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Lattice-Based Dynamic Programming Trajectory Planning in an Intersection Crossing Scenario with a Connected Autonomous Electric Bus and Multiple Human-Driven Vehicles

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Abstract: Building upon authors' two recent papers published in 2025 and 2026 to reduce energy consumption, improve passenger comfort, and ensure trajectory tracking in an intersection crossing scenario involving a Connected Autonomous Electric Bus (CAEB) and a set of Human-Driven Vehicles (HDVs) resulting in problem setting and dynamic models of the CAEB and HDVs, this paper proposes an optimization approach for solving the optimization problem specific to Trajectory Planning (TP) in this scenario. The discrete-time TP for the CAEB is carried out on a spatial lattice obtained by segmenting the trip into road segments and discretizing the lanes and speed levels. The optimization problem is formulated as a lattice-based dynamic programming (DP) problem in the spatial domain, where each node represents a discrete vehicle configuration in relation to a certain road segment, lane index, and speed level, and a Bellman recursion is employed in order to determine the optimal sequence of lane changes and speed decisions that can minimize a segment-wise objective function involving the CAEB energy use, travel time, and penalizing of frequent lane change increments, under traffic lights and safety constraints. The resulting optimal discrete trajectory provides three reference profiles for the CAEB, namely the longitudinal position, the lateral position and the speed, defined at the level of road segments. The open-loop simulation results for six scenarios are reported in order to illustrate the behavior of the CAEB and HDVs, the impact of the traffic light modeling, and the effect of the lattice-based DP TP on the lane change behavior of the CAEB and the evolution of its speed.

Keywords: Connected autonomous electric buses, Energy consumption optimization problems, Human-driven vehicles, Lattice based dynamic programming, Trajectory planning.

1. Introduction

As shown in (e-MATS, 2023), multimodal transport with connected electric transit and shared micro-mobility is vital for sustainable and efficient urban access. A Romania-Sweden-China consortium of research, authorities, and industry advances such systems through electrification, connectivity, and sharing; the authors of this paper represent two partners. In this context, electric public vehicles reduce energy use, ensure comfort, and maintain speed, supported by strategic infrastructure planning, tactical system optimization, and operational control with battery management, while accounting for user needs and behavior.

Current approaches to economical travel at signalized intersections for Connected Autonomous Vehicles (CAVs) emphasize modeling, optimization, and control. Predictive approaches (Dong et al., 2024a) use Bayesian networks and Gaussian processes to improve eco-driving, while dynamic lane-changing with trajectory planning enhances urban mobility (Dong et al., 2024b). Speed optimization via dynamic programming (DP) and Pontryagin's principle (Dong et al., 2021), overtaking strategies (Liu et al., 2022), and combined lateral–longitudinal control (Han et al., 2018) further increase efficiency. The eco-approach and departure techniques reduce stop-and-go driving, including multi-intersection optimization with horizon control and green windows (Hu et al., 2021), and flexible eco-cruising considering lane and speed (Yang et al., 2023a). Platoon-based strategies (Cui et al., 2022) cut fuel use and drag, while dynamic/kinematic models (Liu et al., 2021), (Kong et al., 2015) improve trajectory planning with a smoother braking and acceleration.

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Beyond the eco-approach and eco-cruising strategies at signalized intersections, several works have further explored eco-driving optimal control and DP-based implementations. Methodological aspects of solving eco-driving optimal control problems for conventional vehicles and comparing different optimization paradigms are discussed in (Maamria et al., 2016). Sun et al. (2018) address robust eco-driving control under uncertain traffic signal timing, emphasizing the need to explicitly account for signal variability. A GPU-accelerated implementation of a look-ahead optimal controller for eco-driving based on DP, highlighting the computational challenges of DP in practical applications is proposed in (Zhu et al., 2021). At the same time, recent studies on eco-driving of mixed platoons with CAVs and Human-Driven Vehicles (HDVs) underline the importance of coordination between automated vehicles and HDVs (Yang et al., 2023b), while traffic-based reference SoC planning for plug-in hybrid vehicles further illustrates how energy-related objectives can be embedded in predictive control frameworks (Li et al., 2021). Recent work has specifically addressed eco-driving strategies for connected electric buses at signalized intersections with stations, showing significant potential for reducing energy consumption and travel time by coordinated speed planning (Zhang et al., 2024).

Building upon the authors' recent papers (Precup et al., 2025 and Precup et al., 2026), the contribution of this paper is twofold: (i) a lattice-based DP approach to Stage I trajectory planning problem, i.e. a road segment level trajectory planning approach, framed as a discrete-time optimization problem with traffic constraints, and (ii) its solving using DP in terms of an appropriate optimization algorithm. The goal is to minimize the Connected Autonomous Electric Bus (CAEB) energy use and improve comfort by reducing CAEB's lane changing frequency. The trajectory is a reference input for the CAEB position controller, though solving the optimization problem is beyond this paper.

This contribution, which is based on segment-wise objective functions that combine energy-related terms, travel time, and comfort or penalties for car maneuvers on the road is common in eco-driving formulations for connected vehicles and trains, where DP or other optimal control techniques are used to compute fuel- or energy-efficient speed profiles (Ebrahimi et al., 2024; Dong et al., 2021; Lim et al., 2017; Chen et al., 2024). The structure of the discrete-segment objective that will be presented within the paper, which accounts for CAEB energy consumption, segment travel time, and a penalty on frequent lane changes, follows this line of work but is tailored to the intersection scenario with a connected autonomous electric bus and surrounding HDVs. Although metaheuristic algorithms are successful in solving complicated optimization problems with formulations such as molecular fuzzy genetic algorithms for circular-oriented project investments (Zavadskas et al., 2025) and alternative optimization for maximizing the downlink sum-rate (Meng et al., 2025), this paper focuses on a classical optimization algorithm to establish a solid mathematical foundation.

Methodologically, the road is discretized into longitudinal segments, and the CAEB state at each segment is described by a triplet composed of the segment index, the lane index, and a discretized speed level. The resulting lattice defines a finite set of discrete vehicle configurations, over which a Bellman recursion is applied to determine the optimal sequence of lane change and speed decisions that minimizes a segment-wise objective function reflecting energy use, travel time, and passenger comfort.

The lattice-based DP approach to trajectory planning proposed in this paper provides an eco-driving oriented trajectory planner in the spatial domain, where progression is in relation to distance rather than time and the optimization balances energy efficiency and comfort under traffic light and safety constraints.

Recent eco-driving strategies for connected and automated vehicles at signalized intersections also adopt distance-based formulations, in which the optimization is carried out in relation to the route rather than in absolute time (Ebrahimi et al., 2024; Dong et al., 2022; Lim et al., 2017). These works show that spatial parameterizations are well suited to handle signal locations, speed limits, and road geometry, which motivates the segment-based discretization adopted here.

Similar lattice- or grid-based motion planning structures are widely used in robotics and automated driving, where each node represents a discrete configuration and DP or graph search is applied over the resulting network (Tiger et al., 2021; Wen et al., 2025). In the approach proposed in this paper, the lattice is defined with regard to longitudinal segments, lane indices, and speed levels, which are tailored to eco-driving in an intersection scenario.

The remainder of this paper is organized as follows. Section 2 describes the optimization problem, and Section 3 presents the simulation scenarios. Finally, Section 4 includes the conclusion of this paper and outlines possible future research directions. With regard to the supplementary material related to this paper, the intersection crossing scenario and the vehicle models are described in detail in Section 2 of Bojan-Dragos et al. (2026), whereas the implementation steps of the overall algorithm are summarized in a compact pseudocode in Section 3 of the same document. Further on, additional simulation results are included in Section 4 of Bojan-Dragos et al. (2026).

2. Problem Formulation and Design Approach

This section is dedicated to the formulation of the Stage I optimization problem, which determines the optimal lane decisions and speed profile of the CAEB at the road-segment level and then presents its solution based on a lattice-based DP approach. The resulting reference trajectory will next be used as input for the Stage II, i.e. cruise control design, not addressed in this paper. This section is based on the intersection crossing scenario and vehicle models given in (Bojan-Dragos et al., 2026).

2.1 Optimal Trajectory Planning

In Stage I, the CAEB trajectory planning problem is formulated as a constrained optimization problem that balances energy use, travel time, and passenger comfort at the road-segment level. Unlike eco-driving strategies for connected electric buses that optimize continuous speed trajectories using multiobjective optimal control (Zhang et al., 2024), this approach formulates the Stage I problem on a discrete lattice and defines a segment-wise objective that combines CAEB energy use, segment travel time, and the penalization of frequent lange change increments.

To define the optimization problem involved in trajectory planning, the CAEB trajectory is parameterized along the road using a spatial (segment-based) discretization. During the trip, CAEB's lane choice depends on HDV speed and position, requiring either a left/right lane change or staying in the lane to minimize energy use and improve passenger comfort by keeping the CAEB near the center lane. The trip is divided into N_s road segments (nodes), with the segment index $k = 1 \ldots N_s$:

$$N_s = \lceil D_{Cz} / D_s \rceil,\ N_s \geq 2 \tag{1}$$

each with the length D_s. The segment length D_s depends on the desired yaw angle γ_{rc}:

$$D_s = 1.5 D_\omega / \tan \gamma_{rc} \tag{2}$$

At a low speed (comfort mode), the yaw angle is small and D_s large; in the "sporty" mode, the yaw angle is large and D_s small. At each segment, the CAEB decides whether to change lanes or remain on the same lane (Dong et al., 2024a; Dong et al., 2024b). D_ω is the width of the lane, $D_\omega = 3.75$ m for this problem. As specified in (Dong et al., 2024b), the center of lane 2 (i=2) is defined as the zero longitudinal position, so the centers of lanes 1 (i=1) and 3 (i=3) are negative and positive, respectively.

The driving lane optimization problem is defined as follows in terms of the combination of the objective functions given in (Dong et al., 2024a) and (Dong et al., 2024b), but simplified according to Precup et al. (2025):

$$\begin{bmatrix} U_{opt} \\ V_{opt} \\ (\gamma_{rc})_{opt} \end{bmatrix} = \arg \min_{\substack{u_3^{k\to k+1}, v_{CAEBr}^{[k]} \\ k=1\ldots N_s-1 \\ v_{CAEBr}^{[N_s]}, \gamma_{rc}}} \zeta_e \sum_{k=1}^{N_s} E_c^{[k]} + \sum_{k=1}^{N_s} \zeta_t^{[k]} T_c^{k-1\to k} + \varsigma \sum_{k=1}^{N_s-1} (u_3^{k\to k+1} - u_3^{k-1\to k})^2, \tag{3}$$

where $T_c^{k-1\to k}$, $k = 1 \ldots N_s$ denotes the travel time associated with segment k during both lane-changing and lane-keeping, i.e. $T_c^{0\to 1}$ is the travel time for the first segment, from the moment the CAEB enters in the communication zone to the beginning of second segment and $T_c^{N_s-1\to N_s}$ is

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the travel time for the final segment, from the end of the penultimate segment N_s–1 to the end of the communication zone, during both lane-changing and lane-keeping, and the parameter $\zeta_e > 0$ and the variable $\zeta_t^{[k]} > 0$ convert the energy and time costs into their monetary counterpart, respectively, based on the electricity bill and hourly pay considerations, $\varsigma > 0$ is the normalized coefficient for the penalty of frequent lane changes, and U_{opt} is the optimal sequence of lane-change control inputs:

$$U_{opt} = \{(u_3^{k-1\to k})_{opt} \mid k = 1...N_s\} = \{(L_c^{k-1\to k})_{opt} \mid k = 1...N_s\}, \quad (4)$$

where $(u_3^{k-1\to k})_{opt} = (L_c^{k-1\to k})_{opt}$ is the optimal control input associated with segment k for $k = 1...N_s$. Moreover, an initial control input is applied for the first transition, $u_3^{0\to 1}$, while $u_3^{N_s-1\to N_s}$ denotes the control input applied for the last transition:

$$u_3^{0\to 1} = 0, \; u_3^{N_s-1\to N_s} = 0, \quad (5)$$

These two control input values are taken based on the constraints given in (Precup et al., 2025), which state that no lane changing is allowed for the first and last segments. The set V_{opt} in equation (3) is the optimal sequence of CAEB speeds:

$$V_{opt} = \{(v_{CAEBr}^{[k]})_{opt} \mid k = 1...N_s\}, \quad (6)$$

$(v_{CAEBr}^{[k]})_{opt}$ is the optimal CAEB reference (set-point) speed along the kth segment.

The energy consumption for the kth segment, $E_c^{[k]}$ can be expressed as:

$$E_c^{[k]} = P_b^{[k]} T_c^{k-1\to k}, \; k = 1...N_s, \quad (7)$$

where $P_b^{[k]}$ is the battery power of the kth segment, which is assumed to be constant across all segments, $P_b^{[k]} = P_b$, $k = 1 \ldots N_s$. Consequently, the average value of P_b was determined in (Precup et al., 2025) as $P_b = 1.99 \cdot 10^4$ W.

Regarding the CAEB's speed in the final segment, as specified in (Precup et al., 2025), if the traffic light is green at the final moment (T_f), the CAEB continues at its current speed and adjusts its position accordingly. Conversely, if the traffic light turns red, the CAEB stops, sets its speed to zero, and updates its position. This is expressed as follows by processing the constraints given in (Precup et al., 2025):

$$\begin{cases} v_{CAEBr}^{[N_s]} - v_{CAEBr}^{[N_s-1]} < 0 \text{ and } s_{xr}^{[N_s]} = S - S_{v,S}, \text{ if } P^{[N_s]} = 0, \\ v_{CAEBr}^{[N_s]} = v_{CAEBr}^{[N_s-1]} \text{ and } s_{xr}^{[N_s]} = S, \text{ if } P^{[N_s]} = 1. \end{cases} \quad (8)$$

The first constraint in (8) shows that the CAEB speed only decreases, ensuring a smooth deceleration. This means stopping within the last segment is not enforced. A complete stop will only be imposed in the control stage of continuous time. The second constraint in (8) shows that the CAEB speed is kept constant in the final segment. These are expressed by the following continuous-time counterpart of the constraints in equation (8):

$$\begin{cases} v_{CAEBr}(T_f) = 0 \text{ and } s_{xr}(T_f) = S - S_{v,S}, \text{ if } P(T_f) = 0, \\ v_{CAEBr}(T_f) = v_{CAEBr}^{[N_s]} \text{ and } s_{xr}(T_f) = S, \text{ if } P(T_f) = 1. \end{cases} \quad (9)$$

The variable $T_c^{k-1\to k}$ is computed as follows (Precup et al., 2025):

$$T_c^{k-1\to k} = \begin{cases} \dfrac{2D_s}{v_{CAEBr}^{[k+1]} + v_{CAEBr}^{[k]}}, \text{if } u_3^{k-1\to k} = 0, k = 1...N_s - 1, \\ \dfrac{f}{v_{CAEBr}^{[k]}}, \qquad \text{if } u_3^{k-1\to k} \neq 0, \; k = 1...N_s. \end{cases} \quad (10)$$

where f can be expressed as:

$$f = \frac{1}{\sqrt{d}} \int_0^{D_s} \sqrt{ax^4 - bx^3 + cx^2 + d}\, dx, \quad (11)$$

with the coefficients $a = 256 \tan^6 \gamma_{rc}$, $b = 768 D_\omega \tan^5 \gamma_{rc}$, $c = 576 D_\omega^2 \tan^4 \gamma_{rc}$, and $d = 81 D_\omega^4$.

As specified in (Precup et al., 2025), since the length of the last road segment can be less than D_s, no lane change is allowed in the final road segment for k=N_s. Specifically, if the traffic light is red, namely $P^{[N_s]} = 0$, the vehicle decelerates to a stop before the stop line and the time for the last segment is computed using the distance D_s^{stop}. If the light is green, namely $P^{[N_s]} = 1$, the vehicle keeps a constant speed and reaches the intersection, so the time for the last segment is computed using the distance D_s^{go}. These two situations lead to:

$$T_c^{N_s-1\to N_s} = \begin{cases} \dfrac{2D_s^{stop}}{v_{CAEBr}^{[N_s-1]}}, \; D_s^{stop} = S - S_{v,S} - s_{xr}^{[N_s-1]}, \text{if } P^{[N_s]} = 0 \\ \dfrac{2D_s^{go}}{v_{CAEBr}^{[N_s-1]} + v_{CAEBr}^{[N_s]}}, \; D_s^{go} = S - s_{xr}^{[N_s-1]}, \text{if } P^{[N_s]} = 1 \end{cases} \quad (12)$$

Using equation (7) in equation (3) and considering that the variable $\zeta_t^{[k]}$ is constant across all segments, $\zeta_t^{[k]} = \zeta_t$, $k = 1 \ldots N_s$, the optimization problem that ensures the reduction of the lane change increments and energy reduction becomes:

$$
\begin{aligned}
&\begin{bmatrix} U_{opt} \\ V_{opt} \\ (\gamma_{rc})_{opt} \end{bmatrix} = \arg \min_{\substack{u_3^{k-1\to k}, v_{CAEBr}^{[k]} \\ k=2\ldots N_s-1 \\ v_{CAEBr}^{[N_s]}, \gamma_{rc}}} (\zeta_t + \zeta_e P_b) T_c^{N_s-1\to N_s} \\
&+ \sum_{k=1}^{N_s-1} [(\zeta_t + \zeta_e P_b) T_c^{k-1\to k} + \varsigma (u_3^{k\to k+1} - u_3^{k-1\to k})^2] \\
&= (\zeta_t + \zeta_e P_b) \arg \min_{\substack{u_3^{k-1\to k}, v_{CAEBr}^{[k]} \\ k=2\ldots N_s-1 \\ v_{CAEBr}^{[N_s]}, \gamma_{rc}}} T_c^{N_s-1\to N_s} \\
&+ \sum_{k=1}^{N_s-1} \{T_c^{k-1\to k} + [\varsigma / (\zeta_t + \zeta_e P_b)](u_3^{k\to k+1} - u_3^{k-1\to k})^2\} \\
&= \arg \min_{\substack{u_3^{k-1\to k}, v_{CAEBr}^{[k]} \\ k=2\ldots N_s-1 \\ v_{CAEBr}^{[N_s]}, \gamma_{rc}}} T_c^{N_s-1\to N_s} \\
&+ \sum_{k=2}^{N_s-1} [T_c^{k-1\to k} + \psi (u_3^{k\to k+1} - u_3^{k-1\to k})^2] \qquad (13) \\
&= \begin{cases} \arg \min_{\substack{u_3^{k-1\to k}, v_{CAEBr}^{[k]} \\ k=2\ldots N_s-1 \\ v_{CAEBr}^{[N_s]}, \gamma_{rc}}} T_c^{N_s-1\to N_s} (v_{CAEBr}^{[N_s-1]}, \gamma_{rc}) \\ + \sum_{k=2}^{N_s-1} J_c^{k-1\to k} (v_{CAEBr}^{[k]}, v_{CAEBr}^{[k+1]}, u_3^{k-1\to k}, u_3^{k\to k+1}, \gamma_{rc}), \\ if\ P^{[N_s]} = 0, \\ \arg \min_{\substack{u_3^{k-1\to k}, v_{CAEBr}^{[k]} \\ k=2\ldots N_s-1 \\ v_{CAEBr}^{[N_s]}, \gamma_{rc}}} T_c^{N_s-1\to N_s} (v_{CAEBr}^{[N_s-1]}, v_{CAEBr}^{[N_s]}, \gamma_{rc}) \\ + \sum_{k=2}^{N_s-1} J_c^{k-1\to k} (v_{CAEBr}^{[k]}, v_{CAEBr}^{[k+1]}, u_3^{k-1\to k}, u_3^{k\to k+1}, \gamma_{rc}), \\ if\ P^{[N_s]} = 1, \end{cases} \\
&= \arg \min_{\substack{u_3^{k-1\to k}, v_{CAEBr}^{[k]}, \\ k=2\ldots N_s-1 \\ v_{CAEBr}^{[N_s]}, \gamma_{rc}}} J_c (u_3^{1\to 2}, v_{CAEBr}^{[1]}, \ldots, u_3^{N_s-1\to N_s}, \\
&v_{CAEBr}^{[N_s-1]}, v_{CAEBr}^{[N_s]}, \gamma_{rc}),
\end{aligned}
$$

where the lane-change control inputs on the first and final transitions are fixed by the boundary constraints in equation (5) and are not included in U_{opt}, the first CAEB reference speed is also fixed as shown in the sequel, and the parameter ψ is the weight of the lane change increments versus the monetary counterpart of the time cost (Precup et al., 2025):

$$\psi = \varsigma / (\zeta_t + \zeta_e P_b), \qquad (14)$$

and $J_c^{k-1\to k}$ represents the segment-wise contribution to the discrete-time objective function, combining the travel-time component $T_c^{k-1\to k}$ and the penalty on lane-change increments.

The optimization problem in equation (13) is subject to initial and terminal conditions, CAEB speed and lane bounds, and safety constraints with respect to HDVs and the traffic signal, expressed in equation (15):

$$
\begin{aligned}
&u_3^{k\to k+1} \in \begin{cases} \{0,1\}, & \text{if } i^{[k]} = 1, \\ \{-1,0,1\}, & \text{if } i^{[k]} = 2, \\ \{-1,0\}, & \text{if } i^{[k]} = 3, \end{cases} \\
&|u_3^{k\to k+1} - u_3^{k-1\to k}| \le 1,\ k = 1\ldots N_s - 1, \\
&t^{[1]} = T_i = 0,\ \ t^{[k+1]} = t^{[k]} + T_c^{k\to k+1},\ k = 1\ldots N_s - 1, \\
&s_{xr}^{[1]} = 0, s_{xr}^{[k+1]} = kD_s,\ k = 1\ldots N_s - 2, \\
&s_{xr}^{[N_s]} = \begin{cases} S - S_{v,S}, \text{if } P^{[N_s]} = 0 \\ S, \text{ if } P^{[N_s]} = 1 \end{cases}, s_{yr}^{[1]} = -D_\omega, \\
&s_{yr}^{[k+1]} = s_{yr}^{[k]} + D_\omega \operatorname{sgn}(u_3^{k-1\to k}), k = 1\ldots N_s - 2, \\
&s_{yr}^{[N_s]} \in \{-D_\omega, 0, D_\omega\}, \qquad (15) \\
&\begin{cases} d_{v,ij}^{[k]} - d_{s,ij}^{[k]} \ge 0, \\ d_{v,ij}^{[k]} - d_{s,ij}^{[k]} < 0\ \& \begin{cases} u_3^{k-1\to k} = 0\ \& \\ \begin{cases} |s_{xr}^{[k]} - s_{x,ij}^{[k]}| \ge H_s + v_{CAEBr}^{[k]} \cdot T_h, \\ k = 1\ldots N_s, \\ v_{CAEBr}^{[k]} - v_{CAEBr}^{[k-1]} < 0, k = 2\ldots N_s, \end{cases} \\ u_3^{k-1\to k} \ne 0\ \& \ |s_{yr}^{[k]} - s_{y,ij}^{[k]}| \ge H_s, \\ k = 1\ldots N_s, \\ \qquad |s_{xr}^{[k]} - s_{x,ij}^{[k]}| \ge H_s, \end{cases} \end{cases} \\
&v_{CAEB\min} \le v_{CAEBr}^{[k]} \le v_{CAEB\max}, \\
&4.5D_\omega / D_{Cz} < \tan\gamma_{rc} < 1.5D_\omega / L, u_3^{0\to 1} = 0, \\
&u_3^{N_s-1\to N_s} = 0,\ v_{CAEBr}^{[1]} = v_s,
\end{aligned}
$$

where v_s is the initial speed of the CAEB and S represents the location of the stop line at the intersection, also the traffic light position, the road length and the destination position, S = 300 m in this problem. The detailed physical interpretation of each term can be found in (Precup et al., 2025).

The following 4-tuple $x_{DP}^{[k]}$ defines a lattice node in a three-dimensional discrete DP state space:

$$x_{DP}^{[k]} = (k,\ i^{[k]},\ i_v^{[k]},\ u_{DP}^{[k+1]}), \qquad (16)$$

where the segment index $k = 1\ldots N_s$ is also the discrete progression along the road in the longitudinal direction, $i^{[k]} = 1\ldots 3$ is the lane index, corresponding to the centerlines of the right, middle, and left lane, $i_v^{[k]} = 1\ldots N_v$ is the speed index stored by the DP state at each segment k, while the corresponding physical speed value $v_{CAEBr}^{[k]}$ is obtained through the deterministic mapping:

$$
\begin{aligned}
&v_{CAEBr}^{[k]} = V(i_v^{[k]}) \in \{v_{CAEBr}^{[1]} = v_{CAEBr\min}, v_{CAEBr}^{[2]} > v_{CAEBr}^{[1]}, \\
&\ldots, v_{CAEBr}^{[N_v-1]} > v_{CAEBr}^{[N_v-2]}, v_{CAEBr}^{[N_v]}, v_{CAEBr}^{[Nv]} = v_{CAEBr\max} > v_{CAEBr}^{[N_v-1]}\},
\end{aligned} \qquad (17)
$$

and $u_{DP}^{[k+1]} = u_3^{k\to k+1}$ is the discrete control action at each lattice node corresponding to segment k.

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Not all lane-change actions are admissible in every lane. To avoid infeasible maneuvers (e.g. leaving the roadway) and ensure smooth, limited transitions, the CAEB can either remain in the current lane or move to an adjacent one. Accordingly, the admissible actions are defined as a function of the current lane index in the first three constraints of equation (15). The lane-change control input $u_3^{k\to k+1}$ determines the lane index of the next segment $i^{[k+1]} = i^{[k]} + u_3^{k\to k+1}$ where $i^{[k]}$ represents the current lane.

Using the lattice-based DP notations defined in this section and the last constraint in equation (15), the objective function in equation (13) is transformed into the expression:

$$\begin{aligned} &J_{DP}(u_{DP}^{[2]}, v_s, ..., u_{DP}^{[N_s-1]}, v_{CAEBr}^{[N_s-1]}, v_{CAEBr}^{[N_s]}, \gamma_{rc}) \\ &= J(u_3^{1\to 2}, v_{CAEBr}^{[1]}, ..., u_3^{N_s-1\to N_s}, v_{CAEBr}^{[N_s-1]}, v_{CAEBr}^{[N_s]}, \gamma_{rc}). \end{aligned} \tag{18}$$

Substituting equation (18) into equation (13) yields the equivalent optimization problem. This problem formulates the trajectory planning problem as a finite discrete lattice form defined based on the longitudinal progression, lane index, and vehicle speed can then be rendered as:

$$\begin{aligned} &\begin{bmatrix} U_{opt} \\ V_{opt} \\ (\gamma_{rc})_{opt} \end{bmatrix} \\ &= \arg \min_{\substack{u_{DP}^{[k]}, v_{CAEBr}^{[k]}, \\ k=2...N_s-1 \\ v_{CAEBr}^{[N_s]}, \gamma_{rc}}} J_{DP}(u_{DP}^{[2]}, v_s, ..., u_{DP}^{[N_s-1]}, v_{CAEBr}^{[N_s-1]}, v_{CAEBr}^{[N_s]}, \gamma_{rc}) \\ &= \begin{cases} \arg \min_{\substack{u_{DP}^{[k]}, v_{CAEBr}^{[k]}, \\ k=2...N_s-1 \\ v_{CAEBr}^{[N_s]}, \gamma_{rc}}} T_c^{N_s-1\to N_s}(v_{CAEBr}^{[N_s-1]}, \gamma_{rc}) \\ + \sum_{k=2}^{N_s-1} J_c^{k-1\to k}(x_{DP}^{[k]}, u_{DP}^{k-1\to k}, x_{DP}^{[k+1]}, \gamma_{rc}), if\ P^{[N_s]} = 0, \\ \arg \min_{\substack{u_{DP}^{[k]}, v_{CAEBr}^{[k]}, \\ k=2...N_s-1 \\ v_{CAEBr}^{[N_s]}, \gamma_{rc}}} T_c^{N_s-1\to N_s}(v_{CAEBr}^{[N_s-1]}, v_{CAEBr}^{[N_s]}, \gamma_{rc}) \\ + \sum_{k=2}^{N_s-1} J_c^{k-1\to k}(x_{DP}^{[k]}, u_{DP}^{k-1\to k}, x_{DP}^{[k+1]}, \gamma_{rc}), if\ P^{[N_s]} = 1 \end{cases} \end{aligned} \tag{19}$$

where $P_b^{[k]}$ is the battery power of the kth segment, assumed to be constant across all segments, $P_b^{[k]} = P_b$, $k = 1...N_s$. Consequently, the average value of P_b was determined as $P_b = 1.99 \cdot 10^4$ W.

The desired yaw angle γ_{rc} is kept constant along all road segments (for all segment indices $k = 1...N_s$) during a single iteration of the lattice-based DP optimization, but it is treated as a variable across the iterations of the algorithm, being updated from one iteration to the next.

2.2 Road-segment-based Discretization

In Stage I, the longitudinal position s_x is parameterized by the segment index, k, through the fourth constraint in equation (15), which defines a spatial grid along the road, and the final condition is expressed as the fifth set of two constraints in equation (15). Consequently, the lattice evolution is formulated in the spatial (segment-based) domain rather than in the time domain.

The lateral position is discretized by the lane index i, with the physical mapping:

$$s_y^{[k]} = (i^{[k]} - 2)D_\omega \in \{-D_\omega, 0, D_\omega\}. \tag{20}$$

The lateral position is therefore quantified to lane centers, consistent with lattice-based planning. The combination of the longitudinal segmentation $s_x^{[k]}$ and the lane-center lateral positions $s_y^{[k]}$ thus defines a spatial lattice on which the DP state will be constructed in the next subsection.

2.3 Lattice State, Actions and Transitions

Each lattice node stores one speed level from this grid. Speed is treated as a state variable, not a control input, which is typical in discrete DP formulations for eco-driving and trajectory planning. Distance-domain DP formulations for eco-driving often rely on a discretized speed grid and treat speed as a state component, as in (Tiger et al., 2021; Wen et al., 2025).

The complete lattice state space is therefore given by:

$$\mathrm{X} = \{x_{DP}^{[k]} \mid k = 1...N_s\}. \tag{21}$$

This discrete state space provides the basis for solving the optimization problem in equation (13) by DP. For each fixed yaw angle γ_{rc}, the segment length $D_s(\gamma_{rc})$ in equation (7) determines the corresponding number of road segments $N_s(\gamma_{rc})$, and the lattice state space X in equation (21) is constructed with this value of N_s.

The following CAEB state vector is considered at the kth segment (Dong et al., 2024b):

$$\mathbf{x}_l^{[k]} = [t^{[k]} \quad s_{xr}^{[k]} \quad s_{yr}^{[k]}]^T = [t^{[k]} \quad (k-1)D_s \quad s_{yr}^{[k]}], \tag{22}$$

where $t^{[k]}$ is the initial time moment of the kth segment, and $s_{xr}^{[k]}$ and $s_{yr}^{[k]}$ are the desired longitudinal and lateral position of the CAEB at

the kth segment, respectively. Although the state is discrete, it corresponds to continuous physical quantities via deterministic mapping:

$$x_{DP}^{[k]} = (k,\, i^{[k]}, i_v^{[k]}, u_3^{k-1\to k}) \leftrightarrow \left\{ s_x^{[k]} = (k-1)D_s, s_y^{[k]} = (i^{[k]}-2)D_\omega, v_{CAEBr}^{[k]} = V(i_v^{[k]}). \right. \tag{23}$$

For each discrete lattice state $x_{DP}^{[k]}$, the corresponding physical CAEB state at the segment k is collected in the vector $x_l^{[k]}$ defined in equation (22) and it is obtained from $x_{DP}^{[k]}$ through the deterministic mapping in equation (23). In particular, $s_{rx}^{[k]} = s_x^{[k]}$ and $s_{ry}^{[k]} = s_y^{[k]}$ are lane centers.

The transition travel time $T_c^{k-1\to k}(\gamma_{rc})$, with $k = 1 \ldots N_s$, depends on the yaw angle in relation to the segment length and the resulting discrete longitudinal positions.

The segment-entry time $t^{[k]}$ is propagated along the lattice according to the third constraint in equation (15) which adds the transition travel time $T_c^{k-1\to k}(\cdot)$, $k = 1 \ldots N_s$, to the entry time for segment k–1. Specifically, for a fixed γ_{rc}, the time at the entry on segment k+1 equals the time at the entry on segment k plus the travel time for the transition $k \to k+1$, which depends on the current and subsequent speeds and on the lane-change control input applied on that transition. The travel-time term $T_c^{k-1\to k}(\cdot)$ is given by equations (10) and (12). This propagation ensures that the traffic light constraints in equation (15) are evaluated using a physically consistent arrival time at the stop line. The resulting formulation is a spatially discretized optimal control problem, where continuous dynamics is represented on a lattice in relation to distance, lane, and speed. Such representations are well suited for eco-driving, as DP operates directly on variables aligned with speed limits, traffic lights, and lane-change opportunities.

In order to ensure a smooth longitudinal behavior and to avoid unrealistic speed jumps between consecutive segments, the speed decision at each step is restricted to a local neighborhood of the current speed level. Let $v^{[k]}$ denote the speed associated with the current lattice state at segment k. The admissible speed indices for the next segment are restricted to a neighborhood around $i_v^{[k]}$, $i_v^{[k]} \in N(i_v^{[k]})$, where $N(i_v^{[k]}) = \left\{ i_v^{[k+1]} \in \{1,\ldots,N_v\} \;\middle|\; \left| i_v^{[k+1]} - i_v^{[k]} \right| \le \Delta_v \right\}$.

For the implementation considered in this work, the neighborhood width is set to $\Delta_v = 20$, which means that speed transitions are allowed within a fixed local window around the current speed level. This constraint effectively enforces longitudinal smoothness while preserving the discrete nature of the DP formulation.

2.4 DP Solution and Bellman Recursion

For each fixed candidate yaw angle γ_{rc}, the optimization problem defined in equation (3), subject to the constraints in equation (15), is solved by a finite-horizon DP approach which is adopted by introducing a value function defined on the lattice and applying a backward Bellman recursion. The discrete lattice introduced in subsection 2.2 enables the decomposition of the global objective function into segment-wise contributions, leading to a recursive Bellman formulation. From equation (3), the contribution associated with the transition from segment k to segment k+1 is defined as:

$$\begin{aligned} &J_c^{k-1\to k}(x_{DP}^{[k]}, u_{DP}^{k-1\to k}, x_{DP}^{[k+1]}, \gamma_{rc}) \\ &= T_c^{k-1\to k}(v_{CAEBr}^{[k]}, \gamma_{rc}) + \psi(u_3^{k\to k+1} - u_3^{k-1\to k})^2. \end{aligned} \tag{24}$$

The terminal segment contribution is given by:

$$\begin{cases} J_c^{N_s-1\to N_s}(x_{DP}^{[N_s-1]}, \gamma_{rc}) = 0,\ if\ P^{[N_s]} = 0, \\ J_c^{N_s-1\to N_s}(x_{DP}^{[N_s-1]}, x_{DP}^{[N_s]}, \gamma_{rc}) = 0,\ if\ P^{[N_s]} = 1. \end{cases} \tag{25}$$

The transition time $T_c^{k-1\to k}(\gamma_{rc})$, $k = 1 \ldots N_s$ is computed as in the third constraint of equation (15) and depends parametrically on the yaw angle γ_{rc}. The terminal segment contribution is set to zero since no transition beyond N_s exists; all travel-time terms are accounted for on transitions $k \to k+1$, $k = 1 \ldots N_s - 1$. For a fixed yaw angle γ_{rc}, a segment-wise value function is introduced on the lattice. For each segment index k, lane index $i^{[k]}$, speed-grid index $i_v^{[k]}$, and previous lane-change control input $u_3^{k\to k+1}$, the value function $V^{[k]}(i^{[k]}, i_v^{[k]}, u_3^{k-1\to k})$ is defined as the minimal discrete-time objective function accumulated from segment k to the terminal segment N_s:

$$\begin{aligned} &V^{[k]}(i^{[k]}, i_v^{[k]}, u_3^{k-1\to k}) \\ &= \min_{\substack{u_3^{idx\to idx+1},\\ i_v^{[idx+1]},\\ idx=k\ldots N_s-1}} \sum_{idx=k}^{N_s-1} J_c^{idx\to idx+1}(x_{DP}^{[idx]}, u_{DP}^{idx\to idx+1}, x_{DP}^{[idx+1]}, \gamma_{rc}), \end{aligned} \tag{26}$$

where the minimization is applied to all admissible lane-change control $u_3^{idx\to idx+1}$ and speed indices, $i_v^{[idx+1]}$ for $idx = k \ldots N_s - 1$, that satisfy the safety and traffic-light constraints in equation (15).

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The proposed lattice-based formulation leads to a standard finite-horizon DP problem as it was expressed in (Bertsekas, 2005), with a segment-wise value function $V^{[k]}$ defined over the discrete lattice state as in equation (26). At each segment k, the Bellman recursion expresses the optimal cost from k to N_s as the sum of the segment-wise contribution $J_c^{k\to k+1}$ and the value function of the next segment $V^{[k+1]}$ minimized over all admissible lane-change control inputs and next-segment indices, which yields equation (27) subject to the admissible lane-change and state-transition constraints defined in subsection 2.2.

$$V^{[k]}(i^{[k]},i_v^{[k]},u_3^{k-1\to k}) = \min_{\substack{u_3^{k\to k+1}\in U(i^{[k]})\\ i_v^{[k+1]}\in N(i_v^{[k]})}} [V^{[k+1]}(i^{[k+1]}, i_v^{[k+1]},u_3^{k\to k+1}) + J_c^{k\to k+1}(x_{DP}^{[k]},u_{DP}^{k\to k+1},x_{DP}^{[k+1]},\gamma_{rc})] \quad (27)$$

For a fixed yaw angle γ_{rc}, the minimization in equation (27) is taken with respect to the lane-change control input $u_3^{k\to k+1}$ and the next-speed index $i_v^{[k+1]}$, while the global discrete-time objective function in equation (13) and (20) is recovered by summing the segment-wise contributions $J_c^{k-1\to k}(\cdot,\gamma_{rc})$ along the optimal state–action sequence.

The recursion is initialized at the terminal segment $k = N_s$ by setting V_{N_s} as equal to the terminal segment contribution $J_c^{[N_s]}$, and backward evaluation is performed for it for $k = N_s - 1 \ldots 1$ using the given initial state and fixed initial lane-change index. Once the backward evaluation is completed, the optimal sequences of lane indices, speed indices, and lane-change control inputs are obtained by means of forward backtracking from the initial segment.

In the numerical implementation, the optimization over the yaw angle γ_{rc} is carried out through an outer adaptive grid search. Starting from the admissible interval of γ_{rc} defined by the first of the penultimate constraints in equation (15), at each outer iteration a grid of N_g candidate yaw angles is sampled uniformly on the current interval. For each candidate, the lattice-based DP algorithm is executed to obtain the corresponding objective value and trajectory. This interval is then shrunk around the best grid point and its neighbors. The procedure is repeated until the interval length is reduced before a maximum number of outer iterations is reached, and the optimal yaw angle $(\gamma_{rc})_{opt}$ in equation (19) is taken as the best yaw angle found. The overall procedure, combining the outer search and DP recursion, is summarized in the algorithm presented in (Bojan-Dragos et al., 2026).

3. Simulation of Multi-Vehicle System Behavior

The traffic system is simulated on a real urban route with a CAEB, as it was modeled in (Maamria et al., 2016) and (Sun et al., 2018), and six HDVs. The goal is the autonomous, efficient control with energy management and obstacle avoidance. The lattice-based DP trajectory planning algorithm is implemented offline in MATLAB with the corresponding Simulink version on a PC equipped with an AMD Ryzen 7 5800U processor with Radeon Graphics @ 1.90 GHz CPU and 16 GB RAM. This offline computation yields accurate reference trajectories for Stage II control and reveals the computational effort for a future online implementation. Two initial traffic-light states are considered, namely $I_{in} = 0$ (red at the beginning of the communication zone) and $I_{in} = 1$ (green at the beginning). For each traffic-light state, three initial lateral positions of the CAEB are analyzed, corresponding to the three lane centers, i.e. the initial lateral position on the right, middle, and left lane, respectively. This leads to six simulation scenarios given in (Bojan-Dragos et al., 2026), which jointly illustrate the influence of the initial traffic-light indications and of the initial CAEB lane on the lattice-based trajectory planning outcome.

4. Conclusion

Building upon the authors' recent papers (Precup et al., 2025 and Precup et al., 2026), this paper proposes a lattice-based dynamic programming (DP) approach for solving the optimization problem specific to trajectory planning for CAEBs aiming at energy reduction and improve passenger comfort. Six simulation scenarios were analyzed, incorporating CAEB/HDV dynamics and traffic light modeling, and illustratively showing the decrease of the objective function as the number of iterations for the optimization algorithm increased. This paper focuses on offline trajectory planning, where the lattice-based DP algorithm

computes the CAEB reference trajectories prior to its execution. However, there is a limitation to this approach, which lies in the offline solving of the optimization resuting in offline trajectory planning. Extending the proposed approach to online trajectory planning, with a real-time updating of the reference inputs based on traffic measurements and signal timing, represents a direction for future research.

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