



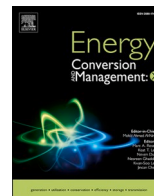
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A modular multilevel converter AC/DC small-signal admittance-based model for stability analysis of hybrid AC/DC grids

Oriol Cartiel^a, Mebtu Beza^{b,*}, Luis Sainz^a, Massimo Bongiorno^b, Juan José Mesas^a

^a Department of Electrical Engineering (DEE), Escola Tècnica Superior d'Enginyeria Industrial de Barcelona (ETSEIB), Universitat Politècnica de Catalunya - BarcelonaTech (UPC), Av. Diagonal 647, 08028 Barcelona, Spain

^b Department of Electrical Engineering, Division of Electric Power Engineering, Chalmers University of Technology, Gothenburg 412 96, Sweden

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ABSTRACT

Modular Multilevel Converters (MMCs) have become a cornerstone technology for high-power applications such as High Voltage Direct Current transmission due to their scalability, low harmonic distortion, and high efficiency. However, their complex internal dynamics and multiple control loops introduce significant challenges in MMC modelling and stability analysis, particularly in multi-terminal hybrid AC/DC transmission grids. To address these challenges, this paper presents a novel frequency-domain MMC AC/DC small-signal admittance-based model that captures the dominant dynamic interactions between the AC and DC sides of the converter. Using dq -complex space vectors, the proposed model incorporates the MMC internal dynamics, circulating-current controller, phase-locked loop, inner current controller, and outer control loops, providing a detailed analytical representation of MMC behaviour. To address the inherently linear time-periodic (LTP) nature of MMC dynamics, the proposed formulation employs a structural reformulation in the dq -complex domain to obtain a compact linear time-invariant representation of the adopted LTP MMC model, without resorting to explicit harmonic expansions, multi-harmonic linearisation, or high-order harmonic state-space approximations. Analytical derivations of the MMC AC- and DC-side input admittances are also presented. The proposed AC/DC model enables frequency-domain stability analysis using approaches such as the Generalised Nyquist Criterion and the Positive-Mode-Damping stability criterion, while preserving the dominant AC/DC coupling structure required for converter-driven stability studies. Validation results and representative instability case studies using MATLAB/Simulink demonstrate the accuracy and robustness of the proposed model, highlighting the importance of simultaneously considering AC and DC dynamics in converter-dominated AC/DC systems.

1. Introduction

The increasing global demand for clean and efficient energy has led to the development and adoption of advanced power electronics technologies. Although two-level Voltage Source Converters (VSCs) have been widely employed [1–5], Modular Multilevel Converters (MMCs) have gained significant attention due to their superior performance in high-power applications, including High Voltage Direct Current (HVDC) transmission grids [6–8]. The MMC's key advantages—such as low harmonic distortion, high efficiency, and scalability—make it an ideal choice for modern energy systems that require robust and efficient solutions to integrate renewable energy technologies into the grid [9]. Originally introduced by Marquardt in 2001, the MMC has since evolved into a critical technology for HVDC transmission grids, employed in

projects worldwide, including Europe, China, and America, with applications ranging from offshore wind integration to point-to-point and multi-terminal HVDC grids [9–13].

Despite the modular structure and high performance of MMCs, their complex internal dynamics and the need for additional control loops—such as capacitor voltage balancing and circulating-current control (CCC)—introduce various challenges in their study, particularly regarding mathematical modelling and stability analysis [7,9,14–18]. In the literature, small-signal stability and dynamic studies of converter-based power systems have traditionally relied on different modelling paradigms, including time-domain state-space variables [19,20] and frequency-domain approaches [21–25], each presenting distinct advantages and limitations. While state-space models provide detailed representations of system eigenvalues, their analytical complexity scales drastically with system size, making them less suitable

* Corresponding author.

E-mail address: meltu.beza@chalmers.se (M. Beza).

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Nomenclature

The upper and lower arms of the converter are indicated by the subscripts “u” and “l”, respectively. Expressions that apply to either arm are represented using the subscript “u,l”.

ω_1	Fundamental angular frequency.
k	Converter phase legs a, b, c.
N	Number of submodules per arm.
C	Submodule capacitance.
L_c	Arm inductance.
R_c	Parasitic arm resistance.
L_f	Inductance of phase reactor.
R_f	Parasitic resistance of phase reactor.
L_t	$= L_f + L_c / 2$; Equivalent phase inductance.
R_t	$= R_f + R_c / 2$; Equivalent phase resistance.
$V_{du,l}$	Per-pole DC-bus voltage.
V_{dc}	$= V_{du} + V_{dl}$; Pole-to-pole DC-bus voltage.
V_{dc}^Δ	$= V_{du} - V_{dl}$; Imbalance DC-bus voltage.

I_{dc}	DC-bus current.
$V_{cu,l}^{k,n}$	$n = 1, \dots, N$; Individual capacitor voltages.
$V_{cu,l}^{\Sigma k}$	$= \sum_{n=1}^N V_{cu,l}^{k,n}$; Sum capacitor voltages.
$V_c^{\Sigma k}$	$= V_{cu}^{\Sigma k} + V_{cl}^{\Sigma k}$; Total capacitor voltage.
$V_c^{\Delta k}$	$= V_{cu}^{\Sigma k} - V_{cl}^{\Sigma k}$; Imbalance capacitor voltage.
$n_{u,l}^k$	$\approx (V_{dc} / 2 - V_{c,r}^k \mp V_{s,r}^k) / V_{dc}$; Insertion indices.
$V_{cu,l}^k$	$= n_{u,l}^k V_{cu,l}^{\Sigma k}$; Inserted voltages.
$I_{u,l}^k$	Arm currents.
I_s^k	$= I_u^k - I_l^k$; AC output current.
$I_{s,r}^k$	Reference for I_s^k .
I_c^k	$= (I_u^k + I_l^k) / 2$; Circulating current.
$I_{c,r}^k$	Reference for I_c^k .
V_s^k	$= (V_{dc}^\Delta - V_{cu}^k + V_{cl}^k) / 2$; Output voltage driving I_s^k .
$V_{s,r}^k$	Reference for V_s^k .
V_c^k	$= (V_{dc} - V_{cu}^k - V_{cl}^k) / 2$; Internal voltage driving I_c^k .
$V_{c,r}^k$	Reference for V_c^k .
V_g^k	Converter AC output voltage.
E^k	Grid voltage.

for large-scale multi-terminal grids. In addition, they are generally less suited to the integration of black-box representations. Conversely, although frequency-domain admittance-based models generally provide a less detailed representation than state-space models, they simplify stability assessment by characterising the interactions between interconnected subsystems, while also enabling the integration of black-box models of power-electronics devices [5,7,26,27]. These frequency-domain formulations commonly rely on dq -domain representations (i.e., dq -real or dq -complex domains [5,28–30]).

In recent years, studies have predominantly focused on the development of separate AC-side [14,16,17,26,27,31,32] or DC-side [18,33] admittance models for MMCs. These decoupled approaches have provided valuable insights into specific aspects of converter stability, yielding compact and computationally efficient models suitable for frequency-domain analysis [16–18,26]. However, these formulations remain limited to isolated AC- or DC-side representations and therefore cannot capture the dynamic interaction mechanisms between the AC and DC sides of the MMC. MMCs inherently interconnect AC and DC grids, meaning that the dynamics of one side may affect the stability of the other. Consequently, comprehensive hybrid AC/DC frequency-domain admittance models have become increasingly important for

the small-signal stability assessment of modern MMC-based HVDC systems, where the converter cannot be accurately represented by independent AC- and DC-port models. Motivated by this need, several recent contributions have addressed AC/DC MMC modelling and multi-port admittance-based representations of MMCs [34–39]. Although these works pursue different objectives and adopt different mathematical reformulations, their derivation generally follows a common two-level modelling framework, illustrated in Fig. 1. First, a physical modelling stage is established by defining the adopted modelling assumptions (e.g., average-value representation, retained internal dynamics, and control-loop representation). Second, a mathematical reformulation stage is employed to address the inherently linear time-periodic (LTP) nature of the resulting MMC small-signal equations and transform these equations into an equivalent linear time-invariant (LTI) frequency-domain system suitable for admittance-based model derivation and small-signal stability analysis. The LTP nature arises from the periodic steady-state operation of the converter, leading to frequency-coupling effects that complicate both analytical derivation and numerical implementation. From the viewpoint of the mathematical reformulation adopted to address these LTP equations, the existing literature can be broadly classified into four representative modelling methods, as illustrated in

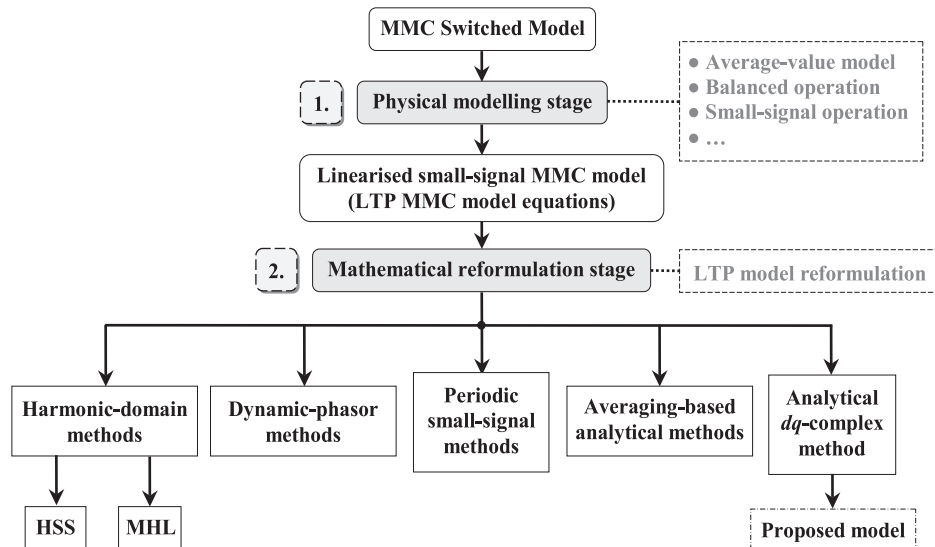


Fig. 1. Flowchart of the MMC small-signal model derivations.

Fig. 1: (i) harmonic-domain methods, (ii) dynamic-phasor methods, (iii) periodic small-signal methods, and (iv) averaging-based analytical methods. Harmonic-domain methods encompass both Harmonic State-Space (HSS) formulations [27,31,32,36,37] and Multi-Harmonic Linearisation (MHL) techniques [35], which explicitly model harmonic coupling through Fourier-series-based Toeplitz matrices. Dynamic-phasor methods [34,38] represent periodic state variables through time-varying Fourier coefficients defined in multiple rotating dq -frames associated with different harmonic frequencies, offering detailed representations of internal dynamics and harmonic interactions up to specific frequencies (e.g., 2ω). Periodic small-signal methods [39] rely on the time-domain periodic linearisation of the system to obtain an LTP model, and then numerical frequency-scanning procedures are applied to measure multi-harmonic admittance matrices. Averaging-based analytical methods [16–18,26] simplify the periodic dynamics through suitable averaging assumptions, yielding a compact analytical LTI formulation while avoiding the explicit representation of higher-order harmonic interactions. Although the representative methods reviewed in [34–39] provide detailed and physically meaningful representations of AC/DC converter dynamics and harmonic interactions, their practical implementation generally involves one or more trade-offs (see Table 1). Depending on the adopted mathematical reformulation, these include the need to retain and truncate harmonic components, increased matrix dimensions associated with harmonic-domain formulations, or numerical frequency-scanning and identification procedures instead of explicit analytical expressions. Consequently, obtaining a compact, explicit analytical admittance-based representation that preserves the dominant AC/DC coupling mechanisms while avoiding explicit harmonic expansions remains challenging, particularly when direct implementation in large-scale nodal-admittance-based small-signal stability studies is required.

The availability of a compact, analytically tractable, and unified AC/DC admittance model is essential for analysing modern power systems characterised by the increasing integration of converter-based renewable energy and the deployment of multi-terminal hybrid AC/DC transmission grids [5,29]. In these systems, stability problems are commonly driven by low-damped resonances and control interactions—often referred to as *converter-driven stability* issues [40]—

with oscillatory modes typically ranging from a few Hz to several kHz. To evaluate these phenomena, various small-signal frequency-domain stability criteria have been proposed for multi-terminal power-electronics-based systems, including the Generalised Nyquist Criterion (GNC) and the Positive-Mode-Damping (PMD) stability criterion. The GNC extends the traditional Nyquist criterion to the eigenvalue loci of the loop transfer function and has been widely adopted for the stability assessment of multi-terminal systems. However, the GNC has several limitations [21–25]: (i) it is conditioned by the source-load subsystem partition, which can lead to erroneous stability conditions; (ii) it does not allow the use of Participation Factors (PFs) to analyse the influence of system and control parameters on stability, as in time-domain eigenvalue analysis; and (iii) it is not intuitive to analyse results. To overcome these limitations, the PMD stability criterion [24] has recently been proposed to extend Resonance Mode Analysis (RMA) [41–43], overcoming the key scalability limitations of traditional time-domain eigenvalue analysis [19,20] and impedance-based frequency-domain methods [23,44]. The PMD stability criterion is well-suited for multi-terminal systems, supports black-box models, and incorporates PFs together with damping margin indicators to locate and quantify instability risks [42,45,46]. In addition to these stability assessment techniques, other complementary frequency-domain approaches have been proposed to analyse power dissipation (passivity) properties [18,47,48], as well as AC/DC interaction characteristics under different control configurations [49]. However, the practical application of these advanced stability-analysis methods strongly depends on the availability of accurate, compact, and analytically tractable converter models that simultaneously capture the dominant AC/DC coupling effects while remaining directly compatible with frequency-domain nodal-admittance-matrix formulations.

This paper presents a compact analytical dq -complex frequency-domain MMC admittance-based model for multi-terminal hybrid AC/DC transmission grids. Unlike approaches that explicitly retain multiple harmonic components, the objective of the proposed formulation is not to maximise harmonic detail, but rather to provide an explicit analytical implementation that captures the dominant AC/DC coupling dynamics while retaining a compact mathematical structure suitable for large-scale frequency-domain stability studies. By extending the admittance-

Table 1
Qualitative comparison of representative AC/DC MMC small-signal modelling frameworks.

Reference	Model Structure & Coordinate Frame	Retained Harmonics & Frequency Range	Control-Loop Representation	Analytical Tractability & Formulation	Target Application & Implementation Focus
Ö.C. Sakinci et al. [34]	Dynamic phasor (multi- dqz -frames) / dq frame	Internal dynamics up to 3ω ; valid up to Nyquist frequency	Detailed AC/DC control loops	Multiport ABCD parameters with an LTI state-space model linearised around equilibrium point via automatic differentiation to derive the 3×3 MIMO admittance matrix	Impedance-based harmonic stability analysis of hybrid AC/DC grids
C. Zhao & Q. Jiang [35]	MHL / pn frame	Truncated harmonic spectrum ($n = 4$ in the reported studies)	Detailed AC/DC control loops	Theoretical frequency-domain matrix formulation using harmonic vectors; evaluated point-by-point	MMC-BTB AC/DC terminal stability analysis and instability-source localisation
D. Wang & K. Zhang [36]	HSS / pn frame	Truncated harmonic spectrum ($h = \pm 1, \pm 2, \dots$)	Detailed AC/DC control loops	Extended Toeplitz matrix formulation based on HSS	Hybrid passivity-based small-signal stability assessment
W. Wu et al. [37]	HSS / dq frame	Truncated harmonic spectrum (typically up to the 3rd harmonic)	Detailed AC/DC control loops	Extended Toeplitz matrix formulation based on HSS	Hybrid AC/DC admittance modelling and impedance-based stability analysis
X. Wu et al. [38]	Dynamic phasor (multi- dq -frames) / dq frame	Reduced harmonic coupling (DC, fundamental and second harmonic)	Detailed AC/DC control loops	Analytical multi-port transfer-function formulation based on dynamic phasors	Multi-port impedance-based stability assessment of MMC-MVDC systems; specifically tailored for LFO analysis in PV-MVDC systems
D. del Giudice et al. [39]	Periodic small-signal model / Time Domain	Multi-harmonic spectrum with frequency up/down conversion	Detailed AC/DC control loops	Purely numerical formulation	Simulation-based admittance measurement; relies on time-domain software engines
Proposed Model	Structural LTI / dq-complex frame	Dominant AC/DC coupled dynamics (focus on fundamental & DC coupling)	Fully integrated (AC/DC controls, PLL & circulating current)	Explicit, closed-form analytical equations	Ready-to-program, three-port self-contained matrix; direct integration into large-scale nodal admittance matrix frameworks

modelling methodology previously developed for VSCs [5], the proposed formulation incorporates the MMC internal dynamics together with the CCC, phase-locked loop (PLL) [50], inner current controller (CC), and outer control loops into a unified AC/DC admittance framework. Compared with mathematical reformulations that explicitly retain multiple harmonic components or rely on numerical identification procedures, the proposed model employs a structural reformulation in the dq -complex domain to obtain an equivalent LTI representation of the adopted LTP MMC model. The resulting formulation provides explicit closed-form admittance matrices that can be directly implemented and incorporated into frequency-domain nodal admittance matrix formulations without requiring additional model-order reduction, harmonic-state truncation, or numerical post-processing. This approach yields a compact analytical derivation of both the AC-side and DC-side input admittances while preserving the dominant AC/DC coupling effects relevant to converter-driven stability phenomena. The proposed model naturally extends the previously reported AC-side [17] and DC-side [18] admittance formulations (i.e., the physical MMC model) by simultaneously representing both subsystems within a unified AC/DC dq -complex framework. Consequently, it enables the direct application of frequency-domain small-signal stability assessment techniques, including the GNC and the PMD stability criterion. Beyond providing an accurate representation of MMC dynamics, the proposed formulation establishes a practical analytical framework that bridges converter modelling and system-level frequency-domain small-signal stability analysis for future MMC-based hybrid AC/DC transmission grids, including the so-called super grids [8,11,12]. To better position the proposed contribution within the current state of the art, Table 1 presents a qualitative comparison of representative AC/DC MMC modelling approaches in terms of model structure, retained harmonics, analytical tractability, computational burden, control-loop representation, modelling accuracy, applicable frequency range, and intended application. As shown, existing approaches adopt different trade-offs between harmonic modelling detail, analytical tractability, implementation complexity, and suitability for large-scale frequency-domain small-signal stability studies. In contrast, the objective of the proposed methodology is not to maximise harmonic modelling detail, but to derive a compact, fully analytical, and explicit dq -complex *three-port* admittance formulation that simultaneously captures the dominant AC- and DC-side dynamics and their associated control loops while remaining directly programmable and readily applicable to large-scale nodal-admittance-based small-signal stability assessment methods. Finally, the accuracy and validity of the proposed analytical model are verified through detailed electromagnetic transient (EMT) time-domain simulations in MATLAB/Simulink [51] under various operating conditions and control configurations.

The paper is structured to ensure that the rigorous analytical derivation, validation, and application of the proposed MMC AC/DC model are presented as clearly as possible. To achieve this, each block of the MMC is examined separately, followed by validation studies and representative stability-analysis applications. The paper is organised as follows: Section 2 presents the internal dynamics of the MMC, where the relationships between the AC- and DC-side currents are derived. Section 3 outlines and models the control strategy adopted for the proposed MMC model. Section 4 derives the complete MMC AC/DC system model, including the calculation of the equivalent admittances seen from both the AC and DC sides. Section 5 presents the validation of the analytical model against MATLAB/Simulink time-domain simulation results under different control schemes. Section 6 introduces the frequency-domain framework adopted for *converter-driven stability* assessment in multi-terminal AC/DC grids. It first presents the assembly of the hybrid AC/DC nodal admittance matrix from the individual component admittance models (including the proposed MMC AC/DC admittance model) and subsequently introduces the PMD stability criterion together with propagation indicators for local-area oscillations. Section 7 applies the proposed model to five representative instability scenarios. In each case,

frequency-domain stability indicators (the PMD stability criterion and AC/DC interaction) are computed and validated through time-domain simulations. Finally, Section 8 concludes the paper, while supplementary theoretical details are provided in the appendices.

2. AC- and DC-side internal dynamics

The first step in deriving the AC/DC small-signal admittance-based model is to formulate the internal dynamics equations along with AC- and DC-side relationships. The configuration in Fig. 2(a) is considered, where the MMC is connected via a phase reactor (R_f , L_f).

According to the k^{th} phase ($k = a, b, c$) of the three-phase MMC in Fig. 2(a) and using the notations in the Nomenclature section, the well-known equations that describe the dynamics of the circulating current, the AC-side output current, and the total and imbalance capacitor voltages are expressed as (see [9,14,17,18] for details)

$$(L_f p + R_t) I_s^k = \frac{V_{s,r}^k}{2V_{dc}} V_c^{\Sigma k} + \left(-\frac{1}{4} + \frac{V_{c,r}^k}{2V_{dc}} \right) V_c^{\Delta k} - E^k \quad (1)$$

$$(L_c p + R_c) I_c^k = \left(-\frac{1}{4} + \frac{V_{c,r}^k}{2V_{dc}} \right) V_c^{\Sigma k} + \frac{V_{s,r}^k}{2V_{dc}} V_c^{\Delta k} + \frac{V_{dc}}{2} \quad (2)$$

$$\frac{C}{N} p V_c^{\Sigma k} = \left(1 - \frac{2V_{c,r}^k}{V_{dc}} \right) I_c^k + \left(\frac{-2V_{s,r}^k}{V_{dc}} \right) \frac{I_s^k}{2} \quad (3)$$

$$\frac{C}{N} p V_c^{\Delta k} = \left(\frac{-2V_{s,r}^k}{V_{dc}} \right) I_c^k + \left(1 - \frac{2V_{c,r}^k}{V_{dc}} \right) \frac{I_s^k}{2}, \quad (4)$$

where $p = d/dt$ represents the derivative operator. It can be observed that the dynamics of the AC-side output current (I_s^k), the circulating current (I_c^k), and the total and imbalance capacitor voltages ($V_c^{\Sigma k}$ and $V_c^{\Delta k}$ respectively) depend on the reference voltages driving I_c^k and I_s^k (i.e., $V_{c,r}^k$ and $V_{s,r}^k$ respectively). These voltages are imposed by the circulating current and AC-side grid current controllers (see Fig. 2(b)), respectively.

The following subsections will develop the MMC small-signal relations for both AC- and DC-side current dynamics based on (1)-(4) relations.

2.1. AC-side current dynamics

Small-signal relations are derived from (1)-(4) by linearising them around a steady-state operating point:

$$(L_f p + R_t) \tilde{I}_s^k = \frac{V_{s0,r}^k}{2V_{dc0}} \tilde{V}_c^{\Sigma k} - \frac{1}{4} \tilde{V}_c^{\Delta k} + \tilde{V}_{s,r}^k - \tilde{E}^k - \frac{V_{s0,r}^k}{V_{dc0}} \tilde{V}_{dc} \quad (5)$$

$$(L_c p + R_c) \tilde{I}_c^k = -\frac{1}{4} \tilde{V}_c^{\Sigma k} + \frac{V_{s0,r}^k}{2V_{dc0}} \tilde{V}_c^{\Delta k} + \tilde{V}_{c,r}^k + \frac{1}{2} \tilde{V}_{dc} \quad (6)$$

$$\frac{C}{N} p \tilde{V}_c^{\Sigma k} = \tilde{I}_c^k - \frac{V_{s0,r}^k}{V_{dc0}} \tilde{I}_s^k - \frac{I_{s0}^k}{V_{dc0}} \tilde{V}_{s,r}^k - \frac{2I_{c0}^k}{V_{dc0}} \tilde{V}_{c,r}^k + \frac{V_{s0,r}^k I_{s0}^k}{V_{dc0}^2} \tilde{V}_{dc} \quad (7)$$

$$\frac{C}{N} p \tilde{V}_c^{\Delta k} = -\frac{2V_{s0,r}^k}{V_{dc0}} \tilde{I}_c^k + \frac{1}{2} \tilde{I}_s^k - \frac{2I_{c0}^k}{V_{dc0}} \tilde{V}_{s,r}^k - \frac{I_{s0}^k}{V_{dc0}} \tilde{V}_{c,r}^k + \frac{2V_{s0,r}^k I_{c0}^k}{V_{dc0}^2} \tilde{V}_{dc}, \quad (8)$$

where the notation $\tilde{x}$ represents the small-signal perturbation around the steady-state operating point x_0 for all the variables in (5)-(8). Considering that the steady-state ripples in $V_{c0}^{\Delta k}$ and $V_{c0}^{\Sigma k}$ are much smaller than the steady-state DC-link voltage V_{dc0} , the following assumptions are adopted in (5)-(8) [17]: i) $V_{c0,r}^k = R_c I_{c0,r}^k \ll V_{dc0}$; ii) $V_{c0}^{\Delta k} = 0$; and iii) $V_{c0}^{\Sigma k} = 2 \cdot V_{dc0}$. Note that the impact of the DC-link voltage perturbation is considered in (5)-(8) to relate the AC- and DC-side voltages and currents, thereby enabling the derivation of the MMC AC/DC small-signal model.

Considering the CCC for phase k in Fig. 2(b) [14,17], where F_{cc} is the

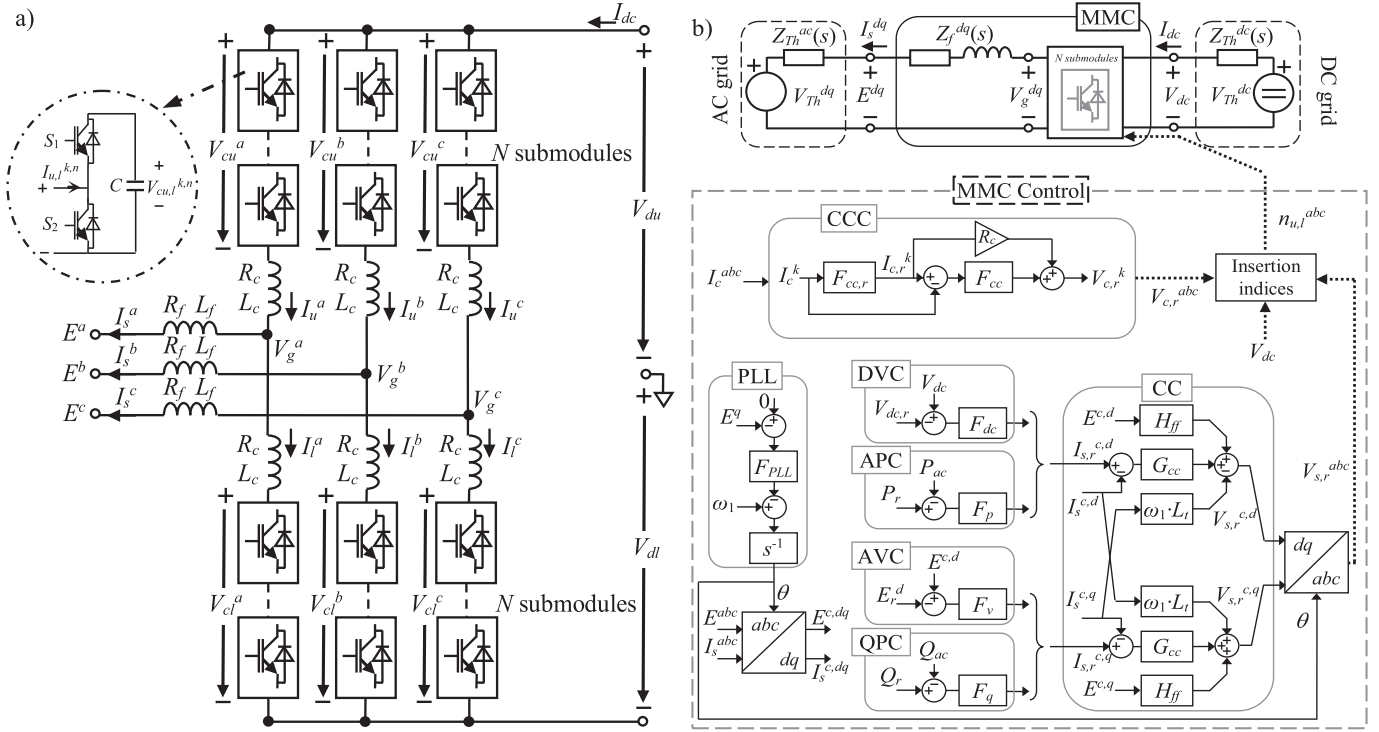


Fig. 2. Grid-connected MMC (with I_s^{dq} outgoing and I_{dc} incoming of the MMC): a) Three-phase general circuit; b) DQ-frame control circuits.

proportional control with steady-state compensation for the leg-arm resistive drop in R_c , and $F_{cc,r}$ represents the reference circulating current estimator, implemented as a low-pass filtering of the measured circulating current [14,17], the perturbation in the reference internal voltage driving I_c^k (i.e., $V_{c,r}^k$) in (6)-(8) is calculated as

$$\begin{aligned} V_{c,r}^k &= \underbrace{F_{cc}}_{\alpha_{ccc} L_c} \left(I_{c,r}^k - I_c^k \right) + R_c I_{c,r}^k, \quad I_{c,r}^k = \frac{F_{cc,r}(p)}{p + 0.1 \alpha_{ccc}} I_c^k \\ &\Rightarrow \tilde{V}_{c,r}^k = - \underbrace{H_d (F_{cc} - (F_{cc} + R_c) F_{cc,r}(p))}_{F_{cc}(p)} \tilde{I}_c^k, \end{aligned} \quad (9)$$

where α_{ccc} is the circulating current controller closed-loop bandwidth and $H_d = e^{-pT_d}$ represents the transfer function for a converter time delay T_d .

The reference voltage driving I_s^k (i.e., $V_{s,r}^k$) depends on the type of AC-side control and will be included later (see Section 3.2). Thus, substituting (9) in (5)-(8), a small-signal model including both internal and AC-side current dynamics is expressed as

$$\underbrace{Z_t(p)}_{(L_t p + R_t)} \tilde{I}_s^k = \frac{V_{s0,r}^k}{2V_{dc0}} \tilde{V}_c^k - \frac{1}{4} \tilde{V}_c^k + \tilde{V}_{s,r}^k - \tilde{E}^k - \frac{V_{s0,r}^k}{V_{dc0}} \tilde{V}_{dc} \quad (10)$$

$$\underbrace{Z_c(p)}_{(L_c p + R_c + F_{cc}(p))} \tilde{I}_c^k = -\frac{1}{4} \tilde{V}_c^k + \frac{V_{s0,r}^k}{2V_{dc0}} \tilde{V}_c^k + \frac{1}{2} \tilde{V}_{dc} \quad (11)$$

$$\frac{C}{N} p \tilde{V}_c^k = \left(1 + \frac{2I_{c0}^k F_{cc}(p)}{V_{dc0}} \right) \tilde{I}_c^k - \frac{V_{s0,r}^k}{V_{dc0}} \tilde{I}_s^k - \frac{I_{c0}^k}{V_{dc0}} \tilde{V}_{s,r}^k + \frac{V_{s0,r}^k I_{c0}^k}{V_{dc0}^2} \tilde{V}_{dc} \quad (12)$$

$$\frac{C}{N} p \tilde{V}_{dc}^k = \left(-\frac{2V_{s0,r}^k}{V_{dc0}} + \frac{I_{c0}^k F_{cc}(p)}{V_{dc0}} \right) \tilde{I}_c^k + \frac{1}{2} \tilde{I}_s^k - \frac{2I_{c0}^k}{V_{dc0}} \tilde{V}_{s,r}^k + \frac{2V_{s0,r}^k I_{c0}^k}{V_{dc0}^2} \tilde{V}_{dc} \quad (13)$$

To proceed, (10)-(13) can be written in three-phase matrix form:

$$\mathbf{Z}_t^{abc}(p) \tilde{\mathbf{I}}_s^{abc} = \frac{1}{2V_{dc0}} \mathbf{V}_{Ms0,r}^{abc} \tilde{\mathbf{V}}_c^{abc} - \frac{1}{4} \tilde{\mathbf{V}}_c^{abc} + \tilde{\mathbf{V}}_{s,r}^{abc} - \tilde{\mathbf{E}}^{abc} - \frac{1}{V_{dc0}} \mathbf{V}_{s0,r}^{abc} \tilde{V}_{dc} \quad (14)$$

$$\mathbf{Z}_c^{abc}(p) \tilde{\mathbf{I}}_c^{abc} = -\frac{1}{4} \tilde{\mathbf{V}}_c^{abc} + \frac{1}{2V_{dc0}} \mathbf{V}_{Ms0,r}^{abc} \tilde{\mathbf{V}}_c^{abc} + \frac{1}{2} \mathbf{1}^{abc} \tilde{V}_{dc} \quad (15)$$

$$\begin{aligned} \mathbf{C}^{abc}(p) \tilde{\mathbf{V}}_c^{abc} &= \left(\mathbf{I}_3 + \frac{2I_{c0}^{abc}}{V_{dc0}} \mathbf{F}_{cc}^{abc}(p) \right) \tilde{\mathbf{I}}_c^{abc} - \frac{1}{V_{dc0}} \mathbf{V}_{Ms0,r}^{abc} \tilde{\mathbf{I}}_s^{abc} - \frac{1}{V_{dc0}} \mathbf{I}_{Ms0}^{abc} \tilde{\mathbf{V}}_{s,r}^{abc} \\ &\quad + \frac{1}{V_{dc0}^2} \mathbf{V}_{Ms0,r}^{abc} \mathbf{I}_{s0}^{abc} \tilde{V}_{dc} \end{aligned} \quad (16)$$

$$\begin{aligned} \mathbf{C}^{abc}(p) \tilde{\mathbf{V}}_{dc}^{abc} &= \left(-\frac{2}{V_{dc0}} \mathbf{V}_{Ms0,r}^{abc} + \frac{1}{V_{dc0}} \mathbf{I}_{Ms0}^{abc} \mathbf{F}_{cc}^{abc}(p) \right) \tilde{\mathbf{I}}_c^{abc} + \frac{1}{2} \mathbf{1}^{abc} - \frac{2I_{c0}^{abc}}{V_{dc0}} \tilde{\mathbf{V}}_{s,r}^{abc} \\ &\quad + \frac{2I_{c0}^{abc}}{V_{dc0}^2} \mathbf{V}_{s0,r}^{abc} \tilde{V}_{dc}, \end{aligned} \quad (17)$$

being $\mathbf{I}_3$ the 3x3 identity matrix, $\mathbf{1}^{abc}$ a 3x1 vector of ones, and

$$\begin{aligned} \mathbf{Z}_t^{abc}(p) &= \underbrace{Z_t(p)}_{(R_t + L_t p)} \mathbf{I}_3, \quad \mathbf{Z}_c^{abc}(p) = \underbrace{Z_c(p)}_{(R_c + F_{cc}(p) + L_c p)} \mathbf{I}_3 \\ \mathbf{C}^{abc}(p) &= \frac{C}{N} p \mathbf{I}_3, \quad \mathbf{F}_{cc}^{abc}(p) = F_{cc}(p) \mathbf{I}_3 \\ \mathbf{A}^{abc} &= \begin{bmatrix} A^a \\ A^b \\ A^c \end{bmatrix}, \quad \mathbf{B}_M^{abc} = \begin{bmatrix} B^a & 0 & 0 \\ 0 & B^b & 0 \\ 0 & 0 & B^c \end{bmatrix}, \end{aligned} \quad (18)$$

where $A \in \{\tilde{I}_c, \tilde{I}_s, I_{s0}, \tilde{V}_{s,r}, V_{s0,r}, \tilde{V}_c, \tilde{V}_{dc}\}$ and $B \in \{I_{s0}, V_{s0,r}\}$ (referring to both bold and italic forms of A and B). It is important to remark that I_{c0}^k is assumed to be considered a DC quantity in (16) and (17), even though there are harmonic oscillations around 100 Hz with a magnitude of about 5–10% of the DC values. Based on this assumption, the three

steady-state k -terms take the same value, i.e., $I_{c0}^k \approx I_{c0}$ ($k = a, b, c$). These assumptions, together with those adopted in (5)-(8), define part of the physical modelling stage of the adopted MMC model.

According to Appendix A, the internal and the AC-side grid current dynamics in (14)-(17) are expressed from the normalised Ku transformation with dqz -complex space vectors as

$$\mathbf{Z}_{\pm t}^{dqz}(p) \tilde{\mathbf{I}}_s^{dqz} = \frac{1}{2V_{dc0}} \mathbf{V}_{Ms0,r}^{dqz} \tilde{\mathbf{V}}_c^{\Sigma dqz} - \frac{1}{4} \tilde{\mathbf{V}}_c^{\Delta dqz} + \tilde{\mathbf{V}}_{s,r}^{dqz} - \tilde{\mathbf{E}}^{dqz} - \frac{1}{V_{dc0}} \mathbf{V}_{s0,r}^{dqz} \tilde{V}_{dc} \quad (19)$$

$$\mathbf{Z}_{\pm c}^{dqz}(p) \tilde{\mathbf{I}}_c^{dqz} = -\frac{1}{4} \tilde{\mathbf{V}}_c^{\Sigma dqz} + \frac{1}{2V_{dc0}} \mathbf{V}_{Ms0,r}^{dqz} \tilde{\mathbf{V}}_c^{\Delta dqz} + \frac{1}{2} \mathbf{1}^{dqz} \tilde{V}_{dc} \quad (20)$$

$$\mathbf{C}_{\pm}^{dqz}(p) \tilde{\mathbf{V}}_c^{\Sigma dqz} = \left(\mathbf{I}_3 + \frac{2I_{c0}}{V_{dc0}} \mathbf{F}_{\pm cc}^{dqz}(p) \right) \tilde{\mathbf{I}}_c^{dqz} - \frac{1}{V_{dc0}} \mathbf{V}_{Ms0,r}^{dqz} \tilde{\mathbf{I}}_s^{dqz} - \frac{1}{V_{dc0}} \mathbf{I}_{Ms0,r}^{dqz} \tilde{\mathbf{V}}_{s,r}^{dqz} + \frac{1}{V_{dc0}^2} \mathbf{V}_{Ms0,r}^{dqz} \mathbf{I}_{s0}^{dqz} \tilde{V}_{dc} \quad (21)$$

$$\mathbf{C}_{\pm}^{dqz}(p) \tilde{\mathbf{V}}_c^{\Delta dqz} = \left(-\frac{2}{V_{dc0}} \mathbf{V}_{Ms0,r}^{dqz} + \frac{1}{V_{dc0}} \mathbf{I}_{Ms0}^{dqz} \mathbf{F}_{\pm cc}^{dqz}(p) \right) \tilde{\mathbf{I}}_c^{dqz} + \frac{1}{2} \tilde{\mathbf{I}}_s^{dqz} - \frac{2I_{c0}}{V_{dc0}} \tilde{\mathbf{V}}_{s,r}^{dqz} + \frac{2I_{c0}}{V_{dc0}^2} \mathbf{V}_{s0,r}^{dqz} \tilde{V}_{dc}, \quad (22)$$

being $\mathbf{1}^{dqz} = [0 \ 0 \ \sqrt{3}]^T$, and

$$\mathbf{Z}_{\pm t}^{dqz}(p) = \text{diag} \left\{ \begin{bmatrix} Z_t(p + j\omega_1) \\ Z_t(p - j\omega_1) \\ Z_t(p) \end{bmatrix} \right\}, \quad \mathbf{Z}_{\pm c}^{dqz}(p) = \text{diag} \left\{ \begin{bmatrix} Z_c(p + j\omega_1) \\ Z_c(p - j\omega_1) \\ Z_c(p) \end{bmatrix} \right\}$$

$$\mathbf{C}_{\pm}^{dqz}(p) = \frac{C}{N} \text{diag} \left\{ \begin{bmatrix} (p + j\omega_1) \\ (p - j\omega_1) \\ p \end{bmatrix} \right\}, \quad \mathbf{F}_{\pm cc}^{dqz}(p) = \text{diag} \left\{ \begin{bmatrix} F'_{cc}(p + j\omega_1) \\ F'_{cc}(p - j\omega_1) \\ F'_{cc}(p) \end{bmatrix} \right\}$$

$$\mathbf{A}^{dqz} = \begin{bmatrix} \mathbf{A}^{dq} \\ (\mathbf{A}^{dq})^* \\ \mathbf{A}^z \end{bmatrix}, \quad \mathbf{B}_{\pm M}^{dqz} = \frac{1}{\sqrt{3}} \begin{bmatrix} \mathbf{B}^z & e^{-j3\theta} (\mathbf{B}^{dq})^* & \mathbf{B}^{dq} \\ e^{j3\theta} \mathbf{B}^{dq} & \mathbf{B}^z & (\mathbf{B}^{dq})^* \\ (\mathbf{B}^{dq})^* & \mathbf{B}^{dq} & \mathbf{B}^z \end{bmatrix}, \quad (23)$$

where $\mathbf{A}^{dq} = A^{dq}/\sqrt{2} = (A^d + jA^q)/\sqrt{2}$ and $\mathbf{A}^z$ are the dq -complex and the zero-sequence terms respectively (similarly for $\mathbf{B}^{dq}$ and $\mathbf{B}^z$ terms), $\mathbf{A} \in \{\tilde{\mathbf{I}}_c, \tilde{\mathbf{I}}_s, \mathbf{I}_{s0}, \tilde{\mathbf{E}}, \tilde{\mathbf{V}}_{s,r}, \mathbf{V}_{s0,r}, \tilde{\mathbf{V}}_c^{\Sigma}, \tilde{\mathbf{V}}_c^{\Delta}\}$ and $\mathbf{B} \in \{\mathbf{I}_{s0}, \mathbf{V}_{s0,r}\}$ (referring to both bold and italic forms of $\mathbf{A}$ and $\mathbf{B}$), and $\text{diag}\{\mathbf{x}\}$ is the diagonal matrix of a column vector $\mathbf{x}$ (i.e., $\text{diag}\{\mathbf{x}\} = \sum_{i=1}^n (\mathbf{e}_i^T \mathbf{x})(\mathbf{e}_i \mathbf{e}_i^T)$), being $\mathbf{e}_i$ the i -th standard basis vector of $\mathbb{R}^n$ and the superscript “T” denotes the transpose). It should be noted that, in the case of a three-phase system without a neutral conductor, the component $\mathbf{I}_s^z$ is equal to zero. However, at this point, it is not disregarded for generalisation purposes. Additionally, it is worth mentioning that $\mathbf{B}_{\pm M}^{dqz}$ (i.e., $\mathbf{V}_{Ms0,r}^{dqz}$ and $\mathbf{I}_{Ms0,r}^{dqz}$) are time-dependent voltage and current matrices, in contrast to the other constant dqz -complex voltage and current vectors (i.e., $\mathbf{A}^{dqz}$ vectors).

The expressions for $\tilde{\mathbf{V}}_c^{\Delta dqz}$ and $\tilde{\mathbf{V}}_c^{\Sigma dqz}$ in (19) should be obtained explicitly in terms of $\tilde{\mathbf{I}}_s^{dqz}$ and $\tilde{\mathbf{V}}_{s,r}^{dqz}$ from (20), (21), and (22) to find the AC/DC relation between voltages and currents [17]. The first step is to include $\tilde{\mathbf{I}}_c^{dqz}$ from (20) within (21) and (22). Subsequently, several algebraic manipulations are performed to combine both resulting expressions, thereby isolating $\tilde{\mathbf{V}}_c^{\Delta dqz}$ and $\tilde{\mathbf{V}}_c^{\Sigma dqz}$ into two separate expressions:

$$\tilde{\mathbf{V}}_c^{\Delta dqz} = \mathbf{H}_{\pm 3}^{dqz}(p) \mathbf{K}_{\pm i1}^{dqz}(p) \tilde{\mathbf{I}}_s^{dqz} + \mathbf{H}_{\pm 3}^{dqz}(p) \mathbf{K}_{\pm v1}^{dqz}(p) \tilde{\mathbf{V}}_{s,r}^{dqz} + \mathbf{H}_{\pm 3}^{dqz}(p) \mathbf{K}_{\pm d1}^{dqz}(p) \tilde{V}_{dc} \quad (24)$$

$$\frac{1}{2V_{dc0}} \mathbf{V}_{Ms0,r}^{dqz} \tilde{\mathbf{V}}_c^{\Sigma dqz} = \mathbf{K}_{\pm i2}^{dqz}(p) \tilde{\mathbf{I}}_s^{dqz} + \mathbf{K}_{\pm v2}^{dqz}(p) \tilde{\mathbf{V}}_{s,r}^{dqz} + \mathbf{K}_{\pm d2}^{dqz}(p) \tilde{V}_{dc}, \quad (25)$$

where $\mathbf{H}_{\pm 3}^{dqz}(p) = (\mathbf{C}_{\pm}^{dqz}(p) + \mathbf{K}_{\pm 2}^{dqz}(p))^{-1}$ and $\mathbf{K}_{\pm 2}^{dqz}(p)$, $\mathbf{K}_{\pm i1}^{dqz}(p)$, $\mathbf{K}_{\pm d1}^{dqz}(p)$, $\mathbf{K}_{\pm i2}^{dqz}(p)$, $\mathbf{K}_{\pm v2}^{dqz}(p)$, and $\mathbf{K}_{\pm d2}^{dqz}(p)$ terms are studied in Appendices B and C.

Then, substituting (24) and (25) into (19), and including the delay $H_d = e^{-pT_d}$ involved in $\tilde{\mathbf{V}}_{s,r}^{dqz}$ [17], results in,

$$\mathbf{Z}_{\pm A}^{dqz}(p) \tilde{\mathbf{I}}_s^{dqz} = \mathbf{F}_{\pm A}^{dqz}(p) \tilde{\mathbf{V}}_{s,r}^{dqz} - \tilde{\mathbf{E}}^{dqz} + \mathbf{R}_{\pm A}^{dqz}(p) \tilde{V}_{dc}, \quad (26)$$

where,

$$\mathbf{Z}_{\pm A}^{dqz}(p) = \mathbf{Z}_{\pm t}^{dqz}(p) + \frac{1}{4} \mathbf{H}_{\pm 3}^{dqz}(p) \mathbf{K}_{\pm i1}^{dqz}(p) - \mathbf{K}_{\pm i2}^{dqz}(p)$$

$$\mathbf{F}_{\pm A}^{dqz}(p) = H_d \left(\mathbf{I}_3 - \frac{1}{4} \mathbf{H}_{\pm 3}^{dqz}(p) \mathbf{K}_{\pm v1}^{dqz}(p) + \mathbf{K}_{\pm v2}^{dqz}(p) \right)$$

$$\mathbf{R}_{\pm A}^{dqz}(p) = -\frac{1}{V_{dc0}} \mathbf{V}_{s0,r}^{dqz} - \frac{1}{4} \mathbf{H}_{\pm 3}^{dqz}(p) \mathbf{K}_{\pm d1}^{dqz}(p) + \mathbf{K}_{\pm d2}^{dqz}(p).$$

It is worth noting that the time-dependent matrices $\mathbf{B}_{\pm M}^{dqz}$ in (23) multiply a transfer function in $\mathbf{K}_{\pm 2}^{dqz}(p)$, $\mathbf{K}_{\pm i1}^{dqz}(p)$, $\mathbf{K}_{\pm v1}^{dqz}(p)$, $\mathbf{K}_{\pm d1}^{dqz}(p)$, $\mathbf{K}_{\pm i2}^{dqz}(p)$, $\mathbf{K}_{\pm v2}^{dqz}(p)$, and $\mathbf{K}_{\pm d2}^{dqz}(p)$. These products make the MMC small-signal equations inherently LTP, since the steady-state variables vary periodically with the fundamental frequency. This introduces coupling between multiple frequency components, complicating the analytical derivation of the MMC model. In the literature, various methods—e.g., harmonic linearisation or HSS using Toeplitz matrices—have been proposed to capture these LTP effects by representing the time-varying interactions through cumbersome harmonic expressions [27,31,32,37]. Other studies have obtained approximations by averaging the LTP expressions in the abc -domain [16,17,26]. Accordingly, once the physical modelling stage has been completed, different mathematical reformulation approaches can be employed to address the inherent LTP behaviour of the adopted MMC model (see Fig. 1). In the present work, this is achieved through a dqz -complex algebraic reformulation of the adopted LTP MMC equations. Consequently, the previous transfer functions are adapted in Appendix B to account for the multiplication of a matrix with time-dependent components by the dqz -complex transfer function. As shown in Appendix B, this algebraic reformulation transforms the LTP MMC equations into an equivalent LTI representation. Only once an equivalent LTI system has been obtained can the differential operator p be replaced by the Laplace variable s . Thus, based on Appendix B and assuming a balanced three-phase system (i.e., the zero-sequence terms are neglected), (26) becomes

$$\mathbf{Z}_{\pm A}^{dq}(s) \tilde{\mathbf{I}}_s^{dq} = \mathbf{F}_{\pm A}^{dq}(s) \tilde{\mathbf{V}}_{s,r}^{dq} - \tilde{\mathbf{E}}^{dq} + \mathbf{R}_{\pm A}^{dq}(s) \tilde{V}_{dc}, \quad (28)$$

being

$$\mathbf{Z}_{\pm A}^{dq}(s) = \mathbf{Z}_{\pm t}^{dq}(s) + \frac{1}{4} \mathbf{H}_{\pm 3}^{dq}(s) \mathbf{K}_{\pm i1}^{dq}(s) - \mathbf{K}_{\pm i2}^{dq}(s)$$

$$\mathbf{F}_{\pm A}^{dq}(s) = H_d \left(\mathbf{I}_2 - \frac{1}{4} \mathbf{H}_{\pm 3}^{dq}(s) \mathbf{K}_{\pm v1}^{dq}(s) + \mathbf{K}_{\pm v2}^{dq}(s) \right)$$

$$\mathbf{R}_{\pm A}^{dq}(s) = -\frac{1}{V_{dc0}} \mathbf{V}_{s0,r}^{dq} - \frac{1}{4} \mathbf{H}_{\pm 3}^{dq}(s) \mathbf{K}_{\pm d1}^{dq}(s) + \mathbf{K}_{\pm d2}^{dq}(s), \quad (29)$$

where $\mathbf{I}_2$ is the 2x2 identity matrix, $\mathbf{H}_{\pm 3}^{dq}(s) = (\mathbf{C}_{\pm}^{dq}(s) + \mathbf{K}_{\pm 2}^{dq}(s))^{-1}$, and $\mathbf{Z}_{\pm t}^{dq}(s)$, $\mathbf{V}_{s0,r}^{dq}$, $\mathbf{C}_{\pm}^{dq}(s)$, $\mathbf{K}_{\pm 2}^{dq}(s)$, $\mathbf{K}_{\pm i1}^{dq}(s)$, $\mathbf{K}_{\pm v1}^{dq}(s)$, $\mathbf{K}_{\pm d1}^{dq}(s)$, $\mathbf{K}_{\pm i2}^{dq}(s)$, $\mathbf{K}_{\pm v2}^{dq}(s)$, and $\mathbf{K}_{\pm d2}^{dq}(s)$ terms are in Appendix C.

2.2. DC-side current dynamics

The three-phase equations in (2) are added together to find an expression to model the DC-side current dynamics [18],

$$\sum_{k=1}^3 (L_c p + R_c) I_c^k = \sum_{k=1}^3 \left(\left(-\frac{1}{4} + \frac{V_{c,r}^k}{2V_{dc}} \right) V_c^{\Sigma k} \right) + \sum_{k=1}^3 \left(\frac{V_{s,r}^k}{2V_{dc}} V_c^{\Delta k} \right) + \sum_{k=1}^3 \left(\frac{V_{dc}}{2} \right). \quad (30)$$

Next, (30) can be rewritten as

$$(L_c p + R_c) I_{dc} = \sum_{k=1}^3 \left(\left(-\frac{1}{4} + \frac{V_{c,r}^k}{2V_{dc}} \right) V_c^{\Sigma k} \right) + \sum_{k=1}^3 \left(\frac{V_{s,r}^k}{2V_{dc}} V_c^{\Delta k} \right) + \sum_{k=1}^3 \left(\frac{V_{dc}}{2} \right), \quad (31)$$

by assuming [18]

$$\sum_{k=1}^3 I_c^k = \sum_{k=1}^3 \left(\frac{I_u^k + I_l^k}{2} \right) = \sum_{k=1}^3 \frac{I_u^k}{2} + \sum_{k=1}^3 \frac{I_l^k}{2} = \frac{I_{dc} + I_{dc}}{2} = I_{dc}. \quad (32)$$

Then, (31) is transformed into small-signal relations as,

$$Z_c(p) \tilde{I}_{dc} = - \sum_{k=1}^3 \frac{1}{4} \tilde{V}_c^{\Sigma k} + \sum_{k=1}^3 \frac{V_{s0,r}^k}{2V_{dc0}} \tilde{V}_c^{\Delta k} + \frac{3}{2} \tilde{V}_{dc}, \quad (33)$$

where $\tilde{V}_{c,r}^k$ from (9) is included, and the following is considered:

$$\left. \begin{aligned} \tilde{V}_{c,r}^k &= -F_{cc}(p) \tilde{I}_c^k \\ \sum_{k=1}^3 I_c^k &= I_{dc} \Rightarrow \sum_{k=1}^3 \tilde{I}_c^k = \tilde{I}_{dc} \end{aligned} \right\} \Rightarrow \sum_{k=1}^3 \tilde{V}_{c,r}^k = -F_{cc}(p) \tilde{I}_{dc}. \quad (34)$$

Note that assumptions in (5)-(8) are also considered in (33).

The three-phase matrix form can be built from (33) as,

$$Z_c(p) \tilde{I}_{dc} = -\frac{1}{4} (\mathbf{1}^{abc})^T \tilde{\mathbf{V}}_c^{\Sigma abc} + \frac{1}{2V_{dc0}} (\mathbf{V}_{s0,r}^{abc})^T \tilde{\mathbf{V}}_c^{\Delta abc} + \frac{3}{2} \tilde{V}_{dc}. \quad (35)$$

Subsequently, the three-phase system is transformed using the normalised Ku transformation (see Appendix A), and the resulting dqz -complex space vectors are given by,

$$Z_c(p) \tilde{I}_{dc} = -\frac{1}{4} (\mathbf{1}^{dqz})^H \tilde{\mathbf{V}}_c^{\Sigma dqz} + \frac{1}{2V_{dc0}} (\mathbf{V}_{s0,r}^{dqz})^H \tilde{\mathbf{V}}_c^{\Delta dqz} + \frac{3}{2} \tilde{V}_{dc}. \quad (36)$$

Substituting (24) and (25) into (36), and accounting for the delay $H_d = e^{-pT_d}$ involved in the reference $\tilde{\mathbf{V}}_{s,r}^{dqz}$ [17], this results in

$$\tilde{I}_{dc} = \mathbf{Z}_{\pm D}^{dqz}(p) \tilde{\mathbf{I}}_s^{dqz} + \mathbf{F}_{\pm D}^{dqz}(p) \tilde{\mathbf{V}}_{s,r}^{dqz} + R_D^{dqz}(p) \tilde{V}_{dc}, \quad (37)$$

being

$$\begin{aligned} \mathbf{Z}_{\pm D}^{dqz}(p) &= \frac{1}{Z_c(p)} \left(\frac{1}{2V_{dc0}} (\mathbf{V}_{s0,r}^{dqz})^H \mathbf{H}_{\pm 3}^{dqz}(p) \mathbf{K}_{\pm i1}^{dqz}(p) - \frac{1}{4} (\mathbf{1}^{dqz})^H \mathbf{L}_{\pm i1}^{dqz}(p) \right) \\ \mathbf{F}_{\pm D}^{dqz}(p) &= \frac{H_d}{Z_c(p)} \left(\frac{1}{2V_{dc0}} (\mathbf{V}_{s0,r}^{dqz})^H \mathbf{H}_{\pm 3}^{dqz}(p) \mathbf{K}_{\pm v1}^{dqz}(p) - \frac{1}{4} (\mathbf{1}^{dqz})^H \mathbf{L}_{\pm v1}^{dqz}(p) \right) \\ R_D^{dqz}(p) &= \frac{1}{Z_c(p)} \left(\frac{1}{2V_{dc0}} (\mathbf{V}_{s0,r}^{dqz})^H \mathbf{H}_{\pm 3}^{dqz}(p) \mathbf{K}_{\pm d1}^{dqz}(p) - \frac{1}{4} (\mathbf{1}^{dqz})^H \mathbf{L}_{\pm d1}^{dqz}(p) + \frac{3}{2} \right), \end{aligned} \quad (38)$$

where $\mathbf{L}_{\pm i1}^{dqz}(p)$, $\mathbf{L}_{\pm v1}^{dqz}(p)$ and $\mathbf{L}_{\pm d1}^{dqz}(p)$ terms are studied in Appendices B and C.

It is worth noting that the time-dependent matrices $\mathbf{B}_{\pm M}^{dqz}$ in (23) multiply a transfer function in $\mathbf{L}_{\pm i1}^{dqz}(p)$, $\mathbf{L}_{\pm v1}^{dqz}(p)$, and $\mathbf{L}_{\pm d1}^{dqz}(p)$ (i.e., LTP products). Thus, based on Appendix B and assuming a balanced three-phase system (i.e., the zero-sequence terms are neglected), the

derivative operator p can be replaced with the Laplace variable s (as the system becomes LTI) and (37) results in

$$\tilde{I}_{dc} = \mathbf{Z}_{\pm D}^{dq}(s) \tilde{\mathbf{I}}_s^{dq} + \mathbf{F}_{\pm D}^{dq}(s) \tilde{\mathbf{V}}_{s,r}^{dq} + R_D^{dq}(s) \tilde{V}_{dc}, \quad (39)$$

being

$$\begin{aligned} \mathbf{Z}_{\pm D}^{dq}(s) &= \frac{1}{Z_c(s)} \left(\frac{1}{2V_{dc0}} (\mathbf{V}_{s0,r}^{dq})^H \mathbf{H}_{\pm 3}^{dq}(s) \mathbf{K}_{\pm i1}^{dq}(s) - \frac{1}{4} \mathbf{L}_{\pm i1}^{dq}(s) \right) \\ \mathbf{F}_{\pm D}^{dq}(s) &= \frac{H_d}{Z_c(s)} \left(\frac{1}{2V_{dc0}} (\mathbf{V}_{s0,r}^{dq})^H \mathbf{H}_{\pm 3}^{dq}(s) \mathbf{K}_{\pm v1}^{dq}(s) - \frac{1}{4} \mathbf{L}_{\pm v1}^{dq}(s) \right) \\ R_D^{dq}(s) &= \frac{1}{Z_c(s)} \left(\frac{1}{2V_{dc0}} (\mathbf{V}_{s0,r}^{dq})^H \mathbf{H}_{\pm 3}^{dq}(s) \mathbf{K}_{\pm d1}^{dq}(s) - \frac{1}{4} \mathbf{L}_{\pm d1}^{dq}(s) + \frac{3}{2} \right), \end{aligned} \quad (40)$$

where $\mathbf{L}_{\pm i1}^{dq}(s)$, $\mathbf{L}_{\pm v1}^{dq}(s)$, $\mathbf{L}_{\pm d1}^{dq}(s)$ terms are in Appendix C.

The validation of the analytical model corresponding to the MMC AC/DC internal dynamics, both with and without the CCC enabled, is presented in Section 5.

3. MMC control strategy

The MMC control strategy is represented by the CCC (9), the PLL, the inner CC, and the outer control loops. The validation of the analytical models developed for the MMC control structure is presented in Section 5.

3.1. PLL relations

The AC grid voltage ($\mathbf{E}^{dq}$), the AC output current ($\mathbf{I}_s^{dq}$), and the reference for the voltage driving the AC output current ($\mathbf{V}_{s,r}^{dq}$) can be expressed using dq -complex space vectors in the converter and grid domains as the following small-signal relations [50],

$$\tilde{\mathbf{I}}_s^{c,dq} = \tilde{\mathbf{I}}_s^{dq} + \mathbf{G}_{\pm plli}^+ \tilde{\mathbf{E}}^{dq}, \quad \mathbf{G}_{\pm plli}^+ = -\mathbf{G}_{\pm plli}^- = -G_{pll}(s) \frac{(I_{s0}^d + jI_{s0}^q)}{2} \quad (41)$$

$$\tilde{\mathbf{E}}^{c,dq} = \mathbf{G}_{\pm pllv}^+ \tilde{\mathbf{E}}^{dq}, \quad \mathbf{G}_{\pm pllv}^+ = 1 - G_{pll}(s) \frac{(E_0^d + jE_0^q)}{2}, \quad \mathbf{G}_{\pm pllv}^- = 1 - \mathbf{G}_{\pm pllv}^+ \quad (42)$$

$$\tilde{\mathbf{V}}_{s,r}^{c,dq} = \tilde{\mathbf{V}}_{s,r}^{dq} + \mathbf{G}_{\pm pllm}^+ \tilde{\mathbf{E}}^{dq}, \quad \mathbf{G}_{\pm pllm}^+ = -\mathbf{G}_{\pm pllm}^- = -G_{pll}(s) \frac{(V_{s0,r}^d + jV_{s0,r}^q)}{2}, \quad (43)$$

where $G_{pll}(s) = F_{pll}(s) / (s + E_0^d F_{pll}(s))$ and $F_{pll}(s)$ is the transfer function of the PLL control. It should be noted that the dq -complex space vectors in the converter dq -domain are denoted with the superscript “c”, while the grid dq -domain variables do not have any superscript.

3.2. AC-side grid current controller – Inner CC loop relations

Considering the dq -frame AC-side grid CC in Fig. 2(b), the control law of the inner CC loop can be expressed as a small-signal relation using dq -complex space vectors by [5],

$$\tilde{\mathbf{V}}_{s,r}^{c,dq} = \mathbf{H}_{\pm ff}^{dq}(s) \tilde{\mathbf{E}}^{c,dq} + \mathbf{Z}_{\pm w}^{dq} \tilde{\mathbf{I}}_s^{c,dq} + \mathbf{G}_{\pm cc}^{dq}(s) \left(\tilde{\mathbf{I}}_{s,r}^{c,dq} - \tilde{\mathbf{I}}_s^{c,dq} \right), \quad (44)$$

where $G_{cc}^+(s) = k_{p,c} + k_{i,c}/s$ ($G_{cc}^-(s) = 0$) represents the AC-side grid CC, $\mathbf{I}_{s,r}^{dq}$ is the reference current, and $H_{ff}^+(s) = \alpha_f / (\alpha_f + s)$ ($H_{ff}^-(s) = 0$) is the filter used as a feed-forward term in addition to the current decoupling component $\mathbf{Z}_w^+ = j\omega_1 L_t$ ($\mathbf{Z}_w^- = 0$).

3.3. Outer control loop relations

The reference currents generated by the outer control loops can be related to the DC voltage and the AC voltage and current at the PCC with

dq -complex space vectors as the following general expression [5]:

$$\tilde{\mathbf{I}}_{s,r}^{c,dq} = -\mathbf{J}_{\pm o}(s)\tilde{\mathbf{V}}_{dc} - \mathbf{G}_{\pm o}(s)\tilde{\mathbf{I}}_s^{c,dq} - \mathbf{Y}_{\pm o}(s)\tilde{\mathbf{E}}^{c,dq}, \quad (45)$$

where the complex transfer matrices $\mathbf{J}_{\pm o}$, $\mathbf{G}_{\pm o}$ and $\mathbf{Y}_{\pm o}$ depend on the chosen MMC outer control loops in Fig. 2(b) to set $\tilde{\mathbf{I}}_{s,r}^{c,d}$ (either by DVC or APC) and $\tilde{\mathbf{I}}_{s,r}^{c,q}$ (either by AVC or QPC). These complex transfer matrices are the sum of each control complex transfer matrix, depending on the MMC operation mode (e.g., $\mathbf{G}_{\pm o} = \mathbf{G}_{\pm o,APC} + \mathbf{G}_{\pm o,QPC}$).

- Direct-voltage controller (DVC):

The control law of the DVC is expressed with dq -complex space vectors as [5],

$$\begin{aligned} \tilde{\mathbf{I}}_{s,r}^{c,dq} &= -\mathbf{J}_{\pm o,DVC}(s)\tilde{\mathbf{V}}_{dc} \\ \mathbf{J}_{\pm o,DVC}(s) &= -\frac{1}{\sqrt{2}} \begin{bmatrix} F_{dc}(s) \\ F_{dc}(s) \end{bmatrix}, \end{aligned} \quad (46)$$

where $F_{dc}(s) = k_{p,dc} + k_{i,dc}/s$, with $k_{p,dc}$ and $k_{i,dc}$ being the proportional and integral gains of the DVC.

- Active-power controller (APC):

Since $E_0^q = 0$, the control law of the APC is expressed with dq -complex space vectors as [5],

$$\begin{aligned} \tilde{\mathbf{I}}_{s,r}^{c,dq} &= -\mathbf{G}_{\pm o,APC}(s)\tilde{\mathbf{I}}_s^{c,dq} - \mathbf{Y}_{\pm o,APC}(s)\tilde{\mathbf{E}}^{c,dq} \\ \mathbf{G}_{\pm o,APC}(s) &= \frac{F_p(s)E_0^d}{2} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad \mathbf{Y}_{\pm o,APC}(s) = \frac{F_p(s)}{\sqrt{2}} \begin{bmatrix} \left(\mathbf{I}_{s0}^{dq}\right)^* & \mathbf{I}_{s0}^{dq} \\ \left(\mathbf{I}_{s0}^{dq}\right)^* & \mathbf{I}_{s0}^{dq} \end{bmatrix}, \end{aligned} \quad (47)$$

where $F_p(s) = k_{p,p} + k_{i,p}/s$, with $k_{p,p}$ and $k_{i,p}$ being the proportional and integral gains of the APC.

- Alternating-voltage controller (AVC):

The control law of the AVC is expressed with dq -complex space vectors as [5],

$$\begin{aligned} \tilde{\mathbf{I}}_{s,r}^{c,dq} &= -\mathbf{Y}_{\pm o,AVC}(s)\tilde{\mathbf{E}}^{c,dq} \\ \mathbf{Y}_{\pm o,AVC}(s) &= -\frac{F_v(s)}{2} \begin{bmatrix} j & j \\ -j & -j \end{bmatrix}, \end{aligned} \quad (48)$$

where $F_v(s) = k_{p,v} + k_{i,v}/s$, with $k_{p,v}$ and $k_{i,v}$ being the proportional and integral gains of the AVC.

- Reactive-power controller (QPC):

Since $E_0^q = 0$, the control law of the QPC is expressed with dq -complex space vectors as [5],

$$\begin{aligned} \tilde{\mathbf{I}}_{s,r}^{c,dq} &= -\mathbf{G}_{\pm o,QPC}(s)\tilde{\mathbf{I}}_s^{c,dq} - \mathbf{Y}_{\pm o,QPC}(s)\tilde{\mathbf{E}}^{c,dq} \\ \mathbf{G}_{\pm o,QPC}(s) &= \frac{F_q(s)E_0^d}{2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad \mathbf{Y}_{\pm o,QPC}(s) = \frac{F_q(s)}{\sqrt{2}} \begin{bmatrix} -\left(\mathbf{I}_{s0}^{dq}\right)^* & \mathbf{I}_{s0}^{dq} \\ \left(\mathbf{I}_{s0}^{dq}\right)^* & -\mathbf{I}_{s0}^{dq} \end{bmatrix}, \end{aligned} \quad (49)$$

where $F_q(s) = k_{p,q} + k_{i,q}/s$, with $k_{p,q}$ and $k_{i,q}$ being the proportional and integral gains of the QPC.

4. Complete model of the MMC AC/DC system

The complete AC/DC small-signal admittance-based model of the grid-connected MMC in Fig. 2 is derived in this section.

4.1. AC/DC small-signal admittance-based model

From the final expressions of the AC- and DC-side current dynamics in (28) and (39), respectively, and the relations of the inner CC loop (44), the PLL (41)-(43), and the outer control loops (45), the relationship between voltages and currents can be directly obtained as,

$$\begin{aligned} \begin{pmatrix} -\tilde{\mathbf{I}}_s^{dq} \end{pmatrix} &= \mathbf{Y}_{\pm M}^{dq}(s)\tilde{\mathbf{E}}^{dq} - \mathbf{J}_{\pm M}^{dq}(s)\tilde{\mathbf{V}}_{dc} \\ \tilde{\mathbf{I}}_{dc} &= \left(\mathbf{Y}_{\pm dc}^{dq}(s) - \mathbf{K}_{\pm dc}^{dq}(s)\mathbf{Y}_{\pm M}^{dq}(s)\right)\tilde{\mathbf{E}}^{dq} + \left(\mathbf{K}_{\pm dc}^{dq}(s)\mathbf{J}_{\pm M}^{dq}(s) + \mathbf{J}_{dc}^{dq}(s)\right)\tilde{\mathbf{V}}_{dc}, \end{aligned} \quad (50)$$

being

$$\begin{aligned} \mathbf{Y}_{\pm M}^{dq}(s) &= \mathbf{Y}_{\pm K}^{dq}(s) \left(\mathbf{I}_2 - \mathbf{F}_{\pm A}^{dq}(s)\mathbf{H}_{\pm o}^{dq}(s) \right) \\ \mathbf{Y}_{\pm dc}^{dq}(s) &= \mathbf{F}_{\pm D}^{dq}(s)\mathbf{H}_{\pm o}^{dq}(s) \\ \mathbf{J}_{\pm M}^{dq}(s) &= \mathbf{Y}_{\pm K}^{dq}(s) \left(\mathbf{R}_{\pm A}^{dq}(s) - \mathbf{F}_{\pm A}^{dq}(s)\mathbf{G}_{\pm cc}^{dq}(s)\mathbf{J}_{\pm o}(s) \right) \\ \mathbf{J}_{dc}^{dq}(s) &= \mathbf{R}_D^{dq}(s) - \mathbf{F}_{\pm D}^{dq}(s)\mathbf{G}_{\pm cc}^{dq}(s)\mathbf{J}_{\pm o}(s) \\ \mathbf{K}_{\pm dc}^{dq}(s) &= \mathbf{Z}_{\pm D}^{dq}(s) - \mathbf{F}_{\pm D}^{dq}(s)\mathbf{Z}_{\pm \pi}^{dq}(s), \end{aligned} \quad (51)$$

where

$$\begin{aligned} \mathbf{Z}_{\pm \pi}^{dq}(s) &= \left(\mathbf{G}_{\pm cc}^{dq}(s) + \mathbf{G}_{\pm cc}^{dq}(s)\mathbf{G}_{\pm o}(s) - \mathbf{Z}_{\pm w}^{dq}(s) \right) \\ \mathbf{H}_{\pm o}^{dq}(s) &= \left(\mathbf{H}_{\pm ff}^{dq}(s) - \mathbf{G}_{\pm cc}^{dq}(s)\mathbf{Y}_{\pm o}(s) \right) \mathbf{G}_{\pm pll v} - \mathbf{Z}_{\pm \pi}^{dq}(s)\mathbf{G}_{\pm plli} - \mathbf{G}_{\pm pllm} \\ \mathbf{Y}_{\pm K}^{dq}(s) &= \left(\mathbf{Z}_{\pm A}^{dq}(s) + \mathbf{F}_{\pm A}^{dq}(s)\mathbf{Z}_{\pm \pi}^{dq}(s) \right)^{-1}, \end{aligned} \quad (52)$$

and $\mathbf{Z}_{\pm A}^{dq}(s)$, $\mathbf{F}_{\pm A}^{dq}(s)$ and $\mathbf{R}_{\pm A}^{dq}(s)$ are in (29), $\mathbf{Z}_{\pm D}^{dq}(s)$, $\mathbf{F}_{\pm D}^{dq}(s)$ and $\mathbf{R}_D^{dq}(s)$ are in (40), $\mathbf{G}_{\pm plli}(s)$, $\mathbf{G}_{\pm pll v}(s)$ and $\mathbf{G}_{\pm pllm}(s)$ are in (41)-(43), $\mathbf{G}_{\pm cc}^{dq}(s)$, $\mathbf{H}_{\pm ff}^{dq}(s)$ and $\mathbf{Z}_{\pm w}^{dq}(s)$ are in (44), and $\mathbf{J}_{\pm o}(s)$, $\mathbf{G}_{\pm o}(s)$ and $\mathbf{Y}_{\pm o}(s)$ are in (45)-(49). It is worth mentioning that the $\tilde{\mathbf{I}}_s^{dq}$ expression in (50) is derived by reversing the dq -current flow in Fig. 2 to work with consumed currents.

Given that $\tilde{\mathbf{I}}_s^{dq} = [\tilde{\mathbf{I}}_s^{dq} \quad \tilde{\mathbf{I}}_s^{dq}]^T$ and $\tilde{\mathbf{E}}^{dq} = [\tilde{\mathbf{E}}^{dq} \quad \tilde{\mathbf{E}}^{dq}]^T$ and based on (50), the AC/DC small-signal admittance matrix of MMCs is represented as

$$\begin{aligned} \begin{bmatrix} \tilde{\mathbf{I}}_{dc} \\ -\tilde{\mathbf{I}}_s^{dq} \\ -\left(\tilde{\mathbf{I}}_s^{dq}\right)^* \end{bmatrix} &= \underbrace{\begin{bmatrix} Y_{cc} & Y_{cp} & Y_{cn} \\ Y_{pc} & Y_{pp} & Y_{pn} \\ Y_{nc} & Y_{np} & Y_{nn} \end{bmatrix}}_{\mathbf{Y}^{AC/DC}(s)} \begin{bmatrix} \tilde{\mathbf{V}}_{dc} \\ \tilde{\mathbf{E}}^{dq} \\ \left(\tilde{\mathbf{E}}^{dq}\right)^* \end{bmatrix} \\ Y_{cc} &= \mathbf{K}_{\pm dc}^{dq}(s)\mathbf{J}_{\pm M}^{dq}(s) + \mathbf{J}_{dc}^{dq}(s) \\ \mathbf{Y}_{12} &= \begin{bmatrix} Y_{cp} & Y_{cn} \end{bmatrix} = \mathbf{Y}_{\pm dc}^{dq}(s) - \mathbf{K}_{\pm dc}^{dq}(s)\mathbf{Y}_{\pm M}^{dq}(s) \\ \mathbf{Y}_{21} &= \begin{bmatrix} Y_{pc} \\ Y_{nc} \end{bmatrix} = -\mathbf{J}_{\pm M}^{dq}(s) \\ \mathbf{Y}_{22} &= \begin{bmatrix} Y_{pp} & Y_{pn} \\ Y_{np} & Y_{nn} \end{bmatrix} = \mathbf{Y}_{\pm M}^{dq}(s). \end{aligned} \quad (53)$$

A detailed description of the validation procedure of the *three-port* model in (53) and the corresponding results is provided in Section 5.

4.2. AC-side small-signal admittance-based model

The MMC AC-side equivalent admittance $\mathbf{Y}^{AC}(s)$ can be easily derived from (53) by imposing the DC-side relation $\tilde{\mathbf{I}}_{dc} = -\mathbf{Y}_{Th}^{dc}\tilde{\mathbf{V}}_{dc}$ in Fig. 2 [5]:

$$\begin{bmatrix} -Y_{Th}^{dc} \tilde{V}_{dc} \\ -\tilde{I}_s^{dq} \end{bmatrix} = \begin{bmatrix} Y_{cc} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \tilde{V}_{dc} \\ \tilde{E}^{dq} \end{bmatrix} \Rightarrow -\tilde{I}_s^{dq} = Y^{AC}(s) \tilde{E}^{dq} \quad (54)$$

$$Y^{AC}(s) = Y_{22} - Y_{21} (Y_{Th}^{dc} + Y_{cc})^{-1} Y_{12}.$$

As seen in (54), disregarding AC/DC interactions in (53) (i.e., $Y_{21} = 0$ and $Y_{12} = 0$) prevents the AC-side expression from capturing DC-side dynamics, including those of Y_{Th}^{dc} .

4.3. DC-side small-signal admittance-based model

The MMC DC-side equivalent admittance $Y^{DC}(s)$ can also be derived directly from (53) by applying the AC-side relation $\tilde{I}_s^{dq} = Y_{\pm Th}^{ac} \tilde{E}^{dq}$ in Fig. 2 [5]:

$$\begin{bmatrix} \tilde{I}_{dc} \\ -Y_{\pm Th}^{ac} \tilde{E}^{dq} \end{bmatrix} = \begin{bmatrix} Y_{cc} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} \tilde{V}_{dc} \\ \tilde{E}^{dq} \end{bmatrix} \Rightarrow \tilde{I}_{dc} = Y^{DC}(s) \tilde{V}_{dc} \quad (55)$$

$$Y^{DC}(s) = Y_{cc} - Y_{12} (Y_{\pm Th}^{ac} + Y_{22})^{-1} Y_{21}.$$

As seen in (55), disregarding AC/DC interactions in (53) (i.e., $Y_{12} = 0$ and $Y_{21} = 0$) prevents the DC-side expression from capturing AC-side dynamics, including those of $Y_{\pm Th}^{ac}$.

5. Model validation

The MMC AC/DC small-signal admittance matrix $Y^{AC/DC}(s)$ is modelled in (53) as a *three-port* system. As demonstrated in [29], the dq -complex space vectors $\tilde{X}^{dq}$ and $(\tilde{X}^{dq})^*$ (with $X \in \{E, I\}$) in (53) can be represented by the *positive*- and *negative*-sequence phasors, $\tilde{X}^p$ and $\tilde{X}^n$, respectively, in the frequency-domain ($s = j\omega$) because there exists a biunique (one-to-one) relation between the dq -complex space vectors and the sequence phasors. That is, the following expression can be derived from (53):

$$\begin{bmatrix} \tilde{I}_{dc} \\ \tilde{I}_s^p \\ \tilde{I}_s^n \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{cc} & Y_{cp} & Y_{cn} \\ Y_{pc} & Y_{pp} & Y_{pn} \\ Y_{nc} & Y_{np} & Y_{nn} \end{bmatrix}}_{Y^{AC/DC}(j\omega)} \begin{bmatrix} \tilde{V}_{dc} \\ \tilde{E}^p \\ \tilde{E}^n \end{bmatrix}. \quad (56)$$

The relation between (53) and (56) provides the formal link between the dq -complex representation and the pn -sequence decomposition of the abc perturbations, which is subsequently used in the validation of the $Y^{AC/DC}(j\omega)$ using frequency-domain measurements (see [29] for details).

To validate the proposed model, a frequency-domain characterisation is carried out in MATLAB/Simulink by using the test circuit shown in Fig. 3. Controlled pn -sequence AC and DC voltage perturbations are injected via the sources $u_{abc,harm}$ and $v_{dc,harm}$, respectively, and the resulting steady-state current, i_{abc} and i_{dc} , responses are recorded. FFT calculations are then applied to extract the complex phasors (i.e., the root-mean-square and phase angle values) of the voltage and current

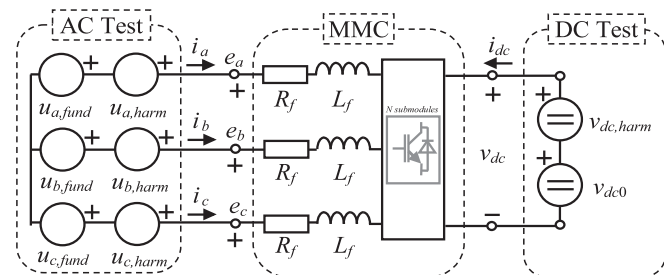


Fig. 3. Grid-connected MMC system for AC and DC admittance measurements.

perturbations, enabling the numerical reconstruction of the admittance matrix entries in (56) from time-domain simulations. The MATLAB/Simulink validation model corresponds to a nonlinear EMT arm-averaged implementation of the MMC, executed in discrete time with a fixed sample time of $T_s = 20 \mu s$ ($f_s = 50$ kHz). Since an arm-averaged model is employed, individual semiconductor switching events and the PWM carrier are not explicitly simulated; instead, the modulation stage is represented through the corresponding averaged arm-voltage relationship. The discrete-time implementation inherently accounts for the effects associated with controller sampling and the corresponding discrete-time execution, while the zero-order hold (ZOH) is explicitly represented by maintaining the control signals constant over each sampling interval. In addition, the current-control and circulating-current-control loops include the control delays specified in Table 4. These delays are retained consistently in the MATLAB/Simulink validation model and in the analytical formulation. No independent PWM update or carrier frequency is defined for the arm-averaged validation model, since the individual switching events and PWM carrier are not explicitly simulated. Apart from the aforementioned arm-averaging assumption, no additional simplifications are introduced, and all converter dynamics and control loops are represented explicitly. The converter employs the same arm-voltage balancing strategy adopted in [9] and [17], based on direct voltage control (direct modulation). This arm-averaged validation framework is adopted because the proposed analytical formulation is derived from the same physical MMC modeling framework used in [17] and [18], where it was validated against detailed switching EMT models, showing essentially identical frequency responses up to frequencies approaching the Nyquist limit associated with the equivalent converter-arm switching frequency.

The admittance terms of the *three-port* matrix $Y^{AC/DC}(j\omega)$ (56) are then calculated based on Ohm's law ($Y = I / V$), following the methodology proposed in [29]. Specifically:

1. A p -sequence perturbed AC voltage, represented by the complex phasor $\tilde{E}^p$, results in:
 - p - and n -sequence perturbed AC currents, represented by the complex phasors $\tilde{I}^p$ and $\tilde{I}^n$, characterising Y_{pp} and Y_{np} , respectively.
 - A perturbed DC current $\tilde{I}_{dc}$ characterising Y_{cp} .
2. An n -sequence perturbed AC voltage, represented by the complex phasor $\tilde{E}^n$, results in:
 - p - and n -sequence perturbed AC currents, represented by the complex phasors $\tilde{I}^p$ and $\tilde{I}^n$, characterising Y_{pn} and Y_{nn} , respectively.
 - A perturbed DC current $\tilde{I}_{dc}$ characterising Y_{cn} .
3. A perturbed DC voltage $\tilde{V}_{dc}$ results in:
 - p - and n -sequence perturbed AC currents, represented by the complex phasors $\tilde{I}^p$ and $\tilde{I}^n$, characterising Y_{pc} and Y_{nc} , respectively.
 - A perturbed DC current $\tilde{I}_{dc}$ characterising Y_{cc} .

This systematic approach allows the reconstruction of the full frequency-dependent admittance matrix $Y^{AC/DC}(j\omega)$ in (56), enabling direct comparison with the analytically derived model in (53). The above methodology inherently captures the *mirror frequency effect* between the p - and n -sequences [29]. All frequency-response measurements presented in the paper have been obtained using the SARA toolbox [51], a MATLAB/Simulink-based framework for automated AC/DC admittance measurement. The toolbox automatically performs the complete frequency-sweep process, including signal injection, steady-state detection, data acquisition, FFT processing, and admittance extraction using a consistent methodology, thereby ensuring reliable and reproducible frequency-domain measurements throughout the validation process. The frequency-domain measurements are performed over the 1–1000 Hz frequency range with a 1 Hz frequency resolution. The injected perturbations have a magnitude of 1% of the corresponding steady-state operating quantity. For each frequency point, the system is

first allowed to reach steady state before the perturbation is applied and the measurement window is initiated. The steady-state condition is assessed individually for each simulation by monitoring the relevant electrical and control variables and ensuring that the operating point has reached a stationary periodic regime before the frequency-domain measurement is performed. The FFT-based extraction is performed over a 1 s measurement window, corresponding to the 1 Hz frequency resolution and containing 50 cycles of the 50 Hz fundamental component. The spectral component corresponding to the injected perturbation frequency is then extracted to calculate the corresponding admittance element. The simulation time step is an independent numerical parameter: the nonlinear MATLAB/Simulink validation models are executed using a fixed time step of 20 μ s, corresponding to a numerical sampling frequency of 50 kHz. Thus, the simulation time step determines the time resolution of the EMT simulations, whereas the measurement-window duration determines the frequency resolution of the admittance extraction. The validation is based on controlled pn -sequence AC perturbations and DC perturbations, from which the complete 3×3 admittance matrix is reconstructed according to (56). Consequently, the validation includes both the self-port and cross-port admittances of the complete AC/DC *three-port* formulation.

According to the previous procedure, the validation of the MMC AC/DC small-signal admittance matrix (53) under various operating conditions, along with a discussion of key observations, is presented below. To quantitatively assess the agreement between the analytical model and the EMT simulations, the peak-normalised absolute error is employed:

$$\varepsilon_i(\%) = 100 \frac{|x_i^{\text{model}} - x_i^{\text{sim}}|}{\max_i |x_i^{\text{sim}}|}, \quad (57)$$

where x_i^{model} and x_i^{sim} denote the analytical-model and MATLAB/Simulink responses at the i -th frequency point, respectively, with the latter taken as the reference response, and $\max_i |x_i^{\text{sim}}|$ denotes the maximum absolute value of the reference response over the considered frequency range. This metric normalises the absolute error by the peak value of the reference response, thereby avoiding artificially large relative errors in frequency regions where the reference response approaches zero. In addition, the normalised root-mean-square error (NRMSE) is employed as a complementary quantitative metric:

$$NRMSE(\%) = 100 \frac{\sqrt{\frac{1}{N_f} \sum_{i=1}^{N_f} |x_i^{\text{model}} - x_i^{\text{sim}}|^2}}{\max_i |x_i^{\text{sim}}|}, \quad (58)$$

where N_f is the number of frequency points. The NRMSE is evaluated separately for the real and imaginary parts of each admittance-matrix element. Unlike the peak-normalised absolute error, which quantifies the point-by-point deviation over the frequency range, the NRMSE provides an overall measure of the RMS discrepancy relative to the peak magnitude of the reference response. The objective is not only to verify and quantify the capability of the proposed model to accurately capture the internal dynamics and control interactions of the MMC, but also to validate, in a progressive manner, each modelling stage developed throughout Sections 2–4 before constructing the complete AC/DC admittance representation. For this purpose, the validation procedure is organised according to the different modelling stages considered during the derivation of the proposed MMC model:

1) Step 1 – Internal dynamics and CCC validation (Section 2):

This first step considers open-loop operation (without inner CC), evaluated both with and without the CCC enabled, to isolate the influence of the MMC internal dynamics and the CCC on the AC/DC frequency response.

Fig. 4 presents the comparison between the analytical model and the simulation results for this first modelling stage. The results validate the accuracy of the developed model in representing the MMC internal dynamics and the effect of the CCC. Under open-loop operation, the internal dynamics introduce characteristic resonance behaviours observable on both the AC and DC sides. Enabling the CCC modifies these resonances and shifts their frequencies, demonstrating the direct impact of the CCC dynamics on the overall converter frequency response. The maximum peak-normalised absolute error obtained for this validation stage is 9.27%, while the maximum NRMSE across all real and imaginary admittance responses is 1.14%, confirming the very good agreement between the analytical model and the EMT simulations.

2) Step 2 – Inner CC validation (Section 3.2): This second step incorporates the inner CC into the model in order to evaluate its influence on the AC- and DC-side dynamics.

Fig. 5 presents the results for this second modelling stage. The comparison with simulation results validates the correct incorporation of the inner CC into the proposed analytical model. In this case, the controller significantly modifies the AC-side dynamics by introducing the expected closed-loop behaviour, while the DC-side dynamics remain comparatively less affected. The maximum peak-normalised absolute error obtained for this validation stage is 9.93%, while the maximum NRMSE across all real and imaginary admittance responses is 4.51%.

3) Step 3 – Full closed-loop MMC validation (Sections 3 and 4):

This final step considers the complete MMC model with all control loops enabled, including the inner CC, PLL, and outer control loops, in order to

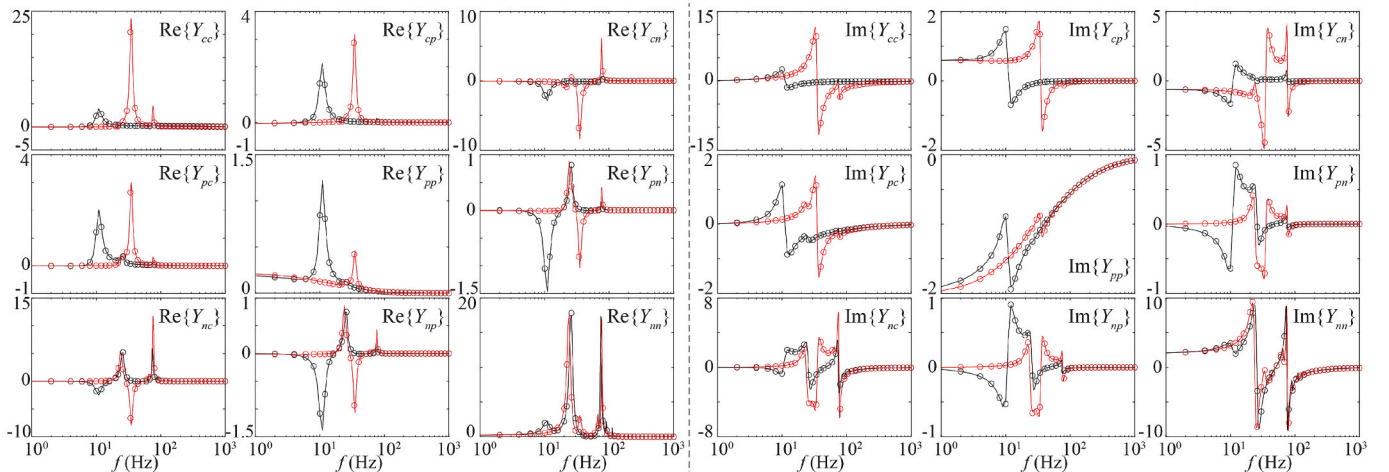


Fig. 4. Real and imaginary parts of the admittance matrix terms from analytical expressions (solid curves) and from MATLAB/Simulink simulations (circles) for the AC/DC admittance-based model of the MMC internal dynamics (in Ω^{-1}), presented without (red plots) and with (black plots) circulating current controller.

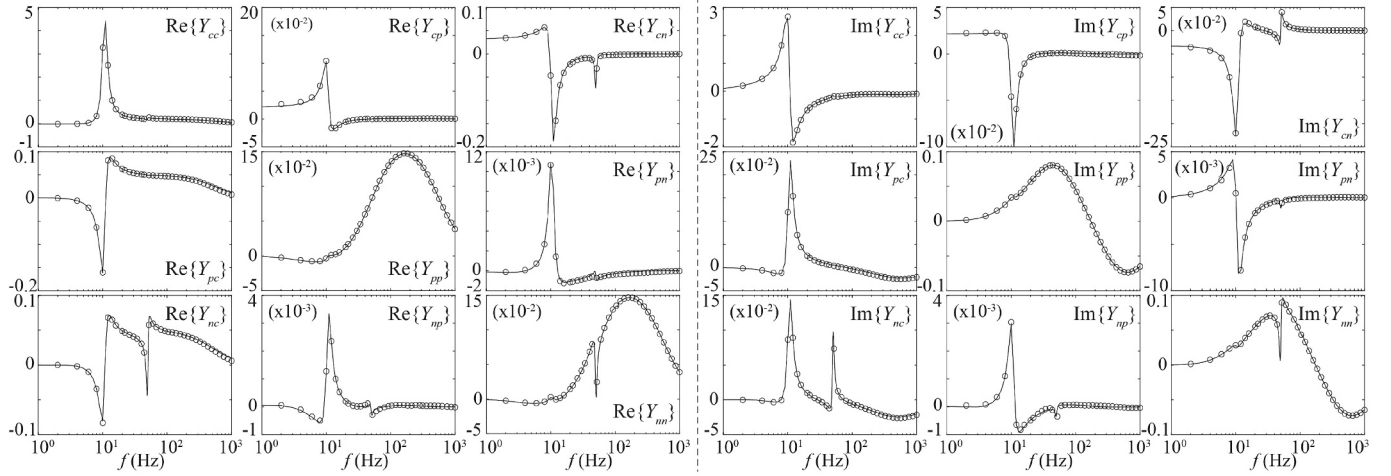


Fig. 5. Real and imaginary parts of the admittance matrix terms from analytical expressions (solid curves) and from MATLAB/Simulink simulations (circles) for the AC/DC admittance-based model of the MMC internal dynamics (in Ω^{-1}), incorporating both circulating current and inner current controllers.

evaluate the full closed-loop system dynamics.

Fig. 6 presents the validation results for this final modelling stage under full control operation. The obtained results validate the correct integration of the PLL and outer control loops into the proposed MMC AC/DC model. For the DVC-QPC operating mode with PLL dynamics, Fig. 6 presents the direct element-by-element comparison of the complete 3×3 admittance matrix, including all self-port and cross-port admittance terms. The remaining outer-control configurations considered in this study are likewise validated through the complete 3×3 admittance matrices using the same quantitative error metrics. For conciseness, however, the corresponding matrix-response comparisons are not shown individually; instead, the real eigenvalues of the Hermitian matrix of the complete admittance matrix are presented in Fig. 7 as a compact representation of the corresponding validation results and associated power-dissipation characteristics. In the DVC-QPC operating mode scenario, the outer control loops predominantly affect the low-frequency response of both the AC and DC sides, reflecting the strong interaction between outer-loop dynamics and the overall converter behaviour. The maximum peak-normalised absolute error obtained during the validation of the DVC-QPC operating mode is 9.06%, while the maximum NRMSE across all real and imaginary admittance responses is 4.53%.

Additionally, following the power-dissipation (passivity) criterion

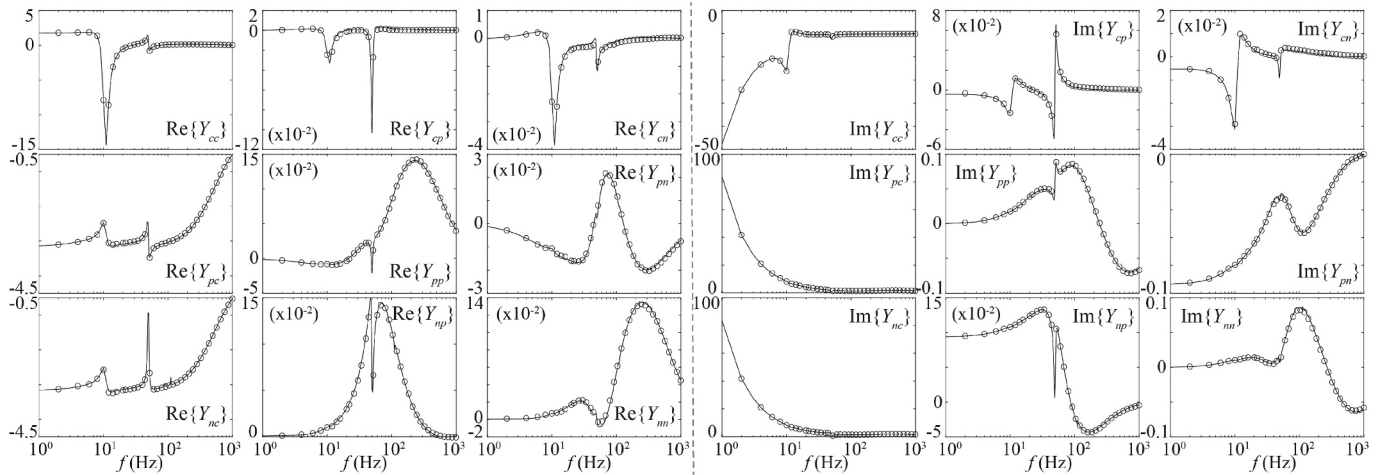


Fig. 6. Real and imaginary parts of the admittance matrix terms from analytical expressions (solid curves) and from MATLAB/Simulink simulations (circles) for the AC/DC admittance-based model of the MMC internal dynamics (in Ω^{-1}), incorporating circulating current and inner current controllers, a phase-locked loop, and DVC with QPC outer control loops.

[47], the real eigenvalues of the Hermitian matrix of the proposed 3×3 MMC AC/DC admittance matrix (i.e., $\mathbf{Y}_p(j\omega) = \frac{1}{2}(\mathbf{Y}^{\text{ACDC}}(j\omega) + \mathbf{Y}^{\text{ACDC}}(j\omega)^H)$) are employed to validate the remaining outer control-loop configurations in a compact manner, without unnecessarily increasing the number of admittance-response plots. Fig. 7 compares the analytical and MATLAB/Simulink frequency responses of these real eigenvalues for the PLL-only, PLL+DVC, PLL+AVC, PLL+APC, and PLL+QPC operating modes. The maximum peak-normalised absolute error among all the eigenvalue responses is 8.67%, while the maximum NRMSE is 5.27%. The corresponding complete 3×3 admittance matrices are also quantitatively assessed element-by-element, yielding a maximum peak-normalised absolute error of 9.44% and a maximum NRMSE of 4.47% across all real and imaginary admittance responses.

The previous validation stages demonstrate the accuracy and consistency of the proposed modelling procedure, validating each intermediate modelling step prior to obtaining the final MMC AC/DC admittance representation. Furthermore, the obtained results illustrate how the different MMC control loops modify the converter frequency response and power-dissipation characteristics on both the AC and DC sides. Depending on the operating condition and control configuration, different control loops may either enhance system damping or introduce non-passive behaviour on the AC side, the DC side, or both. In particular, slight variations in control parameters can significantly modify the

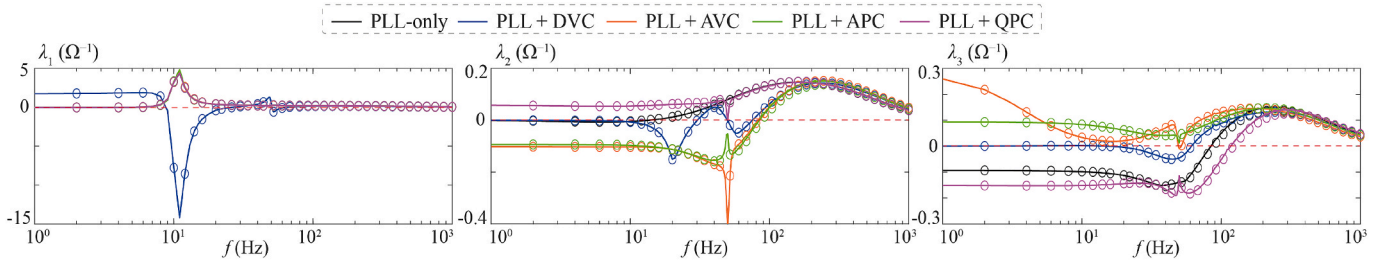


Fig. 7. Analytical (solid curves) and MATLAB/Simulink (circles) frequency responses of the eigenvalues λ_1 , λ_2 , and λ_3 (in Ω^{-1}) of the Hermitian matrix of the MMC AC/DC admittance matrix for different outer control-loop configurations (PLL-only, PLL+DVC, PLL+AVC, PLL+APC, and PLL+QPC).

passivity characteristics of the MMC [18,47,48]. This sensitivity is further investigated in Section 7, where several case studies demonstrate how specific parameter variations may lead to instability phenomena.

All MMC parameters and operating points obtained from the MATLAB/Simulink simulations for each scenario are provided in Appendix D.

It should be noted that the validation presented in this section is focused on assessing the accuracy of the proposed formulation against the nonlinear EMT reference model and is not intended to provide a quantitative benchmark against alternative mathematical reformulation methodologies. Although the proposed reformulation is structurally compact and avoids explicit harmonic-state expansion, as highlighted in Table 1, a direct quantitative comparison with an equivalently implemented HSS or other multiharmonic formulation is not performed in this work. Such a comparison would nevertheless be valuable to systematically assess the relative computational and modelling characteristics of the different approaches. However, it would require consistent physical modelling assumptions, controller structures, harmonic truncation criteria, numerical solvers, and validation conditions. Consequently, the present results are not intended to establish relative model dimension, retained harmonic orders, model-construction time, frequency-sweep time, memory consumption, or frequency-response accuracy with respect to these alternative methodologies.

6. Frequency-domain stability assessment of multi-terminal AC/DC transmission grids

Frequency-domain impedance-based approaches are becoming increasingly attractive for stability assessment as they use less complex systems of equations and provide detailed information with reduced computational effort. In multi-terminal AC/DC grids dominated by power electronics-based converters, these methods enable the analysis of converter-driven oscillatory phenomena and interaction mechanisms over a wide frequency range. As shown in Fig. 8(a), frequency-domain stability studies are commonly performed through the characterisation of the entire grid using the nodal voltage method [5,22],

$$\begin{aligned} \left. \begin{aligned} \mathbf{I}_G(s) &= \mathbf{Y}_G(s)\mathbf{V}_B(s) \\ \mathbf{I}_G(s) &= \mathbf{I}_B(s) - \mathbf{Y}_C(s)\mathbf{V}_B(s) \end{aligned} \right\} \\ \Rightarrow \mathbf{V}_B(s) &= \mathbf{Y}_B^{-1}(s)\mathbf{I}_B(s) \quad (\mathbf{Y}_B(s) = \mathbf{Y}_C(s) + \mathbf{Y}_G(s)), \end{aligned} \quad (59)$$

where $\mathbf{Y}_G(s)$ characterises the nodal admittance matrix of the grid and

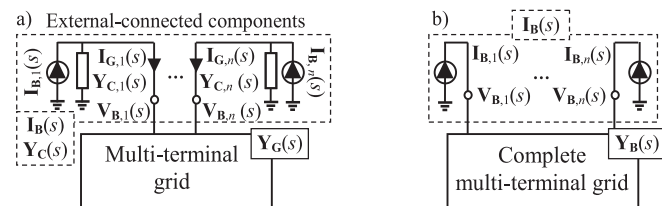


Fig. 8. Multi-terminal grid-connected power electronics-based converters: a) Equivalent circuit of multi-terminal grid with power electronics-based converters. b) Equivalent circuit of complete multi-terminal grid.

$\mathbf{Y}_C(s)$ is a diagonal matrix with the admittance matrices $\mathbf{Y}_{C,i}(s)$ of the external components connected to the grid buses (including power electronics-based converters). $\mathbf{V}_B(s)$ and $\mathbf{I}_B(s)$ represent the bus voltage and injected current vectors, respectively, and $\mathbf{Y}_B(s)$ is the nodal admittance matrix of the system (see Fig. 8(b)) [5,24]. System stability in (59) is commonly assessed by the GNC. However, the PMD stability criterion has recently been proposed for stability assessment of multi-terminal grid-connected power electronics-based converters [24].

Additionally, beyond determining whether the system is stable or unstable, other frequency-domain techniques can also be employed to investigate the propagation of oscillations across interconnected AC/DC subsystems [49].

6.1. AC/DC nodal admittance matrix

The proposed MMC admittance model is intended for frequency-domain small-signal stability assessment of hybrid AC/DC transmission grids through the nodal admittance matrix formulation introduced in (59). To this end, all grid components are represented by admittance matrices relating their terminal voltage and current perturbations, allowing the complete interconnected system to be assembled within a unified mathematical framework. A key advantage of the proposed formulation is that AC-only, DC-only, and hybrid AC/DC components can all be represented using a common dq -complex *three-port* admittance structure. Accordingly, the proposed MMC admittance model can be directly incorporated into the nodal admittance matrix together with the remaining grid components without requiring any additional reformulation.

Based on Fig. 8 and the nodal voltage method (59), the complete transmission grid is represented through the nodal admittance matrix $\mathbf{Y}_B(s)$. This matrix establishes the relationship between the nodal current injection vector $\mathbf{I}_B(s)$ and the nodal voltage vector $\mathbf{V}_B(s)$. The entries of $\mathbf{Y}_B(s)$ are obtained from two fundamental building blocks: the primitive admittance matrix $\mathbf{Y}_R(s)$, which contains the admittance models of all grid components, and the incidence matrix $\mathbf{A}$, which describes the grid topology.

Consider a transmission grid composed of N_b branches and components, the primitive admittance matrix is assembled by placing the admittance matrix of every branch or bus-connected component along its diagonal,

$$\mathbf{Y}_R(s) = \begin{bmatrix} \mathbf{Y}_{R,1}(s) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \mathbf{Y}_{R,N_b}(s) \end{bmatrix}, \quad (60)$$

where $\mathbf{Y}_{R,i}(s)$ denotes the 3×3 dq -complex admittance matrix associated with component (or branch) i .

Thus, every electrical device is represented through a *three-port* AC/DC admittance formulation,

$$\begin{bmatrix} \tilde{I}_{dc} \\ \tilde{I}_s \\ (\tilde{I}_s^{dq})^* \end{bmatrix} = \underbrace{\begin{bmatrix} Y_{cc}(s) & Y_{cp}(s) & Y_{cn}(s) \\ Y_{pc}(s) & Y_{pp}(s) & Y_{pn}(s) \\ Y_{nc}(s) & Y_{np}(s) & Y_{nn}(s) \end{bmatrix}}_{\mathbf{Y}_{R,i}(s)} \begin{bmatrix} \tilde{V}_{dc} \\ \tilde{\mathbf{E}}^{dq} \\ (\tilde{\mathbf{E}}^{dq})^* \end{bmatrix}. \quad (61)$$

This formulation enables AC-only, DC-only, and hybrid AC/DC devices to be incorporated consistently into the same grid model. Accordingly, the components of a multi-terminal hybrid AC/DC transmission grid can be classified into the following three categories:

• AC/DC Three-Port Components:

AC/DC power-electronic converters (e.g., HVDC MMCs) can be expressed in the s -domain through the general admittance (61). Within the primitive admittance matrix, these devices are treated as components incident to a single bus i .

In particular, the proposed MMC admittance model, derived in (53), is incorporated as a bus-connected *three-port* element whose admittance matrix contributes directly to the corresponding block $\mathbf{Y}_{R,i}(s)$ of the primitive admittance matrix $\mathbf{Y}_R(s)$ (60). This enables the AC/DC dynamic coupling captured by the proposed model to be preserved naturally during the assembly of the complete transmission-grid model.

• AC-side Two-Port Components:

AC-side dq -complex two-port terminal component models are expressed according to the 3×3 admittance-based model as follows:

$$\mathbf{Y}_{R,i}(s) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{Y}^+(s) & \mathbf{Y}^-(s) \\ 0 & (\mathbf{Y}^-(s))^* & (\mathbf{Y}^+(s))^* \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{0}_{1 \times 2} \\ \mathbf{0}_{2 \times 1} & \mathbf{Y}^{AC}(s) \end{bmatrix}, \quad (62)$$

where the admittance matrix $\mathbf{Y}^{AC}(s)$ depends on the model of the AC-side transmission grid branch component. For instance, for the common model of a transformer (with no phase shift) is $\mathbf{Y}^+(s) = 1 / (R + L(s + j\omega_1))$ and $\mathbf{Y}^-(s) = 0$ (where R and L are the resistance and inductance, and ω_1 is the fundamental angular frequency).

• DC-side Single-Port Components:

DC-side dq -complex single-port terminal component models are expressed according to the 3×3 admittance-based model as follows:

$$\mathbf{Y}_{R,i}(s) = \begin{bmatrix} Y_{cc}(s) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} Y^{DC}(s) & \mathbf{0}_{1 \times 2} \\ \mathbf{0}_{2 \times 1} & \mathbf{0}_{2 \times 2} \end{bmatrix}, \quad (63)$$

where the admittance $Y^{DC}(s)$ depends on the adopted DC-side component model. For instance, for the common model of a capacitor is $Y_{cc}(s) = C_{dc}s$ (where C_{dc} is the capacitance of the component).

Consequently, every component of the hybrid AC/DC transmission grid is represented using the same mathematical structure, considerably simplifying the assembly procedure while preserving the dynamic coupling introduced by converter-interfaced devices.

The grid topology is represented through the incidence matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{1,1} & \cdots & \mathbf{A}_{1,n} \\ \vdots & \ddots & \vdots \\ \mathbf{A}_{N_b,1} & \cdots & \mathbf{A}_{N_b,n} \end{bmatrix}, \quad (64)$$

where n is the number of buses, and $\mathbf{A}_{r,i}$ are 3×3 submatrices assembled according to the orientation of branch r with respect to bus i :

- $\mathbf{A}_{r,i} = \mathbf{I}_{3 \times 3}$ if the branch originates from bus i ,
- $\mathbf{A}_{r,i} = -\mathbf{I}_{3 \times 3}$ if the branch terminates at bus i ,
- $\mathbf{A}_{r,i} = \mathbf{0}_{3 \times 3}$ if the branch is not connected to bus i .

with $\mathbf{I}_{3 \times 3}$ and $\mathbf{0}_{3 \times 3}$ being the 3×3 identity and zero matrices, respectively.

Finally, by applying Kirchhoff's laws, the complete nodal admittance matrix is obtained through the singular transformation [43]:

$$\mathbf{Y}_B(s) = \mathbf{A}^T \mathbf{Y}_R(s) \mathbf{A}. \quad (65)$$

It is worth noting that, after applying the singular transformation in (65), some rows and columns of $\mathbf{Y}_B(s)$ may become entirely zero. This occurs at buses connected exclusively to AC-only or DC-only components, where the corresponding entries of the *three-port* formulation are zero. Only buses interfacing both AC and DC subsystems retain full 3×3 admittance blocks. Therefore, any rows and columns composed entirely of zeros should be removed before performing the subsequent frequency-domain stability analysis.

6.2. The positive-mode-damping stability criterion

The PMD stability criterion [24] is a frequency-domain method for assessing small-signal stability issues in multi-terminal grids that include power electronics-based converters [40]. This approach is based on the eigenvalue decomposition of the nodal admittance matrix $\mathbf{Y}_B^{(f)}$ at each frequency within the frequency range, using the RMA [41] as:

$$\mathbf{V}_B^{(f)} = \left(\mathbf{Y}_B^{(f)} \right)^{-1} \mathbf{I}_B^{(f)} \Rightarrow \mathbf{V}_m^{(f)} = \left(\Lambda_Y^{(f)} \right)^{-1} \mathbf{I}_m^{(f)} \\ \Lambda_Y^{(f)} = \begin{bmatrix} \lambda_{Y,1}^{(f)} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_{Y,n}^{(f)} \end{bmatrix} = \mathbf{L}^{(f)} \mathbf{Y}_B^{(f)} \mathbf{R}^{(f)}, \quad (66)$$

where $\mathbf{V}_m^{(f)} = \mathbf{L}^{(f)} \mathbf{V}_B^{(f)}$ and $\mathbf{I}_m^{(f)} = \mathbf{L}^{(f)} \mathbf{I}_B^{(f)}$ represent the modal voltage and current vectors, $\Lambda_Y^{(f)}$ is the diagonal eigenvalue matrix of $\mathbf{Y}_B^{(f)}$, and $\mathbf{R}^{(f)}$ and $\mathbf{L}^{(f)} = (\mathbf{R}^{(f)})^{-1}$ are the right (in columns) and left (in rows) eigenvector matrices, respectively.

Stability is assessed by analysing the critical resonance modes $Z_{cm}^{(fr)}$ at each resonance frequency f_r , corresponding to the peak values of the moduli of the modal impedances $|Z_{m,j}^{(fr)}| = |1/\lambda_{Y,j}^{(fr)}|$ (being $j = \{1, \dots, n\}$ the mode number). These critical resonance modes and their associated eigenvectors can be directly obtained, without the need to compute all modal impedances (only the biggest modal impedance is needed) [42]. The real and imaginary parts of the critical resonance modes, $R_{cm}^{(fr)}$ and $X_{cm}^{(fr)}$, respectively, are used to assess stability [24]. The stability condition states that the grid is stable if:

$$\sigma_i^{(fr)} \approx \frac{R_{cm}^{(fr)}}{m_{x,cm}^{(fr)}} < 0 \left(m_{x,cm}^{(fr)} = \left. \frac{\partial X_{cm}^{(fr)}}{\partial f} \right|_{f=f_r} \right) \forall f_r, \quad (67)$$

where $m_{x,cm}$ is the slope of $X_{cm}^{(fr)}$. Notably, the system's damping oscillatory modes relate to this criterion via $\sigma_i^{(fr)}$ [24].

Recent enhancements to the PMD criterion incorporate PF-based sensitivity analysis [42] to improve stability analysis. PFs are computed as [41,42,46]:

$$PF_b^{(fr)} = R_b^{(fr)} L_b^{(fr)}, \quad (68)$$

where $R_b^{(fr)}$ and $L_b^{(fr)}$ are the right and left eigenvectors associated with the critical resonance modes $Z_{cm}^{(fr)}$, and b represents the grid buses. These PFs quantify the influence (observability and excitability) of each bus on system stability, identifying key elements affecting resonance modes [41,42,46].

6.3. Propagation indicators for local-area oscillations

This subsection focuses on the phenomenon of local-area oscillations in hybrid AC/DC grids and reviews propagation indicators to characterise them. In such systems, the control strategies of power electronics

converters may restrict the propagation of oscillations across subsystems, potentially leading to the emergence of local-area oscillations [49]. These are oscillatory modes confined to a limited group of nodes—referred to as the oscillation source (OS) area—without significant interaction with the rest of the grid, or non-oscillation source (NOS) area.

To quantitatively characterise this phenomenon, a Frequency-domain Modal Analysis combined with Block Diagonal Dominance (FMA-BDD) methodology is employed [49]. This approach relies on decomposing the grid admittance matrix at a critical oscillatory frequency (s_m) and classifying the nodes via their PFs. Nodes with high PFs form the OS area, while the remaining nodes are assigned to the NOS area. Once this partition is performed, the system impedance matrix is rearranged into a block structure,

$$\begin{bmatrix} \tilde{\mathbf{V}}_{OS} \\ \tilde{\mathbf{V}}_{NOS} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{OS}(s_m) & \mathbf{Z}_{off1}(s_m) \\ \mathbf{Z}_{off2}(s_m) & \mathbf{Z}_{NOS}(s_m) \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{I}}_{inj(OS)} \\ \tilde{\mathbf{I}}_{inj(NOS)} \end{bmatrix}, \quad (69)$$

from which three key indicators are derived to quantify the coupling between OS and NOS areas:

- Inter-area Controllability:

$$Con_{area} = \frac{\|\mathbf{Z}_{off1}(s_m)\|_2}{\|\mathbf{Z}_{OS}(s_m)\|_2}, \quad (70)$$

- Inter-area Observability:

$$Obs_{area} = \frac{\|\mathbf{Z}_{off2}(s_m)\|_2}{\|\mathbf{Z}_{NOS}(s_m)\|_2}, \quad (71)$$

- Inter-area Participation Factor:

$$PF_{area} = Con_{area} \cdot Obs_{area}. \quad (72)$$

These indicators capture the directional energy exchange between subsystems and serve to assess the extent to which an oscillation mode is confined locally or may spread system-wide [49]. In particular:

- If $PF_{area} = 0$ and $Obs_{area} = 0$, a strictly local oscillation is in the OS area identified.
- If $PF_{area} = 0$ but $Obs_{area} \neq 0$, the oscillation remains confined in terms of participation, but its effects are observable throughout the system.

The above indicators are subsequently employed in Section 7 to analyse the propagation paths of MMC-driven oscillations across the studied hybrid AC/DC test systems.

7. Applications

This section demonstrates the practical small-signal stability capabilities of the proposed *three-port* MMC small-signal admittance-based model by analysing five test cases in the hybrid AC/DC test system shown in Fig. 9, each illustrating a different converter-driven stability scenario. All cases are evaluated analytically using the PMD stability criterion and PFs (see Section 6.2) and subsequently validated against nonlinear time-domain simulations in MATLAB/Simulink. Beyond illustrating the practical applicability of the proposed methodology, these case studies also provide an additional validation of the proposed AC/DC admittance formulation under different converter operating conditions, power-flow directions, control configurations, and oscillatory mechanisms through comparisons with nonlinear EMT simulations. Starting from a stable reference configuration of the MMCs (see Table 2 for data) and the grid (see Fig. 9), the AC/DC transmission system is independently modified in each scenario to provoke specific oscillatory behaviours. The first four test cases are classified according to the affected MMC side (AC or DC) and the oscillation frequency range (near-synchronous or harmonic), whereas the fifth test case has been specifically designed to demonstrate the importance of preserving the complete AC/DC coupling structure for accurate stability prediction. It should be noted that the capacitive elements connected to the AC and DC terminals in Fig. 9 represent the equivalent capacitance of the

Table 2

MMC model parameter values for applications.

Parameter	MMC 1	MMC 2
S_{MMC}	1 GVA	1 GVA
$E_{N,ac}$	220 kV	220 kV
$V_{N,dc}$	440 kV	440 kV
f_1	50 Hz	50 Hz
f_{sw}	1 kHz	1 kHz
k	3	3
N	10	10
C	10 mF	10 mF
L_c	34.5 mH	34.5 mH
R_c	1.1 Ω	1.1 Ω
L_f	38.5 mH	38.5 mH
R_f	0.12 Ω	0.12 Ω
a_{ccc}	600π rad/s	600π rad/s
a_{cc}	1000 rad/s	1000 rad/s
a_f	100 rad/s	100 rad/s
a_{pll}	100 rad/s	100 rad/s
ξ_{pll}	$\sqrt{2}/2$	$\sqrt{2}/2$
a_{dvc}	-	100 rad/s
a_{pc}	100 rad/s	-
a_{avc}	-	-
a_{qc}	100 rad/s	-
T_d	100 μ s	100 μ s
P_o	1 GW	1 GW
Q_o	0 GVar	0 GVar

Note: See Table 4 for parameter definitions.

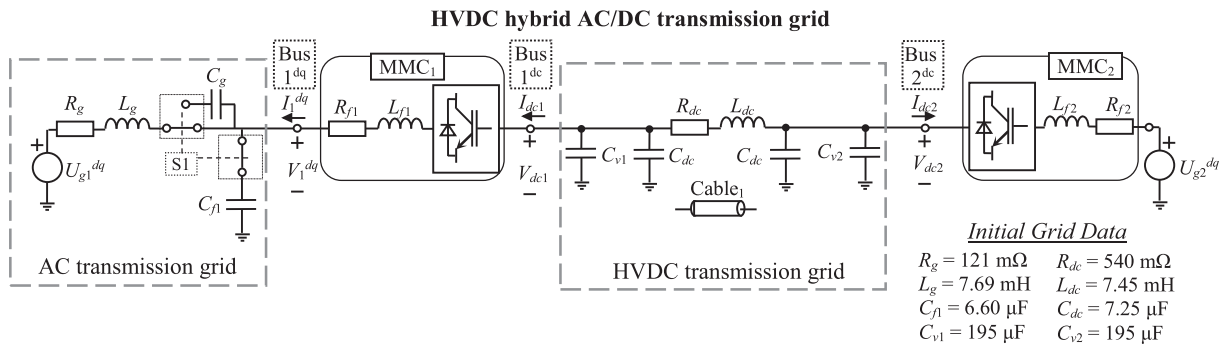


Fig. 9. Multi-terminal HVDC hybrid AC/DC transmission system.

surrounding AC and DC grids (e.g., grid or cable capacitance), included to model external system dynamics that may participate in oscillatory modes. The switching frequency reported in Table 2 corresponds to the switching frequency of each individual MMC submodule. The considered MMC employs $N = 10$ submodules per arm and uses phase-shifted carrier PWM; therefore, the corresponding effective switching frequency at the converter-arm level is approximately $N \cdot f_{sw} = 10$ kHz [15]. Although the arm-averaged validation model does not explicitly simulate the individual semiconductor switching events or the PWM carrier, this effective arm-level switching frequency characterises the switching behaviour of the underlying MMC and, for the adopted arm-averaged representation, defines a corresponding Nyquist limit of 5 kHz. In addition, the nonlinear EMT validation model is executed in discrete time with $T_s = 20$ μ s, corresponding to a digital control sampling frequency of $f_s = 50$ kHz and a corresponding Nyquist frequency of 25 kHz. The two resulting Nyquist frequencies therefore provide upper-frequency references of 5 kHz for the arm-averaged representation and 25 kHz for the discrete-time control implementation. However, the ZOH and digital execution delay introduce an additional phase effect that must be considered when assessing the accuracy of the continuous-frequency analytical formulation relative to the discrete-time validation model. In particular, the effective 30 μ s delay associated with the ZOH and digital execution corresponds to a phase shift of approximately 21.6° at 2 kHz. Accordingly, the frequency range considered appropriate for the present study can be taken to extend approximately from 1 Hz to 2 kHz. This range fully encompasses the 1–1000 Hz frequency range used in the validation and application studies. The control-loop delays T_d specified in Table 2 are consistently represented in both the analytical formulation and the MATLAB/Simulink validation model and therefore do not introduce an additional discrepancy between the two models. These considerations support the consistency between the proposed frequency-domain formulation and the nonlinear discrete-time EMT validation model within the frequency range investigated.

In addition to the PMD-based stability assessment and the PF analysis (used to identify the dominant source of instability), each case includes a calculation of AC/DC interaction indices (see Section 6.3). These indicators are employed to quantify the local-area propagation of oscillations and to assess the importance of preserving the full AC/DC dynamic coupling structure in the proposed MMC model, rather than relying on simplified AC- or DC-side representations alone. The application case considered in this section is intended primarily as a methodological benchmark for assessing the proposed AC/DC modelling and stability-analysis framework and should not be interpreted as a detailed representation of a specific commercial MMC-HVDC installation. In particular, the adopted $N = 10$ configuration represents an aggregated or macro-submodule description of the converter. The number of submodules cannot be varied independently of the remaining converter parameters if physically consistent scaling is intended. For a fixed arm voltage, the nominal submodule voltage approximately scales as $V_{SM} \propto 1/N$, while preserving the total nominal capacitor energy per arm, $E_{arm} = \frac{1}{2} \cdot N \cdot C_{SM} V_{SM}^2$, leads approximately to $C_{SM} \propto N$. In practical designs, the required capacitance is additionally constrained by the allowable capacitor-voltage ripple, which depends on the arm current, transferred power, operating point, and modulation strategy. The rated current is primarily determined by the converter power and voltage ratings and should therefore be scaled consistently with these quantities rather than with N alone. Consequently, extrapolation of the present benchmark to a commercial MMC-HVDC installation would require coordinated scaling of the number of submodules and the associated electrical and control parameters. The numerical values adopted here, while retaining the power (1 GVA) and voltage (220/440 kV) ratings considered in the study, are therefore intended to provide a compact and computationally tractable benchmark that preserves the MMC internal dynamics and AC/DC interactions relevant to the proposed methodology.

7.1. Test #1 – AC near-synchronous oscillation

In this first test case, the original R_g – L_g grid impedance is modified into a series-compensated R_g – L_g – C_g configuration by activating switch S1, which simultaneously disconnects the AC shunt capacitance C_{f1} in Fig. 9. Moreover, the line inductance L_g is increased by a factor of 13 ($L_g \times 13$), and the series-connected capacitance is set to $C_g = 50 \times C_{f1}$.

The PMD stability criterion applied to the AC/DC system in Fig. 10(a) (orange thick curve) reveals an oscillatory mode at 35 Hz driving system instability. The PF analysis clearly identifies Bus 1—associated with MMC₁—as the dominant contributor to this instability, specifically linked to dynamics on the AC side. Based on this diagnosis and according to the local-area oscillation indicators defined in Section 6.3, the AC-side is identified as the oscillation source area (i.e., OS area), the DC-side as the non-oscillation source area (i.e., NOS area), and 35 Hz as the critical oscillation frequency (i.e., s_m). To assess the extent of propagation from the AC side to the DC side, the observability index Obs_{area} and the associated participation factor PF_{area} (see Section 6.3 for details) are evaluated at 35 Hz in Fig. 10(b). The result— $Obs_{area} \approx 0$ and hence $PF_{area} \approx 0$ —confirms that the instability remains confined to the AC-side and does not propagate through the converter into the DC grid. Time-domain simulations in MATLAB/Simulink (Fig. 10(c)) validate these findings. Clear oscillations around 35 Hz appear in the AC power P_{pcc1} , while no such oscillations are observed in the DC power P_{dc1} . It should be noted that the final instability in P_{dc1} result from the collapse of the simulated system, not from the original oscillatory mode. Overall, this test demonstrates the capability of the proposed 3×3 model to accurately: i) detect near-synchronous instabilities originated on the AC side; ii) identify the dominant source of instability; and iii) determine whether such instabilities propagate across AC and DC subsystems. In this scenario, the results verify that the MMC₁ control operating in APC-QPC mode isolates the DC side from the AC-side near-synchronous oscillations. Additional passivity-oriented analyses (not shown for brevity) further indicate that this operating condition introduces non-passive behaviour around 35 Hz on the AC side, which is consistent with the observed instability.

Additional PMD-based stability assessment tests using the AC- and DC-side equivalent models from equations (54) and (55), respectively, are shown in Fig. 10(a) (blue and green thin curves, respectively). This study illustrates that the AC-side model correctly predicts the instability, while the DC-side model does not. This discrepancy arises because reduced models inherently obscure key dynamic behaviours through component grouping [23]. Although the reduced models in (54) and (55) may still provide useful information for subsystem-oriented analyses, they are not sufficient for system-wide stability assessment in strongly coupled hybrid AC/DC systems. In particular, the interaction mechanisms between AC- and DC-side dynamics—which are fully represented only in the complete AC/DC model—may significantly influence the propagation and amplification of oscillatory modes. Consequently, this case demonstrates that relying exclusively on reduced AC- or DC-side models may lead to incomplete or even incorrect stability predictions, especially when oscillations originate in one subsystem and propagate through AC/DC coupling effects.

7.2. Test #2 – AC harmonic oscillation

This second test case explores the impact of increasing the control delay of MMC₁ from 100 μ s in the reference stable configuration to 330 μ s. The delays are set according to the factor $q_d = T_d \times f_{sw}$, whose commonly accepted range is $q_d \in [0.25, 1]$ [47]. The stable case represents a reduced-delay scenario with $q_d = 0.1$. The unstable case considers $q_d = 0.33$, well within the typical range.

The application of the PMD stability criterion to the complete AC/DC system in Fig. 11(a) identifies a harmonic oscillatory mode at 720 Hz driving system instability, with the PF analysis indicating Bus 1 AC-side as the main contributor. Based on this, the AC-side is identified as the OS

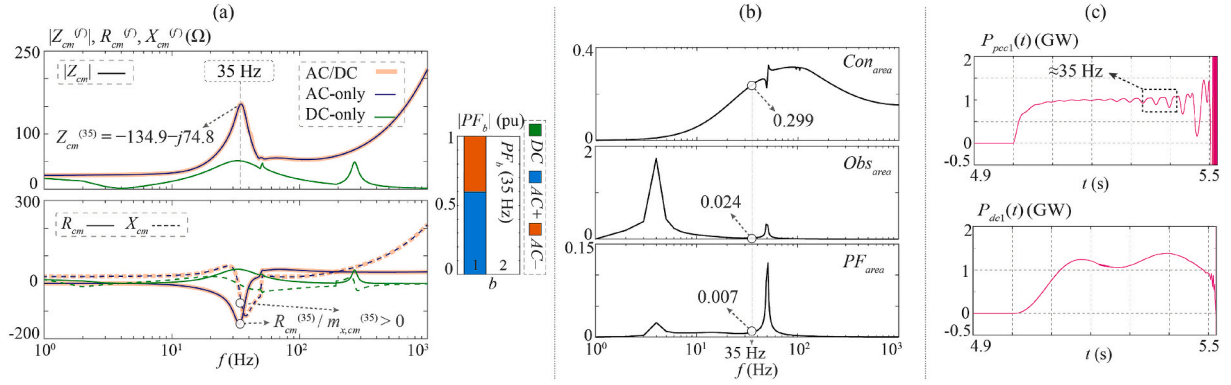


Fig. 10. TEST #1: (a) PMD stability criterion application and PF analysis for the AC/DC (orange lines), AC-only (blue lines), and DC-only (green lines) grid's models. (b) AC/DC propagation indices for MMC₁. (c) MATLAB/Simulink time-domain simulation results of the AC and DC power outputs, P_{pcc1} and P_{dc1} , respectively.

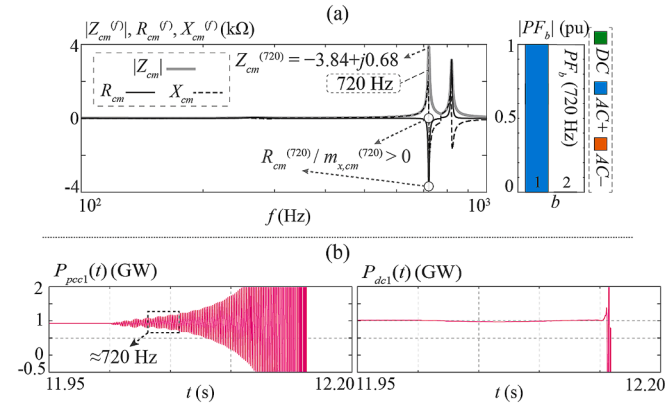


Fig. 11. TEST #2: (a) PMD stability criterion application and PF analysis for the AC/DC grid. (b) MATLAB/Simulink time-domain simulation results of the AC and DC power outputs, P_{pcc1} and P_{dc1} , respectively.

area, the DC-side as the NOS area, and 720 Hz as the s_m . Although the AC/DC propagation study is not shown for brevity, it confirms that the instability remains confined to the AC subsystem, as $Obs_{area} \approx 0$ and $PF_{area} \approx 0$. These findings are validated by time-domain simulations in Fig. 11(b), where harmonic oscillations at ≈ 720 Hz are observed in the AC power P_{pcc1} , while P_{dc1} remains unaffected. It should be noted that the final instability in P_{dc1} are attributed to simulated system collapse rather than the oscillatory mode itself.

Overall, this test confirms that the proposed 3×3 model reliably captures high-frequency AC instabilities induced by control delays. Further studies (not shown for brevity) using the equivalent AC- and DC-side models also highlight—similarly to Test #1—the limitations of DC-side-only monitoring, as such instabilities remain undetected without an integrated AC/DC dynamic model.

7.3. Test #3 – DC near-synchronous oscillation

This third test case investigates instabilities originating in the DC grid. The system is modified by increasing the AC grid resistance ($R_g \times 5$), the DC-link capacitance of MMC₂ ($C_{v2} \times 10$), and the DVC bandwidth of MMC₂ to 150 rad/s.

The application of the PMD stability criterion in Fig. 12(a) identifies an oscillatory mode at 17 Hz driving system instability. The PF results confirm that both DC-side buses 1 and 2 are involved in the instability, with the DC side of Bus 2—associated with MMC₂—being the main contributor. Based on this, the DC-side is identified as the OS area, the AC-side as the NOS area, and 17 Hz as the s_m . The AC/DC interaction indicators reveal $PF_{area} \approx 0$ but $Obs_{area} \neq 0$ for both MMC₁ and MMC₂ at

17 Hz, indicating that the instability may propagate into both AC grids. Time-domain simulations in Fig. 12(b) validate these results, showing clear oscillations ≈ 15 Hz in P_{dc2} , that are also reflected in P_{pcc1} and P_{pcc2} —confirming AC/DC coupling.

Similarly to the previous AC-driven tests, additional studies using the equivalent AC- and DC-side models highlight the limitations of AC-only analysis. These results show that DC-driven instabilities would remain undetected from the AC side without integrated AC/DC modelling—further reinforcing the relevance of the proposed approach.

7.4. Test #4 – DC harmonic oscillation

This fourth test case investigates a high-frequency instability originating on the DC side. The DC resonance frequency is increased by reducing the DC-link resistance ($R_{dc}/10$), inductance ($L_{dc}/2.3$), and MMC₂ capacitance ($C_{v2}/5.5$). Additionally, MMC₂ internal control delay is increased to 500 μ s ($q_d = T_d \times f_{sw} = 0.5$).

The application of the PMD stability criterion in Fig. 13(a) detects a harmonic oscillatory mode at 542 Hz driving system instability. PF analysis clearly indicates the DC side of Bus 2—associated with MMC₂—as the dominant source. Based on this, the DC-side is identified as the OS area, the AC-side as the NOS area, and 542 Hz as the s_m . The AC/DC propagation study results indicate that, despite $PF_{area} \approx 0$, the $Obs_{area} \neq 0$ for both converters at 542 Hz indicates potential propagation

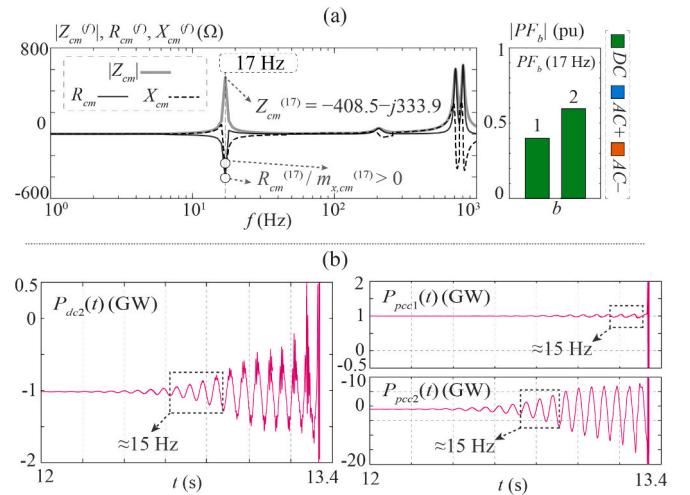


Fig. 12. TEST #3: (a) PMD stability criterion application and PF analysis for the AC/DC grid. (b) MATLAB/Simulink time-domain simulation results of the DC and AC power outputs, P_{dc2} and P_{pcc1} and P_{pcc2} , respectively.

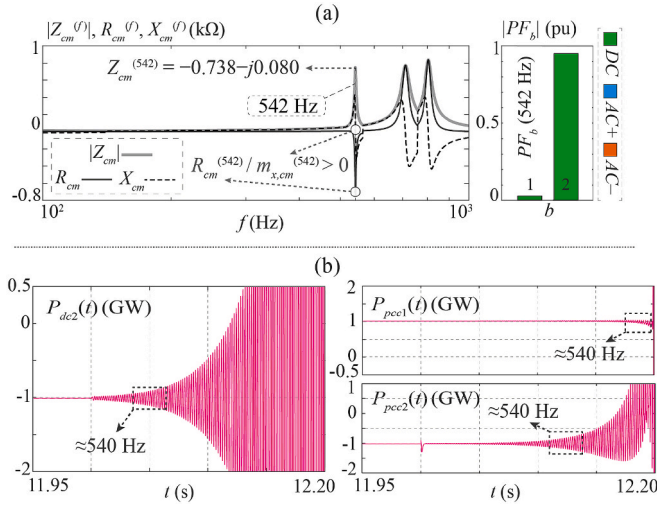


Fig. 13. TEST #4: (a) PMD stability criterion application and PF analysis for the AC/DC grid. (b) MATLAB/Simulink time-domain simulation results of the DC and AC power outputs, P_{dc2} and P_{pcc1} and P_{pcc2} , respectively.

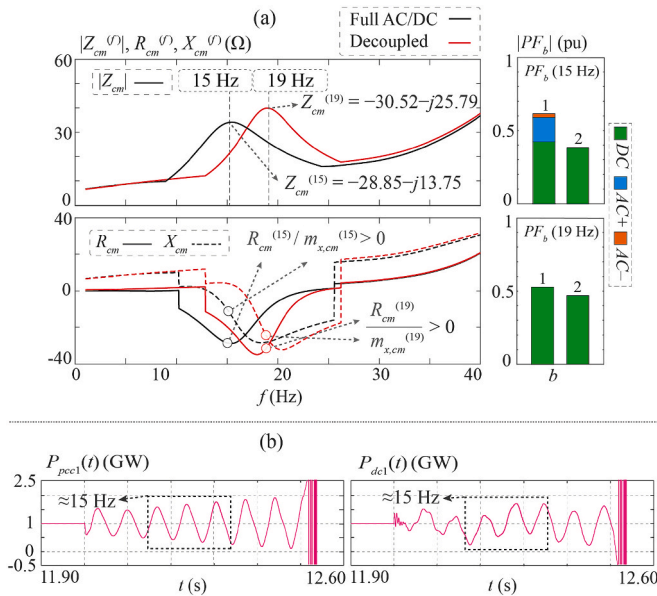


Fig. 14. TEST #5: (a) PMD stability criterion application and PF analysis for the AC/DC grid considering the complete AC/DC model (black curves) and the decoupled AC/DC model (red curves). (b) MATLAB/Simulink time-domain simulation results of the DC and AC power outputs, P_{dc1} and P_{pcc1} , respectively.

toward AC domains. This is further supported by the time-domain simulations in Fig. 13(b), which show oscillations at ≈ 540 Hz in P_{dc2} , along with AC ripples in P_{pcc1} and P_{pcc2} —confirming AC/DC oscillation propagation effects.

As in previous subsections, additional studies using the equivalent AC- and DC-side models show that only the DC-side equivalent correctly predicts the instability. The AC-side model fails to do so—once again exposing the limitations of AC-only assessments in capturing DC-driven phenomena.

7.5. Test #5 – AC/DC subsynchronous oscillation

Unlike the previous four application examples, this additional test has been specifically designed to demonstrate the importance of preserving the complete AC/DC coupling structure of the proposed AC/DC model. Starting from the AC-grid configuration employed in Test #1,

MMC₁ is operated under DVC while MMC₂ operates under APC and QPC. In addition, the DC-side capacitance connected in parallel with MMC₁ is increased to $C_{v1} \times 10$. These modifications intentionally increase the dynamic interaction between the AC and DC subsystems, creating an oscillatory mode simultaneously influenced by both electrical subsystems. To evaluate the influence of the AC/DC coupling terms on the predicted stability, four analytical models are compared: the complete *three-port* AC/DC admittance model, the same *three-port* model after artificially suppressing the coupling terms by imposing $Y_{12} = Y_{21} = 0$, and the AC-side and DC-side equivalent models.

The PMD stability criterion applied to the complete AC/DC model in Fig. 14(a) (black curves) identifies a subsynchronous oscillatory mode at approximately 15 Hz. The PF analysis shows that both the AC and DC terminals of MMC₁ contribute significantly to this mode, confirming that the oscillatory behaviour is jointly influenced by both electrical subsystems. The analytical prediction is validated by the EMT simulations shown in Fig. 14(b), which exhibit oscillations at approximately 15 Hz in both P_{dc1} and P_{pcc1} . The stability study is then repeated after artificially suppressing the AC/DC coupling terms by imposing $Y_{12} = Y_{21} = 0$. Although the decoupled model still predicts an unstable operating condition (red curves in Fig. 14(a)), it no longer reproduces the oscillatory mode correctly. In particular, the predicted oscillation frequency shifts from approximately 15 Hz to 19 Hz, while the predicted damping is also modified, as evidenced by the differences between the black and red curves in Fig. 14(a). Furthermore, the PF analysis indicates that only the DC terminals participate significantly in the oscillatory mode, thereby failing to capture the simultaneous AC/DC interaction observed with the complete formulation. These discrepancies demonstrate that neglecting the AC/DC coupling terms may lead to inaccurate stability predictions whenever strong dynamic interactions exist between the AC and DC subsystems. For completeness, the same analysis is also carried out using the AC-side and DC-side equivalent models. The resulting predictions are summarised in Table 3 (not shown in Fig. 14(a) for brevity). The AC-side equivalent model predicts a stable operating condition and therefore fails to detect the instability, whereas the DC-side equivalent model predicts an oscillatory mode at 19 Hz with characteristics similar to those obtained using the decoupled formulation. These results further demonstrate that the instability is primarily associated with the DC-side dynamics, while its modal characteristics are significantly affected by the interaction with the AC subsystem. Overall, these results (see Fig. 14 and Table 3) show that the instability is primarily driven by the DC-side dynamics, while its modal characteristics are significantly affected by the interaction with the AC subsystem. Consequently, simplified DC-only or decoupled formulations may indicate the existence of instability, but provide inaccurate predictions of its oscillation frequency and damping, whereas AC-only formulations may fail to detect the instability altogether. Only the complete *three-port* AC/DC formulation correctly predicts both the existence of the unstable oscillatory mode associated with the dominant DC-side dynamics and its dynamic characteristics as influenced by the AC/DC interaction.

8. Conclusions

This paper presented a comprehensive contribution to the frequency-domain modelling, validation, and application of an AC/DC small-signal admittance model for MMCs in hybrid transmission systems. A novel AC/DC small-signal admittance-based model was developed, capable of capturing the dynamic interactions between AC and DC sides while incorporating internal dynamics, the circulating current controller, the phase-locked loop, the inner current controller, and the outer control loops. Unlike existing detailed AC/DC MMC models, which are primarily oriented toward analysing the internal converter behaviour and harmonic interactions, the proposed mathematical formulation is specifically designed for small-signal system-level stability assessment of hybrid AC/DC grids. The model was extensively validated through MATLAB/Simulink time-domain simulations, confirming its accuracy in

Table 3

Summary of the stability predictions obtained in Test #5.

Model	Stability prediction	Oscillation frequency	Main modal characteristics	Z_{cm} (Ω)	$ PF_b $ (pu)	
					Bus 1	Bus 2
Complete AC/DC three-port	Unstable	≈ 15 Hz	Significant AC and DC participation	$-28.85 - j13.75$	DC: 0.42 AC+: 0.17 AC-: 0.03	DC: 0.38 AC+: 0 AC-: 0
Decoupled AC/DC ($Y_{12} = Y_{21} = 0$)	Unstable	≈ 19 Hz	DC participation	$-30.52 - j25.79$	DC: 0.53 AC+: 0 AC-: 0	DC: 0.47 AC+: 0 AC-: 0
AC-only equivalent	Stable	-	Instability not detected	-	-	-
DC-only equivalent	Unstable	≈ 19 Hz	DC participation	$-30.52 - j25.79$	DC: 0.53 AC+: 0 AC-: 0	DC: 0.47 AC+: 0 AC-: 0
EMT simulation	Unstable	≈ 15 Hz	Oscillations in both P_{dc1} and P_{pcc1}	-	-	-

reproducing the dynamic behaviour of the MMC under different operating conditions and control configurations. Furthermore, its application to converter-driven stability analysis demonstrated its capability to identify both near-synchronous and harmonic oscillatory modes originating from either the AC or DC side. In each scenario, the proposed model successfully anticipated the onset of instability using the positive-mode-damping stability criterion, identified the dominant buses based on participation factors, and provided further insight into the instability mechanisms through AC/DC interaction indices. The results consistently demonstrated that hybrid unstable oscillations—whether initiated on the AC or DC side—can propagate across subsystems, depending on internal control dynamics and structural interactions within the converter. An additional study further demonstrated that preserving the complete AC/DC coupling structure is essential for accurate stability prediction under strong AC/DC interactions, whereas decoupled and AC- or DC-side formulations may lead to incorrect predictions of the oscillation frequency and damping. Overall, the proposed formulation provides a physically consistent and analytically tractable framework for frequency-domain modelling and small-signal stability assessment of MMC-based HVDC systems. Beyond enabling accurate prediction of converter-driven oscillatory phenomena, it enhances the understanding of AC/DC interaction mechanisms and supports the development of robust stability analysis and controller design methodologies for future multi-terminal power systems. Future work will focus on sensitivity analysis of control parameters, particularly the impact of outer-loop bandwidth and gain tuning on MMC passivity and stability margins, as

well as real-time hardware-in-the-loop validation to further assess the practical performance of the proposed model.

CRedit authorship contribution statement

Oriol Cartiel: Writing – original draft, Validation, Software, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Mehtu Beza:** Writing – original draft, Validation, Resources, Methodology, Investigation, Formal analysis. **Luis Sainz:** Writing – original draft, Supervision, Methodology, Funding acquisition, Conceptualization. **Massimo Bongiorno:** Supervision, Resources, Project administration, Funding acquisition. **Juan José Mesas:** Writing – original draft, Supervision, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Complex transfer functions

Complex transfer functions are used in previous sections to model grid-connected MMC systems with dqz -complex space vectors.

The normalised Ku (dqz -complex) transformation [5,21,29,30] of a three-phase set of voltages and currents $\mathbf{X}^{abc} = [X^a \ X^b \ X^c]^T$, where $X \in \{V, I\}$ (referring to both bold and italic forms of X), is given by $\mathbf{X}^{dqz} = \mathbf{K}(\theta)\mathbf{X}^{abc}$, where

$$\mathbf{K}(\theta) = \frac{1}{\sqrt{3}} \begin{bmatrix} e^{-j\theta} & a \cdot e^{-j\theta} & a^2 \cdot e^{-j\theta} \\ e^{j\theta} & a^2 \cdot e^{j\theta} & a \cdot e^{j\theta} \\ 1 & 1 & 1 \end{bmatrix} \quad (a = e^{j2\pi/3}). \quad (73)$$

Its inverse transformation is $\mathbf{X}^{abc} = \mathbf{K}(\theta)^{-1} \cdot \mathbf{X}^{dqz} = \mathbf{K}(\theta)^H \cdot \mathbf{X}^{dqz}$, where the superscript H denotes the transpose and complex conjugate (Hermitian conjugate). In addition, the set of voltages and currents in dqz -complex is $\mathbf{X}^{dqz} = [\mathbf{X}^{dq} (\mathbf{X}^{dq})^* \ \mathbf{X}^z]^T$, where the dq -complex space vector is defined as $\mathbf{X}^{dq} = \mathbf{X}^{dq}/\sqrt{2} = (\mathbf{X}^d + j \cdot \mathbf{X}^q)/\sqrt{2}$ and $\mathbf{X}^z$ is the zero-sequence term.

According to [29] and assuming a balanced three-phase system (i.e., the zero-sequence terms are neglected), a dq -real domain impedance matrix,

$$\begin{bmatrix} \mathbf{V}^d \\ \mathbf{V}^q \end{bmatrix} = \begin{bmatrix} Z^{dd}(s) & Z^{dq}(s) \\ Z^{qd}(s) & Z^{qq}(s) \end{bmatrix} \begin{bmatrix} \mathbf{I}^d \\ \mathbf{I}^q \end{bmatrix}, \quad (74)$$

can be transformed into the dq -complex domain matrix as [4,5,21,28]

$$\begin{aligned}
\begin{bmatrix} \mathbf{V}^{dq} \\ (\mathbf{V}^{dq})^* \end{bmatrix} &= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ 1 & -j \end{bmatrix} \begin{bmatrix} \mathbf{Z}^{dd}(s) & \mathbf{Z}^{dq}(s) \\ \mathbf{Z}^{qd}(s) & \mathbf{Z}^{qq}(s) \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -j & j \end{bmatrix} \begin{bmatrix} \mathbf{I}^{dq} \\ (\mathbf{I}^{dq})^* \end{bmatrix} \\
\Rightarrow \underbrace{\begin{bmatrix} \mathbf{V}^{dq} \\ (\mathbf{V}^{dq})^* \end{bmatrix}}_{\mathbf{V}^{dq}} &= \underbrace{\begin{bmatrix} \mathbf{Z}^+(s) & \mathbf{Z}^-(s) \\ (\mathbf{Z}^-(s))^* & (\mathbf{Z}^+(s))^* \end{bmatrix}}_{\mathbf{Z}_{\pm}(s)} \underbrace{\begin{bmatrix} \mathbf{I}^{dq} \\ (\mathbf{I}^{dq})^* \end{bmatrix}}_{\mathbf{I}^{dq}}.
\end{aligned} \tag{75}$$

Transformation (75) is also true for any dq -complex domain matrix of dq -real domain space vectors, e.g., admittance matrices. Note that the fully transformed dq -complex matrix is denoted as $\mathbf{Z}_{\pm}(s)$ (i.e., with the subscript “ $\pm$ ”), while the terms within the matrix are denoted by either $\mathbf{Z}^+(s)$ or $\mathbf{Z}^-(s)$ (i.e., with the superscripts “+” or “-”).

The full dqz -complex Ku transformation matrix in (73) is presented because it is necessary for certain steps of the modelling process. However, the final model (53) disregards the zero-sequence component by assuming a balanced three-phase system and, therefore, applies dq -complex expressions.

Appendix B. Coordinate transformation of transfer functions

Some expressions in (26)-(29) and (37)-(40) must be transformed because their terms $\mathbf{K}_{\pm 2}^{dqz}(p)$, $\mathbf{K}_{\pm 11}^{dqz}(p)$, $\mathbf{K}_{\pm v1}^{dqz}(p)$, $\mathbf{K}_{\pm d1}^{dqz}(p)$, $\mathbf{K}_{\pm 12}^{dqz}(p)$, $\mathbf{K}_{\pm v2}^{dqz}(p)$, $\mathbf{K}_{\pm d2}^{dqz}(p)$, $\mathbf{L}_{\pm 11}^{dqz}(p)$, $\mathbf{L}_{\pm v1}^{dqz}(p)$, and $\mathbf{L}_{\pm d1}^{dqz}(p)$ contain products of the time-dependent matrices $\mathbf{V}_{M0,r}^{dqz}$ and $\mathbf{I}_{M0}^{dqz}$ (expressed as $\mathbf{B}_{\pm M}^{dqz}$ in (23)) with dqz -complex diagonal transfer functions $\mathbf{T}_{\pm}^{dqz}(p)$ (i.e., LTP products) as

$$\mathbf{T}_{\pm}^{dqz}(p) = \begin{bmatrix} T(p + j\omega_1) & 0 & 0 \\ 0 & T(p - j\omega_1) & 0 \\ 0 & 0 & T(p) \end{bmatrix}. \tag{76}$$

Three different LTP products involving these dqz -complex time-dependent matrices and dqz -complex diagonal transfer functions are addressed below, with only the final results presented for each case. These results are based on the frequency translation $p \rightarrow p + j\omega_1$, caused when a transfer function is multiplied on both sides by time-dependent components as follows (see the coordinate transformation in [21]),

$$\mathbf{y} = e^{-jm\theta_1} T(p + jk\omega_1) e^{jm\theta_1} \mathbf{u} = T(p + j(k + m)\omega_1) \mathbf{u}, \tag{77}$$

where $\mathbf{y}$ and $\mathbf{u}$ are dq -complex space vectors, $\theta_1 = \omega_1$, $T(p)$ represents a generic transfer function, and $k, m \in \mathbb{Z}$ are coefficients that may multiply the corresponding terms.

Moreover, a balanced three-phase system is assumed for simplicity from this point forward, and the zero-sequence terms in the time-dependent matrices $\mathbf{V}_{M0,r}^{dqz}$ and $\mathbf{I}_{M0}^{dqz}$ in (23) are neglected accordingly,

$$\mathbf{B}_{\pm M}^{dqz} = \frac{1}{\sqrt{3}} \begin{bmatrix} 0 & e^{-j3\theta} (\mathbf{B}^{dq})^* & \mathbf{B}^{dq} \\ e^{j3\theta} \mathbf{B}^{dq} & 0 & (\mathbf{B}^{dq})^* \\ (\mathbf{B}^{dq})^* & \mathbf{B}^{dq} & 0 \end{bmatrix}. \tag{78}$$

The three above-mentioned cases are the following:

- **Case #1** $\rightarrow \mathbf{B}_{\pm M-1}^{dqz} \mathbf{T}_{\pm}^{dqz}(p) \mathbf{B}_{\pm M-2}^{dqz}$

This type of dqz -complex product appears specifically within $\mathbf{K}_{\pm 2}^{dqz}(p)$, $\mathbf{K}_{\pm 11}^{dqz}(p)$, $\mathbf{K}_{\pm v1}^{dqz}(p)$, $\mathbf{K}_{\pm d1}^{dqz}(p)$, $\mathbf{K}_{\pm 12}^{dqz}(p)$, $\mathbf{K}_{\pm v2}^{dqz}(p)$, and $\mathbf{K}_{\pm d2}^{dqz}(p)$ in (24)-(27). After several algebraic manipulations, the transformed equations constitute an LTI system in the dq -complex domain. Consequently, the differential operator $p = d/dt$ can be replaced by the Laplace variable s , yielding the following transfer-function representation:

$$\mathbf{B}_{\pm M-1}^{dqz} \mathbf{T}_{\pm}^{dqz}(p) \mathbf{B}_{\pm M-2}^{dqz} \sim \frac{1}{3} \left(\mathbf{T}_{a\pm}^{dq}(s) \mathbf{B}_1^{dq} (\mathbf{B}_2^{dq})^H + \mathbf{T}_{b\pm}^{dq}(s) \text{diag}\{(\mathbf{B}_1^{dq})^*\} \text{diag}\{\mathbf{B}_2^{dq}\} \right), \tag{79}$$

where

$$\mathbf{B}_i^{dq} = \begin{bmatrix} \mathbf{B}_i^{dq} \\ (\mathbf{B}_i^{dq})^* \end{bmatrix} \quad (i = 1, 2) \quad \mathbf{B}_i^{dq} = \frac{B_i^d + jB_i^q}{\sqrt{2}} \tag{80}$$

$$\mathbf{T}_{a\pm}^{dq}(s) = T(s) \mathbf{I}_2 \quad \mathbf{T}_{b\pm}^{dq}(s) = \text{diag}\left\{ \begin{bmatrix} T(s + j2\omega_1) \\ T(s - j2\omega_1) \end{bmatrix} \right\}.$$

The above transformation results in the transfer functions $\mathbf{K}_{\pm 2}^{dq}(s)$, $\mathbf{K}_{\pm 11}^{dq}(s)$, $\mathbf{K}_{\pm v1}^{dq}(s)$, $\mathbf{K}_{\pm d1}^{dq}(s)$, $\mathbf{K}_{\pm 12}^{dq}(s)$, $\mathbf{K}_{\pm v2}^{dq}(s)$, and $\mathbf{K}_{\pm d2}^{dq}(s)$ in (29).

It should be noted that $\mathbf{B}_{\pm M-i}^{dqz}$ ($i = 1, 2$) can represent either $\mathbf{V}_{M0,r}^{dqz}$ or $\mathbf{I}_{M0}^{dqz}$ (i.e., it is possible that $\mathbf{B}_{\pm M-1}^{dqz} = \mathbf{B}_{\pm M-2}^{dqz}$ or $\mathbf{B}_{\pm M-1}^{dqz} \neq \mathbf{B}_{\pm M-2}^{dqz}$). Thus, $\mathbf{B}_i^{dq}$ represents either $\mathbf{V}_{S0,r}^{dq}$ or $\mathbf{I}_{S0}^{dq}$.

- **Case #2** $\rightarrow \mathbf{B}_{\pm M-i}^{dqz} \mathbf{T}_{\pm}^{dqz}(p) \mathbf{1}^{dqz}$

This type of dqz -complex product with $\mathbf{1}^{dqz} = [0 \ 0 \ \sqrt{3}]^T$ appears specifically within $\mathbf{K}_{\pm d1}^{dqz}(p)$ and $\mathbf{K}_{\pm d2}^{dqz}(p)$ in (24)-(27), and is expressed in the dq -complex domain, with the derivative operator p replaced by the Laplace variable s (as the system becomes LTI), as follows:

$$\mathbf{B}_{\pm M-i}^{dqz} \mathbf{T}_{\pm}^{dqz}(p) \mathbf{1}^{dqz} \sim \mathbf{T}_{a\pm}^{dq}(s) \mathbf{B}_i^{dq}. \tag{81}$$

The above transformation results in the transfer functions $\mathbf{K}_{\pm d1}^{dq}(s)$ and $\mathbf{K}_{\pm d2}^{dq}(s)$ in (29).

• **Case #3** $\rightarrow (\mathbf{1}^{dqz})^T \mathbf{T}_{\pm}^{dqz}(p) \mathbf{B}_{\pm M}^{dqz}$

This type of dqz -complex product appears specifically within $\mathbf{L}_{\pm 11}^{dqz}(p)$, $\mathbf{L}_{\pm v1}^{dqz}(p)$, and $\mathbf{L}_{\pm d1}^{dqz}(p)$ in (38), and is expressed in the dq -complex domain, with the derivative operator p replaced by the Laplace variable s (as the system becomes LTI), as follows:

$$(\mathbf{1}^{dqz})^T \mathbf{T}_{\pm}^{dqz}(p) \mathbf{B}_{\pm M}^{dqz} \sim (\mathbf{B}_1^{dq})^H \mathbf{T}_{\pm}^{dq}(s). \quad (82)$$

The above transformation results in the transfer functions $\mathbf{L}_{\pm 11}^{dq}(s)$, $\mathbf{L}_{\pm v1}^{dq}(s)$, and $\mathbf{L}_{\pm d1}^{dq}(s)$ in (40).

Appendix C. Expressions derived from (29) and (40)

This subsection presents the expressions derived from (29) and (40). These terms provide all the necessary expressions to complete the novel MMC AC/DC small-signal admittance-based model. The expressions are obtained from (27) and (38) by directly applying the coordinate-transformation methodology developed in Appendix B, followed by straightforward symbolic substitutions and algebraic manipulations. Since these intermediate manipulations do not introduce additional modelling concepts, they are omitted for conciseness.

Considering that $\mathbf{I}_2$ is the 2x2 identity matrix, $\mathbf{V}_{s0,r}^{dq} = [\mathbf{V}_{s0,r}^{dq} (\mathbf{V}_{s0,r}^{dq})^*]^T$ and $\mathbf{I}_{s0}^{dq} = [\mathbf{I}_{s0}^{dq} (\mathbf{I}_{s0}^{dq})^*]^T$, the terms $\mathbf{Z}_{\pm}^{dq}(s)$, $\mathbf{C}_{\pm}^{dq}(s)$, $\mathbf{K}_{\pm 2}^{dq}(s)$, $\mathbf{K}_{\pm 11}^{dq}(s)$, $\mathbf{K}_{\pm 12}^{dq}(s)$, $\mathbf{K}_{\pm v2}^{dq}(s)$, and $\mathbf{K}_{\pm d2}^{dq}(s)$ from (29), as well as $\mathbf{L}_{\pm 11}^{dq}(s)$, $\mathbf{L}_{\pm v1}^{dq}(s)$, and $\mathbf{L}_{\pm d1}^{dq}(s)$ from (40), are defined as follows:

$$\begin{aligned} \mathbf{Z}_{\pm}^{dq}(s) &= \text{diag} \left\{ \begin{bmatrix} Z_t(s + j\omega_1) \\ Z_t(s - j\omega_1) \end{bmatrix} \right\} \quad \mathbf{C}_{\pm}^{dq}(s) = \frac{C}{N} \text{diag} \left\{ \begin{bmatrix} s + j\omega_1 \\ s - j\omega_1 \end{bmatrix} \right\} \\ \mathbf{K}_{\pm 2}^{dq}(s) &= \frac{1}{2V_{dc0}^2} (2\mathbf{W}_{\pm A}^{dq}(s) - \mathbf{W}_{\pm B}^{dq}(s) - 2\mathbf{W}_{\pm C}^{dq}(s) + \mathbf{W}_{\pm D}^{dq}(s)) \\ \mathbf{K}_{\pm i1}^{dq}(s) &= \frac{1}{2}\mathbf{I}_2 - \frac{2}{4V_{dc0}^2}\mathbf{W}_{\pm E}^{dq}(s) + \frac{1}{4V_{dc0}^2}\mathbf{W}_{\pm F}^{dq}(s) \\ \mathbf{K}_{\pm i2}^{dq}(s) &= -\frac{1}{2V_{dc0}^2}\mathbf{W}_{\pm G}^{dq}(s) + \frac{1}{V_{dc0}^2}\mathbf{W}_{\pm H}^{dq}(s)\mathbf{H}_{\pm 3}^{dq}(s)\mathbf{K}_{\pm i1}^{dq}(s) \\ \mathbf{K}_{\pm v1}^{dq}(s) &= -\frac{2I_{c0}}{V_{dc0}}\mathbf{I}_2 - \frac{2}{4V_{dc0}^2}\mathbf{W}_{\pm I}^{dq}(s) + \frac{1}{4V_{dc0}^2}\mathbf{W}_{\pm J}^{dq}(s) \\ \mathbf{K}_{\pm v2}^{dq}(s) &= -\frac{1}{2V_{dc0}^2}\mathbf{W}_{\pm K}^{dq}(s) + \frac{1}{V_{dc0}^2}\mathbf{W}_{\pm H}^{dq}(s)\mathbf{H}_{\pm 3}^{dq}(s)\mathbf{K}_{\pm v1}^{dq}(s) \\ \mathbf{K}_{\pm d1}^{dq}(s) &= \frac{1}{2V_{dc0}^3}\mathbf{W}_{\pm E}^{dq}(s)\mathbf{I}_{s0}^{dq} - \frac{1}{4V_{dc0}^3}\mathbf{W}_{\pm F}^{dq}(s)\mathbf{I}_{s0}^{dq} + \frac{2I_{c0}}{V_{dc0}^2}\mathbf{V}_{s0,r}^{dq} - \frac{2}{2V_{dc0}}\mathbf{H}_{\pm 1}^{dq}(s)\mathbf{V}_{s0,r}^{dq} + \frac{1}{2V_{dc0}}\mathbf{F}_{\pm cc}^{dq}(s)\mathbf{H}_{\pm 1}^{dq}(s)\mathbf{I}_{s0}^{dq} + \\ &\quad + \frac{2}{2V_{dc0}}\mathbf{H}_{\pm 1}^{dq}(s)\mathbf{H}_{\pm 2}^{dq}(s)\mathbf{K}_{\pm 1}^{dq}(s)\mathbf{V}_{s0,r}^{dq} - \frac{1}{2V_{dc0}}\mathbf{F}_{\pm cc}^{dq}(s)\mathbf{H}_{\pm 1}^{dq}(s)\mathbf{H}_{\pm 2}^{dq}(s)\mathbf{K}_{\pm 1}^{dq}(s)\mathbf{I}_{s0}^{dq} \\ \mathbf{K}_{\pm d2}^{dq}(s) &= \frac{1}{V_{dc0}^2}\mathbf{W}_{\pm H}^{dq}(s)\mathbf{H}_{\pm 3}^{dq}(s)\mathbf{K}_{\pm d1}^{dq}(s) + \frac{1}{2V_{dc0}^3}\mathbf{W}_{\pm G}^{dq}(s)\mathbf{I}_{s0}^{dq} + \frac{1}{V_{dc0}}\mathbf{H}_{\pm 2}^{dq}(s)\mathbf{K}_{\pm 1}^{dq}(s)\mathbf{V}_{s0,r}^{dq} \\ \mathbf{L}_{\pm i1}^{dq}(s) &= -\frac{1}{V_{dc0}}(\mathbf{V}_{s0,r}^{dq})^H \mathbf{H}_{\pm 2}^{dq}(s) + \frac{2}{V_{dc0}}(\mathbf{V}_{s0,r}^{dq})^H \mathbf{H}_{\pm 2}^{dq}(s)\mathbf{K}_{\pm 1}^{dq}(s)\mathbf{H}_{\pm 3}^{dq}(s)\mathbf{K}_{\pm i1}^{dq}(s) \\ \mathbf{L}_{\pm v1}^{dq}(s) &= -\frac{1}{V_{dc0}}(\mathbf{I}_{s0}^{dq})^H \mathbf{H}_{\pm 2}^{dq}(s) + \frac{2}{V_{dc0}}(\mathbf{V}_{s0,r}^{dq})^H \mathbf{H}_{\pm 2}^{dq}(s)\mathbf{K}_{\pm 1}^{dq}(s)\mathbf{H}_{\pm 3}^{dq}(s)\mathbf{K}_{\pm v1}^{dq}(s) \\ \mathbf{L}_{\pm d1}^{dq}(s) &= \frac{2}{V_{dc0}}(\mathbf{V}_{s0,r}^{dq})^H \mathbf{H}_{\pm 2}^{dq}(s)\mathbf{K}_{\pm 1}^{dq}(s)\mathbf{H}_{\pm 3}^{dq}(s)\mathbf{K}_{\pm d1}^{dq}(s) + \frac{1}{V_{dc0}^2}(\mathbf{V}_{s0,r}^{dq})^H \mathbf{H}_{\pm 2}^{dq}(s)\mathbf{I}_{s0}^{dq} + 6H_2(s)K_1(s), \end{aligned} \quad (83)$$

where $\mathbf{W}_{\pm i}^{dq}$ terms ($i = A$ to K) are derived using (79) as:

$$\begin{aligned}
\mathbf{W}_{\pm A}^{\text{dq}}(s) &= \frac{1}{3} \left(\mathbf{H}_{a\pm 1}^{\text{dq}}(s) \mathbf{V}_{s0,r}^{\text{dq}} (\mathbf{V}_{s0,r}^{\text{dq}})^{\text{H}} + \mathbf{H}_{b\pm 1}^{\text{dq}}(s) \text{diag} \left\{ (\mathbf{V}_{s0,r}^{\text{dq}})^* \right\} \text{diag} \left\{ \mathbf{V}_{s0,r}^{\text{dq}} \right\} \right) \\
\mathbf{W}_{\pm B}^{\text{dq}}(s) &= \frac{1}{3} \left(\mathbf{F}_{a\pm cc}^{\text{dq}}(s) \mathbf{H}_{a\pm 1}^{\text{dq}}(s) \mathbf{I}_{s0}^{\text{dq}} (\mathbf{V}_{s0,r}^{\text{dq}})^{\text{H}} + \mathbf{F}_{b\pm cc}^{\text{dq}}(s) \mathbf{H}_{b\pm 1}^{\text{dq}}(s) \text{diag} \left\{ (\mathbf{I}_{s0}^{\text{dq}})^* \right\} \text{diag} \left\{ \mathbf{V}_{s0,r}^{\text{dq}} \right\} \right) \\
\mathbf{W}_{\pm C}^{\text{dq}}(s) &= \frac{1}{3} \left(\mathbf{H}_{a\pm 1}^{\text{dq}}(s) \mathbf{H}_{a\pm 2}^{\text{dq}}(s) \mathbf{K}_{a\pm 1}^{\text{dq}}(s) \mathbf{V}_{s0,r}^{\text{dq}} (\mathbf{V}_{s0,r}^{\text{dq}})^{\text{H}} + \right. \\
&\quad \left. + \mathbf{H}_{b\pm 1}^{\text{dq}}(s) \mathbf{H}_{b\pm 2}^{\text{dq}}(s) \mathbf{K}_{b\pm 1}^{\text{dq}}(s) \text{diag} \left\{ (\mathbf{V}_{s0,r}^{\text{dq}})^* \right\} \text{diag} \left\{ \mathbf{V}_{s0,r}^{\text{dq}} \right\} \right) \\
\mathbf{W}_{\pm D}^{\text{dq}}(s) &= \frac{1}{3} \left(\mathbf{F}_{a\pm cc}^{\text{dq}}(s) \mathbf{H}_{a\pm 1}^{\text{dq}}(s) \mathbf{H}_{a\pm 2}^{\text{dq}}(s) \mathbf{K}_{a\pm 1}^{\text{dq}}(s) \mathbf{I}_{s0}^{\text{dq}} (\mathbf{V}_{s0,r}^{\text{dq}})^{\text{H}} + \right. \\
&\quad \left. + \mathbf{F}_{b\pm cc}^{\text{dq}}(s) \mathbf{H}_{b\pm 1}^{\text{dq}}(s) \mathbf{H}_{b\pm 2}^{\text{dq}}(s) \mathbf{K}_{b\pm 1}^{\text{dq}}(s) \text{diag} \left\{ (\mathbf{I}_{s0}^{\text{dq}})^* \right\} \text{diag} \left\{ \mathbf{V}_{s0,r}^{\text{dq}} \right\} \right) \\
\mathbf{W}_{\pm E}^{\text{dq}}(s) &= \frac{1}{3} \left(\mathbf{H}_{a\pm 1}^{\text{dq}}(s) \mathbf{H}_{a\pm 2}^{\text{dq}}(s) \mathbf{V}_{s0,r}^{\text{dq}} (\mathbf{V}_{s0,r}^{\text{dq}})^{\text{H}} + \mathbf{H}_{b\pm 1}^{\text{dq}}(s) \mathbf{H}_{b\pm 2}^{\text{dq}}(s) \text{diag} \left\{ (\mathbf{V}_{s0,r}^{\text{dq}})^* \right\} \text{diag} \left\{ \mathbf{V}_{s0,r}^{\text{dq}} \right\} \right) \\
\mathbf{W}_{\pm F}^{\text{dq}}(s) &= \frac{1}{3} \left(\mathbf{F}_{a\pm cc}^{\text{dq}}(s) \mathbf{H}_{a\pm 1}^{\text{dq}}(s) \mathbf{H}_{a\pm 2}^{\text{dq}}(s) \mathbf{I}_{s0}^{\text{dq}} (\mathbf{V}_{s0,r}^{\text{dq}})^{\text{H}} + \right. \\
&\quad \left. + \mathbf{F}_{b\pm cc}^{\text{dq}}(s) \mathbf{H}_{b\pm 1}^{\text{dq}}(s) \mathbf{H}_{b\pm 2}^{\text{dq}}(s) \text{diag} \left\{ (\mathbf{I}_{s0}^{\text{dq}})^* \right\} \text{diag} \left\{ \mathbf{V}_{s0,r}^{\text{dq}} \right\} \right) \tag{84} \\
\mathbf{W}_{\pm G}^{\text{dq}}(s) &= \frac{1}{3} \left(\mathbf{H}_{a\pm 2}^{\text{dq}}(s) \mathbf{V}_{s0,r}^{\text{dq}} (\mathbf{V}_{s0,r}^{\text{dq}})^{\text{H}} + \mathbf{H}_{b\pm 2}^{\text{dq}}(s) \text{diag} \left\{ (\mathbf{V}_{s0,r}^{\text{dq}})^* \right\} \text{diag} \left\{ \mathbf{V}_{s0,r}^{\text{dq}} \right\} \right) \\
\mathbf{W}_{\pm H}^{\text{dq}}(s) &= \frac{1}{3} \left(\mathbf{H}_{a\pm 2}^{\text{dq}}(s) \mathbf{K}_{a\pm 1}^{\text{dq}}(s) \mathbf{V}_{s0,r}^{\text{dq}} (\mathbf{V}_{s0,r}^{\text{dq}})^{\text{H}} + \mathbf{H}_{b\pm 2}^{\text{dq}}(s) \mathbf{K}_{b\pm 1}^{\text{dq}}(s) \text{diag} \left\{ (\mathbf{V}_{s0,r}^{\text{dq}})^* \right\} \text{diag} \left\{ \mathbf{V}_{s0,r}^{\text{dq}} \right\} \right) \\
\mathbf{W}_{\pm I}^{\text{dq}}(s) &= \frac{1}{3} \left(\mathbf{H}_{a\pm 1}^{\text{dq}}(s) \mathbf{H}_{a\pm 2}^{\text{dq}}(s) \mathbf{V}_{s0,r}^{\text{dq}} (\mathbf{I}_{s0}^{\text{dq}})^{\text{H}} + \mathbf{H}_{b\pm 1}^{\text{dq}}(s) \mathbf{H}_{b\pm 2}^{\text{dq}}(s) \text{diag} \left\{ (\mathbf{V}_{s0,r}^{\text{dq}})^* \right\} \text{diag} \left\{ \mathbf{I}_{s0}^{\text{dq}} \right\} \right) \\
\mathbf{W}_{\pm J}^{\text{dq}}(s) &= \frac{1}{3} \left(\mathbf{F}_{a\pm cc}^{\text{dq}}(s) \mathbf{H}_{a\pm 1}^{\text{dq}}(s) \mathbf{H}_{a\pm 2}^{\text{dq}}(s) \mathbf{I}_{s0}^{\text{dq}} (\mathbf{I}_{s0}^{\text{dq}})^{\text{H}} + \right. \\
&\quad \left. + \mathbf{F}_{b\pm cc}^{\text{dq}}(s) \mathbf{H}_{b\pm 1}^{\text{dq}}(s) \mathbf{H}_{b\pm 2}^{\text{dq}}(s) \text{diag} \left\{ (\mathbf{I}_{s0}^{\text{dq}})^* \right\} \text{diag} \left\{ \mathbf{I}_{s0}^{\text{dq}} \right\} \right) \\
\mathbf{W}_{\pm K}^{\text{dq}}(s) &= \frac{1}{3} \left(\mathbf{H}_{a\pm 2}^{\text{dq}}(s) \mathbf{V}_{s0,r}^{\text{dq}} (\mathbf{I}_{s0}^{\text{dq}})^{\text{H}} + \mathbf{H}_{b\pm 2}^{\text{dq}}(s) \text{diag} \left\{ (\mathbf{V}_{s0,r}^{\text{dq}})^* \right\} \text{diag} \left\{ \mathbf{I}_{s0}^{\text{dq}} \right\} \right),
\end{aligned}$$

being

$$\begin{aligned}
Z_t(s) &= R_t + L_t s, \quad Z_c(s) = R_c + F_{cc}(s) + L_c s \\
\mathbf{C}_{a\pm}^{\text{dq}}(s) &= \frac{C}{N} s \mathbf{I}_2, \quad \mathbf{C}_{b\pm}^{\text{dq}}(s) = \frac{C}{N} \text{diag} \left\{ \begin{bmatrix} s + j2\omega_1 \\ s - j2\omega_1 \end{bmatrix} \right\} \\
\mathbf{H}_{a\pm 1}^{\text{dq}}(s) &= \frac{1}{Z_c(s)} \mathbf{I}_2, \quad \mathbf{H}_{b\pm 1}^{\text{dq}}(s) = \left(\text{diag} \left\{ \begin{bmatrix} Z_c(s + j2\omega_1) \\ Z_c(s - j2\omega_1) \end{bmatrix} \right\} \right)^{-1} \\
\mathbf{F}_{a\pm cc}^{\text{dq}}(s) &= F_{cc}(s) \mathbf{I}_2, \quad \mathbf{F}_{b\pm cc}^{\text{dq}}(s) = \text{diag} \left\{ \begin{bmatrix} F_{cc}(s + j2\omega_1) \\ F_{cc}(s - j2\omega_1) \end{bmatrix} \right\} \tag{85} \\
\mathbf{K}_{a\pm 1}^{\text{dq}}(s) &= \left(\frac{1}{4} \mathbf{I}_2 + \frac{I_{c0}}{2V_{dc0}} \mathbf{F}_{a\pm cc}^{\text{dq}}(s) \right) \mathbf{H}_{a\pm 1}^{\text{dq}}(s), \quad \mathbf{K}_{b\pm 1}^{\text{dq}}(s) = \left(\frac{1}{4} \mathbf{I}_2 + \frac{I_{c0}}{2V_{dc0}} \mathbf{F}_{b\pm cc}^{\text{dq}}(s) \right) \mathbf{H}_{b\pm 1}^{\text{dq}}(s) \\
\mathbf{H}_{a\pm 2}^{\text{dq}}(s) &= \left(\mathbf{C}_{a\pm}^{\text{dq}}(s) + \mathbf{K}_{a\pm 1}^{\text{dq}}(s) \right)^{-1}, \quad \mathbf{H}_{b\pm 2}^{\text{dq}}(s) = \left(\mathbf{C}_{b\pm}^{\text{dq}}(s) + \mathbf{K}_{b\pm 1}^{\text{dq}}(s) \right)^{-1}.
\end{aligned}$$

Appendix D. Operating points and control configurations for analytical model validation

This appendix presents a comprehensive summary of the steady-state operating points and associated control structures employed for validating the proposed MMC analytical model. The three validation test scenarios correspond to those presented in [Section 5](#): (i) the internal dynamics and CCC, (ii) the inner CC, and (iii) full control including the PLL and outer control loops. All steady-state values are expressed in the synchronous dq -frame, and the relevant control design parameters are listed in [Table 4](#).

1) AC- and DC-side Internal Dynamics

In this test, the MMC operates in open-loop mode while transferring 15 kW of active power and 15 kVar of reactive power to the AC grid. The steady-state dq -frame operating point from MATLAB/Simulink is:

- o $E_0^{dq} = 400 + j0$ V,
- o $V_{s0}^{dq} = 420.5 + j16.72$ V,
- o $I_{s0}^{dq} = 37.5 - j37.5$ A,
- o $V_{dc0} = 1000$ V,
- o $I_{dc0} = 15$ A (with $I_{c0}^k = I_{dc0}/3$).

To evaluate the impact of internal dynamics and the CCC, two operating conditions are considered:

1. **Without the CCC:** All CCC terms are disabled (i.e., $F_{cc}^k = F_{cc,r}^k = 0$).
2. **With the CCC:** The CCC is enabled (see Table 4 for parameters).

2) Inner CC loop

Maintaining the same power transfer, this test evaluates inner CC dynamics alongside CCC effects. The operating point remains unchanged. The control transfer functions are (see Table 4 for parameters):

- o $H_{ff}^+(s) = \alpha_f / (\alpha_f + s)$,
- o $G_{cc}(s) = k_{p,c} + k_{i,c} / s$, where $k_{p,c} = \alpha_{cc} L_t$ and $k_{i,c} = \alpha_{cc} R_t$, being $L_t = L_f + L_c/2$ and $R_t = R_f + R_c/2$.

3) PLL and Outer Control loops

This scenario considers the complete control structure, including the PLL, DVC, APC, AVC, and QPC, under different operating modes as detailed in Section 5. These are:

1. **DVC–QPC or DVC mode:** The controller objectives are to maintain DC voltage regulation and supply 15 kVar to the grid. The operating point is:
 - o $E_0^{dq} = 400 + j0$ V,
 - o $V_{s0}^{dq} = 418.6 - j1.90$ V,
 - o $I_{s0}^{dq} = 0 - j37.5$ A,
 - o $V_{dc0} = 1000$ V,
 - o $I_{dc0} = -0.08$ A (with $I_{c0}^k = I_{dc0}/3$).
2. **APC or QPC mode:** The controller objectives are to supply 15 kW and 15 kVar to the grid. The steady-state operating point remains identical to that used in the internal dynamics test.
3. **AVC mode:** The controller objective is to regulate the AC voltage while supplying 15 kW to the grid. The operating point is:
 - o $E_0^{dq} = 400 + j0$ V,
 - o $V_{s0}^{dq} = 401.9 + j18.5$ V,
 - o $I_{s0}^{dq} = 37.5 + j0$ A,
 - o $V_{dc0} = 1000$ V,
 - o $I_{dc0} = 15$ A (with $I_{c0}^k = I_{dc0}/3$).

The transfer functions for the PLL and outer controllers are (see Table 4 for parameters):

- o $F_{pll}(s) = k_{p,pll} + k_{i,pll} / s$, where $k_{p,pll} = 2\xi_{pll}\alpha_{pll}/E_0^d$ and $k_{i,pll} = \alpha_{pll}^2/E_0^d$;
- o $F_{dc}(s) = k_{p,dc} + k_{i,dc} / s$, where $k_{p,dc} = \alpha_{dvc} C_{MMC}$ and $k_{i,dc} = \alpha_{dvc}^2 C_{MMC}$ (with $C_{MMC} = 6C/N$);
- o $F_p(s) = k_{p,p} + k_{i,p} / s$, where $k_{p,p} = \alpha_{pc}/(\alpha_{cc} E_0^d)$ and $k_{i,p} = \alpha_{pc}/E_0^d$;
- o $F_v(s) = k_{p,v} + k_{i,v} / s$, where $k_{p,v} = \alpha_{avc}/\alpha_{cc}$ and $k_{i,v} = \alpha_{avc}$;
- o $F_q(s) = k_{p,q} + k_{i,q} / s$, where $k_{p,q} = \alpha_{qc}/(\alpha_{cc} E_0^d)$ and $k_{i,q} = \alpha_{qc}/E_0^d$.

Table 4
MMC model parameter values for validation.

MMC Parameter		Units	Value
S_{MMC}	Power rating	kVA	75
$E_{N,ac}$	Rated AC voltage	kV	0.4
$V_{N,dc}$	Rated DC voltage	kV	1
f_1	Fundamental angular frequency	Hz	50
f_{sw}	Submodule switching frequency	kHz	5
k	Number of converter phase legs a, b, c	-	3
N	Number of submodules per arm	-	4
C	Submodule capacitance	mF	10
L_c	Arm inductance	mH	2
R_c	Parasitic arm resistance	mΩ	50
L_f	Inductance of phase reactor	mH	1
R_f	Parasitic resistance of phase reactor	mΩ	25
α_{ccc}	Circulating current controller bandwidth	rad/s	1000π
α_{cc}	Closed-loop current controller bandwidth	rad/s	1000π
α_f	Grid voltage feedforward low-pass filter bandwidth	rad/s	100π
α_{pll}	PLL bandwidth	rad/s	100π
ξ_{pll}	PLL damping factor	-	$\sqrt{2}/2$

(continued on next page)

Table 4 (continued)

MMC Parameter		Units	Value
α_{dvc}	DVC bandwidth	rad/s	100 π
α_{pc}	APC bandwidth	rad/s	100 π
α_{avc}	AVC bandwidth	rad/s	100 π
α_{qc}	QPC bandwidth	rad/s	100 π
T_d	Converter time delay	μ s	20
P_o	Active power supply	kW	15
Q_o	Reactive power supply	kVAr	15

Note: The switching frequency f_{sw} listed in Table 4 refers to the switching frequency of each MMC submodule. As each converter arm contains N submodules operating with phase-shift carrier PWM [15], the equivalent arm switching frequency is $N \cdot f_{sw}$.

Data availability

Data will be made available on request.

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