Semi-classical analysis of two-mode quadrature squeezing in a high-Q microresonator

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Abstract: We demonstrate that two-mode quadrature squeezing of multiple quantum frequency modes generated in integrated microring resonators can be simulated using a semi-classical approach based on the Ikeda map.

1. Introduction

Integrated microring resonators are key components in both classical and quantum optics, particularly in applications exploiting optical nonlinearities. In the classical regime, they are widely used for the generation of soliton microcombs, whose dynamic is accurately modeled by the Ikeda map [1]. High-Q Kerr nonlinear microresonators also display quantum correlations and entanglement, leading to an exciting new direction in quantum information processing that relies on quantum frequency combs [2]. These phenomena can be well described using established quantum formalisms [3–5], rigorously based on the canonical commutation relations and that naturally include the vacuum noise. However, their computational complexity increases with the number of modes analyzed, and they have challenges incorporating realistic noise sources, such as pump phase and intensity noise and thermorefractive noise in the cavity, which we know in the classical regime affect the fundamental performance of microcombs [6]. In this work, we show that the generation of two-mode quadrature squeezing, that arises in microresonators operated below the parametric oscillation threshold, can be simulated using the same semi-classical Ikeda map approach commonly used for classical microcomb analysis. This is as surprising as convenient. Due to the model structure, we have fast and direct access to all the quantum frequency modes. This approach may provide a compact tool for exploring quantum effects in microcavities using fewer sets of equations, with the potential of expanding the geometrical complexity of the device, and of including additional technical noise sources.

2. Semi-classical model

We simulate the quantum dynamic of frequency modes generated in an overcoupled microresonator pumped just below the optical parametric threshold P_{th}. The normalized pump power is $P_{\text{in}}/P_{\text{th}} = 0.99$ to maximize quadrature squeezing. In this regime, the modulation instability (MI) gain remains lower than the total cavity loss, as shown in Fig.1a). Classically, it implies that the pump is the only oscillating intracavity mode. In contrast, from a quantum perspective, spontaneous four-wave mixing (SFWM) converts two degenerate pump photons into signal and idler photons, populating the vacuum cavity modes. The new modes are in a two-mode quadrature squeezed state [3].

Our numerical simulation framework is based on the Ikeda map [1]. The time evolution of the field envelope propagating in the device is modeled with two steps in each roundtrip: (i) the coupling between the bus waveguide and the ring, described by the standard beam splitter equations with reflection coefficient $|r| = \sqrt{\theta}$, where θ is the coupling rate; (ii) the nonlinear propagation of the intracavity field. The latter is described by a modified nonlinear Schrödinger equation and implemented numerically using the split-step Fourier method.

Although there are several noise sources that characterize a microresonator [6] and that may affect the squeezing level, here we only focus on the effect of vacuum fluctuations to validate our approach. We consider the pump to be shot noise limited and the main loss channel to be the propagation loss. In both cases, the noise is represented by an additional field with amplitude and phase defined by unit normal distributions [7]. We show that this semi-classical noise representation is enough to simulate quadrature squeezing.

After each roundtrip, the ring output field is analyzed in the frequency domain. For every signal and idler pairs, we construct composite modes and quadratures (X_1, X_2) to characterize two-mode quadrature squeezing [8]. The variances of the composite modes are then compared with those obtained from the vacuum signal and idler modes, evaluated using the same simulation framework with no input power.

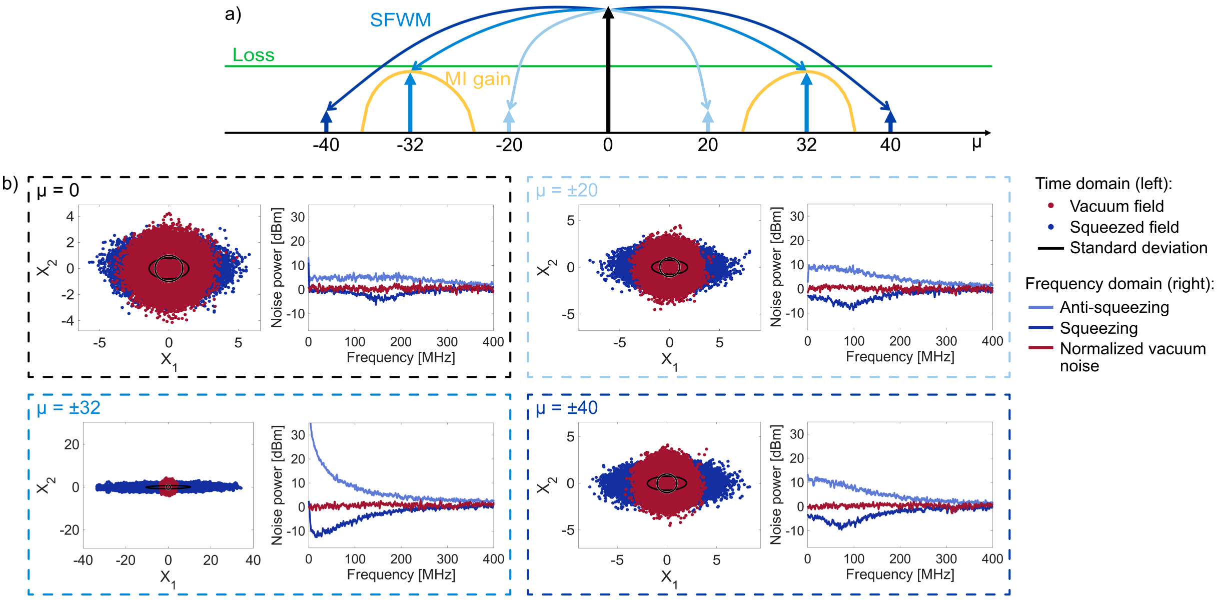

Fig. 1: (a) Schematic of the quantum frequency comb generation. (b) Two-mode quadrature squeezing in the quadrature plane (time domain) and the corresponding normalized noise spectra (frequency domain). The simulation parameters are: free spectral range FSR = 100 GHz, group velocity dispersion $\beta_2 = -42.9\,\mathrm{ps}^2/\mathrm{km}$, detuning parameter $\delta = 0$, nonlinear coefficient $\gamma = 0.787\,\mathrm{W}^{-1}\mathrm{s}^{-1}$, intrinsic and extrinsic quality factors $Q_\mathrm{i} = 20 \cdot 10^6$ and $Q_\mathrm{e} = 1 \cdot 10^6$, $P_\mathrm{th} = 0.0375\,\mathrm{W}$, and $P_\mathrm{in} = 0.037\,\mathrm{W}$.

3. Simulation results

We investigate two-mode quadrature squeezing in three signal and idler pairs and single-mode squeezing of the pump. The analysis is carried out in both the time and frequency domains, as shown in Fig.1b). In the time domain, we examine the field distribution in the quadrature plane. The vacuum field exhibits the typical circular standard deviation (SD), while the squeezed fields show the expected elliptical SD. For each mode pair, the squeezing angle, which defines the orientation of the squeezing ellipse in the plane, is adjusted such that X_2 exhibits squeezing and X_1 anti-squeezing. All fields, except for the vacuum, are displaced to have zero mean amplitude.

By Fourier transforming these time domain fields, we obtain their noise spectra, here normalized to the vacuum noise level. The resulting squeezing and anti-squeezing levels depend on the frequency detuning of the corresponding modes. A maximum squeezing of 12 dB is observed at low frequencies for modes $\mu = \pm 32$, which coincide with the MI gain peaks. Away from these modes, the maximum squeezing level decreases and occurs at different detunings. In contrast, the anti-squeezing always reaches its maximum at DC. These results are in agreement with the quantum formalism laid in [3,5].

In summary, we show that certain quantum properties of microresonators, such as quadrature squeezing, can be simulated using a semi-classical model. We observe a maximum squeezing of 12 dB at the MI gain peak modes and detuning dependent squeezing across additional modes, including the pump. This model can be extended in future works to more complex microresonator geometries and to incorporate other classical noise sources.

References

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