frac{N}{T_{s,\text{mpc}}}-1} \mathbf{u}_{mpc}^T(k)R(k)\mathbf{u}_{mpc}(k) \right),$$

subject to

$$\mathbf{x}(k+1) = f_d(\mathbf{x}(k), \mathbf{u}(k)), \quad \mathbf{x}(0) = \mathbf{x}_{\text{init}}$$

$$\mathbf{u}_{l,\text{mpc}}(k) \leq \mathbf{u}_{mpc}(k) \leq \mathbf{u}_{u,\text{mpc}}(k), \quad k = 1, \dots, \frac{N}{T_{s,\text{mpc}}} - 1,$$

$$\mathbf{r}_{\min}(k) \leq \mathbf{x}(k) \leq \mathbf{r}_{\max}(k), \quad k = 1, \dots, \frac{N}{T_{s,\text{mpc}}}$$

where f_d is the discretized vehicle model, and $Q(k)$ and $R(k)$ are the state and control weighting matrices, respectively. These corrections only clarify the mathematical notation and do not affect the implementation, optimization procedure, or reported results.

5. OPERATING CONDITION-BASED TUNING OF CONTROL OBJECTIVES

1) *Clarification of adaptive MPC weight formulation:* The adaptive weighting formulation in (26)–(29) requires additional clarification. The normalized adaptive weights w_{sc,ω_z} and $w_{sc,\beta}$ are defined assuming symmetric state limits with $\omega_{z,\min} < 0$, $\omega_{z,\max} > 0$, $\beta_{\min} < 0$, and $\beta_{\max} > 0$, ensuring that the normalized weights remain within the range [0,1].

Equation (28) defines the longitudinal tracking weight as $w_{sc,a_x} = w_{sc,v_x} = 1 - \max(w_{sc,\beta}, w_{sc,\omega_z})$, which intentionally reduces longitudinal tracking priority as the lateral stability limits are approached. Furthermore, the weighting matrix in (29) is a normalized scaling of the MPC state costs and is not intended as a dimensionally homogeneous physical cost function.

B. CONTROL ALLOCATOR

1. POWER LOSS MINIMIZATION

1) *Clarification of power loss objective and allocation constraint formulation:* In the originally published paper, minor inconsistencies were present in the notation of the control allocation optimization problem. These inconsistencies are related to the objective function

notation, the definition of the optimization variables, and the description of the associated constraint vectors.

First, the objective function was written using an inconsistent summation index. The corrected power loss minimization problem is expressed as:

$$\mathbf{u}^* = \arg \min_{\mathbf{u}} \left(\sum_{i=1}^n P_{\text{loss,EM}i} + \sum_{i=1}^n P_{\text{loss,Br}ki} \right)$$

where the optimization variable $\mathbf{u}$ represents the individual actuator torque commands.

The original notation $T_{\text{Br}ki}$ was used inconsistently, as it did not clearly distinguish between the actuator torque state, the requested torque, and the torque determined by the allocation optimization. The corrected notation ensures consistency between the optimization variables and the power loss model. The friction brake loss model is evaluated using the allocated brake torque $T_{\text{req,Br}k,i}$, which follows the negative brake torque convention ($T_{\text{req,Br}k,i} \leq 0$), together with positive wheel speed ($\omega_{\text{whl},i} > 0$). Therefore, the brake power loss is given by:

$$P_{\text{loss,Br}ki} = -T_{\text{req,Br}ki} \cdot \omega_{\text{whl},i}$$

Furthermore, the description of the allocation constraints was not consistent with the hierarchical structure of the controller. The torque limits $\mathbf{u}_l$ and $\mathbf{u}_u$ used in the control allocation problem are not the same constraints as the MPC input constraints. The MPC operates in the wheel torque domain, whereas the control allocation problem operates in the actuator torque domain. Therefore, the updated constraints defined in (4) of this paper incorporate the instantaneously achievable actuator torque limits, which account for both actuator dynamic limitations and tyre force constraints transformed into the actuator torque domain.

The corrected allocation optimization formulation is therefore:

$$\mathbf{u}^* = \arg \min_{\mathbf{u}} \frac{1}{2} \mathbf{u}^T \mathbf{H} \mathbf{u} + \mathbf{g}^T \mathbf{u}$$

$$\text{s.t. } \mathbf{B} \mathbf{u} = \mathbf{v}_{\text{req}}$$

$$\mathbf{u}_l \leq \mathbf{u} \leq \mathbf{u}_u$$

These corrections only clarify the mathematical notation and the distinction between wheel-level MPC constraints and actuator-level allocation constraints. The implemented controller structure and the reported results remain unchanged.

2) The allocation matrix $\mathbf{B}$ in (32) contains a notation error in the yaw-moment terms corresponding to the rear electric machines and rear friction brakes. The rear axle track width was incorrectly denoted by t_f ; these terms should instead use the rear track width t_r . The corrected allocation matrix is provided below:

$$\mathbf{B} = 1/r_w \cdot \begin{bmatrix} g r_{\text{crs}} & -\frac{g r_{\text{crs}} t_f}{2} \\ g r_{\text{crs}} & \frac{g r_{\text{crs}} t_f}{2} \\ g r_{\text{stb}} & -\frac{g r_{\text{stb}} t_r}{2} \\ g r_{\text{stb}} & \frac{g r_{\text{stb}} t_r}{2} \\ 1 & -\frac{t_f}{2} \\ 1 & \frac{t_f}{2} \\ 1 & -\frac{t_r}{2} \\ 1 & \frac{t_r}{2} \end{bmatrix}^T$$

This correction is limited to the mathematical notation of the allocation matrix and does not affect the implemented controller.

AUTHORS' DISCLAIMER

The authors used generative artificial intelligence (AI) tools, such as ChatGPT, for grammar and language refinement only. The authors retain full responsibility for the content, scientific contributions, and references of this work. The use of AI tools was performed in accordance with ethical guidelines to ensure the originality and accuracy of the manuscript.

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