Last-Mile Delivery Problem with Flexible Time Slot and Location Options under Stochastic Customer Behavior

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Last-mile delivery problem with flexible time slot and location options under stochastic customer behavior

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ABSTRACT

The growth of e-commerce has led to a significant rise in daily parcel deliveries, placing increasing pressure on logistics services. Although various delivery options, including home and out-of-home delivery, are offered to enhance customer availability, uncertainty in customer preferences regarding delivery locations and time slots further complicates delivery operations. In this paper, we introduce a last-mile delivery problem with flexible delivery options under stochastic customer behavior, formulated as a two-stage stochastic programming model. The novelty of our model lies in the integration of comprehensive delivery options, including various locations and time slots, with pricing decisions. Furthermore, we incorporate heterogeneous customer preferences across mixed delivery options. To efficiently solve the problem, we develop a hybrid adaptive large neighborhood search (h-ALNS) heuristic based on scenario simulations. Extensive numerical experiments demonstrate the advantages of the mixed delivery strategy and confirm the effectiveness of the proposed algorithm. Our numerical analysis offers important managerial insights for optimizing last-mile delivery assortment and pricing, particularly in addressing customer uncertainty.

1. Introduction

In recent years, the expansion of Internet-based retail platforms has transformed consumer behavior, leading to a substantial increase in the volume and frequency of online purchases across a wide range of industries. As the largest e-commerce market in the Nordic region, the Swedish e-commerce market is estimated to generate nearly 15 billion U.S. dollars in revenue in 2025 (Yltävä, 2025). The continued growth of e-commerce has led to a substantial rise in daily parcel deliveries, placing increasing pressure on transportation systems and logistics providers to meet the increasing demand efficiently and reliably.

As the most popular option, home delivery accounts for 79% of online purchases in Europe (Ruth Justen, 2021). It offers maximum convenience by delivering parcels directly to customers' doorsteps, saving them time and effort. However, due to the high volume of delivery requests, retailers often cannot fulfill deliveries within customers' preferred time slots, potentially decreasing customer satisfaction. Moreover, it has been reported that, depending on the data source, 2%–60% of home deliveries fail due to customer absence (Rai et al., 2021), leading to substantial losses for logistics companies and inconvenience for customers.

On the other hand, logistics service providers are increasingly investing in alternative out-of-home (OOH) delivery options to address the above issues. The use of shared delivery locations (SDLs), such as parcel lockers or local grocery stores, enables customers to collect their parcels at their convenience. In Sweden, it has been reported that delivery points and parcel lockers account for 75%

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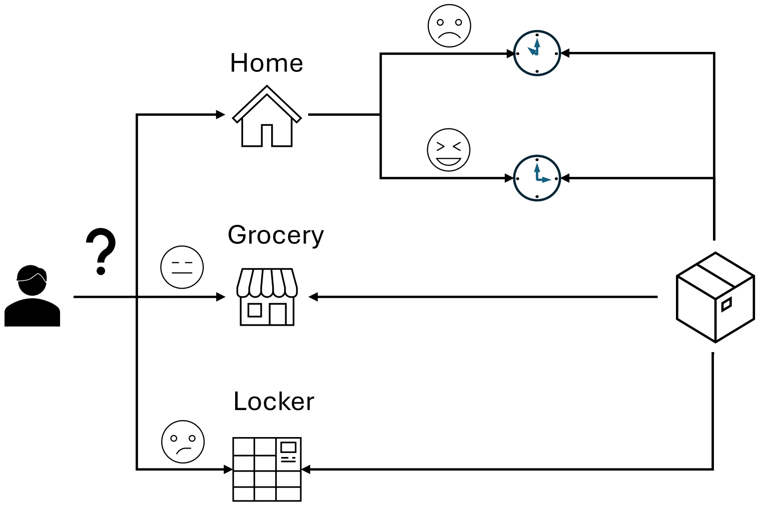


Fig. 1. An instance of a customer's preferences for different delivery locations and time slots.

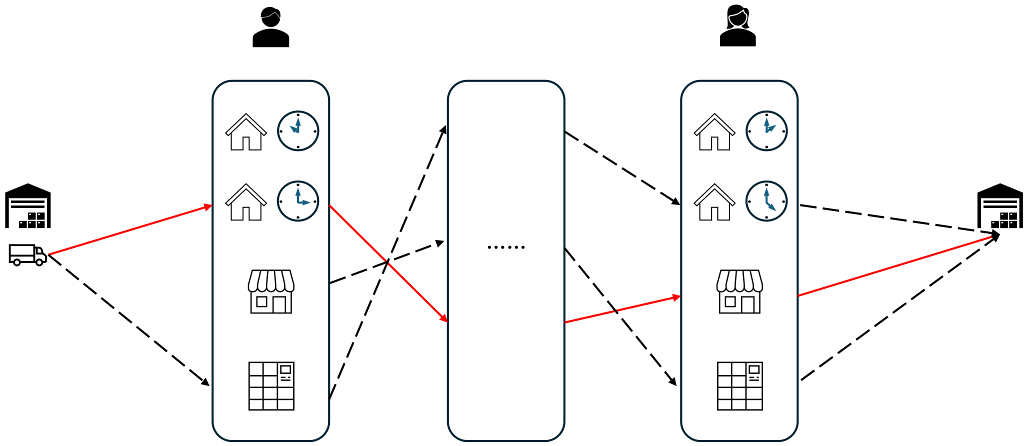


Fig. 2. An instance of a flexible delivery problem from the retailer perspective. For each customer, there are home delivery options with different time slots as well as out-of-home delivery alternatives.

of all parcel deliveries (Vakulenko, 2023). The widespread adoption of pickup point delivery in Sweden can largely be attributed to its perceived advantages in convenience, operational efficiency, and cost-effectiveness. Nonetheless, OOH delivery may be inherently less appealing to customers who are sensitive to pickup distance. Besides, when the shipment involves valuable or perishable items-such as fresh food-or requires payment or a signature, customers often prefer to receive them at home instead.

As shown in Fig. 1, customers can be offered both at-home and OOH delivery options. Each customer may have individual preferences due to heterogeneous characteristics. While the increasing number of alternative options offers greater convenience for customers, it also presents new challenges for retailer companies. Although companies possess valuable data such as customers' locations and historical delivery preferences, significant uncertainty remains regarding preferred delivery time slots and locations. For example, the same customer may make different choices depending on the specific parcels or their schedule. Uncertainty about preferred time slots and delivery locations can lead to inefficient logistics operations, resulting in higher delivery costs and lower customer satisfaction.

Furthermore, as the last-mile delivery problem is typically formulated as a variant of the Vehicle Routing Problem (VRP), the interaction between retailers and customers, namely, the retailer's offering and the customer's confirmation, is often simplified or even overlooked. In many formulations, customers are treated as fixed points where parcels are dropped off, without explicitly modeling the offering and pricing process of retailers and the subsequent confirmation by customers. By neglecting these interactions and the uncertainty in customer behavior, models may underestimate the complexity of last-mile logistics and overlook opportunities to enhance delivery success and customer satisfaction.

To address the above challenges, this paper presents a Vehicle Routing Problem with Flexible Delivery options under Stochastic customer Behavior (VRPFD-SB). Fig. 2 illustrates the problem from the retailer's perspective. Compared to the conventional vehicle routing problem with time windows (VRPTW), this flexible delivery problem features implicit delivery locations and time windows. For instance, each customer can be served either at home during different time slots or at a nearby SDL (e.g., grocery stores or lockers). As illustrated in Fig. 2, the logistics planning involves two categories of decision variables, which interact with customer responses. First, the retailer must decide which combination of delivery options to offer to each customer. Then, based on customers' choices, the VRPFD-SB reduces to a VRPTW, and routing decisions must be made accordingly. The lines connecting different options represent possible vehicle routes, while the red lines indicate the optimal route, as illustrated in Fig. 2. It is important to note that these two decision stages are tightly interdependent: the offering and pricing strategies influence customer choices, and, in turn, the resulting delivery routes feed back into the optimal design of the offering and pricing plan.

In this paper, the VRPFD-SB is formulated as a two-stage stochastic program to capture customer stochasticity and the interactions between retailers and customers. In the first stage, the retailer must decide which delivery options and associated discounts to offer customers, prior to knowing their preferences. Moreover, we employ the mixed logit model to capture the heterogeneity in customer preferences. Once customers make their choices from the offered delivery options based on the utility-maximizing principle, their selected time slots and locations are revealed. Consequently, the second stage corresponds to a conventional VRPTW, where customer demand is derived from the first stage decisions. Furthermore, a tailored heuristic algorithm framework is proposed to solve the formulated problem. We conduct numerical experiments to evaluate the proposed model and algorithms.

To the best of our knowledge, this is the first study to jointly consider offering and pricing for both attended home delivery (AHD) and OOH delivery under stochastic customer behavior. The main contributions are summarized as follows:

- We develop a tactical offering and pricing framework that integrates flexible delivery time-slot and location options with a stochastic customer choice model, formulated as a two-stage stochastic program. Our key contribution is the explicit incorporation of pricing decisions and heterogeneous customer preferences into the joint design of time-slot and location offerings, enabling robust decisions under behavioral uncertainty.
- We develop a hybrid adaptive large neighborhood search (h-ALNS) framework to address the computational challenges for large-scale instances. The proposed method is further compared with both a commercial solver and a state-of-the-art ALNS in terms of computational efficiency and solution quality.
- The findings underline the strength of the proposed h-ALNS and offer concrete managerial insights on pricing and delivery option design, while advancing methods for solving two-stage stochastic optimization problems.

The remainder of the paper is structured as follows. Section 2 presents a literature review. Section 3 describes the mathematical formulation of the VRPFD-SB problem. Next, the heuristic algorithm adapted from the large neighborhood search (LNS) method is introduced in Section 4. The numerical experiments and managerial insights are presented in Section 5. Finally, Section 6 summarizes the main findings and draws the conclusion.

2. Literature review

In this section, we first review the relevant literature on demand management in AHD. Next, we examine studies focused on OOH delivery. Finally, we analyze the literature that incorporates both delivery options.

2.1. *Attended home delivery*

Demand management for AHD aims to generate customer demand, such as customer-specific delivery time slots, and channel it to support the order fulfillment process. Waßmuth et al. (2023) categorizes demand management decisions along two dimensions: planning levels and maneuver levers. Specifically, it encompasses strategic, tactical, and operational planning levels, as well as offering and pricing levers. Within this framework, we focus on relevant studies at the tactical planning level, where decisions are made over a relatively short time horizon but prior to order capture, and on two levers: offering and pricing.

Tactical offering decisions determine the availability of delivery options before customers submit their requests. In this context, early demand management studies focus on aggregated demand and deterministic models. For instance, Cleophas and Ehmke (2014) proposed an iterative algorithm to optimize time slot allocation, aiming to maximize expected revenue across regions. Their results demonstrate that the proposed method can yield substantial profit. Additionally, Mackert (2019) examined the e-grocer's dynamic time slot allocation problem, which seeks to influence customer demand to maximize expected profit. The author proposed a mixed-integer linear program (MILP) that explicitly incorporates customer choice behavior. Simulation results indicate that accounting for customer preferences can enhance profitability, especially when those preferences are highly diverse. On the other hand, later studies began to incorporate customer stochasticity. Côté et al. (2019) considered the uncertainty of customer locations and service times. Visser and Savelsbergh (2019) considered the uncertainty regarding the placement of orders and formulated the tactical time slot management problem as a two-stage stochastic program: the first stage determines priori routes, while the second stage calculates the expected revenue. Their study offers valuable insights into managing time slot availability based on optimal delivery routes. Nonetheless, the aforementioned studies assume aggregated customer demand and overlook individual preferences.

On the other hand, tactical pricing seeks to steer customers toward favorable options by differentiating prices across customer groups and service options. In this venue, Klein et al. (2019) aimed to influence customer choices through differentiated delivery pricing. The authors proposed a MILP that incorporates routing constraints to anticipate delivery costs and models customer behavior using a nonparametric rank-based choice framework. Similarly, their approach assumes aggregated and deterministic demand.

Beyond the above classification, several studies have proposed dynamic pricing approaches to determine incentives that steer customers toward the most profitable time slots. Koch and Klein (2020) proposed a route-based approximate dynamic programming method to address the complex dynamic pricing problem in AHD. Their results show that the method outperforms benchmarks in terms of both profit and the number of customers served. Similarly, Ulmer (2020) studied the same-day delivery problem and presented a dynamic pricing and routing approach designed to incentivize customers to select options that enhance routing efficiency. In addition, Abdollahi et al. (2023) introduced a partially time-windowed dynamic routing method with forecasted orders to tackle the dynamic pricing challenge in AHD.

In this regard, Abdolhamidi and Lurkin (2025) is the study most closely related to ours. They integrate offering and pricing decisions within tactical time slot management and explicitly account for individual customer preferences using a mixed logit model. The problem is formulated as a MILP and solved using a simulation-based ALNS method. Nonetheless, their analysis is restricted to the AHD setting.

2.2. Out-of-home delivery

As an alternative to AHD, OOH delivery is gaining popularity in both industry and academia. Beyond customer-facing benefits, shared locations can improve logistics efficiency through inter-carrier coordination at meet points (Zhou et al., 2024). At the strategic level, most studies (Xu et al., 2021; Lin et al., 2022; Raviv, 2023) focus on determining the optimal location and capacity of OOH facilities. Collectively, these studies suggest that OOH delivery enables logistics providers to manage demand surges more effectively and that OOH facilities should be located in densely populated areas along common delivery routes (Akkerman et al., 2025).

At the tactical and operational levels, the existing literature investigates vehicle routing problem (VRP) variants that incorporate OOH delivery locations. Early work on integrating home and pickup delivery within a unified VRP framework includes Zhou et al. (2018), who study a two-echelon system with alternative delivery locations. However, their model does not incorporate time slots or pricing. Moreover, Dumez et al. (2021) defined the vehicle routing problem with delivery options, which includes multiple potential delivery locations per customer. Pan et al. (2021) proposed a two-level model for designing urban delivery networks using parcel lockers: the upper level optimizes locker locations and capacities through a parcel network flow problem, while the lower level addresses a multi-depot capacitated VRP to manage deliveries between depots and lockers. Arslan (2021) introduces the location-or-routing problem, highlighting the interdependence between selecting delivery locations and routing decisions. Additionally, Janinhoff and Klein (2023) developed a two-stage stochastic location-routing model that integrates delivery options, customer choice, and allows for multiple trips per vehicle. Nonetheless, most of these studies assume simplistic customer choice models: either as a constant function of the distance to OOH locations (Janinhoff and Klein, 2023) or based on a ranked preference list (Dumez et al., 2021).

Recent studies started to explicitly model customer choice in the context of OOH delivery. Zhang et al. (2023) developed a convex optimization model to capture the distribution of customers across different delivery options. Galiullina et al. (2024) focuses on incentivizing customers to select OOH delivery in order to reduce routing costs. In their model, the customer's final choice is represented as a probability that depends on the retailer's incentive.

2.3. Mixed delivery strategy

Several studies have attempted to integrate AHD and OOH delivery methods and expand the range of options available to customers. Mancini and Gansterer (2021) proposed a mixed delivery strategy that combines AHD and OOH delivery, formulating the problem as a variant of VRPTW. Their results show a 40% improvement in solution quality without compromising service levels. Moreover, Dumez et al. (2021) designed a system that allows customers to specify multiple delivery options with different locations, time windows, and associated preference levels. This problem is similarly formulated as a VRP variant and solved using a tailored LNS method. Recent work by Hong et al. (2025) also incorporates self-pickup as an alternative to AHD within an integrated routing framework. Their formulation expands operational flexibility but, similar to earlier studies, does not model customer choice or incorporate pricing decisions. Moreover, Akkerman et al. (2025) proposed a method to dynamically offer OOH options and prices to steer customer behavior in a dynamic setting. Their model explicitly accounts for future customer arrivals and choices using a convolutional neural network. Specifically, customer preferences among delivery options are modeled using a multinomial logit model.

These studies provide valuable insights into offering mixed delivery options to enhance both delivery efficiency and customer satisfaction. Nonetheless, Mancini and Gansterer (2021) and Dumez et al. (2021) focus on static VRP and thus overlook the dynamic interaction between online retailers and customer decision-making. Moreover, their models do not incorporate pricing and ignore the stochastic nature of customer choice behavior. Additionally, Akkerman et al. (2025) focuses solely on OOH delivery and therefore neglects time slot management for home delivery options. To address these gaps, we propose a tactical offering and pricing model that jointly manages delivery time slots and locations, explicitly incorporating customers' stochastic choice behavior.

To position our work within this landscape, Table 1 provides a structured comparison of relevant studies across delivery modes, decision levers, and modeling approaches. Specifically, behavioral assumptions specify whether customer responses are treated deterministically or modeled as stochastic choices, while the choice model type defines how these responses are captured. As illustrated, existing research predominantly focuses on either AHD or OOH in isolation, or addresses mixed delivery strategies primarily from a deterministic perspective without pricing levers. While recent contributions have successfully integrated pricing and stochastic choice models into specific domains, there remains a distinct gap in jointly optimizing assortment and pricing for both time slots

Table 1
Comparison of relevant literature on tactical demand management and last-mile delivery.

Reference	Flexible Options		Decisions			Behavioral	11Choice Model
	Time	*Loc.*	*Offer*	*Price*	*Route*	Assumptions	
Attended Home Delivery (AHD) Focus							
Cleophas and Ehmke (2014)	✓		✓			Deterministic	–
Mackert (2019)	✓		✓			Stochastic	Finite-mixture MNL
Klein et al. (2019)	✓			✓	✓	Deterministic	Rank-based
Visser and Savelsbergh (2019)	✓		✓		✓	Deterministic	Aggregated
Abdollahi et al. (2023)	✓			✓	✓	Stochastic	MNL
Abdolhamidi and Lurkin (2025))	✓		✓	✓	✓	Stochastic	Mixed Logit
Out-of-Home (OOH) Focus							
Zhou et al. (2018)		✓			✓	Deterministic	–
Pan et al. (2021)		✓	✓		✓	Deterministic	–
Janinhoff and Klein (2023)		✓	✓		✓	Stochastic	Distance-based
Zhang et al. (2023)		✓	✓	✓		Deterministic	Convex Opt.
Akkerman et al. (2025)		✓	✓	✓		Stochastic	MNL
Mixed Delivery (AHD + OOH)							
Mancini and Gansterer (2021)	✓	✓			✓	Deterministic	–
Dumez et al. (2021)	✓	✓	✓		✓	Deterministic	Ranked Preference
Hong et al. (2025)	✓	✓			✓	Deterministic	–
This Paper	✓	✓	✓	✓	✓	**Stochastic**	**Mixed Logit**

Note: "Time" = Time Slots; "Loc." = Locations; "Offer" refers to assortment/time slot availability decisions; "Price" refers to pricing decisions; "Route" refers to routing optimization. MNL = Multinomial Logit.

and locations under behavior uncertainty. Our study bridges this gap by proposing a unified framework that simultaneously manages mixed delivery options and pricing decisions to steer stochastic customer demand.

3. Problem description and mathematical model

In the following, we first describe the VRPFD-SB in Section 3.1. Next, we introduce the retailer's offering and pricing decisions in Section 3.2. In Section 3.3, we present the model of the customer decision process. Finally, Section 3.4 details the retailer's routing decisions and presents the complete MILP formulation.

Table A.7 in Appendix A provides a summary of the main notations used in this section for readers' convenience.

3.1. Problem description

The problem concerns an e-commerce retailer or a delivery operator steering customer delivery demand through offering and pricing decisions to improve service quality while reducing logistics costs. We assume that both the customer demand and each customer's default delivery address are known in advance, whereas the customers' final delivery choices remain uncertain and are influenced by the operator's offering and pricing decisions.

This abstraction reflects several common operational settings, such as subscription-based delivery programs in which e-grocery retailers deliver goods on a recurring schedule, or services where all orders and delivery addresses are collected before a cutoff time. In the latter case, providers can send updated delivery options, including possible discounts, for customer confirmation shortly before the final delivery day. For instance, both DHL (2025) and Post NL (2025) utilize Email or mobile application notifications to allow customers to modify their delivery preferences before a specified cutoff time. Under these workflows, retailers know who will be served and where they live, while customers may still confirm or adjust their preferred delivery location or time slot before routing is finalized.

In this paper, we consider a subscription-based delivery setting where customers may cancel their orders. Customer demand and locations are known in advance, and the retailer must pre-select which delivery options to offer and at what prices. These offering and pricing decisions are tactical choices made prior to order realization. Specifically, the retailer must determine the available delivery options, including pickup stations and home deliveries with various time slots, and their associated fees, before customer preferences are revealed. In reality, customer preferences for delivery locations and time slots are difficult to predict, which is a key source of uncertainty. This uncertainty arises from either the variability in customers' personal schedules and movements or concerns related to privacy.

Subsequently, given an assortment of delivery options and prices, customers can freely choose one option or even reject all offers and leave the system, which is captured by the opt-out alternative. Under the subscription setting, an opt-out represents a direct loss of demand. It is worth emphasizing the necessity of introducing the opt-out option. If all delivery options are fully controlled by the retailer, the model may overestimate demand by assuming that all customers will always select from the offered alternatives. Moreover, without an opt-out option, customers are treated as captive, and the problem may become unbounded (Paneque et al., 2021; Haering et al., 2024). Alternatively, the customer's default option can also be designed as the opt-out option. Once all customers have confirmed their final delivery time and locations, the retailer plans the delivery routes, where the problem is reduced to a classic VRPTW.

3.2. The retailers' decision on delivery options

We consider a set of customers I requesting last-mile delivery service. Each customer $i \in I$ is associated with a home delivery address. Although customer locations are known, their final delivery choices, either home delivery within a specific time slot or delivery to an SDL without a time constraint, remain uncertain and depend on the retailer's offering and pricing decisions.

We assume that the retailer simultaneously offers home delivery options with different time slots and OOH alternatives, each associated with different price multipliers. For home delivery options, we divide the service period into identical time slots, indexed by $t \in T$. On the other hand, we denote by F the set of SDLs. Note that each customer i is only compatible with a subset of $F_i \subseteq F$, which depends on the customer's maximum pick-up time δ. Moreover, H denotes the set of discrete price multipliers. Therefore, for each customer i, the retailer must decide which pickup point, time slot, and price multiplier to offer. We use a tuple $(t, h) \in T \times H$ to depict a time slot option associated with a price multiplier h. Similarly, a tuple $(f, h) \in F \times H$ denotes a pickup point alternative, where the parcel is delivered to pickup point f with multiplier h. Finally, we denote a complete set of options by $M = \{T \times H\} \cup \{F \times H\} \cup \{o\}$, where $\{o\}$ represents the opt-out alternative, which is always offered to the customer.

We define the decision variable y_{mi} to capture the retailer's decisions, which equals one if a delivery option m is offered to customer i, and zero otherwise. Specifically, this includes y_{oi}, y_{thi}, and y_{fhi}. Naturally, we have the following constraints to restrict the retailer's decisions:

$$y_{oi} = 1, \quad \forall i \in I \tag{1}$$

$$\sum_{t \in T} \sum_{h \in H} y_{thi} \geq n, \quad \forall i \in I \tag{2}$$

$$\sum_{f \in F} \sum_{h \in H} y_{fhi} \geq k, \quad \forall i \in I \tag{3}$$

$$\sum_{h \in H} y_{thi} \leq 1, \quad \forall t \in T, i \in I \tag{4}$$

$$\sum_{h \in H} y_{fhi} \leq 1, \quad \forall f \in F, i \in I \tag{5}$$

$$y_{thi}, y_{fhi} \in \{0, 1\}, \tag{6}$$

Constraint (1) ensures that the opt-out option $\{o\}$ is always available. Eqs. (2)–(3) guarantee that at least n time slots and k pickup point are provided. While constraints (4)–(5) prevent providing multiple multipliers corresponding to a single time slot or a pickup point. It is trivial to offer the same delivery location with different prices, as rational customers would always opt for the cheapest option.

It is worth mentioning that the use of price multiplier unifies the offering and pricing decisions into a generalized offering problem. Instead of determining the price and which options to offer separately, each option is associated with a distinct multiplier. Consequently, the offering and pricing decisions are jointly captured by y_{mi}.

3.3. Customer choice behavior

In this paper, we apply the mixed logit model to characterize customers' choices. Each option is associated with a utility function, and customers are assumed to select the option with the highest utility. The customers' utility is typically modeled as a random variable to capture the choice uncertainty. Conventionally, the utility that customer i associates with option $m \in M$ can be computed by:

$$U_{mi} = V_{mi} + \epsilon_{mi} \tag{7}$$

where V_{mi} indicates the deterministic part of the utility, such as the inherent attributes of the alternatives and the socio-demographic features of the customer; while ϵ_{mi} reflects the uncertainty that arises from the unobserved factors, such as unobserved individual characteristics or measurement errors (Ben-Akiva and Bierlaire, 2003).

Specifically, we recognize three critical factors that affect customers' delivery choices: the delivery time window is important for home delivery, while travel distance is crucial for pickup delivery; Lastly, the delivery fee plays a significant role in both options. Therefore, the deterministic term of the disutility of customer i is given by:

$$V_{thi} = \beta^1 + \beta^t + \beta_i^f \cdot h \cdot p_1, \quad \forall t \in T, \forall h \in H, \forall i \in I \tag{8}$$

$$V_{fhi} = \beta^2 + \beta_i^d \cdot exp(c_i^{pu}) + \beta_i^f \cdot h \cdot p_2, \quad \forall f \in F, \forall h \in H, \forall i \in I \tag{9}$$

In Eqs. (8)–(9), the parameters β^1 and β^2 reflect the inherent delivery utility of home and non-home delivery; β^t, β_i^d, and β_i^f represent customers' sensitivities for delivery time slot, pickup distance, and delivery fee, respectively. Later two are individual-specific parameters and are assumed to follow normal distributions $\beta_i^d \sim \mathcal{N}(\mu^d, \sigma^d)$, $\beta_i^f \sim \mathcal{N}(\mu^{price}, \sigma^{price})$. Besides, c_i^{pu} denotes the distance between the pickup point and home; and p_1, p_2 are the initial prices for home and OOH delivery, respectively. Therefore, we can derive the probability of customer i select option m as follow (Ben-Akiva and Bierlaire, 2003):

$$P_{mi} = \int_{\beta_i^f}\int_{\beta_i^d} \frac{y_{mi} \cdot e^{V_{mi}}}{\sum_{k\in M} y_{ki} \cdot e^{V_{ki}}} f(\beta_i^d, \beta_i^f \mid \Theta)\, d\beta_i^d\, d\beta_i^f, \quad \forall m \in M, \forall i \in I \tag{10}$$

where $f(\beta_i^d, \beta_i^f)$ is the joint probability density function; and Θ is the sample space of β_i^d and β_i^f.

Nevertheless, integrating the above formulation into the VRP is challenging. Therefore, a simulation-based approach proposed by Paneque et al. (2021) is utilized to approximate the probabilities of customers' choices. The fundamental principle is to generate R draws from the distributions of all individual-specific parameters, including β_i^d, β_i^f, and error terms ϵ_{mir}. Note that the error term ϵ_{mir} is assumed to be independently and identically distributed following the Gumbel distribution $\epsilon_{mir} \sim \mathcal{G}(0,1)$. At each draw, we sample $(\bar{\beta}_i^d, \bar{\beta}_i^f, \bar{\epsilon}_{mir})$ for every customer i, which defines one scenario. Within that scenario, all customer parameters are fixed, so choices are deterministic. We then solve a separate VRPTW for that scenario.

Once the draws have been generated, we can derive the utility that each customer i chooses option m for a given draw r as follows:

$$U_{thir} = \beta^1 + \beta^t + \bar{\beta}_{ir}^f \cdot h \cdot p_1 + \bar{\epsilon}_{thir}, \quad \forall t \in T, \forall h \in H, \forall i \in I, \forall r \in R \tag{11}$$

$$U_{fhir} = \beta^2 + \bar{\beta}_{ir}^d \cdot exp(c_i^{pu}) + \bar{\beta}_{ir}^f \cdot h \cdot p_2 + \bar{\epsilon}_{fhir}, \quad \forall f \in F, \forall h \in H, \forall i \in I, \forall r \in R \tag{12}$$

Notice that each customer may have access to different delivery options, which depend on the retailer's decision variables y_{thi} and y_{fhi}. Hence, the complete utility formulation is given by:

$$U_{thir} = \beta^1 + \beta^t + \bar{\beta}_{ir}^f \cdot h \cdot p_1 + \bar{\epsilon}_{thir} - \mathcal{M}_{ir}(1 - y_{thi}), \quad \forall t \in T, \forall h \in H, \forall i \in I, \forall r \in R \tag{13}$$

$$U_{fhir} = \beta^2 + \bar{\beta}_{ir}^d \cdot exp(c_i^{pu}) + \bar{\beta}_{ir}^f \cdot h \cdot p_2 + \bar{\epsilon}_{fhir} - \mathcal{M}_{ir}(1 - y_{fhi}), \quad \forall f \in F, \forall h \in H, \forall i \in I, \forall r \in R \tag{14}$$

where $\mathcal{M}_{ir} = max_{m\in M} \|U_{mir}\|$ is the largest absolute utility value across all options for customer i under scenario r. Note that using a random number of scenarios may introduce substantial variance and undermine the credibility of the retailer's decisions; therefore, in Section 5.3.1, we examine how many scenarios are needed to sufficiently reduce this noise.

The choice of customer i in scenario r is denoted by the general form w_{mir}, which consists of three sets of binary variables $w_{oir}, w_{thir}, w_{fhir}$. These variables indicate whether the base option $\{o\}$, a time slot (t,h), or a pickup point option (f,h) is chosen by customer i in scenario r, respectively. Therefore, we introduce the following constraints to capture the user's decision-making behavior.

$$\sum_{t\in T}\sum_{h\in H} w_{thir} + \sum_{f\in F}\sum_{h\in H} w_{fhir} + w_{oir} = 1, \qquad \forall i \in I, \forall r \in R \tag{15}$$

$$w_{thir} \le y_{thi}, \qquad \forall t \in T, \forall h \in H, \forall i \in I, \forall r \in R \tag{16}$$

$$w_{fhir} \le y_{fhi}, \qquad \forall f \in F, \forall h \in H, \forall i \in I, \forall r \in R \tag{17}$$

In constraints (15), each customer is forced to select only one option in each scenario. Constraints (16) –(17) guarantee that any alternative not offered to the customer cannot be selected.

As the random utility theory assumes the customer would choose an option associated with the maximum utility, we impose the following constraints to reflect the customers' choice behaviors:

$$U_{mir} \le U_{ir}^u \le U_{mir} + \mathcal{M}_{ir}^U(1 - w_{mir}), \quad \forall m \in M, \forall i \in I, \forall r \in R \tag{18}$$

$$U_{ir}^u \in \mathbb{R}, w_{mir} \in \{0,1\}, \tag{19}$$

where $\mathcal{M}_{ir}^U = 2\mathcal{M}_{ir}$ is a sufficiently large constant. Notice that if $\mathcal{M}_{ir}^U < 2\mathcal{M}_{ir}$, the model might be infeasible (Paneque et al., 2021; Abdolhamidi and Lurkin, 2025). We also introduce a continuous variable U_{ir}^u to represent the highest utility among all available options. Constraints (18) guarantee that the option chosen by each customer is the one with the highest utility U_{ir}^u. Specifically, when U_{mir} is not the highest, these constraints force $w_{mir} = 0$ to ensure it is not being violated. On the contrary, if a customer i chooses m in scenario r, we have $w_{mir} = 1$, which makes $U_{ir}^u = U_{mir}$, indicating that the option with the highest utility is chosen. According to the Sample Average Approximation (SAA) method, the probability in Eq. (10) can be approximated as:

$$P_{mi} \approx \frac{1}{R}\sum_{r\in R} w_{mir}, \quad \forall m \in M, \forall i \in I \tag{20}$$

Now that we have modeled both the retailer and customer choice processes, we will then integrate these with the VRPTW and formulate a complete optimization model.

3.4. Scenario-based routing decisions

Let us define the last-mile delivery network $G = (V_0, E)$, where $V_0 = I \cup F \cup \{0\}$, including all the customers' home locations, pickup points, and the depot. $V = I \cup F$ denotes the set of all nodes excluding the depot. With each edge $e = (i,j)$, where $i \ne j, i, j \in V_0$, we associate a cost c_{ij}, which is assumed to be a constant and known in advance. To reflect the limited capacity of a pickup point, such as the number of available lockers, we associate a capacity B_f with each pickup point f.

For each customer i, if he or she chooses to be served at home, it has to be within the selected time window $[a_t, b_t]$. On the contrary, there is no time limit for pickup point delivery within the planning time horizon. The time window for the depot is $[0, T_{\max}]$, which means all the vehicles must return to the depot before time $T_{\max}$. Note that the service period $[0, T_{\max}]$ is divided into several identical smaller time slots, which are indexed by t, and a_t, b_t are the corresponding lower and upper bounds. A service time s_i is defined for each customer, regardless of whether the service takes place at a pickup point or their home. Moreover, each vehicle used for delivery would incur a fixed cost γ.

We define three sets of routing decision variables. Firstly, the binary variable x_{ijr} indicates whether node j is visited directly after node i in scenario r. Secondly, we denote by $Z_{fr} \in \{0,1\}$ if a pickup point $f \in F$ is visited (1) or not (0) in scenario r. Lastly, the decision variable $\mathcal{T}_{ir}$ is defined for each vertex and denotes the time a vehicle visits vertex $i \in V_0$ in scenario r. The routing constraints are given by:

$$\sum_{i \in V_0} x_{ijr} + \sum_{f \in F_j} \sum_{h \in H} w_{fhjr} + w_{0jr} = 1, \quad \forall j \in I, \forall r \in R \tag{21}$$

$$\sum_{i \in V_0} x_{ijr} = \sum_{i \in V_0} x_{jir}, \quad \forall j \in V_0, \forall r \in R \tag{22}$$

$$Z_{fr} \geq \frac{1}{|I|} \sum_{i \in I} \sum_{h \in H} w_{fhir}, \quad \forall f \in F, \forall r \in R \tag{23}$$

$$\sum_{i \in V_0} x_{ifr} = Z_{fr}, \quad \forall f \in F, \forall r \in R \tag{24}$$

$$\mathcal{T}_{jr} \geq \mathcal{T}_{ir} + t_{ij} + s_j - 2T_{max}(1 - x_{ijr}), \quad \forall j \in V, \forall i \in V_0, \forall r \in R \tag{25}$$

$$-T_{max} \sum_{f \in F} \sum_{h \in H} w_{fhir} + \sum_{t \in T} \sum_{h \in H} a_t \cdot w_{thir} \leq \mathcal{T}_{ir}, \quad \forall i \in I, \forall r \in R \tag{26}$$

$$\mathcal{T}_{ir} \leq \sum_{t \in T} \sum_{h \in H} b_t \cdot w_{thir} + T_{max} \sum_{f \in F} \sum_{h \in H} w_{fhir}, \quad \forall i \in I, \forall r \in R \tag{27}$$

$$\mathcal{T}_{jr} + s_j + t_{j0} \leq T_{max}, \quad \forall j \in V, \forall r \in R \tag{28}$$

$$\sum_{i \in I} \sum_{h \in H} w_{fhir} \leq B_f, \quad \forall f \in F, \forall r \in R \tag{29}$$

$$\sum_{h \in H} y_{fhi} = 0, \quad \forall f \in F : f \notin F_i, \forall i \in I \tag{30}$$

$$x_{ijr}, Z_{fr}, \in \{0,1\}, T_{ir} \geq 0 \tag{31}$$

In constraints (21), we guarantee that each customer is either served at home, served at a pickup station, or chooses to opt out. Constraints (22) ensure the conservation of vehicles. Constraints (23)–(24) ensure that an SDL f is visited if at least one customer selects it in scenario r. Constraints (25) guarantee that the visiting times at each node do not conflict. Constraints (26)–(27) specify the lower and upper bounds on the visiting times at each node. Constraints (28) ensure that each vehicle returns to the depot before the service deadline. Constraints (29) are the capacity constraints. Constraints (30) verify that incompatible SDLs are not offered to customers. Lastly, Constraints (31) are domain constraints.

3.5. The complete mathematical model

In summary, the VRPFD-SB can be formulated as a two-stage stochastic optimization problem, and its compact form is presented as follows:

$$\max_{y} \; f(y) + \mathbb{E}_{\xi}[Q(y, \xi)]. \tag{32}$$

where $y := y_{mi}, m \in M$, represents the first-stage offering and pricing decisions, which are identical across all scenarios; ξ denotes the uncertain customer-preference parameters, with each realization of ξ represented by random draws of β and ϵ; and $Q(y, \xi)$ is the optimal value of the second-stage recourse, which includes customers' realized choices and the resulting VRPTW routing cost. Because $Q(y, \xi)$ does not have a closed-form expression, we use SAA with R scenarios to approximate the expectation:

$$\max_{y, \{w^r, U^r, x^r, T^r, Z^r\}_{r=1}^{R}} \; \frac{1}{R} \sum_{r=1}^{R} \left[\mathcal{R}(y, w^r) - C(x^r, T^r, Z^r) \right]. \tag{33}$$

subject to 1) first-stage constraints on y, shared by all scenarios; 2) second-stage constraints for each scenario r, including utilities, customer choice constraints, and routing constraints. The objective function maximizes the difference between delivery revenue $\mathcal{R}$ and routing costs C over all scenarios. Ultimately, the extensive MILP formulation is given below:

$$\text{maximize} \quad \frac{1}{R} \sum_{r \in R} \left(\sum_{i \in I} \left(\sum_{m \in M \setminus 0} w_{mir} h_m p_m - \zeta \cdot w_{0ir} \right) - \alpha \sum_{i \in V_0} \sum_{j \in V_0} c_{ij} x_{ijr} - \gamma \sum_{j \in V} x_{0jr} \right) \tag{34}$$

$$\text{subject to} \quad \text{Constraints (1)} - -(5), (30) \rightarrow \text{Option availability decisions} \tag{35}$$
$$\text{Constraints (15)} - -(18) \rightarrow \text{Cutomer's utility and choices} \tag{36}$$
$$\text{Constraints (21)} - -(29) \rightarrow \text{Vehicle flexible routing decisions} \tag{37}$$
$$\text{Constraints (6)}, (19), (31), \rightarrow \text{Variables' domains} \tag{38}$$

In the objective function, the first term in the parenthesis is the total delivery revenue, including fees from home and OOH deliveries, as well as penalties from customers who opt out; ζ denotes the unit penalty; the second term represents the total routing cost, scaled by α; and the third term corresponds to the vehicle cost, scaled by γ. Additionally, the aforementioned constraints can be categorized into four classes: retail offering and pricing decisions, customer decisions, vehicle routing decisions, and variable domain constraints. Note that the number of vehicles, denoted by $\sum_{j\in V} x_{0jr}$, is assumed to be flexible, which ensures that the final routing decisions are always feasible.

The model determines y_{mi} first and then, for each realization of ξ, the second-stage decisions are taken independently. Note that we have $f(y) \equiv 0$ in the objective function (32). Because the first stage decision y does not incur any cost directly. It just restricts the customer's decision in the second stage. In principle, we could introduce $f(y) = \sum_{m\in M} \sum_{i\in I} y_{mi}$ in the objective so as to maximize the number of offered options for each customer. Nonetheless, since our focus is on maximizing profit, we intentionally omit any explicit dependence on y in the objective.

The proposed mathmatical model is general and can be readily adapted to settings where some customers are fixed to default delivery options. For example, a customer is only available for home delivery around 18:00. In that case, we can simply impose the following constraints:

$$y_{thi} = 1, \quad t = 2, h = 1, i = 1 \tag{39}$$
$$\sum_{m\in M} y_{mi} = 1, \quad i = 1, M = \{T \times H, F \times H, 0\} \tag{40}$$

which ensure that only the preferred time slot is offered and that the customer can select only this fixed option.

4. Solution approach

The VRPFD-SB is a highly complex two-stage stochastic programming problem that extends the conventional VRP and requires solving scenario-dependent VRPs based on customer decisions. The computational challenge of our model does not stem from the size of the option set, but rather from (i) the integration of offering, pricing, customer choice, and routing decisions into a unified MILP framework, and (ii) the need to solve a large number of scenario-dependent VRPTWs. Solving this problem to optimality is impractical, particularly for large-scale instances. Therefore, we are motivated to develop a computationally efficient algorithm to address computational challenges. As a well-established metaheuristic, ALNS has demonstrated strong performance in solving complex VRP variants, striking a good balance between computational efficiency and solution quality (Pisinger and Ropke, 2018). In this section, we propose a hybrid heuristic that integrates ALNS with an LNS-based matheuristic, along with several supporting subroutines, to efficiently solve the VRPFD-SB. A matheuristic is a class of methods that combine mathematical programming with metaheuristics to exploit the strengths of both approaches (Boschetti et al., 2009).

The overall framework of our h-ALNS is depicted in Fig. 3. We begin by initializing the parameters and constructing a feasible solution, $S_{incumbent}$. In the first phase, various destroy and repair operators are applied to generate a new feasible solution, $S_{current}$. Nonetheless, evaluating the objective value of $S_{current}$ is computationally challenging, as it requires solving n VRPTW instances-one for each customer behavioral scenario. To address this, we employ a routing heuristic to approximate the objective function value for each scenario. Moreover, by exploiting the independence among scenarios, these second-stage subproblems are solved in parallel to improve computational efficiency. Subsequently, we employ the Record-to-Record Travel acceptance criterion to update $S_{incumbent}$, S_{best}, and the operator weights. According to the Record-to-Record Travel criterion, the new solution $S_{current}$ is accepted if its objective function value is within a threshold ϕ of the incumbent solution $S_{incumbent}$. The threshold ϕ decreases linearly throughout the iterations. To distinguish it from the h-ALNS, we refer to the first-phase algorithm as pure ALNS (p-ALNS). which subsequently serves as a benchmark for performance comparisons.

Once the stopping criterion is met, we proceed to the second phase, where an LNS-based matheuristic is applied to refine S_{best}. A small subset of customers, denoted as $S_{flexible}$, is randomly selected, and their decisions are unset. Meanwhile, the remaining variables, S_{fix} are fixed, and the corresponding stochastic problem is solved. Although the number of decision variables is significantly reduced, solving this subproblem to optimality remains computationally expensive. Therefore, we aim to obtain high-quality feasible solutions rather than exact optima, which allows us to find relatively good solutions within a reasonable computation time.

Although the solution framework also consists of two phases, it does not correspond directly to the two-stage problem. Instead, both phases address the problem by destructing and repairing the solution directly. It is worth mentioning that the second phase serves as a refinement process. As observed during implementation, the p-ALNS results in a large gap. The relationship between the two phases is presented in Section 5.3.2.

We next describe the framework in two parts. Section 4.1 presents the implementation details of the p-ALNS algorithm, while Section 4.2 elaborates on the LNS-based matheuristic.

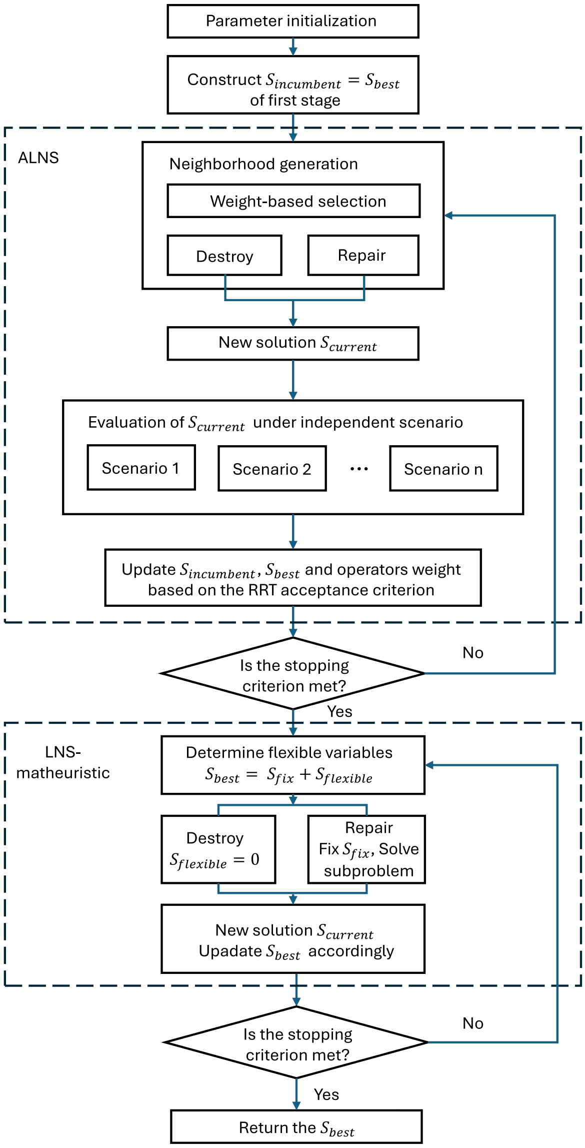


Fig. 3. Flowchart of the h-ALNS algorithm.

4.1. *Constructive phase: p-ALNS*

We first present the procedure for constructing an initial feasible solution in Part 4.1.1. Subsequently, Part 4.1.2 outlines the overall p-ALNS framework. Finally, Part 4.1.3 describes the destroy and repair operators specifically tailored to the VRPFD-SB problem.

4.1.1. *Initial solution*

The greedy heuristic employed to generate high-quality feasible solutions is detailed in Algorithm 1. It adapts the route-first, time-second approach (Abdolhamidi and Lurkin, 2025) to accommodate the inclusion of both time slots and SDL options.

To begin with, customers are clustered using the classical K-Means method. Within each cluster, a route is constructed using the Nearest Neighbor Search method, starting and ending at the depot while visiting all customers in that cluster. Next, for each customer along the route, we apply a forward-backward timing approach to identify all feasible time slots. Specifically, forward timing determines the earliest arrival time by initiating from the depot and cumulatively summing the travel times along the route; conversely, backward timing computes the latest feasible arrival time by working backwards from the end of the route. The key objective is to identify the largest feasible time window for each customer.

Subsequently, we iterate over all feasible time slots and identify the lowest price multiplier. The selected option (t, h_t) must, on average across scenarios, have a higher choice probability than the opt-out alternative. If such an option is found, we compute its

Algorithm 1: Greedy heuristic.

$G \leftarrow$ Cluster customers into groups using K-means;
for *cluster* $\in G$ **do**
  Establish a *route* inside a cluster using the Nearest Neighbor Search;
  for *each customer* $i \in$ *route* **do**
    $\mathcal{T}_i \leftarrow$ Calculate the feasible time slots using the forward and backward timing;
    for *each slot* $t \in \mathcal{T}_i$ **do**
      $h_t \leftarrow$ Find the lowest price multiplier that makes the option (t, h_t) more attractive than the opt-out alternative across various scenarios;
      $opt_t \leftarrow$ Calculate the marginal profit for offering (t, h_t);
    for *each SDL* $f \in \mathcal{F}_i$ **do**
      $h_t \leftarrow$ Find the lowest price multiplier that makes the option (f, h_f) more attractive than the opt-out alternative across various scenarios;
      $opt_f \leftarrow$ Calculate the marginal profit for offering (f, h_f);
    $opt_t^{ranked}, opt_f^{ranked} \leftarrow$ Rank all options based on the profit ;
    $(y_{thi} = 1, y_{fhi} = 1) \leftarrow$ Assign the top n options from $opt_t^{ranked}, opt_f^{ranked}$ to customer i based on the minimum requirements for the number of options;
return $S = (y_{thi}, y_{fhi})$;

marginal profit under the assumption that the customer will select it. A similar process is applied to the selection of SDL options. Note that profit is calculated according to Eq. (34). Once the iteration terminates, we sort the two option sets $opt_t^{ranked}, opt_f^{ranked}$ based on the profits. Finally, the top n options from each set are selected to satisfy constraints (1) and (2), respectively.

4.1.2. p-ALNS framework

Algorithm 2 presents the adapted ALNS framework. First, all parameters are initialized, and an initial solution is generated using Algorithm 1. Then, the roulette wheel selection method is employed to choose a destroy and a repair operator based on their associated weights. These operators are subsequently applied to construct a new solution.

The most challenging aspect lies in evaluating a given solution, as solving the stochastic problem to optimality is computationally intensive and often impractical for large instances. To address this, we adopt a scenario generation approach to capture the problem's stochastic nature. Following the methodology in Abdolhamidi and Lurkin (2025), we incorporate Monte Carlo simulations to evaluate solutions under stochastic customer behaviors. Specifically, we sample the error terms ϵ_{oir} and utility parameters β_{ir} from their respective distributions. Each scenario is thus defined by a particular realization of the vector pair $(\epsilon, \boldsymbol{\beta})$. Given a solution S_c and a sampled scenario, customer utilities are computed accordingly, revealing each customer's delivery location and time window. This enables the formulation of a scenario-dependent VRPTW instance.

Rather than solving each resulting VRPTW instance to optimality, which is computationally prohibitive, we estimate routing costs using the Clarke and Wright savings heuristic (Clarke and Wright, 1964). This heuristic begins by assuming each customer is initially served by a dedicated vehicle. It then generates a savings list by evaluating the cost reduction from merging two routes. The list is ranked based on potential travel cost savings. Finally, we iteratively merge routes from the list, ensuring that vehicle capacity and time window constraints are satisfied. Furthermore, this scenario-based structure facilitates parallel computing, thereby enhancing computational efficiency, particularly for large-scale instances. As shown in line 5, the final objective value obj_c is computed as the average of the objective values across all scenarios.

Once the objective value of the current solution S_c is obtained, we compare it with the incumbent solution and its associated objective value. If the current solution outperforms the incumbent, all relevant quantities are updated accordingly. Alternatively, if the current solution is slightly inferior-specifically, within a threshold ϕ, it is still accepted, but the selected destroy and repair operators receive a lower score. The threshold ϕ decreases linearly by a factor λ, facilitating a transition from exploration to exploitation (Santini et al., 2018). Note that four categories of scores are used to update operator performance, corresponding to distinct conditions: highest, average, low, and lowest. The selection probability for each destroy and repair operator is then adjusted based on these updated scores. Finally, the algorithm terminates if there is no improvement of at least $\delta\%$ over N consecutive iterations.

4.1.3. Destroy and repair operators

Destroy operators apply different rules to identify which customers should be selected for removal, thereby eliminating all associated delivery options. In contrast, repair operators are responsible for restoring feasible delivery options for these customers. This section presents a detailed description of each operator.

Random destroy: This operator randomly selects a small subset of customers and removes all delivery options assigned to them.

Distance destroy: This operator randomly selects a customer and computes the distance from all other customers to the selected one. It then identifies the top n nearest customers and removes all associated delivery options.

Algorithm 2: Scenario-based p-ALNS.

$N, \delta, \phi, \phi_{\min} \leftarrow$ Parameters initialization;
$S_i, S_b, obj_i, obj_b \leftarrow$ Construct the initial solution with Algorithm 1 and compute the objective value;
repeat
  $S_c = r(d(S_i)) \leftarrow$ Construct a new solution S_c: select and apply a destroy and a repair operator using the roulette wheel selection method;
  $obj_c = \frac{1}{R}\sum_r obj_c^r(S_c) \leftarrow$ Compute the objective value of S_c across scenarios in parallel;
  if $obj_c >= obj_i$ **then**
    $S_i, obj_i \leftarrow$ Update the incumbent solution and objective value;
    if $obj_c >= obj_b$ **then**
      $S_b, obj_b \leftarrow$ Update the best solution and its objective value, and assign the highest scores to the selected operators;
    else
      Assign average scores to the selected destroy and repair operators;
  else
    if $obj_c >= obj_i - \phi$ **then**
      $S_i, obj_i \leftarrow$ Update the incumbent solution and its objective value, and assign low scores;
    else
      Assign the lowest scores to the selected destroy and repair operators;
  $\phi \leftarrow \max(\phi_{\min}, \phi - \lambda)$;
  Update the weights of the destroy and repair operators based on their new scores;
until *No δ% improvement in N iterations*;
return S_b;

Profit destroy: Customers are selected based on their contribution to profit. For each scenario, we record the individual profit generated by each customer. We then compute the mean profit across all scenarios for each customer and rank them in ascending order. Finally, the top n customers with the lowest average profit are selected, and all associated delivery options are removed.

SDL destroy: We identify the currently in-use SDLs based on the current solution. These SDLs are then ranked according to the number of customers assigned to each. Finally, the first n customers from the ranked SDL set are selected.

After removing delivery options for certain customers, repair operators are utilized to restore these options, ensuring that the revised solution satisfies all constraints. The operators we used are described as follows.

Random repair: Similar to the random destroy operator, this operator randomly selects n time slots and SDLs to offer. Note that constraints 2–5 and 30 must be respected during the random selection process.

High-utility repair: This operator iterates through all possible options and computes the corresponding utilities across scenarios. The scenario-dependent utilities are compared with the opt-out option in each scenario, and an option is recorded if its utility exceeds that of the opt-out. Next, the home delivery and SDL options are ranked based on the number of scenarios in which their utilities are higher. Finally, the top n options from each set are offered simultaneously.

Utility regret repair: Instead of offering the best option from the ranked utility list, this operator deliberately removes the top option with the highest utility and offers the remaining options. The ranked utility list is computed in the same way as in the high-utility repair operator.

4.2. Improvement phase: matheuristic

The motivation for the second-phase framework is to further refine the solution obtained by the ALNS. As demonstrated later in Section 5.3.2, a significant gap remains between the near-optimal solution and the one obtained solely from the first-stage p-ALNS. To address this issue, we design a matheuristic, detailed in Algorithm 3, to reduce the optimality gap. Inspired by the LNS framework, the method primarily relies on randomly releasing a subset of customers (destroy phase) and solving the resulting reduced problem using a solver (repair phase).

In Lines 4-5, we first select a small number of customers as free customers, meaning that all their associated variables y_{thi} and y_{fhi} are free. In contrast, the remaining variables are fixed according to the best solution S_b. Note that the number of free customers is dynamically adjusted in proportion to the total number of customers. Subsequently, we solve the VRPFD-SB (Eqs. (34)–(38)) with a partially fixed solution y_{mi} and use a solver to repair the solution for the free customers. Due to the restriction on the number of free customers and the predefined time limit, the solver can efficiently find a high-quality feasible solution. Nonetheless, as the number of customers increases-particularly in large instances with hundreds of customers and scenarios-it becomes necessary to tune the parameters to adapt to the problem size, in order to strike a balance between computational efficiency and solution quality.

Naturally, the new solution is accepted if it yields a better objective value. The algorithm terminates upon reaching the time limit or pre-terminates if there is no improvement over N consecutive iterations.

Algorithm 3: LNS-matheuristic.

$T_{\max}, N \leftarrow$ Parameters initialization;
$S_b \leftarrow$ The best solution obtained by Algorithm 2;
repeat
    $P = P_{flex} + P_{fix} \leftarrow$ Randomly selects n flexible customers, while the others remain fixed;
    $S_c^{partial} = S_b(P_{fix}) + S_b(P_{flex}) \leftarrow$ Fix a large number of variables as in the best solution, while leaving the remaining variables free;
    $S_c^{complete} = solver(S_c^{partial}) \leftarrow$ Solve the mathematical model using a solver within a specified time limit;
    if $V(S_c^{complete}) > V(S_b)$ **then**
        $S_b = S_c^{complete} \leftarrow$ update the best solution
until $T_{\max}$ *or no improvement in* N *iterations*;
return S_b;

4.3. Generalizability of the proposed framework

It is worth noting that the proposed h-ALNS framework is highly generalizable to broader classes of time-slot and location offering and pricing problems due to its modular structure. The algorithm consists of four independent components: 1) an ALNS module that updates offering and pricing decisions, 2) a customer-choice simulation module, 3) a VRP-variant solver executed in parallel, 4) a local-search refinement module. Each component can be substituted with state-of-the-art techniques. This modularity allows the framework to accommodate different behavioral models, e.g., MNL, Probit, or deterministic choice, and to support any combination of time-slot design, location choice, pricing, routing, and customer-choice specification. To demonstrate the effectiveness of the overall framework, we benchmark it against two baselines: a commercial MILP solver and a representative p-ALNS without the final matheuristic refinement phase. The latter is adapted from Abdolhamidi and Lurkin (2025) and represents a state-of-the-art ALNS approach.

5. Numerical experiment

This section presents a series of numerical experiments to evaluate the proposed model and algorithm. We begin by describing the instance generation process in Section 5.1, which leverages a publicly available dataset (Mancini et al., 2020). Section 5.3 then assesses the performance of the proposed algorithm. Finally, Section 5.4 investigates the characteristics of the stochastic problem and conducts sensitivity analyses on key parameters, providing managerial insights and underscoring important implications for decision-making.

5.1. Experiment setup

The VRPFD-SB problem involves two types of input data: vehicle routing data and customers' utility parameters. For the VRPTW, we generate five sets of instances, each comprising one depot, five SDLs, and varying numbers of customers ranging from 10 to 100. Following Mancini and Gansterer (2021), the locations of the depot and the SDLs are fixed across all instances, while customers are randomly distributed within a 10×10 km square area, as illustrated in Fig. 4. For all instances, we consider a 720-minute time horizon, which is divided into three equal periods representing the morning, afternoon, and evening, respectively. The travel cost c_{ij} between two arbitrary nodes is calculated as $c_{ij} = 3 * d_{ij}$, where d_{ij} is the Euclidean distance between i and j, and the coefficient reflects a travel speed of 20 km/h. We set the unit penalty for an unserved customer ζ, the travel cost scaler α, and the vehicle cost γ to 2, 2, and 1, respectively.

The second category of input data mainly refers to the customer utility parameters used in Eqs. (13) and (14). To address the absence of real-world data, we calibrate these parameters using the routing data described above. For utility bases β^1, β^2, the distance sensitivity β^d, and the mean price sensitivity β^f, we set a range of possible values with varying increments. In contrast, the remaining parameters are held fixed throughout the simulations. The price multipliers considered are 0% and 15%. For delivery costs, we adopt the standard Amazon delivery fees in Gothenburg: 49 Swedish kronor for home delivery and 29 for pickup point delivery (Amazon, 2025). The β^t is set to $-1.7, -1.3, -2.5$, corresponding to the three time periods. The base utility for opting out is normalized to 0. Note that each option is associated with a random error term ϵ, sampled from the Gumbel distribution $\mathcal{G}(0, 1)$. The objective is to achieve approximately 50–60% home deliveries and 30–40% pickup point deliveries. To this end, the final values of $\beta^1, \beta^2, \beta^d$ are set to 4, 3, and -0.005, respectively. The price sensitivity parameter β^f follows a normal distribution $\mathcal{N}(-0.08, 0.02)$. Although synthetic, the customer-level parameters were not randomly chosen. Instead, they were calibrated to reproduce observed market patterns reported

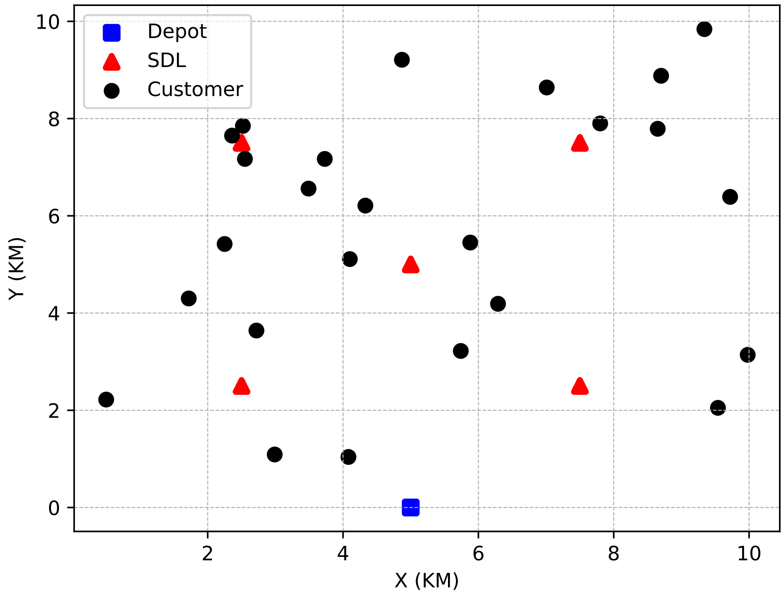


Fig. 4. An instance with one depot (square), 5 SDLs (triangles), and 25 customers (circles).

in recent empirical studies of AHD/OOH usage (Vakulenko, 2023; Akkerman et al., 2025). To avoid drawing conclusions that depend on one fixed synthetic calibration, we also conducted extensive sensitivity analyses on these utility parameters in Section 5.4.

Calibrated via preliminary tuning on small-scale instances to balance exploration quality and computational efficiency, the h-ALNS terminates if improvement falls below $\delta\% = 0.01$ over $N = 100$ iterations. For the Record-to-Record Travel acceptance criterion, the initial threshold ϕ is set to 5% of the initial objective value. The threshold is gradually decreased to a minimum value $\phi_{\min}$, defined as 1% of the initial ϕ. The decay rate is calibrated such that the threshold reaches $\phi_{\min}$ when the search approaches stagnation. The adaptive weight adjustment utilizes a reaction factor of 0.5. The LNS-matheuristic treats 10-20% of customers as flexible and runs for a maximum of $T_{\max} = 5$ or 10 minutes, stopping after 10 non-improving rounds. To balance computational efficiency and solution quality, the Gurobi solver is run with a time limit ranging from 5 to 150 seconds and a MIP gap tolerance between 2% and 7%, depending on the instance size.

The instance is denoted as CXX_RXX, where C represents the number of customers and R denotes the number of scenarios. For example, $C5\_R5$ refers to an instance with five customers and five scenarios. Because fixing customers by imposing constraints (39)–(40) reduces the retailer's ability to influence or adjust customer choices, we conduct experiments only on instances where all customers remain flexible, which also represents the most computationally challenging scenario. Moreover, the solution algorithms are implemented in Python 3.9, and the optimization models are solved using Gurobi 11.0.0. All experiments are performed on a computer equipped with an Intel Core i7-1370P processor and 32 GB of RAM.

Due to the stochastic nature of both the problem and the solution method, each experiment is run five times unless stated otherwise, and only the mean value is reported to ensure result robustness.

5.2. Model characteristics

In this subsection, we investigate the characteristics of the VRPFD-SB problem and the proposed mathematical model. First, we examine how different problem formulations affect computational time. Next, we demonstrate the benefits of incorporating customer stochastic behavior through a two-stage stochastic formulation. Finally, we discuss the advantages of including both time slots and SDL options.

5.2.1. Critical constraints

The constructed model has a certain degree of freedom in terms of designing constraints (2)–(3). For instance, we can change it to the following constraints:

$$\sum_{t\in T}\sum_{h\in H} y_{thi} + \sum_{f\in F}\sum_{h\in H} y_{fhi} \le 1, \quad \forall i \in I \tag{41}$$

$$\sum_{t\in T}\sum_{h\in H} y_{thi} + \sum_{f\in F}\sum_{h\in H} y_{fhi} \ge n, \quad \forall i \in I \tag{42}$$

Table 2
Comparison of different constraints on the number of options provided.

Constraints	C5_R100			C100_R100		
	Obj.Val	Opt.Gap(%)	Time(s)	Obj.Val	Opt.Gap(%)	Time(s)
$T >= 2, F >= 1$	44.31	77.17	7200	–	–	7200
$T + F <= 1$	36.93	0	240.5	1695.53	1.62	7200
$T + F >= 1$	36.60	107	7200	–	–	7200

Notes: Obj.Val denotes the objective value; Opt.Gap(%) denotes the optimality gap; Time(s) denotes the computation time in seconds. The unit of the objective value is Swedish krona; it is omitted in all tables for simplicity.

Table 3
Comparison of VSS and EVPI across different instances.

Instance	Z_d	Z_s	$\frac{VSS}{Z_s}(\%)$	Z_p	$\frac{EVPI}{Z_s}(\%)$	$\frac{VSS}{VSS+EVPI}(\%)$
$C5\_R50$	34.27	45.36	24.4	85.70	88.90	21.5
$C10\_R50$	116.64	155.70	25.1	229.26	47.2	34.7
$C20\_R50$	266.70	357.95	25.5	483.94	35.2	42.0

where Eq. (41) ensures that at most one alternative is offered to each customer, while Eq. (42) requires at least n alternatives to be provided, no matter it is different time slots or SDLs. Note that the explicit parameter k for the number of SDLs is no longer needed, as n already represents an arbitrary integer.

We solve the stochastic model with different constraints on the number of options provided. Two instances, $C5\_R100$ and $C100\_R100$, are tested using Gurobi with a two-hour time limit. The results are summarized in Table 2. For simplicity, we denote the sum of all time slots and SDL options, $\sum_{t,h} y_{thi}$ and $\sum_{f,h} y_{fhi}$, as T and F, respectively. For example, the constraint $T \geq 2, F \geq 1$ indicates that at least two time slots and one SDL option are provided. We observe that restricting the number of options to a maximum of one can significantly improve computational efficiency. When $T + F \leq 1$, Gurobi is able to solve the largest instance, $C100\_R100$, with only a 1.62% optimality gap. In contrast, increasing the number of options substantially complicates the computation, making it difficult for Gurobi to find a tight upper bound or even a feasible solution. Additionally, imposing separate constraints on T and F can positively impact the objective value. Compared to the case where $T + F \leq 1$, the constraint $T \geq 2, F \geq 1$ improves the objective value by 20%. This indicates that offering more options across both time slots and SDL increases the likelihood that customers will select a delivery option under stochastic settings. Consequently, this leads to higher profits. Nonetheless, this inevitably expands the solution space, making it more challenging to reach the optimum. In the following experiments, we adopt the original constraint $T \geq 2, F \geq 1$ to maximize profit.

5.2.2. Benefits of the stochastic model

To demonstrate the benefits of using a stochastic model to capture the stochasticity of customer behavior, we use two critical metrics: the Value of the Stochastic Solution (VSS) and the Expected Value of Perfect Information (EVPI) (Birge and Louveaux, 2011).

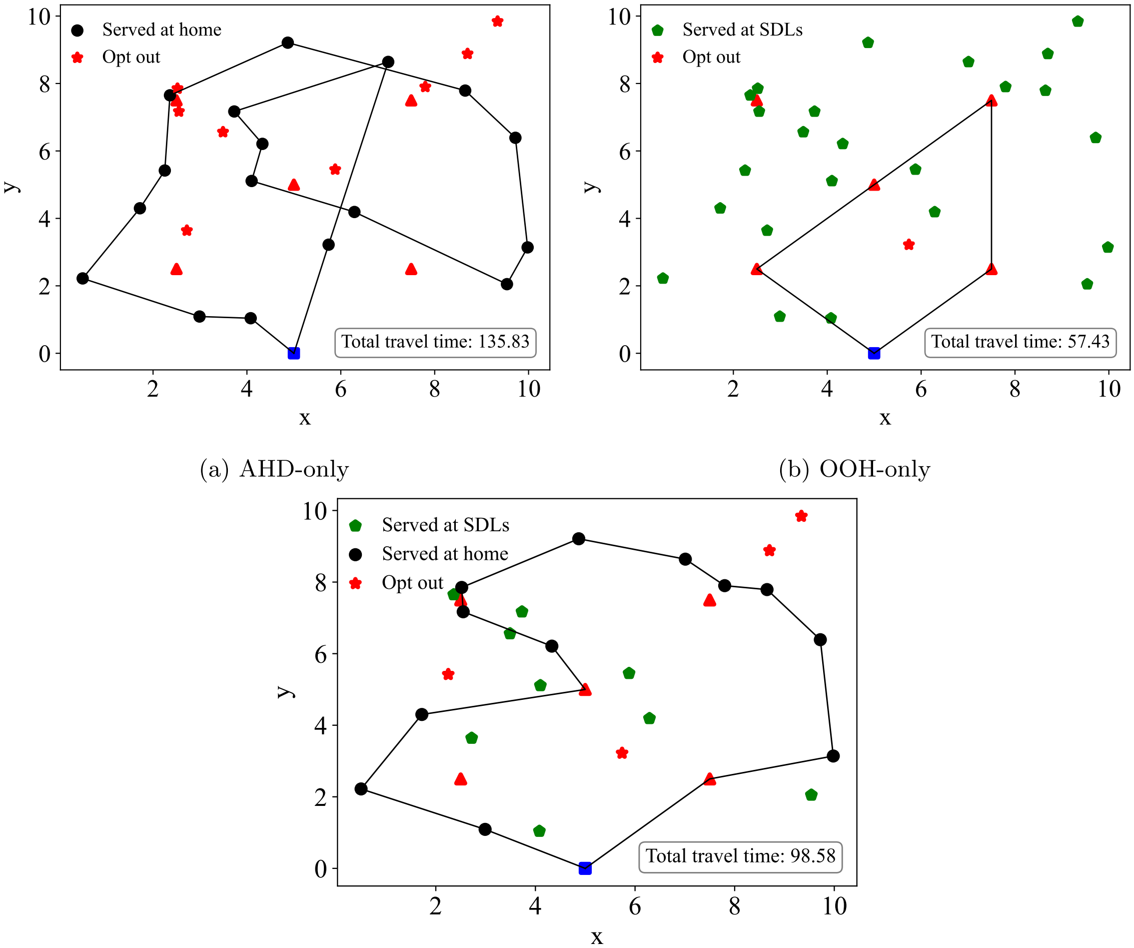
The VSS metric quantifies the gap between the objective values obtained from the deterministic Z_d and stochastic models Z_s. Specifically, the deterministic model corresponds to the original stochastic model but excludes the stochastic errors ϵ_{mir} in constraints (13) and (14). After solving the deterministic model, the resulting solution is re-evaluated using the original stochastic model, incorporating the stochastic errors ϵ_{mir}. The difference between this re-evaluated objective value and the optimal value of the stochastic model defines the VSS. In Table 3, we report the percentage of VSS relative to the objective value of the stochastic model. A gap of approximately 25% is observed across instances between the deterministic and stochastic models, demonstrating the advantage of incorporating customers' stochastic behavior.

On the other hand, EVPI measures the loss in profit due to the presence of uncertainty. We compute the objective value of each scenario assuming perfect information about customers' behavior, and denote by Z_p the average value across all scenarios. Similarly, we report the percentage of EVPI relative to the optimal value of the stochastic model. As shown in Table 3, an average gap of 57.1% is observed between the stochastic value and the optimal solution under perfect information, where uncertainty is eliminated. This highlights the potential benefit of knowing customers' behavior in advance and suggests a promising direction for future research on customer behavior prediction to enhance the efficiency of last-mile delivery.

Nonetheless, the last column of Table 3 shows the percentage of the perfect information value (EVPI + VSS) captured by the stochastic solution. An improvement of approximately 32.7% highlights the significant benefits provided by the stochastic model, emphasizing the importance of employing such models when customer behavior is uncertain-due to privacy concerns, for instance.

5.2.3. Benefits of the mixed delivery strategy

An important contribution of our framework is the integration of time slots management of attended home delivery and shared location delivery. This subsection aims to demonstrate the benefits of this mixed delivery strategy by comparing it with AHD-only and OOH-only delivery strategies. In this context, we consider a single scenario to simplify the analysis and enable direct comparison



(a) AHD-only

(b) OOH-only

(c) Mixed delivery strategy

Fig. 5. Vehicle route for serving 25 customers under different strategies.

of routing outcomes across strategies. These three strategies are evaluated across three customer set sizes in terms of objective value and the percentage of customers choosing home delivery, SDL, or opting out, respectively. The results are summarized in Table 4.

Across different instances, we observe that the mixed strategy consistently outperforms the other two delivery strategies. When the number of customers is 100, the mixed strategy improves the objective value by 44.6% compared to the AHD-only delivery. In Figure. 5, we also compare the vehicle routes for serving 25 customers under different strategies. Compare to AHD-only delivery, the mixed strategy reduces the total travel time by 27.4%, showing its potential in lowering vehicle energy consumption. Moreover, we find that the AHD-only strategy results in the highest percentage of customers opting out, suggesting that relying solely on home delivery may lead to customer dissatisfaction or postponed deliveries. On the other hand, although the OOH-only approach reduces the number of opt-outs and yields the shortest travel time, it also results in lower profits due to the lower delivery fees. In this regard, the mixed strategy combines the advantages of both approaches, achieving a balanced trade-off between home and SDL delivery, yielding fewer opt-outs and the highest profits across all instances.

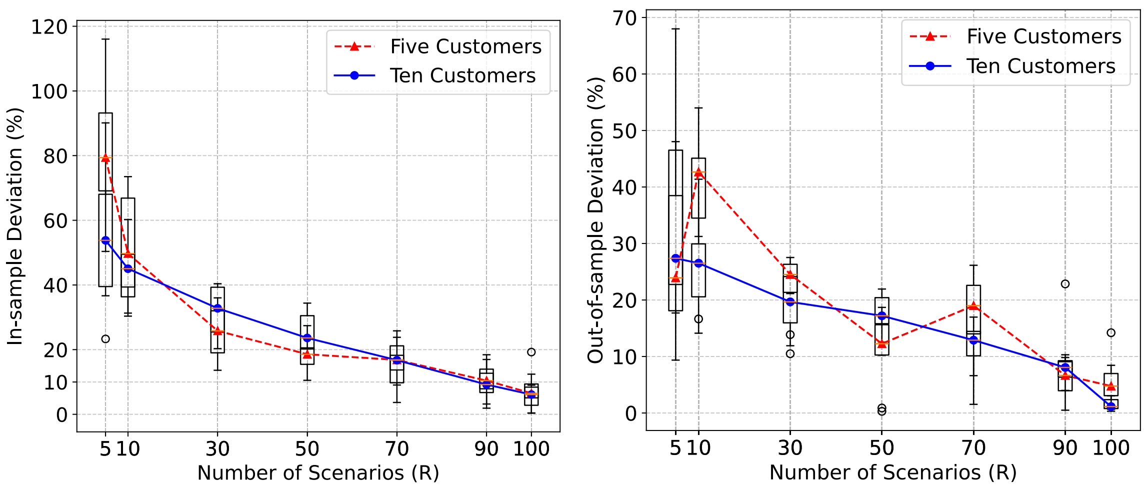
However, when the opt-out option does not represent a direct loss of demand, for example, when it is designated as the customer's default option, the advantage of the mixed strategy becomes weaker. In addition, the trade-off between direct delivery revenue and routing cost influences the objective value. Because the mixed strategy seeks to balance profit and logistics cost, its effect may diminish when profit dominates the objective. In such cases, the e-tailer may incorporate additional cost components, such as service time penalties, to better capture the operational trade-offs.

5.3. *Algorithm performance*

In this subsection, we conduct a series of numerical experiments to evaluate the proposed h-ALNS algorithm.

Table 4
Comparison of different delivery strategies across different instances.

Number of customers	Strategy	Obj.val	Percentage of customers choose (%)		
			Home	SDL	Opt_out
25	AHD-only	485.5	68	0	32
	OOH-only	573.8	0	96	4
	Mixed	635.5	48	36	16
50	AHD-only	1339.2	76	0	24
	OOH-only	980.7	0	80	20
	Mixed	1386.2	46	42	12
100	AHD-only	2272.0	63	0	37
	OOH-only	2378.9	0	90	10
	Mixed	3284.3	63	31	6



(a) In-sample test (b) Out-of-sample test

Fig. 6. Scenario sample test with five and ten customers.

5.3.1. Number of scenarios

The stochasticity in VRPFD-SB arises from Eqs. (13) and (14), which model customer choice behavior. To estimate these choices, we employ Monte Carlo simulations to approximate customer choice probabilities. Therefore, determining an appropriate number of scenarios is critical to accurately capturing heterogeneous customer behavior. The number of scenarios must be sufficiently large to ensure the accuracy and robustness of the model, while remaining computationally tractable. Following Kaut and Stein (2003), Abdolhamidi and Lurkin (2025), we conduct a series of in-sample and out-of-sample tests to determine the scenario sample size.

We use two sets of instances, with five and ten customers respectively, and solve the model using h-ALNS. For the in-sample test, we assess the stability of the scenario-generation method by comparing objective values as the number of scenarios increases. Specifically, we examine the deviation of objective values for varying scenario sizes, ranging from 5 to 100. For each scenario size, the objective values from ten trials, each with different β^f and ϵ, are compared against a reference value defined as the mean objective value obtained under 120 scenarios. As shown in Fig. 6a, the objective values converge as the number of scenarios increases, demonstrating the robustness of the scenario-generation method. When the number of scenarios increases from 5 to 50, deviations decrease significantly from approximately 80% to around 20%. Meanwhile, the standard deviation drops below 5% as the number of scenarios increases from 50 to 100, while the mean gradually decreases.

On the other hand, we re-evaluate the solution obtained from a given β^f and ϵ against alternative ϵ vectors, and report its deviation from the true objective value. As shown in Fig. 6b, the deviation exhibits a similar convergence pattern as the number of scenarios increases. Although the out-of-sample deviation for five-customer instances exhibits greater variability, the overall trend is decreasing and maintains a low deviation level when the number of scenarios exceeds 90. Moreover, when the number of scenarios reaches 100, both in-sample and out-of-sample deviations reduce to approximately 5%, with standard deviations stabilizing below 4%. Therefore, to balance computational efficiency and solution robustness, we adopt 100 scenarios for all subsequent simulations unless stated otherwise.

Table 5
Comparison of Gurobi, p-ALNS, and h-ALNS on different instances.

Instance	BKS^*	Gurobi			p-ALNS (10 runs)			h-ALNS (10 runs)		
		Best Obj (Gap%)	MIP Gap%	Time (s)	Best Obj (Gap%)	Mean BKSGap%	Mean Time(s)	Best Obj (Gap%)	Mean BKSGap%	Mean Time(s)
C5_R5	**56.9**	**56.9** (0)	0.0	32.6	47.4 (16.7)	22.2	1.8	**56.9** (0)	2.6	39.9
C5_R50	**49.4**	45.4 (8.1)	50.3	7200	43.4 (12.2)	18.1	9.4	**49.4** (0)	3.9	292.8
C5_R100	**44.3**	**44.3** (0.0)	77.2	7200	39.4 (11.1)	24.9	13.2	44.1 (0.5)	3.0	623.5
C50_R5	**1272.9**	1262.7 (0.8)	3.4	7200	1081.5 (15.0)	18.7	4.8	**1272.9** (0)	0.9	320.3
C50_R50	**1109.2**	**1109.2** (0)	7.0	7200	970.1 (12.5)	14.5	45.7	1083.2 (2.3)	4.1	676.4
C50_R100	**960.9**	–	–	7200	934.8 (2.7)	3.7	53.3	**960.9** (0)	1.6	714.2
C100_R5	**2719.3**	**2719.3**(0)	3.9	7200	2311.3 (15.0)	17.0	17.8	2716.1 (0.1)	1.8	482.6
C100_R50	**2218.4**	–	–	7200	2073.8 (6.5)	10.2	92,1	**2218.4**(0)	4.4	1257.6
C100_R100	**2085.4**	–	–	7200	1988.4 (4.7)	10.0	274.4	**2085.4**(0)	7.6	1599.4
Avg.	–	–	–	–	–	**15.5**	–	–	**3.3**	–

*The BKS is shown in bold in each row.

Table 6
Comparison of p-ALNS, matheuristic, and h-ALNS on a medium-size instance $C50\_R50$ with fixed time limit 10 mins.

p-ALNS		matheuristic		h-ALNS	
Obj. Val-	BKSGap(%)	Obj. Val-	BKSGap(%)	Obj. Val-	BKSGap(%)
1017.08	8.3	1035.86	6.6	1060.94	4.3
1012.49	8.7	1050.45	5.3	1071.93	3.3
1016.32	8.4	1037.50	6.5	1064.73	4.0
1013.38	8.6	1054.95	4.9	1080.08	2.6
1032.99	6.9	1042.49	6.0	1060.39	4.4
1018.45	**8.2**	**1044.25**	**5.9**	**1067.61**	**3.7**

Compared with the BKS of 1109.2 found by Gurobi.

5.3.2. Performance of the h-ALNS framework

We compare the computational performance of the two proposed algorithms, p-ALNS and h-ALNS, using Gurobi as a benchmark. We define the Best Known Solution (BKS) metric, which is the best objective value found among all runs of all methods under the stated time limits, including Gurobi, p-ALNS, h-ALNS. Besides, for each method, we report both the best objective value z and the mean percentage gap to BKS (Zhou et al., 2025). The gap is defined as:

$$BKSGap(\%) = \frac{BKS - z}{BKS} \times 100 \tag{43}$$

The comparison is conducted on instances with varying numbers of scenarios and problem sizes, and the results reported for each instance are averaged over ten runs. The results are summarized in Table 5

As shown, Gurobi can solve the problem to optimality only for instances with five customers and five scenarios. For larger instances, it often fails to find a feasible solution within the two-hour time limit. In contrast, the p-ALNS can find good-quality solutions within a few minutes, albeit with a BKSgap of nearly 15.5%. As a complement, the h-ALNS significantly reduces the gap, consistently to below 5%, while maintaining a reasonable computation time. In $C5\_R50$ and $C50\_R5$, the h-ALNS even finds better solutions than Gurobi, with an average BKSGap of 3.9 and 0.9, respectively.

To further investigate the relationship between the two phases of h-ALNS, we solve the medium-sized instance $C50\_R50$ using p-ALNS, the matheuristic (Algorithm 3), and h-ALNS. To ensure a fair comparison, we set the time limit to 10 minutes for all algorithms. The results are presented in Table 6. Each algorithm is run five times, and the last row reports the average objective value and the mean BKSgap.

As shown, p-ALNS yields the largest gap, suggesting that a longer time limit does not guarantee high-quality solutions for p-ALNS. In fact, p-ALNS can quickly find good feasible solutions within 36 seconds, while most of the remaining time only improves the solution quality by about 5%. The strength of p-ALNS lies in its computational efficiency and scalability. As evidenced in Table 5, p-ALNS can find a good feasible solution in approximately 4 minutes for the largest instance $C100\_R100$, which involves millions of constraints and binary variables in the mathematical model.

On the other hand, the matheuristic is inherently inferior to p-ALNS in terms of computational efficiency and scalability. Nonetheless, it generally produces better solutions when given sufficient solving time. However, it still yields inferior solutions compared to h-ALNS, demonstrating the effectiveness of combining the two methods. In fact, the matheuristic can significantly reduce the gap between two stages by 12.2%, as shown in Table 5. In summary, p-ALNS efficiently produces good feasible solutions in the first phase. Subsequently, the matheuristic is designed to further refine these solutions and reduce the optimality gap. The combination of the two methods achieves a good balance between computational efficiency and solution quality. Importantly, the delivery strategy analysis and all sensitivity analyses are conducted exclusively on medium-sized instances with 25 and 50 customers, as well as on

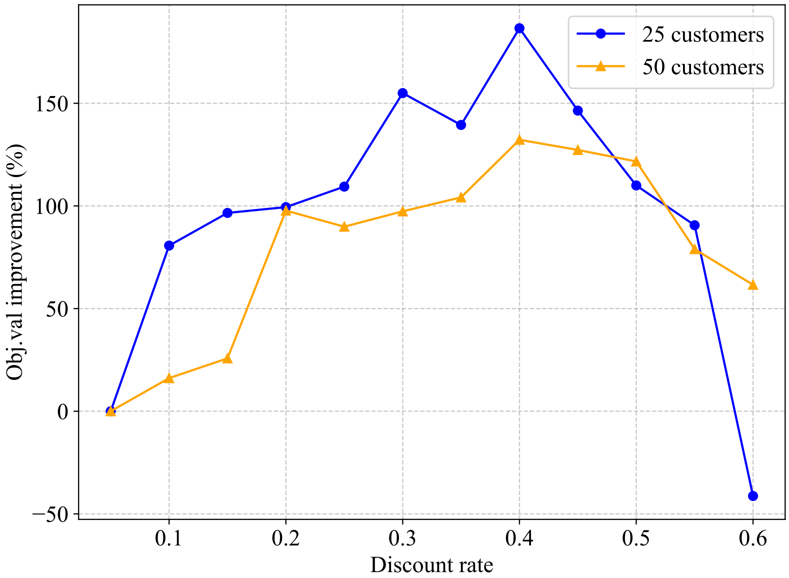


Fig. 7. Impact of discount rates on overall profit improvement for $C25\_R100$ and $C50\_R100$ solved with h-ALNS.

100-customer cases under a single scenario, for which Gurobi provides certified bounds that support the quality of the solutions. We deliberately selected these instance sizes to ensure that the managerial insights derived from our analysis rest on verifiable solution quality guarantees. By contrast, the larger instances for which Gurobi is unable to identify a feasible solution are included solely to demonstrate the scalability and computational efficiency of p-ALNS and h-ALNS.

5.4. Sensitivity analysis

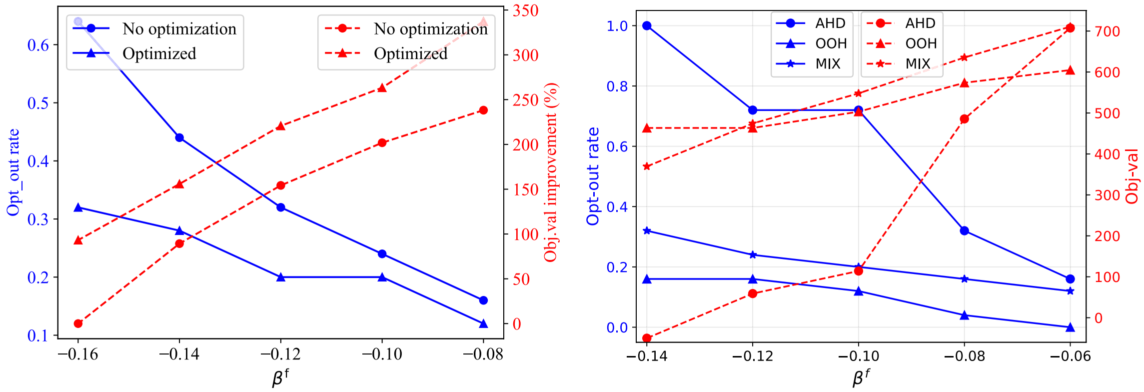
5.4.1. Price discount

In this section, we examine how different prices affect total profit. Since h is interpreted as a price multiplier, the corresponding discount rate is $1-h$. For clearer economic interpretation, we therefore use $1-h$ to represent the discount rate throughout the analysis. We conduct experiments with 25 and 50 customers across 100 scenarios, varying the discount rate from 0.05 to 0.6 in increments of 0.05, and compare the objective value against a baseline of 0.05. The results are illustrated in Fig. 7. We find that a lower penalty and a larger travel cost scaler reduce the impact of customer retention. As a result, profit consistently decreases with increasing discount rates. To ensure a fair comparison, the parameter α is set to 1, and each unserved request incurs a penalty of -20.

Although the number of customers differs, we observe a similar pattern as the discount rate increases. The objective value increases gradually, then decreases sharply after reaching an optimal point. Increasing the discount rate reduces the profit per customer but attracts more customers. When the benefit gained from the additional customers outweighs the losses from the lower prices and increased routing costs, the overall profit increases. In contrast, once the discount rate reaches a critical point, further increases can no longer offset the losses from the steep discount, and total profit begins to decline. Coincidentally, both lines reach their optimal value when the discount rate is 0.4, striking a balance between the revenue from retained customers and the negative costs associated with offering a larger discount. In the 25-customer case, increasing the discount rate to 0.6 results in a negative profit improvement, indicating that excessive discounting can actually reduce profitability, even compared to scenarios with minimal or no discounts. This highlights a fact: profit gains from attracting more customers through discounts are eventually offset by rising operational costs. Hence, there is a clear tradeoff between expanding the customer base and managing delivery expenses. Nonetheless, as mentioned earlier, different parameter settings influence the calculation of the objective value. This suggests that the optimal discount rate is dependent on the specific configuration of the model parameters.

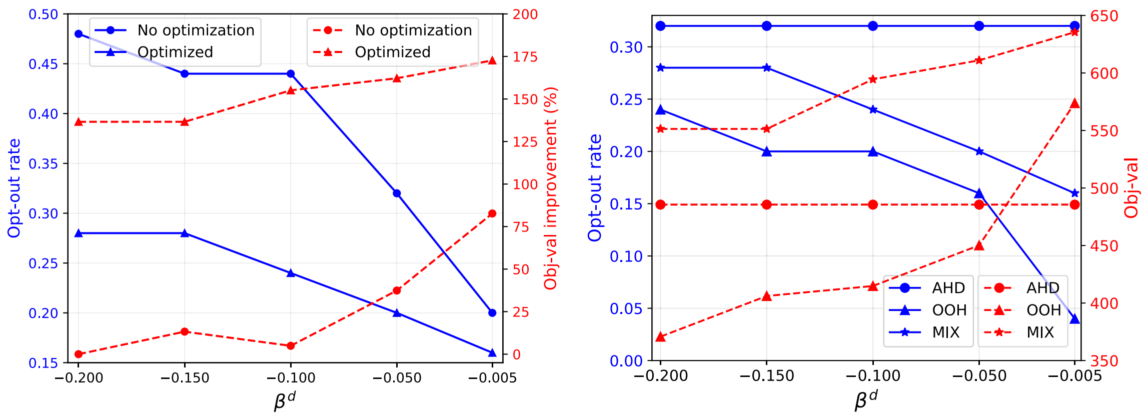
5.4.2. Utility parameters

In Eqs. (13) and (14), we introduce three key utility parameters, β^f, β^d, and β^t, to capture customers' sensitivity to price, pick-up distance, and time slots, respectively. In this section, we analyze how these utility parameters influence the delivery service performance and profit. To facilitate a clear interpretation of behavioral drivers, the analysis is conducted on an instance with 25 customers under a single behavior scenario. For comparison, we design a baseline without optimization, in which all options are offered without discounts. Furthermore, we compare the proposed mixed strategy against two common single-mode delivery strategies: AHD and OOH. To clearly interpret the results and decouple the composite effects of optimization and the mixed strategy, we use two figures for each utility parameter: one illustrating the impact of optimization and the other showing the impact of the mixed strategy. We employ two metrics to evaluate the quality of option decisions. First, we define the opt-out rate as the proportion



(a) Compare with no-optimization benchmark (b) Compare with single delivery mode

Fig. 8. Impact of customers' price sensitivity on opt-out rate and total profit for 25 customers.



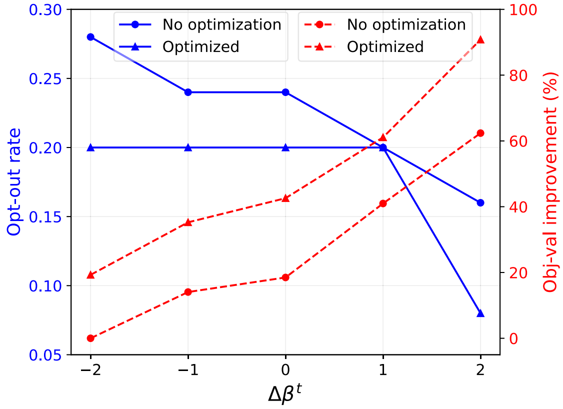
(a) Compare with no-optimization benchmark (b) Compare with single delivery mode

Fig. 9. Impact of customers' pickup distance sensitivity on opt-out rate and total profit for 25 customers.

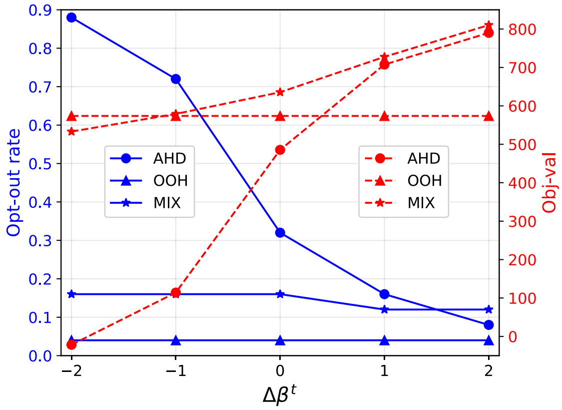
of customers who leave the system without selecting any of the available options. Second, we measure the improvement rate of the objective values achieved by the optimized solutions relative to the baseline. Finally, when comparing different strategies, we evaluate the absolute objective values directly.

Price sensitivity: Fig. 8 illustrates how customers' price sensitivities affect the opt-out rate and system profit. The β^f axis represents customers' sensitivity to price changes, with lower values indicating higher sensitivity. Fig. 8a demonstrates the efficacy of the optimization approach. As β^f increases, the percentage of customers opting out drops significantly: from 60% to 10%. Moreover, we find that the optimized solution substantially mitigates the negative effects of high price sensitivity: when $\beta^f = -0.16$, the number of customers opting out is halved, while profit increases by 20% under the optimization method. Overall, the optimized method consistently outperforms the no-optimization baseline in both profit and customer retention. This improvement indicates that optimized offering and pricing decisions exhibit greater robustness against customers' price sensitivity by reducing the opt-out rate and increasing profitability.

Besides, Fig. 8b illustrates the strength of the mixed strategy compared to single-mode methods. the AHD strategy is highly sensitive to price changes; at $\beta^f = -0.14$, the opt-out rate is 100%, indicating that high prices drive all customers away. In contrast, the OOH strategy proves most effective for retention, maintaining the lowest opt-out rates (below 20%) due to its lower cost. The Mixed strategy strikes a balance: while its opt-out rate is slightly higher than the pure OOH approach, it successfully retains the majority of customers, avoiding the catastrophic customer loss seen in the AHD-only scenario. In terms of profitability, the Mixed strategy demonstrates superior robustness, consistently yielding the highest objective values across most of the sensitivity range. Notably, while AHD is unprofitable under high price sensitivity, it eventually outperforms OOH as customers become less sensitive ($\beta^f > -0.08$). The Mixed strategy capitalizes on this by segmenting the market: it captures high-margin revenue from customers willing to pay for AHD, while simultaneously offering OOH to retain price-sensitive customers who would otherwise opt out.



(a) Compare with no-optimization benchmark



(b) Compare with single delivery mode

Fig. 10. Impact of customers' time slot sensitivity on opt-out rate and total profit for 25 customers.

Distance sensitivity: Fig. 9 depicts the impact of customer pickup distance sensitivity β^d. The horizontal axis ranges from -0.2 to -0.005, where higher values ($\beta^d = -0.005$) indicate lower sensitivity, meaning that customers become relatively indifferent to travel distance. Fig. 9a compares the optimized solution against a non-optimized benchmark. Without optimization, the opt-out rate is static and high (44-48%) across most of the range, only dropping when customers become nearly indifferent to distance ($\beta^d > -0.05$). This suggests the baseline pricing/offering is not attractive enough to overcome the inconvenience of travel until that inconvenience is negligible. In contrast, The optimized solution consistently achieves a much lower opt-out rate. Even when customers are highly sensitive to distance (-0.2), the optimizer manages to retain roughly 20% more customers than the baseline. Besides, the optimization method outperforms the benchmark and maintains a high profit improvement even when customers are highly sensitive to distance.

Fig. 9b contrasts the performance of the Mixed strategy against single-mode benchmarks (AHD and OOH) as customers' sensitivity to pickup distance β^d varies. As expected, the AHD strategy manifests as a horizontal line, confirming that home delivery performance remains invariant to pickup distance parameters. In contrast, the OOH strategy is highly volatile: as distance sensitivity increases, OOH profitability drops sharply and opt-out rates rise, eventually falling below the AHD baseline. The Mixed strategy, however, demonstrates superior robustness, consistently yielding the highest objective value across the entire range. It successfully leverages the stability of AHD when customers are reluctant to travel, while capitalizing on the cost-efficiency of OOH when distance sensitivity diminishes.

Time slot sensitivity: Fig. 10 illustrates customers' sensitivity to time slots. In the experimental setup, we set β^t to -1.7, -1.3, and -2.5, corresponding to three distinct time periods. Rather than analyzing these values separately, we vary them synchronously by adding a small increment Δ, ranging from -2 to 2. When $\Delta\beta^t = -2$, home delivery is less preferred, as its utility becomes relatively lower. Fig. 9a illustrates how the optimization algorithm adds value compared to the fixed benchmark. As $\Delta\beta^t$ increases, the opt-out rate remains stable initially and then drops significantly as $\Delta\beta^t$ approaches 2. This is because when home delivery is uncompetitive, most customers choose non-home delivery options. As $\Delta\beta^t$ increases from -2 to 0, although the opt-out rate remains unchanged, customers gradually switch from non-home to home delivery. Since the latter is inherently more profitable, overall profit increases during this stage. Further increases in $\Delta\beta^t$ encourage even more customers to select home delivery, leading to a significant drop in the opt-out rate and a majority choosing this option. Across all tested settings, the optimization method outperforms the no-optimization baseline and consistently improves customer retention and generates higher profitability, demonstrating the robustness of the optimization-based method.

Fig. 10b illustrates the impact of time-slot sensitivity on the performance of different delivery strategies. As expected, the OOH strategy remains unaffected. Conversely, the AHD strategy is highly sensitive; when AHD is not preferred ($\Delta\beta^t = -2$), the opt-out rate surges to approximately 90%, leading to negligible profit. However, as AHD become more attractive, AHD performance improves rapidly and eventually surpasses the profitability of OOH. The mixed strategy exhibits a high level of adaptability. It reproduces the stability of OOH in high-sensitivity scenarios, thereby mitigating customer loss, but smoothly shifts toward capturing the high-margin potential of AHD as customer flexibility increases. As a result, it consistently achieves a better performance comparable to both single-mode strategies.

5.5. Managerial insights

The numerical experiments provide several managerial insights for practitioners who design delivery menus involving both tem-proal (locations) and spatial (time slot) options.

We find that the mixed strategy is most advantageous in the intermediate range of β^f. When customers are either highly price-sensitive or price-indifferent, the incremental benefit of the mixed strategy diminishes, as the market naturally converges toward a single dominant mode, OOH or AHD, respectively. Consequently, the strategic value of offering mixed options is less pronounced at these extremes. Similarly, when distance and time sensitivities sit at the extremes, the hybrid strategy offers little headroom over the best single mode. Managerially, these findings imply a strategy of targeted, differentiated resource allocation. Operators should avoid over-engineering solutions for segments whose behavior is largely predetermined by extreme sensitivities. In markets with strong price indifference ($\beta^f > -0.06$) or distance indifference ($\beta^d > -0.005$), managers should simply bias capacity and investment toward the naturally dominant mode (AHD and OOH, respectively) to capture efficiency gains with minimal intervention. The primary focus of active demand management, such as dynamic pricing and fine-grained slot adjustments, should be concentrated on segments with moderate sensitivities. It is in this middle ground where customer choices are most malleable and the hybrid strategy's ability to redirect demand via incentives yields the highest return on investment, effectively filtering customers to the most profitable channel based on their specific constraints.

Pricing tests further show an interior optimum: discounts that are too shallow miss demand, while deep discounts negatively impact unit margins; in our runs, profit peaks around a 40% discount, suggesting a practical range of 15-40% is optimal for balancing demand volume with profit margins.

Finally, when hybrid options are offered, the optimization-based approach consistently outperforms the no-optimization benchmark. We identify a critical trade-off in menu design: offering too few options creates a barrier to adoption, while presenting all possible alternatives undermines the impact of offering and pricing levers. This highlights the necessity of the proposed optimization framework: the assortment and prices of AHD and OOH options must be strategically selected rather than randomly determined or fixed. Optimization is therefore essential for determining the appropriate service menu to unlock the full potential of the hybrid strategy.

6. Conclusion

In this paper, we investigate the VRP with uncertain customer demand in delivery time slot assortments and locations. A two-stage stochastic programming model is formulated to capture customers' heterogeneous preferences for mixed delivery options. To address the uncertainties, the sample average approximation method is employed to estimate the optimal solution. Given the complexity of the VRPFD-SB problem, which involves multiple scenarios and customer choice behavior, we propose a scenario-based h-ALNS framework that integrates conventional ALNS with an LNS-based matheuristic method.

Through extensive numerical experiments, we demonstrate the effectiveness of our approach in managing flexible delivery times and locations. The results validate the computational efficiency and solution quality of the h-ALNS compared to both a commercial solver and conventional ALNS. Furthermore, the results indicate that the mixed delivery strategy outperforms pure AHD or OOH delivery. Additionally, the stochastic model enables retailers to improve customer retention and reduce routing costs under imperfect information-an essential feature when customer preferences are unavailable either theoretically (due to inconsistent behavior) or ethically (due to privacy concerns).

There are several promising directions for future research. One limitation of the framework is its collect-first-then-optimize workflow, which fits subscription-based or pre-delivery confirmation settings but is less representative of e-commerce environments where customers select delivery options at checkout. In such cases, the problem becomes a sequential, real-time decision process, for which reinforcement learning methods provide a promising modeling direction. Besides, while the algorithm itself is generalizable, practitioners should re-estimate the utility parameters using their own historical sales data. Incorporating real-world data to validate these parameters would further enhance the accuracy of the modeled customer behavior. Moreover, machine learning techniques can be employed to learn from historical delivery option data. By doing so, the number of possible scenarios can be significantly reduced, thereby enhancing the efficiency of solving the flexible delivery problem. Lastly, exploring pricing strategies with continuous discount rates and developing personalized pricing mechanisms introduce additional computational challenges, but hold great potential for improving both delivery efficiency and profitability, benefiting both customers and logistics retailers.

Declaration of generative AI and AI-assisted technologies in the manuscript preparation process

During the preparation of this work the authors used ChatGPT in order to improve the readability and language of the manuscript. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the published article.

CRediT authorship contribution statement

Ze Zhou: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization; **Fangting Zhou:** Writing – review & editing, Funding acquisition; **Jiaming Wu:** Writing – review & editing, Supervision; **Balázs Kulcsár:** Writing – review & editing, Supervision, Methodology, Funding acquisition, Conceptualization.

Data availability

Data will be made available on request.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A.

Table A.7
Main notations used in the VRPFD-SB in Section 3, organized by sets, parameters, and variables.

Name	Description	Section
I	Set of all customers (indexed by i)	3.2
T	Set of all time slots (indexed by t)	3.2
F	Set of all shared delivery locations (indexed by f)	3.2
H	Set of all discount rates (indexed by h)	3.2
M	Set of all potential alternatives (indexed by m, $m = 0$ denotes the opt-out option)	3.2
R	Number of draws from the distribution of utility error ϵ_{mi} (indexed by r)	3.3
G	Last-mile delivery network	3.4
V_0	$I \cup F \cup 0$ including all customer home addresses, SDLs, and the depot	3.4
V	$I \cup F$ all nodes excluding the depot	3.4
E	Edges in G	3.4
n, k	Minimum numbers of time slots and locations offered	3.2
β	Utility parameter capturing customer sensitivity, including $\beta^1, \beta^2, \beta^t, \beta^d, \beta^f$	3.3
c_i^f	Distance between a SDL f and the customer's home	3.3
p_1, p_2	Initial prices for home and OOH delivery, respectively	3.3
U_{mir}	Utility for customer i choose alternative m under scenario r, including $U_{0ir}, U_{thir}, U_{fhir}$	3.3
V_{mir}	The determinstic part of the utility U_{mir}	3.3
ϵ_{mir}	The unobserved part of the utility U_{mir}	3.3
$\mathcal{M}_{ir}$	The largest absolute utility value across all options for customer i under r	3.3
$\mathcal{M}_{ir}^U$	$2\mathcal{M}_{ir}$, a sufficient large constant	3.3
B_f	Capacity for a SDL $f \in F$	3.4
$T_{\max}$	Service deadline	3.4
a_t	Lower bound on time slot t	3.4
b_t	Upper bound on time slot t	3.4
s_i	Service time at node i	3.4
t_{ij}	Travel time between node i and j	3.4
ζ	Unit penalty for a unserved customer	3.4
α	Weight for total travel cost	3.4
γ	Fixed cost for a vehicle	3.4
ξ	The collection of uncertainties stemming from random customer preference parameters	3.5
y_{mi}	Avalibility of alternative m to customer i, including y_{0i}, y_{thi}, y_{fhi}	3.2
w_{mir}	Choice variable associated with m by i in r, including $w_{0ir}, w_{thir}, w_{fhir}$	3.3
U_{ir}^u	Highest utilty attained by customer i in r, auxiliary variable	3.3
x_{ijr}	Node j is visited directly after node i in scenario r, binary variable	3.4
Z_{fr}	A SDL f is visited in scenario r, binary variable	3.4
$\mathcal{T}_{ir}$	Vehicle's visiting time at node i in scenario r	3.4

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