

Friction-Aware Speed Planning with Gaussian Process Uncertainty and Chance-Constrained Model Predictive Control

Ektor Karyotakis, Nikolce Murgovski, Derong Yang, and Mats Jonasson

Abstract—This paper presents a friction-aware speed control framework that regulates braking feasibility at a prescribed risk level under uncertain road friction conditions. Road friction is modelled as a spatial Gaussian Process (GP) whose nonstationary variance reflects perception confidence. Because the friction work over the braking distance is Gaussian under this model, the braking requirement reduces to a one-sided quantile condition on its mean and variance, which is evaluated outside the optimisation. This chance constraint is incorporated into a spatial-domain model predictive controller (MPC) as a terminal condition beyond the prediction horizon, while braking within the horizon is limited by a GP-based lower friction bound. The same formulation applies to both AEB and ACC through a common available-braking-distance input. Monte Carlo simulations under the assumed friction model show that the empirical Hit Rate (failures to meet the braking requirement) follows the prescribed risk level and stays at or below it in AEB, and that a larger comfort weight, a stricter local friction bound and an actuator buffer each add conservatism. The terminal risk level mainly shapes when braking starts, whereas the local friction bound mainly governs the failure frequency. Under deliberate model mismatch, such as rare low-friction patches or overstated envelope coverage, the Hit Rate exceeds the prescribed level. In ACC scenarios with a snow-ice transition, the Hit Rate depends strongly on the assumed lead-vehicle braking capability; reduced camera confidence makes the controller more conservative, whereas occlusion has a minor effect.

Index Terms—Gaussian Process, chance-constrained control, friction uncertainty, model predictive control, Monte Carlo, probabilistic braking, longitudinal vehicle control.

I. INTRODUCTION

IT is well known that road surface and weather variability lead to large fluctuations in road friction, which in turn significantly increases braking-related accident risk. Existing in-production driver assistance systems are not designed to explicitly account for such uncertainty [1]: automated emergency braking (AEB) typically assumes fixed high friction,

while adaptive cruise control (ACC) prioritises comfort over safety-critical braking performance. As a result, the vehicle's current speed may not remain compatible with the available braking distance under adverse or rare road conditions.

Gaussian Process (GP) models are widely used to represent uncertainty within chance-constrained model predictive control (CC-MPC) formulations for autonomous driving. Most works augment a nominal dynamics model with a GP capturing residual nonlinearities, then propagate the resulting uncertainty using linearisation or moment matching to obtain approximately Gaussian state predictions [2]. Chance constraints on states and inputs can be reformulated in closed form under Gaussian assumptions, following classical stochastic MPC methods such as Gaussian-CDF tightening and Boole-based approximations [3]. GPs have also been applied to environment modelling—e.g., road boundaries or obstacles—where the posterior directly informs probabilistic collision-avoidance constraints [4]. However, these approaches do not explicitly represent friction uncertainty as a spatially correlated random field. Continuous road friction measurements show substantial variation along the road [5], while previous work has also represented predictive friction over road position using Gaussian processes [6].

Friction-related effects have also been addressed through GP-based residual learning. In [7], a GP is trained offline from a double lane-change manoeuvre on snow, augmenting a nominal bicycle model with dynamics corrections reflecting low-friction behaviour. While effective under fixed conditions, such offline residual models cannot incorporate spatially predictive friction information or provide braking-feasibility bounds under varying friction.

An alternative line of work handles dynamics uncertainty due to friction by optimising over multiple discrete hypotheses. Contingency MPC [8] computes both a nominal trajectory and a contingency trajectory anticipating events such as a sudden transition to low friction, with both trajectories sharing the first control input to maintain preparedness for the adverse case. Similarly, [9] optimises over several friction hypotheses, enabling aggressive behaviour on straight segments while remaining conservative in curves. These scenario-based approaches, however, rely on a finite set of pre-selected friction values and cannot represent continuous, spatially varying friction uncertainty.

Friction-adaptive control has also been demonstrated in autonomous racing, where repeated laps allow systems to learn spatial friction patterns for the track surface [10]. Likewise,

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[11] employed an ensemble of online GPs, each corresponding to a different friction level, yielding improved tracking when the correct model is identified. Such methods rely on strong and repeated excitation—a realistic assumption in racing but seldom achievable in everyday driving, where available tire-force excitation is limited [12].

Road surface classification (RSC) provides another avenue for friction assessment. LiDAR-based RSC [13] correlates reflectivity, distance, and incident angle with surface type, though performance degrades with range and automotive-grade sensors suffer from sparse point clouds beyond 50 m [14], [15]. Cameras offer longer range [16] but also exhibit distance-dependent degradation, while surface classes must still be converted into usable friction intervals for use in planning. Consequently, perception-based friction estimates remain uncertain, range-dependent, and must be handled explicitly in the control layer. This limitation is also highlighted in [6], which shows that neither RSC nor local slip-based estimation alone is sufficient for traction-adaptive planning, and that fusing the two yields substantially better performance in critical scenarios.

A related effort in [17] used inflated perception-based friction bounds in a robust NMPC scheme for ACC, which resulted in markedly conservative behaviour. To the best of our knowledge, no existing work formulates braking feasibility over spatially correlated friction uncertainty with an explicit, tunable risk level.

Learning-based longitudinal planners have also been proposed, for example combining reinforcement and imitation learning for personalised driving [18] or multi-objective deep reinforcement learning for safe eco-driving [19]. These approaches optimise driving style or efficiency from data, but they treat safety through learned rewards and do not provide explicit braking-feasibility margins under friction uncertainty.

In summary, there is currently no computationally lightweight longitudinal controller that simultaneously (i) represents friction uncertainty using a spatial GP, (ii) imposes a probability bound on braking feasibility, and (iii) operates consistently across both AEB and ACC scenarios. This paper addresses these limitations, and its main contributions are:

- 1) A probabilistic friction-work formulation that expresses braking feasibility through the distribution of the friction available over the whole braking distance, yielding a one-sided chance constraint that accounts for the spatial correlation of friction uncertainty.
- 2) A longitudinal speed-planning and control framework (Fig. 1) that combines the probabilistic post-horizon friction-work (global) constraint with a conservative local bound on friction-limited acceleration. The same MPC formulation applies to AEB and ACC through a common available-braking-distance input.

II. PRELIMINARIES

A. Assumptions

The following assumptions are adopted throughout the analysis:

a) Upstream environment information and friction envelope validity: An upstream Road & Environment Condition Manager (RCM) provides road condition information in the form of an RSC class together with corresponding spatial friction envelopes and a confidence profile (a friction prior), while situation awareness ahead (e.g., stationary obstacles or lead-vehicle state) is provided by the vehicle's object perception. We treat these outputs as an interface: the analysis assumes that the RSC class is correctly identified and that the realised friction lies within its class-associated envelopes, so the reported risk levels are conditional on this information. The perception and data fusion that generate the RCM outputs are outside the scope; given these outputs, we focus on GP-based friction uncertainty modelling and its integration into CC-MPC. The effect of envelopes that do not contain the realised friction is examined in Section VIII-C.

b) Longitudinal centreline abstraction: Friction and vehicle dynamics are modelled only along the path centreline; lateral motion and lateral dynamics are neglected, as are road slope and rolling resistance. This abstraction is sufficient for longitudinal speed planning and braking feasibility assessment on straight roads.

c) Uncertainties included and excluded: The primary source of uncertainty modelled in this work is road friction. Other uncertainties—such as sensing errors in the lead vehicle's position or velocity, or a lead-vehicle braking capability exceeding the assumed upper friction bound (e.g., due to different tyres or load)—are not included in the ACC formulation.

d) Model validity and conditional risk levels: The probability bounds reported in this work are conditional on the adopted friction uncertainty model. Specifically, the prescribed risk levels apply to the braking-feasibility event under the GP model and have a physical interpretation only when the GP mean and uncertainty match or conservatively bound the true friction statistics. They are not hard guarantees of collision avoidance.

e) Definition of failure event: A failure event in this work is defined differently in AEB and ACC but always arises from insufficient braking capability under the realised friction. In AEB, a failure ("Hit") occurs if the ego vehicle does not reduce its speed below the target terminal speed by the designated braking point. In ACC, a failure occurs if the ego vehicle violates the required standstill gap to the lead vehicle at any point along the trajectory. Other failure modes (e.g., lateral instability, perception dropouts, or incorrect object tracking) are outside the scope of this study.

B. Simulation environment

Figure 1 illustrates the simulation architecture together with the proposed friction-aware framework, comprising (i) an upstream Road & Environment Condition Manager (RCM), (ii) a GP-based friction uncertainty estimator, (iii) a supervisory logic, and (iv) the CC-MPC. The remaining blocks (vehicle plant, torque conversion, PI, ABS) belong to the simulation environment and are used only to evaluate closed-loop performance.

Specifically, the RCM provides a prior description of the road friction ahead—as spatial symmetric friction envelopes

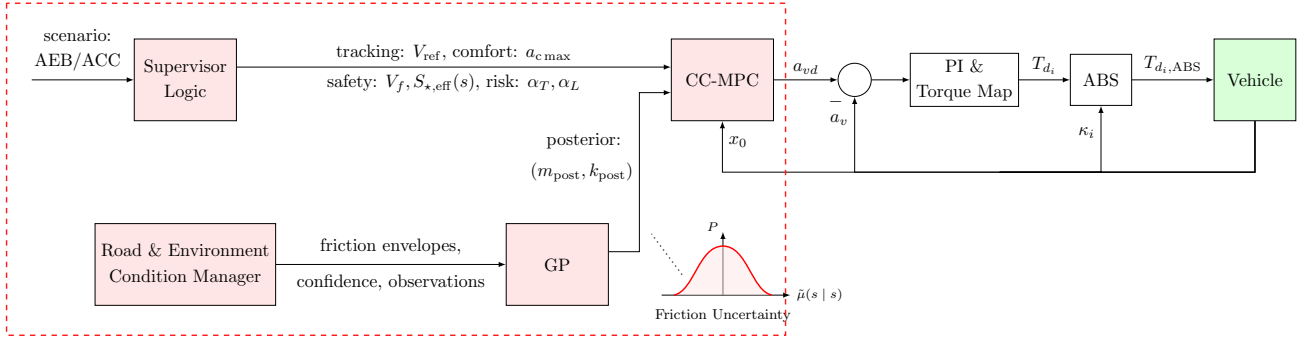


Fig. 1. Block diagram of the simulation architecture including uncertainty-aware friction Gaussian Process (GP) estimation and chance-constrained MPC (CC-MPC). The dashed red outline highlights the proposed friction-aware planning and control framework, consisting of the Road & Environment Condition Manager (RCM), GP estimator, supervisory logic, and CC-MPC.

and a confidence profile—which is passed to the GP-based uncertainty estimator. The resulting GP posterior is provided to the CC-MPC for planning under friction uncertainty. In parallel, object perception supplies situation awareness outputs such as the presence and state of stationary obstacles or a lead vehicle (e.g., relative distance and speed). Based on this information, the supervisory logic generates the CC-MPC references and requirements, including the reference speed, comfort limits, the available braking distance, and an acceptable risk level.

The CC-MPC outputs an acceleration command, which is applied to a double-track vehicle model of a Volvo EX40 with pitch, wheel, and brake actuator dynamics and a Magic Formula Tire (MFT) model; 5-DoF body modelling details are given in [20]. To track the commanded acceleration, a torque conversion followed by a PI controller is used, together with a simple ABS logic that limits wheel torque to prevent excessive slip.

III. PROBLEM STATEMENT

Consider an ego vehicle travelling along the lane centreline with longitudinal position s and speed $V(s)$, and a desired reference speed $V_{\text{ref}}(s)$. Road friction ahead is uncertain and modelled as a spatial GP $\tilde{\mu}(s)$. The objective is to compute a longitudinal speed control policy that tracks V_{ref} subject to comfort and actuation limits, while requiring, at a prescribed risk level, that the ego reduce its speed to a target terminal speed $V_f > 0$ within the available braking distance ahead.

Let $L > 0$ denote the available braking distance from the current position, provided by situation awareness. In AEB, L is computed from the distance to a stationary obstacle, whereas in ACC it is computed from an assumed lead-vehicle braking distance. Thus, the controller formulation is identical in both cases and depends only on L (i.e., the MPC is *mode-agnostic*). Given current speed V_0 and risk level $\varepsilon \in (0, 1)$, the following probabilistic braking condition is required

$$\mathbb{P}(V(L) \leq V_f) \geq 1 - \varepsilon, \quad (1)$$

under the friction GP $\tilde{\mu}(\cdot)$. The above requirement must be achieved without unnecessarily aggressive braking, i.e., while respecting friction-limited actuation, state/input constraints, and comfort.

IV. FRICTION-LIMITED PARTICLE MODEL AT PATH CENTRELINE

The particle's longitudinal dynamics is given by (simplified from [21])

$$\begin{aligned} \dot{V}(t) &= a_v(t) - k_d V^2(t), \\ \dot{a}_v(t) &= \frac{a_{vd}(t) - a_v(t)}{\tau_{av}}, \end{aligned} \quad (2)$$

where V is the vehicle's velocity [m/s], k_d is the aerodynamic coefficient [1/m] modelling the drag force acting on the vehicle, a_v [m/s²] is the tangential (longitudinal) component of acceleration along V , a_{vd} is the acceleration demand, and τ_{av} is the actuator time (lag) constant. The above nonlinear system has the state and input vectors $x = [V \ a_v]^\top$, $u = a_{vd}$.

The dynamic system (2) may also be written in compact form as

$$\dot{x}(t) = f(x, u), \quad (3)$$

where repeated functional arguments, e.g., $f(x(t), u(t)) \rightarrow f(x, u)$, are omitted for brevity.

A. Space-domain formulation

Since friction uncertainty is defined along the road ahead, it is natural to express dynamics in the spatial domain for MPC prediction. The transformation between time and space is given by

$$S_p(t) = \frac{dt}{ds} = \frac{1}{V(t)}, \quad (4)$$

provided the speed remains positive, $V > 0$. Here, $s(t)$ [m] denotes the distance travelled along the path centreline. The dynamics (3) can then be expressed in the space domain as

$$\frac{dx(s)}{ds} = f(x, u) S_p. \quad (5)$$

V. MODELLING ROAD FRICTION AND UNCERTAINTY

Tyre-road friction varies spatially with surface class and environmental conditions, and its estimation is subject to perception uncertainty. Because friction strongly influences braking performance, this uncertainty should be represented explicitly in motion planning. We model the friction coefficient as a spatial GP informed by a perception-based RSC estimator

within the upstream RCM, which provides both a surface class (e.g., snow, wet asphalt, or dry asphalt) and a measure of confidence.

The model reflects two key properties of road friction. First, continuous road friction measurements exhibit structured spatial variation, including localised and repeatable low-friction regions [5]. This motivates representing friction as a spatially correlated process. We use a GP as a convenient probabilistic representation of this dependence, but it is not an experimentally validated model. We adopt a Matérn-3/2 kernel to obtain distance-dependent correlation with finite smoothness [22]. Second, each surface class is associated with characteristic friction levels and physical bounds, typically expressed as lower/upper envelopes. We place the prior mean at the midpoint of these envelopes and use their width to determine the prior variance.

A. Perception-based friction envelopes

The perception module provides a surface class that is matched to friction envelopes along the look-ahead distance. These envelopes encode the expected friction range for the detected surface class and are denoted by $\mu_{\text{envL}}(s)$ and $\mu_{\text{envU}}(s)$ (in this work obtained through a double-sigmoid fit of camera output).

From these envelopes, the GP prior is constructed as follows. The prior mean is taken as the midpoint

$$m(s) = \frac{1}{2}(\mu_{\text{envL}}(s) + \mu_{\text{envU}}(s)). \quad (6)$$

Define the envelope half-width

$$h(s) = \frac{1}{2}(\mu_{\text{envU}}(s) - \mu_{\text{envL}}(s)), \quad (7)$$

and set the prior amplitude

$$a_0(s) = \frac{h(s)}{z_{0.975}}, \quad (8)$$

so that, as a modelling choice, a two-sided 95% Gaussian interval matches the envelope half-width; $z_{0.975}$ is the 97.5th-percentile standard Normal quantile. This choice provides a convenient probabilistic interpretation of the camera-derived envelopes but is not presented as an empirically calibrated coverage probability.

For a camera-based system, the number of pixels covering the road decreases with distance, making near-field friction estimates more reliable than far-field ones. To conservatively capture this loss of epistemic confidence, we inflate the GP variance with distance, acknowledging that the surface class may reasonably change beyond the locally observed region. This affects only *planning*; not the physical variability of friction. To reflect this, we adopt a linear confidence profile

$$c_{\text{cam}}(s) = 1 - \frac{s}{S_{\text{view}}}(1 - c_{\text{view}}) \in (0, 1], \quad s \in [0, S_{\text{view}}], \quad (9)$$

where c_{view} is the confidence at view distance S_{view} . The resulting distance-dependent amplitude is

$$a(s) = \frac{a_0(s)}{c_{\text{cam}}(s)}, \quad (10)$$

which defines the standard deviation of the prior and determines how perceived uncertainty grows with distance ahead.

B. GP Matérn friction model

Let $\tilde{\mu}(s)$ denote the uncertain friction coefficient along the path centreline, modelled as

$$\tilde{\mu}(s) \sim \mathcal{GP}(m(s), k(s, s')), \quad (11)$$

where $m(s)$ is the perception-informed (prior) mean from (6), and $k(s, s')$ is the covariance between friction values at positions s and s' , which is based on the Matérn- $\frac{3}{2}$ kernel

$$k_0(s, s') = \left(1 + \frac{\sqrt{3}}{\ell}|s - s'|\right) \exp\left(-\frac{\sqrt{3}}{\ell}|s - s'|\right). \quad (12)$$

with length-scale $\ell > 0$. To incorporate the distance-dependent uncertainty derived from perception, we scale the kernel amplitude as

$$k(s, s') = a(s)a(s')k_0(s, s'), \quad (13)$$

where $a(s)$ is given by (10). This produces a nonstationary GP whose variance increases with distance, reflecting reduced perception confidence.

a) Posterior GP: Given friction observations (e.g., current position: from wheel slip; farther ahead: cloud data) standard GP regression yields a posterior GP

$$\tilde{\mu}(s) | \mathcal{D} \sim \mathcal{GP}(m_{\text{post}}(s), k_{\text{post}}(s, s')). \quad (14)$$

In regions without friction observations—other than the perception-derived envelopes—which is the typical case ahead of the vehicle, the GP posterior reduces to the prior. In this work, we will only focus on this case; in the following sections, however, all chance constraints use the posterior functions ($m_{\text{post}}, k_{\text{post}}$) for completeness.

b) Application of GP on data: The GP regression with a stationary kernel was applied to offline real-world data acquired from a vehicle performing ABS braking on the Hällered test track, and the results are presented in Fig. 2. For the observations, the longitudinal deceleration was averaged over 5 m spatial bins during ABS activation. The prior bounds were assumed to be ± 0.1 around the expected friction level associated with the known surface class. The resulting posterior indicates that the GP captures the observed spatial trend in this example, provided the prior mean appropriately captures the underlying trend. In practice, friction observations ahead of the ego vehicle may be sparse, in which case the GP naturally reverts towards the prior, thereby producing conservative predictions in unobserved portions of the path.

c) Hyperparameters choice: The GP length scale ℓ was set to 15 m to approximately match the spatial transition scale observed in the Hällered braking data. The signal variance is inferred directly from the perception-derived friction envelopes. These envelopes are wider than the friction range observed in repeated ABS braking on uniform prepared surfaces, which was below 0.1 within a surface, tyre type and speed [23], and thus provide a conservative and physically interpretable prior. A class-level calibration on real roads, however, has not been performed, and the hyperparameters were not tuned for performance.

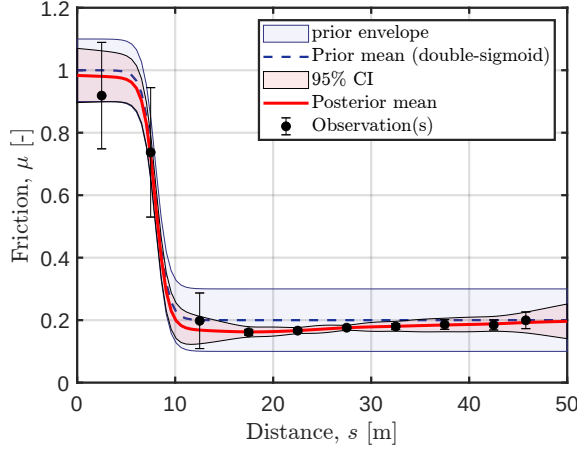


Fig. 2. GPR (Matérn-3/2) with double-sigmoid prior mean applied on offline braking data from Hällered test track.

VI. CHANCE-CONSTRAINED BRAKING FROM ENERGY RELATION

We now derive a probabilistic braking condition corresponding to (1). The derivation uses the work–energy relation in the spatial domain under an idealised friction-limited braking model. For analytical tractability, we neglect actuator dynamics in (2) so that the braking analysis depends linearly on the friction field and the resulting friction-work integral remains Gaussian under the GP model. Aerodynamic drag is also omitted, which is conservative, since drag only adds deceleration.

a) Kinetic energy and work done by friction: Define the kinetic energy per unit mass $[\text{m}^2/\text{s}^2]$

$$K(s) \triangleq \frac{1}{2} V^2(s), \quad (15)$$

as a change of variable $V(s)$. Neglecting aerodynamic drag and actuator dynamics in (2), and assuming friction-limited braking ($a_v(s) = -g \tilde{\mu}(s)$), the kinetic energy (15) satisfies

$$\frac{dK}{ds} = -g \tilde{\mu}(s). \quad (16)$$

Integrating over $[0, L]$ with $L > 0$ yields

$$K(L) = K(0) - W_f(L), \quad W_f(L) \triangleq g \int_0^L \tilde{\mu}(u) du, \quad (17)$$

where $W_f(L)$ is the work done by friction per unit mass $[\text{J/kg}]$ accumulated over distance L . Intuitively, braking is feasible once the work done by friction exceeds the required reduction in kinetic energy.

b) Braking requirement: Define the required kinetic-energy drop

$$K_{\text{req}} \triangleq K(0) - K_f = \frac{1}{2} (V_0^2 - V_f^2). \quad (18)$$

Reaching the target speed V_f by distance L is equivalent to

$$K(L) \leq K_f \iff W_f(L) \geq K_{\text{req}}. \quad (19)$$

c) Gaussianity of the work done by friction: Since $\tilde{\mu}(\cdot)$ is modelled as a GP posterior $\mathcal{GP}(m_{\text{post}}, k_{\text{post}})$, the integral $\int_0^L \tilde{\mu}(u) du$ is a linear functional of a GP and therefore Gaussian. Hence the work done by friction $W_f(L) = g \int_0^L \tilde{\mu}(u) du$ is also Gaussian

$$W_f(L) \sim \mathcal{N}(m_W(L), \sigma_W^2(L)), \quad (20)$$

with mean $[\text{J/kg}]$ and variance $[\text{J}^2/\text{kg}^2]$

$$m_W(L) = g \int_0^L m_{\text{post}}(u) du, \quad (21)$$

$$\sigma_W^2(L) = g^2 \int_0^L \int_0^L k_{\text{post}}(u, u') du du'. \quad (22)$$

For stationary Matérn- $\frac{3}{2}$ kernels with constant amplitude, $\sigma_W^2(L)$ admits the closed-form expression given in Appendix A; for the nonstationary kernel used with distance-dependent perception uncertainty, (22) is evaluated numerically. On a spatial grid, this quadrature reduces to the quadratic form $\mathbf{w}^\top \mathbf{K} \mathbf{w}$ of the posterior covariance matrix with trapezoidal weights $\mathbf{w}$. It is evaluated once per control update outside the optimisation, so the MPC receives only a scalar terminal bound.

d) Chance-constrained braking condition: The chance-constrained braking requirement

$$\mathbb{P}(W_f(L) \geq K_{\text{req}}) \geq 1 - \varepsilon, \quad (23)$$

is equivalent to the one-sided Gaussian quantile constraint

$$m_W(L) - z_{1-\varepsilon} \sigma_W(L) \geq K_{\text{req}}, \quad (24)$$

where $z_{1-\varepsilon}$ is the $(1 - \varepsilon)$ -quantile of the standard Normal distribution. Equivalently,

$$V_0^2 \leq V_f^2 + 2(m_W(L) - z_{1-\varepsilon} \sigma_W(L)) \quad (25)$$

Here, if the right-hand side becomes non-positive, the condition is infeasible for the given distance L , meaning that no admissible initial speed V_0 can satisfy $V(L) \leq V_f$ at the chosen risk level. The resulting one-sided chance constraint forms the global braking requirement, which is embedded as the terminal feasibility condition in the CC-MPC formulation presented next.

VII. CHANCE-CONSTRAINED MPC FOR BRAKING UNDER FRICTION UNCERTAINTY

Friction uncertainty affects braking feasibility through two complementary conditions: a global terminal constraint and a local friction-limited acceleration constraint. The global chance constraint bounds, under the adopted GP model, the probability that sufficient friction work remains to reach a nonzero target speed within the chosen distance. The local constraint uses a conservative pointwise friction bound to maintain feasibility of the planned input. The two conditions address different events and are therefore assigned separate risk parameters.

A. Braking requirement distance for AEB/ACC

Let $S_*(s)$ denote the available distance from the current position s to reach the target speed $V_f > 0$. This quantity is obtained from situation awareness and defines the terminal braking-feasibility requirement. For AEB, $S_*(s)$ is the remaining distance to a slow-moving or stationary obstacle. For ACC, $S_*(s)$ is constructed from an assumed lead-vehicle braking position and the required standstill gap. No separate AEB/ACC terminal logic is needed inside the MPC; only $S_*(s)$ changes. The resulting feasibility statement remains conditional on the validity of the corresponding obstacle or lead-vehicle information.

a) *AEB mode*: In Autonomous Emergency Braking, a stationary obstacle at position s_{obs} yields

$$S_*^{\text{AEB}}(s) = (s_{\text{obs}} - s) - \Delta d_{\min}, \quad (26)$$

where $\Delta d_{\min}$ is a standstill gap. The target speed $V_f > 0$ is required by the spatial-domain formulation and marks the end of the friction-limited braking phase, below which low-speed tyre and ABS effects make friction-based prediction unreliable. The residual stopping distance $V_f^2/(2g\mu)$, at most about 1 m for $V_f = 5$ km/h and $\mu \geq 0.1$, can be included in $\Delta d_{\min}$. The distance $S_*(s)$ may shrink over time as the vehicle approaches the obstacle.

b) *ACC mode*: In Adaptive Cruise Control, the ego vehicle must retain braking feasibility behind a lead vehicle that may decelerate abruptly. Let (s, V) and $(s_{\text{lead}}, V_{\text{lead}})$ denote the positions and speeds of the ego and lead vehicle, respectively, and let $\Delta d_{\min}$ be the required standstill gap. We model lead-vehicle braking using the same idealised work-energy relation as in (17). Let $m_{\text{lead}}(s)$ denote the friction profile assumed when predicting the lead vehicle's braking capability. Using the GP-derived upper friction envelope yields the shortest predicted lead braking distance and is therefore conservative for the ego vehicle. Over the braking distance S_{lead} , the lead vehicle's speed satisfies

$$\begin{aligned} V_{\text{lead}}^2(S_{\text{lead}}) &= V_{\text{lead},0}^2 - 2W_{f,\text{lead}}(S_{\text{lead}}), \\ W_{f,\text{lead}}(S_{\text{lead}}) &= g \int_0^{S_{\text{lead}}} m_{\text{lead}}(u) du. \end{aligned} \quad (27)$$

The distance S_{lead} is obtained numerically by evaluating the cumulative integral of $m_{\text{lead}}(\cdot)$ over a spatial grid and inverting the above work-energy relation. This calculation yields a deterministic prediction of the lead vehicle's braking distance under the assumed braking-capability profile. When occlusion prevents friction estimation beyond the lead vehicle, the current value $m_{\text{lead}}(s_{\text{lead}})$ is held constant over the unobserved segment.

The ego vehicle's available braking distance is then

$$S_*^{\text{ACC}}(s) = ((s_{\text{lead}} + S_{\text{lead}}) - s) - \Delta d_{\min}, \quad (28)$$

which is recomputed at each control update from the current lead state, so the lead's braking manoeuvre is accounted for through S_*^{ACC} .

B. Actuator-lag distance buffer

The chance constraint (24) is derived for idealised instantaneous friction-limited deceleration. To preserve tractability while accounting for actuation lag, we conservatively reduce the available braking distance by a speed-dependent buffer

$$\Delta S_{\text{act}} = V \tau_{\text{buf}}, \quad (29)$$

where $\tau_{\text{buf}} \geq 0$ is a buffer time. A natural choice is the actuator time constant, $\tau_{\text{buf}} = \tau_{av}$, which approximates the delay of the lumped first-order response; larger values add margin when the actual delay exceeds this approximation. The *effective* braking distance is then

$$S_{*,\text{eff}}(s) = S_*(s) - \Delta S_{\text{act}}. \quad (30)$$

The buffer is a heuristic and its effect on the empirical Hit Rate is evaluated in Section VIII-B.

C. Tail chance constraint at the end of the control horizon

Let S denote the MPC control horizon length, and let $\rho \in [0, S]$ denote the local distance along the horizon measured from the current position s . The perception system provides friction estimates over a longer look-ahead interval $[0, S_{\text{view}}]$ with coordinate ρ_v . Both ρ and ρ_v represent forward distance along the path, but the friction posterior $(m_{\text{post}}, k_{\text{post}})$ is defined on the perception coordinate ρ_v and evaluated at absolute positions $s + \rho_v$.

Define the *tail interval*

$$\mathcal{T}(s) = [S, S_{*,\text{eff}}(s)], \quad S_{*,\text{eff}}(s) > S, \quad (31)$$

i.e., the segment beyond the MPC horizon over which the probabilistic target-speed requirement is imposed. Let V_S denote the predicted speed at the end of the horizon. Applying the chance-constrained braking condition (24) to the tail interval yields the terminal condition

$$V_S^2(s) \leq V_f^2 + 2(m_{W,\text{tail}}(s) - z_{1-\alpha_T} \sigma_{W,\text{tail}}(s)), \quad (32)$$

where $\alpha_T \in (0, 1)$ is the chosen *terminal risk level*, and $m_{W,\text{tail}}$ and $\sigma_{W,\text{tail}}$ are the mean and standard deviation of the friction work accumulated over the tail

$$m_{W,\text{tail}}(s) = g \int_{\mathcal{T}(s)} m_{\text{post}}(s + \rho_v) d\rho_v, \quad (33)$$

$$\sigma_{W,\text{tail}}^2(s) = g^2 \int_{\mathcal{T}(s)} \int_{\mathcal{T}(s)} k_{\text{post}}(s + \rho_v, s + \rho'_v) d\rho_v d\rho'_v. \quad (34)$$

For stationary Matérn- $\frac{3}{2}$ kernels with constant amplitude, the closed-form expression in (43) may be evaluated with the corresponding tail length; otherwise (34) is computed numerically.

Constraint (32) imposes probability at least $1 - \alpha_T$ that the friction work available from the end of the MPC horizon to the target point exceeds the remaining kinetic-energy reduction required to reach V_f . As stated in the assumptions, this probability is defined under the adopted GP model.

D. Discrete MPC formulation

The spatial horizon $[s, s + S]$ is discretised into N intervals of length $\delta s = S/N$ with nodes $\rho_i = i \delta s$, $i = 0, \dots, N$. At node $s + \rho_i$, the MPC state and input are $x_i = [V_i \ a_{v,i}]^\top$ and $u_i = a_{vd,i}$. The dynamics are discretised using RK4 over δs , denoted by $x_{i+1} = F(x_i, u_i)$.

1) *Acceleration/Braking Comfort*: Comfort is handled by a blended constraint that allows the controller to exceed the comfort limit $a_{c \max}$ only when the slack $\epsilon_{a,i}$ is activated

$$|a_{v,i}| \leq a_{c \max} + \left(m_{\text{lower},i} g - a_{c \max}\right) \epsilon_{a,i}, \quad (35)$$

where $a_{c \max} = 2 \text{ m/s}^2$, and $0 \leq \epsilon_{a,i} \leq 1$; the slack is penalised in the cost. Friction feasibility is enforced through a hard bound on the commanded acceleration

$$|u_i| \leq g m_{\text{lower},i}, \quad (36)$$

using the GP *lower* bound

$$m_{\text{lower}}(\cdot) = m_{\text{post}}(\cdot) - z_{1-\alpha_L} \sqrt{k_{\text{post}}(\cdot, \cdot)}, \quad (37)$$

where $\alpha_L \in (0, 1)$ is the pointwise lower posterior quantile parameter used to define the conservative in-horizon friction margin. The corresponding *upper* bound, used for the lead vehicle's braking, is defined symmetrically as

$$m_{\text{upper}}(\cdot) = m_{\text{post}}(\cdot) + z_{1-\alpha_L} \sqrt{k_{\text{post}}(\cdot, \cdot)}. \quad (38)$$

The local parameter α_L controls the pointwise friction margin used to ensure in-horizon actuation feasibility, whereas α_T specifies the risk level of the global tail-work event in (32). These parameters are treated separately because the local and global constraints address distinct feasibility events.

2) *Terminal chance constraint*: When an effective tail interval exists, i.e., $S_{*,\text{eff}}(s) > S$, terminal feasibility is enforced via the tail chance constraint (32). If the required braking point lies within the prediction horizon, $S_{*,\text{eff}}(s) \leq S$, no tail segment remains and we instead impose the deterministic in-horizon braking condition $V(\rho^*) \leq V_f$ with $\rho^* = S_{*,\text{eff}}(s)$ (implemented at the nearest grid node at ρ^*).

When the remaining distance is too short (e.g. $S_{*,\text{eff}} < 10$ m), the controller switches to deterministic full braking $u_i = -g m_{\text{lower},i}$ until the target speed V_f is reached to avoid infeasible MPC solutions. Although one could instead command full braking and rely on ABS for potentially higher deceleration, this strategy is not pursued in this work: the empirical Hit Rate is intended to reflect the planned friction margins set by α_T and α_L . In a production system, such a fallback would act as a separate safety layer and would further reduce the failure rate.

3) *Speed constraints*: The speed is bounded between a lower speed $V_{\min} > 0$ to avoid the space-domain singularity and a maximum speed $V_{\max,i} > V_{\text{ref},i}$ that may be dynamic

$$V_{\min} \leq V_i \leq V_{\max,i}. \quad (39)$$

4) *Cost and MPC formulation*: The cost function balances reference-speed tracking, control effort, and comfort. Tracking performance is weighted by W_V , control effort by W_U , and comfort violations by W_a through the slack variables $\epsilon_{a,i}$. A jerk regularisation term with weight W_j promotes smooth control variations along the horizon. The discrete MPC formulation is given by

$$\begin{aligned} \min_{\{x_i, u_i, \epsilon_{a,i}\}} \quad & J = W_V (V_N - V_{\text{ref},N})^2 \delta s \\ & + \sum_{i=0}^{N-1} \left[W_V (V_i - V_{\text{ref},i})^2 + W_a \epsilon_{a,i}^2 + W_U u_i^2 \right] \delta s \\ & + \sum_{i=0}^{N-2} W_j (u_{i+1} - u_i)^2 / \delta s \end{aligned} \quad (40a)$$

$$\text{s.t. } x_{i+1} = F(x_i, u_i), \quad (40b)$$

$$V_{\min} \leq V_i \leq V_{\max,i}, \quad (40c)$$

$$|a_{v,i}| \leq a_{c \max} + \left(m_{\text{lower},i} g - a_{c \max}\right) \epsilon_{a,i}, \quad (40d)$$

$$|u_i| \leq g m_{\text{lower},i}, \quad (40e)$$

$$V_N^2 \leq V_f^2 + 2 \left(m_{W,\text{tail}} - z_{1-\alpha_T} \sigma_{W,\text{tail}} \right) \quad (40f)$$

$$x_0 = x_{\text{init}}. \quad (40g)$$

The nonlinear program is solved using multiple shooting in CasADi [24] with IPOPT [25].

Unless stated otherwise, the numerical studies set $\alpha_L = \alpha_T$, so that a single parameter governs conservatism. No analytical mapping from (α_T, α_L) to an overall braking-failure probability is derived; instead, the effect of decoupling the two levels is evaluated empirically in Section VIII-B.

VIII. NUMERICAL EVALUATION

A. Matched-model AEB verification

In this section, we verify the proposed friction-aware framework through Monte Carlo (MC) simulations under a braking scenario with spatially uniform friction uncertainty. Because the friction prior (mean and envelope) is constant along distance in this test case, the local bound is a scalar and the terminal chance constraint is a quantile condition on a stationary process. This study verifies the construction under matched friction statistics. The friction model should still be validated against real-world data (see Sections VIII-C and IX).

In each simulation, a random friction curve is sampled while the MPC plans using the corresponding lower-quantile friction bound ($\alpha_L = \alpha_T$), which is scalar because the bound is constant over distance. The objective is to reach the target speed within the available distance. A Hit is recorded when the vehicle fails to reach that speed at the designated point; sustained realisations below the assumed bound are a principal cause. Over many MC trials, the Hit Rate is expected to approach the common risk level $\alpha_L = \alpha_T$, up to finite-sample variation, control-plant and actuator-model mismatch, and the influence of the MPC horizon and cost terms.

TABLE I
MONTE CARLO RESULTS FOR A DAMP ASPHALT BRAKING SCENARIO
($N = 1000$). STANDARD ERROR $\sqrt{\hat{p}(1 - \hat{p})/N}$ OF THE HIT RATE IN
PARENTHESES.

Risk $\alpha_L = \alpha_T$	Hit Rate [%]	Pre-braking Speed [km/h]	Friction Bound [-]	Impact Speed [km/h]
1%	0.7 (0.3)	57.5	0.56	9.9
5%	3.2 (0.6)	59.5	0.63	13.1
10%	6.2 (0.8)	60.5	0.67	13.7
50%	35.3 (1.5)	64.5	0.8	16.1
99%	94 (0.8)	69	1.04	22.4

We use 1,000 Monte Carlo runs, which provide sufficient resolution for this verification study (for a 10% event probability, the standard error is $\sigma_{MC} = \sqrt{p(1-p)/N} \approx 0.9$ percentage points). The scenario conditions are: AEB mode, specified braking distance $S_* = 30$ m, horizon distance $S = 10$ m with step $\delta s = 0.5$ m, reference speed $V_{ref} = 70$ km/h, initial speed $V_0 = 60$ km/h, and a damp road with $\mu_{mean} = 0.8$, $\mu_{lower} = 0.6$, $\mu_{upper} = 1$. Braking begins at 50 m to allow initial conditions to settle, and the vehicle targets $V_f = 5$ km/h at 80 m, i.e., 30 m ahead. Weights are set as: $W_V = 1$, $W_U = 0.25$, $W_a = 10$, and $W_j = 10 \delta s^2$. Further, the distance buffer in (30) is set to zero so that the Hit Rate includes the effect of ignoring actuator and vehicle-body response times.

The simulations are implemented in MATLAB using CasADi for nonlinear optimisation and IPOPT as the solver, and run on an Intel Core Ultra 7 165U processor (1.70 GHz) and 32 GB of RAM, with the average solve time per MPC iteration in 20–30 ms.

Table I reports Monte Carlo results showing that, with $\alpha_L = \alpha_T$, the Hit Rate follows the prescribed risk ordering and stays below it, under the examined MPC settings. Additional conservatism can be achieved by increasing the comfort weight or enlarging the actuator distance buffer, as quantified in Section VIII-B.

The table also lists the average cruising speed prior to initiating braking, defined from here onward as the pre-braking speed. Its magnitude reflects both the assumed friction bound and the cost weights used in the MPC objective. For the displayed friction realisation, the difference in pre-braking speed between the 1% and 10% risk settings is approximately 3 km/h. Finally, the average impact speed for the hits is included (reported at the braking point), which relates directly to crash severity and can be mapped to published injury-risk curves by interested readers.

An example in which the vehicle fails to reach the target speed is presented in Fig. 3. Braking starts 30 m before the target, and the vehicle initially remains below the feasible-speed envelope; once the realised friction falls below the assumed bound, the speed exceeds this envelope and the target speed is not reached.

B. Sensitivity to CC-MPC parameters

The qualitative effect of each CC-MPC parameter on the empirical Hit Rate needs to be clarified. To do this, additional Monte Carlo runs were conducted in an AEB scenario with

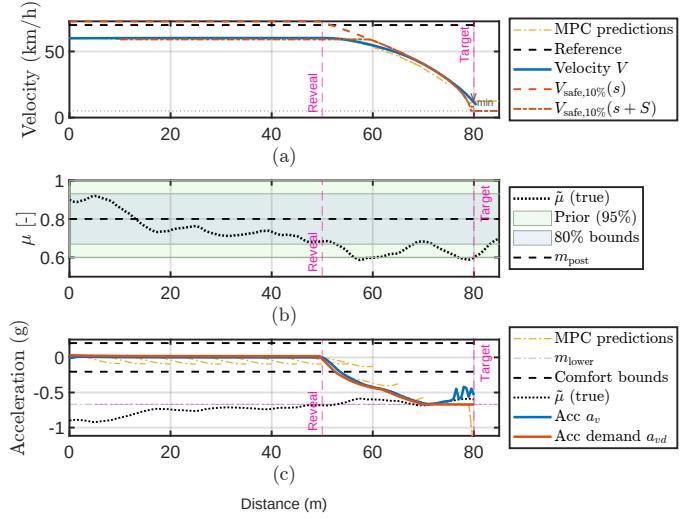


Fig. 3. Example of a failed braking manoeuvre under uncertain friction ($\alpha_T = 10\%$), where friction falls below the assumed bound. (a) Speed (blue), MPC predictions every 10 m (yellow), feasible-speed envelope (red dashed) and terminal speed bound at the horizon end (red dash-dotted); (b) friction profile along the route; (c) longitudinal acceleration.

a longer braking distance: $S_* = 100$ m, horizon $S = 20$ m, and otherwise the same conditions as in Table I. Each setting uses $N = 200$ runs with identical friction realisations, so settings are compared pairwise using the exact sign test on the b runs with a hit under only one setting ($p = 2 \cdot 0.5^b$ when all differ in the same direction). Unless specified otherwise, the settings are $\alpha_T = \alpha_L = 30\%$, $W_a = 10$, without an actuator buffer. A risk level of 30% was chosen so that differences become statistically visible with a moderate number of runs. The results are summarised in Table II.

a) Common risk level: With $\alpha_T = \alpha_L$ varied between 5% and 70%, the Hit Rate increases monotonically with the prescribed risk and stays below it at every level (2% to 39.5%). As in the short-distance case (Table I), the combined effect of the finite horizon, the comfort penalty and the pointwise lower friction bound makes the closed loop conservative with respect to the prescribed terminal risk.

b) Comfort weight: Increasing the comfort weight from $W_a = 0$ to $W_a = 100$ reduced the Hit Rate from 18.5% to 8.5% ($p < 10^{-5}$). The difference between $W_a = 0$ and $W_a = 10$ (18.5% vs. 17%) was, although declining, within sampling error. A larger comfort penalty therefore adds conservatism to the closed loop, because braking is initiated earlier and milder.

c) Local versus terminal risk: With $\alpha_T = 30\%$ fixed, decreasing the local risk level α_L from 50% to 5% reduced the Hit Rate from 25.5% to 3%. A smaller α_L forces the MPC to plan with lower friction-bounded deceleration, and consequently, to brake earlier. Conversely, with $\alpha_L = 10\%$ fixed, increasing α_T from 1% to 50% increased the Hit Rate only slightly from 4% to 5.5%, while braking initiated later (from 42.5 m to 36 m before the target). The two levels thus act on different parts of the manoeuvre: α_T decides when the vehicle starts to brake, whereas α_L sets the margin of the in-horizon braking, which here, mainly determines the Hit Rate. For the edge case $[\alpha_L = 1\%, \alpha_T = 30\%]$ all runs failed, as

TABLE II

SENSITIVITY OF THE HIT RATE TO CC-MPC PARAMETERS (AEB, $S_* = 100$ M, $S = 20$ M, $N = 200$). DEFAULT: $\alpha_T = \alpha_L = 30\%$, $W_a = 10$, $\tau_{\text{buf}} = 0$. STANDARD ERROR $\sqrt{\hat{p}(1-\hat{p})/N}$ IN PARENTHESES.

Varied parameter	Value	Hit Rate [%]
$\alpha_T = \alpha_L$	5% / 10% / 30% 50% / 70%	2 (1) / 4 (1.4) / 17 (2.7) 26 (3.1) / 39.5 (3.5)
W_a	0 / 10 / 100	18.5 (2.7) / 17 (2.7) / 8.5 (2)
α_L ($\alpha_T = 30\%$)	5% / 10% 30% / 50%	3 (1.2) / 5 (1.5) 17 (2.7) / 25.5 (3.1)
α_T ($\alpha_L = 10\%$)	1% / 5% / 10% 30% / 50%	4 (1.4) / 4 (1.4) / 4 (1.4) 5 (1.5) / 5.5 (1.6)
τ_{buf} [s]	0 / 0.07 ($= \tau_{av}$) 0.25 / 0.5	17 (2.7) / 5 (1.5) 0 / 0

the tail constraint admitted a speed that could not be reduced to V_f under the much lower local deceleration bound, making the MPC infeasible as the target approached. Very small α_L relative to α_T should therefore be avoided.

d) *Actuator buffer*: Introducing the buffer (29) with $\tau_{\text{buf}} = \tau_{av} = 0.07$ s (approximately 1.4 m at the 70 km/h pre-braking speed) reduced the Hit Rate from 17% without a buffer to 5%, while no hits were recorded for $\tau_{\text{buf}} \geq 0.25$ s. The actuator and vehicle body dynamics neglected in the idealised braking model are responsible for part of the Hit Rate. Adding a small distance buffer compensates for their effect in the examined scenario.

C. Robustness to friction model mismatch

The preceding results use the same world-friction and controller-assumed GP model. Since the reality might differ, the consequences of a friction model mismatch need to be quantified. Two mismatch cases were simulated in the long AEB scenario of Section VIII-B with $\alpha_T = \alpha_L = 10\%$, $W_a = 10$ and $\tau_{\text{buf}} = 0$ (with $N = 200$ under identical underlying friction realisations). The controller remains unaware of the mismatch. The following cases are examined: (i) *rare low-friction patches*: patches of 5 m length with $\mu = 0.3$, i.e., half the lower envelope, are placed at random within the braking zone at a mean rate λ_p per 100 m; and (ii) *overstated envelope coverage*: the spread of the realised friction around the mean is scaled by $k_\sigma \geq 1$ relative to the assumed GP, so that the nominal 95% envelope contains only $2\Phi(1.96/k_\sigma) - 1$ of the realisations.

Table III shows that the Hit Rate increases monotonically in both cases. In the first case, a patch rate of $\lambda_p = 0.5$ per 100 m—at least one patch lies in the braking zone in about 39% of the runs—raised the Hit Rate from 4% to 14%, which is above the prescribed risk of 10%. Not every patch caused a hit, since its effect depends on where it occurs relative to the braking phase. In the second case, an envelope with an actual coverage of 88% instead of 95% raised the Hit Rate to 13%, and a coverage of 67% to 28%. Notably, the latter envelope means the friction coefficient can lie between 0.4–1.1 (capped), which is essentially a snow–dry asphalt coverage; a quite rare case. Unsurprisingly, the envelope needs to cover at

TABLE III

HIT RATE UNDER FRICTION MODEL MISMATCH (AEB, $S_* = 100$ M, $S = 20$ M, $\alpha_T = \alpha_L = 10\%$, $W_a = 10$, $N = 200$). STANDARD ERROR IN PARENTHESES.

Mismatch	Setting	Hit Rate [%]
None (matched)	–	4 (1.4)
Low-friction patches	$\lambda_p = 0.5$ per 100 m $\lambda_p = 1$ per 100 m $\lambda_p = 2$ per 100 m	14 (2.5) 23 (3) 37 (3.4)
Envelope coverage (k_σ)	88% ($k_\sigma = 1.25$) 81% ($k_\sigma = 1.5$) 67% ($k_\sigma = 2$)	13 (2.4) 15.5 (2.6) 28 (3.2)
Remedy: $\alpha_T = \alpha_L = 5\%$	88% coverage	6.5 (1.7)

least its nominal fraction (95%) of the real friction range for the prescribed risk level not to be exceeded. When uncertain, a lower common risk level $\alpha_L = \alpha_T$ or wider envelopes add conservatism. As demonstrated in the last row of the table, lowering the risk level to 5% under the 88% coverage case reduced the Hit Rate from 13% to 6.5%, compensating for the understated prior spread.

D. ACC application under spatially varying friction uncertainty

In this scenario, we model a transition from snow to ice in ACC mode. The lead vehicle brakes at a predetermined position using the true friction limit, while the ego vehicle brakes according to the conservative lower friction bound prescribed by the MPC design. Then, we make a sensitivity study of three factors that affect the controller’s performance and evaluate the Hit Rate.

The simulation conditions are: 1,000 Monte Carlo runs, ACC mode, horizon length $S = 10$ m, reference speed $V_{\text{ref}} = 70$ km/h, ego initial speed $V_0 = 70$ km/h, initial ego–lead distance 50 m, and lead speed $V_{\text{lead}} = 60$ km/h. Road friction follows a snow–ice transition with a mean profile $\mu_{\text{mean}}(s) : 0.4 \rightarrow 0.2$ and symmetric bounds of ± 0.1 around the mean. As before, the distance buffer in (30) is set to zero so that the observed Hit Rate reflects purely the effect of friction uncertainty (actuator and body dynamics are intentionally not compensated here). The ego risk level is set to 10%. The lead vehicle follows the particle model time dynamics (2), begins braking at 175 m, and stops at $V_f = 5$ km/h. A simulation example of a successful stop is presented in Fig. 4.

E. Sensitivity study in ACC mode

We consider three categorical factors: camera confidence, occlusion, and the assumed lead-vehicle friction bound, yielding $2^3 = 8$ scenarios. In this context, occlusion is defined as the situation in which friction information is available only up to the position of the lead vehicle; consequently, the most recently observed friction value is assumed to remain valid for the road segment ahead.

Table IV reports the Monte Carlo Hit Rates, grouped by camera confidence, occlusion, and the assumed lead braking friction. Under the posterior-mean (here the mean-profile due

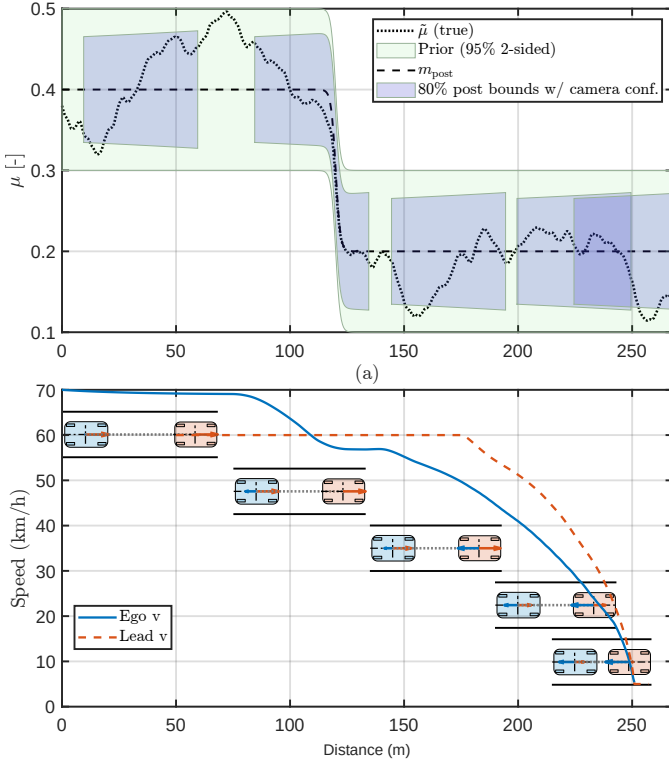


Fig. 4. Simulation example in ACC mode. Camera-based friction envelopes and vehicle states plotted at five positions along the route. (a) True friction bounds together with the GP posterior envelopes and the camera-based confidence profile (shown over a 50 m segment for clarity). (b) Ego (blue) and lead (red) vehicle states at the same five positions. Vehicle outlines are scaled for visualisation and do not represent actual size. Blue arrows indicate longitudinal force vectors and red arrows denote velocity vectors

to no observations) friction assumption for the lead vehicle, the MPC yields a high Hit Rate. With the mean assumption, the lead's predicted braking distance is too long whenever it brakes on friction above the mean, so S_{*}^{ACC} is overestimated and the ego plans too little margin. The ACC result is therefore conditional on the adopted lead-vehicle braking assumption: the computed available distance is valid only when the assumed lead braking position is conservative for the realised manoeuvre. In contrast, when the lead braking capability is modelled using the conservative GP upper friction bound, the MPC Hit Rate is close to the selected terminal risk level (10.8% and 11.2%, within about one standard error of 10%). These results show how standstill-gap violations depend on the friction model and the lead-vehicle braking assumption.

The pre-braking relative distance (reported when the lead starts braking) and the relative impact speed (reported at the first violation of the standstill gap) are also reported and illustrate the conservativeness and risk of the controller. Larger distances before initiating braking generally correspond to lower Hit Rates and larger impact speeds.

Although not designed under a specific jerk limit, for this examined case, the controller achieves an RMS jerk about 1 m/s^3 and the highest 95th percentile lies around 3 m/s^3 which is within typical comfort ranges for ACC.

TABLE IV
HIT RATE FROM 1000 MONTE CARLO SIMULATIONS IN ACC MODE UNDER A SNOW-ICE TRANSITION SCENARIO, GROUPED BY THREE FACTORS: CAMERA CONFIDENCE, OCCLUSION, AND LEAD-VEHICLE BRAKING LIMIT ASSUMPTION

Occlusion	m_{lead} assumed bound	Hit Rate [%]	Pre-braking relative distance [m]	Relative impact speed [km/h]
Camera confidence = 100%				
No	Upper	10.8 (1.0)	33.6	8.1
No	Mean	89.7 (1.0)	28.7	16
Yes	Upper	11.2 (1.0)	34.1	8
Yes	Mean	90.2 (0.9)	29.1	16
Camera confidence at $S_{\text{view}} = 60\%$				
No	Upper	4.1 (0.6)	35.2	6.3
No	Mean	80.4 (1.3)	29.3	14.3
Yes	Upper	4.9 (0.7)	35.3	6.7
Yes	Mean	81 (1.2)	29.8	14.4

IX. DISCUSSION

The proposed friction-aware framework (Fig. 1) provides a structured way to reason about friction uncertainty in longitudinal control. Key insights follow:

a) Adaptive risk tuning in AEB: Adjusting the specified braking distance and risk level in AEB mode naturally yields different speed profiles adapted to upcoming friction and traffic context. For example, around school areas one would impose short braking distances and near-zero risk, whereas on a country road a larger braking distance and a nonzero risk level may be acceptable.

b) MPC cost influences braking behaviour: Control effort and comfort penalties make the controller more conservative, leading to earlier and milder braking and lower Hit Rates (Section VIII-B). The prediction horizon also shapes closed-loop behaviour through preview, as a longer horizon allows deceleration to be distributed in anticipation of the terminal requirement. The cost structure and horizon length jointly tune the trade-off between comfort and the empirical braking-feasibility margin.

c) Extension to lateral motion: Although this work focuses on friction variation along the path centreline, the framework can be extended to lateral manoeuvres by augmenting the vehicle model to two spatial dimensions (s, e) as in [21] and defining a GP over $\mathbb{R}^2$ (e.g., a Matérn kernel), yielding a spatially varying $\tilde{\mu}(s, e)$ suitable for combined braking and steering.

This study is limited to:

d) Failure-mode coverage: Only longitudinal infeasibility is quantified; lateral failure modes are not accounted for, except indirectly through speed reduction before curves (cf. [17]).

e) Perception-to-friction mapping: The coupling between the RSC output and the friction envelopes used in this work is based on offline exploration and has not yet been

validated in real-world driving. Nevertheless, the proposed approach is generic: it remains applicable to any friction-range specification and any sensor-confidence model, provided that these can be represented as spatial envelopes for the GP prior.

f) *Conditional probability bounds*: As with any chance-constrained scheme, the braking-feasibility probability applies only under the assumed GP prior and noise model. Unmodelled perception errors, such as misclassification, and friction distributions outside the prior are not explicitly handled. Interpreting α_T as a real-world risk would require calibrating the GP uncertainty, in particular the probability of rare low-friction events, against independent friction measurements; the consequences of a miscalibrated prior are illustrated in Section VIII-C.

g) *Computational aspects and experimental validation*: While the MPC solution times are lightweight for the scenarios tested, further work is needed to benchmark the framework on embedded hardware and to validate it experimentally on a vehicle.

h) *Closed-loop friction estimation*: The framework assumes no friction observations ahead of the vehicle. Incorporating online friction updates in closed loop, and propagating their effect through the chance constraints, is left for future work.

X. CONCLUSION

This paper presented a friction-aware longitudinal speed-planning framework for AEB and ACC that regulates braking feasibility under uncertain road friction at a prescribed risk level. Perception-derived road surface information is mapped to spatial friction envelopes and embedded in a spatial Gaussian Process model, while the probability of sufficient remaining friction work is expressed through a one-sided chance constraint derived from the work–energy relation and integrated into a spatial-domain model predictive controller through a terminal tail constraint.

The framework was verified through Monte Carlo simulations that compared empirical Hit Rates with the prescribed model risk under matched stochastic assumptions. In AEB, the Hit Rates followed the expected risk ordering and remained below the prescribed level, owing to the finite horizon, comfort and control-effort penalties, and the vehicle plant. Increasing the comfort weight, decreasing the local risk level or adding an actuator buffer each reduced the Hit Rate further, making these explicit tuning parameters for conservatism. The Hit Rate should therefore be viewed as an empirical indicator rather than a hard bound on collision risk. Separate sweeps showed that the terminal level α_T mainly shapes braking onset, while the local level α_L largely determines the Hit Rate in the examined scenario. Furthermore, when friction was deliberately mismatched, through rare low-friction patches outside the envelope or an envelope with lower coverage than assumed, the Hit Rate exceeded the prescribed risk, confirming that the risk level is meaningful only if the friction prior is calibrated.

In the ACC snow–ice scenarios, the dominant factor affecting standstill-gap violations was the assumed braking capability of the lead vehicle. Using a conservative upper friction

bound produced Hit Rates close to the selected model risk in the examined cases, whereas a mean-friction assumption was overly optimistic and led to frequent violations. Reduced camera confidence increased planning conservatism and lowered Hit Rates by tightening the GP-based friction bounds, while occlusion had a comparatively minor effect in the examined scenarios.

We conclude that the proposed global chance constraint on the kinetic energy–work relation combined with GP lower-bound constraints on in-horizon braking offers a transparent approach for friction-aware longitudinal control, and can serve as a foundation for future extensions including steering manoeuvres and closed-loop friction estimation.

APPENDIX

For stationary Matérn- $\frac{3}{2}$, set $k(s, s') = \kappa(|s - s'|)$, $\phi = |s - s'|$, (12) becomes

$$\kappa(\phi) = \sigma^2(1 + \iota\phi) \exp(-\iota\phi), \quad \iota = \frac{\sqrt{3}}{\ell} \quad (41)$$

For the work done by friction $W_f(L) = g \int_0^L \tilde{\mu}(u) du$, the variance is

$$\sigma_W^2(L) = g^2 2 \int_0^L (L - \phi) \kappa(\phi) d\phi. \quad (42)$$

Carrying out the convolution yields the closed form

$$\sigma_W^2(L) = g^2 \frac{2\sigma^2}{\iota^2} \left[2\iota L - 3 + (\iota L + 3)e^{-\iota L} \right]. \quad (43)$$

CONFLICT OF INTEREST

Ektor Karyotakis and Derong Yang are employees of Volvo Car AB (Volvo Cars). Mats Jonasson is affiliated with Chalmers University of Technology and works part time as a consultant for companies in Gothenburg, including Volvo Cars and Volvo Group. The authors declare no other financial or personal relationships that could have influenced the work reported in this paper.

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TABLE V
NOTATION USED THROUGHOUT THE PAPER

Symbol	Unit	Meaning
<i>Dynamics and spatial domain</i>		
s	m	Position along lane centreline
ρ	m	Local spatial coordinate along MPC horizon
$V(s), V_0$	m/s	Ego speed / initial ego speed
V_f	m/s	Target terminal speed
$V_{\text{ref}}(s)$	m/s	Reference speed
a_v, a_{vd}	m/s ²	Acceleration state / demand (input)
τ_{av}	s	Actuator lag time constant
τ_{buf}	s	Buffer time for the actuator distance buffer
k_d	1/m	Aerodynamic drag coefficient
S_p	s/m	Time-space conversion factor
$x = [V \ a_v]^\top, u = a_{vd}$	–	State and input
<i>Energy and braking feasibility</i>		
$K = \frac{1}{2} V^2$	J/kg	Kinetic energy per unit mass
K_{req}	J/kg	Required energy drop to reach V_f
$W_f(L)$	J/kg	Friction work over distance L
$m_W(L), \sigma_W(L)$	J/kg	Mean and Std. of $W_f(L)$
g	m/s ²	Gravitational acceleration
<i>Braking distances</i>		
L	m	Generic braking distance (problem statement)
$S_*, S_{*,\text{eff}}$	m	Available / effective braking distance
Δd_{min}	m	Standstill gap
ΔS_{act}	m	Actuator distance buffer
$S, \mathcal{T}(s)$	m	MPC horizon length / tail interval
<i>GP friction model</i>		
$\bar{\mu}(s)$	–	Random friction coefficient
$\mu_{\text{env}L}, \mu_{\text{env}U}$	–	Perception-derived envelopes
$m(s)$	–	GP prior mean
$a_0(s), a(s)$	–	Prior amplitude / distance-scaled amplitude
$c_{\text{cam}}(s)$	–	Camera confidence profile
$k(s, s'), \ell$	–, m	GP covariance (Matérn-3/2) / length scale
$m_{\text{post}}, k_{\text{post}}$	–	Posterior mean / covariance
$m_{\text{lower}}, m_{\text{upper}}$	–	GP lower / upper friction bound
<i>Risk levels</i>		
ε	–	Generic braking-risk requirement
α_T	–	Global terminal risk parameter
α_L	–	Local (in-horizon) pointwise lower GP posterior quantile
$z_{1-\alpha}$	–	Standard Normal $(1 - \alpha)$ -quantile
<i>Discrete MPC</i>		
$N, \delta s$	–, m	Number of intervals / spatial step
x_i, u_i	–	State and input at node i
$\epsilon_{a,i}$	–	Comfort constraint slack
J	–	Objective function value
W_V, W_U, W_a, W_j	–	MPC cost weights
V_S, V_N	m/s	Speed at horizon end (continuous / discrete)

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