Embodied experiences of the equivalence principle and the temptation of inertial forces

Citation for the original published paper (version of record):
Isacsson, A., Pendrill, A., Sundberg, L. (2026). Embodied experiences of the equivalence principle and the temptation of inertial forces. Eurasia Journal of Mathematics, Science and Technology Education, 22(7). http://dx.doi.org/10.29333/ejmste/18964

(article starts on next page)

MODESTUM EURASIA Journal of Mathematics, Science and Technology Education, 2026, 22(7), em2872
ISSN:1305-8223 (online)
OPEN ACCESS Research Paper https://doi.org/10.29333/ejmste/18964

Embodied experiences of the equivalence principle and the temptation of inertial forces

Andreas Isacsson [1], Ann-Marie Pendrill [2,3*], Linus Sundberg [2]

[1] Department of Physics, Chalmers University of Technology, Göteborg, SWEDEN
[2] Department of Physics, University of Gothenburg, Göteborg, SWEDEN
[3] Department of Physics, Lund University, Lund, SWEDEN

Received 20 October 2025 ▪ Accepted 11 June 2026

Abstract
According to the equivalence principle, acceleration cannot be distinguished from a gravitational field in the opposite direction. Although conceptually different, using inertial forces in a consistent way gives mathematically equivalent descriptions of the interactions required for acceleration. However, we find that while most new students have learned to avoid 'centrifugal forces', many of them are still tempted to draw or choose force diagrams including an outward force in some cases of circular motion, but referring to a 'centripetal force', and many others prefer a diagram with a centripetal force as a real force. Invoking the weak equivalence principle, which applies to objects with mass, can be a way to connect students' bodily understanding with formal mechanics. In this way embodied experiences of acceleration during everyday life, sports and play can be used as resources for conceptual understanding. Including kinetic diagrams in the construction of free-body diagrams is a reminder that force equilibrium does not apply for accelerating bodies. The weak equivalence principle is a consequence of classical mechanics. It inspired Einstein to develop the general theory relativity.

Keywords: inertial forces, centrifugal, reaction force, equivalence principle, embodied experiences, kinetic diagrams

INTRODUCTION

Why do the swings move out as a chain flyer ride (**Figure 1**) starts moving? Maybe you hear a parent or teacher tell a child or student about the centrifugal force. Should we, as physics educators, discourage the use of the term? Although many professions, including engineers and earth scientists use inertial forces unapologetically, many textbooks tell students to avoid centrifugal forces.

Note how the angle of the swings in the chain flyer is the same whether the swing is empty or not. This mass independence is a beautiful example of the equivalence between gravitational mass (in mg) and inertial mass (in ma). Due to this equivalence acceleration is experienced as a gravitational field in the opposite direction and free fall is experienced as weightlessness. It is a key to understanding the experiences of the body in accelerated motion.

Following d'Alembert, Newton's second law, $\boldsymbol{F} = m\boldsymbol{a}$, can be rewritten as $\boldsymbol{F} - m\boldsymbol{a} = 0$, where the term $-m\boldsymbol{a}$ is treated as a force vector, representing inertial forces. Although mathematically equivalent, it is conceptually quite different, making a dynamic situation appear like equilibrium.

Common difficulties concerning circular motion are well documented as reviewed later. We have encountered them in our teaching in physics, engineering, and teaching programs, as well as in outreach activities. The equivalence principle and its consequences are discussed later as background to the research questions (RQs).

Circular Motion Controversies

'Once again the controversy regarding circular motion and how it shall be taught has erupted in the pages of periodical literature of interest to teachers' Struthers (1963) notes and presents a number of

✉ Andreas.Isacsson@chalmers.se ✉ Ann-Marie.Pendrill@physics.gu.se **(*Correspondence)** ✉ Linus.Sundberg@physics.gu.se

Contribution to the literature

- We find that many new students enter university with a view that the centrifugal force is a 'force that must not be named', as physics textbooks avoid the term.
- Using kinetic diagrams together with free-body diagrams gives a geometric representation of the relation between force and acceleration and ensures that acceleration is remembered in discussions of force situations.
- We suggest that embodied experiences of acceleration can be used to teach Newton's second law already in middle school, by introducing the equivalence principle, which gives a physical interpretation of inertial forces.

Figure 1. Which forces act on a swing in a chain flyer ride (sometimes referred to as rotating swings or wave swinger)? (Photo: Ann-Marie Pendrill)

exchanges, 'sometimes spirited', in the professional journals from 1916 and onwards. Discussions about how the dynamics in circular motion should be taught keep erupting along the same lines as a century ago!

'Physics for the inquiring mind' by Rogers (1960) devotes more than 10 pages to a discussion of experiences and forces in circular motion. He lists four different opinions on centrifugal forces, inviting the reader to 'choose according to your taste'

I. A phony force, imagined through a misinterpretation of evidence, confusing agent and victim.

II. A delusion arising from living in the rotating system and trying to forget it.

III. The engineer's 'headache cure', enabling the conversion to a statics problem, by adding an imaginary force to obtain equilibrium.

IV. Relativity.

Many years later Roche (2001) identified "at least three interpretations of centrifugal force in the literature: a valid meaning in physics, an entirely different but equally valid meaning in engineering, and a cluster of false meanings".

Physics textbooks typically describe the centrifugal force as a pseudo-force, which can be used to describe motion in a non-inertial system. Engineers on the other hand may use it to analyze the stresses that develop, e.g., in rapidly rotating turbine blades, treating centripetal and centrifugal forces as an action-reaction pair, emphasizing that 'centrifugal and centripetal force are equal and opposite here but do not balance because they act on different bodies'. Roche (2001) concludes that 'true centrifugal force exists only as a reaction to macroscopic contact or binding forces' and recommend that the 'fictional centrifugal force of physics should not even be mentioned' and that 'centrifugal forces should be left to the engineers'. He noted that the centrifugal force 'has now virtually disappeared from school and undergraduate physics textbooks because it can be highly confusing', agreeing with Rogers (1960) who recommended to use only the centripetal force. Rogers (1962) also looked into science books for young readers and concluded that those responsible for editorial control have not ensured that the texts are scientifically correct.

Struthers (1963) reviewed various treatments of centrifugal and centripetal force. Looking for possible sources for the various opinions of physics teachers, he identified multiple different uses in textbooks, including *reaction* and *inertial* centrifugal forces. Struthers (1963) also suggested that 'simple deletion of the term centrifugal from all discussions as practiced by some does not seem advisable in the light of its two useful and well-defined meanings and its firmly entrenched general usage.' Still, this practice has continued, e.g., Young and Freedman (2020, p. 178) promise 'we won't mention this term again, and we strongly advise you to avoid it.' ending a 15-line paragraph **caution: avoid using 'centrifugal force'**. A recent review of 20 undergraduate textbooks by Taibu and Mataka (2025) found that only 11 of them included 'centrifugal force' in the index.

However, the term 'centripetal' is also found to be confusing (Smith, 1992). As noted by many teachers, students are tempted to use it 'in the same way as weight, normal forces, friction force or tension in a rope' (De Jong, 1988). Talking instead of the force required for *centripetal acceleration* is a reminder that circular motion involves acceleration.

Student difficulties concerning circular motion have not gone away (e.g., Arons, 1981; Shaffer & McDermott, 2005; Viennot, 1985). Controversies about forces in circular motion are not about the physics involved, only about the ways we choose to describe the forces, or about the best way to teach circular motion.

What Is That Force?

Viennot (1985) notes that physicists encountering an outward force in a student diagram for a body in circular motion, are likely to conclude 'that the student has changed the frame of reference without warning, or [...] that the student was not even aware of the existence of frames of reference.' Viennot (1985) continues with the observation that 'there are several indications that centrifugal force is not just an inertial force transposed from one frame to another, or at least that it is not always the case'. She emphasizes that 'there is a centrifugal force when a stone moves in a circular motion in a Galilean frame, but it is acting on the string. [...] the point here is that students do not shift from one frame or reference to another, but from one object on which the force is supposed to act to another one.'

Viennot (1985) finds that centrifugal forces are more common when students may see a need to explain why a mass doesn't fall. The intuitive desire to add a centrifugal force was also seen in the group discussions analyzed by Eriksson et al. (2020) for a car moving in a vertical circle, as well as for a swing in a chain flyer ride. A few of these students had probably used centrifugal forces in their previous physics courses, whereas most of them had learned to avoid the term. Even when they obtain the correct numerical results, the discussions reveal confusion about what forces are involved, and on what system they act and what name should be used.

Regardless of dynamic situation, some students start their free-body diagrams by drawing an upward arrow to compensate for gravity, naming it 'normal'. Students sometimes need reminding that this is not the meaning of the word 'normal' in a physics context (Pendrill et al., 2019, 2023).

The Equivalence Principle

> Of the suppositions used by Newton as the foundations of his theory of gravitation, the most important is the one which claims that the gravitation produced by the earth on an earth-bound body is proportional to the mass of the body and is independent of the structure of the substance composing it.

Thus, Eötvös (1889) opened his presentation to the Hungarian Academy of Sciences on 20 January 1889, where he proposed experiments to test this proportionality with higher accuracy than achieved by Newton and later by Bessel (Kilenyi, 2019; von Eötvös et al., 1922). This remarkable property of gravity, laying the foundation for the general theory of relativity, was thus questioned and investigated already by Newton.

The chain flyer illustrates the principle of the Eötvös' (1889) experiments performed more than a century ago, with the earth as a 'carousel'. Any difference in gravitational attraction for different materials on the two sides of a torsion pendulum would lead to a small shift in equilibrium angle between day and night (Dicke, 1961; Eötvös, 1889). Later tests of gravity have been reviewed by Will (2010). Modern torsion-balance tests of the weak equivalence principle have been performed by the Eöt-Wash group (Wagner et al., 2012).

The term 'equivalence principle' was coined by Einstein (1920) who discusses 'the equality of inertial and gravitational mass as an argument for the general postulate of relativity'. The *weak* equivalence principle involving massive objects holds also in classical mechanics. Roeder (1975) analyzes amusement rides and notes that for a chain flyer 'The chains suspending the chairs will hang – like plumb lines – in the direction of the net gravitational field' and invokes general relativity to introduce the concept of 'artificial gravity'.

The weak equivalence principle can be illustrated by a number of situations where mass essentially does not influence the motion (if air resistance is neglected), e.g., sliding blocks (Pendrill et al., 2014a), falling objects (Pendrill et al., 2014b), swings (Mårtensson-Pendrill, 2023) or rolling balls (Pendrill, 2020a) on a playground.

If the equivalence between gravitational and inertial mass is mentioned at all in undergraduate textbooks, it is likely to be in connection with the introduction to gravity (e.g., Fishbane et al., 2005; Mazur, 2015; Wolfson & Pasachoff, 1999), but not in relation to inertial forces. Young and Freedman (2020) mentions the principle of relativity but not the equivalence between inertial and gravitational mass. The textbook by Fishbane et al. (2005) also mentions Eötvös' (1889) experiments (p. 356). In our convenience sample of textbooks we also noted that the d'Alembert principle (rewriting Newton's second law as $\boldsymbol{F}\text{-}m\boldsymbol{a} = 0$) was mentioned only in the engineering dynamics books (Bedford & Fowler, 2005; Hibbeler, 2004; Meriam & Kraige, 2008).

Gravitational Forces, Weight, and G-Forces

There is no escape from gravitational forces. Still you cannot measure the gravitational force, only the acceleration it leads to for objects in free fall. What you perceive as weight is actually the force(s) opposing the gravitational force, e.g., from the floor or a bathroom scale. Einstein (1920) has described it as his happiest thought when he realized that in free fall you feel as if you had no weight. Without external reference, it is impossible to distinguish acceleration from a gravitational field in the opposite direction. Extending this principle to include light gives the *strong* equivalence principle, leading to general relativity (Einstein, 1920).

The vector $\boldsymbol{W} = m(\boldsymbol{g}-\boldsymbol{a})$ can be used as an operational definition of weight (Hecht, 2024; Stein et al., 2023; Taibu et al., 2015), where $(\boldsymbol{g}-\boldsymbol{a})$ corresponds to the 'net gravitational field' used by Roeder (1975). Still, many textbooks prefer to describe $m(\boldsymbol{g}-\boldsymbol{a})$ as an 'apparent weight' since it includes a fictive inertial force.

The popular concept of 'G-force' can be defined mathematically as the vector $\boldsymbol{G} = (\mathbf{a}-\mathbf{g}) = -\boldsymbol{W}/m$, which is what is measured by an inertial accelerometer sensor, e.g., in your smartphone (Staacks et al., 2018; Vieyra et al., 2020).

The body in physics education

The term 'body' in Newton's laws neither implies nor excludes human bodies. The forces experienced by the human body are real, although in textbook problems the experiences are more prominent in the form of exerting forces by pulling ropes or pushing boxes than of accelerating in a swing. The common tacit avoidance of the experience of students' bodies may be due to fear of confusion related to inertial forces.

Kinetic diagrams are an important tool for analyzing and understanding the forces acting on an accelerating body. Yet, research on alternative representations of interactions and forces typically focus on equilibrium situations (e.g., Garcia-Lladó & Lòpez, 2020).

Challenging students' incomplete understanding of Newton's laws can lead to interesting discussions, e.g., when students quietly assume that maximum or minimum speed implies zero acceleration, thus forgetting centripetal acceleration. Thought experiments can help students learn dynamics concepts (Sharifi et al., 2025). Tabor-Morris (2015) suggests that students can 'sense' the forces on an object by 'remembering what it feels like on a roller coaster, merry-go-round/carousel or Ferris wheel' and use these recalled experiences to make sense of force situations, as well as rotations of axes for interpreting sensor data. The task to reconcile the experience of the body with the mathematical description can also involve additional representations, including data from smartphone sensors or tracker video analysis of the motion (e.g., Mårtensson-Pendrill, 2023).

Invariant Interactions

Interactions with the surroundings are, of course, invariant with respect to our mathematical choices, as can be illustrated with the forces on a chain flyer swing (or the analogous situation of a conical pendulum), analyzed in some detail, e.g., by Roeder (1975) and Pendrill (2016).

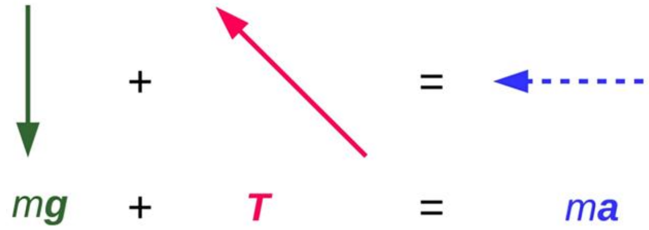

Figure 2. Graphical illustration of Newton's second law for a swing in the chain flyer ride (subtracting $m\boldsymbol{a}$ from both sides of the equation corresponds to adding the inertial 'centrifugal force', resulting in a zero left-hand side) (the authors' own elaboration)

The forces acting on a swing in a chain flyer ride in **Figure 1** are the gravitational force $m\boldsymbol{g}$ and the force $\boldsymbol{T}$ from the chains. Here we neglect air resistance and can let m include the mass of a person in the swing. Applying Newton's second law, we get

$$m\boldsymbol{g} + \boldsymbol{T} = m\boldsymbol{a}. \tag{1}$$

A swing in a chain flyer ride moving with constant speed in a circle in a horizontal plane accelerates horizontally towards the center of the circle. For this case, Newton's second law in Eq. (1) can be represented graphically as in **Figure 2**.

The tension in the string is related to the gravitational force as $T = mg/\cos\theta$ for chains making an angle θ to the vertical. It should be noted that for younger students, a geometric approach can be applied, drawing a triangle combining the three vectors in **Figure 2**. The ratio between the lengths of the horizontal and diagonal lines to the vertical line gives the ratios a/g and T/mg. This enables you to figure out how much heavier you feel than normal when the chains make an angle θ to the vertical – without the need for trigonometry.

The length and direction of force arrows matter. For a pendulum or playground swing, the acceleration in the turning points of pendulum motion is orthogonal to the chain and the tension in the string is, instead, smaller than mg.

The right hand side in **Figure 2** can be referred to as a 'kinetic diagram' (e.g., Hibbeler, 2004; Meriam & Kraige, 2008). In their recent comparison between mechanics teaching in engineering and physics, Moelter et al. (2025) noted that the use of kinetic diagrams was part only of their engineering mechanics courses.

Subtracting the term $m\boldsymbol{a}$ from both sides of the equation, following d'Alembert, gives a mathematically identical equation. This can be seen as working in a non-inertial system and adding the term $-m\boldsymbol{a}$ as an inertial force to compensate for the acceleration of the system. It can also be seen as using the operationally defined ('apparent') weight $m(\boldsymbol{g}-\boldsymbol{a})$.

Savage and Williams (1989) note that 'finding a satisfactory explanation for students' subjective experience of accelerated motion is not a simple matter'.

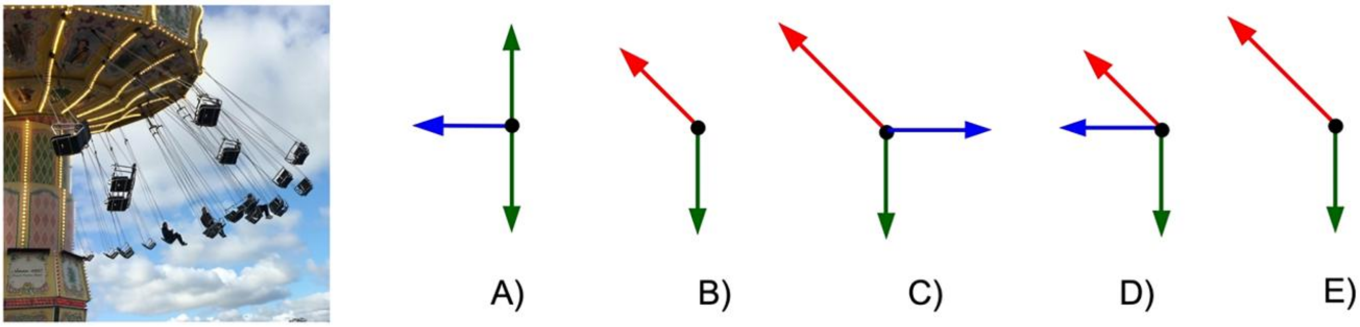

Figure 3. Which diagram gives the best representation of the forces acting on a chain flyer swing? (the authors' own elaboration)

After analyzing the effect of different parts of the body for a number of situations they suggest that an alternative to introducing non-inertial frames of reference and inertial forces 'is to establish that the effect of an acceleration on the human body can be identical to that produced in a static situation by a body force (e.g., gravity) acting in the opposite direction' – implicitly invoking the equivalence principle. They emphasized that 'the generalized or alternative theory applying inertial forces in an accelerated frame is equally valid mathematically and scientifically'.

Some situations are easier to describe using centrifugal forces, e.g., the shape of the earth may be intuitively described as a 'centrifugal bulge', with the ellipsoid surface is orthogonal to the 'net force' (gravitational plus centrifugal). The shape may be calculated as an equipotential of the gravitational potential together with a centrifugal potential term $R\Omega^2 \cos \lambda$ depending on the latitude λ (Greenstein, 2022a, 2022b). The shape can also be derived from Newton's second law in an inertial reference system using a 'local plumb line', orthogonal to the surface (Mohazzabi & James, 2000).

Research Questions

In this work, we address the following RQs:

RQ1: What forces do new students want to include in a free-body diagram for a swing in a chain flyer ride?

RQ2: What terms do new students use to describe the forces in the free-body diagram?

To address RQs, we analyze Mentimeter responses to questions about forces in a chain flyer ride from 50 first-year physics students during an introductory course.

METHOD

The Mentimeter response system was used to collect student responses to a quiz about forces in different rides in a nearby amusement park. The questions in the quiz were based on experiences over many years of informal discussions during problem-solving classes as well as student responses to open-ended questions, which have revealed many different strategies (Pendrill, 2022). The Mentimeter quiz aimed to expose more formally some problems we had found to be common.

The quiz was used as part of a lecture in the university physics program. Before the physics questions, we had asked the students for permission to use their anonymous responses. Only one student declined. The analysis is based on the responses from the 50 students who granted permission.

The lecture took place during the first half of the first semester in October 2024, as part of an introductory physics course taught in parallel with mathematics, before the first mechanics course. In principle, the students had, of course, learned Newton's laws in high school.

The first two of the physics questions involved forces in the chain flyer ride shown in **Figure 1**. Students were first asked to choose the diagram in **Figure 3** that they think best represents the forces on a swing in the ride in **Figure 1**. The alternatives are based on responses to an open-ended question by previous cohorts of students (Pendrill, 2020b, 2022).

As a follow-up question, students were asked to write down the forces they had included, with responses generating a word cloud. The distribution of responses and the resulting word cloud was not shown to the class until everyone had entered their responses. As part of the analysis, the individual responses to the multiple choice question in **Figure 3** were combined with the forces mentioned.

RESULTS: FORCES IN A CHAIN FLYER RIDE AS PERCEIVED BY NEW STUDENTS

The force situation for a swing in a chain flyer ride is shown in **Figure 2**. Only the force of gravity and the force from the chains act (neglecting air resistance), and the force from the chain combines with the gravitational force, resulting in a horizontal inward force, as in diagram E in the multiple-choice question in **Figure 3**. Diagram B includes the correct forces but with incorrect length for the force arrow from the chain. The force from the chain has the correct length in diagram C, but the horizontal outward arrow leads to a net zero force.

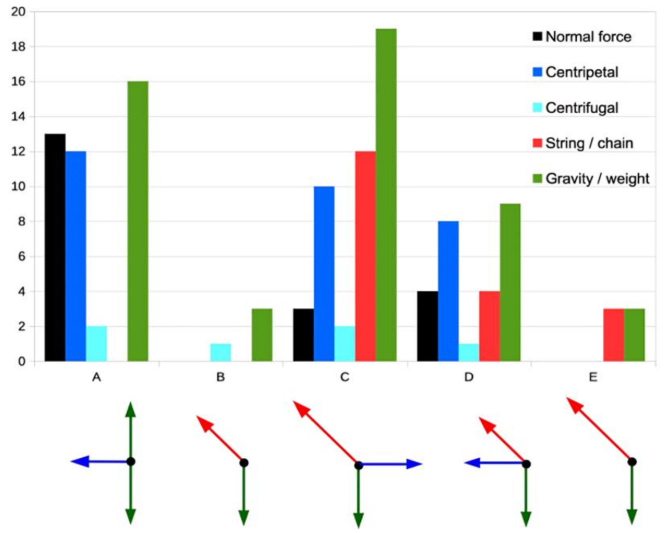

Figure 4. Forces mentioned by respondents who had chosen the different alternatives in **Figure 3** (gravity or weight was mentioned by all respondents) (the authors' own elaboration)

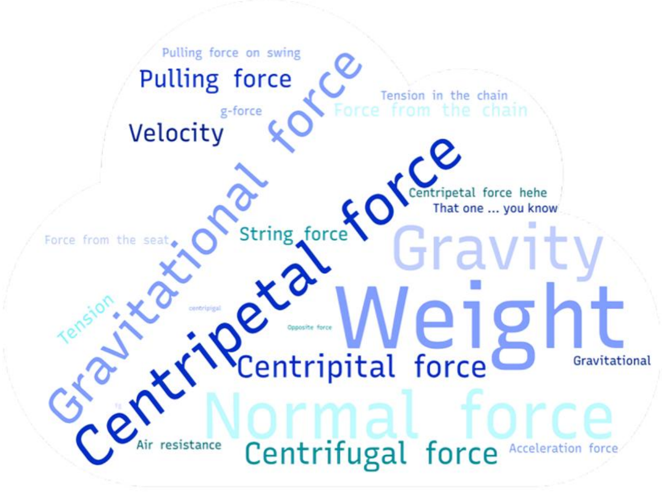

Figure 5. Word cloud based on translated responses to an open question 'Which forces did you include for the chain flyer?' (the authors' own elaboration)

Diagram D is analogous to diagram B, but with an added inward arrow, found also in diagram A, which also includes an upward arrow cancelling gravity. It should be noted that the length of the arrow for the tension $\boldsymbol{T}$ obtained using a centrifugal force arrow to obtain an 'equilibrium' is the same as for alternative E, where the sum of the force vectors is horizontal, resulting in a horizontal acceleration. Although conceptually different, the approaches are mathematically equivalent. A and C were the most popular choices for this group of 50 respondents, chosen by 16 and 19 students, respectively, followed by D chosen by 9 students while B and E both attracted only 3 responses.

What Forces?

After answering the multiple choice question, the students were asked what forces they had included. **Figure 4** shows what forces were mentioned by students who had chosen the different diagrams in the multiple choice question in **Figure 3**. Although 19 students had picked alternative C which included a centrifugal force, only 2 of them mentioned 'centrifugal', whereas 10 mentioned centripetal.

Most of the students who chose diagram A or diagram D, which include a horizontal force to the left, did mention the centripetal force. Diagram A, which also included an upward force counteracting gravity, was chosen by 16 students, 13 of them mentioning a 'normal force'. Diagram E was chosen by only 3 students, who also mentioned only the force from the chain and the gravitational force, which are the only interactions involved.

The free text responses about what forces were used resulted in a word cloud (**Figure 5**). (A more detailed word count is given in **Appendix A**) Clearly, gravitational forces and/or weight were very common responses, as were 'normal force' and centripetal force (sometimes misspelled). Most of the students in this group avoided the term 'centrifugal'. One student even referred to 'that one ... you know'.

Summary

Only a small minority of the students chose a diagram involving only the force from the chains in addition to the gravitational force. The most popular diagram included a centrifugal force and a correct force from the chains, but most of these respondents claimed that they had included a centripetal force. The diagrams with an inward force arrow also tempted many students, and most of them mentioned 'centripetal' as a force they had included.

DISCUSSION

Students enter university physics programs from many different high schools, but all should have learned about Newtonian mechanics and forces in circular motion. Still, the responses indicate that a significant number of new students chose alternative C which includes an outward force. As noted by Viennot (1985), including a centrifugal force in a free-body diagram is particularly tempting in situations where it may seem necessary to explain why a mass doesn't fall.

A majority of the students who resisted the temptation of an outward arrow indicating centrifugal force instead chose diagrams with an inward arrow and also mentioned a centripetal force. Indeed, the centripetal force can be confusing and it seems like these students treated it as a force in its own right, without considering its origin. As noted by De Jong (1988), 'you cannot call upon angels to provide centripetal force when needed'.

Figure 6. Examples of angles related to horizontal acceleration (an object on a string in an airplane taking off and in a teacup amusement ride, the banking of a roller coaster curve, as well as a skater in a wing in team figure skating) (the authors' own elaboration)

Acceleration is often invisible when students are discussing forces, after years of working with forces for equilibrium situations, where the force of gravity must always be compensated exactly. The responses in **Figure 4** show that 13 of the 50 students referred to a 'normal force' after choosing the diagram which includes an upward force cancelling gravity. This incorrect strategy was identified, e.g., in connection with discussions of forces in uniform circular motion and in swings and roller coaster loops (Pendrill et al., 2019, 2023).

Explicitly including a 'kinetic diagram' for ma on the right-hand side of a geometrical equation with forces on the left hand side can be one way of increasing students' awareness of acceleration as a vector quantity and of the equivalence between inertial and gravitational mass. This also emphasizes that the 'centripetal force' should not be treated as a regular force, as it was drawn in the diagrams A and D in **Figure 3**.

Acceleration: From Kinematics to Embodied Experiences of Motion in Three Dimensions

Many textbooks introduce acceleration through kinematics and the mathematical relations between position, velocity and acceleration, without any forces involved. Bowden et al. (1992) found that success in mechanical, quantitative problem solving in kinematics can mask inadequate understanding of basic concepts. Student difficulties to connect graphs of the motion with physics are well documented (e.g., McDermott & Redish, 1999; McDermott et al., 1987). To help students develop an understanding of kinematic graphs, complementary representations of acceleration can be obtained with sensors, providing graphs of motion in the classroom, as suggested, e.g., by Volkwyn et al. (2020).

Hake (1987) demonstrated that a non-calculus-based introductory course could be effective in promoting student 'crossover to the Newtonian world'. Thornton and Sokoloff (1990) noted how immediate feedback of a microcomputer-based laboratory system contributed to making the initially abstract acceleration concept concrete for middle school or high-school students as well as for physics majors or non-majors at university.

In addition to the outside perspective of kinematics, acceleration can also be represented with relation to the forces required. Horizontal acceleration can be measured with an object on a string. Move around and watch it stay behind as you speed up or move ahead as you stop or to the side when you turn. Take it along in an airplane or a small amusement ride and observe the motion. Watch skaters lean as they move in circles around a center in a figure skating wing (Malmqvist & Pendrill, 2022), make students run around in a circle (Heilig, 2026) or watch the slope of a banked roller coaster curve (**Figure 6**). These examples help students develop a view of velocity and acceleration as vector quantities and see the centripetal acceleration, e.g., in the chain flyer ride (**Figure 1**), as just a special case of horizontal acceleration.

Frames of Reference and Inertial Forces

Embodied experiences of acceleration can be added to more conventional representations. However, many textbooks emphasize the need to include inertial forces when we are in a non-inertial reference frame but also discourage the use of inertial forces (e.g., the review by Taibu & Mataka, 2025).

What does it mean to be in a non-inertial frame of reference? Do we have to think of ourselves in a non-inertial frame of reference when we move about our daily life? Does it matter if we can look out from an elevator as it accelerates up or down? Can we not merge the inside and outside views. Videos comparing the points of view from stationary and non-inertial frames of references are discussed, e.g., by Wagner et al. (2006) and Grossman (2012).

We have encountered colleagues who insist that you should describe the forces acting on your own body as a sum of real forces and fictive forces, (i.e., $\Sigma \boldsymbol{F}\text{-}m\boldsymbol{a}$), which should equal zero since you are at rest in your own coordinate system. Similar views were expressed by the Phenomenologist in a delightful discussion of embodied experiences between different traditions (Kersting et al., 2024). However, we have not encountered this view among our new students, and we have never seen students or textbook examples adding an upward inertial force on a person jumping from a diving board.

The conical pendulum and embodied experiences

The 'flying pig' is a common classroom demonstration of a conical pendulum where the forces are analogous to the chain flyer ride. The comoving observer in a conical pendulum experiences an 'effective gravitational field' $\boldsymbol{g}\text{-}\boldsymbol{a}$, related to an operational definition of weight (Hecht, 2024; Stein et al., 2023; Taibu et al., 2015), as discussed before.

A conical pendulum is used by Taibu and Mataka (2025) to illustrate the difference between inertial and non-inertial observers. However, the drawing shows the non-inertial observer aligned with the acceleration of gravity, rather than with the chains and the 'local plumb line' as in the photo in **Figure 1**.

Neglect of the embodied experience of the person in circular motion can be found also in online problems as support for exam preparation provided by Pearson (2026) for several of their textbooks. Among the problems with uniform circular motion we find a carousel with two wires attached to the seat, stating that the horizontal wire maintains a radius of the circle at 7.50 m and the other wires forms an angle 40° to the vertical, as the swings move around the carousel at 20.0 rpm (revolutions/minute). Students are asked to determine the tension in each cable if the seat weighs 200 N and a person weighing 850 N is seated in it. The image gives an impression of realism, although the seat and person are shown as vertical, rather than aligned with the chain as in the photo of the real people in a carousel in **Figure 1**. Inserting numerical values leads to a horizontal acceleration of *3.3 g*, which exceeds the limit of *2 g* for a person oriented as in **Figure 1**, sitting upright, facing away from the center of the ride. For humans, orientation matters: If the seat had been realistically tilted, the force from the ride would have been in the direction of the spine, where the total force from ride is allowed to be up to *6 mg* (Eager et al., 2016). Although we did not do a systematic search in textbooks, we note that a similar problem, (5.51), is found in the textbook by Young and Freedman (2020), where the ride instead moves with 28 rpm, giving an acceleration twice as large, thereby exposing the rider to forces outside permissible limits independent of orientation. It may have been adapted from an earlier version of the problem, with forces on a machine part, rather than a human. This type of pseudo-realistic problems are likely to contribute to students' views that the numbers they obtain are interesting only in comparison with the answer provided. Shut up and calculate!

Arons (1981, p. 169) notes that 'there is only low correlation between correctness of the force diagram and that of the correctness of the numerical application of Newton's second law. Many students memorize calculational procedures in which they get correct results in simple problems, without having any appreciable understanding of the phenomenology of the forces'. Feynman et al. (1964), in the preface to the lectures write about problems assigned to students:

> I think one way we could help the students more would be by putting more hard work into developing a set of problems which would elucidate some of the ideas in the lectures. Problems give a good opportunity to fill out the material of the lectures and make more realistic, more complete, and more settled in the mind the ideas that have been exposed.

Conceptual Questions for Interactive Engagement

The importance of 'interactive engagement' for developing conceptual understanding is well established (Hake, 1998). We have found that using 'peer instruction' (Crouch & Mazur 2001) during lectures following interactive conceptual questions about forces in different situations can be a very productive way to help students sort out conflicting mental concepts. Displaying Mentimeter responses is often a good start for peer discussions, when students have discovered that there are many views in the class and need to elaborate their initial thoughts. The word cloud in **Figure 5** was used as an introduction to a discussion about forces and interactions – and about what is 'normal' about a normal force. Demonstrations with a spiral toy and an object on a string were part of the follow-up discussions.

CONCLUSIONS

Limitations

The results presented in this paper are based on a limited study which involved only 50 students in one introductory course and responses to only one situation. We aimed to capture into Mentimeter responses what we have previously experienced informally in teaching, when a systematic documentation of responses and discussions as they happened was not possible. The paper is intended as encouragement for the reader to try out these and similar questions for their own classes or for larger populations. Other examples of relevant questions can be found in Pendrill (2020b, 2022).

We presented only a few examples of unrealistic problems for students, but encourage the reader to reflect on problems they assign for their classes: Do they mainly require imitation and insertion of numbers or do they also challenge conceptual understanding? Are the results interesting beyond the comparison with the correct numerical result provided? Pendrill (2016) presents a large number of possible assignments for chain flyer rides using authentic data and Pendrill et al. (2023) discuss different possible assignments involving motion in vertical planes.

Future Work

We would like to explore in more detail situations where students are tempted by inertial forces and when they are not.

We encourage teachers to try conceptual questions together with naming forces or other intervention strategies. Qualitative interview studies and small-group interviews would be worthwhile research projects, as would more quantitative approaches.

A systematic study of textbook and exam problems on circular motion would be a valuable topic for educational researchers to investigate: Can the problems be solved by simply recalling facts, or inserting numbers, (give or take a sign)? Do the problems reflect conceptual difficulties, tiptoe around them or challenge them? Are realistic situations included where students can build on embodied experiences?

Implications for Teaching

The human body is an underused resource in physics education. This may be related to fear of confusing students with inertial forces. The common recommendation to avoid centrifugal forces seems to have been largely successful in ensuring that students avoid the term, as a force that must not be named, but less successful in developing an understanding that circular motion is a special case of acceleration. Telling students about the etymology of the words and the meaning of 'fugal' and 'petal' (Binder, 2023) may help eliminate some of the confusion. Teachers also need to be aware that many students treat centripetal forces as real forces to be included in free-body diagrams.

Relating students' subjective experience of accelerated motion to textbook descriptions can be challenging. We suggest that students should learn that the equivalence principle offers a reconciliation between a Newtonian description and the inertial forces experienced by our bodies. Rather than banning the use of 'centrifugal forces', students should learn the term, which is used outside the physics classroom, but they should also learn why it is not part of a Newtonian description. Many amusement parks offer science days, attracting thousands of students who can experience consequences of the weak equivalence principle including free-fall weightlessness in drop towers and roller coaster and feeling heavy in valleys and turns. Many parks also offer teacher preparation days and support material.

Newton's second law, linking force to acceleration is much more intuitive than the first, which seems to contradict everyday experience. Instead of introducing the study of force and motion through non-motion, we could start already in grade school with focusing the attention to the forces acting on our bodies in a variety of real-life situations – not restricted to uniform motion in one dimension, and starting at $x = 0$. Newton's third law can help students discern forces acting on your own body from forces you exert on your surroundings. Including kinetic diagrams as part of an equation together with free-body diagrams (as in **Figure 2**) ensures that acceleration is not forgotten and demonstrates the mathematical validity of the conceptually challenging inertial forces.

Teachers need to be open to the possibility that a centrifugal force arrow introduced by a student may refer to a different system. Although we have taught introductory physics to many different student groups, before this study all of us were unaware of the use of the centrifugal force as a reaction force to the centripetal force, found in a few older texts, e.g., Rogers (1960), Viennot (1985), and Roche (2001). In this tradition, it is you who exert a centri*fugal* force on a turning car, as it exerts a centri*petal* force on you, to make you change the direction of motion.

Although we acknowledge that there are situations where inertial centrifugal forces give simpler treatment and a more intuitive interpretation, e.g., in a chain flyer or for describing the shape of the earth, we are not advocating for its use. After many years of teaching mechanics at physics and engineering programs, we prefer to avoid inertial forces and are able to treat complex motions in 1, 2, and 3 dimensions without being tempted to invoke inertial forces – although for some situations we may resort to Lagrange methods. However, we also conclude that students need to learn that a consistent use of inertial forces gives rise to the same interaction forces as a Newtonian description – a consequence of the equivalence principle.

The weak equivalence principle fits well in introductory courses in classical mechanics. It can be explored in many accessible and fascinating situations already in middle school, e.g., on a playground. It could also be applied to discussions about gravitational effects on antimatter (Anderson et al., 2023) – and about the generalization to light (Einstein, 1920). The work on Einsteinian-science curricula for early school years (e.g., Kaur et al., 2024) could aim to include embodied experiences of how acceleration is experienced like a gravitational force in the opposite direction. Embodied experiences of forces experienced by students' own bodies and connecting them to the weak equivalence principle can be part of a solid conceptual foundation before introducing students to curved space-time. The equivalence between gravitational and inertial mass inspired Einstein to develop the general theory of relativity.

Author contributions: **AI:** conceptualization, methodology, validation, writing – review & editing; **A-MP:** conceptualization, methodology, validation, writing – original draft, writing – review & editing; **LS:** conceptualization, validation, writing – review & editing. All authors agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: Part of this study presented in method section involved university students (all aged over 16) participating in an educational activity conducted as part of their regular physics curriculum. The project did not require formal approval from an institutional ethics committee, as no identifiable personal data were collected and the activity constituted standard educational practice. The participants provided informed consent within the questionnaire for the anonymous use and publication of their data.

AI statement: No generative AI was used in preparing this article.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Anderson, E., Baker, C., Bertsche, W., Bhatt, N. M., Bonomi, G., Capra, A., Carli, I., Cesar, C. L., Charlton, M., Christensen, A., Collister, R., Mathad, A. C., Quiceno, D. D., Eriksson, S., Evans, A., Evett, N., Fabbri, S., Fajans, J., Ferwerda, A., … Wurtele, J. S. (2023). Observation of the effect of gravity on the motion of antimatter. *Nature, 621*, 716-722. https://doi.org/10.1038/s41586-023-06527-1

Arons, A. (1981). Thinking, reasoning and understanding in introductory physics courses. *The Physics Teacher, 19*(3), 166-172. https://doi.org/10.1119/1.2340737

Bedford, A., & Fowler, W. (2005). *Engineering mechanics: Dynamics*. Pearson.

Binder, P.-M. (2023). The origins of physics words. *European Journal of Physics, 45*(2), Article 025801. https://doi.org/10.1088/1361-6404/ad0f9b

Bowden, J., Dall'Alba, G., Martin, E., Laurillard, D., Marton, F., Masters, G., Ramsden, P., Stephanou, A., & Walsh, E. (1992). Displacement, velocity, and frames of reference: Phenomenographic studies of students' understanding and some implications for teaching and assessment. *American Journal of Physics, 60*(3), 262-269. https://doi.org/10.1119/1.16907

Crouch, C. H., & Mazur, E. (2001). Peer instruction: Ten years of experience and results. *American Journal of Physics, 69*(9), 970-977. https://doi.org/10.1119/1.1374249

De Jong, M. L. (1988). What name should be used for the force required to move a mass in a circle? 1. Centripetal force 2. Centrifugal force 3. None of the above. *The Physics Teacher, 26*(7), 470-471. https://doi.org/10.1119/1.2342581

Dicke, R. H. (1961). The Eötvös experiment. *Scientific American, 205*, 84-95. https://doi.org/10.1038/scientificamerican1261-84

Eager, D., Pendrill, A.-M., & Reistad, N. (2016). Beyond velocity and acceleration: Jerk, snap and higher derivatives. *European Journal of Physics, 37*(6), Article 065008.

Einstein, A. (1920). The equality of inertial and gravitational mass as an argument for the general postulate of relativity. *Marxists Internet Archive*. https://www.marxists.org/reference/archive/einstein/works/1910s/relative/ch20.htm

Eötvös, L. (1889). On the gravitation produced by the earth on different substances. *Progress in Physics*. https://zelmanov.progress-in-physics.com/papers/zj-2008-02.pdf

Eriksson, M., Eriksson, U., & Linder, C. (2020). Using social semiotics and variation theory to analyse learning challenges in physics: A methodological case study. *European Journal of Physics, 41*(6), Article 065705. https://doi.org/10.1088/1361-6404/abb0a2

Feynman, R. P., Leighton, R. B., & Sands, M. (1964). *The Feynman lectures on physics, volume 1*. Addison-Wesley.

Fishbane, P. M., Gasiorowicz, S. G., & Thornton, S. T. (2005). *Physics – For scientists and engineers, with modern physics*. Pearson.

Garcia-Lladó, À., & Lòpez, V. (2020). Beyond recurring free-body force diagrams: Educational pros and cons of alternative means of representing forces and interactions. *The Physics Teacher, 58*(7), 504-508. https://doi.org/10.1119/10.0002073

Greenstein, G. (2022a). An intuitive approach to earth's centrifugal bulge. *The Physics Teacher, 60*(5), 369-371. https://doi.org/10.1119/5.0030646

Greenstein, G. (2022b). The shape of earth: An early test of Newtonian physics. *The Physics Teacher, 60*(5), 365-368. https://doi.org/10.1119/5.0035142

Grossman, J. (2012). Frames of reference in the classroom. *The Physics Teacher, 50*(9), 548-549. https://doi.org/10.1119/1.4767490

Hake, R. R. (1987). Promoting student crossover to the Newtonian world. *American Journal of Physics, 55*(10), 878-884. https://doi.org/10.1119/1.14945

Hake, R. R. (1998). Interactive-engagement versus traditional methods: A six-thousand-student survey of mechanics test data for introductory physics courses. *American Journal of Physics, 66*(1), 64-74. https://doi.org/10.1119/1.18809

Hecht, E. (2024). On defining weight and weightlessness. *The Physics Teacher, 62*(6), 429-434. https://doi.org/10.1119/5.0130542

Heilig, S. J. (2026). An intuitive approach to centripetal acceleration. *The Physics Teacher, 64*(3), 186-187. https://doi.org/10.1119/5.0227043

Hibbeler, R. C. (2004). *Engineering mechanics: Dynamics*. Prentice Hall.

Kaur, T., Kersting, M., Blair, D., Adams, K., Treagust, D., Santoso, J., Lonshakova, A., Boublil, S.,Zadnik, M., Ju, L., Wood, D., Horne, E., & McGoran, D. (2024). Developing and implementing an Einsteinian science curriculum from years 3-10: A. concepts, rationale and learning outcomes. *Physics Education, 59*(6), Article 065008. https://doi.org/10.1088/1361-6552/ad66a7

Kersting, M., Amin, T. G., Euler, E., Gregorcic, B., Haglund, J., Hardahl, L. K., & Steier, R. (2024). What is the role of the body in science education? A conversation between traditions. *Science and Education, 33*, 1171-1210. https://doi.org/10.1007/s11191-023-00434-7

Kilenyi, E (2019). The Eötvös experiment in its historical context. *REAL-EOD*. https://real-eod.mtak.hu/8266/

Malmqvist, K., & Pendrill, A.-M. (2022). From skating rink to physics assignment – Viewing a photo from a mechanics perspective. *Physics Education, 57*(5), Article 055003. https://doi.org/10.1088/1361-6552/ac7311

Mårtensson-Pendrill, A.-M. (2023). Serious physics on a playground swing – With toys, your own body, and a smartphone. *The Physics Teacher, 61*(5), 355-359. https://doi.org/10.1119/5.0074171

Mazur, E. (2015). *Principles & practice of physics*. Pearson.

McDermott, L. C., & Redish, E. F. (1999). Resource letter: PER-1: Physics education research. *American Journal of Physics, 67*(9), 755-767. https://doi.org/10.1119/1.19122

McDermott, L. C., Rosenquist, M. L., & van Zee, E. H. (1987). Student difficulties in connecting graphs and physics: Examples from kinematics. *American Journal of Physics, 55*(6), 503-513. https://doi.org/10.1119/1.15104

Meriam, J. L., & Kraige, L. G. (2008). *Engineering mechanics: Dynamics*. John Wiley & Sons.

Moelter, M. J., Self, B., Vokos, S., Bello, D. D., & Youngblood, B. (2025). Mechanics in engineering and physics: Differences and lessons. *The Physics Teacher, 63*(9), 752-756. https://doi.org/10.1119/5.0191267

Mohazzabi, P., & James, M. C. (2000). Plumb line and the shape of the earth. *American Journal of Physics, 68*(11), 1038-1041. https://doi.org/10.1119/1.1285947

Pearson. (2026). Study prep in Pearson+. *Pearson*. https://www.pearson.com/channels/physics/exam-prep/centripetal-forces-gravitation/centripetal-forces

Pendrill, A.-M. (2016). Rotating swings – A theme with variations. *Physics Education, 51*(1), Article 015014. https://doi.org/10.1088/0031-9120/51/1/015014

Pendrill, A.-M. (2020a). Balls rolling down a playground slide: What factors influence their motion? *Physics Education, 56*(1), Article 015005. https://doi.org/10.1088/1361-6552/abbb5a

Pendrill, A.-M. (2020b). Forces in circular motion: Discerning student strategies. *Physics Education, 55*(4), Article 045006. https://doi.org/10.1088/1361-6552/ab8047

Pendrill, A.-M. (2022). Teacher interventions and student strategies for circular motion problems: A matrix representation. *Physics Education, 57*(3), Article 035003. https://doi.org/10.1088/1361-6552/ac4987

Pendrill, A.-M., Ekström, P., Hansson, L., Mars, P., Ouattara, L., & Ryan, U. (2014a). Motion on an inclined plane and the nature of science. *Physics Education, 49*(2), 180-186. https://doi.org/10.1088/0031-9120/49/2/180

Pendrill, A.-M., Ekström, P., Hansson, L., Mars, P., Ouattara, L., & Ryan, U. (2014b). The equivalence principle comes to school – Falling objects and other middle school investigation. *Physics Education, 49*(4), 425-430. https://doi.org/10.1088/0031-9120/49/4/425

Pendrill, A.-M., Eriksson, M., Eriksson, U., Svensson, K. & Ouattara, L. (2019). Students making sense of motion in a vertical roller coaster loop. *Physics Education, 54*(6), Article 065017. https://doi.org/10.1088/1361-6552/ab3f18

Pendrill, A.-M., Eriksson, M., Eriksson, U., & Linder, C. (2023). Round and round in circles – Shifting relevance structures as students discuss acceleration and forces during circular motion in a vertical plane. *European Journal of Physics, 44*(5), Article 055008. https://doi.org/10.1088/1361-6404/ace945

Roche, J. (2001). Introducing motion in a circle. *Physics Education, 36*(5), 399-405. https://doi.org/10.1088/0031-9120/36/5/305

Roeder, J. L. (1975). Physics and the amusement park. *The Physics Teacher, 13*(6), 327-332. https://doi.org/10.1119/1.2339173

Rogers, E. M. (1960). *Physics for the inquiring mind: The methods, nature and philosophy of physical science*. Princeton University Press.

Rogers, E. M. (1962). Responsibility and science writing. *Science, 136*(3515), 499-501. https://doi.org/10.1126/science.136.3515.499

Savage, M. D., & Williams, J. S. (1989). Centrifugal force: Fact or fiction? *Physics Education, 24*(3), 133-140. https://doi.org/10.1088/0031-9120/24/3/304

Shaffer, P. S., & McDermott, L. C. (2005). A research-based approach to improving student understanding of the vector nature of kinematical concepts. *American Journal of Physics, 73*(10), 921-931. https://doi.org/10.1119/1.2000976

Sharifi, F., Ahmadi, F., & Meshkat, M. (2025). Investigating the effect of teaching dynamic concepts with the help of thought experiments on the academic progress and cognitive skills of students. *Physics Education, 60*(4), Article 045003. https://doi.org/10.1088/1361-6552/adcdb7

Smith, P. A. (1992). Let's get rid of "centripetal force." *The Physics Teacher, 30*(5), 316-317. https://doi.org/10.1119/1.2343554

Staacks, S., Hütz, S., Heinke, H., & Stampfer, C. (2018). Advanced tools for smartphone-based experiments: Phyphox. *Physics Education, 53*(4), Article 045009. https://doi.org/10.1088/1361-6552/aac05e

Stein, B., Stein, H., & Galili, I. (2023). The concept of observer in science teaching in middle school: Pre-instructional knowledge as a lever for learning rather than an obstacle. *Education Sciences, 13*(1), Article 95. https://doi.org/10.3390/educsci13010095

Struthers, J. A. (1963). Teaching the dynamics of uniform circular motion. *The Physics Teacher, 1*(1), 27-31. https://doi.org/10.1119/1.2350559

Tabor-Morris, A. E. (2015). Thinking in terms of sensors: Personification of self as an object in physics problem solving. *Physics Education, 50*(2), 203-209. https://doi.org/10.1088/0031-9120/50/2/203

Taibu, R., & Mataka, L. M. (2025). Textbook presentation of circular motion dynamics: Centrifugal force controversy & implications for teaching. *Electronic Journal for Research in Science & Mathematics Education, 29*(1), 60-77.

Taibu, R., Rudge, D., & Schuster, D. (2015). Textbook presentations of weight: Conceptual difficulties and language ambiguities. *Physical Review Physics Education Research, 11*, Article 010117. https://doi.org/10.1103/PhysRevSTPER.11.010117

Thornton, R. K., & Sokoloff, D. R. (1990). Learning motion concepts using real-time microcomputer-based laboratory tools. *American Journal of Physics, 58*(9), 858-867. https://doi.org/10.1119/1.16350

Viennot, L. (1985). Analyzing students' reasoning: Tendencies in interpretation. *American Journal of Physics, 53*(5), 432-436. https://doi.org/10.1119/1.14194

Vieyra, R., Vieyra, C., Pendrill, A.-M., & Xu, B. (2020). Gamified physics challenges for teachers and the public. *Physics Education, 55*(4), Article 045014. https://doi.org/10.1088/1361-6552/ab8779

Volkwyn, T. S., Airey, J., Gregorcic, B., & Linder, C. (2020). Developing representational competence: Linking real-world motion to physics concepts through graphs. *Learning: Research and Practice, 6*(1), 88-107. https://doi.org/10.1080/23735082.2020.1750670

von Eötvös, V. R., Pekàr, D., & Fekete, E. (1922). Beiträge zum Gesetze der Proportionalität von Trägheit und Gravität [Contributions to the law of proportionality of inertia and gravity]. *Annalen der Physik, 373*(9), 11-66. https://doi.org/10.1002/andp.19223730903

Wagner, A., Altherr, S., Eckert, B., & Jodl, H. J. (2006). Multimedia in physics education: A video for the quantitative analysis of the centrifugal force and the Coriolis force. *European Journal of Physics, 27*(5), Article L27. https://doi.org/10.1088/0143-0807/27/5/L01

Wagner, T. A., Schlamminger, S., Gundlach, J. H., & Adelberger, E. G. (2012). Torsion-balance tests of the weak equivalence principle. *Classical and Quantum Gravity, 29*(18), Article 184002. https://doi.org/10.1088/0264-9381/29/18/184002

Will, C. M. (2010). Resource letter PTG-1: Precision tests of gravity. *American Journal of Physics, 78*(12), 1240-1247. https://doi.org/10.1119/1.3481700

Wolfson, R., & Pasachoff, J. M. (1999). *Physics with modern physics for scientists and engineers*. Addison-Wesley.

Young, H. D., & Freedman, R. A. (2020). *University physics with modern physics*. Pearson.

APPENDIX A

Table A1. Responses from 50 first-year students to the question about what forces were acting on a person in the chain flyer ride in **Figure 1**, discussed in results section

Responses	Number of students
Gravity/weight	
Gravitational force	19
Weight	19
Gravitational	1
Fg	1
Gravity	10
Centrifugal force	5
That one ... you know	1
Centripetal force	23
Centripital force	5
Centripetal force hehe	1
Centripigual force	1
String/chain	
Pulling force	4
Pulling force on swing	1
Force from the chain	1
Force in the chain	1
Force of the chain	2
Force in the string to the left	1
String force	1
Interaction with the string	1
Tension	3
Tension in the chain	1
Pulling force on swing	1
Force from the seat	1
Normal force	19
Fn	1
Other	
Air resistance	2
Velocity	2
g-force	1
Acceleration force	1
Opposite force	1

Note. Each student could list several items; The text color corresponds to the bars for the categories in **Figure 4**; & The last group ('other') was not included in **Figure 4**