



Role of Spin-Isospin Symmetries in Nuclear β -Decays

Downloaded from: <https://research.chalmers.se>, 2026-07-29 09:51 UTC

Citation for the original published paper (version of record):

Li Muli, S., Djarv, T., Forssén, C. et al (2026). Role of Spin-Isospin Symmetries in Nuclear β -Decays. Physical Review Letters, 136(24). <http://dx.doi.org/10.1103/vk51-b476>

N.B. When citing this work, cite the original published paper.

Role of Spin-Isospin Symmetries in Nuclear β -Decays

Simone Salvatore Li Muli^{1,*}, Tor R. Djärv^{2,†}, Christian Forssén^{1,‡}, and Daniel R. Phillips^{3,1,§}

¹Department of Physics, Chalmers University of Technology, SE-412 96 Göteborg, Sweden

²Computing and Computational Sciences Directorate, Oak Ridge National Laboratory, Oak Ridge, Tennessee, USA

³Department of Physics and Astronomy and Institute of Nuclear and Particle Physics, Ohio University, Athens, Ohio 45701, USA



(Received 31 March 2025; revised 1 December 2025; accepted 2 April 2026; published 17 June 2026)

A century ago, Wigner's SU(4) symmetry was introduced to explain the properties of atomic nuclei. Despite recent revived interest, its impact on nuclear structure, transitions, and reactions has not been fully explored. Here, we show that a variety of high-fidelity nuclear interactions predict nuclear states that have $\geq 90\%$ probability of being in a single SU(4) irreducible representation. Meanwhile, our analysis of axial current operators in chiral effective field theory (χ EFT) reveals that one-body currents at low momentum transfer act only within SU(4) irreducible representations, while two-body currents connect different representations. These selection rules interfere with the expected convergence pattern of the χ EFT expansion and explain key phenomenological observations, e.g., the unnaturally large two-body corrections to the axial-current matrix elements in eight-body nuclei.

DOI: [10.1103/vk51-b476](https://doi.org/10.1103/vk51-b476)

Introduction—Nuclear β -decay transmutes atoms of one element into atoms of a different element. Over the last decades, first-principles calculations of β -decay have deepened our understanding of this low-energy manifestation of the standard model's weak interactions [1–12]. In Ref. [6], *ab initio* many-body calculations using a low-energy effective field theory of QCD explained observed β -decay rates in elements from tritium to tin—without the need for the empirical quenching factors employed in the nuclear shell model. Recent *ab initio* calculations like these facilitate the use of precise measurements of β -decay as an arena for tests of the standard model [13–19].

In this Letter, we show that the pattern of β -decay matrix elements seen in these forefront calculations is driven by a symmetry identified in one of the earliest attempts to understand the systematics of nuclear structure: Wigner's SU(4) spin-isospin symmetry [20,21]. Interest in SU(4) symmetry [from now on, we label Wigner's SU(4) as SU(4) for brevity] in light nuclei was revived in the mid-1990s due to the observation that a (contracted) SU(4) symmetry of nuclear forces emerges in the large- N_c (large number of colors) limit of QCD [22–24]. More recently, lattice QCD

calculations at the SU(3)-flavor symmetric evinced SU(6) symmetry in baryon-baryon scattering amplitudes [25]. Lee *et al.* showed that chiral EFT (χ EFT) potentials have SU(4) symmetry if evaluated at a particular resolution scale and that this symmetry is visible in matrix elements of the nucleon-nucleon (NN) interaction in ^{30}P [26]. Moreover, the leading order (LO) interactions in an EFT for nuclei in which pions have been integrated out are SU(4) invariant [27], and Lin *et al.* recently demonstrated that this explains why certain amplitudes for the M1 threshold capture of photons in the $A = 3$ system are vanishingly small [28]. Such suppressions are not predicted by the power counting of χ EFT, because SU(4) is an emergent symmetry in that EFT.

The relevance of SU(4) to β -decay stems from a long-established consequence of the group's structure: the Gamow-Teller operator, being a generator of the SU(4) algebra, cannot induce transitions between different irreducible representations (irreps) [21,29–31]. While this selection rule is well known, its physical utility hinges on the degree to which nuclei computed from first principles are themselves pure representations of SU(4) symmetric states. To investigate this, we performed *ab initio* no-core shell model (NCSM) calculations for nuclei with $A = 3$ –8. Following a strategy similar to earlier shell-model work in the sd shell [31], we decomposed our resulting wave functions into SU(4) irreps, using an efficient Lanczos-based algorithm well suited to large model spaces [32].

Our novel finding is that *ab initio* wave functions for light nuclei based on a number of sophisticated χ EFT forces are strongly aligned with a single SU(4) irrep. This holds for χ EFT forces constructed at LO as well as NLO and N2LO in the χ EFT expansion. It also holds under a

*Contact author: simone.limuli@chalmers.se

†Contact author: djarvtr@ornl.gov

‡Contact author: christian.forssen@chalmers.se

§Contact author: phillid1@ohio.edu

variety of fitting strategies and regardless of whether a three-nucleon (3N) force is included in the calculation or not. Moreover, our analysis of the axial-current operator derived from χ EFT reveals a specific pattern: while the dominant one-body part of the current is largely block diagonal in SU(4) irreps, two-body currents do not share this property. SU(4) symmetry thus imposes approximate selection rules on low-energy electroweak processes.

The SU(4) group and its irreducible representations—To introduce our original work, we first give a summary of relevant results obtained already by many authors; see, for instance, Refs. [20,21,29–31,33–35]. SU(4) symmetry is invariance under the group of special unitary transformations in spin-isospin space. The fifteen group generators are $T_a = \frac{1}{2} \sum_k \tau_a^{(k)}$, $S_b = \frac{1}{2} \sum_k \sigma_b^{(k)}$, and $G_{ab} = \frac{1}{2} \sum_k \tau_a^{(k)} \sigma_b^{(k)}$, with $a, b = 1, 2, 3$, and k a single-particle index. The quadratic Casimir, C_2 , commutes with all these generators, and so is invariant under SU(4) transformations:

$$C_2 = T^a T_a + S^b S_b + G^{ab} G_{ab}, \quad (1)$$

where the Einstein convention of summing repeated indices is used. States populating identical irreps of SU(4) have the same expectation value of C_2 . They also belong to the same irreps of the permutation group, and so can be classified using Young tableaux. In Table II in the End Matter, we list the irreps of SU(4) with the lowest values for the quadratic Casimir operator in systems of $A = 2$ –8 nucleons. We identify them by their degeneracy $[n]$, the value of the quadratic Casimir $\langle C_2 \rangle$, and the Young tableau specifying the permutational symmetry.

SU(4) structure of nuclear wave functions—We seek to expand a nuclear state $|\Psi_{JM_T}\rangle$, with good total angular momentum J and isospin projection M_T , in terms of irreps of the SU(4) group. Our representation of SU(4) states is based on Ref. [29]; key results from this reference are summarized in the End Matter for convenience. The basis states are constructed as the tensor product of a many-body spatial state $|\tilde{f}; \alpha LM_L\rangle$ and a coupled spin-isospin state $|[f]; STM_S M_T\rangle$. Here, $[f]$ is a Young tableau with dual tableau $[\tilde{f}]$; L, S , and T are the total orbital, spin, and isospin quantum numbers with projections M_L, M_S , and M_T , respectively; while α includes spatial quantum numbers beyond L and M_L . The states $|[f]; STM_S M_T; \alpha LM_L\rangle \equiv |[f]; STM_S M_T\rangle \otimes |\tilde{f}; \alpha LM_L\rangle$ form a complete basis; nuclear states are expanded as

$$|\Psi_{JM_T}\rangle = \sum_{[f]} \sum_{\substack{\alpha LM_L \\ SM_S T}} \langle [f]; STM_S M_T; \alpha LM_L | \Psi_{JM_T} \rangle \times |[f]; STM_S M_T; \alpha LM_L\rangle. \quad (2)$$

To calculate the probability, $P([f])$, of populating a specific irrep $[f]$, one needs to evaluate several scalar products of $|\Psi_{JM_T}\rangle$ with eigenstates of the Casimir. Since the Casimir operator is independent of the nucleons' orbital degrees of freedom, diagonalization in terms of nuclear many-body states increases the degeneracy of the operator factorially. Explicit decomposition becomes prohibitively expensive in large model spaces like those used in many *ab initio* techniques.

The Lanczos algorithm offers an alternative and efficient way to achieve this decomposition [36]. This method has been applied to investigate nuclear properties such as the spin-orbit decomposition of nuclear states [32] and spatial symmetries [37–39]. In this approach, we start the Lanczos iteration to diagonalize the quadratic Casimir using $|\Psi_{JM_T}\rangle$ as the pivot vector $|K_1\rangle$. The Lanczos algorithm then generates a Krylov space $\{|\Psi_{JM_T}\rangle, |K_j\rangle; j = 2, 3, \dots\}$, and diagonalization of the tridiagonal matrix that is the representation of C_2 in this space yields the eigenvalues of C_2 , as well as its eigenvectors, expressed as linear combinations of the Krylov vectors. This allows us to simply read off the amplitude for $|\Psi_{JM_T}\rangle$ to be in any of the states with eigenvalue $\langle C_2 \rangle$. Notably, this decomposition accumulates the probabilities of populating irreps that share the same value for $\langle C_2 \rangle$. This feature does not create problems in our Letter, since all the analyzed irreps have distinct values for $\langle C_2 \rangle$; see Table II in the End Matter.

SU(4) structure of nuclear axial currents—Following Ref. [40], the LO axial-current operator in χ EFT is

$$j_{S,a,b}^{\text{LO}}(\mathbf{q}) = -\frac{g_A}{2} \sum_k e^{i\mathbf{q}\cdot\mathbf{r}^{(k)}} \tau_a^{(k)} \sigma_b^{(k)}, \quad (3)$$

where g_A is the nucleon axial coupling constant, and k is a nucleon label. Up to and including N3LO (order Q^3 relative to leading) in the χ EFT expansion, the only one-body correction to Eq. (3) is a relativistic effect with a prefactor of $1/(8M_N^2)$ and a size of $< 1\%$ relative to LO. The operator in Eq. (3) transforms as a $[211]$ tensor under SU(4). This structure connects states populating different SU(4) irreps, as is shown explicitly for a selection of transitions in the section *SU(4) tensor products* of the End Matter. However, in the long-wavelength limit, the current reduces to the A -body Gamow-Teller operator, a special case of a $[211]$ tensor without orbital structure. Being a generator of SU(4), the Gamow-Teller operator is unable to transition states to different irreps. The approximate SU(4) symmetry of nuclear states therefore leads to approximate selection rules for transitions mediated by the one-body operator in the $\mathbf{q} \rightarrow 0$ limit.

Two-body currents also appear in the χ EFT expansion of the nuclear axial current and enter at N2LO (N3LO) in the version of χ EFT with (without) an explicit $\Delta(1232)$. The complete current up to N3LO can be found in Eqs. (2.5)–(2.11) of Ref. [40] and includes both a short-distance piece

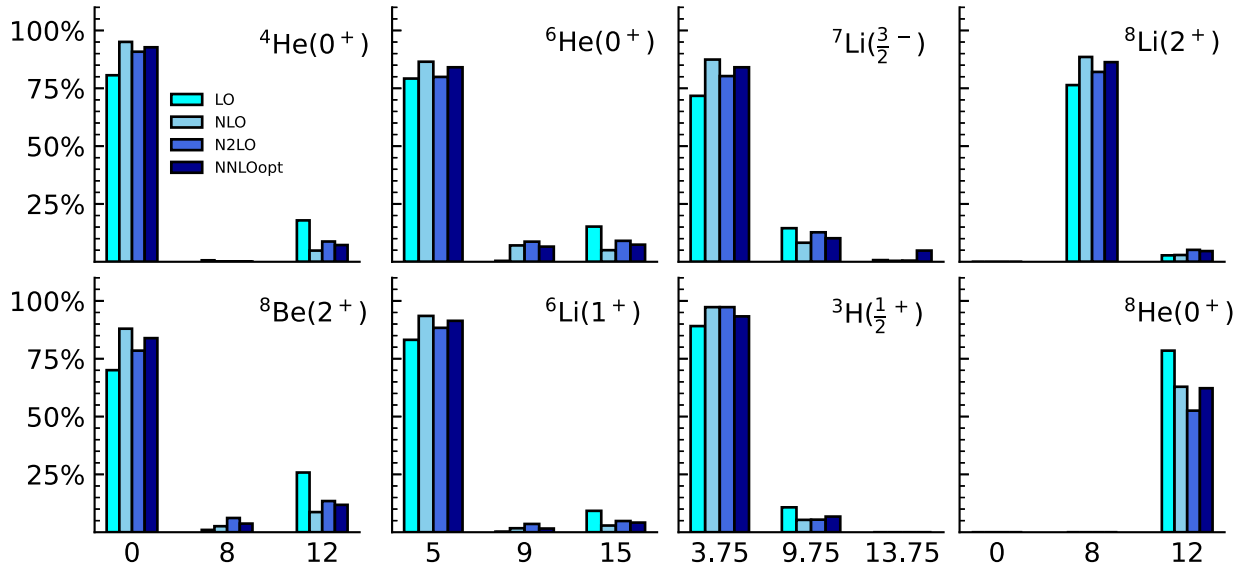


FIG. 1. Probabilities of selected nuclei to populate SU(4) irreps. The irreps are identified by the corresponding value of the quadratic Casimir; see Table II. Nuclear wave functions are obtained from χ EFT interactions at LO (cyan), NLO (light blue), and N2LO (blue) [42]. Results for the NNLOopt [48] interaction—which does not include three-body forces—are in dark blue.

and a (spatial) tensor structure. The spin-isospin structure of the dominant pieces of the two-body current can be reexpressed in terms of two-body SU(4) generators; see Eqs. (C1) and (C2) in the End Matter. As a result, two-body axial current operators are block diagonal in SU(4) irreps of two-particle states. They are not, however, block diagonal in the A -body SU(4) irreps. Thus, while SU(4) symmetry may still generate certain selection rules for two-body transitions, their impact must be studied on a case-by-case basis in A -body systems. In the section *SU(4) tensor products* of the End Matter, we show a selection of SU(4) transitions induced by two-body currents. In future work, we will decompose χ EFT two-body axial currents into SU(4) tensors, thereby yielding more specific selection rules.

Methods—Weak operators and nuclear potentials are calculated using χ EFT. In this framework, nuclear operators are expanded in powers of the small parameter $Q \equiv \max(m_\pi, p)/\Lambda_b$, where m_π is the pion mass, p is the characteristic momentum of nucleons in the nucleus, and $\Lambda_b \approx 500\text{--}600$ MeV is the breakdown momentum of the EFT [41,42]. Previously, we discussed the axial current up to relative order Q^3 in this expansion. To construct nuclear Hamiltonians at the same order, N2LO potentials are needed; these include both NN and 3N forces. χ EFT potentials must be regularized and fit to the data. Both aspects of this procedure produce variability in predictions, even among potentials that are nominally of the same χ EFT order.

We solve the many-body Schrödinger equation using the NCSM [43]. In the NCSM, the Hamiltonian is projected on a harmonic-oscillator basis and becomes a large but sparse matrix. The diagonalization is performed iteratively using

the Lanczos algorithm. The NCSM includes all nucleons as dynamical degrees of freedom, but the full Hilbert space is truncated on total harmonic-oscillator excitations, parametrized by N_{max} . Here, we employ the pAntoine [44–46] and JupiterNCSM [47] codes. The latter implementation handles full 3N forces.

Results—We use the Lanczos method to decompose nuclear states for $A = 3\text{--}8$ in SU(4) irreps. The probabilities, $P([f])$, are shown in Fig. 1, where we label SU(4) irreps by the corresponding value of C_2 . Despite the presence of explicit SU(4)-symmetry breaking in the Coulomb and nuclear forces—and different amounts of breaking in different force implementations—the decompositions of nuclear eigenstates are largely insensitive to the choice of nuclear Hamiltonian and chiral order. We study the sequence of LO (cyan), NLO (light blue), and N2LO (blue) χ EFT interactions from Ref. [42] and the NNLOopt interaction without 3N forces (dark blue) from Ref. [48]. All produce essentially the same SU(4) decompositions. The main components in the SU(4) decomposition also converge quickly with N_{max} . Nuclear states for $A = 3\text{--}8$ have more than 90% of their normalization in a specific SU(4) irrep. When 3N forces are included, as in the N2LO interaction from Ref. [42], the probability of the main component decreases slightly, indicating that 3N operators produce stronger symmetry breaking.

We use our results to explain the β -decay patterns noted in *ab initio* calculations; see, for instance, Ref. [7]. Gamow-Teller matrix elements can be evaluated analytically in the SU(4) limit [29], where nuclear states have L , S , and T as additional good quantum numbers. Following the approach described in the End Matter, we obtain

TABLE I. Reduced Gamow-Teller matrix elements obtained with Eq. (4) compared to several *ab initio* results: our NCSM results with the same LO, NLO, and NNLOopt interactions as in Fig. 1 and the VMC calculation of Ref. [7] with the NV2 + 3-Ia interaction. The second to last column gives the NCSM model space $N_{\max}$, while $\hbar\omega = 16$ MeV for ${}^3\text{H}$, ${}^3\text{He}$, and $\hbar\omega = 20$ MeV for all others. The second column indicates if the transition stays within the same SU(4) irrep.

Transition	Within irrep	Eq. (4)	This Letter (NCSM)				VMC [7]
			LO	NLO	NNLOopt	$N_{\max}$	
${}^3\text{H}(\frac{1}{2}^+) \rightarrow {}^3\text{He}(\frac{1}{2}^+)$	✓	2.449	2.267	2.332	2.313	18	...
${}^6\text{He}(0^+) \rightarrow {}^6\text{Li}(1^+)$	✓	2.449	2.178	2.259	2.260	14	2.200
${}^7\text{Be}(\frac{3}{2}^-) \rightarrow {}^7\text{Li}(\frac{3}{2}^-)$	✓	2.582	2.301	2.375	2.357	12	2.317
${}^7\text{Be}(\frac{3}{2}^-) \rightarrow {}^7\text{Li}(\frac{1}{2}^-)$	✓	2.309	2.086	2.178	2.175	12	2.157
${}^8\text{Li}(2^+) \rightarrow {}^8\text{Be}(2^+)$	✗	0.0	0.018	0.081	0.093	10	0.147
${}^8\text{He}(0^+) \rightarrow {}^8\text{Li}(1^+)$	✗	0.0	0.066	0.364	0.335	10	0.386

$$\begin{aligned}
 &\langle \Psi'_{\text{SU}(4)} | GT_{\pm} | \Psi_{\text{SU}(4)} \rangle \\
 &= (-1)^{L+J+S'+1} \sqrt{8(2J+1)(2S'+1)C_2([f])} \\
 &\quad \times C_{JM,10}^{J'M} C_{TM_T,1\pm 1}^{T'M_T'} W_{[f]ST,[211]11}^{[f']S'T'} \left\{ \begin{matrix} S & L & J \\ J' & 1 & S' \end{matrix} \right\} \delta_{[f'] [f]}, \quad (4)
 \end{aligned}$$

where $W_{[f]ST,[211]11}^{[f']S'T'}$ are reduced Wigner coefficients that generalize Clebsch-Gordan coefficients to the coupling of SU(4) states, $C_2([f])$ is the quadratic Casimir operator of SU(4) evaluated in the $[f]$ irrep, and the last two terms are a Wigner 6- j coefficient and a Kronecker delta on the SU(4) irreps.

In Table I, we compare matrix elements evaluated using Eq. (4) to *ab initio* NCSM calculations using LO, NLO, and NNLOopt χ EFT interactions and the variational Monte Carlo (VMC) calculations of Ref. [7]. For SU(4)-allowed transitions, two observations are noteworthy. First, Eq. (4) gives results remarkably close to the *ab initio* computations—the relative difference for these transitions is less than 10%, irrespective of the interaction. Second, the Gamow-Teller matrix element is very stable with respect to the order of the χ EFT interaction used. For example, in ${}^6\text{He}$, the result with a leading-order χ EFT wave function is within 4% of the results at $O(Q^3)$ (albeit without three-body forces). This happens because the matrix element is symmetry protected. The emergence of approximate SU(4) symmetry means that all models yield matrix elements for Gamow-Teller operators that are close to the prediction of Eq. (4).

Furthermore, our findings also explain the suppression of Gamow-Teller matrix elements in $A = 8$ nuclei observed in Refs. [6,7], because these transitions are SU(4) forbidden. While the wave function of ${}^8\text{Be}(2^+)$ mostly populates the $[000]$ irrep, see Fig. 1, the two isobaric partners ${}^8\text{Li}(2^+)$ and ${}^8\text{B}(2^+)$ have zero overlap with this representation. The decays ${}^8\text{Li}(2^+) \rightarrow {}^8\text{Be}(2^+)$ and ${}^8\text{B}(2^+) \rightarrow {}^8\text{Be}(2^+)$ primarily occur within the irrep $[211]$ of SU(4) and are

suppressed due to the small weight of this component in ${}^8\text{Be}(2^+)$. Similar arguments apply to ${}^8\text{He}(0^+) \rightarrow {}^8\text{Li}(1^+)$, with the exception that this transition occurs mostly in the irrep $[220]$.

This elucidates the reason for the large two-body corrections ($\approx 30\%$) observed in the nuclear matrix elements of $A = 8$ decays [7]: SU(4) symmetry suppresses one-body, but not two-body, matrix elements. In fact, a component of the spin-isospin two-body axial current—which is linear in the two-body generator $G_{ab}^{(2)}$, see Eq. (C1) in the End Matter—has a tensor structure $[211]$. The dominant component of the wave function of ${}^8\text{Be}(2^+)$ has a tensor structure $[000]$, while the dominant parts of ${}^8\text{Li}(2^+)$ and ${}^8\text{B}(2^+)$ wave functions have structure $[211]$. These large components are, thus, connected by two-body axial currents, resulting in a nonsuppressed contribution.

In Refs. [5,7], the suppression of Gamow-Teller transitions in the $A = 8$ system is attributed to them connecting large and small orbital components [49] of nuclear states. In fact, since the total wave function is antisymmetric, the orbital wave function populates orbital irreps with Young tableaux that are dual to the irreps of the spin-isospin wave function. The spatial-structure explanation for the suppression of Gamow-Teller matrix elements adduced in Ref. [7] and references therein is, therefore, dual to the SU(4)-content analysis carried out here (see also Ref. [50]). However, analyzing spin-isospin symmetries offers some advantages compared to spatial symmetries due to the nature of the underlying groups. In a basis expansion, the spin-isospin wave functions populate irreps of U(4), a group of fixed dimension. In contrast, the spatial wave functions populate irreps of U(Ω), where Ω is the dimension of the corresponding single-particle basis. While this distinction does not guarantee that the spin-isospin analysis is simpler—the spin-isospin symmetry may be more strongly broken than in the cases considered here—it generally involves fewer irreps, and those have smaller dimensions. An approximate spin-isospin symmetry then

constrains the allowed irreps of $U(\Omega)$, because of the total antisymmetry of the wave function. This, in turn, facilitates the mapping of spatial states to relevant groups for collective dynamics, such as Elliott's $SU(3)$ [39] or the noncompact symplectic group $Sp(3, \mathbb{R})$ [38,51].

Conclusions—In this Letter, we showed that *ab initio* computed eigenstates of light nuclei have as much as 90% of their normalization in a single $SU(4)$ irrep. The symmetry is broken only weakly in these nuclear many-body states. This might seem surprising in light of the large difference between the s-wave NN scattering lengths in the spin-one and spin-zero channels [23]. However, several authors have observed that, although the s-wave NN phase shifts are quite different at very low energies, they are similar for relative momenta of order 100 MeV [52]. $SU(4)$ invariant interactions built on this observation describe the structure of the trinucleons [53,54] and ^4He [55] well. Indeed, Ref. [55] argued that the relevant symmetry group for these nuclei is associated with proximity to the unitary limit [56]. It is an important and open question whether the $SU(4)$ symmetry emerges for that reason, because of the contracted $SU(4)$ symmetry of hadronic interactions in the large- N_c limit of QCD [26,57,58], as a consequence of entanglement suppression [59,60], or for some other reason.

Since the Gamow-Teller operator is a generator of $SU(4)$, it follows that Gamow-Teller transitions between nuclear states that are in different $SU(4)$ irreps are $SU(4)$ forbidden. However, two-body axial currents are diagonal only when restricted to two-body systems, so in such cases, the effects of two-body currents are enhanced compared to the standard χEFT power counting.

In contrast, when the Gamow-Teller transition is $SU(4)$ allowed, the result for the one-body matrix element is remarkably robust with χEFT order, changing by $< 5\%$ as the order of the interaction is increased from LO to NNLO. $SU(4)$ thus essentially guarantees that χEFT will reproduce these matrix elements to within 10%, regardless of χEFT order, because the pure $SU(4)$ prediction for the reduced matrix element agrees with *ab initio* results at this level—a phenomenon that was already noted for tritium β -decay and pp fusion in the lower-resolution pionless EFT [61]. We note that the $SU(4)$ decomposition of nuclear states is not a direct observable; a unitary transformation of the Hamiltonian could alter the weights found in this Letter. Such a transformation would necessarily change the balance between one- and two-body electroweak operators to ensure all nuclear observables remain constant. However, the fact that our calculated decompositions exhibit remarkable stability across four distinct χEFT interactions suggests that no simple unitary transformation exists that could simultaneously preserve the long-range component of the nuclear force and, at the same time, significantly modify the balance of one- to two-body operators.

These findings impact nuclear magnetic dipole (M1) transitions as well. Key parts of the M1 one-body operator can be written in terms of $SU(4)$ generators [35]. Lin *et al.* [28] have shown that the approximate $SU(4)$ symmetry of three-nucleon wave functions explains the otherwise surprisingly small threshold rate of $nd \rightarrow \gamma t$; Lin and Vanasse subsequently explored a dual EFT and $SU(4)$ expansion for $\gamma t \rightarrow nd$ [62].

$SU(4)$ symmetry is key to quantitatively explaining Gamow-Teller matrix elements, imposing patterns that sit atop any convergence in the χEFT expansion. Characterizing the $SU(4)$ content of nuclear states is, thus, a prerequisite for the rigorous uncertainty quantification of β -decay amplitudes, as is needed in searches for physics beyond the standard model. In future work, we will use statistical tools to analyze the convergence of the χEFT series for these amplitudes [41,63]. We expect Wigner's $SU(4)$ symmetry to remain relevant in heavier systems, although admixtures of different irreps might become more prominent there [31].

Acknowledgments—We thank Garrett King for a clarifying conversation regarding the results in Table I. This research was supported by the Swedish Research Council via a Tage Erlander Visiting Professorship, Grant No. 2022-00215 (D. R. P. and S. S. L. M.), and a Research Project Grant, No. 2021-04507 (C. F. and S. S. L. M.), as well as by the U.S. Department of Energy under Contract No. DE-FG02-93ER40756 (D. R. P.) and the National Science Foundation under Award No. PHY-2402275. S. S. L. M. acknowledges travel support from the Barbro Osher Endowment, Scholarship No. 90401181, and the Royal Swedish Academy of Sciences, Scholarship No. PH2024-0090. S. S. L. M. is grateful for hospitality provided by Oak Ridge National Laboratory (ORNL) and Ohio University, where part of this work was done. The computations and data handling were partly enabled by resources provided by the National Academic Infrastructure for Supercomputing in Sweden (NAISS) and partially funded by the Swedish Research Council through Grant Agreement No. 2022-06725. This research used resources from the Oak Ridge Leadership Computing Facility located at ORNL, which is supported by the Office of Science of the Department of Energy under Contract No. DE-AC05-00OR22725.

This work was supported (in part) by the U.S. Department of Energy, Office of Science, Office of Advanced Scientific Computing Research, and Office of Nuclear Physics, Scientific Discovery through Advanced Computing (SciDAC) program (SciDAC-5 NUCLEI) and by the U.S. Department of Energy, Office of Science. This manuscript has been authored, in part, by UT-Battelle, LLC, under Contract No. DE-AC05-00OR22725 with the U.S. Department of Energy (DOE). The U.S. government retains and the publisher, by accepting the article for

publication, acknowledges that the U.S. government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this manuscript, or allow others to do so, for U.S. government purposes. DOE will provide public access to these results of federally sponsored research in accordance with the DOE Public Access Plan [64].

Data availability—The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.

- [1] P. Navrátil, B. R. Barrett, and W. E. Ormand, Large-basis shell-model calculation of the $^{10}\text{C} \rightarrow ^{10}\text{B}$ fermi matrix element, *Phys. Rev. C* **56**, 2542 (1997).
- [2] R. Schiavilla and Robert B. Wiringa, Weak transitions in $A = 6$ and $A = 7$ nuclei, *Phys. Rev. C* **65**, 054302 (2002).
- [3] P. Maris, J. P. Vary, P. Navrátil, W. E. Ormand, H. Nam, and D. J. Dean, Origin of the anomalous long lifetime of ^{14}C , *Phys. Rev. Lett.* **106**, 202502 (2011).
- [4] A. Ekström, G. R. Jansen, K. A. Wendt, G. Hagen, T. Papenbrock, S. Bacca, B. Carlsson, and D. Gazit, Effects of three-nucleon forces and two-body currents on Gamow-Teller strengths, *Phys. Rev. Lett.* **113**, 262504 (2014).
- [5] S. Pastore, A. Baroni, J. Carlson, S. Gandolfi, Steven C. Pieper, R. Schiavilla, and R. B. Wiringa, Quantum Monte Carlo calculations of weak transitions in $A = 6$ –10 nuclei, *Phys. Rev. C* **97**, 022501 (2018).
- [6] P. Gysbers *et al.*, Discrepancy between experimental and theoretical β -decay rates resolved from first principles, *Nat. Phys.* **15**, 428 (2019).
- [7] G. B. King, L. Andreoli, S. Pastore, M. Piarulli, R. Schiavilla, R. B. Wiringa, J. Carlson, and S. Gandolfi, Chiral effective field theory calculations of weak transitions in light nuclei, *Phys. Rev. C* **102**, 025501 (2020).
- [8] G. H. Sargsyan, K. D. Launey, M. T. Burkey, A. T. Gallant, N. D. Scielzo, G. Savard, A. Mercenne, T. Dytrych, D. Langr, L. Varriano, B. Longfellow, T. Y. Hirsh, and J. P. Draayer, Impact of clustering on the ^8Li β decay and recoil form factors, *Phys. Rev. Lett.* **128**, 202503 (2022).
- [9] Ayala Glick-Magid, Christian Forssén, Daniel Gazda, Doron Gazit, Peter Gysbers, and Petr Navrátil, Nuclear *ab initio* calculations of ^6He β -decay for beyond the standard model studies, *Phys. Lett. B* **832**, 137259 (2022).
- [10] G. B. King, A. Baroni, V. Cirigliano, S. Gandolfi, L. Hayen, E. Mereghetti, S. Pastore, and M. Piarulli, *Ab initio* calculation of the β -decay spectrum of ^6He , *Phys. Rev. C* **107**, 015503 (2023).
- [11] B. Longfellow, A. T. Gallant, G. H. Sargsyan, M. T. Burkey, T. Y. Hirsh, G. Savard, N. D. Scielzo, L. Varriano, M. Brodeur, D. P. Burdette, J. A. Clark, D. Lascar, K. D. Launey, P. Mueller, D. Ray, K. S. Sharma, A. A. Valverde, G. L. Wilson, and X. L. Yan, Improved tensor current limit from ^8B β decay including new recoil-order calculations, *Phys. Rev. Lett.* **132**, 142502 (2024).
- [12] Michael Gennari, Mehdi Drissi, Mikhail Gorchtein, Petr Navrátil, and Chien-Yeah Seng, *Ab initio* strategy for taming the nuclear-structure dependence of V_{ud} extractions: The $^{10}\text{C} \rightarrow ^{10}\text{B}$ superallowed transition, *Phys. Rev. Lett.* **134**, 012501 (2025).
- [13] Nathal Severijns, Marcus Beck, and Oscar Naviliat-Cuncic, Tests of the standard electroweak model in beta decay, *Rev. Mod. Phys.* **78**, 991 (2006).
- [14] Martin González-Alonso, Oscar Naviliat-Cuncic, and Nathal Severijns, New physics searches in nuclear and neutron β decay, *Prog. Part. Nucl. Phys.* **104**, 165 (2019).
- [15] Vincenzo Cirigliano, Alejandro Garcia, Doron Gazit, Oscar Naviliat-Cuncic, Guy Savard, and Albert Young, Precision beta decay as a probe of new physics, [arXiv:1907.02164](https://arxiv.org/abs/1907.02164).
- [16] M. T. Burkey *et al.*, Improved limit on tensor currents in the weak interaction from ^8Li β decay, *Phys. Rev. Lett.* **128**, 202502 (2022).
- [17] M. Brodeur *et al.*, Nuclear β decay as a probe for physics beyond the Standard Model, [arXiv:2301.03975](https://arxiv.org/abs/2301.03975).
- [18] Vincenzo Cirigliano, Wouter Dekens, Jordy de Vries, Stefano Gandolfi, Martin Hoferichter, and Emanuele Mereghetti, Radiative corrections to superallowed β decays in effective field theory, *Phys. Rev. Lett.* **133**, 211801 (2024).
- [19] Vincenzo Cirigliano, Wouter Dekens, Jordy de Vries, Stefano Gandolfi, Martin Hoferichter, and Emanuele Mereghetti, *Ab initio* electroweak corrections to superallowed β decays and their impact on V_{ud} , *Phys. Rev. C* **110**, 055502 (2024).
- [20] E. Wigner, On the consequences of the symmetry of the nuclear hamiltonian on the spectroscopy of nuclei, *Phys. Rev.* **51**, 106 (1937).
- [21] E. P. Wigner, On coupling conditions in light nuclei and the lifetimes of beta-radioactivities, *Phys. Rev.* **56**, 519 (1939).
- [22] David B. Kaplan, Martin J. Savage, and Mark B. Wise, Nucleon–nucleon scattering from effective field theory, *Nucl. Phys.* **B478**, 629 (1996).
- [23] A. Calle Cordón and E. Ruiz Arriola, Wigner symmetry, large N_c , and renormalized one-boson exchange potentials, *Phys. Rev. C* **78**, 054002 (2008).
- [24] Daniel R. Phillips and Carlos Schat, Three-nucleon forces in the $1/N_c$ expansion, *Phys. Rev. C* **88**, 034002 (2013).
- [25] Michael L. Wagman, Frank Winter, Emmanuel Chang, Zohreh Davoudi, William Detmold, Kostas Orginos, Martin J. Savage, and Phiala E. Shanahan, Baryon-Baryon interactions and spin-flavor symmetry from lattice quantum chromodynamics, *Phys. Rev. D* **96**, 114510 (2017).
- [26] Dean Lee, Scott Bogner, B. Alex Brown, Serdar Elhatisari, Evgeny Epelbaum, Heiko Hergert, Morten Hjorth-Jensen, Hermann Krebs, Ning Li, Bing-Nan Lu, and Ulf-G. Meißner, Hidden spin-isospin exchange symmetry, *Phys. Rev. Lett.* **127**, 062501 (2021).
- [27] Thomas Mehen, Iain W. Stewart, and Mark B. Wise, Wigner symmetry in the limit of large scattering lengths, *Phys. Rev. Lett.* **83**, 931 (1999).
- [28] Xincheng Lin, Hersh Singh, Roxanne P. Springer, and Jared Vanasse, Cold neutron-deuteron capture and Wigner-SU(4) symmetry, *Phys. Rev. C* **108**, 044001 (2023).
- [29] K. T. Hecht and Sing Chin Pang, On the Wigner supermultiplet scheme, *J. Math. Phys. (N.Y.)* **10**, 1571 (1969).
- [30] N. C. Mukhopadhyay and F. Cannata, The persistence of supermultiplet selection rules in nuclear weak and electromagnetic transitions, *Phys. Lett.* **51B**, 225 (1974).

- [31] P. Vogel and W. E. Ormand, Spin-isospin SU(4) symmetry in sd- and fp-shell nuclei, *Phys. Rev. C* **47**, 623 (1993).
- [32] Calvin W. Johnson, Spin-orbit decomposition of *ab initio* nuclear wave functions, *Phys. Rev. C* **91**, 034313 (2015).
- [33] L. A. Radicati, Symmetry properties of nuclei, in *Proceedings of the 15th Solvay Conference on Physics* (Gordon and Breach, New York, 1974).
- [34] I. Talmi, *Simple Models of Complex Nuclei: The Shell Model and Interacting Boson Model*, Beitrage Zur Wirtschaftsinformatik (Harwood Academic Publishers, Chur, Switzerland, 1993).
- [35] Pham Tri Nang, M1 transitions and the validity of SU(4) symmetry in light nuclei, *Nucl. Phys. A* **185**, 413 (1972).
- [36] R. R. Whitehead, Moment methods and lanczos methods, in *Theory and Applications of Moment Methods in Many-Fermion Systems* (Springer, New York, 1980), pp. 235–255.
- [37] V. G. Gueorguiev, J. P. Draayer, and C. W. Johnson, SU(3) symmetry breaking in lower fp-shell nuclei, *Phys. Rev. C* **63**, 014318 (2000).
- [38] Anna E. McCoy, Mark A. Caprio, Tomáš Dytrych, and Patrick J. Fasano, Emergent $\text{Sp}(3, \mathbb{R})$ dynamical symmetry in the nuclear many-body system from an *ab initio* description, *Phys. Rev. Lett.* **125**, 102505 (2020).
- [39] Anna E. McCoy, Mark A. Caprio, Pieter Maris, and Patrick J. Fasano, Intruder band mixing in an *ab initio* description of ^{12}Be , *Phys. Lett. B* **856**, 138870 (2024).
- [40] A. Baroni, R. Schiavilla, L. E. Marcucci, L. Girlanda, A. Kievsky, A. Lovato, S. Pastore, M. Piarulli, Steven C. Pieper, M. Viviani, and R. B. Wiringa, Local chiral interactions, the tritium Gamow-Teller matrix element, and the three-nucleon contact term, *Phys. Rev. C* **98**, 044003 (2018).
- [41] J. A. Melendez, S. Wesolowski, and R. J. Furnstahl, Bayesian truncation errors in chiral effective field theory: Nucleon-nucleon observables, *Phys. Rev. C* **96**, 024003 (2017).
- [42] S. Wesolowski, I. Svensson, A. Ekström, C. Forssén, R. J. Furnstahl, J. A. Melendez, and D. R. Phillips, Rigorous constraints on three-nucleon forces in chiral effective field theory from fast and accurate calculations of few-body observables, *Phys. Rev. C* **104**, 064001 (2021).
- [43] Bruce R. Barrett, Petr Navrátil, and James P. Vary, *Ab initio* no core shell model, *Prog. Part. Nucl. Phys.* **69**, 131 (2013).
- [44] Etienne Caurier and Frederic Nowacki, Present status of shell model techniques, *Acta Phys. Pol. B* **30**, 705 (1999), <https://ui.adsabs.harvard.edu/abs/1999AcPPB..30..705C>.
- [45] P. Navrátil and E. Caurier, Nuclear structure with accurate chiral perturbation theory nucleon-nucleon potential: Application to ^6Li and ^{10}B , *Phys. Rev. C* **69**, 014311 (2004).
- [46] C. Forssén, B. D. Carlsson, H. T. Johansson, D. Sääf, A. Bansal, G. Hagen, and T. Papenbrock, Large-scale exact diagonalizations reveal low-momentum scales of nuclei, *Phys. Rev. C* **97**, 034328 (2018).
- [47] T. Djärv, A. Ekström, C. Forssén, and H. T. Johansson, Bayesian predictions for $A = 6$ nuclei using eigenvector continuation emulators, *Phys. Rev. C* **105**, 014005 (2022).
- [48] A. Ekström, G. Baardsen, C. Forssén, G. Hagen, M. Hjorth-Jensen, G. R. Jansen, R. Machleidt, W. Nazarewicz, T. Papenbrock, J. Sarich, and S. M. Wild, Optimized chiral nucleon-nucleon interaction at next-to-next-to-leading order, *Phys. Rev. Lett.* **110**, 192502 (2013).
- [49] R. B. Wiringa, Pair counting, pion-exchange forces and the structure of light nuclei, *Phys. Rev. C* **73**, 034317 (2006).
- [50] William G. Dawkins, J. Carlson, U. van Kolck, and Alexandros Gezerlis, Clustering of four-component unitary fermions, *Phys. Rev. Lett.* **124**, 143402 (2020).
- [51] T. Dytrych, K. D. Launey, J. P. Draayer, D. J. Rowe, J. L. Wood, G. Rosensteel, C. Bahri, D. Langr, and R. B. Baker, Physics of nuclei: Key role of an emergent symmetry, *Phys. Rev. Lett.* **124**, 042501 (2020).
- [52] I Tews *et al.*, Nuclear forces for precision nuclear physics: A collection of perspectives, *Few-Body Syst.* **63**, 67 (2022).
- [53] Sebastian König, Harald W. Griedhammer, H. W. Hammer, and U. van Kolck, Effective theory of ^3H and ^3He , *J. Phys. G* **43**, 055106 (2016).
- [54] Jared Vanasse and Daniel R. Phillips, Three-nucleon bound states and the Wigner-SU(4) limit, *Few Body Syst.* **58**, 26 (2017).
- [55] Sebastian König, Harald W. Griedhammer, H. W. Hammer, and U. van Kolck, Nuclear physics around the unitarity limit, *Phys. Rev. Lett.* **118**, 202501 (2017).
- [56] Eric Braaten and H.-W. Hammer, Universality in few-body systems with large scattering length, *Phys. Rep.* **428**, 259 (2006).
- [57] Roger F. Dashen, Elizabeth Ellen Jenkins, and Aneesh V. Manohar, The $1/N(c)$ expansion for baryons, *Phys. Rev. D* **49**, 4713 (1994); **51**, 2489(E) (1995).
- [58] David B. Kaplan and Martin J. Savage, The spin-flavor dependence of nuclear forces from large- N QCD, *Phys. Lett. B* **365**, 244 (1996).
- [59] Silas R. Beane, David B. Kaplan, Natalie Klco, and Martin J. Savage, Entanglement suppression and emergent symmetries of strong interactions, *Phys. Rev. Lett.* **122**, 102001 (2019).
- [60] Alma L. Cavallin, Oliver Thim, and Christian Forssén, Entanglement and accidental symmetries in the nucleon-nucleon system, *Phys. Rev. C* **113**, 014005 (2026).
- [61] Hilla De-Leon and Doron Gazit, Theoretical evaluation of solar proton-proton fusion reaction rate and its uncertainties, *Phys. Lett. B* **844**, 138093 (2023).
- [62] Xincheng Lin and Jared Vanasse, Two-body triton photodisintegration and wigner-su(4) symmetry, *Phys. Rev. C* **112**, 024001 (2025).
- [63] R. J. Furnstahl, N. Klco, D. R. Phillips, and S. Wesolowski, Quantifying truncation errors in effective field theory, *Phys. Rev. C* **92**, 024005 (2015).
- [64] <https://www.energy.gov/doe-public-access-plan>.
- [65] The only exception is the irrep [321], see Table II, but our conclusions are unchanged even in that case.
- [66] Note that the isospin raising and lowering operators are defined as $\tau_{\pm} = \tau_x \pm i\tau_y$.

End Matter

SU(4) structure of nuclear wave functions—In the most general irreducible representations of unitary groups, state vectors can be labeled by the quantum numbers of the canonical subgroup chain $U(4) \supset U(3) \supset U(2) \supset U(1)$. Unfortunately, in this representation, neither the total spin nor the isospin can be associated with quantum numbers of the chain. Here, we follow Ref. [29] and apply the subgroup chain $SU(4) \supset SU(2)_S \otimes SU(2)_T$. In this approach, we label state vectors as $|[f]; STM_S M_T; \omega\varphi\rangle$. If the spin and isospin subgroups are contained in the $SU(4)$ irrep with multiplicity greater than 1, the quantum numbers ω and φ are required to uniquely identify spin-isospin states. (This is the origin of what is sometimes referred to as the “state-labeling problem.”) However, for most of the $SU(4)$ irreps considered in this Letter [65], this multiplicity is, at most, 1, and $SU(4)$ states can be identified as $|[f]; STM_S M_T\rangle$. These spin-isospin states must be coupled with orbital states $|\tilde{f}]; \alpha LM_L\rangle$. Here, the set of space quantum numbers other than L and M_L has been abbreviated to α . This orbital state has the permutation symmetry of the Young tableau $[\tilde{f}]$. Antisymmetry of the total wave function implies that the orbital wave function populates the dual Young tableaux $[\tilde{f}]$ of the spin-isospin tableau $[f]$.

TABLE II. The three lowest irreps for systems made with $A = 2$ –8 $SU(4)$ degrees of freedom. (Note that two-particle spin-isospin states are described by just two irreps.) The third column gives the value of the quadratic Casimir, the fourth the Young tableau of the spin-isospin wave function, and the last column the dual Young tableau.

Particles	$[n]$	$\langle C_2 \rangle$	Young tableau	Dual tableau
2	$[6]$	5	$[110]$	$[200]$
	$[10]$	9	$[200]$	$[110]$
3	$[4]$	3.75	$[111]$	$[300]$
	$[20]$	9.75	$[210]$	$[210]$
	$[20]$	15.75	$[300]$	$[111]$
4	$[1]$	0	$[000]$	$[400]$
	$[15]$	8	$[211]$	$[310]$
	$[20]$	12	$[220]$	$[220]$
6	$[6]$	5	$[110]$	$[420]$
	$[10]$	9	$[200]$	$[411]$
	$[64]$	15	$[321]$	$[321]$
7	$[4]$	3.75	$[111]$	$[430]$
	$[20]$	9.75	$[210]$	$[421]$
	$[36]$	13.75	$[322]$	$[331]$
8	$[1]$	0	$[000]$	$[440]$
	$[15]$	8	$[211]$	$[431]$
	$[20]$	12	$[220]$	$[422]$

The states $|[f]; STM_S M_T; \alpha LM_L\rangle \equiv |[f]; STM_S M_T\rangle \otimes |\tilde{f}]; \alpha LM_L\rangle$ form a complete basis; Eq. (2) is, then, the expansion of a state of good J^2 and T_3 on this basis. The probability $P([f])$ of populating the irrep $[f]$ can be obtained as follows:

$$P([f]) = \sum_{\substack{T'S'M'_S \\ \alpha'L'M'_L}} |\langle [f]; S'T'M'_S M'_T; \alpha'L'M'_L | \Psi_{JM_T} \rangle|^2. \quad (A1)$$

The Lanczos decomposition [32,37] accumulates the probabilities of populating irreps with the same $\langle C_2 \rangle$ value. Since, in this Letter, we consider only $SU(4)$ irreps with distinct Casimir eigenvalues, see also Table II, the decomposition isolates $P([f])$. We have verified that the Lanczos algorithm reproduces Eq. (A1) by directly diagonalizing the Hamiltonian and the Casimir operator for a small NCSM model space ($N_{\max} = 2$) in ${}^6\text{He}$ and ${}^6\text{Li}$, evaluating the scalar products between the relevant eigenstates and summing the square of the scalar products associated with identical eigenvalues of the Casimir operator. The Lanczos decomposition method and the exact diagonalization produce identical results.

Gamow-Teller matrix elements under exact SU(4) symmetry—Under exact $SU(4)$ symmetry, nuclear wave functions have good quantum numbers $\{[f]JMLSTM_T\}$. The expansion [Eq. (2)] then simplifies to a sum over α , M_L , and M_S :

$$|\Psi_{SU(4)}\rangle = \sum_{\alpha M_L M_S} \langle [f]; STM_S M_T; \alpha LM_L | \Psi_{SU(4)} \rangle \times |[f]; STM_S M_T; \alpha LM_L\rangle. \quad (B1)$$

Since the long-wavelength Gamow-Teller operator is a generator of $SU(4)$, this decomposition allows its matrix elements to be obtained analytically in the $SU(4)$ -symmetric limit. We write $GT_{\pm} = \sum_k \sigma_z^{(k)} \tau_{\pm}^{(k)}$ [66] and obtain, by using the expansion, Eq. (B1), twice,

$$\begin{aligned} \langle \Psi'_{SU(4)} | GT_{\pm} | \Psi_{SU(4)} \rangle &= \sum_{\substack{M_L M_S \\ M'_S}} C_{SM_S, LM_L}^{JM} C_{S'M'_S, LM_L}^{J'M'} \\ &\times \langle [f]; S'T'M'_S M'_T | GT_{\pm} | [f]; STM_S M_T \rangle, \end{aligned} \quad (B2)$$

where the Clebsch-Gordan coefficients originate from the scalar products $\langle [f]; STM_S M_T; \alpha LM_L | \Psi_{SU(4)} \rangle$ and $\langle \Psi'_{SU(4)} | [f]; S'T'M'_S M'_T; \alpha LM_L \rangle$. Meanwhile, the operator $GT_{\pm}$ is diagonal in M_L and α , and the sum over the quantum numbers α is then removed using completeness of the basis $\{|\alpha LM_L\rangle\}$ for orbital states. The last matrix

element can be evaluated using group theory arguments [29], which yield Eq. (4). To compare our results with the recent calculation of Ref. [7], we follow their notation and define the Gamow-Teller reduced matrix element (RME) as

$$\text{RME} = \frac{\sqrt{2J'+1}}{2} \frac{\langle \Psi'_{JM'_T} | GT_{\pm} | \Psi_{JM_T} \rangle}{C_{JM,10}^{J'M}}. \quad (\text{B3})$$

Using Eqs. (B3) and (4) and the reduced Wigner coefficients from the tables in [29], we obtain the results shown in Table I.

Axial-current χ EFT operator as $SU(4)$ generator—Up to N3LO in χ EFT, the two main spin-isospin structures of the two-body axial current operator can be written as linear combinations of products of two-body $SU(4)$ generators:

$$(\tau^{(1)} \wedge \tau^{(2)})_a (\sigma^{(1)} \wedge \sigma^{(2)})_b = 2\epsilon_{ace} \epsilon_{bdf} G_{cd}^{2B} G_{ef}^{2B} + 4G_{ab}^{2B}, \quad (\text{C1})$$

$$(\tau^{(1)} \wedge \tau^{(2)})_a \epsilon_{ijk} \sigma_j^{(1)} \sigma_l^{(2)} + (1 \rightleftharpoons 2) \\ = \epsilon_{ijk} \epsilon_{abc} G_{bj}^{2B} G_{cl}^{2B} - 2\delta_{il} G_{ka}^{2B}. \quad (\text{C2})$$

These operators are multiplied by a function of the relative coordinate between the two particles, so their matrix elements between two-body $SU(4)$ -symmetric states are straightforwardly evaluated. However, the spatial dependence of the current in the A -body context means the A -body matrix element cannot be evaluated in the same way. The two-body current also contains a spin-momentum structure proportional to the nucleon momentum, $\mathbf{p}_i$. However, this was evaluated in Ref. [7] and found to be numerically small.

Irreducible representations of $SU(4)$ for few-body states and tensor products—In Table II, we identify the lowest irreps of $SU(4)$ for $A = 2$ –8 via the total number of spin-isospin states $[\mathbf{n}]$, the value of the quadratic Casimir $\langle C_2 \rangle$, and the Young tableau of the spin-isospin states. The dual tableau, corresponding to the symmetry of the associated spatial wave function, is also listed.

$SU(4)$ tensor products—Indicating the single-nucleon quantum numbers with $\alpha \equiv \{nlm_i m_s m_t\}$, creation and annihilation operators transform under $SU(4)$ as [29]

$$a_{\alpha_1}^\dagger = [100]; \quad a_{\alpha_2} = [111]. \quad (\text{E1})$$

It is possible to derive the tensor structure of general one- and two-body operators:

$$a_{\alpha_1}^\dagger \otimes a_{\alpha_2} = [000] \oplus [211], \quad (\text{E2})$$

$$a_{\alpha_1}^\dagger \otimes a_{\alpha_2}^\dagger \otimes a_{\alpha_3} \otimes a_{\alpha_4} = 2 \times [000] \oplus 4 \times [211] \oplus [220] \\ \oplus [310] \oplus [422] \oplus [332]. \quad (\text{E3})$$

We use these general expressions to derive selection rules in nuclei based on Wigner $SU(4)$ symmetry. The trivial tensor product,

$$[000] \otimes [f] = [f], \quad (\text{E4})$$

results in selection rules for nuclei with $Z = 2n$ protons and $N = 2n$ neutrons, with n any integer; under $SU(4)$ symmetry, these systems are expected to have a dominant $[000]$ structure, and transition matrix elements are allowed if the final state has the same $SU(4)$ structure as the transition operator. We consider next the two dominant irreps for systems with nucleon number $A = 4n + 2$ and maximal isospin $T = 1$. From the following products,

$$[110] \otimes [211] = [200] \oplus [110] \oplus [222] \oplus [321], \quad (\text{E5})$$

$$[110] \otimes [220] = [110] \oplus [330] \oplus [321], \quad (\text{E6})$$

$$[110] \otimes [310] = [200] \oplus [420] \oplus [411] \oplus [321], \quad (\text{E7})$$

$$[110] \otimes [422] = [321] \oplus [411] \oplus [433] \oplus [532], \quad (\text{E8})$$

$$[110] \otimes [332] = [222] \oplus [321] \oplus [433] \oplus [442]. \quad (\text{E9})$$

the transition $[110] \rightarrow [200]$ is allowed only by tensors $[211]$ and $[310]$. In cases where the transition operator does not contain these structures, the transition is $SU(4)$ forbidden. Finally, we consider systems with $A = 4n \pm 1$ and maximal isospin $T = 3/2$. From the tensor products

$$[111] \otimes [211] = [111] \oplus [210] \oplus [322], \quad (\text{E10})$$

$$[111] \otimes [220] = [210] \oplus [331], \quad (\text{E11})$$

$$[111] \otimes [310] = [210] \oplus [300] \oplus [321], \quad (\text{E12})$$

$$[111] \otimes [422] = [322] \oplus [421] \oplus [533], \quad (\text{E13})$$

$$[111] \otimes [332] = [331] \oplus [322] \oplus [443], \quad (\text{E14})$$

only the tensors $[211]$, $[220]$, and $[310]$ generate transitions between the two dominant irreps $[111]$ and $[210]$.