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We analyze the precision of generic currents in a multiterminal quantum-transport setting. Employing scattering theory, we show that the precision of such currents is limited by a function of the particle-current noise that can be interpreted as the activity in the classical limit. We thereby establish a kinetic uncertainty relation for quantum transport. In the full quantum limit, we find precision bounds with modified activity constraints depending on whether the system is fermionic or bosonic. We expect these bounds to be suitable as guidelines for any transport process aiming at high precision.

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Precision—namely the ratio between the square of average currents and their fluctuations—plays a key role in the performance of small-scale devices, e.g., when acting as thermodynamic machines. While in equilibrium [1,2] or in specific nonequilibrium situations [3,4] fluctuation-dissipation theorems directly relate average quantities of interest to their fluctuations, such relations are more challenging to find under general nonequilibrium conditions [5–7]. Based on fluctuation relations [8–11], which hold even far from equilibrium but are often limited to classical Markovian or weak-coupling scenarios, a number of inequalities have been developed setting constraints on the achievable precision [12,13]. These fundamental bounds include the thermodynamic uncertainty relation (TUR), constraining precision by the entropy production [14,15] and being most predictive close to equilibrium, and the kinetic uncertainty relation (KUR), constraining precision by the dynamical activity [16–18] and being most predictive far from equilibrium, as well as combinations of those [19,20]. Recently, there have been extensive efforts to extend the TUR [21] as well as the KUR [22–27] to the quantum regime, mostly exploiting Lindblad jump operators for the weak coupling limit.

However, especially when dealing with energy-converting devices, where the goal is to generate currents of significant magnitude, strongly coupled systems under nonequilibrium conditions are of interest. In this strong-coupling limit, which has been studied in TURs [28–30], but also in recently developed fluctuation-dissipation bounds [31], quantum statistics can play an important role. Kinetic uncertainty relations for quantum transport [32], valid for strong coupling, are missing.

This is the gap that we fill in this Letter. We analyze stationary quantum transport in a generic multiterminal coherent setup (see Fig. 1), where the contacts are described either by fermionic or bosonic, possibly nonthermal, distributions. For a large set of transport currents, such as particle, energy, or entropy currents that are also relevant in the context of thermodynamics, we show that the precision is limited by the particle-current fluctuations. In the weak-transmission limit—corresponding to negligible quantum correlations—the particle-current fluctuations can be interpreted as a *local activity*. Thus, our findings represent a quantum-transport version of the KUR. However, the presence of quantum correlations impacts bosonic and fermionic systems strongly and very differently due to bunching or antibunching. For this

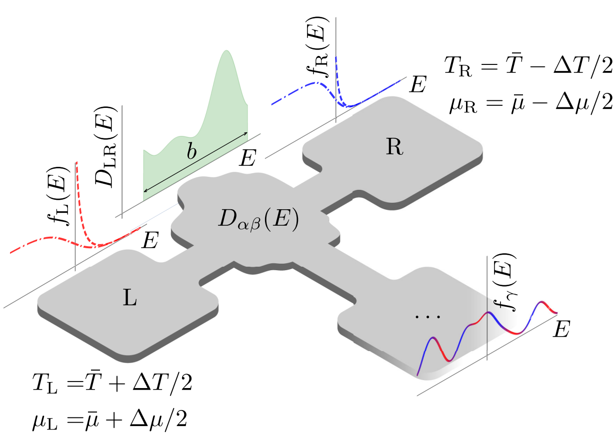

FIG. 1. Sketch of a multiterminal setup with contacts $\alpha = \mathrm{L}, \mathrm{R}, \ldots$, described by thermal fermionic (dashed-dotted lines in examples) or bosonic (dashed lines) distributions, or even nonthermal distributions (f_γ, full line). The central scattering region is characterized by transmission probabilities $D_{\alpha\beta}(E)$ (green area) with width b in energy.

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general case where both classical and quantum fluctuations are present, we establish precision bounds, where the activity is replaced by a combination of measurable transport quantities. We demonstrate our findings at the experimentally relevant examples of bosonic or fermionic two-terminal systems and show that the discovered bounds can indeed be tight far from equilibrium.

For our analysis of quantum transport in this generic multiterminal setting, we resort to scattering theory [33,34], which is valid as long as the scattering region can effectively be described by a Hamiltonian that is quadratic in the field operators. This allows us to model currents and noise in any transport setting with interactions treated up to the mean-field level [35–37], including the possibility to model inelastic effects via Büttiker probes [38]. The setup shown in Fig. 1 is hence characterized by arbitrary transmission probabilities $D_{\alpha\beta}(E)$ between any contacts α, β; no time-reversal symmetry is required. Contacts are either described by standard Bose or Fermi functions, or even by nonthermal distributions, meaning that they need not be described by a temperature or an (electro)chemical potential. Indeed, the only constraint on these distributions is that they fulfill $0 \leq f_\alpha(E)$ for bosons and $0 \leq f_\alpha(E) \leq 1$ for fermions, imposed by exchange statistics. With these ingredients, we write the currents of interest into contact α as

$$I_\alpha^{(\nu)} = \frac{1}{h}\int_0^\infty dE x_\alpha^{(\nu)} \sum_\beta D_{\alpha\beta}(E)[f_\beta(E) - f_\alpha(E)]. \quad (1)$$

Here, the superscript ν indicates the type of current. The real-valued variable $x_\alpha^{(\nu)}$ for particle currents, $\nu = N$, is $x_\alpha^{(N)} = 1$; for energy currents, $\nu = E$, it is $x_\alpha^{(E)} = E$; and for entropy currents [39,40], $\nu = \Sigma$, it is $x_\alpha^{(\Sigma)} = k_B \log[(1 \pm f_\alpha(E))/f_\alpha(E)]$. From now on, the upper and lower signs refer to bosonic and fermionic systems, respectively. We are particularly interested in the precision of these currents. We therefore evaluate their fluctuations, focusing on the zero-frequency autocorrelator $S_{\alpha\alpha}^{(\nu)} = \int dt \langle \delta\hat{I}_\alpha^{(\nu)}(t)\delta\hat{I}_\alpha^{(\nu)}(0)\rangle$, with $\delta\hat{I}_\alpha^{(\nu)}(t) = \hat{I}_\alpha^{(\nu)}(t) - I_\alpha^{(\nu)}$ and $I_\alpha^{(\nu)} = \langle \hat{I}_\alpha^{(\nu)}(t)\rangle$. See Ref. [40] for a detailed treatment of the entropy-current fluctuations. Most previous KUR studies deal with classical processes, and for comparison it is therefore helpful to divide the full fluctuations into a "classical" and a "quantum" contribution, $S_{\alpha\alpha}^{(\nu)} = S_{\alpha\alpha}^{(\nu)\mathrm{cl}} + S_{\alpha\alpha}^{(\nu)\mathrm{qu}}$. The classical contribution

$$S_{\alpha\alpha}^{(\nu)\mathrm{cl}} = \frac{1}{h}\int dE[x_\alpha^{(\nu)}(E)]^2 \left\{\sum_{\beta\neq\alpha} D_{\alpha\beta}(F_{\alpha\beta}^\pm + F_{\beta\alpha}^\pm)\right\} \quad (2a)$$

is linear in the transmission probabilities, and we defined $F_{\alpha\beta}^\pm \equiv f_\alpha(1 \pm f_\beta)$. While quantum properties can enter $D_{\alpha\beta}(E)$ via *interference* effects, the classical part of the fluctuations only contains "single-particle" effects due to the linear dependence on $D_{\alpha\beta}(E)$. Quantum *correlations* are, in contrast, included in the quantum contribution

$$S_{\alpha\alpha}^{(\nu)\mathrm{qu}} = \pm\frac{1}{h}\int dE[x_\alpha^{(\nu)}(E)]^2 \left\{\sum_{\beta\neq\alpha} D_{\alpha\beta}(f_\alpha - f_\beta)\right\}^2, \quad (2b)$$

which is quadratic in the transmission probabilities. The latter is purely due to nonequilibrium; i.e., it vanishes for $f_\alpha = f_\beta$. Importantly, it has opposite signs for bosonic and fermionic systems, meaning that it reduces the total noise for fermions while increasing it for bosons. We emphasize that while the splitting into classical and quantum fluctuations is useful from a mathematical standpoint, in experiments, the full fluctuations are measured.

We start by analyzing the classical noise alone, which is the dominant contribution in the limit of weak transmissions $D_{\alpha\beta}(E) \ll 1$ or for small biases, namely, where $|f_\alpha - f_\beta|$ is small. In order to establish bounds on this classical noise, we use the inequalities

$$x^2 + \frac{1}{4} \geq |x|, \quad (3a)$$

$$F_{\alpha\beta}^\pm(E) + F_{\beta\alpha}^\pm(E) \geq |f_\alpha(E) - f_\beta(E)|, \quad (3b)$$

which are valid for an arbitrary real number x [(3a)] and for bosonic as well as for fermionic arbitrary (possibly nonthermal) distribution $f_\alpha(E)$ and $F_{\alpha\beta}^\pm(E)$ [(3b)]. Using inequalities (3), we find [40,41] the semidefinite positive quadratic form $S_{\alpha\alpha}^{(\nu)\mathrm{cl}} + (y^2/4)S_{\alpha\alpha}^{(N)\mathrm{cl}} - y|I_\alpha^{(\nu)}| \geq 0$, relating noise and current, where y has units of $x_\alpha^{(\nu)}$ and takes on positive, real values. The optimal value of y that minimizes the quadratic form is $y = 2|I_\alpha^{(\nu)}|/S_{\alpha\alpha}^{(N)\mathrm{cl}}$, which gives the bound on the classical precision $\mathcal{P}_\alpha^{(\nu)\mathrm{cl}}$,

$$S_{\alpha\alpha}^{(N)\mathrm{cl}} \geq \frac{\left(I_\alpha^{(\nu)}\right)^2}{S_{\alpha\alpha}^{(\nu)\mathrm{cl}}} \equiv \mathcal{P}_\alpha^{(\nu)\mathrm{cl}}. \quad (4)$$

It is valid for any current [42], $I_\alpha^{(\nu)}$, in either bosonic or fermionic, possibly nonthermal, systems [41], whenever the current and the noise can be written in the form of Eqs. (1) and (2), respectively. This bound, which can be interpreted as a *kinetic uncertainty relation,* is the first central result of this Letter. For this bound (4) on the *classical* component of the noise, we establish a direct relation between the particle-current noise and the *local activity* $\mathcal{K}_\alpha$ with respect to contact α of the system. See Refs. [25,43] for connections between fluctuations and activity in the weak transmission regime or for closed systems. We identify this local activity as

$$S_{\alpha\alpha}^{(N)\mathrm{cl}} = \Gamma_\alpha^\rightarrow + \Gamma_\alpha^\leftarrow \equiv \mathcal{K}_\alpha, \quad (5)$$

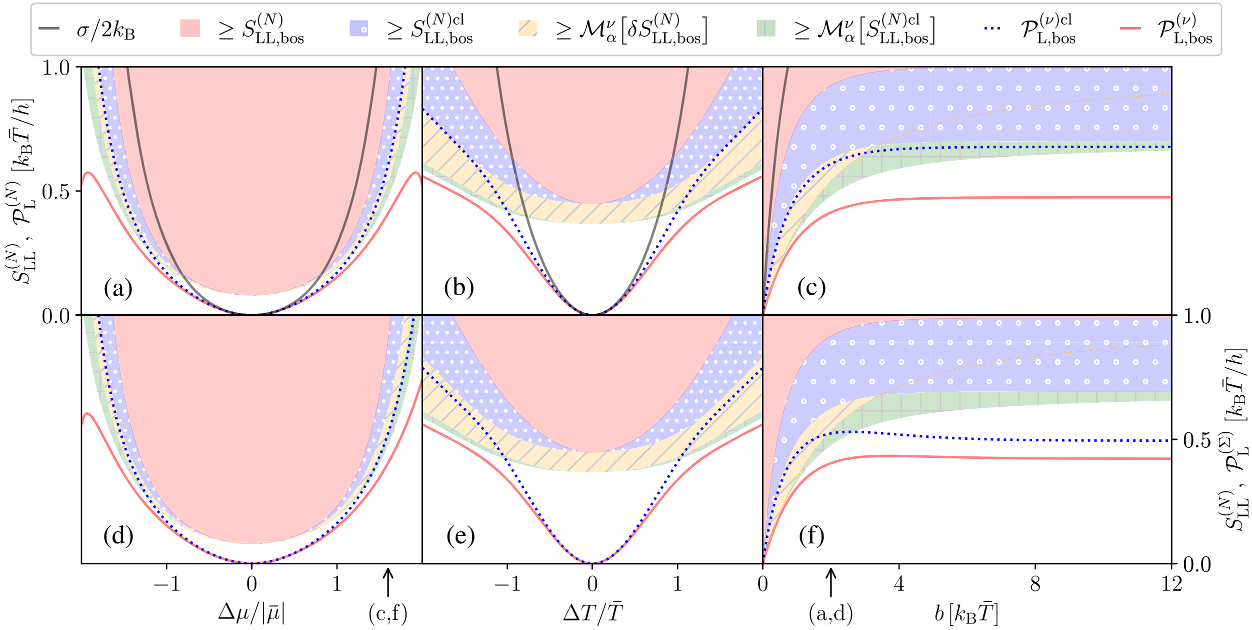

FIG. 2. KUR and KURL; see bounds (4), (6), (8), and (9) in a bosonic two-terminal system. Red and blue lines show precision functions, filled areas are the regions excluded by the bounds. Black lines show the total entropy production $\sigma = I_\mathrm{L}^{(\Sigma)} + I_\mathrm{R}^{(\Sigma)}$ constraining precision via the TUR. Upper or lower row: bounds on *particle*- or *entropy*-current precision as function of (a) and (d) potential bias at $\Delta T = 0$, (b),(e) temperature bias at $\Delta\mu = 0$, and (c),(f) the bandwidth b. In panels (a),(b) and (d),(e), we choose $B_\mathrm{L}^{(\nu)} = b = 2k_\mathrm{B}\bar{T}$, panels (a) and (c), (d) and (f) have $\bar{\mu} = -3k_\mathrm{B}\bar{T}$ with $\Delta T = 0$, panels (b) and (e) have $\bar{\mu} = -1.5k_\mathrm{B}\bar{T}$, $\Delta\mu = 0$, and (c) and (f) have $\Delta\mu = 1.6|\bar{\mu}|$. Arrows show values of bandwidth and potential bias chosen for different plots. For all plots, $D_0 = 1$ and $E_0 = 0.1k_\mathrm{B}\bar{T}$.

with $\Gamma_\alpha^{\rightarrow} = (1/h)\sum_{\beta\neq\alpha}\int dE D_{\alpha\beta}(E)F^{\pm}_{\alpha\beta}(E)$ and $\Gamma_\alpha^{\leftarrow} = (1/h)\sum_{\beta\neq\alpha}\int dE D_{\alpha\beta}(E)F^{\pm}_{\beta\alpha}(E)$. The difference of these rates equals the particle current defined in Eq. (1), and their sum in Eq. (5) is a measure of how active the system is with respect to particle exchange with contact α. The activity, $\mathcal{K}_\alpha$, hence limits the precision of a current as $\mathcal{K}_\alpha \geq \mathcal{P}_\alpha^{(\nu)\mathrm{cl}}$. The result for this KUR bound is shown in blue in Fig. 2, for a two-terminal conductor with thermal bosonic distribution functions with $T_\mathrm{L} = \bar{T} + \Delta T/2, T_\mathrm{R} = \bar{T} - \Delta T/2$, $\mu_\mathrm{L} = \bar{\mu} + \Delta\mu/2, \mu_\mathrm{R} = \bar{\mu} - \Delta\mu/2$, and with a boxcar-shaped transmission probability $D_\mathrm{LR}(E) \equiv D(E) = D_0[\theta(E - E_0) - \theta(E - E_0 - b)]$. Here, b is the width of the transmission window and E_0 is the onset of the region with constant transmission $D_0 \neq 0$. Transmission functions approaching this boxcar shape can be realized, e.g., by wavelength-selective mirrors for optical systems, by multipole Purcell filters in circuit QED [44], or by chains of quantum dots [45] in electronic conductors. We show results both for the particle-current precision $\mathcal{P}_\mathrm{L}^{(N)\mathrm{cl}}$ and the entropy-current precision $\mathcal{P}_\mathrm{L}^{(\Sigma)\mathrm{cl}}$ (dotted blue lines), as compared to the bound set by the activity $\mathcal{K}_\mathrm{L}$ (where the region excluded by $\mathcal{K}_\mathrm{L}$ is colored in light blue). Note that since we here choose the contact distributions to be thermal, entropy currents and their fluctuations are directly related to heat currents and their fluctuations via $x_\alpha^\Sigma \to (E - \mu_\alpha)/T_\alpha$. The plots show that the classical precision $\mathcal{P}_\mathrm{L}^{(\nu)\mathrm{cl}}$ is large when either of the biases is large or when the bandwidth approaches the voltage bias, namely when current and activity are large. While for large biases, the precision approaches the bound set by the activity, a large bandwidth increases the distance between the precision and its bound, as is particularly visible for the entropy current in the lower row of Fig. 2. The reason for this is that (3b) approaches equality when the distribution functions differ strongly, one of them being zero; this situation coincides with the activity approaching the magnitude of the particle current, $\mathcal{K}_\alpha \to \Gamma_\alpha^{\rightleftharpoons}$. However, an increased bandwidth $b > \Delta\mu, T_\alpha$ necessarily involves energy intervals in which the thermal contact distribution functions approach each other. The tightness of the *contact-selective* KUR of Eq. (4) in various parameter regimes differs from the kinetic uncertainty relations for Markovian quantum jump processes [46], which were recently demonstrated not to be tight [26] or only when the activity gets modified [47]. Note that while the KUR is in general tight far away from equilibrium, constraints set by entropy production (TUR) [14,15,48], indicated by the black line, are most relevant close to equilibrium.

In general, it is the *full* noise—including both the classical *and* the quantum part—that is experimentally accessible and that influences the precision. Since the quantum contribution to the noise has an opposite overall sign for fermionic compared to bosonic systems, meaningful extensions of (4) are of different nature for these two cases. Indeed, for bosonic systems, where the quantum contribution to the noise is always positive, one finds that the classical kinetic uncertainty relation (4) continues to hold for the full noise

$$S^{(N)}_{\alpha\alpha,\mathrm{bos}} \geq \mathcal{K}_\alpha \geq \frac{\left(I^{(\nu)}_{\alpha,\mathrm{bos}}\right)^2}{S^{(\nu)}_{\alpha\alpha,\mathrm{bos}}} \equiv \mathcal{P}^{(\nu)}_{\alpha,\mathrm{bos}}. \tag{6}$$

The reason for the inequality to still hold is that quantum fluctuations in bosonic systems decrease the precision on the right hand side of the inequality while increasing the particle-current fluctuations on the left hand side. While relation (6) has the advantage of being valid for the full, experimentally accessible fluctuations, it has the drawback of being a *loose* bound beyond the classical limit, see the red lines and surfaces in Fig. 2.

In order to find an insightful KUR bound for bosonic quantum systems, we identify a general function of measurable transport quantities, which is representative for the activity in the quantum limit. As a first step, we estimate the magnitude of the quantum contribution to the noise identifying the important role of the transport bandwidth. We note that the quantum-noise integrand, Eq. (2b), equals the square of the current integrand, Eq. (1). This integrand can be trivially extended by the indicator function of its support, namely $\zeta^\nu_\alpha(E) = 1$ if $E \in \mathrm{supp}\{x^{(\nu)}_\alpha \sum_\beta D_{\alpha\beta}(f_\alpha - f_\beta)\}$ and $\zeta^\nu_\alpha(E) = 0$ otherwise. Using Cauchy-Schwarz inequality for square-integrable functions [41], we find a bound on the *quantum contribution* to the noise only,

$$S^{(\nu)\mathrm{qu}}_{\alpha\alpha,\mathrm{bos}} \geq \frac{h}{B^\nu_\alpha}\left(I^{(\nu)}_\alpha\right)^2. \tag{7}$$

Apart from the average current $I^{(\nu)}_\alpha$, it involves the bandwidth $B^\nu_\alpha = \int_0^\infty dE \zeta^\nu_\alpha(E)$. This shows that quantum bunching effects also constrain the full precision, $B^\nu_\alpha/h \geq (I^{(\nu)}_\alpha)^2/S^{(\nu)\mathrm{qu}}_{\alpha\alpha,\mathrm{bos}} \geq (I^{(\nu)}_\alpha)^2/S^{(\nu)}_{\alpha\alpha,\mathrm{bos}}$. The intuitive reason for this is that the smaller the bandwidth, the larger the relative weight of equal-energy states subject to bunching. Consequently, a small bandwidth leads to large quantum noise at a given current and hence to a reduced precision. Thus, to optimize precision in bosonic systems, it is beneficial to spread out transport over a large energy interval. Infinite bandwidth would make the bound on the quantum noise trivial; however, in a realistic setup, we always expect the energy interval contributing to transport to be finite. Inequality (7) reaches equality when the current integrand is constant over the bandwidth, e.g., when the bandwidth is much smaller than the contact temperatures.

Combining (4) and (7), we capture the limit on the full precision, $\mathcal{P}^{(\nu)}_{\alpha,\mathrm{bos}}$, by a function of the classical particle-current noise, $\mathcal{K}_{\alpha,\mathrm{bos}} = S^{(N)\mathrm{cl}}_{\alpha\alpha,\mathrm{bos}}$, and of the bandwidth,

$$\mathcal{M}^\nu_\alpha[\mathcal{K}_{\alpha,\mathrm{bos}}] \equiv \frac{\mathcal{K}_{\alpha,\mathrm{bos}}}{1 + \frac{h}{B^\nu_\alpha}\mathcal{K}_{\alpha,\mathrm{bos}}} \geq \mathcal{P}^{(\nu)}_{\alpha,\mathrm{bos}}. \tag{8}$$

In this KUR-type bound, the structure of the function $\mathcal{M}^\nu_\alpha[x]$ takes into account the reduction of precision due to bunching in the presence of a limited bandwidth, while the classical particle-current noise still serves as an activity quantifying the number of single-particle transfers in and out of reservoir α. The bound (8) on the full precision is displayed as the shaded green areas in Fig. 2, where it shows clear improvements with respect to the loose bound (6). In particular, it is tight for small bandwidths.

The drawback of the KUR bound (8) is that the classical activity is not experimentally accessible since it comprises only the classical noise contribution. As a next step, we therefore estimate the activity from measurable observables [49] (current, full particle-current noise, and bandwidth) exploiting the relation (7). Concretely, we introduce the function $\delta S^{(\nu)}_{\alpha\alpha,\mathrm{bos}} \equiv S^{(\nu)}_{\alpha\alpha,\mathrm{bos}} - h(I^{(\nu)}_\alpha)^2/B^\nu_\alpha$, which we find to be a general upper bound to the activity and which equals the classical activity when the bandwidth is small compared to temperature, even far from equilibrium, for $\nu = N$. We find the following KUR-like bound (KURL)

$$\mathcal{M}^\nu_\alpha\left[\delta S^{(N)}_{\alpha\alpha,\mathrm{bos}}\right] \geq \mathcal{M}^\nu_\alpha[\mathcal{K}_{\alpha,\mathrm{bos}}] \geq \mathcal{P}^{(\nu)}_{\alpha,\mathrm{bos}}, \tag{9}$$

allowing us to limit the precision of an arbitrary current $I^{(\nu)}_\alpha$ in terms of measurable particle-transport observables, which give an estimate of the activity in the full quantum case. Together with the KUR bound (8) in terms of classical activity it constitutes the second main result of this Letter. The KURL bound (9) is shown in Fig. 2 as shaded yellow areas. Notably, there is a crossing in panels (c) and (f) between the activity setting the bound in (4) and the function $\mathcal{M}^\nu_\alpha[\delta S^{(N)}_{\alpha\alpha,\mathrm{bos}}]$ setting the bound in (9). This is because $\mathcal{M}^\nu_\alpha[x]$ accounts for bunching effects, relevant for small bandwidths, which the classical activity fails to do. For small biases ΔT or $\Delta\mu$, the quantum part of the noise is negligible, meaning that the classical and full precisions, $\mathcal{P}^{(\nu)\mathrm{cl}}_\alpha$ and $\mathcal{P}^{(\nu)}_\alpha$, are the same and $S^{(\nu)\mathrm{cl}}_{\alpha\alpha,\mathrm{bos}}$ and the respective bounds coincide.

For *fermionic* systems, an inequality of the type of (6) is generally not valid since it can be broken by the quantum contribution to the noise. Indeed, anti-bunching decreases the particle-current noise entering (4), while increasing precision. Bounds accounting for anti-bunching in fermionic systems, expressed in terms of experimentally accessible quantities are hence desirable. In order to establish a relation between the fermionic quantum noise and the current, we estimate [40]

$$S^{(\nu)\mathrm{qu}}_{\alpha\alpha,\mathrm{fer}} \geq -\frac{1}{h}\int dE[x^{(\nu)}_\alpha]^2[1-D_{\alpha\alpha}]\sum_{\beta\neq\alpha}D_{\alpha\beta}|F^-_{\beta\alpha}+F^-_{\alpha\beta}|, \quad (10)$$

which approaches equality far from equilibrium, namely when the fermionic distribution functions differ maximally from each other. Introducing the minimum reflection probability inside the bias window, namely in the energy interval A where transport happens, $R_\alpha \equiv \inf_{E\in A} D_{\alpha\alpha}(E)$, we find a fermionic KURL for the full fluctuations [41]

$$\frac{1}{R_\alpha}\frac{S^{(N)}_{\alpha\alpha,\mathrm{fer}}}{R_\alpha} \geq \frac{1}{R_\alpha}S^{(N)\mathrm{cl}}_{\alpha\alpha,\mathrm{fer}} \geq \frac{\left(I^{(\nu)}_\alpha\right)^2}{S^{(\nu)}_{\alpha\alpha,\mathrm{fer}}} \equiv \mathcal{P}^{(\nu)}_{\alpha,\mathrm{fer}}. \quad (11)$$

In the limit of vanishing reflection probabilities, the precision is *not* bounded. Indeed, for a fully transmitting scattering region and large potential bias, currents are known to be noiseless due to fermionic antibunching, leading, e.g., to a breaking of the TUR [28,31,50,51]. Similarly to the bosonic case, $S^{(N)\mathrm{cl}}_{\alpha\alpha,\mathrm{fer}}$ still takes the role of an activity quantifying the number of single-particle transfers in and out of reservoir α. The upper bound $S^{(N)\mathrm{cl}}_{\alpha\alpha,\mathrm{fer}}/R_\alpha$ is thus a function of the classical activity and a direct extension of the KUR accounting for antibunching. Again, we aim to express this bound in terms of the measurable, full particle-current noise, and we therefore exploit $S^{(N)}_{\alpha\alpha,\mathrm{fer}}/R_\alpha \geq S^{(N)\mathrm{cl}}_{\alpha\alpha,\mathrm{fer}}$. The leftmost bound of (11) is hence both taking into account the effect of antibunching as well as an estimate of the true activity from the full fluctuations.

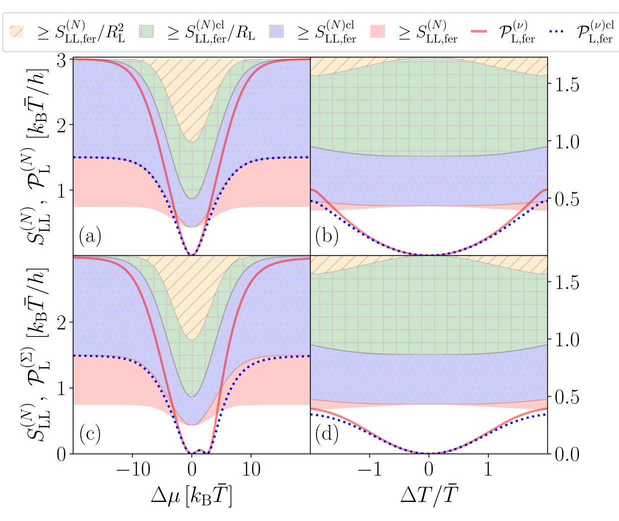

FIG. 3. KUR in a fermionic two-terminal system. The lines show the precision functions and the filled areas the regions excluded by the bounds; see bounds (4), (11). Upper or lower row: bounds on *particle-* or *entropy*-current precision as function of (a) and (c) chemical potential bias $\Delta\mu$ and (b) and (d) temperature bias ΔT. In all panels we use $\bar{\mu} \equiv 0$ and $D_0 = 0.5$, $E_0 = 0.1k_{\mathrm{B}}\bar{T}$, $b = 1k_{\mathrm{B}}\bar{T}$ for the boxcar transmission function. Panels (a) and (c) have $\Delta T = 0$, while panels (b) and (d) have $\Delta\mu = 0$.

The fermionic KUR for classical fluctuations, (4), as well as the full fermionic KURL, (11), are shown in Fig. 3 for particle- and entropy-current precision in a thermal two-terminal system with boxcar-shaped transmission probability. The blue surface bounds the classical precision from above; see Eq. (4). However, in contrast to the bosonic case, the full particle-current noise is decreased with respect to the classical one. As soon as temperature or potential bias are of the order of the average temperature, the precision exceeds the classical, or even the full, particle-current noise, breaking the classical KUR as expected. The precision is instead bounded by the particle-current noise modified by the minimum reflection probability R_{L}, as given in (11). These bounds become tight for large potential biases, when one of the distributions is zero inside the interval of nonzero transmission. In addition, a transmission function which is flat within the relevant energy window is required to make (11) tight. The main difference in the precision for the entropy current (lower row) compared to the one for the particle current (upper row) is the occurrence of an additional zero. It appears at finite bias, when μ_{L} is aligned with the center of the boxcar transmission such that the full entropy production happens in the right contact.

In summary, we have established general precision bounds for quantum transport valid for any setup that can effectively be described by scattering theory. While these bounds take the form of a kinetic uncertainty relation bounded by a *local* activity in the classical limit, we here provide analogous constraints for the precision in the presence of quantum fluctuations. These bounds are given by functions of the classical activity accounting for quantum statistics at arbitrary transmission, thereby filling a gap that can not be addressed by standardly used weak-coupling approaches. As a result of quantum statistics, the precision of bosonic systems benefits from large bandwidths, and the precision of fermionic systems benefits from low reflections. We further reformulate these quantum KURs in terms of measurable particle-current noise and transmission properties. The obtained bounds are expected to serve as guidelines for the design of any multi-terminal device aiming at precision. Additionally, the bounds could serve as inference tools for estimating fluctuations in heat and entropy currents.

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