



Reflectarrays and reconfigurable intelligent surfaces (RIS) are engineered panels that steer and shape radio waves to improve future high-frequency wireless networks. However, accurately simulating large, curved, or reconfigurable surfaces is computationally expensive, especially during repeated optimization or tuning. This thesis develops fast electromagnetic modeling methods for all-metal and cavity-backed reflectarrays. It combines full-wave accuracy for individual elements with efficient array-level techniques, including modular domain decomposition, physical optics, macro basis functions, and fast matrix updates. The proposed methods reduce simulation time while maintaining accuracy, particularly for reconfigurable arrays where previously computed element data can be reused. Overall, the work supports faster reflectarray and RIS design, dataset generation, and data-driven optimization for next-generation wireless applications.

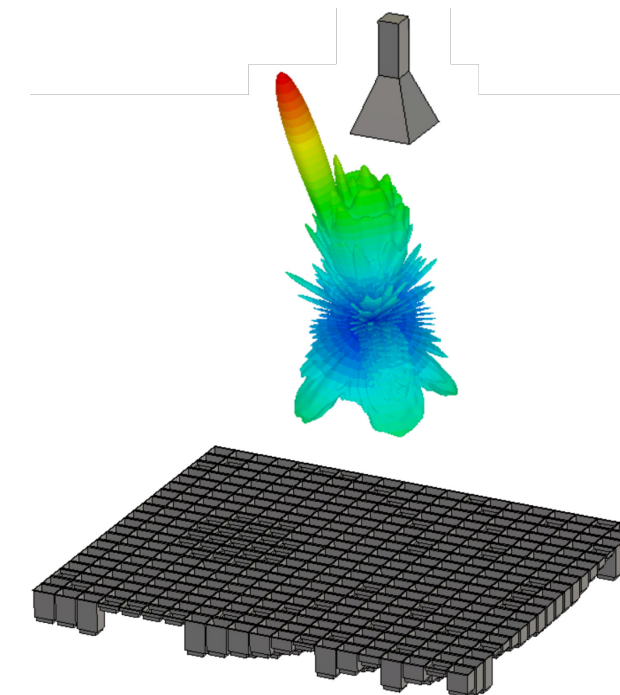


CHALMERS
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Fast Modeling of Finite Reflectarrays and Reconfigurable Intelligent Surfaces for Next-Generation Wireless Applications

2026



Fast Modeling of Finite Reflectarrays and Reconfigurable Intelligent Surfaces

for Next-Generation Wireless Applications

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THESIS FOR THE DEGREE OF LICENTIATE OF ENGINEERING

Fast Modeling of Finite Reflectarrays and Reconfigurable Intelligent Surfaces for Next-Generation Wireless Applications

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CHALMERS UNIVERSITY OF TECHNOLOGY
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Modulated 20x20 reflectarray scanning to 15°

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Abstract

Reflectarrays and reconfigurable intelligent surfaces (RIS) enable low-profile, high-gain wavefront shaping with electrically large apertures in high-frequency wireless systems. However, accurate full-wave analysis of finite, conformal, and reconfigurable reflectarrays remains computationally expensive, especially when many configurations must be evaluated for design, tuning, optimization, or electromagnetic dataset generation.

This thesis develops fast numerical modeling techniques for all-metal and cavity-backed reflectarrays that preserve local full-wave accuracy while accelerating array-level analysis. A hybrid Method-of-Moments–Physical-Optics (MoM–PO) approach is first introduced, where an all-metal reflectarray is represented by a perfect-electric-conductor (PEC) equivalent surface excited by MoM-derived aperture magnetic currents. A domain-decomposition framework is then developed for open-cavity-based array elements, in which unit cells are characterized separately and finite-array mutual coupling is modeled through over-the-air generalized admittance interactions. The formulation is further extended to conformal cavity-backed reflectarrays by coupling interior cavity and exterior radiation domains through generalized aperture-admittance operators.

For electrically large and reconfigurable arrays, the thesis introduces a hybrid domain-decomposition/PO framework using characteristic-mode macro basis functions (MBFs). This enables efficient multi-state analysis by reusing pre-characterized element operators and reducing the number of aperture unknowns. Different MBF constructions, including characteristic mode analysis (CMA), the characteristic basis function method (CBFM), and singular value decomposition (SVD), are compared to quantify trade-offs between accuracy, compression, and conditioning. Finally, Woodbury-identity-based updates and Jacobi-preconditioned Generalized Minimal Residual (GMRES) solvers are used to accelerate recomputation under localized element changes. The resulting methods support scalable analysis, electromagnetic dataset generation, and data-driven optimization of reflectarray and RIS architectures.

Keywords: Reflectarrays; Reconfigurable Intelligent Surfaces; Cavity-Backed Antennas; Computational Electromagnetics; Method of Moments; Domain Decomposition; Physical Optics; Macro Basis Functions; Generalized Admittance; Fast Electromagnetic Solvers.

*Many roads lead to the destination we seek.
Only at the end do the answers speak.*

*Aan mijn familie, waar jullie ook zijn: bedankt voor jullie liefde en
aanmoediging tijdens deze reis.*

家人是远行路上最温暖的牵挂，也是心中最坚定的力量。

*To my family, wherever you may be, thank you for your love and
encouragement throughout this journey.*

List of Publications

This thesis is based on the following publications:

- [A] **Dijun Lin**, Lars Manholm, Oskar Talcotth, Sam Agneessens, Parisa Aghdam, Rob Maaskant, “Fast Scattering Analysis of Large Open-Cavity Based Reflectarrays Using a Hybrid Method of Moment and Physical Optics Approach”. In *2025 International Conference on Electromagnetics in Advanced Applications (ICEAA)*, 8-12 Sept 2025, Palermo, Italy.
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- [C] **Dijun Lin**, Parisa Aghdam, Rob Maaskant, “Modeling of Conformal Reflectarrays Through Domain Decomposition Using a Generalized Admittance Approach”. In *2025 Asia-Pacific Microwave Conference (APMC)*, 2-5 Dec 2025, Jeju Island, Korea.
- [D] **Dijun Lin**, Lars Manholm, Oskar Talcotth, Parisa Aghdam, Rob Maaskant, “Hybrid Domain-Decomposition/Physical-Optics Analysis of Finite Reconfigurable Reflectarrays Using Characteristic-Mode Macro Basis Functions”. *IEEE Antennas and Wireless Propagation Letters (AWPL)*, May 2026.
- [E] **Dijun Lin**, Lars Manholm, Oskar Talcotth, Rob Maaskant, “Comparison of Macro-Basis-Function Construction Methods for Reduced-Order Analysis of Cavity-Backed Reflectarrays”. Accepted at *2026 International Conference on Electromagnetics in Advanced Applications (ICEAA)*, 14-18 Sept 2026, Toyama, Japan.
- [F] **Dijun Lin**, Lars Manholm, Oskar Talcotth, Parisa Aghdam, Rob Maaskant, “Fast MoM-Based Solvers for Cavity-Backed Reflectarrays and RIS Design”. Accepted at the *20th the European Conference on Antennas and Propagation (EuCAP)*, 19-24 April 2026, Dublin, Ireland.

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Acronyms

EM:	Electromagnetics / Electromagnetic
E-field:	Electric Field
H-field:	Magnetic Field
PEC:	Perfect Electric Conductor
RA:	Reflectarray
RIS:	Reconfigurable Intelligent Surface
5G/6G:	Fifth/Sixth Generation (Mobile Networks)
MoM:	Method of Moments
PO:	Physical Optics
DDM:	Domain Decomposition Method
MBF:	Macro Basis Functions
CBFM:	Characteristic Basis Function Method
CMA:	Characteristic Mode Analysis
CM:	Characteristic Mode
EFIE:	Electric Field Integral Equation
MFIE:	Magnetic Field Integral Equation
DoF:	Degrees of Freedom
GMRES:	Generalized Minimal Residual (Method)
SVD:	Singular Value Decomposition
S-parameters:	Scattering Parameters
Y-parameters:	Admittance Parameters

Z-parameters:	Impedance Parameters
CST:	CST Studio Suite
ML:	Machine Learning
DL:	Deep Learning
LHS:	Left-Hand Side
RHS:	Right-Hand Side

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Part I

Overview

CHAPTER 1

General Introduction

1.1 Motivation

Reflectarrays and reconfigurable intelligent surfaces (RIS) are promising electromagnetic (EM) architectures for shaping radiation patterns with high gain and low profile. They are increasingly relevant for high-frequency and large-aperture wireless systems, where compact, efficient, and configurable aperture technologies are required. In practical designs, especially when low loss, high efficiency, and mechanical robustness are important, all-metal and cavity-backed reflectarray configurations are attractive alternatives to dielectric-substrate-based implementations.

A key challenge is that realistic reflectarrays and RIS often involve electrically large apertures, finite-array effects, conformal geometries, mutual coupling, and many possible unit-cell configurations. Accurate full-wave simulation of such structures is computationally demanding. For Method-of-Moments (MoM) formulations, the impedance matrix is dense, and the computational cost and memory requirements grow rapidly with the number of unknowns. This makes repeated evaluation for design iteration, optimization, reconfiguration, and data generation impractical if conventional full-

array MoM formulations are used directly.

This challenge becomes even more important in reconfigurable and data-driven workflows. RIS and tunable reflectarrays may require the analysis of many element-state configurations, while learning-based surrogate modeling, inverse design, and optimization require large volumes of reliable radiation data. Therefore, there is a need for numerical methods that preserve full-wave accuracy where it is essential, such as in the local unit-cell, aperture, or cavity region, while reducing the cost of array-level analysis and repeated recomputation.

This thesis addresses these challenges by developing fast and scalable numerical modeling techniques for all-metal, open-cavity-based, and cavity-backed reflectarrays and related RIS structures. The central theme is to combine accurate local modeling with efficient global analysis through hybridization, domain decomposition, generalized admittance modeling, macro-basis-function reduction, and fast matrix-update strategies.

1.2 Scope and Problem Statement

The focus of this thesis is on computationally efficient modeling of finite reflectarrays, with emphasis on all-metal, open-cavity-based, and cavity-backed implementations. The work considers planar and conformal arrays, as well as reconfigurable configurations where only a subset of elements may change state between simulations.

The practical requirements motivating the proposed methods are:

- **Large-scale analysis:** efficiently predicting radiation patterns, scattering responses, and key antenna metrics for finite electrically large reflectarrays.
- **Cavity-backed and open-cavity element modeling:** retaining full-wave accuracy for local cavity physics, resonant behavior, aperture fields, and element-level responses.
- **Finite-array mutual coupling:** accounting for over-the-air coupling and the different local environments experienced by interior and edge elements.
- **Conformal implementations:** supporting curved or faceted concave

and convex reflectarray surfaces, where local element orientation and geometry complicate the analysis.

- **Reduced-order modeling:** decreasing the number of unknowns through macro basis functions (MBFs) while preserving the dominant electromagnetic behavior.
- **Local reconfiguration:** rapidly updating solutions when only a subset of elements changes state, as is typical in tunable reflectarrays and RIS.
- **Dataset generation:** enabling fast and accurate EM field generation for data-driven design, surrogate modeling, and optimization.

The main problem addressed in this thesis can therefore be stated as follows: *how can finite cavity-backed reflectarrays and related RIS structures be modeled accurately while reducing computational cost sufficiently to enable large arrays, conformal geometries, reconfigurable states, mutual-coupling-aware finite-array analysis, and repeated simulations?*

1.3 Research Objectives

The main objective of this thesis is to develop numerical modeling frameworks that reduce the solving time for reflectarray/RIS analysis while maintaining accuracy comparable to trusted full-wave references. More specifically, the thesis aims to:

1. formulate efficient **hybrid** approaches that combine detailed unit-cell or aperture-level modeling with fast array-level computations;
2. develop **domain decomposition** strategies that separate interior element/cavity physics from exterior radiation and scattering through aperture/interface quantities;
3. model **finite-array mutual coupling** through generalized admittance formulations, including the local coupling environment experienced by edge and interior elements;
4. investigate **macro-basis-function** representations for reduced-order analysis of cavity-backed and reconfigurable reflectarrays;

5. enable **multi-state and reconfigurable analysis** by reusing pre-characterized element information and updating only the portions of the model that change;
6. enable **rapid recomputation** under localized element changes using matrix-update identities and iterative solvers;
7. validate the proposed techniques on representative finite planar, conformal, and reconfigurable reflectarray/RIS configurations.

1.4 Key Contributions

This thesis makes the following contributions:

- **Hybrid MoM–Physical Optics framework for fast array analysis:** A large three-dimensional all-metal reflectarray is modeled using a perfect-electric-conductor (PEC) equivalence excited by impressed aperture magnetic currents. These currents are extracted from isolated unit-cell MoM analysis, while the induced currents on the large PEC-equivalent aperture are computed using Physical Optics (PO). This reduces the cost of array-level analysis while preserving accurate local unit-cell information.
- **Domain decomposition framework for open-cavity-based array elements:** A modular domain decomposition method (DDM) is developed for finite reflectarrays and RIS, where a single reflecting unit cell can be characterized separately from its embedded array environment. Mutual coupling between unit cells is modeled through over-the-air interactions using a generalized admittance matrix formulation. The method is validated for transmitting and scattering problems against CST Microwave Studio and conventional MoM results, and provides a roadmap toward large-scale reflectarray/RIS analysis.
- **MoM-based domain decomposition for conformal cavity-backed reflectarrays:** A domain decomposition formulation is developed for finite conformal cavity-backed reflectarrays. The method separates the interior cavity region from the exterior radiation region through aperture/interface quantities, using equivalent aperture magnetic currents

and a generalized aperture-admittance formulation. This enables accurate modeling of concave and convex conformal reflectarray geometries.

- **Hybrid domain-decomposition/physical-optics analysis using characteristic-mode MBFs:** A hybrid DDM-PO framework is developed for finite large-scale reconfigurable reflectarrays and RIS. The exterior response is approximated using PO, while element behavior is captured through pre-characterized admittance operators. Scalability to large arrays and many tuning states is achieved using a characteristic-mode MBF expansion of the aperture current, allowing multi-state analysis by reusing element operators and updating only changed elements.
- **Comparison of MBF construction methods for reduced-order analysis:** A controlled comparison is performed between characteristic mode analysis (CMA), the characteristic basis function method (CBFM), and a singular-value-decomposition (SVD)-based basis for reduced-order MoM analysis of cavity-backed reflectarrays. The comparison uses identical geometry, discretization, partitioning, and solver settings to isolate the impact of the MBF construction method on current reconstruction, far-field accuracy, number of retained unknowns, and conditioning.
- **Fast reconfiguration updates using Woodbury identity and iterative solvers:** A fast EM field data generation framework is developed by combining domain decomposition with Woodbury-identity-based iterative updates. The method avoids full matrix reassembly under localized element changes and introduces a Jacobi-type preconditioner to accelerate Generalized Minimal Residual method (GMRES) convergence for large-scale repeated solves.
- **Validation for scalable and data-driven design workflows:** The proposed methods are validated on representative finite reflectarray configurations, including planar, open-cavity, conformal, and reconfigurable arrays. The results demonstrate reductions in memory usage and simulation time while preserving accuracy, supporting scalable analysis, optimization, and dataset generation for reflectarray and RIS architectures.

1.5 Thesis-by-Publication Format

This thesis is written in a thesis-by-publication format, where each main technical part corresponds to a self-contained paper. To improve readability and consistency, common background material, notation, numerical concepts, and validation metrics are introduced before the included papers. Acronyms are listed in the front matter to minimize repetition.

The included papers form the following methodological progression:

1. Paper A: hybrid MoM–PO modeling for fast analysis of large all-metal reflectarrays;
2. Paper B: DDM analysis of open-cavity-based array elements as a roadmap toward large-scale finite reflectarray and RIS modeling;
3. Paper C: MoM-based domain decomposition for conformal cavity-backed reflectarrays;
4. Paper D: hybrid DDM–PO analysis of finite reconfigurable reflectarrays using characteristic-mode MBFs;
5. Paper E: comparison of MBF construction methods for reduced-order analysis of cavity-backed reflectarrays;
6. Paper F: fast MoM data generation using domain decomposition, Woodbury updates, and iterative solvers.

1.6 Organization of the Thesis

The remainder of this thesis is organized as follows:

- Chapter 2 introduces the physical and numerical background, including reflectarrays, RIS, cavity-backed all-metal unit cells, conformal reflectarrays, equivalence principles, MoM, PO, and relevant acceleration concepts.
- Chapter 3 presents the common numerical modeling framework and acceleration techniques used throughout the thesis, including aperture equivalence, domain decomposition, generalized aperture admittance, hybrid MoM–PO and DDM–PO modeling, MBF-based reduction, Woodbury updates, and iterative solution strategies.

- Chapter 4 describes the common validation workflow, benchmark configurations, and evaluation metrics used to assess accuracy and computational performance across the included papers.
- Chapter 5 provides a roadmap of the included papers and explains how the individual contributions relate to one another.
- Papers A–F present the included papers as self-contained contributions. Each paper states the specific problem addressed, describes the proposed method, presents numerical results, and includes a paper-specific bibliography.
- The final chapter 6 summarizes the overall findings, discusses limitations, and outlines directions for future research in fast modeling, reconfigurable reflectarray/RIS analysis, reduced-order methods, and data-driven EM design.

CHAPTER 2

Background on Reflectarrays, Cavity-Backed Elements, and Key EM Concepts

2.1 Reflectarrays and Reconfigurable Intelligent Surfaces

Reflectarrays are planar or conformal apertures composed of periodically arranged unit cells that reradiate an incident electromagnetic field with a prescribed spatial phase distribution, thereby forming a desired far-field beam [1–3]. Compared with classical parabolic reflectors, reflectarrays can provide high-gain aperture operation while offering a lower-profile structure and potentially reduced mass and integration complexity.

Reconfigurable intelligent surfaces (RIS) extend this concept by enabling the unit-cell response to be tuned, for example by switching between discrete states or continuously varying a controllable parameter [4–6]. This allows the reflected pattern to adapt to changing propagation or system requirements. From an electromagnetic viewpoint, both reflectarrays and RIS can be interpreted as engineered boundary conditions that impose a desired complex

reflection response over an aperture:

$$\Gamma(\mathbf{r}) = |\Gamma(\mathbf{r})|e^{j\phi(\mathbf{r})}, \quad (2.1)$$

where $\Gamma(\mathbf{r})$ is the local reflection coefficient and $\phi(\mathbf{r})$ is the local phase shift required to synthesize the target radiation pattern.

Beam-Forming Principle and Required Phase Profile

For a feed illuminating the aperture, the design objective is to compensate the spatial phase delay from the feed to each unit cell and to add the phase required for the desired outgoing direction $\hat{\mathbf{k}}_0$. A commonly used phase compensation condition is

$$\phi(\mathbf{r}) \approx -k_0 R(\mathbf{r}) + k_0 \hat{\mathbf{k}}_0 \cdot \mathbf{r} + \phi_0, \quad (2.2)$$

where $k_0 = 2\pi/\lambda_0$ is the free-space wavenumber, $R(\mathbf{r})$ is the distance from the feed phase center to the point $\mathbf{r}$ on the reflectarray surface, and ϕ_0 is a constant reference phase. In practice, (2.2) is implemented using a finite set of unit-cell geometries or states. This discretization introduces phase quantization, finite-size effects, and edge-element effects, all of which influence the final radiation pattern.

Finite-Array and Reconfiguration Considerations

Ideal reflectarray synthesis often relies on local periodic or isolated-cell assumptions, where each element is characterized independently and then placed in the array. This provides a practical design route, but finite arrays introduce additional effects that are not fully captured by infinite-periodic or isolated-cell models. These include nonuniform feed illumination, changes in the local element environment, and mutual coupling between apertures or cells. Edge elements may experience a different coupling environment than elements in the interior of the array because they have fewer neighboring elements. This edge-element effect is distinct from explicit edge-diffraction modeling, which is not the primary focus of this thesis.

For reconfigurable reflectarrays and RIS, an additional challenge is that the same physical aperture may need to be evaluated for many different state distributions. A direct full-wave recomputation for every configuration is often

computationally impractical. This motivates modular modeling approaches in which local element or aperture responses are pre-characterized and then reused or updated efficiently at the array level.

2.2 Cavity-Backed and All-Metal Unit Cells

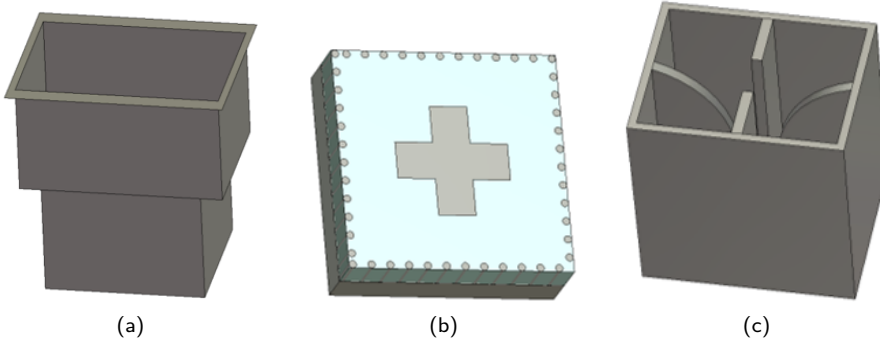


Figure 2.1: Representative cavity-backed reflectarray elements: (a) an all-metal waveguide-based cavity-backed element, (b) a dielectric cavity-backed patch element formed using vias connected to the ground plane, and (c) a ridged cavity-backed element that provides additional control of bandwidth and polarization response.

Cavity-backed reflectarray elements consist of an aperture opening backed by a metallic cavity or waveguide-like region. All-metal implementations are attractive at high frequencies because they can reduce dielectric losses and provide improved thermal and mechanical robustness. The cavity introduces a resonant response that can be exploited to realize a controllable reflection phase. Depending on the design, phase control may be achieved through geometric parameters such as cavity depth, aperture shape, metallic posts, ridges, or other perturbations. The aim is typically to obtain a wide phase range while maintaining a high reflection magnitude.

From a modeling standpoint, cavity-backed and open-cavity elements naturally separate into two regions:

- an **interior domain** or local element domain, where the fields satisfy

the local boundary conditions and may exhibit resonant behavior, and

- an **exterior domain**, where radiation, scattering, and coupling to other array elements occur.

This interior/exterior separation motivates domain decomposition approaches, where coupling across the aperture is enforced using equivalent currents and/or generalized admittance relationships.

2.3 Conformal Reflectarrays

Conformal reflectarrays are mounted on curved or faceted surfaces, such as concave or convex platforms, to satisfy integration constraints, realize specific radiation characteristics, or improve performance aspects such as scan behavior or bandwidth compared with planar configurations [7–11]. Compared with planar apertures, curvature introduces additional modeling and design challenges. First, each element has a different local surface normal, which changes the effective illumination, polarization response, and projected aperture. Second, the required phase distribution must be evaluated on a curved surface rather than on a plane, so the position vector $\mathbf{r}$ in (2.2) must follow the conformal geometry. Third, finite-size and mutual-coupling effects become more difficult to model because the element orientations and local environments vary across the surface.

Equivalence-based interface modeling and domain decomposition are well suited to this setting. They allow each cavity-backed element, or group of elements, to be treated in a local coordinate frame while enforcing global coupling through aperture/interface conditions.

2.4 Equivalence Principle and Aperture Equivalent Currents

Many modeling techniques rely on equivalence principles that replace a complex three-dimensional region by equivalent sources on a reference surface [12–14]. In this thesis, a particularly useful representation is based on equivalent magnetic surface currents on an aperture surface S_a [15–17]:

$$\mathbf{M}_s(\mathbf{r}) = \mathbf{E}_t(\mathbf{r}) \times \hat{\mathbf{n}}, \quad \mathbf{r} \in S_a, \quad (2.3)$$

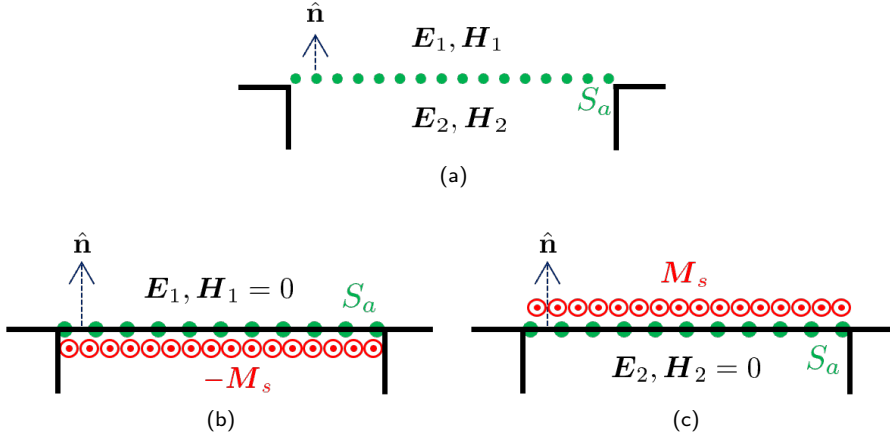


Figure 2.2: Aperture-equivalence principle at the cavity-free-space interface. (a) Original configuration, where the cavity is open to free space through the aperture S_a . At the aperture, the tangential fields are continuous, i.e., $\mathbf{E}_{t,1} = \mathbf{E}_{t,2}$ and $\mathbf{H}_{t,1} = \mathbf{H}_{t,2}$. (b) Equivalent interior problem. The aperture tangential electric field is represented by an equivalent magnetic surface current $-\mathbf{M}_s$, and the aperture is closed so that the interior cavity problem can be treated separately. (c) Equivalent exterior problem. The exterior region is treated separately by setting the interior fields to zero and representing the aperture tangential electric field by the corresponding equivalent magnetic surface current $\mathbf{M}_s$.

where $\hat{\mathbf{n}}$ is the outward unit normal, taken from the interior cavity toward the exterior region, and $\mathbf{E}_t$ is the tangential electric field on the aperture. With this sign convention, $\mathbf{M}_s = \mathbf{E}_t \times \hat{\mathbf{n}} = -\hat{\mathbf{n}} \times \mathbf{E}_t$.

The physical interpretation is that, if the tangential aperture field is known or accurately approximated, the exterior scattered or radiated field can be computed using radiation integrals or asymptotic methods without explicitly resolving the full interior geometry at the array level. In unit-cell-based workflows, $\mathbf{E}_t$ and therefore $\mathbf{M}_s$ may be obtained from detailed full-wave analysis of an isolated element under representative excitation. In domain-decomposition workflows, the aperture quantities are instead solved as interface unknowns coupled to both the interior and exterior domains.

For cavity-backed and open-cavity elements, the aperture is therefore not only a physical opening but also a natural computational interface between the local element/cavity region and the exterior radiation region. This viewpoint motivates the generalized aperture-admittance and domain-decomposition formulations used later in the thesis, where the detailed local response is retained while the exterior problem is treated separately.

2.5 Integral-Equation Modeling and the Method of Moments

The Method of Moments (MoM) is a standard full-wave technique for solving electromagnetic integral equations [12, 13, 18]. In surface-integral-equation formulations, the unknowns are currents defined on material interfaces, conducting surfaces, or equivalent aperture surfaces. This is particularly attractive for open-region radiation and scattering problems because the radiation condition is included through the Green's function.

Assuming time dependence $e^{j\omega t}$, the free-space scalar Green's function is

$$G(\mathbf{r}, \mathbf{r}') = \frac{e^{-jk_0|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}, \quad (2.4)$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ is the free-space wavenumber. Following the operator notation commonly used in MoM formulations with electric and magnetic surface currents [19], the electric-field and magnetic-field radiation operators

are written as

$$\begin{aligned} \mathcal{L}\{\mathbf{X}\}(\mathbf{r}) = & -j\omega\mu_0 \int_S \mathbf{X}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') dS' \\ & + \frac{1}{j\omega\epsilon_0} \nabla \int_S \nabla'_s \cdot \mathbf{X}(\mathbf{r}') G(\mathbf{r}, \mathbf{r}') dS', \end{aligned} \quad (2.5)$$

and

$$\mathcal{K}\{\mathbf{X}\}(\mathbf{r}) = \int_S \mathbf{X}(\mathbf{r}') \times \nabla' G(\mathbf{r}, \mathbf{r}') dS', \quad (2.6)$$

where $\mathbf{X}$ denotes either an electric or magnetic surface-current density and $\nabla'_s \cdot$ is the surface divergence with respect to the source coordinate $\mathbf{r}'$.

For equivalent electric and magnetic surface currents $\mathbf{J}$ and $\mathbf{M}$, the tangential fields can be expressed as

$$\mathbf{E}_{\text{tan}}(\mathbf{J}, \mathbf{M}) = [\mathcal{L}\{\mathbf{J}\} - \mathcal{K}\{\mathbf{M}\}]_{\text{tan}} - \frac{\hat{\mathbf{n}} \times \mathbf{M}}{2}, \quad (2.7)$$

$$\mathbf{H}_{\text{tan}}(\mathbf{J}, \mathbf{M}) = \left[\mathcal{K}\{\mathbf{J}\} + \frac{1}{\eta_0^2} \mathcal{L}\{\mathbf{M}\} \right]_{\text{tan}} + \frac{\hat{\mathbf{n}} \times \mathbf{J}}{2}, \quad (2.8)$$

where $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is the free-space wave impedance. The jump terms involving $\hat{\mathbf{n}} \times \mathbf{M}/2$ and $\hat{\mathbf{n}} \times \mathbf{J}/2$ appear when the observation point approaches the current-supporting surface. When the observation and source surfaces are distinct, these limiting terms are not present.

Electric-Field Integral Equation

For a conducting object or conducting aperture structure, the electric current $\mathbf{J}$ is usually unknown and must be solved from the boundary condition on the tangential electric field. For a perfect electric conductor (PEC), the tangential total electric field vanishes:

$$[\mathbf{E}^i + \mathbf{E}^s]_{\text{tan}} = \mathbf{0}, \quad \mathbf{r} \in S. \quad (2.9)$$

Using $\mathbf{E}^s = \mathcal{L}\{\mathbf{J}\}$ for the field scattered by the induced electric current, the electric-field integral equation (EFIE) becomes

$$[\mathcal{L}\{\mathbf{J}\}]_{\text{tan}} = -[\mathbf{E}^i]_{\text{tan}}. \quad (2.10)$$

In aperture-fed formulations, an impressed magnetic current $\mathbf{M}$ may act as the excitation. From (2.7), the electric field produced by $\mathbf{M}$ is $-\mathcal{K}\{\mathbf{M}\}$. If

a surface impedance Z_s is used to model a lossy conductor, the impedance boundary condition is

$$\mathbf{E}_{\text{tan}} = Z_s \mathbf{J}. \quad (2.11)$$

The corresponding EFIE can then be written as

$$[\mathcal{L}\{\mathbf{J}\}]_{\text{tan}} - Z_s \mathbf{J} = [\mathcal{K}\{\mathbf{M}\}]_{\text{tan}}. \quad (2.12)$$

For a PEC conductor, $Z_s = 0$, and the equation reduces to

$$[\mathcal{L}\{\mathbf{J}\}]_{\text{tan}} = [\mathcal{K}\{\mathbf{M}\}]_{\text{tan}}. \quad (2.13)$$

This form is particularly useful when aperture fields are represented by impressed magnetic currents.

Magnetic-Field Integral Equation

The magnetic-field integral equation (MFIE) is obtained from the magnetic-field boundary condition on a closed conducting surface. The electric surface current on a PEC boundary is related to the total magnetic field by

$$\mathbf{J} = \hat{\mathbf{n}} \times \mathbf{H}^{\text{tot}} = \hat{\mathbf{n}} \times (\mathbf{H}^i + \mathbf{H}^s). \quad (2.14)$$

Using the magnetic-field operator in (2.8), and considering the limiting value on the conducting surface, the MFIE can be written in compact operator form as

$$\left(\frac{1}{2} \mathcal{I} + \mathcal{K} \right) \{\mathbf{J}\} = \hat{\mathbf{n}} \times \mathbf{H}^i, \quad \mathbf{r} \in S, \quad (2.15)$$

where $\mathcal{I}$ is the identity operator. The sign of the $\mathcal{K}$ term depends on the chosen normal direction and on the precise definition of the magnetic-field operator. In this thesis, the sign convention is fixed by the aperture-equivalence and generalized-admittance formulations used in the paper chapters.

The MFIE is mainly used for closed conducting surfaces and is often better conditioned than the EFIE. However, because the cavity-backed and aperture-based formulations in this thesis naturally involve aperture magnetic currents and interface unknowns, the MFIE appears most prominently in the generalized aperture-admittance and hybrid DDM-PO formulations.

MoM Discretization

To solve either the EFIE or MFIE numerically, the unknown current is expanded in a finite set of basis functions. For triangular surface meshes, Rao–Wilton–Glisson (RWG) basis functions are commonly used:

$$\mathbf{J}(\mathbf{r}) \approx \sum_{n=1}^{N_J} I_n^J \mathbf{f}_n^J(\mathbf{r}), \quad \mathbf{M}(\mathbf{r}) \approx \sum_{n=1}^{N_M} V_n^M \mathbf{f}_n^M(\mathbf{r}), \quad (2.16)$$

where I_n^J and V_n^M are expansion coefficients for electric and magnetic currents, respectively.

Applying Galerkin testing with the same basis functions gives a finite-dimensional matrix equation. For the EFIE in (2.12), testing with $\mathbf{f}_m^J$ yields

$$\sum_{n=1}^{N_J} Z_{mn}^{J,J} I_n^J = b_m^J, \quad m = 1, 2, \dots, N_J, \quad (2.17)$$

where

$$Z_{mn}^{J,J} = \langle \mathbf{f}_m^J, \mathcal{L}\{\mathbf{f}_n^J\} - Z_s \mathbf{f}_n^J \rangle, \quad (2.18)$$

and

$$b_m^J = \langle \mathbf{f}_m^J, \mathcal{K}\{\mathbf{M}\} \rangle = \sum_{n=1}^{N_M} \langle \mathbf{f}_m^J, \mathcal{K}\{\mathbf{f}_n^M\} \rangle V_n^M. \quad (2.19)$$

Using the symmetric product

$$\langle \mathbf{a}, \mathbf{b} \rangle = \int_S \mathbf{a}(\mathbf{r}) \cdot \mathbf{b}(\mathbf{r}) \, dS, \quad (2.20)$$

the tested EFIE can be written in matrix form as

$$\mathbf{Z}^{J,J} \mathbf{I}^J = \mathbf{b}^J. \quad (2.21)$$

Similarly, testing the MFIE in (2.15) gives

$$\sum_{n=1}^{N_J} Z_{mn}^{\text{MFIE}} I_n^J = b_m^{\text{MFIE}}, \quad m = 1, 2, \dots, N_J, \quad (2.22)$$

with

$$Z_{mn}^{\text{MFIE}} = \left\langle \mathbf{f}_m^J, \left(\frac{1}{2} \mathcal{I} + \mathcal{K} \right) \{\mathbf{f}_n^J\} \right\rangle, \quad (2.23)$$

and

$$b_m^{\text{MFIE}} = \langle \mathbf{f}_m^J, \hat{\mathbf{n}} \times \mathbf{H}^i \rangle. \quad (2.24)$$

Both the EFIE and MFIE therefore lead to the generic MoM system

$$\mathbf{Z}\mathbf{x} = \mathbf{b}, \quad (2.25)$$

where $\mathbf{Z}$ is the MoM system matrix, $\mathbf{x}$ contains the unknown current expansion coefficients, and $\mathbf{b}$ is the tested excitation vector. In the EFIE, $\mathbf{Z}$ represents electric-field testing of the unknown electric current. In the MFIE, $\mathbf{Z}$ represents magnetic-field testing and includes the identity term.

For electrically large finite arrays, $\mathbf{Z}$ is dense because the Green's function couples distant basis functions. As a result, storage and matrix filling typically scale as $\mathcal{O}(N^2)$, dense matrix-vector products scale as $\mathcal{O}(N^2)$ per iteration, and conventional dense direct solution scales as approximately $\mathcal{O}(N^3)$. These costs motivate the acceleration strategies used in this thesis, including aperture equivalence, generalized admittance, domain decomposition, Physical Optics, macro basis functions, low-rank updates, and iterative solvers.

Iterative Solvers and Preconditioning

For large-scale systems, iterative Krylov subspace methods such as the Generalized Minimal Residual method (GMRES) [20] are widely used to solve (2.25). Their convergence depends strongly on the spectral properties and conditioning of the system matrix $\mathbf{Z}$, as well as on the effectiveness of the chosen preconditioner. In this thesis, lightweight preconditioning is used to reduce iteration counts and improve scalability, especially when combined with matrix-update techniques for localized configuration changes.

2.6 Physical Optics as an Array-Level Approximation

Physical Optics (PO) is an asymptotic high-frequency approximation for estimating the induced current on an electrically large conducting surface illuminated by an incident field [21, 22]. The main assumption is that, in the illuminated region, the surface behaves locally like an infinite tangent PEC plane. Under this approximation, the induced electric surface current density

is written as

$$\mathbf{J}_{\text{PO}}(\mathbf{r}) \approx 2\hat{\mathbf{n}} \times \mathbf{H}^i(\mathbf{r}), \quad \mathbf{r} \in S_{\text{ill}}, \quad (2.26)$$

where $\hat{\mathbf{n}}$ is the outward unit normal, $\mathbf{H}^i$ is the incident magnetic field on the conducting surface, and S_{ill} denotes the illuminated part of the surface. In the shadow region, the PO current is typically assumed to be negligible:

$$\mathbf{J}_{\text{PO}}(\mathbf{r}) \approx \mathbf{0}, \quad \mathbf{r} \in S_{\text{shadow}}. \quad (2.27)$$

Once the approximate surface current is known, the radiated or scattered field can be computed using standard radiation integrals. For a sufficiently large and smooth surface, PO can therefore provide accurate main-beam and dominant scattering predictions with substantially lower computational effort than a full-wave global solve [23].

In the hybrid modeling strategies used in this thesis, PO is applied at the array or exterior-domain level after accurate local information has been extracted from full-wave unit-cell, aperture, or cavity-level solutions. For example, aperture fields or equivalent magnetic currents may first be obtained from MoM or a generalized admittance formulation, and PO is then used to approximate the induced current on the electrically large PEC-equivalent exterior surface. This preserves the essential element-scale physics while keeping the global computation efficient.

The PO approximation does not explicitly resolve all edge-diffraction, shadow-boundary, or multiple-scattering effects. Therefore, its use in this thesis is limited to cases where the exterior response is expected to be dominated by electrically large PEC scattering in the illuminated region, and the resulting patterns are validated against full-wave references in the paper chapters.

2.7 Overview of MoM Acceleration Strategies

The high accuracy of MoM comes with a significant computational cost for electrically large reflectarrays and RIS. Since the discretized integral equation leads to a dense matrix, storage and matrix fill typically scale as $\mathcal{O}(N^2)$. Dense matrix–vector products in iterative solvers also scale as $\mathcal{O}(N^2)$ per iteration, while conventional dense direct solvers typically scale as $\mathcal{O}(N^3)$ [24]. For finite arrays, this becomes particularly limiting when many similar configurations must be evaluated during design optimization, reconfiguration, or electromagnetic dataset generation.

A broad range of MoM acceleration strategies has therefore been developed. Some approaches accelerate the matrix–vector product by restructuring far interactions, as in the multilevel fast multipole method (MLFMM) [24, 25] or the adaptive integral method (AIM) [26]. Other approaches reduce memory and computational cost by compressing low-rank interaction blocks, for example using adaptive cross approximation (ACA) [27–29] or hierarchical matrices ($\mathcal{H}$ - and $\mathcal{H}^2$ -matrices) [30]. A different class of methods reduces the number of unknowns through macro basis functions or higher-order basis functions [31, 32]. Domain decomposition methods divide the problem into coupled subdomains [33, 34], which is especially natural for cavity-backed structures. Finally, high-frequency approximations such as PO [21] and geometrical optics (GO) [13] avoid the global full-wave solve by approximating the relevant currents or fields directly.

The broad acceleration families are summarized in Table 2.1. The listed methods are not exhaustive, and many combined or problem-specific variants exist. The table is therefore intended as a compact overview of the main families into which many practical acceleration techniques fall. The listed complexities should be interpreted as indicative, since the actual cost depends on implementation, geometry, frequency, rank behavior, and solver convergence.

In this thesis, these acceleration ideas are used in a targeted way rather than as a broad comparison of all available solver families. Full-wave MoM is retained for local unit-cell and cavity physics, domain decomposition and aperture equivalence provide a natural interior/exterior coupling, generalized admittance formulations account for finite-array coupling, hybrid MoM–PO and hybrid DDM–PO modeling accelerate array-level scattering from electrically large PEC-like apertures, MBFs reduce the number of unknowns in selected reduced-order formulations, and low-rank update strategies enable fast recomputation under localized element changes. These thesis-specific methods are described in detail in Chapter 3 and in the corresponding paper chapters.

Method	Main idea	Typical scaling
Standard MoM	Dense all-to-all interactions in $\mathbf{Z}$.	Storage/mat-vec: $\mathcal{O}(N^2)$; direct solve: $\mathcal{O}(N^3)$
MLFMM	Multilevel Fast Multipole Method; uses multipole expansions to accelerate far-field interactions.	Often $\mathcal{O}(N \log N)$, sometimes near-linear
AIM	Adaptive Integral Method; uses FFT-based acceleration with near-field correction.	Typically $\mathcal{O}(N \log N)$
ACA	Adaptive Cross Approximation; approximates far-field matrix blocks with low-rank structure.	$\mathcal{O}(kN)$ – $\mathcal{O}(kN \log N)$, rank-dependent
$\mathcal{H}$ -matrix	Hierarchical matrix; stores the dense matrix in hierarchical low-rank block form.	Typically $\mathcal{O}(N \log N)$
$\mathcal{H}^2$ -matrix	Nested hierarchical matrix; reuses basis information across the hierarchy.	Best case: $\mathcal{O}(N)$

Table 2.1: General Method-of-Moments acceleration families included for computational context. The listed complexities are indicative and depend on implementation, geometry, frequency, rank behavior, and solver convergence.

2.8 Summary and Link to the Included Papers

This chapter introduced the physical and numerical background required for the thesis. Reflectarrays and RIS were described as engineered apertures that implement a prescribed reflection phase profile. Finite-array effects, mutual coupling, and reconfigurable-state changes were discussed as key reasons why repeated full-wave analysis becomes computationally demanding. Cavity-backed and open-cavity elements were introduced as unit cells with a natural interior/exterior separation. The equivalence principle was then used to motivate aperture magnetic currents as compact interface quantities for array-level modeling. MoM was introduced as the full-wave baseline, PO as an efficient high-frequency array-level approximation, and common MoM acceleration families were briefly reviewed to position the methods used in the thesis.

These concepts motivate the included papers, where hybrid MoM–PO modeling, open-cavity and conformal domain decomposition, MBF-based reduced-order modeling, hybrid DDM–PO analysis of reconfigurable arrays, and fast Woodbury-identity updates are developed and validated for finite planar, conformal, and reconfigurable cavity-backed reflectarrays and RIS.

CHAPTER 3

Numerical Modeling Framework and Acceleration Techniques

3.1 Overview

Chapter 2 introduced the electromagnetic background and the broader landscape of Method-of-Moments (MoM) acceleration techniques. This chapter summarizes the numerical framework that connects the included papers in this thesis [35–40]. The common objective is to retain full-wave accuracy where it is most important, namely in the local unit-cell, aperture, and cavity regions, while reducing the cost of finite-array analysis and repeated recomputation.

The framework combines the following ingredients:

- **MoM-based integral-equation modeling** for accurate unit-cell, aperture, and subdomain solutions [12, 18];
- **aperture equivalence** using equivalent magnetic surface currents on cavity or open-cavity apertures [13–16, 41];
- **generalized admittance and domain decomposition** to couple local element/cavity domains to the exterior array domain [36, 37];

- **Physical Optics (PO)** to approximate the exterior response of electrically large PEC-like surfaces [21, 23, 35, 38];
- **Macro Basis Functions (MBFs)** to reduce the number of aperture-current unknowns in selected reduced-order formulations [32, 38, 39, 42, 43];
- **Woodbury identity updates** and **GMRES preconditioning** for fast recomputation when only a subset of elements changes state [20, 40].

The chapter is intended as a common methodological bridge. It does not replace the detailed derivations in the paper chapters; instead, it defines the shared notation and explains how the different modeling components relate to one another.

3.2 Methodological Roadmap

The included papers use these numerical components in a progressive way. Paper A introduces a hybrid MoM–PO strategy in which isolated unit-cell aperture currents excite an equivalent PEC sheet. Paper B develops a generalized-admittance domain-decomposition formulation for open-cavity-based array elements, where unit cells are characterized separately and coupled through over-the-air interactions. Paper C applies the generalized-admittance DDM framework to conformal cavity-backed reflectarrays. Paper D combines DDM, PO, and characteristic-mode MBFs to analyze finite reconfigurable reflectarrays and RIS. Paper E compares different MBF construction strategies within a common reduced-order framework. Paper F uses Woodbury updates and preconditioned GMRES to accelerate repeated array simulations for data generation. The overall computational workflow is illustrated in Fig. 3.1.

3.3 MoM Formulation for Conducting Structures

For perfectly electrically conducting (PEC) objects, surface integral equations relate the tangential fields on the conductor to unknown surface currents. After discretization with suitable basis and testing functions, for example Rao–Wilton–Glisson (RWG) functions on triangular meshes, MoM yields a

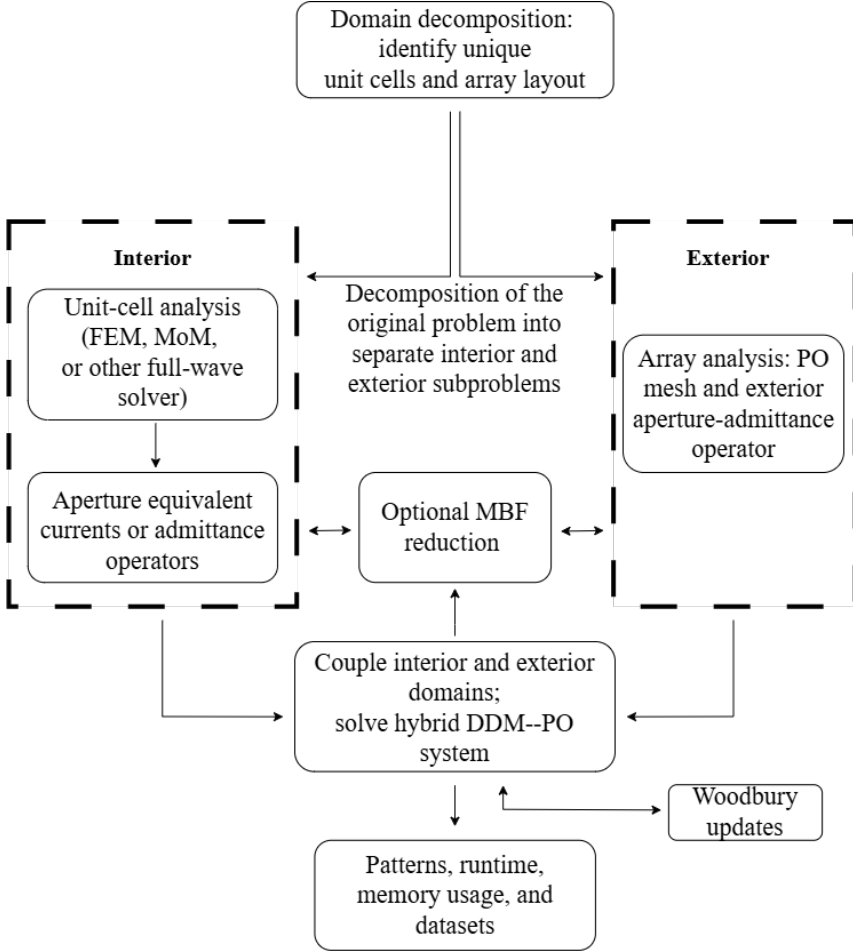


Figure 3.1: Computational workflow of the modular domain-decomposition framework. Unique unit cells are analyzed locally to obtain aperture currents or admittance operators, while the exterior array response is modeled using an equivalent sheet with PO mesh and exterior aperture-admittance operator. The interior and exterior problems are coupled in a hybrid DDM–PO system, optionally reduced using MBFs and updated efficiently with Woodbury updates for reconfigurable cases.

dense linear system

$$\mathbf{Z}\mathbf{x} = \mathbf{b} \quad (3.1)$$

where $\mathbf{Z} \in \mathbb{C}^{N \times N}$ is the impedance matrix, $\mathbf{x} \in \mathbb{C}^N$ contains the unknown current expansion coefficients, and $\mathbf{b} \in \mathbb{C}^N$ represents the tested excitation.

For finite reflectarrays, direct full-array MoM becomes expensive because the matrix is dense and the number of unknowns grows with electrical size. In this thesis, MoM is therefore used in three main ways:

- as a **local solver** for isolated unit cells, cavity interiors, or aperture operators;
- as a **subdomain solver** inside a domain-decomposition formulation;
- as a **full-order reference** for validating hybrid, reduced-order, and update-based methods.

Two computational issues motivate the acceleration techniques in this thesis. The first is electrical size: large apertures require many unknowns and high memory usage. The second is repeated recomputation: reconfigurable arrays, optimization loops, and dataset-generation workflows require many closely related simulations, often with only a small number of changed elements. The methods below address these two issues by separating local element physics from global coupling, by approximating the exterior response when appropriate, by reducing the basis size, and by updating matrices locally rather than recomputing them from scratch.

3.4 Aperture Equivalence and Generalized Admittance

Equivalent Magnetic Aperture Current

A central idea in the thesis is to replace a three-dimensional cavity-backed or open-cavity element by equivalent quantities on its aperture. Let S denote the aperture surface and let $\hat{\mathbf{n}}$ be the unit normal directed from the interior region toward the exterior region. With the sign convention used in this thesis, the equivalent magnetic surface current is

$$\mathbf{M}_s(\mathbf{r}) = \mathbf{E}_t(\mathbf{r}) \times \hat{\mathbf{n}} = -\hat{\mathbf{n}} \times \mathbf{E}_t(\mathbf{r}), \quad \mathbf{r} \in S, \quad (3.2)$$

where $\mathbf{E}_t$ is the tangential electric field on the aperture. The aperture is then used as the interface between an interior domain, where the local cavity response is captured, and an exterior domain, where radiation, scattering, and array-level coupling occur.

Aperture MFIE

In the generalized-admittance formulation, the original scattering problem is decomposed into an exterior region a and an interior region b separated by S . Enforcing tangential magnetic-field continuity on the aperture and applying field equivalence gives the aperture MFIE [15, 16, 36, 41]

$$\begin{aligned} [\mathbf{H}^a(\mathbf{M}_s) + \mathbf{H}^b(\mathbf{M}_s)]_{\text{tan}} + [\mathbf{H}^a(\mathbf{J}_s^a) - \mathbf{H}^b(\mathbf{J}_s^b)]_{\text{tan}} \\ = [\mathbf{H}^{b,i} - \mathbf{H}^{a,i}]_{\text{tan}}. \end{aligned} \quad (3.3)$$

Here, $\mathbf{J}_s^a$ and $\mathbf{J}_s^b$ are equivalent electric currents on the exterior and interior sides of the aperture, respectively. The operators $\mathbf{H}^a(\cdot)$ and $\mathbf{H}^b(\cdot)$ are the corresponding magnetic-field integral operators, and $\mathbf{H}^{a,i}$ and $\mathbf{H}^{b,i}$ are impressed magnetic fields in the two regions.

After discretization and testing, the domain contributions can be expressed in matrix form as a generalized admittance system

$$[\mathbf{Y}_{\text{tot}}^a + \mathbf{Y}_{\text{tot}}^b] \mathbf{V} = \mathbf{I}, \quad (3.4)$$

where $\mathbf{V}$ contains the expansion coefficients of $\mathbf{M}_s$ and $\mathbf{I}$ is the tested excitation vector. The exterior matrix $\mathbf{Y}_{\text{tot}}^a$ represents radiation and over-the-air coupling, while $\mathbf{Y}_{\text{tot}}^b$ represents the local interior or cavity response.

Open-Cavity Unit-Cell Characterization

In Paper B, the generalized-admittance formulation is used to characterize open-cavity-based array elements independently from the full array [36]. The unit-cell response is represented by aperture modes or equivalent magnetic currents, while coupling between cells occurs through the exterior domain. This is particularly useful for open-cavity reflectarray or RIS elements where the cavity or via-fenced design suppresses substrate-mediated coupling, so that mutual coupling is predominantly over the air.

For an array with multiple cells, the interior contribution is block diagonal when each element is treated as a separate local domain. The exterior contribution contains the self and mutual coupling between aperture modes. This creates a modular finite-array formulation: repeated element types can reuse the same interior operator, while the exterior admittance matrix accounts for the finite-array environment, including the different coupling experienced by edge and interior elements. Explicit edge-diffraction corrections are not introduced; edge behavior enters through the finite-array coupling model.

Conformal Arrays

For conformal reflectarrays, the same aperture-admittance idea is applied after assigning each element a local coordinate frame and aperture normal. The interior admittance operator is associated with the local cavity or element design, while the exterior admittance depends on the global position and orientation of the aperture modes. This allows the method to handle concave and convex faceted arrays without relying on periodic boundary conditions [37].

3.5 Hybrid MoM–PO Modeling

Geometry Reduction and Array-Level PO

The hybrid MoM–PO method in Paper A starts from isolated unit-cell analyses. For each element, the aperture field is sampled and converted into an equivalent magnetic current. The array is then represented by a PEC-equivalent sheet excited by the collection of aperture magnetic currents. Under the assumption that rear-surface currents are negligible, the three-dimensional conductor can be reduced, from the exterior perspective, to a two-dimensional PEC sheet in the aperture plane [35].

The induced electric surface current on the exterior PEC surface is then approximated by PO. For an illuminated PEC surface,

$$\mathbf{J}_{\text{PO}}(\mathbf{r}) \approx 2\hat{\mathbf{n}} \times \mathbf{H}_{\text{inc}}(\mathbf{r}), \quad \mathbf{r} \in S_{\text{ill}}, \quad (3.5)$$

where S_{ill} is the illuminated part of the surface and $\mathbf{H}_{\text{inc}}$ is the incident magnetic field on the equivalent PEC surface. In the hybrid reflectarray model, this incident field may include both the feed field and the field radiated by the aperture magnetic currents.

The method is efficient because the full array does not need to be solved with a dense MoM system. However, when aperture currents are extracted from isolated cells, the method does not fully account for mutual coupling effects that modify aperture currents in the embedded finite-array environment. This limitation motivates the generalized-admittance DDM formulations introduced in the later papers.

Applicability and Limitations

PO is most appropriate when the exterior surface is electrically large, predominantly PEC-like, and not dominated by fine edge-diffraction or shadow-boundary effects. In this thesis, PO is used as an exterior-domain acceleration method. The detailed local element response is still obtained from MoM or admittance operators, while the global exterior response is approximated asymptotically. The paper chapters validate the resulting patterns against full-wave references.

3.6 Hybrid DDM–PO Formulation

Paper D combines the generalized-admittance DDM formulation with a PO approximation of the exterior region [38]. Starting from the aperture MFIE in (3.3), the exterior current is split into two PO-driven contributions,

$$\mathbf{J}_s^a = \mathbf{J}_{\mathbf{M}_s}^a + \mathbf{J}_{\mathbf{H}^{a,i}}^a, \quad (3.6)$$

where $\mathbf{J}_{\mathbf{M}_s}^a$ is induced by the field radiated by $\mathbf{M}_s$, and $\mathbf{J}_{\mathbf{H}^{a,i}}^a$ is induced by the impressed exterior magnetic field. Under the PO approximation, the corresponding tangential magnetic-field contributions in the illuminated region are approximated as

$$[\mathbf{H}^a(\mathbf{J}_{\mathbf{M}_s}^a)]_{\text{tan}} \approx [\mathbf{H}^a(\mathbf{M}_s)]_{\text{tan}}, \quad (3.7)$$

$$[\mathbf{H}^a(\mathbf{J}_{\mathbf{H}^{a,i}}^a)]_{\text{tan}} \approx [\mathbf{H}^{a,i}]_{\text{tan}}. \quad (3.8)$$

By superposition,

$$[\mathbf{H}^a(\mathbf{J}_s^a)]_{\text{tan}} \approx [\mathbf{H}^a(\mathbf{M}_s) + \mathbf{H}^{a,i}]_{\text{tan}}. \quad (3.9)$$

Substituting (3.9) into (3.3) yields the MFIE–PO relation

$$[2\mathbf{H}^a(\mathbf{M}_s) + \mathbf{H}^b(\mathbf{M}_s)]_{\text{tan}} - [\mathbf{H}^b(\mathbf{J}_s^b)]_{\text{tan}} = [\mathbf{H}^{b,i} - 2\mathbf{H}^{a,i}]_{\text{tan}}. \quad (3.10)$$

For the exterior-excited cases considered in the paper, no impressed source is assumed in the interior region, so $\mathbf{H}^{b,i} = \mathbf{0}$.

After RWG discretization and Galerkin testing with the product

$$\langle \mathbf{u}, \mathbf{H}(\mathbf{v}) \rangle \triangleq \int_S \mathbf{u} \cdot \mathbf{H}(\mathbf{v}) \, dS, \quad (3.11)$$

the MFIE–PO formulation gives

$$[\mathbf{Y}_{\text{tot}}^a + \mathbf{Y}_{\text{tot}}^b] \mathbf{V} = \mathbf{I}, \quad (3.12a)$$

$$[\mathbf{Y}_{\text{tot}}^a]_{mn} = 2 \langle \mathbf{f}_m^M, \mathbf{H}^a(\mathbf{f}_n^M) \rangle, \quad (3.12b)$$

$$[\mathbf{I}]_m = -2 \langle \mathbf{f}_m^M, \mathbf{H}^{a,i} \rangle, \quad (3.12c)$$

$$[\mathbf{Y}_{\text{tot}}^b]_{mn} = \langle \mathbf{f}_m^M, \mathbf{H}^b(\mathbf{f}_n^M) \rangle - \langle \mathbf{f}_m^M, \mathbf{H}^b(\mathbf{J}_s^b(-\mathbf{f}_n^M)) \rangle. \quad (3.12d)$$

Here, $\mathbf{f}_n^M$ is an RWG basis function for the aperture magnetic current, $\mathbf{V}$ collects the coefficients of $\mathbf{M}_s$, and $\mathbf{J}_s^b(-\mathbf{f}_n^M)$ is the interior electric current induced by the oppositely oriented magnetic-current basis function on the interior side. Its construction follows the underlying DDM formulation in Paper B.

3.7 Interior Operator Reuse

For an array containing L elements drawn from P unique element types, element l is assigned a type index $\tau(l) \in \{1, \dots, P\}$. The interior admittance matrix can then be assembled as

$$\mathbf{Y}_{\text{tot}}^b = \text{blkdiag}(\mathbf{Y}^{b,(\tau(1))}, \dots, \mathbf{Y}^{b,(\tau(L))}). \quad (3.13)$$

Thus, each unique element type or tuning state can be characterized once and reused throughout the array. This is a key advantage for reconfigurable reflectarrays and RIS, where many configurations are built from a small library of element states.

3.8 Macro Basis Function Reduction

Reduced Aperture-Current Expansion

The aperture-current unknowns can be further reduced using MBFs. Let $\mathbf{V} \in \mathbb{C}^N$ collect the RWG coefficients of $\mathbf{M}_s$. The MBF approximation is

$$\mathbf{V} = \mathbf{V}_{\text{MBF}} \mathbf{w}, \quad (3.14)$$

where $\mathbf{V}_{\text{MBF}} \in \mathbb{C}^{N \times K}$ contains the retained MBFs in the original RWG basis, and $\mathbf{w} \in \mathbb{C}^K$ contains the MBF weights. If K_l MBFs are retained for element l , then $K = \sum_{l=1}^L K_l$.

Because the element apertures are non-overlapping, the global MBF matrix can be assembled element-wise as

$$\mathbf{V}_{\text{MBF}} = \text{blkdiag}\left(\mathbf{V}_{\text{MBF}}^{(\tau(1))}, \dots, \mathbf{V}_{\text{MBF}}^{(\tau(L))}\right). \quad (3.15)$$

Substitution of (3.14) into (3.12a) gives the reduced system

$$\left(\mathbf{V}_{\text{MBF}}^T [\mathbf{Y}_{\text{tot}}^a + \mathbf{Y}_{\text{tot}}^b] \mathbf{V}_{\text{MBF}}\right) \mathbf{w} = \mathbf{V}_{\text{MBF}}^T \mathbf{I}. \quad (3.16)$$

The transpose T is used to obtain a symmetric matrix for computation convenience. After solving for $\mathbf{w}$, the full-resolution aperture-current vector is reconstructed using (3.14).

Characteristic-Mode MBFs

In Paper D, characteristic-mode MBFs are used to reduce the aperture-current unknowns. For each unique element type, a local composite admittance matrix is formed from the exterior and interior contributions. Its Hermitian real and imaginary parts are used to define a characteristic-mode eigenproblem, and the modes are ranked using the modal significance

$$\text{MS}_n = \frac{1}{|1 + j\lambda_n|} = \frac{1}{\sqrt{1 + \lambda_n^2}}. \quad (3.17)$$

The most significant modes are retained as local MBFs. The exact eigenproblem convention depends on the admittance sign convention used in the corresponding paper chapter, but the role of the modes is the same: they provide an excitation-independent reduced basis for the aperture current.

CBFM and SVD-Based MBFs

Paper E compares characteristic-mode MBFs with two alternative constructions [39]. In the Characteristic Basis Function Method (CBFM), MBFs are constructed from current responses generated by representative excitations. Primary responses are obtained from isolated-element excitations, while enrichment responses can be generated from local subarrays to capture higher-order coupling effects. An SVD truncation of the response matrix removes redundant functions.

In the SVD-based construction, the basis is obtained directly from the local system operator or its inverse. This provides an excitation-independent numerical basis. Compared with CMA and CBFM, the SVD basis is often well conditioned but may require more retained MBFs to reach a prescribed far-field accuracy.

The MBF comparison paper shows that basis construction affects both electromagnetic accuracy and numerical stability. CBFM can provide strong low-rank accuracy, but enriched bases may produce nonmonotonic solution-error behavior. CMA benefits from explicit metric orthogonalization. SVD gives well-conditioned reduced systems, but it may require a larger reduced dimension.

Method	Strategy	Computational effect	Role in this thesis
Macro Basis Functions (MBFs)	Represent fine-scale aperture currents using a reduced set of larger basis functions.	Reduces the number of unknowns from N to K , with $K \ll N$ desired.	Used for reduced-order analysis of cavity-backed and reconfigurable reflectarrays.
Domain Decomposition Method (DDM)	Separates local element/cavity domains from the exterior radiation domain.	Allows local operators to be reused while finite-array coupling is handled at the array level.	Supports open-cavity, conformal, and reconfigurable reflectarray modeling.
Physical Optics (PO)	Approximates induced currents on electrically large PEC-like surfaces.	Avoids a dense full-wave exterior solve when the exterior surface is suitable for PO.	Used for array-level acceleration in hybrid MoM-PO and DDM-PO models.
Hybrid MoM-PO / DDM-PO	Combines local full-wave or admittance-based models with PO exterior modeling.	Keeps local accuracy while reducing exterior-domain cost.	Main fast-analysis route for large finite reflectarrays and RIS.
Woodbury updates	Updates the inverse or solution after localized matrix changes.	Avoids full matrix recomputation for small state changes.	Used for fast recomputation in dataset-generation and reconfiguration workflows.
GMRES with preconditioning	Uses Krylov iteration with lightweight preconditioning.	Reduces solve time when direct inversion is not efficient.	Used for large and repeated MoM/DDM solves.

Table 3.1: Acceleration concepts most directly used in this thesis.

MBF construction	Strategy	Computational effect	Role in this thesis
Characteristic Mode Analysis (CMA)	Uses characteristic currents as physically motivated aperture-current basis functions.	Provides an excitation-independent basis; conditioning can require metric orthogonalization.	Used in the hybrid DDM-PO framework and compared in the MBF study.
Characteristic Function (CBFM)	Constructs basis functions from primary and enriched current responses.	Can be accurate at low rank, but enriched bases may show nonmonotonic solution error.	Compared against CMA- and SVD-based bases.
Singular Value Decomposition (SVD) basis	Constructs basis functions from singular vectors of a local operator or response matrix.	Can produce well-conditioned reduced systems, but may require more retained functions.	Used as an algebraic reduced-basis benchmark.

Table 3.2: Macro-basis-function construction methods considered in the reduced-order analysis paper.

3.9 Woodbury Updates for Localized Reconfiguration

In reconfigurable reflectarrays and RIS, many simulations differ only by a localized element-state change. In the admittance formulation, a change in element m modifies only the corresponding block of the interior matrix,

$$\mathbf{Y}' = \mathbf{Y} + \Delta\mathbf{Y}, \quad \mathbf{Y} = \mathbf{Y}^a + \mathbf{Y}^b, \quad (3.18)$$

where $\Delta\mathbf{Y}$ has nonzero entries only in the affected local block or blocks. If the update is represented in low-rank form,

$$\Delta\mathbf{Y} = \mathbf{U}\mathbf{C}\mathbf{W}^H, \quad (3.19)$$

then the Woodbury identity gives

$$(\mathbf{Y} + \mathbf{U}\mathbf{C}\mathbf{W}^H)^{-1} = \mathbf{Y}^{-1} - \mathbf{Y}^{-1}\mathbf{U}(\mathbf{C}^{-1} + \mathbf{W}^H\mathbf{Y}^{-1}\mathbf{U})^{-1}\mathbf{W}^H\mathbf{Y}^{-1}. \quad (3.20)$$

This avoids a full inverse or factorization when the update rank is small. Paper F uses this idea to accelerate repeated simulations for data generation, especially when only one or a small number of elements changes state [40].

3.10 Iterative Solvers and Preconditioning

For larger systems or repeated right-hand sides, iterative solvers such as GMRES can be used instead of direct inversion. Since the systems arising from integral-equation and admittance formulations are generally non-Hermitian, GMRES is a suitable Krylov method. Its convergence depends on the spectrum and conditioning of the system matrix and on the quality of the preconditioner.

A lightweight option used in the thesis is a Jacobi-type or block-Jacobi-type preconditioner based on the diagonal or block-diagonal part of the system matrix. In simplified notation,

$$\mathbf{P}^{-1} \approx \text{diag}(\mathbf{Y})^{-1}, \quad (3.21)$$

or, when element blocks are used, $\mathbf{P}^{-1}$ is built from the inverse of the self-interaction blocks. This choice is inexpensive to apply, naturally parallelizable, and compatible with the non-overlapping aperture-current supports used in the DDM formulation.

3.11 Complexity and Scalability Considerations

The methods in this thesis reduce the dominant costs of finite reflectarray and RIS analysis in complementary ways:

- local full-wave modeling is retained only where it is needed, such as in unit cells, cavities, or aperture operators;
- generalized admittance matrices separate reusable interior operators from exterior finite-array coupling;
- PO replaces a full-wave exterior solve when the exterior PEC-like surface is electrically large;
- MBFs reduce the number of aperture unknowns in selected formulations;
- Woodbury updates avoid full recomputation for localized state changes;
- preconditioned GMRES reduces iterative solution time for repeated or large systems.

Together, these techniques provide a modular computational framework for finite planar, conformal, and reconfigurable cavity-backed reflectarrays and RIS.

3.12 Summary and Link to the Paper Chapters

This chapter described the computational backbone used throughout the thesis: MoM as the full-wave baseline, aperture equivalence, generalized admittance, domain decomposition, hybrid MoM–PO and DDM–PO acceleration, MBF-based reduced-order modeling, Woodbury updates, and preconditioned GMRES. The next chapter defines the common validation setup and metrics. The subsequent paper chapters then present each technique in its full formulation, implementation, validation, and computational context.

CHAPTER 4

Common Simulation Setup, Validation Workflow, and Error Metrics

4.1 Overview

This thesis contains several paper chapters, each addressing a specific modeling, reduction, or acceleration technique for finite reflectarrays and related reconfigurable intelligent surfaces (RIS). This chapter summarizes the shared simulation conventions, validation workflow, reference comparisons, and evaluation quantities used across the thesis. Paper-specific details, such as a particular conformal geometry, open-cavity test case, tuning-state distribution, MBF construction, or localized-update scenario, are given in the corresponding paper chapter.

The validation strategy has two parts. First, electromagnetic accuracy is assessed by comparing the proposed methods with trusted references, including CST Microwave Studio, conventional MoM formulations, high-fidelity in-house MoM implementations, or full-order versions of the same DDM framework. Second, computational efficiency is assessed using runtime, speed-up, memory usage, number of unknowns, update cost, and, where relevant, system

conditioning. The principal field comparisons are shown using E- and H-plane radiation or scattering patterns. The quantitative error metrics used throughout the thesis are limited to peak-normalized pattern error and current-vector norm error.

4.2 Geometry Definitions and Coordinate Conventions

Global Coordinate System

Unless otherwise stated, a right-handed Cartesian coordinate system (x, y, z) is used. For planar reflectarrays, the aperture is placed near a reference plane, and the outward unit normal of the PEC reference surface is denoted by $\hat{\mathbf{n}}$. Spherical angles (θ, φ) follow the standard antenna convention, where θ is measured from the $+z$ axis and φ is the azimuth angle in the x - y plane.

Array Classes

The thesis considers several related classes of structures:

- **Planar reflectarrays**, where the aperture is discretized into a finite $N_x \times N_y$ grid of unit cells.
- **Open-cavity-based arrays**, where individual reflecting cells are characterized separately and finite-array coupling is modeled through the exterior region.
- **Conformal reflectarrays**, where the aperture lies on a curved or faceted surface and each element has its own local orientation.
- **Reconfigurable reflectarrays/RIS**, where each element can take one of several geometrical, loaded, or tuned states.

For conformal arrays, local tangent frames and local surface normals are defined per element. The exact element placement, surface definition, and orientation convention are stated in the corresponding paper chapter.

4.3 Unit-Cell Modeling and Element Libraries

Local Element or Cavity Characterization

A recurring part of the workflow is the extraction of aperture quantities from a unit-cell, open-cavity, or cavity-backed element analysis. The local problem may be excited by a feed approximation, plane-wave illumination, a Huygens source, or another excitation specified in the paper chapter. The tangential aperture electric field is represented by an equivalent magnetic surface current,

$$\mathbf{M}_s(\mathbf{r}) = -\hat{\mathbf{n}} \times \mathbf{E}_t(\mathbf{r}), \quad \mathbf{r} \in S_a, \quad (4.1)$$

where S_a is the aperture and $\mathbf{E}_t$ is the tangential electric field.

Depending on the method, the local characterization produces aperture fields, equivalent magnetic currents, generalized admittance operators, MBFs, or reduced-order operators. These quantities can then be reused in array-level simulations, which is essential for reconfigurable arrays where the same element states appear repeatedly.

Generalized Admittance Operators

In the DDM-based papers, local unit cells or cavities are represented by generalized admittance operators. The interior operator captures the local element or cavity response, while the exterior operator captures radiation and over-the-air coupling. For open-cavity-based arrays, the local element response is characterized separately, but the finite-array mutual coupling is retained through the exterior admittance matrix. For conformal arrays, the same principle is used after accounting for the position and orientation of each aperture.

Element States

For reflectarrays, an element state may correspond to a different geometry, cavity depth, aperture shape, or tuning parameter that controls the reflected phase. For RIS, an element state may correspond to a discrete switching or bias condition. The state-to-response mapping is implemented by selecting the appropriate precomputed aperture quantity, admittance block, or reduced-order operator from the element library.

4.4 Reduced-Order and MBF Representations

In the MBF-based papers, the fine-scale aperture-current coefficient vector is approximated using a reduced basis. If $\mathbf{V}$ denotes the full RWG coefficient vector of the aperture magnetic current, the reduced representation is

$$\mathbf{V} \approx \mathbf{V}_{\text{MBF}} \mathbf{w}, \quad (4.2)$$

where $\mathbf{V}_{\text{MBF}}$ contains the retained MBFs and $\mathbf{w}$ contains the reduced coefficients. The MBF comparison paper evaluates CMA-, CBFM-, and SVD-based bases under the same geometry, discretization, partitioning, and solver settings so that differences in current reconstruction, far-field accuracy, and conditioning can be attributed mainly to the basis construction.

4.5 Excitation, Radiation, and Scattering Problems

Illumination Models

The array may be excited by a horn feed, a Huygens source, a plane wave, or another prescribed source. The excitation model is stated in each paper chapter. In the hybrid and DDM formulations, the field incident on the global PEC-equivalent surface include the feed field and the field radiated by aperture magnetic currents.

Transmitting and Scattering Cases

Both transmitting and scattering problems are used. In transmitting cases, the structure is excited by a source or feed and the radiated field is evaluated. In scattering cases, the structure is illuminated by an external field and the scattered field is computed.

E- and H-Plane Pattern Comparisons

The most common pattern-level validation is a comparison of E- and H-plane radiation or scattering patterns. These plots show whether the proposed method reproduces the main beam, side-lobe region, and overall angular behavior of the reference solution. Pattern plots are therefore used as the main qualitative validation tool, while the peak-normalized pattern error defined below provides the quantitative comparison.

4.6 Reference Solvers and Validation Workflow

Reference Comparisons

The reference solution depends on the paper. The hybrid MoM–PO paper uses CST and full/sheet MoM comparisons. The open-cavity DDM paper compares transmitting and scattering cases with CST Microwave Studio and a plain MoM formulation. The conformal DDM paper compares convex and concave arrays with a full-wave EFIE-based MoM solver. The hybrid DDM–PO paper compares 8×8 cases with MoM and the larger 20×20 case with CST MLFMM. The MBF comparison paper uses the RWG solution as reference. The fast-update paper compares conventional MoM, DDM, Woodbury updates, and preconditioned GMRES.

Typical Workflow

A typical validation workflow is:

1. define the geometry, array size, element states, and excitation;
2. compute local quantities such as aperture fields, admittance operators, or MBFs;
3. assemble the array-level model, including exterior coupling or PO approximation;
4. solve for aperture currents, MBF weights, or updated currents;
5. compute the radiation or scattering pattern;
6. compare E- and H-plane patterns with the reference;
7. report the relevant pattern error, current-vector error, runtime, memory usage, number of unknowns, and conditioning when applicable.

4.7 Error Metrics

Only the quantitative error metrics used in the paper chapters are defined here. Main-lobe gain error, field-magnitude error in dB, and compression ratio are not introduced as common metrics because they are not used consistently across the included papers.

Peak-Normalized Pattern Error

Let x_i and x_i^{ref} denote sampled radiation or scattering pattern values in linear scale. The pointwise peak-normalized pattern error is

$$e_i = \frac{|x_i - x_i^{\text{ref}}|}{\max_j |x_j^{\text{ref}}|}. \quad (4.3)$$

This definition normalizes the error by the maximum value of the reference pattern. It avoids artificially large relative errors near deep nulls. Depending on the paper, e_i is evaluated along an E-plane cut, an H-plane cut, or a two-dimensional angular grid.

When a single scalar value is needed, the sampled errors may be averaged as

$$\bar{e} = \frac{1}{N_\Omega} \sum_{i=1}^{N_\Omega} e_i, \quad (4.4)$$

where N_Ω is the number of sampled angular points. Some paper chapters also report peak or median values of the same peak-normalized error over the considered angular samples.

Current-Vector Norm Error

For reduced-order MBF studies, the current reconstruction error is evaluated by comparing the reconstructed reduced-order current vector with the full-order reference vector. If $\mathbf{V}$ is the full-order aperture-current coefficient vector and $\mathbf{V}_{\text{MBF}}\mathbf{w}$ is the reconstructed reduced-order vector, the normalized current-vector error is

$$\varepsilon_{\text{rel}} = \frac{\|\mathbf{V}_{\text{MBF}}\mathbf{w} - \mathbf{V}\|_2}{\|\mathbf{V}\|_2}. \quad (4.5)$$

This metric is used to assess whether the reduced basis captures the local or array-level aperture-current distribution.

4.8 Computational Metrics

Runtime and Speed-Up

Runtime is reported for the relevant stages of each method, such as local characterization, matrix assembly, update operations, iterative solution, direct

solution, and far-field evaluation. A speed-up factor is defined as

$$S = \frac{T_{\text{baseline}}}{T_{\text{proposed}}}, \quad (4.6)$$

where T_{baseline} is the runtime of the reference workflow and T_{proposed} is the runtime of the accelerated method. For localized updates, the baseline is usually recomputation of the full solution, while the proposed time corresponds to the update-based computation.

Memory Usage and Number of Unknowns

Memory usage and number of unknowns are reported where they are relevant to scalability. For example, the hybrid DDM-PO and MBF-based formulations report reduced memory usage and reduced unknown counts compared with full-wave references. These quantities are interpreted together with the pattern and current errors, since a smaller system is useful only if the electromagnetic accuracy remains acceptable.

Conditioning

For reduced-order systems, the spectral condition number may be reported as

$$\kappa_2(\mathbf{A}) = \|\mathbf{A}\|_2 \|\mathbf{A}^{-1}\|_2 = \frac{\sigma_{\max}(\mathbf{A})}{\sigma_{\min}(\mathbf{A})}, \quad (4.7)$$

where $\sigma_{\max}$ and $\sigma_{\min}$ are the largest and smallest singular values. In the MBF comparison paper, conditioning is used to compare the numerical robustness of CMA-, CBFM-, and SVD-based reduced systems.

4.9 Reproducibility and Reporting

Each paper chapter reports the parameters needed to reproduce its results, including:

- frequency, array size, unit-cell pitch, and relevant geometric parameters;
- excitation model, feed/source location, and polarization;
- whether the case is transmitting, scattering, or both;

- element states, modulation patterns, or reconfiguration scenarios;
- conformal surface definition and local element orientations, when applicable;
- discretization details, basis functions, number of unknowns, and partitioning;
- admittance-operator, aperture-current, or MBF construction settings;
- solver tolerance, maximum iterations, and preconditioning settings;
- E- and H-plane cuts or other angular observation regions;
- runtime and hardware information when speed-up is reported.

4.10 Summary

This chapter defined the shared simulation conventions and evaluation quantities used across the thesis. The common validation approach is based on reference comparisons, E- and H-plane pattern plots, peak-normalized pattern error, current-vector norm error, runtime, speed-up, memory usage, number of unknowns, and conditioning where applicable. The next chapter gives a roadmap of the included papers and explains how the individual contributions relate to one another.

CHAPTER 5

Overview of Included Papers

5.1 Overview and Research Narrative

This thesis is written in a thesis-by-publication format. Each included paper addresses one part of the same broader problem: how to analyze finite, electrically large cavity-backed reflectarrays and related reconfigurable intelligent surfaces (RIS) accurately while reducing the computational cost of large-scale, conformal, reconfigurable, and repeated-simulation workflows.

The papers form a methodological progression. Paper A introduces a hybrid MoM–PO approach for fast array-level analysis of large all-metal reflectarrays. Paper B develops a generalized-admittance domain-decomposition framework for open-cavity-based array elements, where unit cells are characterized separately and coupled through over-the-air interactions. Paper C applies the domain-decomposition approach to conformal cavity-backed reflectarrays. Paper D combines domain decomposition, PO, aperture admittance, and characteristic-mode MBFs for finite reconfigurable reflectarrays and RIS. Paper E compares CMA-, CBFM-, and SVD-based MBF constructions under a common reduced-order framework. Paper F introduces Woodbury-based localized updates and Jacobi-preconditioned GMRES for fast repeated recom-

putation and electromagnetic dataset generation.

Together, the papers move from fast array-level approximation, to modular finite-array domain decomposition, to conformal modeling, to reconfigurable-array acceleration, to reduced-basis comparison, and finally to fast localized recomputation for data-driven design.

Common background, notation, numerical concepts, and validation metrics are introduced in Chapters 2–4. The paper chapters then present the complete formulations, numerical results, and paper-specific bibliographies.

5.2 List of Included Papers

The thesis includes the following papers:

1. **Paper A: Hybrid MoM–PO Modeling for Fast Analysis of Large Reflectarrays**

This paper introduces a hybrid technique in which an all-metal reflectarray is represented by a PEC-equivalent sheet excited by impressed aperture magnetic currents. The aperture currents are obtained from isolated unit-cell MoM analysis, while the induced current on the large sheet is approximated using PO. The method reduces array-level cost while retaining detailed local unit-cell information.

2. **Paper B: Domain Decomposition Analysis of Open-Cavity-Based Array Elements — Roadmap Toward Analyzing Large-Scale Reflectarrays and Reconfigurable Intelligent Surfaces**

This paper develops a DDM-based generalized-admittance framework for finite reflectarrays and RIS. A single open-cavity unit cell can be characterized separately from the embedded finite array, while mutual coupling between cells is represented through over-the-air interactions. The method is validated for both transmitting and scattering problems against CST Microwave Studio and conventional MoM results. The author’s contributions to Paper B consisted of testing the numerical implementation, contributing to the theoretical development through discussions and literature review, and helping to position the method with respect to existing DDM and generalized-admittance approaches.

3. **Paper C: Modeling of Conformal Reflectarrays Through Domain Decomposition Using a Generalized Admittance Approach**

This paper extends the generalized-admittance DDM framework to conformal cavity-backed reflectarrays. The interior cavity region and exterior radiation region are separated through aperture quantities and coupled using an aperture-admittance formulation. The method is validated on concave and convex faceted reflectarray geometries.

4. **Paper D: Hybrid Domain-Decomposition/Physical-Optics Analysis of Finite Reconfigurable Reflectarrays Using Characteristic-Mode Macro Basis Functions**

This paper combines DDM and PO for finite large-scale reconfigurable reflectarrays and RIS. The exterior response is approximated using PO, while the interior element response is captured by pre-characterized admittance operators. Characteristic-mode MBFs reduce the aperture-current unknowns and enable efficient multi-state analysis by reusing element operators and updating only changed elements.

5. **Paper E: Comparison of Macro-Basis-Function Construction Methods for Reduced-Order Analysis of Cavity-Backed Reflectarrays**

This paper compares three MBF construction methods within the same reduced-order MoM framework: CMA, CBFM with primary and enriched responses, and an SVD-based basis. The comparison isolates how the basis construction affects current reconstruction, far-field accuracy, reduced-system conditioning, and required number of retained basis functions.

6. **Paper F: Fast MoM-Based Solvers for Cavity-Backed Reflectarrays and RIS Design**

This paper introduces a fast data-generation framework based on DDM, Woodbury identity updates, and Jacobi-preconditioned GMRES. The method accelerates recomputation when only a local subset of elements changes state and supports large-scale electromagnetic dataset generation for learning-based reflectarray and RIS design.

5.3 Connection Between the Papers

The papers share a common modeling philosophy. Local full-wave accuracy is retained through MoM-based unit-cell, cavity, or aperture operators. These

local models are connected to the finite-array environment through aperture equivalence and generalized admittance. When the exterior problem becomes electrically large, PO is introduced to avoid a full dense exterior solve. When the aperture-current basis becomes too large, MBFs reduce the number of unknowns. When many related configurations must be solved, Woodbury updates and preconditioned GMRES reduce the recomputation cost.

The role of each paper can be summarized as follows. Paper A establishes the first hybrid MoM–PO route and identifies isolated-cell aperture-current extraction as an efficient but approximate strategy. Paper B addresses finite-array coupling more directly through a generalized-admittance DDM formulation. Paper C shows that the same DDM concept can handle conformal cavity-backed arrays. Paper D combines DDM with PO and MBFs to target larger and reconfigurable arrays. Paper E studies which MBF construction is most suitable for reduced-order reflectarray analysis. Paper F focuses on repeated simulations and localized element changes, which are central for dataset generation and optimization.

5.4 Contribution Map

Table 5.1 summarizes the role of each included paper in the overall thesis.

5.5 Reading Guide

Readers interested in fast analysis of large planar all-metal reflectarrays may start with Paper A. Readers interested in open-cavity domain decomposition and finite-array coupling may start with Paper B. For conformal cavity-backed reflectarrays, Paper C is the most relevant entry point. Readers focused on reconfigurable arrays and RIS should continue with Paper D. Readers interested in reduced-order modeling and MBF construction should read Paper E. Finally, readers interested in localized updates, repeated recomputation, and dataset generation should read Paper F.

For the full methodological progression, the recommended reading order is

Paper A $\rightarrow$ Paper B $\rightarrow$ Paper C $\rightarrow$ Paper D $\rightarrow$ Paper E $\rightarrow$ Paper F.

Paper	Main method	Main contribution	Role in thesis
Paper A	Hybrid MoM-PO	Uses MoM-derived aperture magnetic currents and PO to accelerate array-level analysis of all-metal reflectarrays.	Introduces the first fast hybrid array-level modeling route.
Paper B	DDM with generalized admittance	Characterizes open-cavity unit cells separately while modeling finite-array mutual coupling through over-the-air admittance interactions.	Provides the modular DDM roadmap for large-scale reflectarray/RIS analysis.
Paper C	MoM-based conformal DDM	Couples interior cavity and exterior radiation domains for concave and convex conformal reflectarrays.	Extends the DDM framework to conformal cavity-backed geometries.
Paper D	Hybrid DDM-PO with CMA-MBFs	Combines aperture-admittance operators, PO, and characteristic-mode MBFs for efficient multi-state analysis.	Enables scalable analysis of finite reconfigurable arrays and RIS.
Paper E	MBF construction comparison	Compares CMA, CBFM, and SVD bases under identical benchmark conditions.	Clarifies tradeoffs between accuracy, number of retained basis functions, and conditioning.
Paper F	Woodbury updates and preconditioned GMRES	Avoids full matrix recomputation for localized element changes and accelerates repeated solves.	Supports fast electromagnetic dataset generation and optimization workflows.

Table 5.1: Contribution map of the included papers and their roles in the thesis.

5.6 Summary

This chapter provided a roadmap of the included papers and explained how they jointly contribute to fast and accurate modeling of cavity-backed reflectarrays and RIS. The papers progress from hybrid MoM–PO array-level modeling, to generalized-admittance domain decomposition, to conformal cavity-backed analysis, to hybrid DDM–PO reconfigurable-array modeling, to MBF construction comparison, and finally to Woodbury-based fast updates for repeated simulations. The subsequent paper chapters present these contributions in self-contained form.

Conclusions and Future Work

6.1 Concluding Remarks

This thesis developed fast numerical modeling techniques for finite cavity-backed reflectarrays and related reconfigurable intelligent surfaces (RIS). The central objective was to reduce the computational cost of full-wave electromagnetic analysis while preserving the accuracy needed to capture local unit-cell, aperture, and cavity physics. This objective is motivated by practical reflectarray and RIS scenarios involving electrically large apertures, finite-array coupling, conformal geometries, many possible element states, and repeated simulations for design, optimization, reconfiguration, or data generation.

The first contribution introduced a hybrid MoM–Physical Optics (PO) approach for fast scattering analysis of all-metal cavity-backed reflectarrays. The method replaces the three-dimensional reflectarray by a PEC-equivalent sheet excited by impressed aperture magnetic currents obtained from isolated unit-cell MoM analyses. The induced current on the sheet is then approximated using PO. The results show that this geometry-reduction strategy can reproduce the main radiation behavior with good accuracy while scaling more favorably than full-array MoM. The main limitation is that aperture currents

extracted in isolated cell environments do not fully capture embedded mutual coupling between apertures.

The second contribution developed a generalized-admittance domain-decomposition method for open-cavity-based array elements. In this framework, a unit cell is characterized separately from the finite embedded array, while mutual coupling between cells is represented through over-the-air interactions in the exterior domain. The method was validated for both transmitting and scattering problems against CST Microwave Studio and conventional MoM solutions. This paper provides the roadmap for combining local element solvers with fast exterior-domain methods in large-scale reflectarray and RIS analysis.

The third contribution extended the domain-decomposition-based generalized admittance approach to conformal cavity-backed reflectarrays. By separating the interior cavity domains from the exterior radiation domain and coupling them through aperture quantities, the method can handle convex and concave faceted configurations. Validation against a conventional full-wave MoM solver for a 27-element conformal reflectarray demonstrated high accuracy in both E- and H-plane pattern comparisons. The modular formulation also supports efficient modification of repeated cell types or array layouts.

The fourth contribution presented a hybrid DDM-PO framework for large-scale reconfigurable reflectarrays and RIS. The formulation couples interior and exterior domains through a generalized aperture-admittance system and uses PO to approximate the electrically large exterior response. Characteristic-mode macro basis functions (CMA-MBFs) reduce the aperture-current unknowns and allow pre-characterized element operators to be reused across many tuning states. Results for 8×8 and 20×20 arrays showed close agreement with full-wave references while substantially reducing memory usage and simulation time.

The fifth contribution provided a controlled comparison of MBF construction methods for reduced-order analysis of cavity-backed reflectarrays. CMA, CBFM, and SVD-based bases were compared using the same formulation, geometry, discretization, partitioning, and solver settings. The results show that CBFM can be attractive at low rank but may exhibit nonmonotonic solution-error behavior when enriched functions are retained. Explicit metric orthogonalization improves CMA conditioning and current reconstruction.

The SVD-based basis gives well-conditioned reduced systems but may require more retained basis functions to reach a prescribed far-field accuracy.

The sixth contribution introduced a fast modular MoM framework for electromagnetic field data generation. The method combines the DDM formulation with Woodbury identity updates and a Jacobi-type preconditioner for GMRES. Localized element changes can be handled without full matrix recomputation, and the resulting speed-ups make the method suitable for generating large, high-quality electromagnetic datasets for deep-learning and optimization workflows.

Taken together, the thesis shows that efficient reflectarray/RIS analysis benefits from combining several complementary ideas. MoM remains essential for accurately capturing local electromagnetic behavior. Aperture equivalence and generalized admittance formulations provide compact interfaces between local cavity physics and global radiation. PO enables efficient exterior-domain modeling for electrically large PEC-like surfaces. MBFs reduce the number of unknowns while preserving dominant aperture-current behavior. Woodbury updates and preconditioned iterative solvers make repeated simulations practical when only part of the array changes. The combined result is a modular and scalable modeling framework for finite planar, conformal, and reconfigurable cavity-backed reflectarrays.

6.2 Main Conclusions

The main conclusions of the thesis are:

- **Hybrid MoM–PO modeling enables fast array-level analysis.**
Replacing the full three-dimensional all-metal structure by aperture magnetic currents on a PEC-equivalent sheet allows PO to be used for the global exterior response while retaining local MoM-derived element information.
- **Generalized admittance provides a useful finite-array interface.**
The DDM formulation separates local element or cavity behavior from exterior coupling and allows separately characterized unit cells to be connected through an admittance-based system.
- **Open-cavity designs are well suited for modular modeling.**
when coupling between elements occurs primarily over the air, local

unit-cell operators can be reused while the finite-array environment is represented by the exterior admittance matrix.

- **The DDM framework extends naturally to conformal reflector arrays.** Local element orientations and curved or faceted geometries can be handled by defining aperture quantities in local frames and coupling them through the exterior domain.
- **Hybrid DDM–PO improves scalability for electrically large reconfigurable arrays.** Combining aperture-admittance operators with PO and MBF reduction enables efficient analysis of larger arrays and multiple element states.
- **MBF construction affects both accuracy and conditioning.** CMA, CBFM, and SVD bases exhibit different tradeoffs between basis size, current reconstruction, far-field accuracy, and reduced-system conditioning. Basis selection must therefore be treated as part of the numerical design.
- **Woodbury updates are effective for localized state changes.** When only a small subset of elements changes, low-rank update strategies avoid full matrix recomputation and can substantially reduce simulation time.
- **Fast high-fidelity data generation is feasible.** The proposed update and preconditioning strategies support large-scale generation of electromagnetic training data without relying only on low-fidelity surrogate simulations.

6.3 Limitations

Several limitations remain. First, the PO-based exterior models are most reliable for electrically large, smooth, PEC-like surfaces and dominant illuminated-region behavior. Strong edge diffraction, shadow-boundary effects, or array-level resonances may require more accurate exterior modeling or diffraction corrections.

Second, MBF reduction depends on the chosen basis construction and truncation strategy. Adding more basis functions does not always improve the final

solution monotonically, especially for enriched CBFM bases. Conditioning, orthogonalization, and accuracy must therefore be considered together.

Third, Woodbury-based updates are most effective when the configuration change is localized, such that the resulting matrix perturbation remains low rank. If a large fraction of the array changes simultaneously, the update rank increases and the computational advantage over a full recomputation may diminish. In such cases, a full rebuild or alternative update strategy may be more appropriate.

Finally, the thesis validation is primarily numerical and based on comparisons with full-wave solvers or full-order formulations. Experimental validation of fabricated all-metal or cavity-backed reflectarray prototypes would be an important next step.

6.4 Future Work

Adaptive MBF Selection

The MBF comparison shows that no single basis construction is universally optimal. Future work should investigate adaptive basis selection, automatic truncation criteria, and hybrid bases that combine the physical interpretability of CMA, the excitation efficiency of CBFM, and the numerical robustness of SVD-based constructions.

Broadband and Multi-Frequency Extensions

The methods in this thesis are mainly validated at selected operating frequencies. Extending the framework to broadband and multi-frequency analysis would be valuable for practical antenna and RIS design. Possible directions include frequency-parametric admittance operators, broadband MBFs, and update schemes that reuse information across frequency samples.

Extension to Transmitarrays and Transmissive RIS

A natural extension of the present framework is the analysis of transmitarrays and transmissive RIS. Unlike the PEC-backed reflectarray structures considered in most of this thesis, transmitarrays are two-sided apertures in which energy is coupled from an incident side to a transmitted side. The generalized

admittance formulation would therefore need to be extended from a one-sided reflecting aperture to a two-sided aperture or multiport representation that couples incident-side and transmitted-side fields.

In such a formulation, each unit cell could be treated as an interior domain connected to two exterior regions. The same concepts developed in this thesis—domain decomposition, aperture equivalence, operator reuse, MBF reduction, and localized matrix updates—could then be applied to scalable transmitarray modeling. However, the equivalent-source representation would need to account for both reflection and transmission, potentially requiring both electric and magnetic equivalent currents or a Huygens-type aperture formulation. This extension would broaden the framework from reflecting surfaces to general wavefront-transforming metasurfaces.

Learning-Based Optimization and Dataset Generation

The fast data-generation framework can be integrated with adaptive deep-learning models for beamforming, array thinning, sparse array optimization, and inverse design of passive cavity-backed reflectarrays and RIS. The proposed solvers can provide high-quality training data on demand, enabling active-learning or uncertainty-aware optimization strategies.

Extension to Cavity-Backed Phased Arrays

The domain-decomposition, update-based, and preconditioned iterative frameworks can also be extended to cavity-backed phased array configurations. This would test the generality of the proposed methods beyond passive reflectarrays and RIS.

Experimental Validation

Experimental validation remains an important future step. Fabricated prototypes would allow comparison between simulated and measured reflection phase, radiation pattern, gain, and reconfigurable-state behavior. Measurements would also quantify effects not fully included in the present models, such as losses, surface roughness, fabrication tolerances, assembly imperfections, and connector or bias-network effects.

6.5 Final Remarks

This thesis demonstrates that fast and accurate analysis of finite cavity-backed reflectarrays and RIS can be achieved by combining physically motivated electromagnetic modeling with targeted computational acceleration. The proposed methods retain full-wave accuracy at the local element and cavity level while reducing the cost of global array analysis and repeated recomputation. Aperture equivalence, domain decomposition, generalized admittance, PO-based exterior modeling, MBF reduction, Woodbury updates, and preconditioned iterative solvers together provide a flexible foundation for scalable reflectarray/RIS simulation, optimization, and data-driven electromagnetic design. The same modular principles also provide a promising path toward future analysis of transmitarrays, transmissive RIS, and other advanced wavefront-transforming aperture systems.

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Part II

Papers

**Fast Scattering Analysis of Large Open-Cavity Based Reflectarrays
Using a Hybrid Method of Moment and Physical Optics Approach**

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The layout has been revised.

Abstract

This paper introduces a hybrid modeling approach for the fast analysis of large reflectarrays. The scattering from a 3-D all-metal reflectarray is modeled by a Perfect Electric Conductor (PEC) sheet with impressed aperture magnetic current sources. These currents are obtained by analyzing the aperture fields of isolated unit cells through the method of moments. The induced electric current on the large PEC sheet is then computed through the Physical Optics (PO) approach, thereby reducing the computational complexity from $\mathcal{O}(N^3)$ for classical MoM to $\mathcal{O}(N)$ for PO. The results show, compared to CST, that the total scattered field yields an error of max. 0.32 dB in the main lobe direction for a 5×5 reflectarray with peak gain of 21.8 dB. Furthermore, the relative error related to the PO approach is less than 0.13 dBi, which confirms the validity of the hybrid modeling approach.

1 Introduction

Electromagnetic surfaces, such as reflectarrays, are typically used for (multi)beam-forming and side-lobe-level control due to their ability to manipulate electromagnetic waves [1–3]. Initial designs often rely on the use of periodic boundary conditions implying infinitely large arrays under plane wave incidence. While this approach speeds up simulations, it fails to accurately model mutual coupling, edge effects and a non-planar phase front, leading to inaccuracies in the scattered field pattern.

Reflectarrays are often very large, making them difficult to solve directly due to their increased complexity. In order to reduce the problem size, domain decomposition methods (DDMs) are often proposed that are based on surface equivalence principles [4, 5]. Many Method-of-Moment (MoM) based DDMs have been proposed employing electric field integral equations (EFIEs) and magnetic field integral equations [6–9]. Although these methods can solve large problems and have produced accurate results, MoM methodologies typically require Gaussian elimination, which renders it an $\mathcal{O}(N^3)$ complexity problem [10].

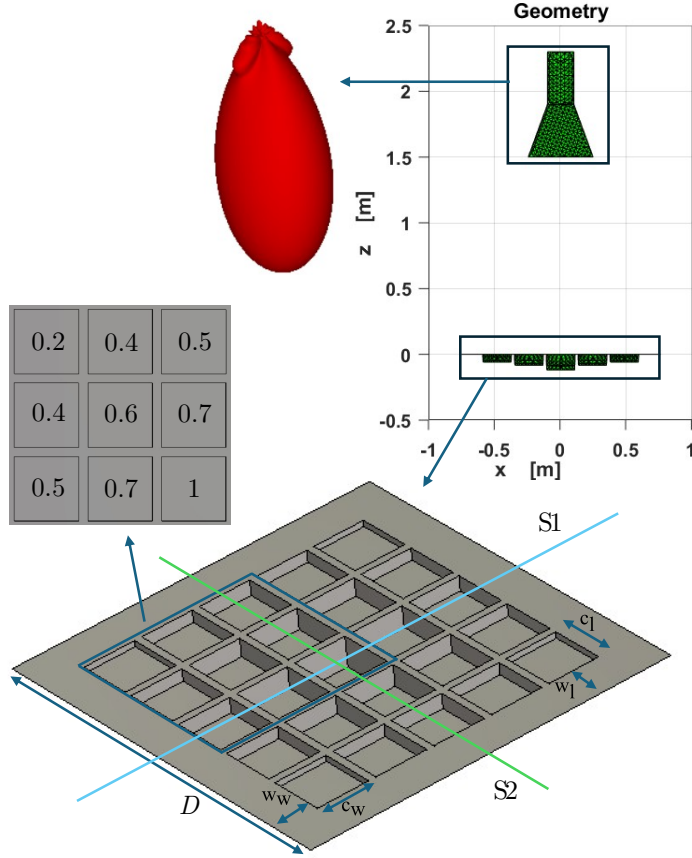


Figure 1: The CST geometric model (bottom) used to validate the proposed model (top-right). The dimensions are $D = 5C_l + 2W_l + 0.4\lambda$, $C_l = C_w = \lambda/\sqrt{2}$, $W_l = W_w = 0.55\lambda$ and the height for the centre element is 0.397λ . All other elements are modulated in height with quadrant symmetry by symmetry lines S1 and S2, and the subarray shows the relative height modulation coefficient, where the cell height is relative modulation $\times 0.397\lambda$.

This paper proposes to apply Schelkunoff's equivalence for modeling cavity array scattering problems, such as reflectarrays with waveguide cells [11–

13]. We propose to characterize the unit cell in isolation before modeling the behaviour of the array. Unlike an infinite periodic array methodology, which is limited to uniformly spaced array elements, our approach can model non-uniformly distributed and unique reflectarray elements. Additionally, our method differs from the aforementioned MoM-based methodologies by using Schelkunoff's equivalence combined with zero back current leading to a geometry dimension reduction. The so-obtained PEC sheet will prevent internal resonance issues, which are known to occur when only EFIE is used for closed cavities [14, p. 29]. Additionally, using the *physical equivalent* [15, p. 337], the PEC body is removed and replaced by an electric source current representing the induced PEC current. This current can be approximated by the Physical Optics (PO) current to solve for the fields on the front side due to the feed illumination and aperture radiation [16]. PO eliminates the need to calculate matrix inverses, reducing the problem complexity from $\mathcal{O}(N^3)$ to $\mathcal{O}(N)$.

The remainder of this article is organized as follows: In Sec. II, the general methodology is explained, including the setup of the EFIE for single cell analysis, geometry dimension reduction, and Physical Optics boundary conditions. In Sec. III, the validation results are discussed from our in-house MoM solver against those obtained from CST's integral equation (IE) solver. Finally, Sec. IV concludes and provides recommendations for future work.

2 Methodology

Fig. 2a shows a simplified visualization of a cavity based reflectarray with $\mathbf{E}^i$ the electric field from an external source (feed) incident on the reflectarray and $\mathbf{E}_n^s$ is the scattered field from only the n th array element (see blue dashed box). This isolated element approach significantly reduces the problem size for the EFIE. This fast analysis is repeated for all unit cells, which is explained in detail later. The final goal of the methodology is to represent the original array in Fig. 2a using Schelkunoff's equivalence principle followed up by a geometry dimension reduction step and the application of the *physical equivalent* theorem to arrive at the array model as shown in Fig. 2b.

The scattered field $\mathbf{E}^s(\mathbf{J}, \mathbf{M})$ in the final configuration in Fig. 2b is due to the total impressed magnetic current $\mathbf{M}$, which is the array of magnetic currents representing the aperture fields, and the electric (source) current $\mathbf{J}$ representing the induced current on the PEC in the presence of $\mathbf{M}$ and the

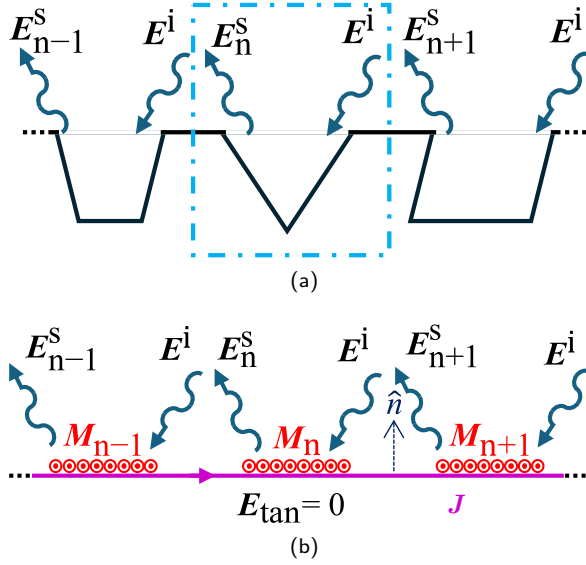


Figure 2: 2a Original reflectarray configuration. 2b Final configuration with set of aperture magnetic currents and induced electric current on a sheet with imposed PEC boundary condition (all sources radiate in free space).

feed. The scattered field is given by [17]¹

$$\mathbf{E}^s(\mathbf{J}, \mathbf{M}) = \mathcal{L}(\mathbf{J}) - \mathcal{K}(\mathbf{M}) + \frac{\hat{\mathbf{n}} \times \mathbf{M}}{2} \quad (\text{A.1a})$$

$$\mathbf{H}^s(\mathbf{J}, \mathbf{M}) = \frac{1}{\eta_0^2} \mathcal{L}(\mathbf{M}) + \mathcal{K}(\mathbf{J}) - \frac{\hat{\mathbf{n}} \times \mathbf{J}}{2} \quad (\text{A.1b})$$

where the operators

$$\mathcal{L}(\mathbf{X}) = -j\omega\mu_0 \text{ p.v.} \iint_s \mathbf{X}(\mathbf{r}') G(\mathbf{r} - \mathbf{r}') dS' + \frac{1}{j\omega\epsilon_0} \nabla \text{ p.v.} \iint_s \nabla' \cdot \mathbf{X}(\mathbf{r}') G(\mathbf{r} - \mathbf{r}') dS' \quad (\text{A.2a})$$

$$\mathcal{K}(\mathbf{X}) = \text{p.v.} \iint_s \mathbf{X}(\mathbf{r}') \times \nabla' G(\mathbf{r} - \mathbf{r}') dS' \quad (\text{A.2b})$$

¹Eq. (2) in [17] contains a minus sign error in front of the normal vector.

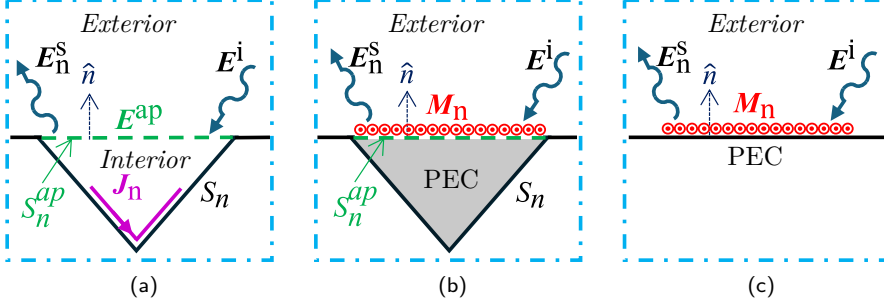


Figure 3: 3a Original unit cell extracted from 2a with E^{ap} sampled at aperture S_n . 3b Equivalence principle converting E^{ap} at the aperture to a magnetic current source M_n for a single cell by closing the cavity with PEC. 3c After removal of the back geometry, effectively flattening the structure, M_n represents the same cell scattering behavior on an infinitely thin PEC sheet

and where $G = e^{-jk_0 R}/(4\pi R)$. Furthermore, $R = |\mathbf{r} - \mathbf{r}'|$ and the wavenumber $k_0 = \omega\sqrt{\mu_0\epsilon_0}$. The operators integrate over the support of the surface current.

To compute the induced electric current $\mathbf{J}$ on the PEC sheet, the PEC boundary condition $\hat{\mathbf{n}} \times \mathbf{E} = \mathbf{0}$ is imposed for which, using the superposition principle, $\mathbf{E}_{tan} = \mathbf{E}_{tan}^i + \mathbf{E}_{tan}^s(\mathbf{J}) + \mathbf{E}_{tan}^s(\mathbf{M})$ results in

$$-\hat{\mathbf{n}} \times \mathbf{E}_{tan}^s(\mathbf{J}) = \hat{\mathbf{n}} \times \mathbf{E}_{tan}^i + \hat{\mathbf{n}} \times \mathbf{E}_{tan}^s(\mathbf{M}) \quad (\text{A.3})$$

This is an equation for the electric field where $\mathbf{J}$ is the unknown current, while the right-hand side is known.

, the EFIE can then be formulated as follows in matrix form:

$$[\mathbf{L}] [\mathbf{J}] = [-\mathbf{E}_{tan}^i] + [\mathbf{K}] [\mathbf{M}] - \frac{\hat{\mathbf{n}} \times \mathbf{M}}{2} \quad (\text{A.4})$$

where matrices $\mathbf{L}$ and $\mathbf{K}$ are obtained by discretizing $\mathcal{L}$ and $\mathcal{K}$ in (A.1).

2.1 Isolated Cell Analysis

In an isolated cell (blue dashed box), see Fig. 3a, only the geometry of a single cell is considered. To compute the induced electric current $\mathbf{J}_n$ on the

interior region of the n th cell, the PEC boundary condition $\mathbf{E}_{\text{tan},n} = \mathbf{E}_{\text{tan}}^i + \mathbf{E}_{\text{tan},n}^s(\mathbf{J}_n) = \mathbf{0}$ is imposed, i.e.,

$$-\mathbf{E}_{\text{tan},n}^s(\mathbf{J}_n) = \mathbf{E}_{\text{tan}}^i \quad \mathbf{r} \in S_n \quad (\text{A.5})$$

Next, $\mathbf{J}_n$ is expanded into $P^{[n]}$ RWG basis functions $\{\mathbf{f}_p^{[n]}\}_{p=1}^{P^{[n]}}$ [18], i.e.,

$$\mathbf{J}_n(\mathbf{r}) = \sum_{p=1}^{P^{[n]}} I_p^{[n]} \mathbf{f}_p^{[n]}(\mathbf{r}) \quad (\text{A.6})$$

where $\{I_p^{[n]}\}$ is the associated set of expansion coefficients. Following Galerkin's moment method, the EFIE in (A.5) is tested $P^{[n]}$ times through the symmetric product $\langle \mathbf{a}, \mathbf{b} \rangle = \iint \mathbf{a} \cdot \mathbf{b} \, dS$ and (A.5) results in the matrix equation

$$\mathbf{Z}^{[n]} \mathbf{I}^{[n]} = \mathbf{V}^{[n]} \quad (\text{A.7})$$

where

$$Z_{pq}^{[n]} = -\langle \mathbf{f}_p^{[n]}, \mathcal{L}(\mathbf{f}_q^{[n]}) \rangle \quad (\text{A.8a})$$

$$V_p^{[n]} = \langle \mathbf{E}^i, \mathbf{f}_p^{[n]} \rangle \quad (\text{A.8b})$$

for $q = 1, 2, \dots, P^{[n]}$. After obtaining the coefficient vector $\mathbf{I}^{[n]}$ by solving (A.7), one can compute the electric field on the aperture $\mathbf{E}^{\text{ap},[n]} = \mathbf{E}^s(\mathbf{J}_n) + \mathbf{E}^i$, where $\mathbf{J}_n$ is known and expanded in terms of RWGs as in (A.6). Using Schelkunoff's equivalence, as seen in Fig. 3b, the magnetic current on the n th aperture is then determined by

$$\mathbf{M}_n = \begin{cases} \mathbf{E}^{\text{ap},[n]} \times \hat{\mathbf{n}} & \mathbf{r} \in S_n^{\text{ap}} \\ \mathbf{0} & \text{otherwise} \end{cases} \quad (\text{A.9})$$

This nullifies the field in the interior region since it is filled with PEC material. Moreover, under the assumption of zero current on the backside of the geometry, one can reduce the geometry to a thin PEC sheet. This allows each cell to be represented as a PEC sheet with impressed $\mathbf{M}_n$ as seen in Fig 3c.

2.2 Array Synthesis

After obtaining the equivalent model representation of the unit cells, Fig. 3c, the cells can be aggregated as seen in Fig 2b. To calculate $\mathbf{J}$ on the PEC sheet,

or array, the EFIE can be formulated using the PEC boundary condition $\mathbf{E}_{\text{tan}} = \mathbf{E}_{\text{tan}}^{\text{i}} + \mathbf{E}_{\text{tan}}^{\text{s}}(\mathbf{M}) + \mathbf{E}_{\text{tan}}^{\text{s}}(\mathbf{J}) = \mathbf{0}$. Substituting (A.1) in this boundary condition results in

$$-\underbrace{\mathcal{L}(\mathbf{J})|_{\text{tan}}}_{\mathbf{E}_{\text{tan}}^{\text{s}}(\mathbf{J})} = \mathbf{E}_{\text{tan}}^{\text{i}} - \underbrace{\mathcal{K}(\mathbf{M})|_{\text{tan}} + \frac{\hat{\mathbf{n}} \times \mathbf{M}}{2}}_{\mathbf{E}_{\text{tan}}^{\text{s}}(\mathbf{M})} \quad (\text{A.10})$$

After expanding the entire array/sheet current $\mathbf{J}$ in P RWG basis functions, and testing the EFIE using Galerkin's approach, one obtains

$$\mathbf{Z}\mathbf{I} = \mathbf{V} \quad (\text{A.11})$$

where

$$Z_{pq} = -\langle \mathbf{f}_p, \mathcal{L}(\mathbf{f}_q) \rangle \quad (\text{A.12a})$$

$$V_p = \langle \mathbf{E}^{\text{i}} + \mathbf{E}^{\text{s}}(\mathbf{M}), \mathbf{f}_p \rangle \quad (\text{A.12b})$$

for $q = 1, 2, \dots, P$, and $\mathbf{M}$ is known following Eq. (A.9).

Alternatively, $\mathbf{J}$ can be found faster and more directly using the Physical Optics method, i.e.,

$$\mathbf{J} = 2\hat{\mathbf{n}} \times (\mathbf{H}^{\text{i}} + \mathbf{H}^{\text{s}}(\mathbf{M})) \quad (\text{A.13})$$

where the error incurred by the PO approximation, which is largest at the array edge, is limited by an upper bound error in directivity [19]:

$$\Delta G \approx 0.22 \left(\frac{\lambda}{D} \right)^2 \text{ [dB]} \quad (\text{A.14})$$

where D represents the smallest diameter of the array. This error diminishes if the reflectarray is significantly larger than the free space wavelength λ . After determining $\mathbf{J}$, the overall radiation behaviour of the reflectarray system can be expressed through the total field

$$\mathbf{E} = \mathbf{E}^{\text{i}} + \mathbf{E}^{\text{s}}(\mathbf{M}) + \mathbf{E}^{\text{s}}(\mathbf{J}) \quad (\text{A.15})$$

3 Results

The proposed methodology is evaluated on a full metal 5×5 waveguide-based reflectarray with a horn feed as shown in Fig. 1. To demonstrate the

methodology's capability to handle various cells, waveguide cells with varying heights are used. For simplicity, the coupling between the horn and the array, as well as blockage effects, were neglected. Reference validation was performed using a full MoM solver (=EFIE for the full geometric model), treating the horn as a far-field source. Additionally, CST results were included to confirm the validity of the full MoM solver.

The results are provided for MoM (sheet model), PO (sheet model) and two references; CST (full model), MoM (full model). Fig. 4a illustrates the magnetic currents on the apertures, where one can see the non-uniform illumination of the aperture due to the feed. Fig. 4b displays the induced current on the PEC sheet, calculated using the MoM (sheet model). The PO calculated current is shown in Fig. 4c. The currents in Fig. 4b and 4c match well, except around the edges where PO modeling is less accurate, as expected. This similarity is also observed in the radiation patterns in the E- and H-planes in Fig. 5a and 5b. The scattered field from a 5×5 reflectarray as computed by full MoM approach differs only by 0.043 dB in peak gain (21.1 dBi) w.r.t. CST's integral equation solver. The MoM sheet model increases the error to max. 0.32 dB. Finally, the physical-optics approximation is observed to not increase this error (< 0.129 dB), which shows that the PO approximation yields accurate results once magnetic aperture currents can be accurately determined including coupling effects. Finally, Fig. 6 compares the computation times for the full-wave MoM, reduced model, and PO, indicating that the PO method scales more favorable with increasing number of cells.

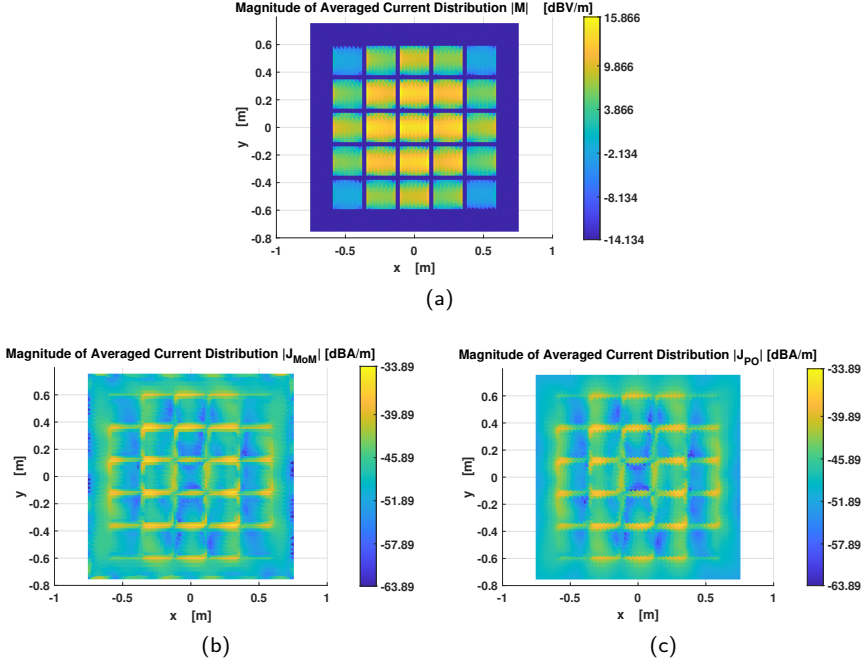
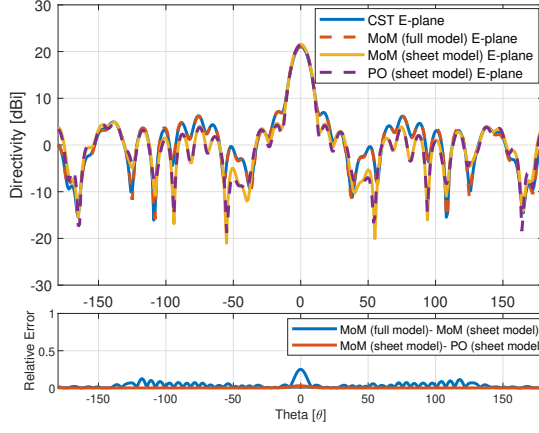


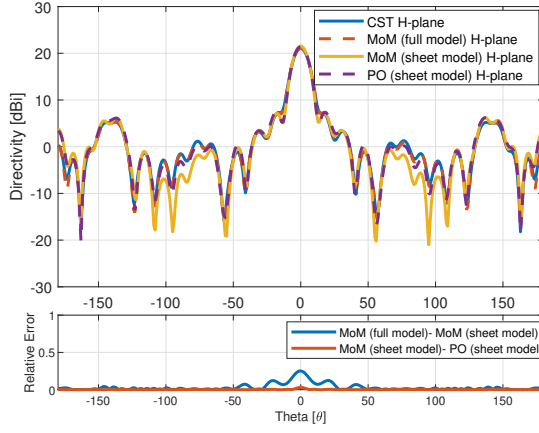
Figure 4: 4a The aperture magnetic current obtained by sampling single cells. 4b The induced current on the PEC ground plane due to incident fields from the feed and the apertures calculated through MoM. 4c The induced current on the PEC ground plane due to incident fields from the feed and the apertures calculated through PO.

4 Conclusion

The results have demonstrated that the proposed hybrid PO-MoM approach scales more favorable with the increasing complexity of the problem while maintaining good accuracy. Additionally, the geometry reduction to a sheet has little effect on the radiation pattern, as evidenced by the well-matched pattern in the backside direction of the array. This suggests that the discrepancy is directly related to the aperture currents obtained through isolated cell analyses. Overall, the results confirm the validity of using a geometry reduction strategy combined with PO and the MoM for fast calculation of cavity-based reflectarrays.



(a)



(b)

Figure 5: 5a E-plane and 5b H-plane radiation pattern of the array with their respective relative error per angle of θ in linear scale.

The present method does not account for full mutual coupling effects between apertures, due to determination of magnetic aperture currents in iso-

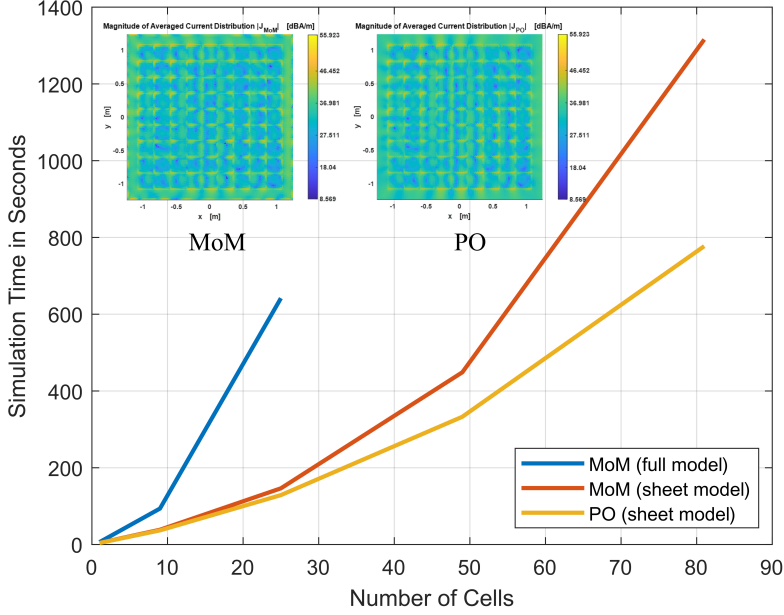


Figure 6: Time benchmark comparing the simulation time using the full model MoM, the sheet MoM and sheet PO.

lated environment. This issue will be addressed in the future using a generalized admittance matrix approach combined with domain decomposition.

Acknowledgment

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**Domain Decomposition Analysis of Open-cavity-based Array
Elements — Roadmap Toward Analyzing Large-Scale
Reflectarrays and Reconfigurable Intelligent Surfaces**

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The layout has been revised.

Abstract

This paper describes a numerical modeling framework toward analyzing large-scale finite-sized reflectarrays and reconfigurable intelligent surfaces. A domain decomposition method (DDM) is proposed, where a single reflecting unit cell can be characterized in isolation from its embedded structure. The mutual coupling between unit cells is occurring only over-the-air and is modeled through a generalized admittance matrix approach. Both the radiated and scattered fields, which are computed for a transmitting and scattering problem, respectively, are in excellent agreement with those computed by CST Microwave Studio and a plain MoM approach. Recommendations and future directions are provided as part of the herein presented roadmap, among which the hybridization with fast methods and high-frequency techniques capable of handling the large-scale exterior domain, and the use of a finite difference/element method for modeling the interior domain of complex unit cells.

1 Introduction

The research on reconfigurable intelligent surfaces (RIS) has grown tremendously in recent years due to their transformative potential in applications such as 5G and beyond wireless networks, radar systems, and electromagnetic wave control. These technologies promise enhanced performance, energy efficiency, and spatial reconfigurability [1].

The analysis of large-scale arrays typically assume periodic boundary conditions (PBC), by treating the array cells as part of an infinitely large periodic structure [2, 3]. An interesting example is the macroscale modeling of such arrays using ray-based approaches [4]. However, these approximations become inaccurate when the coupling between cells depends heavily on finite array effects or when the surface reflectivity becomes aperiodic.

To address these challenges, an efficient numerical approach that can accurately model mutual coupling and multiscale array effects is essential. Numerous studies have proposed frameworks that combine different techniques

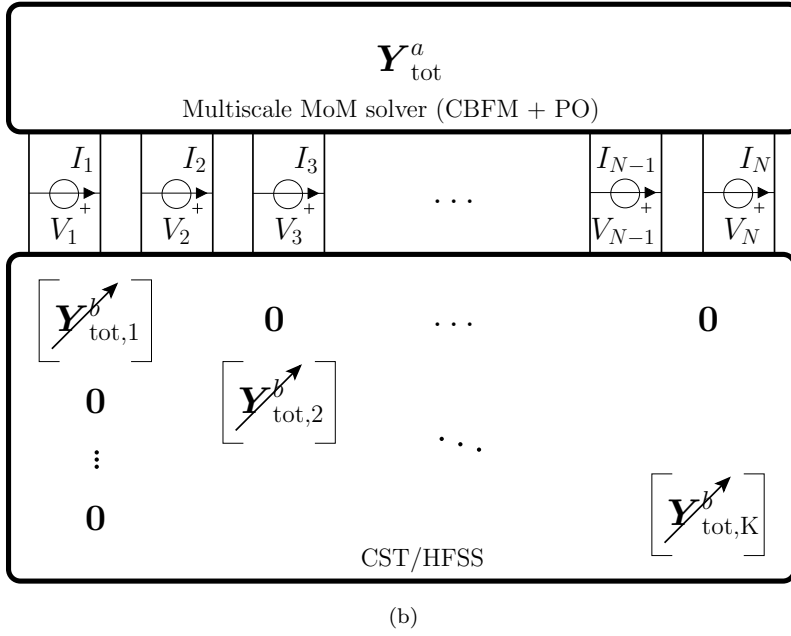
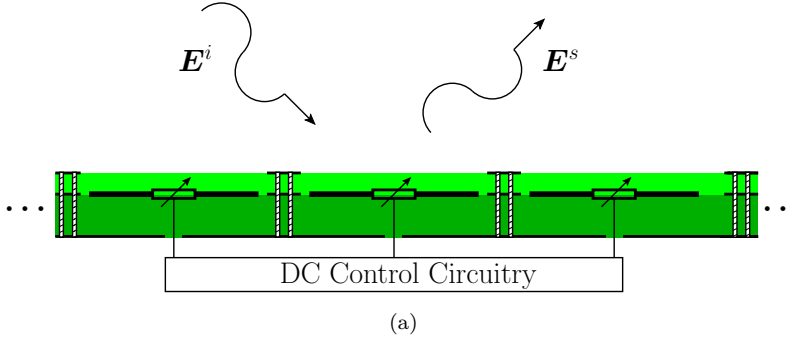


Figure 1: (a) Printed circuit board employing via-fences to realize open-cavity-based reflectarray elements with backside control circuitry, and; (b) Generalized admittance matrix equivalent network representation for the multi-solver multi-mode domain decomposition method. A total of N aperture modes are distributed over K reflectarray cells.

to handle these complexities[5–9]. The present paper contributes to this body of work by introducing a hybrid modeling framework, which synergistically integrates various methods to efficiently address these problems.

The baseline reflectarray geometry in this study is shown in Fig. 1(a), consisting of a (multilayer) printed circuit board (PCB). The PCB employs vias to create open-ended cavities that house the reconfigurable reflectarray elements. This way, the mutual coupling between reflectarray elements occurs only through over-the-air coupling, not via the substrate. This slight limitation in design flexibility significantly reduces the computational complexity of the problem, as we now can model the cells as metal cavities with apertures, distinguishing our approach from methods that model substrate-based coupling through open boundaries [5, 6].

Furthermore, each reflectarray cell can now be modeled independently from their surrounding environment through the use of surface equivalence principles. In fact, a null-field is assumed in the complementary/exterior domain. This leads to both electric and magnetic surface currents at the domain interface. The perfect electric conductor (PEC) boundary condition is imposed on this fictitious surface. Equivalent magnetic current only reside on apertures, not on metal boundaries; aperture field modes are modeled by corresponding magnetic currents, which are then used as a sources to construct a generalized admittance matrix for that domain. This is repeated for each domain after which the K cells and outer region domain are interconnected by interconnecting multiport admittance matrices in a circuit-like manner as shown in Fig. 1(b). The voltage at each port is the amplitude of the corresponding magnetic current modeling an aperture field mode. Fig. 1(b) shows N aperture magnetic currents/modes in total, thus N/K aperture modes per cell, provided that equal cells are used. Each aperture mode can be represented by an entire-domain basis function for modeling the respective magnetic current. This will drastically reduce the number of current modes relative to employing subsectional basis functions and suits the Characteristic Basis Function Method (CBFM) well [7, 8, 10, 11].

Our method belongs to a class of so-called domain decomposition methods (DDMs). In most of the DDMs, such as the generalized transition matrix (GTM) approach [9], Love’s equivalence principle is employed. It is often favored because it leads to well-conditioned numerical systems and integrates well with hybrid computational techniques. In our case, for the exterior equiv-

alence, the entire array structure can be modeled as a PEC structure. Moreover, for large arrays the backside current is small, so that the PEC array of closed cavities can then be reduced to a PEC sheet. We can subsequently use high-frequency techniques, such as the Physical Optics (PO) approximation, to find the induced PEC current. In turn, the admittance matrix of each interior cavity domain can be characterized by a commercial solver employing a finite difference/element technique. Furthermore, by reducing the array of volumetric PEC cavities to a single sheet, it is believed that the relatively ill-conditioning of the proposed method, e.g. due to the interior resonance problem, will then be mitigated. The solver hybridization and geometrical dimension reduction as described above is subject of future study, but it is still mentioned here to motivate our currently preferred choice of equivalence principle. Also, the DDM as proposed in this paper will be at the heart of the RIS modeling framework and will be used to validate the integration of different solvers in the future. In terms of the hybridization of different solvers in a DDM context, the reader is kindly referred to [6].

Finally, it is worth pointing out that the above-proposed DDM framework constitutes a valuable tool for optimizing the global response of active metasurfaces [12, 13]. It allows for the optimization of array configurations by leveraging a database of pre-characterized reconfigurable unit cells. Machine Learning techniques combined with optimization algorithms are then employed to optimize the distribution of these cells [14].

This paper is organized as follows. Sec. 2 details the mathematical formulation of the herein proposed DDM. The numerical results validating the methodology are described in Sec. 3. The paper ends by providing the conclusions in Sec. 4.

2 Formulation

The present method is based on the generalized aperture admittance matrix approach as described in [15], albeit without the need to introduce a correction current.

In the original problem, in Fig. 2(a), the tangential components of the E- and H-fields are continuous *across the aperture* surface S_{ap} , i.e.,

$$\mathbf{E}^a|_{\text{tan}} = \mathbf{E}^b|_{\text{tan}} \quad \text{and} \quad \mathbf{H}^a|_{\text{tan}} = \mathbf{H}^b|_{\text{tan}}, \quad (\text{B.1})$$

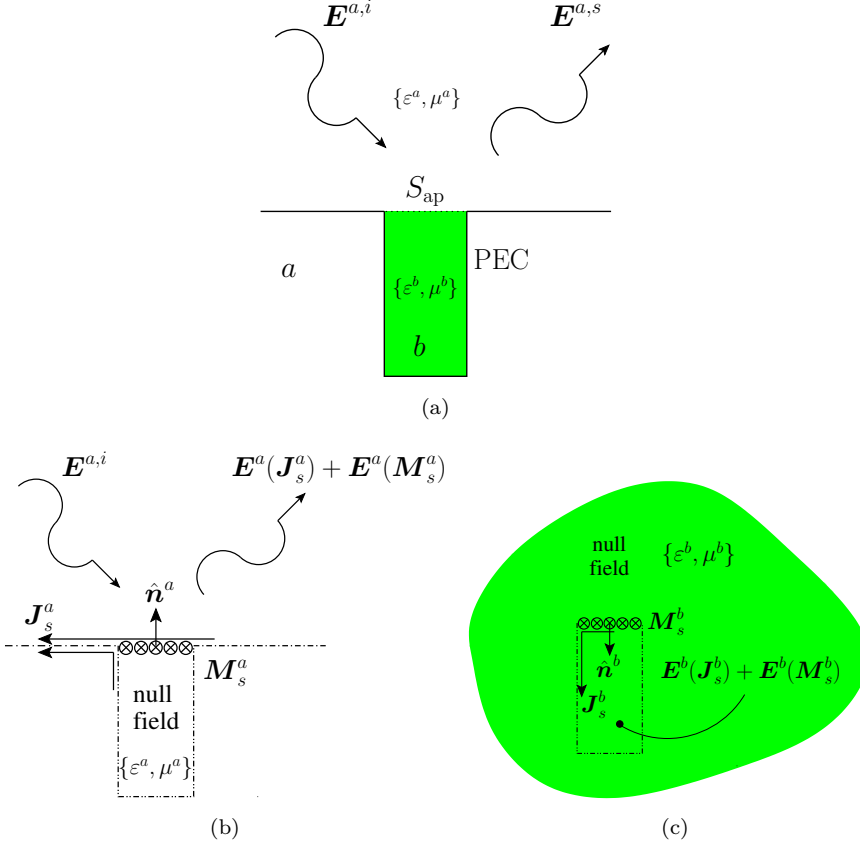


Figure 2: Scattering problem: (a) Original configuration, split into: (b) exterior equivalent fields in region a ; (c) interior equivalent fields in region b .

where a denotes the exterior region and b the interior waveguide region. For the exterior problem, in Fig. 2(b), we assume the original fields in region a and zero fields in region b . The jump in $\mathbf{E}^a|_{\text{tan}} = -\hat{\mathbf{n}}^a \times \hat{\mathbf{n}}^a \times \mathbf{E}^a$ must now be accompanied by an aperture surface magnetic current through the boundary condition $\mathbf{E}^a|_{\text{tan}} = \hat{\mathbf{n}}^a \times \mathbf{M}_s^a$. The jump in $\mathbf{H}^a|_{\text{tan}}$ leads to a surface electric current in the aperture. Furthermore, the PEC surface is removed and replaced by an electric source current $\mathbf{J}_s^a$ representing the induced current

across the exterior PEC surface as indicated in Fig. 2(b). The null-field interior region can now be filled with the exterior medium material a , so that all currents radiate in a homogeneous medium. The total exterior field in region a is therefore

$$\mathbf{E}^a = \mathbf{E}^{a,i} + \mathbf{E}^a(\mathbf{M}_s^a) + \mathbf{E}^a(\mathbf{J}_s^a) \quad (\text{B.2a})$$

$$\mathbf{H}^a = \mathbf{H}^{a,i} + \mathbf{H}^a(\mathbf{M}_s^a) + \mathbf{H}^a(\mathbf{J}_s^a). \quad (\text{B.2b})$$

Similarly, equivalence principles can be applied to obtain the total interior fields in region b , as also shown in Fig. 2(c), i.e.,

$$\mathbf{E}^b = \mathbf{E}^{b,i} + \mathbf{E}^b(\mathbf{M}_s^b) + \mathbf{E}^b(\mathbf{J}_s^b) \quad (\text{B.3a})$$

$$\mathbf{H}^b = \mathbf{H}^{b,i} + \mathbf{H}^b(\mathbf{M}_s^b) + \mathbf{H}^b(\mathbf{J}_s^b), \quad (\text{B.3b})$$

where the interior current $\mathbf{J}_s^b$ is different from the exterior current $\mathbf{J}_s^a$. However, because of Eq. (B.1), where

$$\mathbf{E}^{a,b}|_{\text{tan}} = \hat{\mathbf{n}}^{a,b} \times \mathbf{M}_s^{a,b} \quad (\text{B.4})$$

as well as that $\hat{\mathbf{n}}^a = -\hat{\mathbf{n}}^b$ in the aperture, results in the single magnetic current

$$\mathbf{M}_s^a = -\mathbf{M}_s^b = \mathbf{M}_s. \quad (\text{B.5})$$

An equation for $\mathbf{M}_s$ can be found by substituting (B.5) in both (B.3b) and (B.2b), and then in (B.1), yielding

$$\begin{aligned} & [\mathbf{H}^a(\mathbf{M}_s) + \mathbf{H}^b(\mathbf{M}_s)]_{\text{tan}} \\ & + [\mathbf{H}^a(\mathbf{J}_s^a) - \mathbf{H}^b(\mathbf{J}_s^b)]_{\text{tan}} = [\mathbf{H}^{b,i} - \mathbf{H}^{a,i}]_{\text{tan}} \quad \mathbf{r} \in S_{\text{ap}}, \end{aligned} \quad (\text{B.6})$$

where $\mathbf{r}$ denotes the observation point. The exterior and interior currents $\mathbf{J}_s^{a,b}$ depend on $\mathbf{M}_s$. These relations follow from the E-field boundary conditions in (B.4), which must hold across the entire boundary surface, including the aperture. Note that, outside of the support of $\mathbf{M}_s$, the PEC boundary condition for the E-field is correctly enforced. The interior and exterior electric currents will be resolved first, after which the unknown current $\mathbf{M}_s$ is solved through (B.6).

Substituting (B.3a) and (B.2a) in (B.4) and using (B.5), gives the Electric Field Integral Equations (EFIE)

$$[\mathbf{E}^{a,i} + \mathbf{E}^a(\mathbf{M}_s) + \mathbf{E}^a(\mathbf{J}_s^a)]_{\text{tan}} = \hat{\mathbf{n}}^a \times \mathbf{M}_s \quad (\text{B.7a})$$

$$[\mathbf{E}^{b,i} - \mathbf{E}^b(\mathbf{M}_s) + \mathbf{E}^b(\mathbf{J}_s^b)]_{\text{tan}} = -\hat{\mathbf{n}}^b \times \mathbf{M}_s. \quad (\text{B.7b})$$

The current-dependent fields must be interpreted in terms of the integro-differential operators [16, Eq. (2.127)]:

$$\mathbf{H}^{a,b}(\mathbf{J}_s^{a,b}) = \mathcal{K}^{a,b}(\mathbf{J}_s^{a,b}) - \frac{1}{2}\hat{\mathbf{n}}^{a,b} \times \mathbf{J}_s^{a,b}, \quad (\text{B.8})$$

where

$$\mathcal{K}^{a,b}(\mathbf{X}_s) = \int_S \mathbf{X}_s(\mathbf{r}') \times \nabla' G^{a,b}(\mathbf{r} - \mathbf{r}') dS', \quad (\text{B.9})$$

and where the scalar Green's function for a homogeneous medium

$$G^{a,b}(\mathbf{r} - \mathbf{r}') = \frac{e^{-jk^{a,b}|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r} - \mathbf{r}'|}. \quad (\text{B.10})$$

The wavenumber in the medium is given by its material constitutive parameters, i.e., $k^{a,b} = \omega\sqrt{\mu^{a,b}\varepsilon^{a,b}}$, and $\mathbf{r}'$ is a point in the source domain. Furthermore,

$$\mathbf{E}^{a,b}(\mathbf{J}_s^{a,b}) = \mathcal{L}^{a,b}(\mathbf{J}_s^{a,b}), \quad (\text{B.11})$$

where

$$\begin{aligned} \mathcal{L}^{a,b}(\mathbf{X}) = & -j\omega\mu^{a,b} \int_S \mathbf{X}_s(\mathbf{r}') G^{a,b}(\mathbf{r} - \mathbf{r}') dS' \\ & + \frac{1}{j\omega\varepsilon^{a,b}} \nabla \int_S \nabla' \cdot \mathbf{X}(\mathbf{r}') G^{a,b}(\mathbf{r} - \mathbf{r}') dS'. \end{aligned} \quad (\text{B.12})$$

Finally, using duality,

$$\mathbf{H}^{a,b}(\mathbf{M}_s^{a,b}) = \frac{1}{\eta_{a,b}^2} \mathcal{L}^{a,b}(\mathbf{M}_s) \quad (\text{B.13a})$$

$$\mathbf{E}^{a,b}(\mathbf{M}_s^{a,b}) = -\mathcal{K}^{a,b}(\mathbf{M}_s) + \frac{1}{2}\hat{\mathbf{n}}^{a,b} \times \mathbf{M}_s^{a,b}. \quad (\text{B.13b})$$

These surface integrations need to be interpreted in the Cauchy principal value sense. The EFIEs in (B.7) are now written as

$$-\mathcal{L}^a(\mathbf{J}_s^a)|_{\text{tan}} = [-\mathcal{K}^a(\mathbf{M}_s) + \mathbf{E}^{a,i}]_{\text{tan}} - \frac{1}{2}\hat{\mathbf{n}}^a \times \mathbf{M}_s \quad (\text{B.14a})$$

$$-\mathcal{L}^b(\mathbf{J}_s^b)|_{\text{tan}} = [\mathcal{K}^b(\mathbf{M}_s) + \mathbf{E}^{b,i}]_{\text{tan}} + \frac{1}{2}\hat{\mathbf{n}}^b \times \mathbf{M}_s \quad (\text{B.14b})$$

for the exterior and interior electric surface currents, respectively, given an $\mathbf{M}_s$. We can solve the EFIEs through the method of moments using the following discretizations

$$\begin{aligned} \mathbf{J}_s^a(\mathbf{r}) &= \sum_{n=1}^{N^a} I_n^a \mathbf{i}_n^a(\mathbf{r}), \quad \mathbf{J}_s^b(\mathbf{r}) = \sum_{n=1}^{N^b} I_n^b \mathbf{i}_n^b(\mathbf{r}), \\ \mathbf{M}_s(\mathbf{r}) &= \sum_{n=1}^N V_n \mathbf{m}_n(\mathbf{r}) \end{aligned} \quad (\text{B.15})$$

Testing (B.14) by the basis functions $\{\mathbf{i}_m^{a,b}\}$, using the symmetric product $\langle \mathbf{a}, \mathbf{b} \rangle = \int \mathbf{a} \cdot \mathbf{b} dS$, yields

$$\sum_{n=1}^{N^a} Z_{mn}^a I_n^a = \sum_{n=1}^N D_{mn}^a V_n + V_m^{a,i} \quad (\text{B.16})$$

$$\sum_{n=1}^{N^b} Z_{mn}^b I_n^b = - \sum_{n=1}^N D_{mn}^b V_n + V_m^{b,i} \quad (\text{B.17})$$

for $m = 1, 2, \dots, N^{a,b}$, respectively, where

$$Z_{mn}^{a,b} = -\langle \mathbf{i}_m^{a,b}, \mathcal{L}^{a,b}(\mathbf{i}_n^{a,b}) \rangle \quad (\text{B.18a})$$

$$D_{mn}^{a,b} = -\langle \mathbf{i}_m^{a,b}, \mathcal{K}^{a,b}(\mathbf{m}_n) \rangle - \frac{1}{2} \langle \mathbf{i}_m^{a,b}, \hat{\mathbf{n}}^{a,b} \times \mathbf{m}_n \rangle \quad (\text{B.18b})$$

$$V_m^{a,i} = \langle \mathbf{i}_m^a, \mathbf{E}^{a,i} \rangle \quad \text{and} \quad V_m^{b,i} = \langle \mathbf{i}_m^b, \mathbf{E}^{b,i} \rangle, \quad (\text{B.18c})$$

or simply

$$\mathbf{Z}^a \mathbf{I}^a = \mathbf{D}^a \mathbf{V} + \mathbf{V}^{a,i} \quad (\text{B.19a})$$

$$\mathbf{Z}^b \mathbf{I}^b = -\mathbf{D}^b \mathbf{V} + \mathbf{V}^{b,i}. \quad (\text{B.19b})$$

Thus, the solution becomes

$$\mathbf{I}^a = (\mathbf{Z}^a)^{-1} (\mathbf{V}^{a,i} + \mathbf{D}^a \mathbf{V}) \quad (\text{B.20a})$$

$$\mathbf{I}^b = (\mathbf{Z}^b)^{-1} (\mathbf{V}^{b,i} - \mathbf{D}^b \mathbf{V}). \quad (\text{B.20b})$$

Similarly, applying the same discretization process to the Magnetic Field Integral Equation (MFIE) in (B.6) and testing by the basis functions $\{\mathbf{m}_m\}$,

yields

$$\sum_{n=1}^N [Y_{mn}^a + Y_{mn}^b] V_n + \sum_{n=1}^{N^b} C_{mn}^b I_n^b - \sum_{n=1}^{N^a} C_{mn}^a I_n^a = I_m^{a,i} - I_m^{b,i}, \quad (\text{B.21})$$

where

$$Y_{mn}^{a,b} = -\frac{1}{\eta_{a,b}^2} \langle \mathbf{m}_m, \mathcal{L}^{a,b}(\mathbf{m}_n) \rangle \quad (\text{B.22a})$$

$$C_{mn}^{a,b} = \langle \mathbf{m}_m, \mathcal{K}^{a,b}(\mathbf{i}_n^{a,b}) \rangle - \frac{1}{2} \langle \mathbf{m}_m, \hat{\mathbf{n}}^{a,b} \times \mathbf{i}_n^{a,b} \rangle \quad (\text{B.22b})$$

$$I_m^{a,i} = \langle \mathbf{m}_m, \mathbf{H}^{a,i} \rangle \quad \text{and} \quad I_m^{b,i} = \langle \mathbf{m}_m, \mathbf{H}^{b,i} \rangle, \quad (\text{B.22c})$$

or simply

$$[\mathbf{Y}^a + \mathbf{Y}^b] \mathbf{V} + \mathbf{C}^b \mathbf{I}^b - \mathbf{C}^a \mathbf{I}^a = \mathbf{I}^{a,i} - \mathbf{I}^{b,i}. \quad (\text{B.23})$$

One can now substitute (B.20) in (B.23) to finally arrive at

$$[\mathbf{Y}_{\text{tot}}^a + \mathbf{Y}_{\text{tot}}^b] \mathbf{V} = \mathbf{I}, \quad (\text{B.24})$$

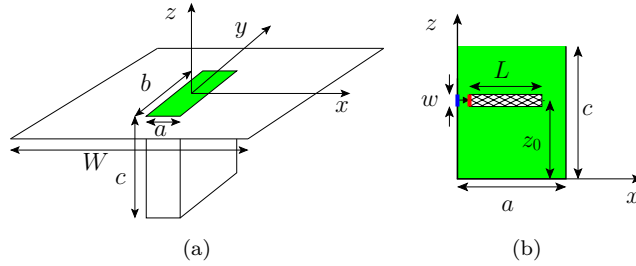
where

$$\mathbf{Y}_{\text{tot}}^a = \mathbf{Y}^a - \mathbf{C}^a (\mathbf{Z}^a)^{-1} \mathbf{D}^a \quad (\text{B.25a})$$

$$\mathbf{Y}_{\text{tot}}^b = \mathbf{Y}^b - \mathbf{C}^b (\mathbf{Z}^b)^{-1} \mathbf{D}^b \quad (\text{B.25b})$$

$$\mathbf{I} = [\mathbf{I}^{a,i} + \mathbf{C}^a (\mathbf{Z}^a)^{-1} \mathbf{V}^{a,i}] - [\mathbf{I}^{b,i} + \mathbf{C}^b (\mathbf{Z}^b)^{-1} \mathbf{V}^{b,i}], \quad (\text{B.25c})$$

and where the matrix elements are computed using (B.22) and (B.18). Once $\mathbf{V}$ is solved for an incident field $\mathbf{I}$, the exterior magnetic current $\mathbf{M}_s$ is known through the basis function expansion (B.15), as well as the exterior electric current $\mathbf{J}_s^a$ through the expansion coefficient vector (B.20a). These currents radiate the exterior scattered field in a homogeneous medium $\{\mu^a, \varepsilon^a\}$, and together with the incident field, the total field is obtained as per (B.2a). Note how Fig. 1(b) visualizes Eq. (B.24), with the difference that the voltage sources are now replaced by current sources that are dependent on the field excitation.



a	b	c	W	L	w	z_0	ϵ_r
9	21	12	23.1	7.5	0.1	7.5	2.2

Figure 3: Dimensions in cm: (a) Open waveguide; (b) Monopole strip and port.

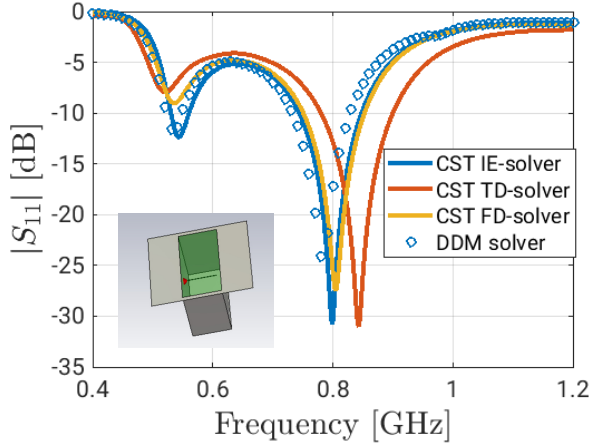
3 Numerical Results

For the purpose of validation, consider the rectangular waveguide element in Fig. 3(a). The element is of size $a \times b \times c$, where its aperture opening is in the $z = 0$ PEC ground plane. The ground plane is of size $W \times W$. We consider two scenarios: (i) a dielectric filled waveguide ($\epsilon_r = 2.2$) considered in the transmitting situation that is excited by a monopole PEC feed as shown in Fig. 3(b), and; (ii) a 2×1 reflectarray of empty air-filled waveguides, excited by a Huygens point source placed along the z -axis and radiating into the negative z -direction.

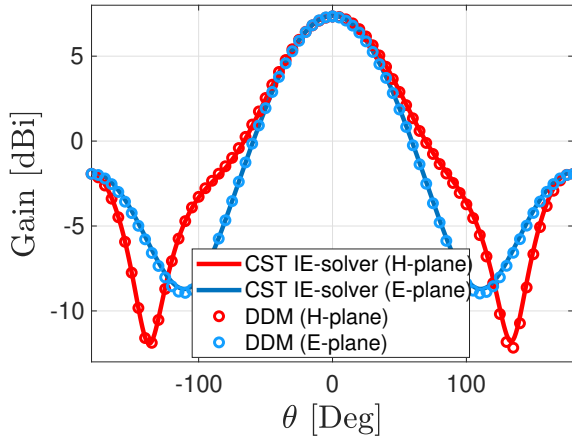
Our objective is to validate the DDM-computed input impedance and radiation pattern for the probe-excited waveguide using both CST Microwave Studio (2020) and a plain EFIE approach. Additionally, the DDM-computed scattering characteristics of the 2×1 reflectarray are compared to a plain MoM approach (EFIE only). In all our simulations we have employed Rao-Wilton-Glisson (RWG) basis and test functions (Galerkin's approach) for both the electric and magnetic currents.

One can conclude from Fig. 4(a) that the magnitude of the port reflection coefficient $|S_{11}|$ as computed by CST's integral solver, time-domain solver, and frequency-domain solver are in good agreement with our proposed DDM results. The shift in resonance frequency is seen to be dependent on CST's solver type (default solver and mesh settings are used). CST's integral equation (IE) solver agrees best with our MoM-based DDM. Furthermore, the E-

and H-field radiation patterns in Fig. 4(b) are visually almost indistinguishable from the DDM-computed patterns. Note that only CST's IE-solver patterns



(a)



(b)

Figure 4: DDM vs. CST radiation characteristics comparison: (a) Port reflection coefficient; (b) Power pattern in two principal planes @0.85 GHz.

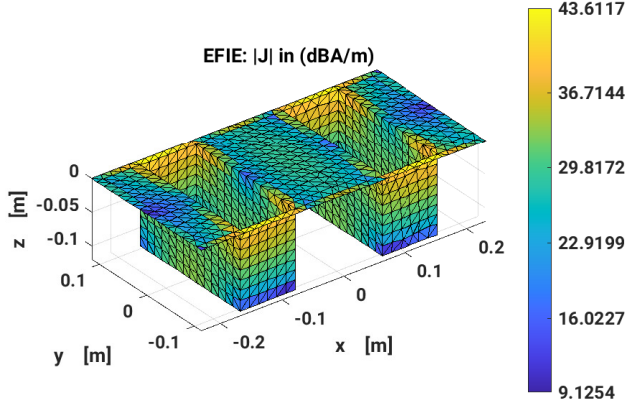
results are shown, since CST's other solvers yield visually identical patterns and are therefore omitted. The DDM mesh density is $\lambda/12$ @1 GHz. At this high mesh density the predicted resonance frequency is only weakly dependent on the mesh. The results generally confirm that the proposed DDM is working as expected.

Next, the 2×1 reflectarray in Fig. 5 is considered. A Huygens point source with unity electric dipole moment $\hat{\mathbf{x}}$ [Am] and magnetic dipole moment $-\eta_a \hat{\mathbf{y}}$ [Vm] are placed at $(x = 0, y = 0, z = \lambda)$ and radiates in the negative z -direction. The reference induced surface current is shown in Fig. 5(a) as computed by a plain MoM approach (EFIE only). The induced electric surface current as computed by the DDM is shown in Fig. 5(b). The exterior and interior current are summed to form the total electric current, so that this total current in Fig. 5(b) can be compared to the MoM current in Fig. 5(a). The current distributions agree very well, despite that the total electric current in the aperture in Fig. 5(b) does not fully vanish. The latter, however, does not have a detrimental effect on the E- and H-plane scattering patterns as shown in Fig. 6(a). The relative pattern error is computed as

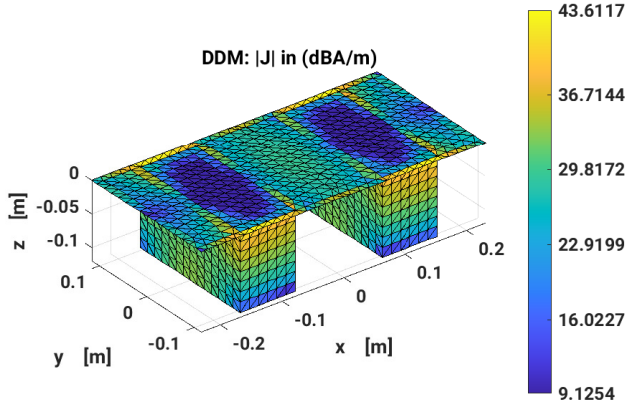
$$\epsilon = \frac{\|\mathbf{U}^{\text{EFIE}} - \mathbf{U}^{\text{DDM}}\|_{\text{Fro}}}{\|\mathbf{U}^{\text{EFIE}}\|_{\text{Fro}}} \times 100\% \quad (\text{B.26})$$

where “Fro” denotes the Frobenius norm and where $\mathbf{U}$ is a vector of uniformly sampled radiation intensities across a far-field sphere.

We noticed that an unusually dense mesh had to be taken to get accurate results, as shown in Fig. 6(b). This may be due to the non-vanishing aperture current in Fig. 5(b), requiring a denser mesh near the edges. Furthermore, MATLAB issued a high condition number warning for the densest mesh ($\lambda/24$). This may be attributed to the well-known interior resonance problem this formulation is susceptible to. We expect not to have this problem in the future, since large reflectarrays will have negligible current on the backside, so that the potentially resonant cavities can be removed when solving the exterior problem as a sheet problem, which also significantly alleviates the computational complexity. For the interior problem we will then use a commercial solver (CST/HFSS), enabling us to model more complex and realistic unit cells. The DDM as presented in this paper will be used to interconnect the multi-solver domains and also to validate the overall obtained solution.



(a)



(b)

Figure 5: Current distribution of 2×1 array of air-filled waveguides as computed by the DDM and a plain EFIE approach (a) EFIE current; (b) DDM current.

4 Conclusion

A domain decomposition method for open-cavity based reflectarrays is proposed. The surface equivalence principle is employed to model the exterior

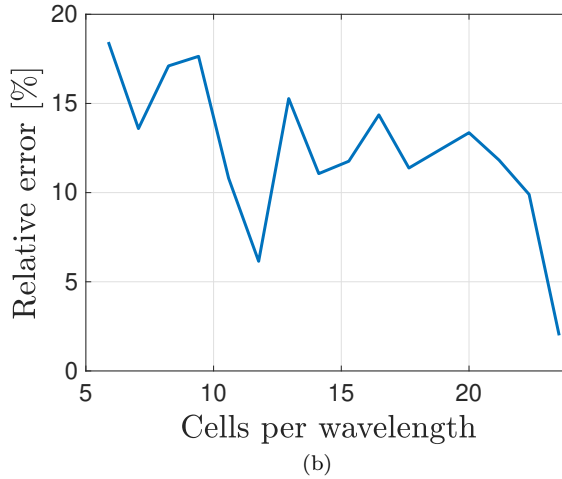
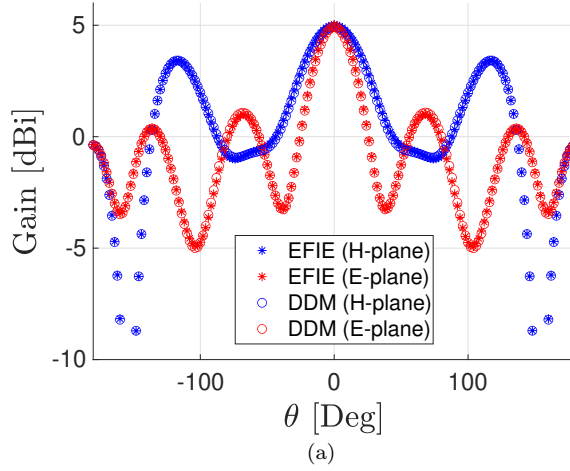


Figure 6: (a) Comparison EFIE vs. DDM of E- and H-plane gain pattern; (b) 3-D pattern error for DDM as a function of mesh density relative to reference solution (EFIE with $\lambda/24$ mesh density).

problem as a full metal structure for which, in the future, fast methods can be used (e.g. CBFM + PO). Furthermore, if the backside current can be neglected, it is believed that the exterior structure may even be reduced to a metal sheet. The interior problem (i.e. the reflectarray element) is characterized by a generalized admittance matrix through an in-house MoM solver. The present paper has shown that the domains, each of which is characterized by a generalized admittance matrix, can be interconnected in a circuit-type manner. The overall array response has been validated for: (i) a radiating dielectric-filled waveguide, and; (ii) a 2×1 reflectarray element with Huygens source excitation. The successful validation with CST and a plain MoM approach of both the radiation and scattering characteristics show that the proposed DDM approach can serve as the future interface for connecting commercial field solvers to fast integral-equation based methods for solving large-scale reflectarray and reconfigurable intelligent surface problems. This hybrid solution method will also eliminate the interior resonance problem that the current formulation is susceptible to.

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**Modeling of Conformal Reflectarrays Through Domain
Decomposition Using a Generalized Admittance Approach**

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The layout has been revised.

Abstract

This paper presents a method-of-moments-based domain decomposition technique for analyzing conformal cavity-backed reflectarrays. Scattering from a 3D conformal all-metal reflectarray is modeled using the perfect electric conductor (PEC) equivalence, where impressed aperture magnetic current sources on the cavity apertures separate the interior and exterior cavity domains via a generalized aperture admittance approach. The methodology is evaluated for two representative conformal shapes, concave and convex. Numerical results demonstrate high accuracy, with average relative errors ranging from 0.5% to 2.2%, confirming the validity of the proposed method for finite conformal cavity-backed reflectarrays.

1 Introduction

Reconfigurable intelligent surfaces (RIS) have recently attracted significant research interest due to their promising features, such as beamforming and the ability to enhance propagation channels through controlled reflections [1, 2]. Reflectarrays are a notable example of RIS, offering controllable reflections via adjustable phase responses on the array surface. Conventional flat reflectarrays face key limitations, including narrow bandwidth and restricted beam-scanning capability, primarily due to phase errors and element pattern degradation at large off-boresight angles. To address these limitations, conformal arrays are being actively investigated [3–6].

Reflectarrays are increasingly considered for applications in satellite communications, aircraft, and UAVs. In these contexts, conformal designs offer advantages, such as enhanced bandwidth and improved side lobe control [7, 8]. However, modeling conformal reflectarrays presents a greater challenge as they are inherently non-periodic. This complexity is further amplified in special cases, such as partially embedded reflectors or SIW-based wideband elements, where the array size and additional features (e.g., via-fencing) significantly increase computational demands [9, 10].

This work builds on our prior research [11, 12] on PEC-equivalent modeling of cavity-backed reflectarray elements in periodic and aperiodic configurations.

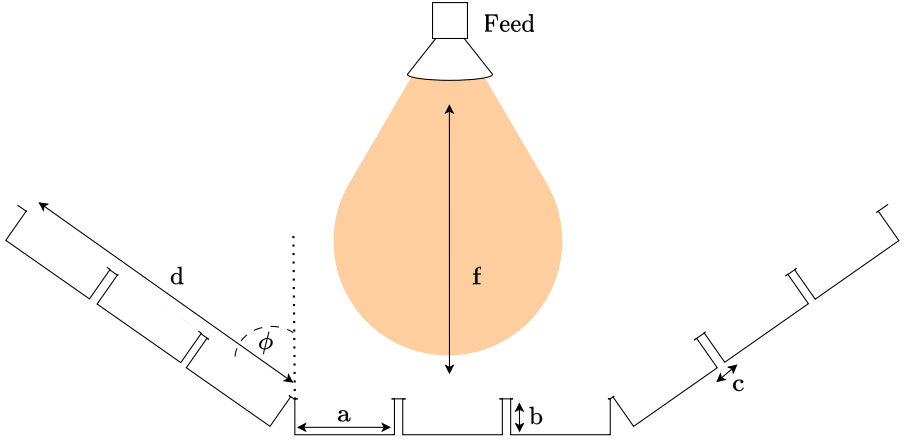


Figure 1: Baseline configuration: a 3-facet reflectarray illuminated by a center-fed single-pol horn at 1 GHz. Parameters are $a = 0.055$ m, $b = \lambda/2$, $c = \lambda/10$, $d = 1.8\lambda$, $f = 2$ m.

Here, we illustrate the general setup and modeling methodology for electrically connected *conformal* reflectarrays, demonstrating the effectiveness of the approach using both concave and convex faceted examples.

The remainder of this paper is organized as follows. Sec. II details the domain-decomposition-based modeling methodology for conformal arrays. Sec. III presents validation results for both concave and convex faceted reflectarray examples, and Sec. IV concludes the paper and discusses recommendations for future work.

2 Methodology

The modeling methodology used in this work builds on the framework described in [11]. In brief, to model the fields in the exterior region, the metal cavities shown in Fig. 1 are closed with perfect electric conductor (PEC) material. The aperture fields are then represented by equivalent magnetic currents placed on the PEC apertures, as shown in Fig. 2. This procedure is general and can be applied regardless of the unit cell's position or orientation, allowing

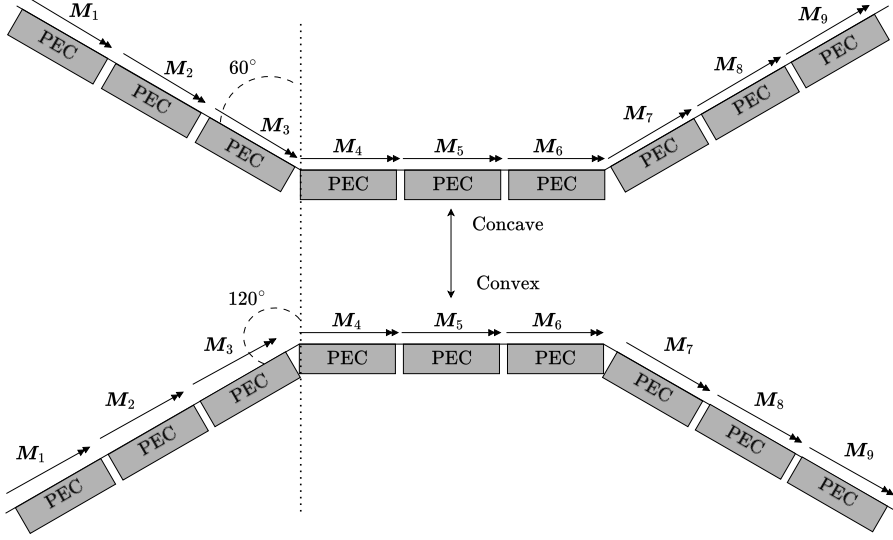


Figure 2: Exterior PEC-equivalent model for convex and concave reflectarrays.

analysis of both convex and concave reflectarrays.

The over-the-air coupling between aperture field modes (or their equivalent magnetic current representations) is captured by the generalized admittance matrix $\mathbf{Y}^a$, where each entry represents the self- or mutual admittance between a pair of modes. In Fig. 2, $\mathbf{M}_n$ denotes the set of K_n equivalent aperture currents corresponding to the K_n field modes at aperture n , for $n = 1, \dots, N$.

The coupling among the K_n aperture modes inside the N individual cavities is modeled by the interior admittance matrix, represented as the sparse block-diagonal matrix

$$\mathbf{Y}^b = \begin{bmatrix} \mathbf{Y}_1^b & 0 & \cdots & 0 \\ 0 & \mathbf{Y}_2^b & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{Y}_N^b \end{bmatrix} \quad (\text{C.1})$$

By enforcing field continuity across the apertures, the interior and exterior

regions are connected via the admittance formulation [11, Eq. (24)]

$$\left[\mathbf{Y}^a + \mathbf{Y}^b \right] \mathbf{V} = \mathbf{I}, \quad (\text{C.2})$$

where $\mathbf{V}$ is the vector of aperture field mode amplitudes, and $\mathbf{I}$ is the magnetic excitation vector due to an external feed.

In this decomposition, $\mathbf{Y}^a$ depends on the geometry and positioning of the cells, while $\mathbf{Y}^b$ remains fixed for a given unit cell design. As a result, the method supports flexible array configurations and is well-suited for modeling conformal reflectarrays.

The total magnetic current distribution $\mathbf{M}$, as well as each individual mode in the above formulation, radiates in the presence of the surrounding PEC structure. To compute the total scattered field, the PEC surface is replaced by its induced electric surface current $\mathbf{J}$. This current is found by solving the electric field integral equation (EFIE):

$$\mathbf{E}_{\text{tan}} = \mathbf{E}_{\text{tan}}^{\text{i}} + \mathbf{E}_{\text{tan}}^{\text{s}}(\mathbf{J}) = \mathbf{0}, \quad (\text{C.3})$$

evaluated at observation points on the PEC surface. Here, $\mathbf{E}_{\text{tan}}^{\text{i}} = \mathbf{E}_{\text{tan}}(\mathbf{M}) + \mathbf{E}_{\text{tan}}^{\text{Feed}}$ is the total incident field resulting from the aperture magnetic currents and the feed. The total scattered field in the exterior region thus arises from both the magnetic current $\mathbf{M}$ and the induced electric current $\mathbf{J}$.

3 Results

The proposed methodology is validated on a 27-element reflectarray in both convex and concave configurations. Each array consists of three 3×3 panels, with the side panels rotated as shown in Fig. 1. In this setup, feed blockage effects are neglected. Both configurations, illustrated in Fig. 2, are selected for their relevance in achieving higher bandwidth and improved wide-angle scanning performance.

Results are compared against a full-wave EFIE-based method-of-moments solver using an identical fixed mesh, enabling direct and consistent benchmarking.

For the convex array, the equivalent magnetic currents are shown in Fig. 3, and the corresponding far-field radiation pattern is plotted in Fig. 4. In the E-plane, the configuration yields an average error of 0.76%, a peak error of

3.91%, and a median of 0.49%. In the H-plane, the average, peak, and median errors are 0.34%, 2.14%, and 0.054%, respectively.

For the concave array (Fig. 5), the radiation pattern is shown in Fig. 6. In the E-plane, the average error is 2.21%, with a peak of 16.43% and a median of 0.90%. In the H-plane, the average, peak, and median errors are 0.53%, 3.57%, and 0.31%, respectively.

We found that the error in the concave array decreases as the mesh density increases, likely due to stronger coupling and more complex current modes that are not adequately captured with the same mesh as the convex array. This trend is also observed between the H- and E-planes, with the latter exhibiting the highest coupling. Overall, these results confirm that the proposed methodology maintains high accuracy across different conformal geometries. Most errors remain below 1% on average, with particularly low median values in both principal planes, demonstrating the robustness of the approach.

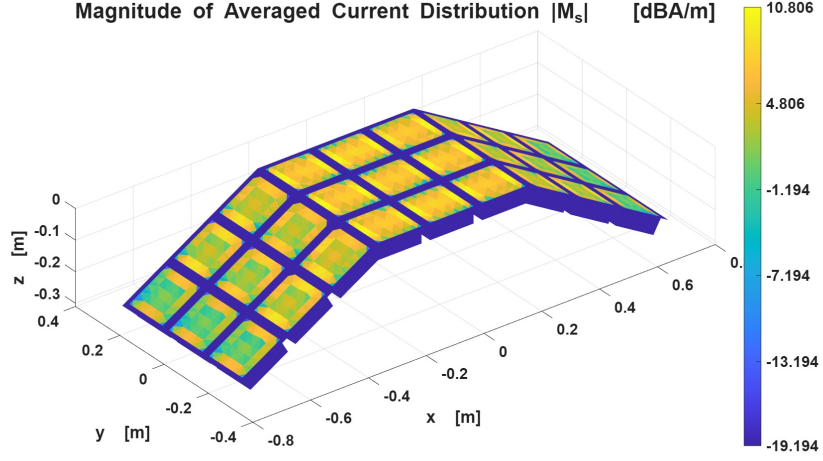


Figure 3: Equivalent convex array with computed magnetic current distribution

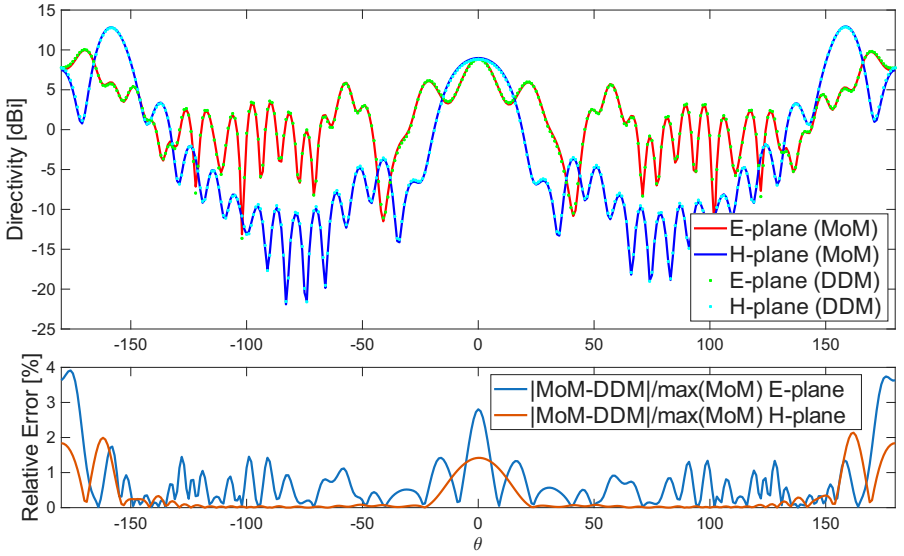


Figure 4: Radiation pattern and the relative error for the convex array

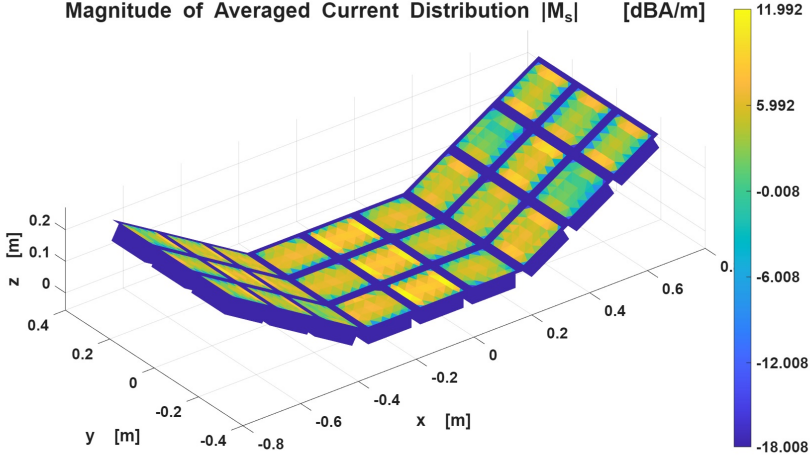


Figure 5: Equivalent concave array with computed magnetic current distribution

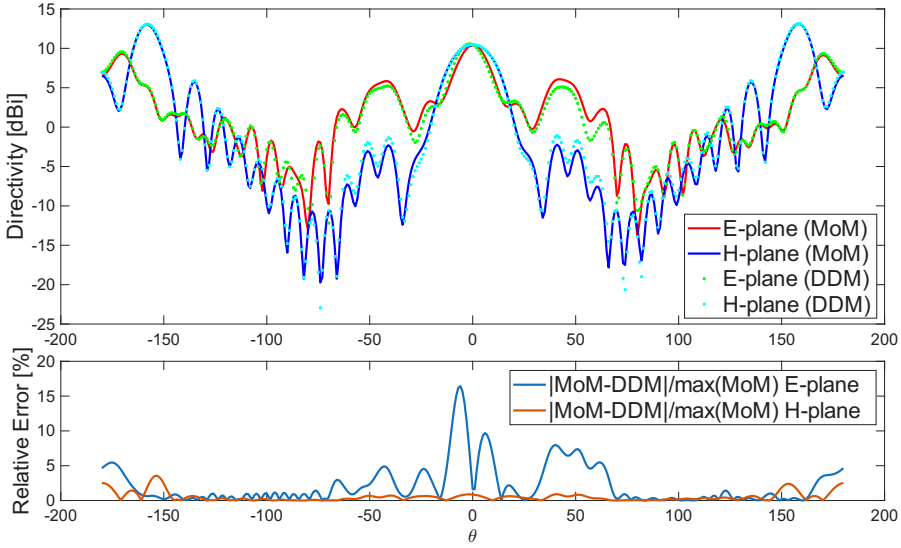


Figure 6: Radiation pattern and the relative error for the concave array

4 Conclusion

This paper validates a domain-decomposition-based generalized admittance matrix approach for the analysis of conformal reflectarrays. High accuracy is demonstrated through comparison with a conventional full-wave method-of-moments solver for a 27-element reflectarray in both convex and concave configurations.

The proposed modular modeling strategy enables efficient reflectarray optimization. For example, in arrays composed of repeated cell types, only partial matrix updates are required to reflect changes in the array lattice. Similarly, when only the cell types are modified, field solutions can be obtained with minimal computational overhead.

Future work will focus on hybridizing this method with high-frequency techniques, such as physical optics, in combination with macro-domain basis functions for current modeling. This hybridization aims to further reduce solution time and memory requirements, enabling efficient simulation of electrically large conformal reflectarrays.

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The first author gratefully acknowledges the memory of his mother for her constant support and encouragement.

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**Hybrid Domain-Decomposition/Physical-Optics Analysis of Finite
Reconfigurable Reflectarrays Using Characteristic-Mode Macro
Basis Functions**

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The layout has been revised.

Abstract

This letter presents an efficient hybrid domain-decomposition/physical-optics framework for analyzing finite, large-scale reflectarrays and reconfigurable intelligent surfaces. The structure is decomposed into exterior and interior domains coupled through a generalized aperture-admittance formulation. The exterior response is approximated using physical optics, while element behavior is captured through admittance operators. Scalability to large arrays and many tuning states is achieved via a macro basis function expansion of the aperture current, which enables multi-state analysis by reusing pre-characterized element operators and updating only the elements that change state. Results for 8×8 and 20×20 arrays under different illuminations closely match full-wave references while substantially reducing memory usage and simulation time, demonstrating suitability for fast analysis and optimization of reconfigurable arrays.

1 Introduction

Reconfigurable intelligent surfaces (RIS) and reflectarrays manipulate electromagnetic wave propagation through controlled reflection responses, and they have been widely studied as potential cost effective solutions for beam steering and coverage enhancement in beyond-5G and 6G scenarios. Reflectarrays achieve phased-array-like behavior using passive elements, whereas RIS extends this concept by allowing the element response to be tuned in operation [1–4].

Accurate modeling of *finite* reflectarray and RIS apertures remains challenging. Periodic boundary conditions and infinite-array assumptions are computationally efficient, but their accuracy degrades for finite or aperiodic layouts, where edge effects and spatially varying mutual coupling become significant [5–8]. In addition, cavity-backed elements have received increased attention due to their broadband potential [9–13]. This trend motivates scalable solvers that can explicitly represent an interior cavity region. While full-wave volumetric methods can capture these effects, they often become prohibitively

expensive for large arrays and for analyses of many tuning states. Surface-integral-equation formulations, particularly the method of moments (MoM), are attractive for open-region problems because they require only surface discretization [14–18]. However, for large reconfigurable apertures, updating element states may still require repeated solves and matrix updates, which can limit scalability.

This letter proposes a hybrid domain-decomposition/physical-optics (DDM–PO) framework for efficient analysis of finite, large-scale reconfigurable reflectarrays and RIS. Two contributions are introduced. First, a magnetic-field integral equation (MFIE–PO) aperture-admittance formulation replaces the exterior MFIE operator with a physical-optics (PO) substitution, avoiding an explicit exterior integral-equation solve for electrically large arrays [19–23]. Second, the interior domain is modeled using element admittance operators together with a Macro Basis Function (MBF) representation, enabling multi-state analysis by reusing precomputed element operators and updating only the matrix blocks associated with elements that change state [24].

The paper is organized as follows. Section II presents the formulation. Section III reports validation results and benchmarks. Section IV concludes the letter.

2 Generalized Admittance Formulation

The scattering problem for a cavity-backed reflectarray (Fig. 1a) is decomposed into *exterior* and *interior* domains separated by the aperture surface S (Fig. 1b). Enforcing tangential magnetic-field continuity on S and applying field equivalence yields the MFIE on the aperture [19, 21, 25, 26]:

$$\begin{aligned} [\mathbf{H}^a(\mathbf{M}_s) + \mathbf{H}^b(\mathbf{M}_s)]_{\text{tan}} + [\mathbf{H}^a(\mathbf{J}_s^a) - \mathbf{H}^b(\mathbf{J}_s^b)]_{\text{tan}} \\ = [\mathbf{H}^{b,i} - \mathbf{H}^{a,i}]_{\text{tan}}. \end{aligned} \quad (\text{D.1})$$

Here, $\mathbf{M}_s$ is the equivalent aperture magnetic current and is oppositely oriented on the two sides of S . The quantities $\mathbf{J}_s^a$ and $\mathbf{J}_s^b$ denote the equivalent electric currents on the exterior and interior sides, respectively. The operators $\mathbf{H}^a(\cdot)$ and $\mathbf{H}^b(\cdot)$ are the corresponding magnetic-field integral operators. The terms $\mathbf{H}^{a,i}$ and $\mathbf{H}^{b,i}$ represent any impressed magnetic fields in the exterior and interior regions. For reference, Fig. 1c illustrates an exterior-only formulation obtained by setting the interior fields to zero; by the equivalence principle,

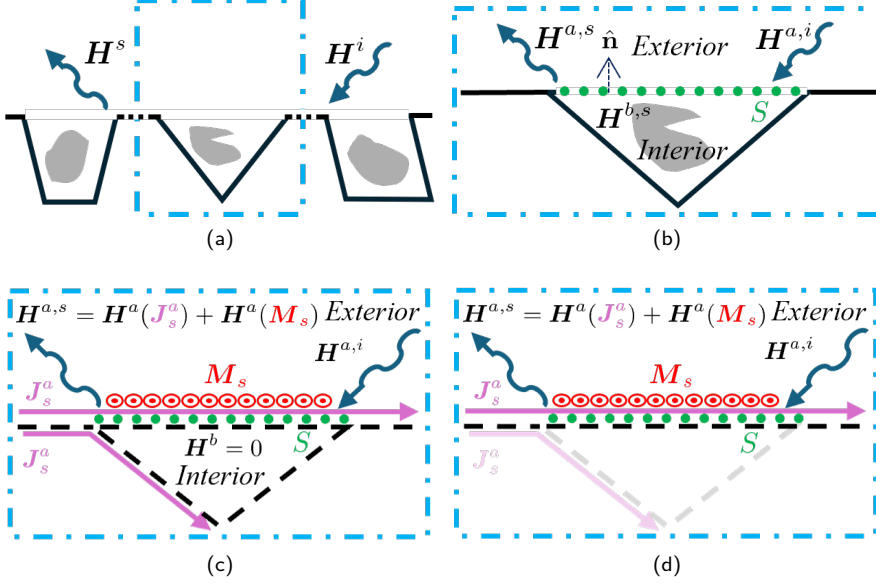


Figure 1: Formulation steps for a cavity-backed reflectarray. (a) Scattering problem with H^i and H^s . (b) Aperture S splits the exterior region a and interior cavity region b . (c) Equivalence principle on S introduces M_s and sets the fields in b to zero. (d) Neglecting rear-surface currents, the 3-D conductor is replaced by a 2-D PEC sheet in the aperture plane.

this yields the same exterior fields as the original scattering configuration in Fig. 1a.

2.1 Physical-Optics Approximation and MFIE-PO substitution

The structure is assumed to be backed by a perfect electric conductor (PEC), such that fields on the rear surface can be neglected. Under this assumption, the original three-dimensional conducting geometry is reduced to an equivalent two-dimensional PEC sheet coincident with the aperture plane (seen from the exterior region) [22], as illustrated in Fig. 1d. This reduction enables an efficient treatment of the exterior domain using PO.

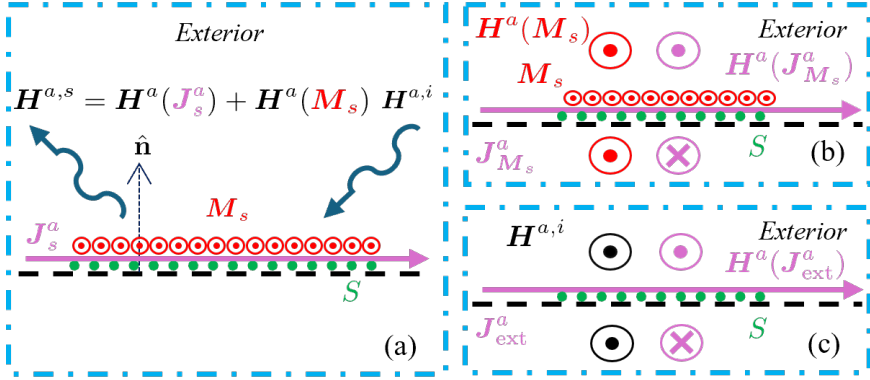


Figure 2: Superposition used for the PO treatment of the exterior domain. (a) Reduced configuration on the aperture with equivalent magnetic current M_s . (b) Contribution driven by M_s , which induces the PO current $J_{M_s}^a$. (c) Contribution driven by the incident field $H^{a,i}$, which induces $J_{H^{a,i}}^a$. The total exterior field is the sum of (b) and (c).

For a PEC boundary, the induced electric surface current density on the exterior side is

$$J_s^a = \hat{n} \times H_{tot}^a, \quad (D.2)$$

where $\hat{n}$ is the unit normal directed into the exterior region and $H_{tot}^a = H^{a,i} + H^{a,s}$ denotes the total magnetic field evaluated on the surface. In the PO approximation, the illuminated portion of a smooth, electrically large PEC surface is modeled locally as an infinite tangent plane. The shadow region is assumed to carry negligible current [23, 27, 28].

Starting from the MFIE (D.1) on the reduced PEC sheet, see Fig. 2a, we treat the two PO-driven excitation cases separately and then combine them by superposition. The exterior equivalent current is written as

$$J_s^a = J_{M_s}^a + J_{H^{a,i}}^a, \quad (D.3)$$

where $J_{M_s}^a$ is the exterior current induced by the field radiated by M_s (Fig. 2b) when $H^{a,i} = 0$, and $J_{H^{a,i}}^a$ is the exterior current induced by $H^{a,i}$ when $M_s = 0$ (Fig. 2c).

Case 1 (Fig. 2b, driven by M_s): Under the PO approximation, the field $H^a(M_s)$ cancels $H^a(J_{M_s}^a)$ in the immediate shadow region. Consequently,

in the illuminated region,

$$[\mathbf{H}^a(\mathbf{J}_{M_s}^a)]_{\text{tan}} \approx [\mathbf{H}^a(\mathbf{M}_s)]_{\text{tan}}. \quad (\text{D.4})$$

Case 2 (Fig. 2c, driven by $\mathbf{H}^{a,i}$): Similarly, the PO approximation in the illuminated region yields,

$$[\mathbf{H}^a(\mathbf{J}_{H^{a,i}}^a)]_{\text{tan}} \approx [\mathbf{H}^{a,i}]_{\text{tan}}. \quad (\text{D.5})$$

By linearity, the total exterior tangential field due to $\mathbf{J}_s^a$ (Fig. 2a) is obtained by superposition,

$$[\mathbf{H}^a(\mathbf{J}_s^a)]_{\text{tan}} = [\mathbf{H}^a(\mathbf{M}_s) + \mathbf{H}^{a,i}]_{\text{tan}}. \quad (\text{D.6})$$

where (D.3) has been substituted in the LHS of (D.6) leading to the RHS when using (D.4) and (D.5).

Substituting (D.6) into (D.1) yields the MFIE-PO

$$\begin{aligned} [2\mathbf{H}^a(\mathbf{M}_s) + \mathbf{H}^b(\mathbf{M}_s)]_{\text{tan}} - [\mathbf{H}^b(\mathbf{J}_s^b)]_{\text{tan}} \\ = [\mathbf{H}^{b,i} - 2\mathbf{H}^{a,i}]_{\text{tan}}. \end{aligned} \quad (\text{D.7})$$

A matrix representation is then obtained by discretizing (D.7) and applying Galerkin testing with Rao–Wilton–Glisson (RWG) basis functions, leading to tested terms of the form $\langle \mathbf{u}, \mathbf{H}(\mathbf{v}) \rangle \triangleq \int_S \mathbf{u} \cdot \mathbf{H}(\mathbf{v}) dS$, where $\mathbf{u}$ and $\mathbf{v}$ denote RWG expansions of equivalent currents on the aperture surface S .

No impressed sources are assumed to exist in the interior region, $\mathbf{H}^{b,i} = \mathbf{0}$. Expanding $\mathbf{M}_s = \sum_{n=1}^{N_M} V_n \mathbf{f}_n^M$ in (D.7) and testing with $\mathbf{f}_m^M$ yields

$$\begin{aligned} 2 \sum_{n=1}^{N_M} V_n \langle \mathbf{f}_m^M, \mathbf{H}^a(\mathbf{f}_n^M) \rangle + \sum_{n=1}^{N_M} V_n \langle \mathbf{f}_m^M, \mathbf{H}^b(\mathbf{f}_n^M) \rangle \\ - \sum_{n=1}^{N_M} V_n \langle \mathbf{f}_m^M, \mathbf{H}^b(\mathbf{J}_s^b(-\mathbf{f}_n^M)) \rangle = -2 \langle \mathbf{f}_m^M, \mathbf{H}^{a,i} \rangle, \end{aligned} \quad (\text{D.8})$$

for $m = 1, 2, \dots, N_M$, where the induced current $\mathbf{J}_s^b(-\mathbf{f}_n^M)$ due to $-\mathbf{M}_s$ on the interior is given in [19, Eq. (18)].

Finally, this yields the generalized aperture-admittance system of linear

equations

$$[\mathbf{Y}_{\text{tot}}^a + \mathbf{Y}_{\text{tot}}^b] \mathbf{V} = \mathbf{I}, \quad (\text{D.9a})$$

$$[\mathbf{Y}_{\text{tot}}^a]_{mn} = 2 \langle \mathbf{f}_m^M, \mathbf{H}^a(\mathbf{f}_n^M) \rangle, \quad (\text{D.9b})$$

$$[\mathbf{I}]_m = -2 \langle \mathbf{f}_m^M, \mathbf{H}^{a,i} \rangle. \quad (\text{D.9c})$$

$$[\mathbf{Y}_{\text{tot}}^b]_{mn} = \langle \mathbf{f}_m^M, \mathbf{H}^b(\mathbf{f}_n^M) \rangle - \langle \mathbf{f}_m^M, \mathbf{H}^b(\mathbf{J}_s^b(-\mathbf{f}_n^M)) \rangle \quad (\text{D.9d})$$

with

$$[\mathbf{Y}_{\text{tot}}^b]_{mn} = - \sum_{n=1}^{N_M} V_n \langle \mathbf{f}_m^M, \mathbf{H}^b(\mathbf{f}_n^M) \rangle - \langle \mathbf{f}_m^M, \mathbf{H}^b(\mathbf{J}_s^b) \rangle \quad (\text{D.10})$$

and

where $\mathbf{V} = [V_1, \dots, V_{N_M}]^T$ collects the RWG coefficients of $\mathbf{M}_s$ and $\mathbf{I}$ is the tested excitation.

2.2 Interior Domain and Macro-Basis Representation

The interior domain is modeled by the cavity-admittance matrix $\mathbf{Y}_{\text{tot}}^b$, which is block diagonal with one self-admittance block per array element. Suppose that the array contains L elements drawn from P unique element types. Element l is assigned a type index $\tau(l) \in \{1, \dots, P\}$, so

$$\mathbf{Y}_{\text{tot}}^b = \text{blkdiag}(\mathbf{Y}^{b,(\tau(1))}, \dots, \mathbf{Y}^{b,(\tau(L))}), \quad (\text{D.11})$$

i.e., each unique type is analyzed once and reused across the array [24]. For MBF generation, each element is treated as a non-overlapping unit-cell analysis region; no buffer or overlap region is introduced. After assembly, inter-element interactions are included through the global aperture-admittance system, with array-level coupling provided by the MFIE–PO exterior formulation.

To reduce the number of unknowns without changing the MFIE–PO formulation, the aperture magnetic current is expanded through MBFs [29, 30]. Let $\mathbf{V} \in \mathbb{C}^N$ collect the RWG coefficients of $\mathbf{M}_s$ on the aperture.

The MBF reduction is written as $\mathbf{V} = \mathbf{V}_{\text{MBF}} \mathbf{w}$, where $\mathbf{V}_{\text{MBF}} \in \mathbb{C}^{N \times K}$ contains the MBFs in the original basis. The vector $\mathbf{w}$ stacks the MBF weights for all elements, with K_l MBFs retained for element l . Accordingly, $K =$

$\sum_{l=1}^L K_l$, and $\mathbf{V}_{\text{MBF}}$ is assembled element-wise as a block-diagonal matrix with blocks $\mathbf{V}_{\text{MBF}}^{(\tau(1))}, \dots, \mathbf{V}_{\text{MBF}}^{(\tau(L))}$.

Substituting this expansion into (D.9a) yields the reduced system

$$\left(\mathbf{V}_{\text{MBF}}^T [\mathbf{Y}_{\text{tot}}^a + \mathbf{Y}_{\text{tot}}^b] \mathbf{V}_{\text{MBF}} \right) \mathbf{w} = \mathbf{V}_{\text{MBF}}^T \mathbf{I}. \quad (\text{D.12})$$

Once $\mathbf{w}$ is solved, the full-resolution current follows from $\mathbf{V} = \mathbf{V}_{\text{MBF}} \mathbf{w}$.

MBF Generation via Characteristic Mode Analysis

MBFs are computed once for each unique element type. For P element types we form the local composite admittance $\mathbf{Y}^{(\tau(p))} = \mathbf{Y}_{\text{tot}}^{a,(\tau(p))} + \mathbf{Y}_{\text{tot}}^{b,(\tau(p))}$ and write $\mathbf{Y}^{(\tau(p))} = \mathbf{G}^{(\tau(p))} + j\mathbf{B}^{(\tau(p))}$, where $\mathbf{G}^{(\tau(p))} = \frac{1}{2}(\mathbf{Y}^{(\tau(p))} + \mathbf{Y}^{(\tau(p))H})$ and $\mathbf{B}^{(\tau(p))} = \frac{1}{2j}(\mathbf{Y}^{(\tau(p))} - \mathbf{Y}^{(\tau(p))H})$. The characteristic currents then satisfy

$$\mathbf{B}^{(\tau(p))} \mathbf{J}_n^{(\tau(p))} = \lambda_n^{(\tau(p))} \mathbf{G}^{(\tau(p))} \mathbf{J}_n^{(\tau(p))}, \quad (\text{D.13})$$

and are ranked using the modal significance $\text{MS}_n^{(\tau(p))} = (1 + (\lambda_n^{(\tau(p))})^2)^{-1/2}$ [31–33]. The K_p most significant modes form the local MBF matrix $\mathbf{V}_{\text{MBF}}^{(\tau(p))}$.

MBF Generation via Singular Value Decomposition

Alternatively, for each unique type p , we compute the SVD of the local inverse composite admittance,

$$(\mathbf{Y}^{(p)})^{-1} = \mathbf{U}^{(p)} \mathbf{\Sigma}^{(p)} \mathbf{D}^{(p)H}, \quad (\text{D.14})$$

and form the reduced basis by retaining the K_p dominant left singular vectors,

$$\mathbf{V}_{\text{MBF}}^{(p),\text{SVD}} = \mathbf{U}^{(p)}(:, 1 : K_p), \quad (\text{D.15})$$

The number K_p is chosen near the knee of the singular-value decay curve, rather than from a universal fixed threshold, since the current-reconstruction error is generally not known a priori; more singular vectors may be retained for higher current accuracy.

Both Characteristic Mode Analysis (CMA)- and singular value decomposition (SVD)-based MBFs are computed once per unique element type and reused across the array. CMA yields physically interpretable, excitation-independent modes, whereas SVD provides an excitation-independent numerical orthonormal basis based on singular values. Both approaches are validated on a single element in Section 3.

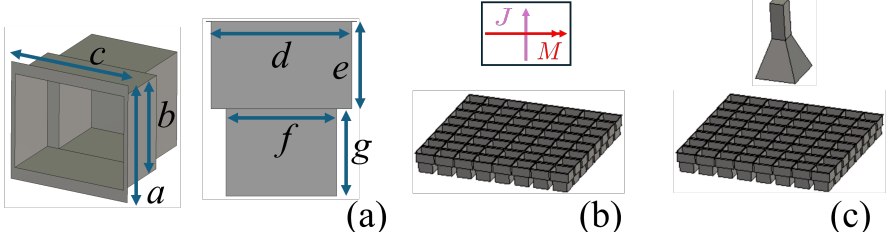


Figure 3: (a) Reference element from [9] used to evaluate the proposed methodology, with $a = 6$ mm, $b = 5.6$ mm, $c = 5$ mm, $d = 4$ mm, $e = 3$ mm, $f = 4.4$ mm, and $g = 3$ mm. (b) 8×8 array configuration with $e = g = 3$ mm under Huygens-source illumination at 10 cm. (c) Same 8×8 array under horn-antenna illumination at 10 cm.

Table 1: Benchmarks (CMA, $K_l = 10$ MBFs per element) on i9-12900K/128 GB/RTX 3080. Times are CPU/GPU seconds; total runtime is their sum. In the MBF+PO column, $A + B$ means (A) reduced-system MBF unknowns plus (B) PO degrees of freedom used for the exterior PO-current evaluation. Here, $k = 10^3$ and $M = 10^6$.

Array	Case	Mem	Time (s)		Unknowns MBF+PO	$\bar{e}$ (%)
			CPU	GPU		
8×8	DDM-PO / Huy.	7 MB	< 1	64	640+35k	1.3
8×8	DDM-PO / Horn	7 MB	< 1	63	640+35k	0.6
8×8	MoM	35 GB	594	—	48k	—
20×20	DDM-PO / Horn	269 MB	171	1017	4k+219k	0.06
20×20	CST (MLFMM)	100 GB	5316	12206	1.23M	—

2.3 Exterior Scattering

After solving for the aperture magnetic current $\mathbf{M}_s$ (via $\mathbf{V}$ or $\mathbf{w}$), the exterior scattering is evaluated from the PO current $\mathbf{J}_{\text{PO}} = 2\hat{\mathbf{n}} \times (\mathbf{H}^{a,i} + \mathbf{H}^a(\mathbf{M}_s))$. With MBF reduction, solving for $\mathbf{M}_s$ scales as $\mathcal{O}(K^3)$, and the subsequent PO current evaluation scales as $\mathcal{O}(N)$.

3 Results

Validation is carried out at 32 GHz for an unmodulated 8×8 reflectarray and a modulated 20×20 reflectarray. The element geometry is shown in Fig. 3a and is based on [9] with minor parameter adjustments. A Huygens source is

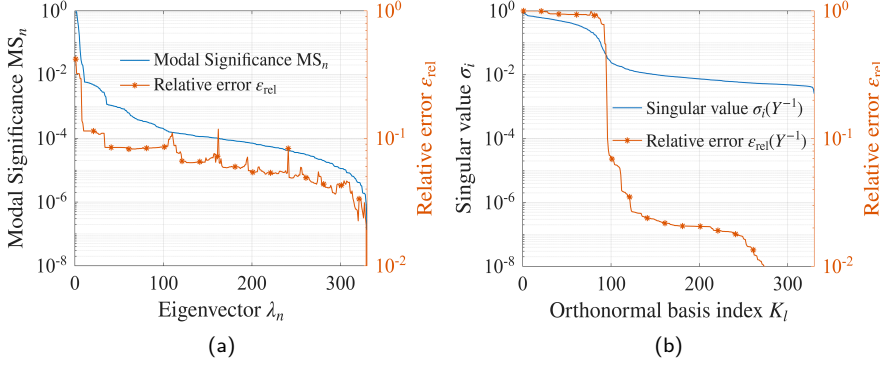


Figure 4: (a) CMA modal significance MS_n and the corresponding reconstruction error ε_{rel} versus the number of retained characteristic modes. (b) Singular values σ_i obtained from $\mathbf{Y}^{-1}$.

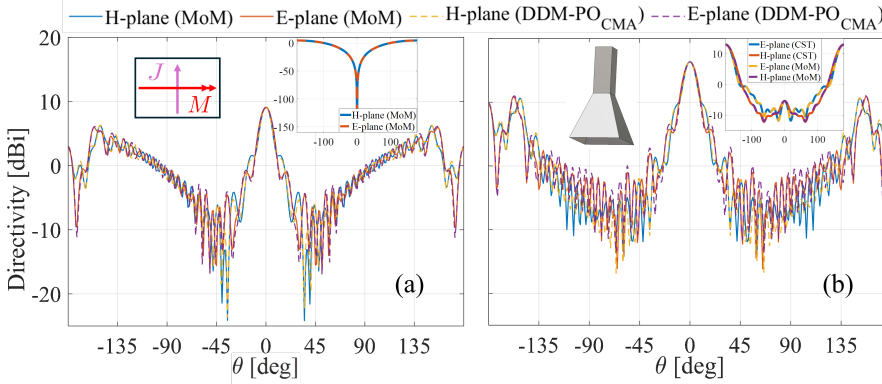


Figure 5: Radiation patterns of the 8×8 array under (a) Huygens-source and (b) horn illumination at 10 cm ($\approx 10.7\lambda$). DDM-PO with $K_i = 10$ CMA-MBFs per element is compared with the MoM reference. Insets show the corresponding feed patterns.

used to realize minimal edge taper, while a high-gain horn provides a strongly tapered excitation. In all cases, the feed is modeled as a far-field source and feed-array coupling is neglected.

The impact of MBF reduction is first assessed on a single element illumi-

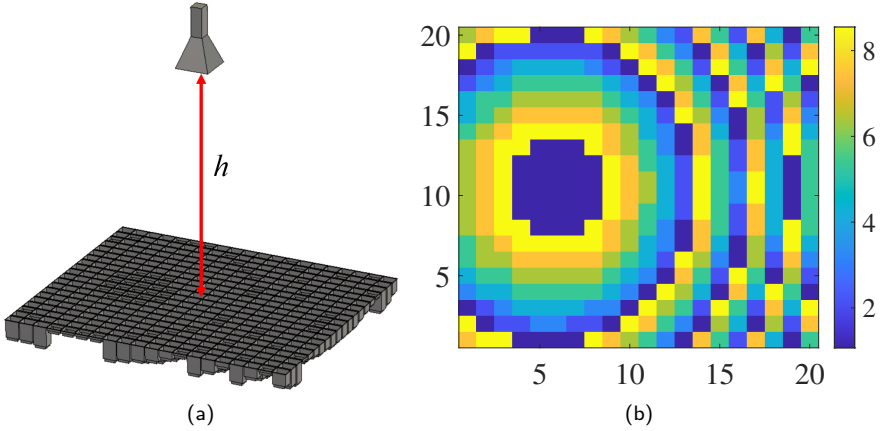


Figure 6: (a) 20×20 reflectarray scanning to $\theta = -15^\circ$, illuminated by a horn at $h = 10$ cm. (b) Element-wise modulation of g (in mm) using eight discrete states, with e fixed to 1 mm.

nated by the Huygens source at a distance of 10 cm ($\approx 10.7\lambda$). Figure 4a reports the CMA modal significance, while Fig. 4b shows the singular spectrum obtained from $\mathbf{Y}^{-1}$ and the corresponding reconstruction error. For both constructions, the reconstruction error is quantified by the relative error:

$$\varepsilon_{\text{rel}} = \frac{\|\mathbf{V}_{\text{MBF}}\mathbf{w} - \mathbf{V}\|_2}{\|\mathbf{V}\|_2}, \quad (\text{D.16})$$

where $\mathbf{V}$ denotes the solved RWG coefficient vector of $\mathbf{M}_s$. Based on these results, subsequent array evaluations use CMA-based MBFs, which provide lower current-reconstruction error for small K_l in the considered example while remaining physically interpretable.

The 8×8 array is evaluated under both illumination conditions using $K_p = 10$ CMA-MBFs for the element type. As shown in Figs. 5a and 5b, the proposed approach closely matches the MoM reference in both the E - and H -planes. The computational benefits are summarized in Table 1, showing orders-of-magnitude lower memory and runtime relative to MoM (CPU+GPU time). The average relative error $\bar{e}$ is computed from the sampled radiation-

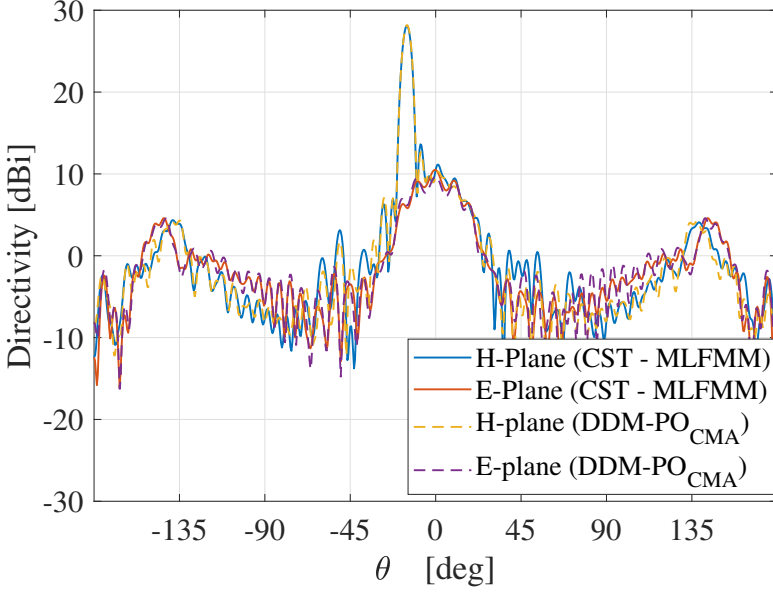


Figure 7: Radiation patterns of the 20×20 array scanning to $\theta = -15^\circ$, computed by DDM-PO with $K_p = 10$ CMA-MBFs per state and compared with CST MLFMM.

pattern values x_i (linear scale) over the E - and H -planes as

$$\bar{e} = \frac{1}{N} \sum_{i=1}^N e_i, \quad e_i = \frac{|x_i - x_i^{\text{ref}}|}{\max_j |x_j^{\text{ref}}|}, \quad (\text{D.17})$$

and remains within 0.6% to 1.3% for both Huygens and horn illumination.

Finally, the method is validated on a 20×20 modulated array configured to scan to $\theta = -15^\circ$. Eight discrete element states are used, and each state is represented using $K_p = 10$ MBFs. The array geometry and modulation are shown in Fig. 6. The resulting radiation pattern is compared with CST (MLFMM, solver tol. 10^{-4} , level = 6) [34] in Fig. 7, since a full MoM reference would require prohibitive memory resources (about 1 TB) that are not available. The results show good agreement, while Table 1 indicates an over two-order-of-magnitude reduction in memory usage and a $> 10\times$ reduction in total runtime (CPU+GPU) relative to CST. Figure 8 shows the computed

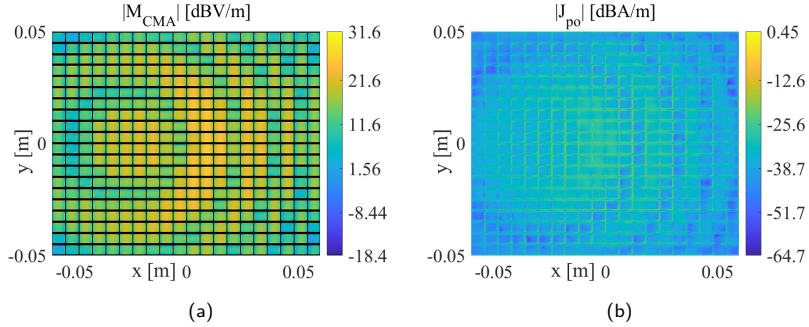


Figure 8: (a) Magnetic current $\mathbf{M}_s$ using 80 CMA-MBFs (10 per state). (b) Induced PO current on the PEC.

$\mathbf{M}_s$ and induced $\mathbf{J}_{\text{PO}}$, illustrating how the modulation shapes the current distribution across the array.

4 Conclusion

This letter presented a DDM-PO framework for analyzing large-scale reconfigurable reflectarrays and RIS. The formulation couples the interior and exterior domains through a generalized aperture-admittance system and employs physical optics to approximate the exterior response, enabling efficient evaluation of electrically large arrays. CMA-MBFs further reduce unknowns by reusing pre-characterized element operators and updating only those elements that change state. Results for 8×8 and 20×20 arrays under different illuminations show close agreement with full-wave references while substantially reducing memory usage and simulation time. Future work will add PO edge-diffraction corrections for finite arrays.

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**Comparison of Macro-Basis-Function Construction Methods for
Reduced-Order Analysis of Cavity-Backed Reflectarrays**

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The layout has been revised.

Abstract

This letter presents a controlled comparison of three macro-basis-function (MBF) constructions for reduced-order method-of-moments analysis of cavity-backed reflectarrays: characteristic mode analysis (CMA), the characteristic basis function method with primary and enriched responses (CBFM), and a singular-value-decomposition (SVD)-based basis. The comparison is performed within a common reduced-order formulation using identical geometry, discretization, partitioning, and solver settings, so that differences in current reconstruction, far-field accuracy, compression, and conditioning can be attributed mainly to the MBF construction. A single element and an identical-element 8x8 array are used as benchmarks at 32 GHz. The results show that CBFM provides attractive low-rank accuracy but exhibits nonmonotonic solution error as enriched functions are retained. Explicit G-orthogonalization substantially improves the conditioning and accuracy of CMA, whereas the SVD basis gives the best-conditioned reduced systems but requires more MBFs to reach the prescribed far-field accuracy. The array results preserve the main single-element current-reconstruction and conditioning trends, while the far-field error is less sensitive to the retained basis size.

1 Introduction

Large-scale reflectarray simulations based on the method of moments (MoM) often lead to dense or ill-conditioned linear systems, resulting in substantial computational cost, especially when repeated solutions are required for multiple excitations or parameter sweeps. Macro-basis functions (MBFs) reduce the number of unknowns by constructing a low-dimensional subspace that captures the dominant current or field behavior, thereby accelerating the solution while maintaining good accuracy [1, 2].

This paper extends the cavity-backed reflectarray framework introduced in [3–7] by isolating the effect of MBF construction within that formulation. Whereas the earlier work established the underlying MFIE–PO framework

and used MBFs as an enabling ingredient, it did not examine in a controlled manner how the basis-construction strategy itself affects compression, conditioning, and field accuracy. The present paper therefore provides a controlled comparison of characteristic mode analysis (CMA) [8–13], the characteristic basis function method (CBFM) [14–23], and singular value decomposition (SVD), with multi-order CBF enrichment treated within the CBFM framework [24–26].

This letter presents a controlled benchmark of CMA-, CBFM-, and SVD-based MBF constructions for cavity-backed reflectarray analysis, isolating the effect of the basis construction by keeping the geometry, discretization, partitioning, and reduced-order solver fixed. The considered approaches are evaluated in terms of reconstructed-current accuracy, far-field accuracy, compression efficiency, and numerical stability through the condition number of the reduced system. In addition, the study examines how closely single-element behavior predicts finite-array performance. The main contributions are a unified reduced-order framework for CMA-, CBFM-, and SVD-based MBFs, a controlled benchmark of their accuracy-versus-rank and conditioning behavior, and an assessment of how single-element trends carry over to the array case.

The remainder of this paper is organized as follows. Section 2 briefly summarizes the underlying formulation and introduces the considered MBF constructions. Section 3 presents the numerical results. Finally, Section 4 concludes the paper.

2 Macro-Basis Functions and System Compression

2.1 MFIE–PO Formulation

The underlying cavity-backed reflectarray formulation follows [3–7] and is briefly summarized here for completeness. The scattering problem is decomposed into exterior and interior domains separated by the aperture surface S , as illustrated in Fig. 1a. Applying the equivalence principle on S introduces the equivalent magnetic current $\mathbf{M}_s$ and suppresses the fields in the interior region, leading to the reduced problem shown in Fig. 1b. Enforcing tangential

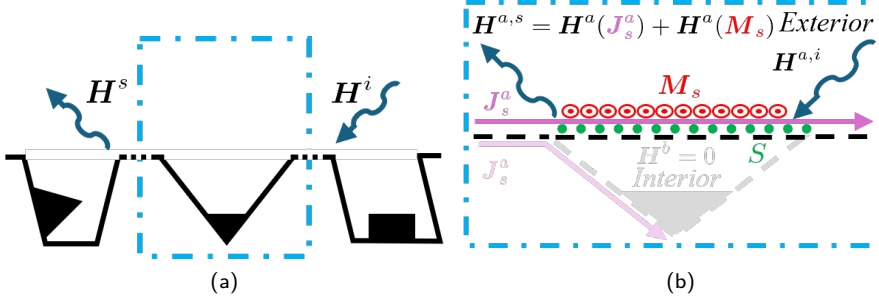


Figure 1: Formulation steps for a cavity-backed reflectarray. 1a Original scattering problem with incident and scattered magnetic fields, H^i and H^s . 1b Application of the equivalence principle on S , introducing the equivalent magnetic current M_s and suppressing the fields in region b . The reduced problem is then obtained by neglecting rear-surface currents, so that the three-dimensional conductor is replaced by a two-dimensional PEC sheet in the aperture plane.

magnetic-field continuity on S yields the MFIE on the aperture [6, 27, 28]:

$$\begin{aligned} [H^a(M_s) + H^b(M_s)]_{\tan} + [H^a(J_s^a) - H^b(J_s^b)]_{\tan} \\ = [H^{b,i} - H^{a,i}]_{\tan}. \end{aligned} \quad (\text{E.1})$$

Here, M_s denotes the equivalent aperture magnetic current, while J_s^a and J_s^b denote the equivalent electric currents on the exterior and interior sides of the aperture, respectively. The operators $H^a(\cdot)$ and $H^b(\cdot)$ denote the corresponding magnetic-field integral operators.

For the reduced problem in Fig. 1b, the MFIE is combined with the proposed PO boundary condition [7] and discretized using Galerkin testing with Rao-Wilton-Glisson (RWG) basis functions [29]. With the testing inner product $\langle \mathbf{a}, \mathbf{b} \rangle = \int_S \mathbf{a} \cdot \mathbf{b} dS$, the resulting system becomes [7]

$$[\mathbf{Y}_{\text{tot}}^a + \mathbf{Y}_{\text{tot}}^b] \mathbf{V} = \mathbf{I}, \quad (\text{E.2a})$$

$$[\mathbf{Y}_{\text{tot}}^a]_{mn} = 2 \langle \mathbf{f}_m^M, H^a(\mathbf{f}_n^M) \rangle, \quad (\text{E.2b})$$

$$[\mathbf{I}]_m = -2 \langle \mathbf{f}_m^M, H^{a,i} \rangle, \quad (\text{E.2c})$$

$$[\mathbf{Y}_{\text{tot}}^b]_{mn} = \langle \mathbf{f}_m^M, H^b(\mathbf{f}_n^M) \rangle - \langle \mathbf{f}_m^M, H^b(J_s^b(-\mathbf{f}_n^M)) \rangle. \quad (\text{E.2d})$$

Here, $\mathbf{f}_m^M$ denotes the RWG basis/testing function with local support on S associated with the aperture magnetic current, $\mathbf{V} \in \mathbb{C}^{N \times 1}$ collects the unknown coefficients, $\mathbf{Y}_{\text{tot}}^a$ is the dense exterior-domain matrix, and $\mathbf{Y}_{\text{tot}}^b$ is the cavity-admittance matrix associated with the interior domain.

For an array of L elements drawn from P unique element types, the interior matrix is block diagonal,

$$\mathbf{Y}_{\text{tot}}^b = \text{blkdiag}(\mathbf{Y}^{b,(\tau(1))}, \dots, \mathbf{Y}^{b,(\tau(L))}), \quad (\text{E.3})$$

where $\tau(l) \in \{1, \dots, P\}$ denotes the type index of element l . Thus, each unique element type is characterized only once and reused throughout the array [3].

2.2 MBF Compression

The problem is well suited for MBF compression because the apertures of different elements are physically separated, allowing an element-wise reduced basis with nonoverlapping supports. Let $\mathbf{V}_{\text{MBF}} \in \mathbb{C}^{N \times K}$ denote the MBF matrix, whose columns contain the MBFs expressed in the original RWG basis, and let $\mathbf{w} \in \mathbb{C}^K$ collect the corresponding MBF weights. The aperture current is expanded as

$$\mathbf{V} = \mathbf{V}_{\text{MBF}} \mathbf{w}. \quad (\text{E.4})$$

If K_l MBFs are retained for element l , the total number of MBFs is $K = \sum_{l=1}^L K_l$, and the global basis matrix is assembled as

$$\mathbf{V}_{\text{MBF}} = \text{blkdiag}(\mathbf{V}_{\text{MBF}}^{(\tau(1))}, \dots, \mathbf{V}_{\text{MBF}}^{(\tau(L))}). \quad (\text{E.5})$$

With $\mathbf{Y} = \mathbf{Y}_{\text{tot}}^a + \mathbf{Y}_{\text{tot}}^b$, substitution of (E.4) into (E.2a) and multiplying by $\mathbf{V}_{\text{MBF}}^T$, yields

$$\tilde{\mathbf{Y}} \mathbf{w} = \tilde{\mathbf{I}}, \quad (\text{E.6})$$

where

$$\tilde{\mathbf{Y}} = \mathbf{V}_{\text{MBF}}^T \mathbf{Y} \mathbf{V}_{\text{MBF}}, \quad \tilde{\mathbf{I}} = \mathbf{V}_{\text{MBF}}^T \mathbf{I}.$$

Because the same nonconjugated testing product $\langle \cdot, \cdot \rangle$ is used in the projection, the superscript T denotes transpose rather than Hermitian transpose.

After solving for $\mathbf{w}$, the full-resolution current follows from (E.4). All methods considered here use the same reduced formulation and differ only in the construction and ordering of the columns of $\mathbf{V}_{\text{MBF}}$.

MBFs may be excitation-dependent or excitation-independent. Excitation-dependent MBFs are often more efficient but may be biased toward the used excitation set, whereas excitation-independent MBFs may generalize better across excitations. In the present comparison, CBFM is an excitation-dependent construction, whereas CMA and the system matrix-based SVD (called SVD hereafter) are excitation-independent.

2.3 Compression and Conditioning

To assess the numerical robustness of the compressed MBF representation, the spectral condition number of the reduced matrix $\tilde{\mathbf{Y}}$ is monitored [30, 31]:

$$\kappa_2(\tilde{\mathbf{Y}}) = \|\tilde{\mathbf{Y}}\|_2 \|\tilde{\mathbf{Y}}^{-1}\|_2 = \frac{\sigma_{\max}(\tilde{\mathbf{Y}})}{\sigma_{\min}(\tilde{\mathbf{Y}})}, \quad (\text{E.7})$$

where $\sigma_{\max}(\cdot)$ and $\sigma_{\min}(\cdot)$ denote the largest and smallest singular values, respectively.

To separate self-interaction from inter-element coupling, the reduced matrix is written as

$$\tilde{\mathbf{Y}} = \mathbf{D} + \mathbf{C}, \quad \mathbf{D} = \text{blkdiag}(\tilde{\mathbf{Y}}_{11}, \dots, \tilde{\mathbf{Y}}_{LL}), \quad (\text{E.8})$$

where $\mathbf{D}$ contains the block-diagonal self-interaction terms and $\mathbf{C}$ contains the off-diagonal coupling blocks. If $\mathbf{D}$ is nonsingular, then

$$\tilde{\mathbf{Y}} = \mathbf{D}(\mathbf{I} + \mathbf{D}^{-1}\mathbf{C}), \quad (\text{E.9})$$

and therefore

$$\kappa_2(\tilde{\mathbf{Y}}) \leq \kappa_2(\mathbf{D}) \kappa_2(\mathbf{I} + \mathbf{D}^{-1}\mathbf{C}). \quad (\text{E.10})$$

Thus, the conditioning of the full reduced system depends on both the conditioning of the local reduced blocks and the strength of the inter-element coupling represented by $\mathbf{D}^{-1}\mathbf{C}$. For weak coupling, the conditioning of the local reduced blocks can therefore provide a useful a priori indicator of the array-level conditioning.

A large condition number indicates increased sensitivity to numerical perturbations and may also degrade the convergence behavior of iterative solvers, although it does not by itself determine the final solution accuracy. The practical basis size is therefore governed by a tradeoff between approximation accuracy and numerical robustness, since adding more MBFs may eventually increase sensitivity more than it improves the approximation.

2.4 MBF Types Considered

Three MBF construction strategies are considered: characteristic mode analysis (CMA), the characteristic basis function method (CBFM), and an SVD-based construction. For each unique element type p , a local basis matrix $\mathbf{V}_{\text{MBF}}^{(p)}$ is constructed from the corresponding isolated-element matrix and mapped to all array elements of the same type through the global assembly described above.

Each construction is used with its natural normalization or truncation procedure. Thus, the comparison evaluates the reduced bases as generated by each method, rather than enforcing a common Euclidean normalization. A purely Euclidean normalization of the CMA modes would define a modified CMA basis and is not considered here.

Characteristic-Mode-Based MBFs

CMA provides an excitation-independent MBF construction in which the basis functions are derived from the intrinsic current modes of each element geometry. For each unique element type p , let

$$\mathbf{Y}^{(p)} = \mathbf{G}^{(p)} + j\mathbf{B}^{(p)}, \quad (\text{E.11})$$

with

$$\mathbf{G}^{(p)} = \frac{1}{2} \left(\mathbf{Y}^{(p)} + \mathbf{Y}^{(p)\text{H}} \right), \quad \mathbf{B}^{(p)} = \frac{1}{2j} \left(\mathbf{Y}^{(p)} - \mathbf{Y}^{(p)\text{H}} \right).$$

In the admittance-based formulation adopted here, the characteristic currents are obtained from [12]

$$\mathbf{B}^{(p)} \mathbf{v}_n^{(p)} = \lambda_n^{(p)} \mathbf{G}^{(p)} \mathbf{v}_n^{(p)}, \quad (\text{E.12})$$

where $\lambda_n^{(p)}$ and $\mathbf{v}_n^{(p)}$ denote the characteristic value and eigenvector of mode n , respectively. The corresponding modal significance is

$$\text{MS}_n^{(p)} = \frac{1}{\sqrt{1 + \left(\lambda_n^{(p)}\right)^2}}. \quad (\text{E.13})$$

The modes with the largest modal significance values are retained as MBFs. Let

$$\mathbf{V}_{\text{raw}}^{(p)} = \left[\mathbf{v}_{n_1}^{(p)}, \mathbf{v}_{n_2}^{(p)}, \dots, \mathbf{v}_{n_{m_p}}^{(p)} \right] \quad (\text{E.14})$$

denote the matrix of selected raw characteristic eigenvectors, where m_p is the number of retained modes for element type p .

To improve numerical robustness, the selected raw characteristic modes are normalized and re-orthogonalized in the G -metric. Define

$$\mathbf{S}^{(p)} = \mathbf{V}_{\text{raw}}^{(p)\text{H}} \mathbf{G}^{(p)} \mathbf{V}_{\text{raw}}^{(p)}, \quad (\text{E.15})$$

with eigendecomposition

$$\mathbf{S}^{(p)} = \mathbf{Q}^{(p)} \mathbf{D}_g^{(p)} \mathbf{Q}^{(p)\text{H}}. \quad (\text{E.16})$$

Here, $\mathbf{Q}^{(p)}$ contains the eigenvectors of $\mathbf{S}^{(p)}$ and $\mathbf{D}_g^{(p)}$ its eigenvalues. Since these eigenvalues may be negative or very small, the local CMA basis is formed as

$$\mathbf{V}_{\text{CMA}}^{(p)} = \mathbf{V}_{\text{raw}}^{(p)} \mathbf{Q}^{(p)} \left| \mathbf{D}_g^{(p)} \right|^{-1/2}, \quad (\text{E.17})$$

where very small eigenvalue magnitudes may be thresholded in practice. This step compensates for numerical scaling and loss of orthogonality, yielding a G -orthogonalized basis. If the raw characteristic modes are already strictly G -orthogonal, then $\mathbf{S}^{(p)}$ is diagonal and the procedure reduces to G -metric normalization.

The main advantage of CMA is that it provides a physically interpretable and excitation-independent reduced space. However, because the basis is not tailored to a particular excitation set, additional modes may be required to capture excitation-specific current features.

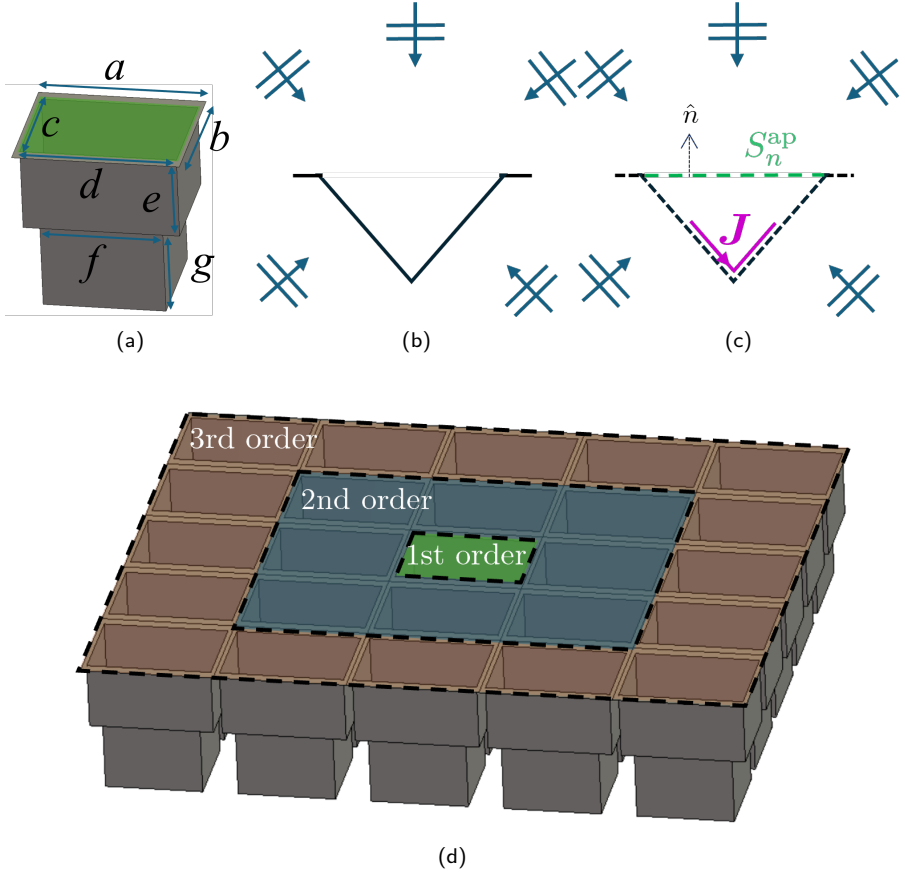


Figure 2: Construction of the CBF basis. 2a Unit-cell geometry with $a = 6$ mm, $b = 5.6$ mm, $c = 5$ mm, $d = 4$ mm, $e = 3$ mm, $f = 4.4$ mm, and $g = 3$ mm. 2b Schematic illustration of the simplified model used for primary CBF generation under plane-wave excitation. 2c Schematic illustration of the corresponding aperture field at S_n^{ap} obtained from the equivalent-current formulation under plane-wave illumination. 2d Local-subarray enrichment used to capture higher-order coupling effects.

CBFM-Based MBFs

CBFM constructs excitation-dependent MBFs from current responses to a prescribed set of excitations [16, 17, 20–23]. For each element type p , the primary CBFs are generated by illuminating an isolated element with a representative set of plane waves, as illustrated in Fig. 2b, and solving

$$\mathbf{Z}^{(p)} \mathbf{I}^{(p)} = \mathbf{v}^{(p)}, \quad (\text{E.18})$$

where $\mathbf{Z}^{(p)}$ denotes the local impedance matrix, $\mathbf{I}^{(p)}$ the induced current vector, and $\mathbf{v}^{(p)}$ the excitation vector.

The induced currents are then used to extract the electric field at the aperture of the central element, as illustrated in Fig. 2c. The equivalent aperture magnetic current is then obtained as $\mathbf{M}_s = \mathbf{E} \times \hat{\mathbf{n}}$ [32], where $\hat{\mathbf{n}}$ denotes the outward unit vector, normal to the aperture. The resulting aperture responses define the primary CBFs used in the reduced formulation.

To enrich the basis, the same procedure is repeated for local subarrays centered on the considered element. In this work, 3×3 and 5×5 subarrays are used to generate additional responses that capture secondary and tertiary coupling effects, as illustrated in Fig. 2d. The primary and enrichment responses are collected into a response matrix $\mathbf{R}^{(p)} = [\mathbf{r}_1^{(p)}, \dots, \mathbf{r}_{q_p}^{(p)}]$, where q_p is the total number of responses for element type p . An SVD-based truncation,

$$\mathbf{R}^{(p)} = \mathbf{U}_R^{(p)} \mathbf{\Sigma}_R^{(p)} \mathbf{W}_R^{(p)H}, \quad (\text{E.19})$$

is then applied to retain only the dominant left singular vectors. The local CBFM basis is therefore

$$\mathbf{V}_{\text{CBFM}}^{(p)} = \mathbf{U}_R^{(p)}(:, 1:m_p) = [\mathbf{b}_1^{(p)}, \mathbf{b}_2^{(p)}, \dots, \mathbf{b}_{m_p}^{(p)}], \quad (\text{E.20})$$

where $\mathbf{b}_k^{(p)}$ denotes the k -th retained basis function for element type p .

Thus, isolated-element responses define the primary CBFs, whereas subarray responses provide enrichment functions. The SVD step removes redundancy and controls the basis size, improving compactness at additional offline cost. CBFM provides efficient compression when the selected excitations are representative of the operating conditions; however, its performance may degrade for unseen excitation scenarios. For arrays with multiple element types, the enrichment procedure also becomes increasingly expensive because

the number of possible local environment configurations grows combinatorially. In the worst case, for P distinct element types and a local environment containing n elements, the number of type combinations scales as P^n . Consequently, the array-level CBFM results reported in Section 3 are restricted to the identical-element benchmark.

SVD-Based MBFs

In the SVD-based approach, the MBFs are constructed directly from the inverse local system operator rather than from selected excitation responses, as in CBFM. For each unique element type p , let

$$\mathbf{Y}^{(p)} = \mathbf{U}^{(p)} \mathbf{\Sigma}^{(p)} \mathbf{W}^{(p)H} \quad (\text{E.21})$$

be the singular value decomposition of the local system matrix. Then

$$\left(\mathbf{Y}^{(p)}\right)^{-1} = \mathbf{W}^{(p)} \left(\mathbf{\Sigma}^{(p)}\right)^{-1} \mathbf{U}^{(p)H}, \quad (\text{E.22})$$

so that the dominant left singular vectors of $\left(\mathbf{Y}^{(p)}\right)^{-1}$ are the right singular vectors of $\mathbf{Y}^{(p)}$ associated with its smallest singular values. If N_p denotes the number of local aperture unknowns and $\mathbf{w}_k^{(p)}$ the k -th column of $\mathbf{W}^{(p)}$, the local SVD basis is

$$\mathbf{V}_{\text{SVD}}^{(p)} = \left[\mathbf{w}_{N_p-m_p+1}^{(p)}, \dots, \mathbf{w}_{N_p}^{(p)} \right]. \quad (\text{E.23})$$

This yields an excitation-independent algebraic reduced basis. However, unlike CBFM, it is not tailored to a particular excitation class and does not in general admit a direct physical interpretation.

2.5 Error Metrics

The following error metrics are reported as functions of the basis size m or the compression ratio m/N :

- Relative current reconstruction error:

$$\varepsilon_{\text{rel}} = \frac{\|\mathbf{V}_{\text{MBF}} \mathbf{w} - \mathbf{V}\|_2}{\|\mathbf{V}\|_2}. \quad (\text{E.24})$$

Table 1: CBFM basis-generation settings for primary and enriched CBFs. Both θ - and ϕ -polarized plane-wave excitations are included for each sampled direction.

CBF Type	Responses	Time	θ Step	ϕ Step
Primary CBF	180	0.6 s	20°	45°
Secondary CBF	180	4 s	20°	45°
Tertiary CBF	180	42 s	20°	45°

- Peak-normalized mean radiation-pattern error, computed over all sampled angular points of the full radiation pattern using the sampled pattern values x_i in linear scale:

$$e_i = \frac{|x_i^{\text{ref}} - x_i|}{\max_j |x_j^{\text{ref}}|}, \quad \bar{e} = \frac{1}{N_\Omega} \sum_{i=1}^{N_\Omega} e_i, \quad (\text{E.25})$$

where N_Ω denotes the total number of sampled angular points. This normalization avoids disproportionately large relative errors near deep pattern nulls.

Together, these metrics provide a consistent basis for comparing the considered MBF constructions in terms of current reconstruction and radiation-pattern accuracy.

3 Results

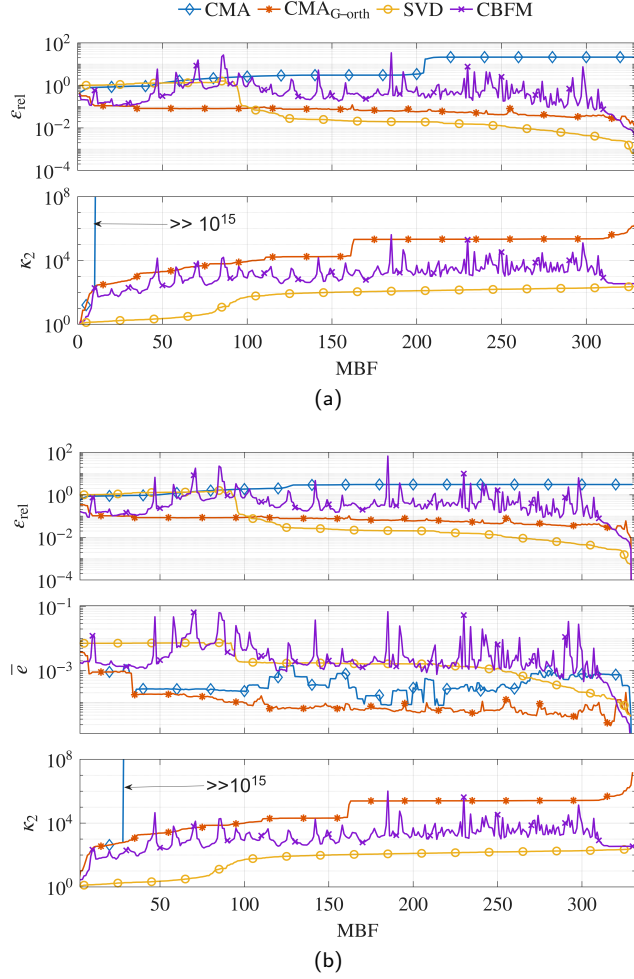


Figure 3: Comparison of CMA, $\text{CMA}_{\text{G-orth}}$, CBFM, and SVD as functions of the number of retained MBFs. 3a Single-element results: relative current reconstruction error and condition number. 3b 8×8 array results: relative current reconstruction error, peak-normalized mean far-field error, and condition number.

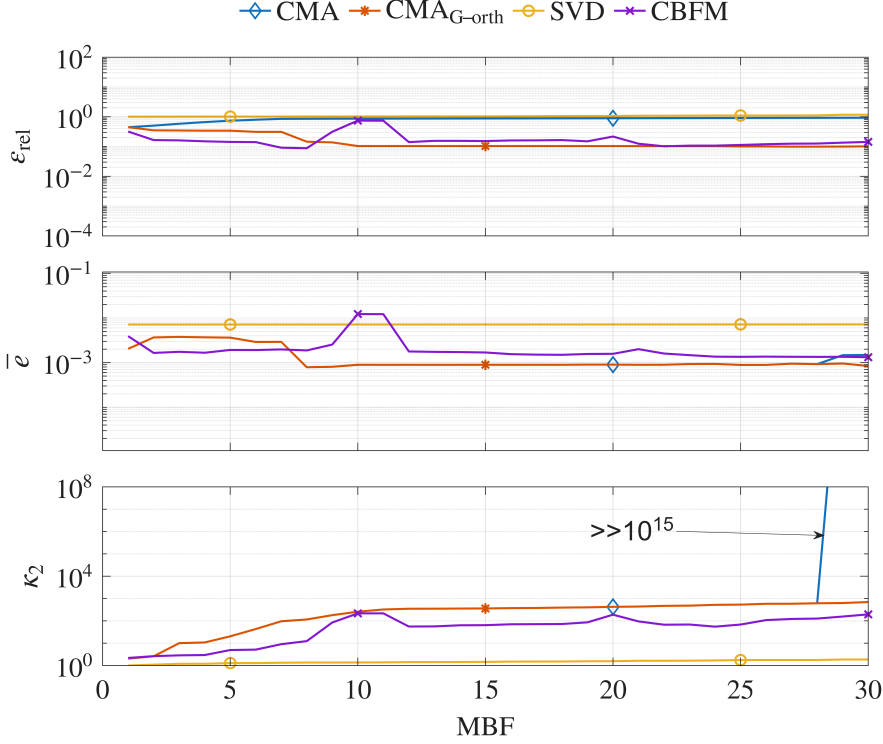


Figure 4: Zoomed 8×8 array comparison of CMA, CMA_{G-orth}, CBFM, and SVD for 0–100 retained MBFs, showing current error, peak-normalized mean far-field error, and condition number.

The different MBF constructions are evaluated for both a single element and an identical-element 8×8 array under identical formulation, discretization, and solver settings. Validation uses a horn illumination located approximately 10.7λ above the array center. The unit-cell geometry is shown in Fig. 2a, and the full-order RWG solution is used as the reference. The operating frequency is 32 GHz, matching the benchmark in [7].

For CBFM, Table 1 summarizes the number of generated candidate responses, generation time, and angular sampling of the plane-wave excitation set. Both θ - and ϕ -polarized incident plane waves are included for each sampled direction. The reported CBFM results use combined primary and enriched CBFs. At the array level, enrichment is restricted to identical-element

Table 2: Minimum number of retained MBFs per element and corresponding reduced dimension K required to reach an array-level far-field error below 10^{-3} . The full-order array system contains $64 \times 330 = 21120$ unknowns.

Method	Retained MBFs	K	$\bar{\epsilon}$	ϵ_{rel}	κ_2
CMA	8	512	0.79×10^{-3}	0.86	116
CMA _{G-orth}	8	512	0.79×10^{-3}	0.15	116
CBFM	120	7680	0.98×10^{-3}	0.26	358
SVD	253	16192	0.96×10^{-3}	0.01	157

arrays, since nonidentical local environments become computationally prohibitive. For the present single-type benchmark, the CMA- and SVD-based bases are generated from the isolated-element matrix, whereas CBFM requires separate response simulations for the plane-wave excitation set and enrichment configurations.

Figures 3a and 3b show the full-range single-element and 8×8 array results, respectively. The single-element results include relative current reconstruction error and the local reduced-block condition number $\kappa_2(\mathbf{D})$, whereas the array results additionally include the peak-normalized mean far-field error. Figure 4 shows a zoomed view of the array results in the practically relevant low-rank range. In all plots, the horizontal axis denotes the number of retained MBFs per element. For the identical-element 8×8 array, this corresponds to $K = 64 \times (\text{retained MBFs per element})$.

For the single-element case, CBFM offers the best accuracy at very low basis sizes, but its error curves are nonmonotonic as additional basis functions are retained. This does not contradict the SVD truncation, since the plotted quantity is the reduced MoM solution error rather than the response-space projection error. Additional retained functions may also affect conditioning and excitation coupling. CMA without explicit G -orthogonalization shows rapidly deteriorating conditioning once 11 MBFs are kept, which is mirrored by poorer current reconstruction than for CMA_{G-orth}. The SVD-based basis is least competitive at very small ranks, but remains the most stable construction as the basis size grows.

The identical-element 8×8 benchmark exhibits the same qualitative ordering, with condition-number trends and current-reconstruction errors largely following the single-element behavior. This is consistent with the decomposi-

tion in Section 2: when the scaled coupling contribution $\mathbf{D}^{-1}\mathbf{C}$ is moderate, the array-level conditioning remains strongly influenced by the local block conditioning $\kappa_2(\mathbf{D})$. The main exception is the nonorthogonalized CMA basis, whose instability is delayed to larger basis sizes, with rapid condition-number growth starting once 29 MBFs are retained. CBFM again provides attractive low-rank compression but remains nonmonotonic as the basis is enriched. By contrast, the far-field error is less sensitive than the current error to the number of retained MBFs, since the radiation operator does not weight all current-error components equally.

Table 2 summarizes the minimum number of retained MBFs per element and the corresponding reduced dimension K required to reach an array-level far-field error below 10^{-3} . It directly quantifies the reduction from the full-order dimension of 21120 unknowns to the reduced dimension required by each MBF construction. The table highlights the practical low-rank tradeoff: CBFM requires more retained MBFs than CMA in this benchmark, whereas SVD requires the largest basis size but yields a well-conditioned reduced system.

Each element aperture supports 330 RWG basis functions. For visual clarity, the results are shown only up to 329 retained MBFs per element, since the full-order case at 330 MBFs is excluded for all methods. At this point, the far-field error reaches the numerical floor and causes clipping in the plots.

4 Conclusion

This letter has presented a controlled benchmark comparison of CMA-, CBFM-, and SVD-based MBF constructions within a common reduced-order framework for cavity-backed reflectarray analysis. Since all methods were evaluated using the same formulation, discretization, partitioning, and solver settings, the observed differences can be attributed primarily to the MBF construction.

For the considered benchmark, CBFM achieves high accuracy with relatively few MBFs, but exhibits nonmonotonic solution-error behavior as the basis is enriched. Explicit G -orthogonalization substantially improves both the conditioning and current-reconstruction accuracy of the CMA basis. The SVD-based basis produces the best-conditioned reduced systems, but requires more MBFs to reach the prescribed far-field accuracy.

The array results follow the main single-element current-reconstruction and condition-number trends, indicating that single-element analyses can serve

as useful screening tools, although they are not substitutes for array-level validation. Overall, the results highlight the tradeoff between compression efficiency, numerical stability, and far-field accuracy in MBF-based reflectarray analysis. Future work will address heterogeneous arrays and reduced-order efficiency.

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**Fast MoM-Based Solvers for Cavity-Backed Reflectarrays and RIS
Design**

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The layout has been revised.

Abstract

This paper presents a fast electromagnetic field data generation framework based on the Method of Moments (MoM) combined with a Domain Decomposition Method and Woodbury identity-based iterative updates. The approach enables rapid recomputation of reflectarray and reconfigurable intelligent surface (RIS) configurations without the need for full matrix reassembly. A Jacobi-type preconditioner is also introduced to accelerate convergence of the Generalized Minimal Residual Method solver for large-scale problems. The proposed techniques significantly reduce computation time, achieving up to 249x speed-up compared to conventional MoM formulations for single element change, while preserving accuracy. Validation on a 5x5 cavity-backed reflectarray demonstrates that the framework efficiently supports localized element updates and is well suited for large-scale dataset generation in deep learning applications. The method provides a scalable foundation for intelligent design and optimization of reflectarray and RIS architectures.

1 Introduction

Reflectarrays and reconfigurable intelligent surfaces (RIS) have emerged as promising technologies for next-generation wireless communication systems, offering unprecedented flexibility in the control of electromagnetic (EM) wave propagation. A reflectarray consists of an array of passive or semi-active elements that impose spatially varying phase shifts to form directive beams, providing phased-array-like capabilities at reduced cost and complexity. RIS extends this concept by employing a grid of tunable unit cells whose reflection or transmission characteristics can be dynamically adjusted in real time. This adaptive control enables efficient manipulation of the propagation environment, improving coverage, spectral efficiency, and energy performance in beyond-5G and 6G networks [1–3].

The electromagnetic modeling and optimization of large-scale cavity backed reflectarray and RIS structures are computationally demanding but they do

bring performance enhancement such as bandwidth and angle-of-arrival insensitivity [4–6]. Such structures require detailed modeling to accurately capture mutual coupling in its array environment. Traditional full-wave solvers, including finite-element and finite-difference methods, often lead to prohibitive computational costs when simulating large arrays with many elements.

The Method of Moments (MoM) has therefore been widely adopted as a compromise between accuracy and efficiency. However, standard MoM formulations require remeshing and partial matrix reconstruction whenever element geometries changes, making them inefficient for modeling the many discrete element states typically employed in reflectarrays and RIS for phase modulation and beam steering [7, 8]. This limitation also hinders their use in generating the large electromagnetic (EM) datasets needed for emerging data-driven optimization methods.

Recent advances in machine learning (ML) and deep learning (DL) have shown strong potential for reflectarray and RIS design and optimization [9–13]. Nevertheless, the success of such models depends on the availability of large, diverse, and accurately labeled EM datasets—resources that are expensive and time-consuming to obtain through conventional full-wave simulations. This motivates the development of specialized MoM-based solvers capable of efficiently generating EM data tailored for ML- and DL-driven array design, such as those introduced in [14, 15].

This paper proposes a fast MoM-based solver framework built on the Domain Decomposition Method (DDM) [14, 15] for efficient EM data generation in cavity-backed reflectarray and RIS configurations. The proposed approach combines physics-based modeling with procedural dataset generation strategies, providing both direct and iterative formulations. The framework bridges the gap between high-fidelity EM modeling and data-driven optimization, enabling rapid and scalable dataset generation for intelligent reflectarray and RIS architectures.

The remainder of this paper is organized as follows. Section 2 describes the proposed numerical framework, detailing the modular update mechanism and solver acceleration schemes. Section 3 presents validation benchmarks and performance comparisons against conventional MoM and DDM implementations. Finally, Section 4 concludes the paper and outlines potential directions for extending the proposed approach to large-scale intelligent surface design and deep learning integration.

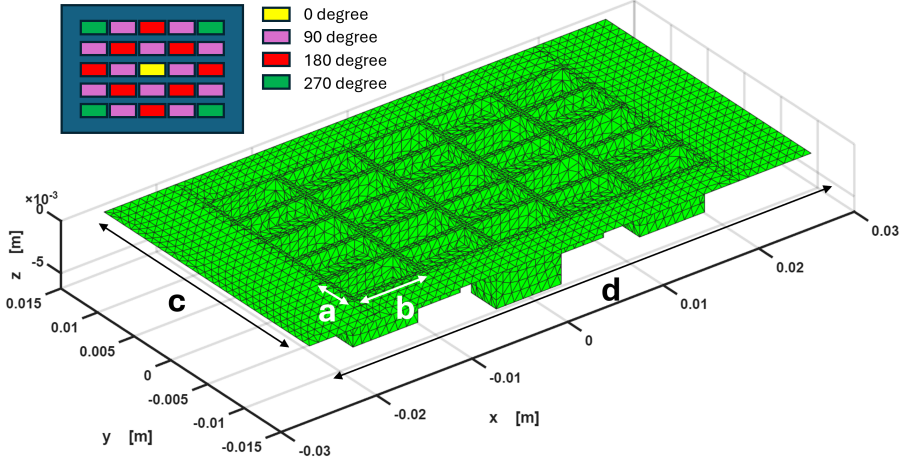


Figure 1: Baseline configuration: a 5×5 reflectarray composed of identical waveguide elements, illuminated by a y -polarized Huygens source placed at a fixed distance of 10 mm from the surface at 32 GHz. The array is modulated according to the scheme shown in the top-left inset, where each color represents a distinct 2-bit phase state of an element. The element (waveguide) dimensions are $a = 3.4$ mm and $b = 6.8$ mm, with depths of [3.231, 1.615, 6.462, 4.846] mm corresponding to phase responses of $[0^\circ, 90^\circ, 180^\circ, 270^\circ]$, respectively. The array dimensions are $c = 14$ mm and $d = 52.5$ mm, and the element spacing is 7.5 mm along x and 4 mm along y .

2 Dataset Generation Concept

A high-quality dataset is essential for training any deep learning or machine learning model. In the case of reflectarrays and reconfigurable intelligent surfaces (RIS), the reflection behavior of each element depends on its geometry or electronic termination, both of which determine the phase response. In this work, we consider the geometric modulation approach, where the unit cell phase is controlled by varying its physical dimensions—such as the element size or cavity depth to achieve a desired phase shift. This approach is particularly relevant for simulation-based data generation, as geometric variations directly alter the array structure and electromagnetic interactions. Nevertheless, the proposed methodology is equally applicable to port-based or electronically

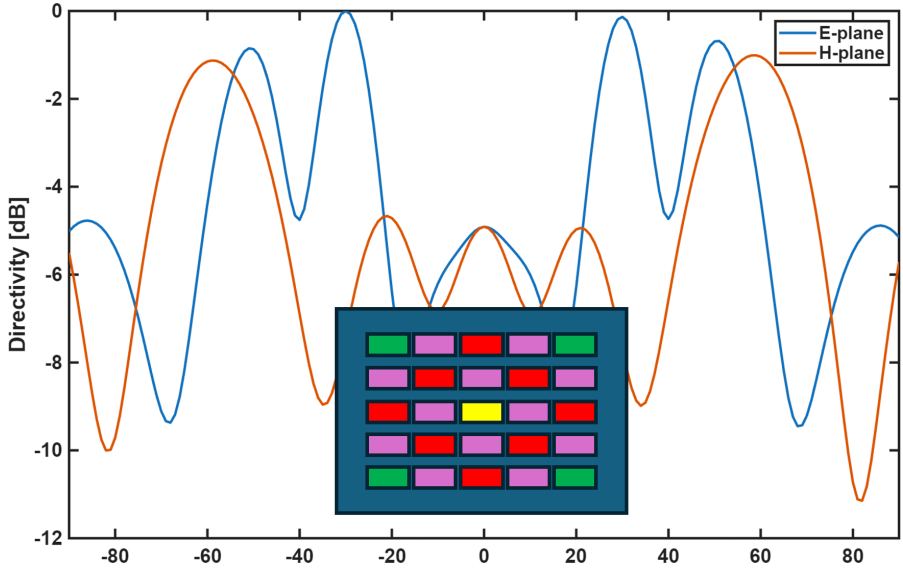


Figure 2: Radiation patterns of a non-optimized modulated baseline configuration.

tuned modulations.

In this work, a simple waveguide-based reflecting element is considered, where the phase response is modulated by varying the cavity depth. The waveguide geometric model provides a controllable and physically interpretable way to represent discrete phase states, which can later be mapped to the learning model.

Figure 1 illustrates the 5×5 array configuration used to generate the dataset. Each element can assume four discrete phase states, resulting in 4^{25} possible combinations, which defines the degrees of freedom of the array. Because phase modulation is achieved through geometric variation, mutual coupling among elements becomes a function of both geometry and position. Therefore, an accurate electromagnetic model is required to correctly represent the array response under all modulation configurations.

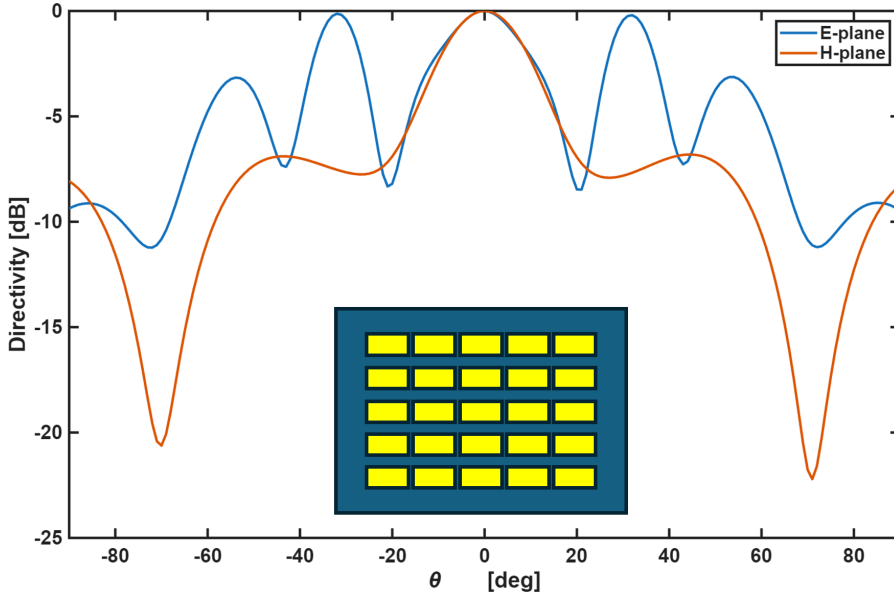


Figure 3: Radiation patterns of the non-modulated baseline configuration, demonstrating a strong change in the pattern due to a different phase response of the elements compared with the modulated configuration.

2.1 Fast Modular MoM Update Framework

The numerical model used for data generation is based on the DDM formulation proposed in [14]. This approach has demonstrated high accuracy, with an average modeling error ranging from 0.76% to 2.21% when compared with a plain MoM solution for complex conformal configurations [16], thereby confirming the high fidelity of the method. The primary objective of this formulation is to compute the equivalent aperture magnetic current for all array elements through the basis-function expansion coefficient vector $\mathbf{V}$. This formulation inherently accounts for both the element geometries and the surrounding array environment when the structure is illuminated by an external source, which gives rise to the excitation vector $\mathbf{I}$:

$$\mathbf{V} = (\mathbf{Y}^a + \mathbf{Y}^b)^{-1} \mathbf{I}, \quad (\text{F.1})$$

where $\mathbf{Y}^a$ and $\mathbf{Y}^b$ denote the admittance matrices associated with the exterior (array) and interior (element) domains, respectively. The exterior domain is efficiently modeled using a physical-optics (PO) formulation, as described in [17]. This method is tailored to cavity-backed elements, assumes zero back current on the array, and considers only the forward radiation of the array.

The exterior admittance $\mathbf{Y}^a$ forms a dense matrix capturing mutual coupling among the N array elements:

$$\mathbf{Y}^a = \begin{bmatrix} \mathbf{Y}_{1,1}^a & \mathbf{Y}_{1,2}^a & \cdots & \mathbf{Y}_{1,N}^a \\ \mathbf{Y}_{2,1}^a & \mathbf{Y}_{2,2}^a & \cdots & \mathbf{Y}_{2,N}^a \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{Y}_{N,1}^a & \mathbf{Y}_{N,2}^a & \cdots & \mathbf{Y}_{N,N}^a \end{bmatrix}. \quad (\text{F.2})$$

In contrast, the interior admittance matrix $\mathbf{Y}^b$ is sparse and block-diagonal where each element exist locally in the matrix because open cavity-based structures only couple via the exterior region:

$$\mathbf{Y}^b = \begin{bmatrix} \mathbf{Y}_1^b & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{Y}_2^b & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{Y}_N^b \end{bmatrix}, \quad (\text{F.3})$$

where $\mathbf{Y}_n^b \in \mathbb{C}^{K \times K}$ represents the self-admittance matrix of the n th unit cell, where K is the total number of magnetic current basis functions per unit cell.

Efficient inverse update via Woodbury identity

A state change in element m , modifies only $\mathbf{Y}_m^b$ to $\mathbf{Y}_m^b + \Delta\mathbf{Y}_m^b$, leaving the remaining submatrices unchanged. Defining the overall system matrix as

$$\mathbf{Y} = \mathbf{Y}^a + \mathbf{Y}^b, \quad (\text{F.4})$$

after an update it becomes

$$\mathbf{Y}' = \mathbf{Y} + \Delta\mathbf{Y}, \quad (\text{F.5})$$

where

$$\Delta\mathbf{Y} = \text{blkdiag}(\mathbf{0}_1, \dots, \mathbf{0}_{m-1}, \Delta\mathbf{Y}_m^b, \mathbf{0}_{m+1}, \dots, \mathbf{0}_N). \quad (\text{F.6})$$

Since $\Delta \mathbf{Y}$ affects only a small local block, it is factorized in low-rank form using the Singular Value Decomposition (SVD) as

$$\Delta \mathbf{Y} = \mathbf{U} \mathbf{S} \mathbf{V}^H, \quad (\text{F.7})$$

where $\mathbf{U}$, $\mathbf{S}$ and $\mathbf{V}^H$ are sparse matrices. Using the Woodbury matrix identity [18], the system inverse can be updated efficiently as

$$(\mathbf{Y}')^{-1} = \mathbf{Y}^{-1} - \mathbf{Y}^{-1} \mathbf{U} (\mathbf{S}^{-1} + \mathbf{V}^H \mathbf{Y}^{-1} \mathbf{U})^{-1} \mathbf{V}^H \mathbf{Y}^{-1} \quad (\text{F.8})$$

For L block updates, the above procedure is repeated for L times, resulting in a total computational cost of $\mathcal{O}(LK^3)$, which is much lower than the $\mathcal{O}((LK)^3)$ cost of recomputing the full inverse for all L element changes simultaneously. This sequential strategy provides significant computational savings and allows reuse of the updated inverse for multiple right-hand-side excitations, such as different feed positions, polarizations, or incidence angles.

Jacobi-type preconditioner for iterative solvers

For cases where iterative solvers such as the Generalized Minimal Residual (GMRES) method are employed, convergence can be further accelerated using a Jacobi-type block-diagonal preconditioner $\mathbf{P}$. Through the strategic choice of modeling the array element as non-overlapping aperture current supports in the array [14, 17], enhances the usage of a Jacobi-type preconditioner [19]. The preconditioner is defined as

$$\mathbf{P} = \begin{bmatrix} (\mathbf{Y})_{1,1}^{-1} & 0 & \cdots & 0 \\ 0 & (\mathbf{Y})_{2,2}^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (\mathbf{Y})_{N,N}^{-1} \end{bmatrix}, \quad (\text{F.9})$$

which is computationally inexpensive, trivially parallelizable, and remains effective across successive block updates. It provides an efficient initialization for GMRES-based solvers, enabling faster convergence when solving large systems and facilitating seamless integration with the far-field pattern evaluation discussed next.

2.2 Pattern Retrieval Framework

The total far-field pattern of a reflectarray or RIS can be expressed as the superposition of the individual contributions from the feed, the specular reflection of the surface, and the response of the reflecting elements:

$$\mathbf{G}_{\text{tot}} = \mathbf{G}_{\text{feed}} + \mathbf{G}_{\text{specular}} + \mathbf{G}_{\text{elements}}, \quad (\text{F.10})$$

The overall far-field can be separated into *static* and *dynamic* components. The static term comprises two parts: $\mathbf{G}_{\text{feed}}$, representing the contribution from the feed radiating in free space, and $\mathbf{G}_{\text{specular}}$, corresponding to surface scattering from the flat reference plane in the absence of magnetic currents (a benefit of the DDM formulation). These components remain constant across different modulation states. The dynamic term, $\mathbf{G}_{\text{elements}}$, represents the contribution of the individual elements and depends on the equivalent magnetic aperture currents, which vary with the element modulation states. The total element contribution can, in turn, be expressed as a superposition of localized magnetic current sources. This representation makes the far-field computation highly parallelizable and well-suited for GPU acceleration, enabling rapid field updates for a large number of modulation combinations.

2.3 Input–Output Definition

For deep learning (DL) model training, the input may consist of radiation patterns, which can be represented either as planar cuts or as full 3D radiation data. The corresponding output could represent the discrete element state configuration of the array. The model may be trained in a one-way fashion: given a desired radiation pattern or envelope, it predicts the element configuration required to reproduce it, as illustrated in Fig. 4. Alternatively, the same framework can be adapted for the inverse problem, where the model predicts the expected radiation pattern from a given element configuration.

However, this paper focuses on the generation of such datasets, which serve as the paired input–output training data for the DL framework.

3 Performance Evaluation

The validation of the simulation performance was carried out by comparing the proposed DDM based solver with a conventional MoM implementation in

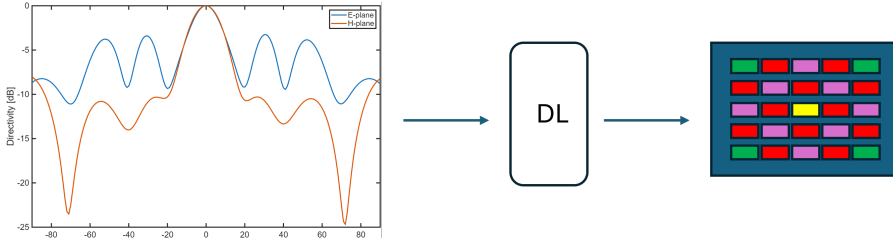


Figure 4: Overview of a possible DL framework. Radiation patterns serve as the input features, while the model output corresponds to the predicted array configuration that reproduces the provided excitation pattern.

MATLAB. Both approaches were evaluated using GMRES and direct inverse solvers, while a Jacobi-type preconditioner and the Woodbury-based update method were additionally tested for performance enhancement. To ensure comparability between direct and iterative methods, a residual tolerance of 10^{-13} is used for the iterative approaches. Furthermore, the primary focus is on simulation time, as the proposed methods do not alter the underlying electromagnetic formulation of the problem. It rather exploits the mathematical properties of the existing formulation to accelerate the simulations, thereby preserving the inherent model accuracy.

The results demonstrate that the Woodbury update applied to a single element modification achieves a speed-up of approximately $18.5\times$ relative to the DDM inverse solver and about $249\times$ compared with the conventional MoM inverse. The Jacobi-preconditioned GMRES solver yields an improvement of roughly $200\times$ over the conventional MoM GMRES and approximately $20\times$ over the DDM GMRES. The Woodbury formulation is particularly effective for partial array updates, where its low-rank structure can be fully exploited. When extended to simultaneous updates of all elements, additional computational overhead associated with repeated block updates is introduced; however, this behavior is expected and does not detract from its strong performance in localized modification scenarios. Similar trends are observed for the larger 8×8 array, where the single-element Woodbury update achieves speed-ups of approximately $44.5\times$ and $535\times$ relative to the DDM inverse and conventional MoM inverse solvers, respectively. For the corresponding iterative solvers, speed-ups of approximately $29\times$ and $163\times$ are obtained compared with the DDM GMRES and MoM GMRES, respectively.

Table 1: Computation time comparison for different solver configurations validated on two array sizes

Method	5×5 (s)	8×8 (s)
Conventional MoM (GMRES)	189.07	1702.50
Conventional MoM (Inverse)	189.31	2261.27
DDM (GMRES)	19.43	306.18
DDM (Inverse)	14.07	188.11
DDM (Precond. GMRES)	0.94	10.44
DDM (Woodbury, all elems.)	17.51	308.32
DDM (Woodbury, single elem.)	0.76	4.22

Overall, the proposed solvers provides a substantial reduction in computation time, making it well suited for large-scale data generation required in DL model training. This improvement directly enhances the feasibility of producing extensive datasets for DL-based reflectarray and RIS optimization. With the observed speed-up, thousands of array configurations can be simulated within practical time frames, enabling efficient and scalable training of data-driven models.

4 Conclusion

This paper presented a fast modular MoM framework that integrates an iterative Woodbury identity-based update scheme and a Jacobi-type preconditioner within a DDM formulation. The proposed approach is particularly suited for multiple right-hand-side problems, such as varying feed positions, polarizations, or illumination angles. The proposed methods were validated using a 25-element array and benchmarked against both DDM and conventional MoM formulations, demonstrating a substantial reduction in computation time over more than 249 and 200 times using Woodbury and Jacobi preconditioned GMRES, respectively. This capability allows efficient modeling of diverse geometric element configurations within complex array environments. Which is particularly valuable for generating large, high-quality EM datasets required for DL model training, without resorting to approximate or surrogate modeling techniques.

Future work will focus on developing adaptive DL models capable of performing beamforming, array thinning, and sparse array optimization in passive cavity-backed reflectarray and RIS systems of varying sizes. In addition, the proposed data generation methodology will be extended to cavity-backed phased array configurations, further emphasizing the generality and scalability of the presented framework.

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