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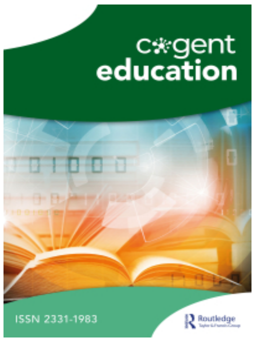

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Pre-service secondary school teachers' mathematical identity

Laura Fainsilber

Department of Mathematical Sciences, Chalmers University of Technology and the University of Gothenburg, Gothenburg, Sweden

ABSTRACT
Teacher educators should know how students relate to the subject they are learning to teach. This study explores how graduating secondary school mathematics teachers describe the evolution of their relation to mathematics during a 5-year teacher education program. It builds upon a thematic analysis of in-depth interviews with four students at the very end of their program. The themes generated in the analysis shed light on the participants' mathematical identity, showing a strengthening of their interest in the subject, a new view of the field as a connected whole, heightened self-efficacy in problem-solving, and growing appreciation for a variety of mathematical representations and for mathematical language and notations. The results improve understanding of what matters to students in mathematics teacher education.

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Introduction

How do we know how teacher students view their studies and how they forge their identity in relation to the subject they will be teaching? Research supports a narrative where students who have chosen to study mathematics are interested or even passionate about the subject, with examples both among students majoring in mathematics (Bartholomew et al., 2011) and among prospective mathematics teachers (Beccuti et al., 2024). When a student asked me: 'Is it true, what they say about the teacher program, that the reason we have to study so many hard math courses is that you want to select only students who are really interested in mathematics?', I realized that I knew little about how the students see themselves in relation to their mathematic studies. I had expected to field questions about the challenges of learning mathematics, as presented in Beccuti et al. (2024), about how 'smart' students needed to be, or how well-prepared, but had assumed that the choice of becoming a mathematics teacher rested upon an identity that included a strong interest in the subject. However, the quote above, from a pre-service secondary school teacher in mathematics in Sweden illustrates the need to complement the narrative on student identity, in order to understand a wider variety of students and especially of pre-service secondary school teachers.

There is a large body of research on teacher identity in general and pre-service teacher identity, which has expanded in the past 20 years and become well-established (Zhang & Wang, 2021). The present study focuses on one facet of the identity of pre-service secondary school teachers: their relation with the subject matter of mathematics. This facet relates to the pre-service teacher as a learner of mathematics and reflects the person's learner identity rather than their identity as a teacher. In terms of teacher professional identity as described by Beijaard et al. (2000), as a combination of being a subject

CONTACT Laura Fainsilber laura@chalmers.se Department of Mathematical Sciences, Chalmers University of Technology and the University of Gothenburg, Gothenburg SE-412 96, Sweden

matter expert, a didactical expert, and a pedagogical expert, this study focuses on becoming a subject matter expert. Mathematical identity has been explored for some groups of teachers, in particular for pre-service elementary teachers (e.g. Jones et al., 2000; Kaasila et al., 2012; Lutovac & Kaasila, 2011), while studies on pre-service secondary school teachers have mostly focused on aspects of identity related to their role as a teacher and not directly related to the subject (e.g. Losano et al., 2018; Marschall, 2022; Mitiche, 2024; Weller, 2022).

This study explores graduating secondary school mathematics teachers' mathematical identity by investigating how four participants retroactively talk about the evolution of their relationship with mathematics during a 5-year university program for upper secondary school teaching. The program, set in Sweden, offers a combination of pedagogical courses, subject courses, and in-school practice. Preliminary results from this study were presented in a short communication at the MADIF14 conference (Fainsilber, 2024) and at the NORMA20 conference in June, 2024.

Aim and research question

The aim of this study is to contribute to the understanding of pre-service teachers' experience of studying mathematics and bring to light their developing relation to the subject.

The research objective is to identify themes that constitute mathematical identity for pre-service secondary school mathematics teachers.

In line with these objectives, the following research question was investigated: How do graduating secondary school teachers in mathematics describe the evolution of their mathematical identity during their university studies?

A theoretical framework for exploring identity

Pre-service teachers' study of mathematics is a well-established research field, primarily regarding cognitive aspects but also from the point of view of attitudes, beliefs and identity (Hannula et al., 2016). The affect-related aspects of mathematics learning are notoriously complex and difficult to define (Sfard & Prusak, 2005b). Among these aspects, identity approaches the relation between learning and the cultural context, from the perspective of the individual. In a critical overview of articles focusing on learners' and teachers' identity in research journals in mathematics education, Darragh (2016) noted several different approaches to the notion of identity, ranging from the sociological to the psychological. Some of the articles grappled with issues of belonging to a particular group of people (e.g. a gender or ethnic group) (Gholson & Martin, 2019), some focused on the individual's belonging in a community of practice (Biza et al., 2014), some explored changes in teacher's expectations, e.g. in relation to using digital tools (Goos, 2005), while others zoomed in on individuals' relation to learning mathematics and on their views of what mathematics is (Hernandez-Martinez et al., 2011; Petocz et al., 2007; Reid et al., 2005; Wood et al., 2012). Corresponding to the different foci, a wide range of theories, mostly based on a socio-constructivist perspective, are used to study teacher and student identity, as described in the review by Graven and Heyd-Metzuyanim (2019). The present study uses a discursive perspective, relying on informants' utterances about their relation to mathematics.

Darragh (2016) discusses the difficulties related to formulating a clear definition of identity and promotes regarding identity as an action, as an individual's work to make sense of his or her own activity in mathematics. Lutovac and Kaasila (2018) also underline the complexity involved in defining identity as a general concept and follow Sfard and Prusak (2005b) in suggesting to start with a working definition, based on discourse, to bring to the foreground expressions of 'who one is'. Sfard and Prusak (2005b) put this into practice and define identity as 'those narratives about individuals that are reifying, endorsable, and significant'. *Reifying* refers to statements about how the person is, rather than about what they do or think, *endorsable* means that the person in question would agree that it describes them, and *significant* means that the statement brings up an important and relevant characteristic. Learning can be seen here as the process of moving from an actual identity to a designated identity, where the latter reflects both cultural and personal images of who the individual is expected to become. Note that in this framework, as seen in the examples presented in Heyd-Metzuyanim & Sfard, (2012), the word ‘narrative’ can refer to a short utterance, and does not necessarily tell a whole story. Identity consists of the narratives themselves, not of mental properties that the narratives would be an expression for. This working definition gives a base to explore students’ mathematical identity in their own words.

Sfard and Prusak’s framework was referred to by a large portion of the studies on identity in mathematics education surveyed by Darragh (2016). For instance, Heyd-Metzuyanim (2015) traced a middle school student’s development of mathematical difficulties using a detailed analysis of the student’s identity statements embedded within mathematical discourse. Bjuland et al. (2012) also build upon Sfard and Prusak’s notion of identity and adapted the framework for their study of one elementary teacher’s identity as a mathematics teacher. They used differences in the teacher’s reflective narratives from five different occasions over a 2-year period to show indications of identity development during the period.

The present study adopts Sfard and Prusak (2005b) as its guiding theoretical framework for identity. It collects and analyses utterances of graduating secondary school mathematics teachers and examines their mathematical identity and their recollection of its evolution.

Research on mathematical identity

While mathematical identity in the present study is defined, as explained above, using reified and endorsable narratives that are significant in the sense that they refer to important aspects of the person’s relation to mathematics, other research has focused on mathematical identity using a variety of frameworks. This section offers an overview of some relevant articles focusing on mathematical identity in the wider sense of individuals’ (both students’ and teachers’) relationship with mathematics.

Students’ conceptions of mathematics and their views of its role in their future plans were investigated in a large international questionnaire study of university students, including mathematics majors and engineering students (Petocz et al., 2007; Wood et al., 2012). Student conceptions ranged from a narrow view of mathematics as working with numbers, to a view of mathematics as a problem-solving toolbox, as a modelling tool, or as an abstract structure, to a wide view of mathematics as an approach to life. While the study was not formulated in terms of identity, it focuses on student views of what mathematics means to them, which can be interpreted to reflect identity. In another context, Hernandez-Martinez et al. (2011) argue for viewing the transition from school to university mathematics, with its difficulties and possibilities for growth, as an issue of student identity.

The mathematical identity of university students has also been researched with qualitative studies, examining, for instance, student narratives on what their participation in mathematics meant to them personally and how they dealt with difficulties in the transition from secondary to tertiary education (Bartholomew et al., 2011; Di Martino & Gregorio, 2019). Beccuti et al. (2024) analyzed essays entitled ‘What mathematics is to me’ written by students in a master’s program in mathematics, many of whom envision a teaching career, and described an identity based on ascetic discipline, the challenges of learning, the comfort of concentrated focus, and the sense of belonging to an exclusive group.

Sfard and Prusak (2005b) presented their definition of identity with an example considering the role that mathematical fluency played in two groups of 17-year-old’s identity and ambitions, as a means of ‘closing the gap between actual and designated identities’. According to their analysis, differences in the two groups’ social identities explain some differences in how they engaged in learning mathematics and in how they succeeded.

In their survey article on research on mathematics-related teacher identity, Lutovac and Kaasila (2018) note major differences in relation to mathematics between elementary teachers, who teach the subject as one of many and may not have chosen it specifically, and secondary teachers, who have chosen mathematics as a speciality. They call for linking research on the two groups but so far, studies on the two groups often differ in their focus. Many studies of elementary teachers consider (problematic) relations to the subject per se (Jones et al., 2000; Kaasila et al., 2012; Lutovac & Kaasila, 2011), and some show effects of the teachers’ personal experience of mathematics on their teaching (Xenofontos & Andrews, 2023). In contrast, studies of secondary teachers mostly consider relation to teaching as a profession (Losano et al., 2018; Marschall, 2022; Mitiche, 2024, 2025; Weller, 2022), leaving the subject of mathematics as a backdrop. Even in the study by Lutovac and Havia (2024), where pre-service secondary teachers draw on their experience of success and failure in learning mathematics, the focus is not on mathematical identity but on how the experiences contribute to the formation of a collective teacher identity. Similarly, in the PIK-model presented by Piñero Charlo and Canto López (2026), which seeks to integrate teacher professional knowledge and identity in mathematics education, the knowledge side of the model balances mathematical knowledge with knowledge for teaching, while the identity side of the model is geared towards teaching, ignoring mathematical identity. Recently, Arslan et al. (2025) attempted to bridge considerations of mathematical identity and of mathematics teaching identity for both elementary and middle-school pre-service teachers by classifying identity profiles of over 800 pre-service teachers. Their study is based on a questionnaire with parallel questions about beliefs, self-efficacy, emotions, motivation, and self-image in relation to mathematics and to mathematics teaching (e.g. 'What does mathematics mean to you?', What is your primary objective in teaching mathematics?) They found common profiles for elementary and middle school teachers, but a higher proportion of elementary school teachers represented in the profile with more negative beliefs, emotions and motivation. Profiles combine aspects of mathematics and mathematics teaching rather than distinguishing between them.

Overall, while some studies on primary school teachers focus on mathematical identity, most studies on secondary teachers focus on teacher identity and take for granted the mathematical engagement of secondary teachers, leaving their mathematical identity unproblematized. Pre-service secondary school mathematics teachers differ from pre-service elementary school teachers but also from other university mathematics students in that they have actively chosen mathematics as a subject but don't necessarily picture themselves as budding mathematicians.

Lacking is research into secondary school pre-service teachers' mathematical identity. This group's mathematical identity is particularly important as it plays a role for completion of teacher education as well as for how long mathematics teachers stay in the profession, and because the teachers will in turn influence their students' knowledge, interest, motivation, and identity. This makes it particularly appropriate to examine how secondary school teachers' mathematical identity develops during teacher education

Methodology and context

This section describes the methodological choices for gathering and analysing data in this study, describes the teacher education program, and introduces the informants.

Empirical data in many earlier studies on identity consist of interviews or surveys, while some use classroom observation (e.g. Heyd-Metzuyanim & Sfard, 2012) or student essays (e.g. Di Martino & Zan, 2010). Given the objective of identifying themes that constitute the mathematical identity of pre-service teachers, I chose to collect data using semi-structured individual interviews for the present study. Semi-structured interviews offer opportunities to explore the complexity of identity, to reflect over contexts and relationships, and to probe thoughts and representations that informants may not be aware of or have formulated clearly (Arksey & Knight, 1999).

In analysing the data, I was interested in exploring significant themes, bringing together common experiences, rather than in following the story of each individual. I therefore chose to use thematic analysis, even though the material consists of narratives, disregarding the overall story to explore content-related themes. While coding for thematics analysis runs the risk of fragmenting the data, as underlined in Bryman, (2016), it enables bringing together the narratives of several informants and to find overarching themes. In the categories described in the review of methods used in research on teacher identity by Lutovac and Kaasila (2019), the analysis for this study could have been classified as *Framework-driven thematic/content analysis*, where Sfard and Prusak's definition of identity provides a frame for coding and theme development.

As described in more detail in the sections below, this study builds upon a thematic analysis of four semi-structured interviews with pre-service teachers graduating from the same 5-year integrated university program leading to teaching credentials for upper secondary school in mathematics.

Data collection

The population of interest consists of pre-service mathematics teachers whose curriculum integrates mathematics, pedagogical subjects and school practice, as in the Swedish 5-year teacher education program. For convenience and coherence, and in order to be able to use my in-depth knowledge of the program and students, I sampled interviewees from one particular university and one particular year. This yields the advantage of keeping the curriculum constant to focus on participant identity development in relation to the context, but it also weakens the claims to generalize to other cohorts. I contacted all the students in the graduating group of upper secondary school teachers who had mathematics as their main subject (12 students) and four students agreed to participate. Since this study is essentially exploratory, seeking themes of interest and concern for the interviewees, without quantitative claims or comparisons between different groups, and since the group is homogeneous as far as teacher education is concerned, four in-depth interviews was deemed to offer sufficient information power (Malterud et al., 2016) and to provide an appropriate balance between quantity of data and analysis possibilities, allowing in-depth interviews and detailed analysis (Arksey & Knight, 1999).

The first part of each interview was based on a mathematical task, while the second part had a retrospective focus. The participants worked on the mathematical problem for approximately 30 minutes, after which they reasoned about how they used their mathematical knowledge in working on the problem and finally about how they looked back on their mathematical development during their university studies, for approximately 30 more minutes. The interviews were conducted in Swedish and recorded.

Task-based interviews are common in mathematics education research (Maher & Sigley, 2020), with Piaget's interviews of children as a prototypical example, but they are usually used to explore students' mathematical thinking. In this study, the task plays the role of a vignette (Barter & Renold, 1999), anchoring the conversation in the context of mathematics and in the participant's personal experience of working with mathematics. The problem, shown in the Appendix of this paper, involved only mathematical content covered in a secondary school calculus course but was challenging as far as problem-solving is concerned and required computational dexterity and the use of both algebraic and graphical representations. It was selected from a book providing problem-solving challenges for upper secondary school Calculus (Petersson, 2017). The choice of a problem with relevance for secondary school enabled both a student's perspective and a teacher's perspective in the interview. The participants were asked what they thought they learned from working with the problem, in which circumstances they, as a teacher, might assign such a problem to pupils, where the challenges lie, and what pupils could learn. They were asked about details of how they had worked with the problem, which concepts were involved and which methods they used.

The interview then shifted to a retrospective part, discussing how the participants thought their current approach differed from how they would have worked at the beginning of their university studies, when in their study program they thought they had built up the competencies that they used in solving the problem and when they thought they developed a teacher's perspective. They were specifically asked to which courses or tasks they could trace their development and whether studying together with others had contributed to it. The retrospective questions are a means to let participants talk about changes in identity and to connect identity development to events in their studies, as used by Chapman (2002) to study beliefs and mathematics teacher growth.

The prepared questions were followed up with individual questions to clarify answers and deepen the conversation.

Data analysis

The data was subjected to a thematic analysis, following the phases described by Braun and Clarke (2006): 1) familiarizing yourself with your data, 2) generating initial codes, 3) searching for themes, 4) reviewing themes, 5) defining and naming themes, 6) producing the report.

After transcribing the entire recordings, I repeatedly listened to and read through the interviews, selecting utterances that contributed to participants' mathematical identity. The relevant excerpts here consist of participant statements about themselves that are reifying in the sense of Sfard and Prusak (2005b), i.e. that characterize them as individuals, and that offer significant insight in how the participants talk about themselves in relation to mathematics. Identifying expressions of identity involved some choices and interpretations. Some statements were clearly both reifying, endorsable and significant, e.g.:

> So it feels like a natural thing to do, so I think it's neat to, like, challenge myself (Adam).

> When I help someone with a task they're on, that I have a good grip of, then I notice that I learn more (Darin).

Other utterances could be read as focusing on mathematical practice but expressed identity as well. For instance, the quote:

> Checked it out graphically, but then I realized, I would have needed to plot a lot. Ah, but that's a drag, so I went on analytically (Charlie).

illustrates Charlie's attitude towards perseverance in working with mathematical problems. It also shows his use of different representations and his propensity to switch between different representation forms to handle mathematical objects. On the other hand, an utterance such as:

> You can just learn inverse functions mechanically, without understanding (Darin).

was not classified as *endorsable*, as it lacked direct reference to the speaker.

While identifying relevant utterances in the transcriptions, using the NVivo software package, I generated codes in a theory-driven process for Phase 2 according to Braun & Clarke, (2006), summarizing the excerpts in terms of mathematical identity, with a focus on *reifying*, i.e. on how a statement expresses how the person is (Sfard & Prusak, 2005b). For example, the first statement above was coded as *attitude: challenge*, the second as *learn by helping others*, the third as both *attitude: stuck* and as *representation (graphical, algebraic)*. Note that some excerpts were labelled with several codes to reflect different aspects. This process led to 31 codes, of which 12 were attached to 6 or more excerpts each.

In Phase 3, related codes from all the interviews were gathered to generate common themes. For instance, the code *Self-confidence* labelled 7 utterances, *Attitude: challenge* labelled 3, and *OK to be wrong* labelled 12. They formed the ground for the wider theme of *Self-efficacy*. The emphasis here was not on distinguishing between individuals, but rather on finding common components in the different participants' identities, retaining only themes common to several informants as significant. For instance, considerations about using multiple representations, found in Charlie's quote cited above, were also expressed by the other participants and gave grounds for one of the common themes. On the other hand, an utterance such as:

> I feel that I'm a bit unsure about when it's okay to simplify when something is 0 over 0, when it's the same and when it's not okay (Charlie).

shows an insecurity and a focus on arithmetic computations that Charlie expressed several times. Since other participants failed to display insecurity in that sense, or bring up computations, this theme was not considered a significant issue and is not included in the main results of this paper. In this phase, some codes were re-examined, for instance the by far most frequent, *concept*, which labelled as many as 25 excerpts, was disregarded for being insufficiently related to identity.

In Phase 4, reviewing each theme led to more deliberation. I presented a choice of statements and preliminary themes for discussion with colleagues in a seminar to increase the reliability of the analysis and reduce interpretation bias. Relevance of the theme *Mathematical communication,* gathering codes *Math language, representations (graphical, algebraic), formulate,* was questioned, as it was deemed to refer to skills rather than to identity. Re-examining the relevant quotes, for instance:

> I think that the difference would have been that if it had been me in high school, I would have, I wouldn't have known this, several words I wouldn't have grasped them. (Darin).

> Now I didn't think it was problematic to write it like that, that you write it like that, *f* and then the inverse, "minus one". (Adam)

confirmed that the theme exhibited an aspect of what kind of person the participants expressed that they had become and thereby contributed to their identity.

In Phase 5, I re-examined the themes, delimiting the scope of each one and renaming some of them. For instance, *Mathematical communication,* was replaced by *Using notation and representations* to lift the identity aspect of the theme. Each theme was described, and the characterisations form the base of the results section of this paper. The new descriptions of the themes were again presented and discussed with colleagues in a seminar to mitigate interpretation bias.

Once the final themes were formulated, illustrative quotes were chosen, with an effort to represent each of the four participants' individuality. Selected quotes were translated from Swedish to English for this paper. Finally, in phase 6, this report was prepared, to formulate, discuss and communicate the study.

The teacher education program

The program admits students with a diploma from upper secondary school including advanced calculus and leads to certification for teaching in Swedish secondary school. During the 5 years, students alternate taking courses in pedagogical subjects, courses in mathematics, equivalent to three semesters including teaching and learning mathematics, courses in another chosen school subject – here physics, chemistry, or biology – and four periods of approximately one month each of in-school teaching practice in mathematics. They also write two undergraduate theses: a literature survey and an empirical study on a topic related to mathematics and teaching. In most courses, pre-service teachers studied mathematics separately from other groups of mathematics students.

University education in Sweden is run by the State and free of tuition. Many of those who choose the teaching profession do so after working or studying in another field. The country foresees a lack of qualified teachers and makes efforts to recruit teacher students, especially in STEM subjects. Competition to attend teacher education is rather low. Selection is mainly based on grades from Upper Secondary School and students are admitted with fair grades, in mathematics and on average. The drop-out rate, ca. 50% over the 5-year program, is comparable with that of other programs of the same length, such as engineering.

Participants

The four participants have different backgrounds and relations to teacher education, which can bear significance for the way they form identity. They are briefly presented here, using pseudonyms.

Adam, a 24-year-old man, started the teacher education program directly after high school, with subjects mathematics and physics. He was especially assiduous and performed strongly in coursework and in school practicum.

Bridget, a 28-year-old woman, studied social sciences in high school, added science and mathematics courses in adult education and studied chemical engineering before switching to teacher education, in mathematics and chemistry. She showed strong engagement, had slight difficulties the first year but found herself at home in the practicum classroom and leading STEM outreach activities.

Charlie, a man, was 30 years old when he started teacher education. Before that, he had tried out three different engineering programs over a period of five years. The first three years in the teacher education program went smoothly but were followed by some difficulties. At the time of the interviews, he was 35 years old, had completed his mathematics courses but lacked the equivalent of one year's studies, including some physics credits, a practicum and a master's thesis. Although he had not completed the full degree, he had finished the mathematics component and could reflect on his development in relation to mathematics studies. He was therefore accepted as a participant in this study.

Darin was a 24-year-old man who came directly from high school to teacher education and studied mathematics and biology. His studies proceeded smoothly. At the time of the interview, he had secured employment as a teacher to start after the summer.

Ethical considerations

As a director of studies, teacher and examiner for some courses and thesis work, I was in contact with the participants all along their studies, with authority over decisions concerning grades, prerequisites and advising. I also acted as employer for some participants working in outreach programs. I had detailed knowledge about the program and previous informal observation of student development, between their first semester and first contact with school practice, via mathematical coursework, to the final presentation of their thesis. The close contact gave me some of the advantages of an observer in an ethnographic study (Bryman, 2016). However, it entailed the risk that the research study would interfere with my role in the program as a course examiner or director of study. To avoid such interference, I chose to restrict data collection to a period after the participants had finished the degree requirements in mathematics. Another option would have been to choose participants outside the program, or to ask another researcher to conduct the interviews, providing me with anonymized transcriptions. Such a scheme would however have prevented me from using my knowledge about the program, courses and students that was valuable in making connections and interpreting the data.

Participation was voluntary and the participant was informed about the possibility of leaving at any time. Data was kept on secure university servers and presented only with pseudonyms so as not to be traceable to the participants. No ethical review was required according to national guidelines.

Results

The themes presented in this section gather the most significant reified narratives where participants expressed their relation to mathematics (Sfard & Prusak, 2005b). In fact, one notable feature of the narratives is that the participants offer many observations about themselves and fewer about mathematics per se or about teaching.

Four themes were formulated in the analysis, that show the participants' expressions of change and development in their identity as mathematics practitioners. The first three themes, *interest in mathematics, conception of mathematics,* and *mathematical self-efficacy as a student and as a teacher*, pertain to the students' identity in relation to the subject as a whole. The last one, *using mathematical representations and notations*, reflects how participants talk about their fluency in the language of mathematics.

Interest in mathematics

All participants describe a shift in their interest in mathematics, from a focus on computing answers to exercises, to an engagement in understanding mathematical contexts and curiosity for mathematics per se.

> So, you develop a will to not just look at the question, but more look at the problem. What more does it contain, what can I do about it, what parts did I actually need, that type of things. That's what I think (Adam).

One participant expresses a growth in interest in mathematics in general, without specifying what she regards as mathematical:

> But oh, now I think actually I got a more, like, nerdy interest in mathematics during my studies, that it's fun to think about math for its own sake. Before, I wasn't really there (Bridget).

Students' interest for mathematics may stretch to mathematical curiosity outside of course content, but in the interviews, it was mostly exemplified with assignments and school curriculum or closely related question.

> Now I can take some time to understand for instance how the quadratic formula works, anyhow, instead of just memorizing how I use it (Darin).

> If I throw a six, a six four times in a row, somehow now I can think about if the dice is loaded or not. I have, like, tools to start figuring out an answer to that question (Charlie).

All four participants express satisfaction and interest in mathematics as a subject. All four have also studied a natural science subject (physics, chemistry, or biology); however, none of them mention modelling for applications in the sciences, nor do they mention interest in deeper theory or training in logical thinking or problem solving.

Conception of mathematics

The participants express a view of the field as a connected whole, contrasting with a memory of viewing it as a collection of separate, context-bound methods.

> Just that mathematics isn't methods and algorithms, instead it's, like, things that are connected. There was that sort of a change in perspective, I think (Charlie).

The image they convey of their earlier view of solving problems or of teaching is that of going down a narrow, compulsory path, while their new identity places them in a more open landscape, with alternatives and the possibility of tracing their own way.

> And now, I see more, what is this actually, what tools are there that you can use, instead of seeing, "How do they do it in the book?" To see it like it's all part of a whole somehow. And there doesn't have to be one specific method that's bound to just that assignment, you can use your knowledge in general somehow to solve it (Bridget).

Seeing mathematical problems and methods as part of a whole empowers the participant to find her own way to a solution and supports self-confidence.

Mathematical self-efficacy as a student and as a teacher

Participants claimed a heightened self-confidence as problem solvers, both accepting that they can temporarily feel stuck with a problem and knowing that they will eventually find a way to handle it. This corresponds to Bandura's concept of self-efficacy (Bandura, 1997) and is an important part of students' identity.

The progression that Darin expressed as a change in interest in mathematics, from applying a formula to understanding how it works, serves as a base for Charlie's increased self-confidence. While high-school students are required to use formulas but not always expected to know why they hold, future teachers learn to reflect over their computations and transform a 'shot in the dark' to a well-connected, justified action.

> I'm much more confident in knowing if what I do is right, and how I can test if it's correct, what I got. Then if, if I get somewhere, I feel more sure that it really is like that. Because, somehow, it's a bit less of a shot in the dark. It's more: I know that you can do like this (Charlie).

Participants now feel confident that they have tools to explore and handle problems even when they don't find a solution method from the start.

> Whereas now, if I for example didn't know something, then I would still try to get around the problem, or to find some way to get some understanding. You can test quite a lot, see if it works, and in the end, I would have been satisfied with something that feels good (Darin).

Self-confidence also supports an identity as a person who can use mathematics as a tool for modelling:

> If I'm curious about something, well, I can gather some data, and then see what it says. Maybe I didn't feel that self-confidence before (Charlie).

Stepping into a teacher identity, Bridget can discuss problems with pupils without going into specific solutions. The sense of connectedness in mathematics, as lifted above, plays a role, as opposed to a view of mathematics as a set of isolated methods:

> I know that when I had my first school practices, I couldn't help students who worked with tasks that I hadn't trained on before. Because I didn't know what I should tell them. And then I have, like, even if I don't know, I can reason with them. And we can get somewhere together, even if you don't know exactly how the book does it (Bridget).

Bridget was the only one of the four interviewees who framed her own development from a teacher's perspective. For her as a teacher, the possibility of reasoning together with students lightens the burden of responsibility for having all the answers and instills self-efficacy. It also connects to the way Adam expressed the change in his interest in mathematics, that the value of working with a task lies not only in solving it but also in considering the issues that it raises. It shifts the view of where the teacher's focus should be, from providing an answer, or a step on the right path, to helping develop the student's thinking.

Using notation and representations

The problem that was used as a starting point for the interview invited both graphical and algebraic interpretations. Participants used both forms and discussed their use of several representations simultaneously, as well as using mathematical language and notations. This fluency in the use of representations can be seen in terms of competence, as a pure mathematical skill, but the participants also express that they have changed and have become persons who use mathematical language as an instrument in the sense of Verillon & Rabardel, (1995), which makes the fluency in this language a part of their identity.

> I started with a graph. I don't think I would have used that before (Adam).

> That you test it then with another representation form to see if it's right (Bridget).

Even in commenting courses, participants showed appreciation for the work of translating between different representations as a way to learn new properties and develop a sense of whole.

> In the analysis course, there was a part where we worked with different sorts of curves, where we needed to, like, imagine what a saddle looked like … There was quite a lot to think about, how the algebra corresponds to the graphical representation (Adam).

The participants pointed out repeatedly that they developed an ease in using mathematical language and notations. In the problem used in the interview, the notation f^{-1} was used for an inverse function. This prompted reflections about differences in attitude while dealing with unfamiliar notation, from giving up to embracing new expressions.

> I think that the difference would have been that if it had been me in high school, I would have, I wouldn't have known this, several words I wouldn't have grasped them. So, I feel that I would have come to a wall and then just stopped there (Darin).

Going beyond the affective aspect of being confronted with unfamiliar mathematical language, students can use different strategies in coping with the situation.

> Now I didn't think it was problematic to write it so. Just that you write like this, f, and then "minus one". I don't think it would have been so before, to write it like that. I think it would have felt uncomfortable (Adam).

The participants have both acquired new representations and integrated them in their own ways of exploring and expressing mathematics.

Recollections of the development process

Beyond the themes described above, the interviews also yielded some recollections of the learning process associated to identity development. When describing the dynamics of their maturing process, the participants emphasized the complex and long-term nature of their development as mathematical thinkers. They described developing in small steps, in everyday routines rather than in single Aha!-moments.

> It's hard to find an isolated thing. It feels like, functions, it's central all over the place. […] We worked with it more and more, working with it more and more. (Adam)

Participants often pointed to the capstone course on mathematical modelling, meant to foster problem-solving skills and connections, where they worked in pairs with modelling assignments for which they did not have set methods, and where they were free to use all their mathematical background to find ways to grapple with problems. Several also mentioned a course in Number Theory which was tailored to the teacher education program in the sense that students systematically give oral presentations for each other. However, almost every mathematics course in the program, including school practice, was mentioned by at least one of the participants, which indicates a long-term development happening over different terrain, benefiting from different approaches and different content.

The changes described here are gradual modifications and do not carry a strong emotional charge, contrasting with results from several studies of elementary teachers whose designated identity involves a radically new relation to mathematics for teaching and is presented in contrast to an earlier negative identity (Jones et al., 2000; Kaasila et al., 2012; Lutovac & Kaasila, 2011; McCulloch et al., 2013).

Together, the participants' expressions paint a picture of a slow but significant maturing process in the students' mathematical identity, with a clear engagement in mathematics, a changed approach to learning and doing mathematics, heightened self-efficacy and ease in the use of representations and notations.

Discussion

The aim of the present study was to contribute to the understanding of pre-service teachers' experience of studying mathematics and bring to light their developing relation to the subject. With only four participants, the study cannot claim to represent most students nor to dig deep or show variations in mathematical identity. However, as an explorative study, it does give elements to answer the research question: How do graduating secondary school teachers in mathematics describe the evolution of their mathematical identity during their university studies?

The following paragraphs discuss results in relation to findings and discussions in earlier research and discuss limitations and methodological challenges faced in this study.

View of mathematics

The conception of mathematics that participants exhibit here, both as increased interest and in the view of mathematics as a connected whole, indicates a change in orientation, as defined by Reid et al. (2005). In their new conception of mathematics, the participants describe a development from the intention of acquiring tools and skills (in an orientation focused on techniques) to an approach focused on mathematical elements, considering mathematics as a subject per se. In the description of Reid et al. (2005), this approach is associated with both the intention to understand mathematics and the intention to help others with mathematics, an appropriate orientation for a teacher. Note that the most abstract category of learning orientation in Reid et al. (2005), *Life*, with its focus on understanding beyond mathematics and an ambition for developing a mathematical way of thinking, was not displayed by any of the participants in this study.

However, the picture of mathematical identity described here is in sharp contrast to the identities described by students in Bartholomew et al. (2011), where the idea of 'natural ability' dominates the students' discourse and some describe losing their mathematical identity, or longing for it, picturing a static state and not the possibility of developing in relation to mathematics. None of the participants here even mentioned personal ability, nor personal difficulties for that matter. The results of this study also diverge from the study on transition from secondary to tertiary mathematics studies by Di Martino and Gregorio (2019), where confrontation with university mathematics is described as a deep crisis, both by successful students and by student who had dropped out. Even in the autobiographical essays of Beccuti et al. (2024), written by students who were successful in their studies, mathematics is described as an arduous challenge requiring ascetic discipline, a view never mentioned by the participants here. It is difficult to explain the differences between the results above and the current study. They could be due to the teacher program offering less challenging mathematics, or to fundamental differences in student identity between pre-service teachers and other types of students.

School mathematics vs. university mathematics

Note also that the examples mentioned by participants showing increased interest in mathematics, such as the quadratic formula, come mostly from school mathematics and not from the advanced courses and abstract mathematics that they had most recently studied. By thinking of school mathematics and university mathematics as different domains, as in Di Martino & Gregorio, (2019), the students here may have kept their focus on school mathematics even though they studied university mathematics, and thus perhaps steered clear from some of the traumatic aspects of the transition to university mathematics studies. The integrated teacher education program, with its mix of pedagogical and subject courses and school practice, leaves a possibility to keep school mathematics in focus while developing in one's way of working with mathematics. Other set-ups, where students start with intensive bachelor's studies in mathematics, probably put higher pressure on students to make a complete transition to university mathematics.

Actual and designated mathematical identity

Sfard and Prusak (2005a) distinguish actual and designated identity, with 'actual identity, consisting of stories about the actual state of affairs, and designated identity, composed of narratives presenting a state of affairs which, for one reason or another, is expected to be the case'.

The participants in this study showed an increased interest in mathematics, a view of mathematics as a connected whole, heightened self-efficacy as a student and as a teacher, and felt they had become competent at using mathematical language and notations. These themes, constructed from the participants' own expressions, contribute to their actual mathematical identity after teacher education.

In their analysis of two groups of 11th-graders' mathematical learning, Sfard and Prusak (2005a) related differences in the groups' learning process to differences in the designated identity of the two groups, using knowledge from interviews of the students and their parents, as well as knowledge about their communities. In the present study, no independent data on designated identity is available. One way to inform designated identity for pre-service mathematics teachers is to turn to research on faculty expectations and on student identity in a wider context. Regarding faculty expectations, the themes appearing in this study's results differ from the qualities that faculty members and researchers use to characterize mathematical maturity (Faulkner et al., 2019; Simelane & Engelbrecht, 2024). The use of symbols and mathematical language figure in both, but the other categories of this study do not, while modelling real-world phenomena, seeing and abstracting patterns, logical reasoning or problem solving, which feature prominently in faculty expectations, are not mentioned here. In fact, participants describe their personal relation to working with mathematics and the changes in their perception of how they do mathematics, rather than talking about acquiring knowledge or about its content and form, which are the aspects of designated identity emphasized by the curriculum and by faculty. This points to a gap between faculty goals and expectations on the one hand, and student preoccupations on the other. The differences outlined here, between the designated identity expressed in faculty expectations of mathematical maturity and this study's results, show that the approach taken here to study mathematical identity from a student's perspective, can yield knowledge complementary to that obtained by faculty within regular courses and examination. It can help teacher educators and university faculty understand how their students view their studies.

Limitations and methodological challenges

The small sample in this study, as well as the homogeneity of the participants, coming from the same program, limit the scope of the conclusions that can be drawn. This study is by no means exhaustive, but it gives indications that can be used to design further research.

Given the dropout rate of approx. 50% in the teacher education program, the students who completed all mathematics courses and agreed to be interviewed here can be counted amongst the stronger and more motivated students from the group that started teacher education five years earlier and are not representative of the whole entering class in that sense. Nevertheless, insights into how these specific students evolved during the program and how they attend to the subject can help teacher educators focus teaching and coaching for future classes, to help more students achieve a similar development. Parallels can be drawn to studies that took into account both successful students and students who had dropped out of their programs, such as (Di Martino & Gregorio, 2019).

As mentioned earlier, my position as interviewer in this study was not neutral: the participants knew me as a teacher, examiner, director of studies and in some cases as an employer. This may have affected the interviews positively in enabling the participants to express themselves personally, but it entailed a risk that the participants tailor their answers to what they thought I wished to hear. As noted by Graven and Heyd-Metzuyanim (2019), the context and audience influence all narratives. The established relationship, in addition to the choice of task and the way I formulated the questions, molded the conversation. For instance, it is difficult to interpret the fact that participants omitted issues that could be important in their mathematical identity, e.g. about the mathematical methods and concepts that they have learned, or about using mathematics in their other teaching subject (physics, chemistry, biology). It may be an effect of their figuring that I already knew about course content or that I would not be so interested in how they reasoned outside of my field. On the other hand, my detailed knowledge of the program and familiarity with the students enabled me to formulate and follow up questions and interpret responses with a closeness comparable to that of a teacher conducting action research (Bryman, 2016), and with much more information than what an external observer could have done.

My knowledge of the program and students, while it had advantages as noted above, also introduced a risk for bias in my interpreting the interviews. In order to mitigate the risk of over-interpreting participant narratives based on impressions acquired outside of the study, I made a point of consulting with researcher colleagues not involved in the program to check interpretations during the theme-development phases of the analysis.

Another difficulty concerns capturing the process of change in identity. The narratives used here are retrospective, based on how the participants see themselves and how they remember their experiences in the program. They are essentially subjective, and are secondary, in the sense that they are based on participants' interpretations of their own memories, in a way similar to the mathematical life stories used by Drake (2006) to link teacher's mathematical history to their teaching practice. This may or may not accurately represent what actually motivated them or determined how they worked with mathematics earlier. Some of the informants may have already described themselves in secondary school as having a wide interest in mathematics and in understanding problems, not just finding answers, for example, but now attributed this interest to their university studies. This study captures traces of the participant's development, as interpreted by them at the time of the interview. One can argue that the retrospective view yields a weak proxy for how the development process actually took place, but one can also value the retrospective narrative as such. It is the narrative that forms the actual identity, the perceived experience that is the basis for how the person relates to mathematics now and in the future.

Listening to the participants' own words, I could hear some of what matters to them in their studies. The results shed light on issues that are not usually brought up explicitly in teacher education but are at the heart of students' development in relation to mathematics.

Implications

The results of this study give some insight into mathematical identity of secondary school pre-service teachers and its development during teacher education. The findings show that participants described the evolution of their mathematical identity in terms of increased interest in mathematics, of a wider conception of what mathematics is, of self-efficiency in doing mathematics and in teaching, and of becoming a comfortable user of mathematical language and representations. This section discusses implications for teacher education and for future research.

Kaasila et al. (2012) found that students had developed a more multi-faceted narrative during their studies, including self-development and gain of insight. They conclude with a recommendation that pre-service elementary school teachers be given opportunities to reflect on their own experience and search for new perspectives on their views of mathematics. The present study suggests that such opportunities could be useful even for secondary school teachers. Teacher educators could consider interventions to support identity development during pre-service or in-service programs, both to help students such as those interviewed here to come further in their development, and to help students at risk of dropping out to develop appropriate aspects of their identity. For instance, the themes appearing in the results of this article could be used as a starting point for a reflective essay or a seminar discussion. Neumayer-Depiper (2013), McCulloch et al. (2013) and Lutovac and Kaasila (2011) also give examples of how students can work explicitly with identity issues, using autobiographical narratives.

Data in this study comes from an integrated 5-year teacher education program where subject studies alternate with pedagogical parts and practicum. The selection of students attending such a program can differ from those who choose to start by studying mathematics as such (Bengmark et al., 2025). In particular, students who describe themselves as not having been directly interested in the subject from the start, as expressed here in the results, may not have taken the opportunity to become mathematics teachers in a structure that required first majoring in the subject. However, the combination of studies in the program and the comparatively long period of time afforded opportunities to develop both interest and self-efficacy. In the context of Sweden, as in many other countries suffering from a lack of qualified mathematics teachers, this yields an argument in favor of offering integrated programs for the preparation of secondary school teachers.

While an integrated teacher education program is a natural place to start exploring the mathematical identity of pre-service teachers, more research is needed to see to what extent results generalize to other forms of teacher education, and especially to students who first focus entirely on the subject before specializing in teacher education. If their mathematical identity shows similar patterns to those in the present study, contrasting with the passionate image found in (Bartholomew et al., 2011; Beccuti et al., 2024), it may be appropriate to re-consider the curriculum for students aiming to become teachers, to emphasize engagement and deep understanding of fundamental part of mathematics, as illustrated in Liping Ma's analysis of primary school teachers' knowledge (Ma, 2010).

To complement this study, it would also be interesting to conduct a longitudinal study, following students through teacher education, to describe how the development actually takes place. Logistical challenges, related to the length of the program and the high attrition rate, could be handled if students were surveyed as part of an early course in the program for example, as in the work of Kaasila et al. (2012). Challenges related to synthesizing the data may be more difficult to tackle. Bjuland et al. (2012), whose data included material from five different situations during a 2-year period, note that the situated nature of identity and data limited to isolated occasions make it difficult to trace change and development.

A more fruitful approach may be to focus on one or a few participants' narrative as a whole, using narrative analysis. Data in the present study used participant narratives only to cut out excerpts, which were subjected to thematic analysis. Now that we have an empirically based picture of some central themes occurring in pre-service teachers' mathematical identity, it would be interesting to undertake a new study focusing on narratives as a whole, to analyse the process (or the retrospective view of the process) by which interest in mathematics, view of the field, self-efficacy and fluency in mathematical language as well as other aspects of students' mathematical identity, developed.

In conducting this study, I came closer to some of my students and can now paint a richer and more accurate picture of their identity and motivation. I can also understand the background to the question my student asked about whether mathematics courses were meant 'to select only students who are really interested in mathematics'. Listening attentively to who they are, has made me a more attentive teacher and I can warmly recommend other faculty to engage in a similar process.

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CRediT: **Laura Fainsilber**: Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Writing – original draft, Writing – review & editing.

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ORCID

Laura Fainsilber http://orcid.org/0000-0002-2516-1420

Data availability statement

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References

Arksey, H., & Knight, P. (1999). *Interviewing for social scientists*. SAGE Publications, Ltd. https://doi.org/10.4135/9781849209335

Arslan, O., Çelikdemir, K., & Haser, Ç. (2025). Mathematics-related teacher identity profiles of preservice elementary school teachers and middle school mathematics teachers: A latent class analysis. *Teaching and Teacher Education*, *165*, 105159. https://doi.org/10.1016/j.tate.2025.105159

Bandura, A. (1997). *Self-efficacy the exercise of control*. W. H. Freeman.

Barter, C., & Renold, E. (1999). The use of vignettes in qualitative research. *Social Research Update*, *25*(9), 1–6.

Bartholomew, H., Darragh, L., Ell, F., & Saunders, J. (2011). I'm a natural and I do it for love!': exploring students' accounts of studying mathematics. *International Journal of Mathematical Education in Science and Technology*, *42*(7), 915–924. https://doi.org/10.1080/0020739X.2011.608863

Beccuti, F., Valero, P., & Robutti, O. (2024). Stories of devoted university students: The mathematical experience as a form of ascesis. *Educational Studies in Mathematics*, *115*(1), 51–67. https://doi.org/10.1007/s10649-023-10259-4

Beijaard, D., Verloop, N., & Vermunt, J. D. (2000). Teachers' perceptions of professional identity: An exploratory study from a personal knowledge perspective. *Teaching and Teacher Education*, *16*(7), 749–764. https://doi.org/10.1016/S0742-051X(00)00023-8

Bengmark, S., Fainsilber, L., & Gustafsson, T. (2025). What graduates want in teacher education. *Scandinavian Journal of Educational Research*, *69*(3), 524–536. https://doi.org/10.1080/00313831.2024.2318651

Biza, I., Jaworski, B., & Hemmi, K. (2014). Communities in university mathematics. *Research in Mathematics Education*, *16*(2), 161–176. https://doi.org/10.1080/14794802.2014.918351

Bjuland, R., Cestari, M. L., & Borgersen, H. E. (2012). Professional mathematics teacher identity: Analysis of reflective narratives from discourses and activities. *Journal of Mathematics Teacher Education*, *15*(5), 405–424. https://doi.org/10.1007/s10857-012-9216-1

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Bryman, A. (2016). *Social research methods*. Oxford university press.

Chapman, O. (2002). Belief structure and inservice high school mathematics teacher growth. In *Beliefs: A hidden variable in mathematics education?* (pp. 177–193). Springer.

Darragh, L. (2016). Identity research in mathematics education. *Educational Studies in Mathematics*, *93*(1), 19–33. https://doi.org/10.1007/s10649-016-9696-5

Di Martino, P., & Gregorio, F. (2019). The mathematical crisis in secondary–tertiary transition. *International Journal of Science and Mathematics Education*, *17*(4), 825–843. https://doi.org/10.1007/s10763-018-9894-y

Di Martino, P., & Zan, R. (2010). Me and maths': Towards a definition of attitude grounded on students' narratives. *Journal of Mathematics Teacher Education*, *13*(1), 27–48. https://doi.org/10.1007/s10857-009-9134-z

Drake, C. (2006). Turning points: Using teachers' mathematics life stories to understand the implementation of mathematics education reform. *Journal of Mathematics Teacher Education*, *9*(6), 579–608. https://doi.org/10.1007/s10857-006-9021-9

Fainsilber, L. (2024, March 19–20). Teacher students' identity as mathematics practitioners. In *MADIF 14: The fourteenth research conference of the Swedish Society for Research in Mathematics*, Education, Örebro. http://matematikdidaktik.org/wp-content/uploads/2024/10/18_MADIF14.pdf

Faulkner, B., Earl, K., & Herman, G. (2019). Mathematical maturity for engineering students. *International Journal of Research in Undergraduate Mathematics Education*, *5*(1), 97–128. https://doi.org/10.1007/s40753-019-00083-8

Gholson, M. L., & Martin, D. B. (2019). Blackgirl face: Racialized and gendered performativity in mathematical contexts. *ZDM*, *51*(3), 391–404. https://doi.org/10.1007/s11858-019-01051-x

Goos, M. (2005). A sociocultural analysis of the development of pre-service and beginning teachers' pedagogical identities as users of technology. *Journal of Mathematics Teacher Education*, *8*(1), 35–59. https://doi.org/10.1007/s10857-005-0457-0

Graven, M., & Heyd-Metzuyanim, E. (2019). Mathematics identity research: The state of the art and future directions. *ZDM*, *51*(3), 361–377. https://doi.org/10.1007/s11858-019-01050-y

Hannula, M. S., Di Martino, P., Pantziara, M., Zhang, Q., Morselli, F., Heyd-Metzuyanim, E., Lutovac, S., Kaasila, R., Middleton, J. A., & Jansen, A. (2016). *Attitudes, beliefs, motivation and identity in mathematics education: An overview of the field and future directions*. Springer Nature. https://doi.org/10.1007/978-3-319-32811-9

Hernandez-Martinez, P., Williams, J., Black, L., Davis, P., Pampaka, M., & Wake, G. (2011). Students' views on their transition from school to college mathematics: Rethinking 'transition' as an issue of identity. *Research in Mathematics Education*, *13*(2), 119–130. https://doi.org/10.1080/14794802.2011.585824

Heyd-Metzuyanim, E. (2015). Vicious cycles of identifying and mathematizing: A case study of the development of mathematical failure. *Journal of the Learning Sciences*, *24*(4), 504–549. https://doi.org/10.1080/10508406.2014.999270

Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: On learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research*, *51–52*, 128–145. https://doi.org/10.1016/j.ijer.2011.12.015

Jones, L., Brown, T., Hanley, U., & McNamara, O. (2000). An enquiry into transitions: From being a 'learner of mathematics' to becoming a 'teacher of mathematics'. *Research in Education*, *63*(1), 1–10. https://doi.org/10.7227/RIE.63.1

Kaasila, R., Hannula, M. S., Laine, A. (2012). 'My personal relationship towards mathematics has necessarily not changed but … ' Analyzing preservice teachers' mathematical identity talk. *International Journal of Science and Mathematics Education*, *10*(4), 975–995. https://doi.org/10.1007/s10763-011-9308-x

Losano, L., Fiorentini, D., & Villarreal, M. (2018). The development of a mathematics teacher's professional identity during her first year teaching. *Journal of Mathematics Teacher Education*, *21*(3), 287–315. https://doi.org/10.1007/s10857-017-9364-4

Lutovac, S., & Havia, J. (2024). Prospective mathematics teachers' collective identity work: Navigating failure experiences and concerns about relating to students. *NOMAD Nordic Studies in Mathematics Education*, *29*(3-4), 19–40. https://doi.org/10.7146/nomad.v29i3-4.150751

Lutovac, S., & Kaasila, R. (2011). Beginning a pre-service teacher's mathematical identity work through narrative rehabilitation and bibliotherapy. *Teaching in Higher Education*, *16*(2), 225–236. https://doi.org/10.1080/13562517.2010.515025

Lutovac, S., & Kaasila, R. (2018). Future directions in research on mathematics-related teacher identity. *International Journal of Science and Mathematics Education*, *16*(4), 759–776. https://doi.org/10.1007/s10763-017-9796-4

Lutovac, S., & Kaasila, R. (2019). Methodological landscape in research on teacher identity in mathematics education: A review. *ZDM*, *51*(3), 505–515. https://doi.org/10.1007/s11858-018-1009-2

Ma, L. (2010). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States*. Routledge.

Maher, C. A., & Sigley, R. (2020). Task-based interviews in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 821–824). Springer International Publishing. https://doi.org/10.1007/978-3-030-15789-0_147

Malterud, K., Siersma, V. D., & Guassora, A. D. (2016). Sample size in qualitative interview studies: Guided by information power. *Qualitative Health Research*, *26*(13), 1753–1760. https://doi.org/10.1177/1049732315617444

Marschall, G. (2022). The role of teacher identity in teacher self-efficacy development: The case of Katie. *Journal of Mathematics Teacher Education*, *25*(6), 725–747. https://doi.org/10.1007/s10857-021-09515-2

McCulloch, A. W., Marshall, P. L., DeCuir-Gunby, J. T., & Caldwell, T. S. (2013). Math autobiographies: A window into teachers' identities as mathematics learners. *School Science and Mathematics*, *113*(8), 380–389. https://doi.org/10.1111/ssm.12041

Mitiche, A. (2024). Redefining professional and academic identities turning points in pre-service and novice mathematics teachers' development of lived experience in and after short-track teacher education. *Cogent Education*, *11*(1), 2417626. https://doi.org/10.1080/2331186X.2024.2417626

Mitiche, A. (2025). Pre-service mathematics teachers' professional identities: A systematic literature review. *Review of Education*, *13*(2), e70078. https://doi.org/10.1002/rev3.70078

Neumayer-Depiper, J. (2013). Teacher identity work in mathematics teacher education. *For the Learning of Mathematics*, *33*(1), 9–15.

Petersson, H. (2017). *Undersökande matematik – Differentierade problem* (1st ed.). Studentlitteratur.

Petocz, P., Reid, A., Wood, L. N., Smith, G. H., Mather, G., Harding, A., Engelbrecht, J., Houston, K., Hillel, J., & Perrett, G. (2007). Undergraduate students' conceptions of mathematics: An international study. *International Journal of Science and Mathematics Education*, *5*(3), 439–459. https://doi.org/10.1007/s10763-006-9059-2

Piñero Charlo, J., & Canto López, M. (2026). Teacher knowledge and teacher identity in mathematics education: An interdependent model [version 2; peer review: 2 approved]. *Open Research Europe*, *5*, 246. https://doi.org/10.12688/openreseurope.21108.3

Reid, A., Wood, L. N., Smith, G. H., & Petocz, P. (2005). Intention, approach and outcome: University mathematics students' conceptions of learning mathematics. *International Journal of Science and Mathematics Education*, *3*(4), 567–586. https://doi.org/10.1007/s10763-004-5818-0

Sfard, A., & Prusak, A. (2005a). Identity that makes a difference: Substantial learning as closing the gap between actual and designated identities. *International Group for the Psychology of Mathematics Education*, *1*, 37–52.

Sfard, A., & Prusak, A. (2005b). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14–22. https://doi.org/10.3102/0013189X034004014

Simelane, B., & Engelbrecht, J. (2024). Measuring the mathematical maturity of students in an Academic Development Programme. *International Journal of Research in Undergraduate Mathematics Education*, *10*(2), 577–606. https://doi.org/10.1007/s40753-023-00222-2

Verillon, P., & Rabardel, P. (1995). Cognition and artifacts: A contribution to the study of though in relation to instrumented activity. *European Journal of Psychology of Education*, *10*(1), 77–101. https://doi.org/10.1007/BF03172796

Weller, J. (2022). Secondary pre-service teachers' voices of becoming mathematics teachers. *Teachers and Teaching*, *28*(7), 843–858. https://doi.org/10.1080/13540602.2022.2137136

Wood, L. N., Petocz, P., & Reid, A. (2012). What do mathematics students say about mathematics internationally? In L. N. Wood, P. Petocz, & A. Reid (Eds.), *Becoming a mathematician: An international perspective* (pp. 53–74). Springer Netherlands. https://doi.org/10.1007/978-94-007-2984-1_4

Xenofontos, C., & Andrews, P. (2023). The experiential construction of mathematics teacher identity and the impact of early mathematical failure. *Frontiers in Education*, *8*. https://doi.org/10.3389/feduc.2023.1158973

Zhang, Y., & Wang, P. (2021). Twenty years' development of teacher identity research: A bibliometric analysis. *Frontiers in Psychology*, *12*. https://doi.org/10.3389/fpsyg.2021.783913

Appendix

Mathematical problem given during the interviews

The problem gives a parametrized family of functions, with instructions to find the intersection between the graph of the function and the graph of its inverse, first in easy special cases, then in general (Peterson, 2017). It was translated for this paper by the author.

Inverses and intersection points

Let a denote a positive number and consider the function:

$$f(x) = \frac{x}{x-a}$$

defined for $x > a$.

This function has an inverse, and this problem is about exploring possible intersection points between the graphs for f and its inverse f^{-1}.

a. Find an expression for f^{-1} and find its domain, when $a = 2$.
b. Are there intersection points between the graphs for f and f^{-1} when $= 2$?
c. Are there intersection points between the graphs for f and f^{-1} when $= 1$?
d. Explore how intersection points for the graphs for f and f^{-1} depend on the value of a.