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Active learning in a large calculus class improves student satisfaction and performance

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Abstract
Active learning strategies during large lectures promote student activity and are known to improve student performance and engagement. In this work, an array of cognitive-science-based and psychological active learning strategies are introduced in large lectures in a first-year undergraduate course in single-variable calculus and analytical geometry. The implemented active learning strategies correspond to preclass quizzes and in-class problem solving followed by student-student discussion using Mentimeter, demonstrations avoiding the curse of knowledge and giving context before content, a popular science-style inspiration lecture, promoting cycles of positivity in teaching and the use of motivational nudges. To evaluate the efficiency of these active learning strategies, the satisfaction and performance of the students engaged in active learning are compared to students attending traditional lectures in the same calculus course during the previous two years. Diagnostic tests before the calculus course reveal that the prerequisite skills of the students of all years are similar. The final grades in the course based on final test scores and grades given by students in course surveys after the course reveal that active learning strategies improve student performance and satisfaction compared to traditional lectures. In addition, the students engaged in active learning greatly enjoyed the active learning strategies implemented and reported high levels of motivation and psychological safety in the course survey. Together with other studies, this work cements the efficiency of active learning strategies and provides practical guidance on how these strategies can be implemented.

Keywords Active learning, Traditional lectures, Interactive teaching, Mentimeter, Undergraduate calculus, Nudging

1 Introduction

My first experiences with large lectures in mathematics were quite frankly daunting, to say the least. As a 19-year-old student, I had never before attended lectures in a large lecture hall with 150 other young students, and in my case the students were enrolled in three different undergraduate programs in chemical engineering, chemical engineering with engineering physics and biotechnology. We were attempting to learn single variable calculus and analytical geometry by copying the lecturers' notes that were written in chalk on large blackboards at a rapid pace. Struggling to keep up, our heads were

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nodding up and down as we were looking up at the blackboards and down in our notebooks where we were ferociously writing down equations, theorems and proofs. Despite this startling first impression, I later switched subject to mathematics and eventually earned a doctorate in applied mathematics. Fifteen years later, as a postdoctoral research fellow, I was given the opportunity to be the lecturer and examiner in the same course that I once took, an offer which I accepted with eagerness and excitement. To my dismay, I discovered that student satisfaction and overall performance in the course, which were already quite low when I took it, had unfortunately worsened over the decades. Determined to improve student engagement, satisfaction and performance, I studied the literature on active learning strategies.

With limited time to prepare for the course, I restricted myself to improving the traditional lectures typically implemented in class. To my delight, previous studies demonstrated that changing traditional lectures by incorporating activities promoting student engagement, the very essence of active learning, had resulted in great improvements. One of the most notable examples from 2011 that illustrates the power of active learning in large lectures is that of a large-enrolment undergraduate physics class at the University of British Columbia in Canada. By directly comparing the test scores of a control group of 267 students attending *traditional lectures* (TL) to another experimental group of 271 students taking the same course using *active learning* (AL) strategies, the authors recorded the highest effect size of any educational intervention at that point in time, as the test scores of the experimental group improved with 2.5 standard deviations compared to the control group [1]. Subsequently, similar interventions in large lecture classes have resulted in substantial improvements in student engagement and learning, both in the case of a course in computer science and mathematics [2] as well as in the so-called "calculus for life sciences courses" [3], to name but a few examples. Using these case studies as inspiration, I modified the structure of the traditional lectures in my single variable calculus and analytical geometry course in the spirit of active learning, while still preserving all other aspects of the course such as the content, the course book, the recommended exercises, the structure of the written exam, etc. In order to compare the performance of my teaching approach based on active learning strategies with the counterpart based on traditional lectures, I set out to quantify the learning outcomes of the students in my course and those of the students in the same course based on traditional lectures given the two previous years.

Given these circumstances, we answer four questions. What were the prerequisite skills of the students in my course based on active learning strategies compared to the prerequisite skills of the students of the two previous years? How did the test scores on the final exam of the students engaged in active learning compare to the test scores the previous two years? How satisfied were the students engaged in active learning compared to the students attending traditional lectures the previous two years? What were the students' opinions on the implemented active learning strategies? The succinct answer we give here is that *students with similar prerequisite skills both learn more and are more satisfied when taught single variable calculus and analytical geometry by active learning strategies compared to traditional lectures*. The presentation of this work is structured as follows: first, we present a *Background* describing the course and the data used in this work, we describe the *Implementation of active learning strategies* in the

course, we present *Results* providing answers to our research questions, and lastly, we consider these findings in a larger context in the *Discussion*.

The novelty of this study lies in the incorporation of active learning strategies in a large-class traditional lecture setting based on the chalk-on-blackboard method. Rather than replacing the classical lecture format entirely, this work investigates how active learning strategies such as in-class problem solving and pre-class quizzes can be implemented during large-class traditional lectures in a first-year calculus course.

2 Background

The undergraduate course of interest is called "single variable calculus and analytical geometry", and it is given during the first year of three different undergraduate programs at Chalmers University of Technology in Sweden. The course is taught in Swedish and is the first university course that students take, and the students are enrolled in the undergraduate programs, with respective abbreviations, named chemical engineering (K), chemical engineering with engineering physics (Kf), and biotechnology (Bt). Here, we describe the background of the course in two parts: the course structure and the data used in this work.

2.1 Course structure

The content of the course is divided into two parts: single variable calculus and analytical geometry. Three quarters of the course are devoted to single variable calculus and the contents correspond roughly to chapters 1–4 in [4]. The topics of the calculus part of the course are limits and continuity, differentiation, transcendental functions and applications of differentiation. The remaining quarter of the course is dedicated to analytical geometry and here the contents correspond roughly to Chap. 10 in [4] as well as sections 1.1 and 1.2 in [5]. The topics of the analytical geometry part of the course are vectors, coordinate geometry in 3-space and an introduction to linear algebra.

There are two forms of classes in the course: lectures and demonstration classes. Traditional lectures are two hours long, three lectures are given per week, and between 100 and 150 students typically attend the lectures. During demonstration classes, students are divided into smaller groups where a teaching assistant solves tasks from the course books on blackboards. The demonstration classes are two hours long, and two demonstration classes are given per week.

2.2 The data consist of diagnostic test scores, final grades and course surveys

To quantify the effect of introducing active learning into traditional lectures, we compare three types of data for three groups of students. The two control groups of students took the same course in 2022 and 2023 when the course was taught using *traditional lectures* (TL) and the last experimental group of students was taught by this author in 2024 using *active learning* (AL) strategies. Provided these three groups of students, we quantify three metrics of prerequisite skills before the course, learning outcomes after the course, and student satisfaction after the course.

The metric to quantify prerequisite skills corresponds to the average scores on the diagnostic tests. Before the course starts each year, students take an unannounced diagnostic test consisting of nine questions on high school level mathematics. In addition to the average diagnostic test scores of the experimental and control groups, we add the

average diagnostic test score from 2014 (TL) since the diagnostic test this year was identical to the one given in 2024 (AL).

The metric quantifying the mathematical knowledge of the students after the course corresponds to the grades on the final exam. The grade is determined by the score on the final exam, where the maximum score on the exam is 50 points and 20 points are required to pass the exam. There are two voluntary but highly recommended online tests that students can take during the course which can result in two bonus points on the final exam. The four grades and their corresponding test scores are: "fail" denoted by U (0–19 points), 3 (20–29 points), 4 (30–39 points) and 5 (40–50 points).

The metric that quantifies student satisfaction is the grades of various aspects of the course given by the students in the course surveys. After the final exam, students are asked to answer a course survey in which they can evaluate how satisfied they were with the course. For eight of the questions in the course survey, students are asked to grade a statement about, say, the teaching on a scale from 1 meaning "completely disagree" to 5 meaning "completely agree".

3 Implementation of active learning strategies

To improve student performance, I changed course teaching to promote active learning in class. Before describing the exact modifications of large lectures in particular and teaching in general, let us begin by defining what the term active learning means. To this end, we use the following working definition of active learning by Michael Prince [6].

> *The core elements of active learning are student activity and engagement in the learning process. Active learning is often contrasted to the traditional lecture where students passively receive information from the instructor.*

In light of this definition, we consider the various elements of active learning strategies that were implemented, and they fall into two categories, namely the cognitive-science based elements and the psychological elements. Subsequently, we describe three different cognitive-science based elements of active learning followed by two psychological elements.

3.1 Promoting interactive learning using the web-based student response system Mentimeter

The main inspiration for the structure of the large lectures in the calculus course comes from the seminal work by Deslauriers, Schelew and Wieman at the University of British Columbia in Canada. Their teaching based on active learning involved "preclass reading quizzes, in-class clicker questions with student-student discussion" and "targeted in-class instructor feedback" [1]. In such classes, students are frequently engaged in problem solving outside their comfort zone, and they are guided by the feedback from instructors. These are, in fact, some of the essential components of what is known as *deliberate practice* which is the main technique to acquire expertise in any domain [7]. Motivated by their approach, I implemented three new teaching elements in association with large lectures.

First, I introduced preclass reading quizzes. The purpose of these quizzes was to encourage students to read the relevant sections of the course books before class. Second, in-class problem solving was introduced in conjunction with a student-student

discussion followed by in-class clicker questions. Specifically, the students were asked to solve mathematical problems in groups, consisting of students sitting close to each other in the lecture hall, before voting in multiple choice polls with possible answers to the problems of interest. Third, the students received feedback from the instructor in the class. Following the voting in the multiple choice polls, I presented my solutions to the mathematical problems at hand to the whole class. A challenge in implementing these three elements of active learning is deciding which format the lectures should follow.

To account for these elements of active learning, I changed the standard "chalk-written notes on the blackboard" format of traditional mathematics lectures. However, this decision was met with skepticism from some faculty members, particularly by a senior lecturer. This senior lecturer expressed their reservations about active learning strategies and they stated that the best method of teaching mathematics is to have traditional lectures using only chalk-written notes on blackboards. Despite this skepticism, I decided to change the format from merely chalk-written notes on blackboards to a hybrid format alternating between slides and hand-written notes.

Technically, the interactive elements of the lectures were implemented in the web-based student response system Mentimeter. Mentimeter is a web-based platform which allows the instructor to create interactive presentations. Using a code that is displayed on the slides of the Mentimeter presentations, students can join the presentation in real time using their smart phones or laptops, and most importantly they can participate in the presentation by, for instance, voting in multiple choice questions. Also, it is straightforward to write mathematical equations in Mentimeter as it supports LaTeX. Numerous studies show that Mentimeter is an effective tool for promoting student engagement in teaching [8], and based on this I decided to implement a hybrid-format combining Mentimeter presentations with chalk-written notes on blackboards. A challenge that arises from the usage of a hybrid format is to decide on the number of slides one should include in each presentation.

A hybrid format can benefit from the best aspects of hand-written notes and slide-based presentations. The advantages of hand-written notes is that the instructor can elaborate on details in a way that is more difficult to achieve using slide-based presentations. A possible disadvantage with restricting oneself to chalk-written notes on blackboards is that the gaze of the students is fixated on their notebooks for extended periods of time while they are copying what is written on blackboards, and this reduces eye contact between instructor and students. Also, the course content is covered at a slower pace during traditional lectures based solely on hand-written notes compared to lectures where slides are used. Aside from the fact that presentation slides allow the instructor to cover course material faster, another advantage is that the students can look up at the slides and make eye contact with the lecturer instead of constantly looking down in their notebooks. On the other hand, a risk with using too many slides as an instructor is that the students can become overwhelmed by vast quantities of information which is a phenomenon known as "Death by PowerPoint" [9]. Consequently, a hybrid format of lectures must account for the strengths and weaknesses of hand-written notes on the one hand and slide-based presentations on the other.

The three main elements of the implemented hybrid lectures were slides, interactive problem solving by students and hand-written demonstrations by the instructor. To avoid "Death by PowerPoint" each Mentimeter presentation normally consisted of

between 10 to 15 slides which were covered during a two-hour lecture. These slides were typically used to introduce various topics as well as presenting the problems that the students were asked to solve in class. Each two-hour lecture were usually interrupted three or four times when the students were asked to discuss and attempt to solve a problem for five to ten minutes together with other students sitting next to them in the lecture hall. Then, the students were asked to vote on possible answers to the problems of interest in a multiple choice poll displayed on a Mentimeter slide. After the voting had finished, the instructor presented their solution to the problem using chalk-written notes on blackboards. Aside from presenting solutions to mathematical problems on blackboards, I also presented theorems and proofs this way. To make some of the proofs more interactive, I extracted key steps of the proofs and formulated them as in-class multiple choice problems. The preclass quizzes were also Mentimeter presentations where each slide presented a question in terms of a multiple choice poll, and the codes required for accessing these preclass quizzes were sent out to the students by email a few days before the relevant lectures. Moreover, the same senior lecturer that passionately discouraged me to use slide-based presentations during lectures also disagreed with another practice that was implemented.

All of the preclass quizzes, Mentimeter presentations and my own hand-written lecture notes were posted on the course web page. This implies that the students could access all of the material covered in class after each lecture, and making this material accessible turned out to be a contentious issue. The same senior lecturer who advised against using slide-based presentations in class, also strongly disagreed with posting lecture notes, or any material for that matter, on the course web page. The reason for this was that if all lecture notes were made available online, then the students would not attend the lectures. Nevertheless, from the perspective of active learning, posting lecture notes online enables students to interact with course material after class [2] which furthermore is known to be a key component of successful learning referred to as *reflection* [10]. In light of this, I decided to publish all of the material covered in class on the course web page. Other findings from the science of learning give us valuable insights into how to best present information during lectures from a cognitive-science perspective.

3.2 Avoiding the curse of knowledge and giving context before content

The psychologist Stephen Pinker reveals two indispensable components of clear writing which account for how information is stored and processed neurologically. These are known as *avoiding the curse of knowledge* and *providing context before content* [11]. These principles are so profound that they are also key to successful learning [10]. Accordingly, these two principles tell us how we should design lectures, in general, and presentation slides, in particular.

The *curse of knowledge* is a great obstacle to clear communication and successful learning. It is defined as the inability to understand that other people do not always share the same knowledge that you have. It is especially important that teachers are aware of the curse of knowledge as they generally discuss topics that are second nature to them but at the same time these very topics are unfamiliar to many of their students. In preparation for the calculus course, I tried to foresee situations in which the curse of knowledge would prevent me from communicating clearly, and an illustrative example of how to avoid miscommunication immediately springs to mind.

New occurrences for most university students taking their first course in mathematics are theorems and proofs. In truth, many students do not know what these terms mean. Usually, instructors in the first undergraduate mathematics course present theorems and their proofs without any explanation of what theorems and proofs are in the first place, and this leaves many students confused at first. To avoid this confusion, I explained what theorems and proofs are using the following analogy with logical reasoning.

> *A logical argument consists of one or several assumptions together with one or several conclusions following from the assumptions. Consider the following logical argument consisting of two assumptions and one conclusion.*
>
> (A1) *: Human beings are mortal,*
> (A2) *: Socrates is a human being,*
> (C) *: Socrates is mortal.*
>
> *In mathematics, theorems are mathematical results that contain a set of assumptions together with a set of conclusions following from those assumptions. Proofs, in turn, are the logical steps or calculations which take us from the assumptions to the conclusions.*

By explaining this analogy between logical reasoning and theorems and proofs, the instructor avoids unclear communication with the students by being aware of the curse of knowledge. Furthermore, as an instructor I proceeded by explaining that theorems are immensely powerful tools for analysing, say, mathematical models of physical systems which leads us to the second essential principle.

A crucial component of clear communication and successful learning is to *give context before content*. Neurologically speaking, we learn new knowledge by associating it to knowledge that is already stored in memory [10]. As an instructor, it is therefore helpful for students to explain why a certain topic is relevant before presenting the topic itself. It is difficult to overstate the importance of this principle, and I used it extensively throughout my lectures. Let's consider a concrete example of this principle at work from the calculus course.

A topic within single variable calculus is that of limits at infinity. Normally, an instructor would just begin by presenting an informal definition of limits at infinity akin to the one presented in Definition 3 on page 73 in [4]. If f is a function, x is the independent variable and a and L are two numbers then an informal definition of a limit at infinity is given by the following excerpt from [4].

> *If f is a function that is defined on an interval (a, ∞) for some $a > 0$ and if we can ensure that $f(x)$ is as close to the number L we want by taking x large enough then we say that $f(x)$ approaches the limit L as x approaches infinity and we write*
>
> $$\lim_{x \to \infty} f(x) = L\,."$$

Now the question is, how can we give students some context before presenting the informal definition of limits at infinity above? As an instructor, my answer was to present the following real-life example which all students can relate to.

> *Suppose that we cultivate bacteria in a flask containing a limited amount of sugar. Then, the size of the population of bacteria will increase over time initially as the*

bacteria consume sugar, and eventually the population size will reach a plateau when all of the sugar has been consumed. Given this situation, suppose that the size of the initial population at time $t = 0$ *is given by* $N_0 > 0$ *number of bacteria. Also, suppose that the growth rate measured in "per minute" is given by the number* $r > 0$ *and that the flask has a carrying capacity given by the number* $K > 0$*, which is the maximum population size restricted by the availability of sugar in the flask and the volume of the flask itself. Then, the growth of the population over time is described by the function*

$$N(t) = \frac{K}{1 + \left(\frac{K - N_0}{N_0}\right) e^{-rt}},$$

where $N(t)$ *is the number of bacteria at time* t*. Considering the above equation, we ask ourselves how large will the population be after "a very long time"? In other words, what is* $N(t)$ *when time goes to infinity? In fact, we have the following limit*

$$\lim_{t \to \infty} N(t) = K,$$

stating that the population size approaches the carrying capacity K *when time goes to infinity. It is often of interest to know the so-called limits at infinity.*

Another advantage of giving context before content is that the topics of concern are presented in a different way during lectures compared to how they are presented in the course book. This implies that students must make an effort to relate the information given during lectures to the relevant text in the course book, and this is known to be an efficient method of learning [10]. Next, we discuss a method related to giving context before content in order to promote student engagement.

3.3 Inspiration as a catalyst for student engagement

When I was sixteen years old, I read the book "The selfish gene" by the evolutionary biologist Richard Dawkins [12] that made me fall in love with science. This beautifully written masterpiece gave me an immense hunger to learn and appreciate the beauty of scientific and philosophical thought. Therefore, I believe that inspiration can be a powerful catalyst for student engagement, and this is especially true when an inspiring topic provides context to the course content.

On this basis, I decided to make one of the scheduled lectures in the calculus course into a so-called inspiration lecture. Essentially, this was a popular science presentation about some of the research projects and articles on which I have worked as a researcher in mathematical biology. Importantly, after each topic of the popular science presentation, I listed all mathematical concepts of the calculus course that were related to the research topic of interest. Moreover, a great deal of emphasis was placed on discussing big picture ideas and fundamental research questions in applied mathematics. Thus, the two goals of the inspiration lecture were to show that the mathematics the students learned in the course is useful and that scientific research is deeply interesting and thought provoking.

Until now, we have discussed the cognitive-science aspects of active learning that were implemented in the course. Next, we present the psychological aspects of active learning that we accounted for using findings from relationship science.

3.4 Promoting cycles of positivity in teaching

In management, an effective way for a leader to ensure participation, learning effectiveness, and productivity in a team is to cultivate psychological safety in the work environment [13]. In workplaces pervaded by psychological safety, employees feel safe to speak up, to learn by making mistakes, and to contribute by being engaged at work. Profound levels of psychological safety are products of good relationships between leader and employees, and similarly, a necessity for psychological safety in the classroom is a good relationship between teacher and students. This is especially true in the context of teaching children, a sentiment captured by the following quote by the clinical psychologist Gordon Neufeld and the physician Gabor Maté [14].

> *On close scrutiny, four essential qualities are primary in determining a child's teachability: a natural curiosity, an integrative mind, an ability to benefit from correction, and a relationship with the teacher.*

Admittedly, university students are not children, but the relationship between instructor and students is still paramount at this level as well. In my experience, this central relationship is often overlooked in a large-lecture setting, so I set out to find a strategy to build good teacher-student relationships in my calculus course. To achieve this aim, we answer the following question: How can we establish good instructor-student relationships in a large-lecture setting?

In the realm of marriages, the science of healthy relationships reveals the defining features of stable marriages. These features were elucidated as a result of the pioneering work of psychologist John Gottman. In the early 1970s, Gottman thought that the psychological study of relationships between spouses was relatively unempirical at that point in time. To render this research field more data-driven, he set up his research laboratory where his team interviewed married couples, and after two decades, his work "reaped enormous reward" [15]. During the interviews of spouses, Gottman's associates measured physiological responses of the spouses as well as their heart rates, and they videotaped the interviews, enabling the researchers to study microexpressions during arguments, for example. After hundreds of hours of interviews, the researchers had collected vast amounts of data. What is especially appealing with Gottman's work to a person that is fond of mathematical beauty is that the researchers distilled all of these data into a simple, elegant and counterintuitive mathematical formula with a high predictive accuracy.

The formula for predicting whether a marriage is stable or unstable is known as *Gottman's five to one ratio*. This magic ratio states that in stable marriages there are at least five positive interactions for every negative interaction [15], and in unstable marriages, conversely, there are fewer than five positive interactions for every negative interaction. Using this five-to-one ratio, Gottman and his team could predict with 90% precision which couples would stay together after four years, namely the couples in stable marriages, and which couples would divorce each other, namely the couples in unstable marriages [15]. Gottman's five-to-one ratio was surprising at the time it was proposed as it allows for many different relationship styles to form stable marriages. For example, both volatile couples with a high frequency of conflicts on the one extreme and avoidant couples who avoid arguments on the other, can both form stable marriages as long as

they satisfy the magic five-to-one ratio [15]. Furthermore, Gottman's findings reveal that positivity breeds positivity and negativity breeds negativity in marital relationships.

Interactions between spouses in stable marriages are characterised by cycles of positivity. This implies that positive interactions such as validation, compliments, or expressions of empathy generate more positive interactions. Similarly, interactions between spouses in unstable couples are characterised by cycles of negativity. Unstable couples often find themselves in a downward spiral defined by what Gottman describes as the *four horsemen of the apocalypse* where *criticism* leads to *contempt* which leads to *defensiveness* which leads to *stonewalling* [15]. Better still, Gottman's findings are not limited to marriages.

The five-to-one ratio also defines stable and unstable relationships between parents and their children. The psychologist Martin Forster has adapted Gottman's framework to the context of parenting, and in his book[1], which title translates to "Five times more love" [16], he gives advice on how to break cycles of negativity and replace them by cycles of positivity. On account of the fact that stable relationships between spouses as well as between parents and their children satisfy the five-to-one ratio, it is not far fetched to assume that the same mechanisms characterise stable relationships between teachers and students, in general, and lecturers and students in a large-lecture setting, in particular. In light of these findings, we postulate that active learning is indirectly promoted by creating cycles of positivity in class. This is due to the fact that stable relationships between the instructor and the students yield secure students experiencing high levels of psychological safety, which, in turn, leads to high levels of student engagement, the very hallmark of active learning. Vice versa, we postulate that student engagement is reduced in classes characterised by cycles of negativity, which leads to a discussion about the calculus courses taught the previous years by means of traditional lectures.

Unfortunately, I argue that the calculus course in previous years, possibly going back decades, was characterised by cycles of negativity. Many students did not enjoy the teaching in previous years and expressed sentiments such as that "the instructor never smiles" in the course surveys. This kind of negativity is not a recent occurrence as I even remember one time when I was a student in the same course decades ago, when a classmate of mine asked the lecturer a question during one of the first lectures of the course. The lecturer scoffed "That's absurd!" in reply. After that, student engagement was minimal for the remainder of the course. The same negativity was reciprocated by previous lecturers I talked to before the course, who were discontent with student engagement. As part of this cycle of negativity, the lecturers resisted from uploading lecture notes to the course website in fear that students would stop attending lectures. A lecturer even told me that despite the construction of easier final exams in the course so that more of the students could pass, the proportion of students failing the calculus course was still close to half of all students taking the final exam. Provided this negative environment in class, a major goal of mine was to break cycles of negativity and instead foster positive instructor-student relationships.

An important strategy to improve student engagement in class is to promote cycles of positivity. Four effective ways for the instructor to achieve this are compliments, creating a sense of we-ness, general declarations of goodwill towards the students, and

[1] The title of Martin Forster's book is "Fem gånger mer kärlek" in Swedish, and to my knowledge it has not been translated into English.

expressions of empathy with the students. Regarding compliments, if a student asks a question containing a mathematical error, instead of correcting the error immediately, we as instructors can compliment the effort or the fact that the student asked a question in the first place. A sense of we-ness is often linked to cycles of positivity, whereas in classes plagued by cycles of negativity there is a clear sense of two teams: instructor against the students. Having open discussions about the implementation of active learning and its principles in class is not only beneficial to learning [10], but also instils the feeling that this is *our course* rather than the course of the students. Plainly stating that "we are in this together" and "your success is my success" helps in this regard as well. Another example of positivity is declarations of goodwill towards the students. Some phrases I used in the calculus course were "I believe in you" and "I want you to succeed". Lastly, expressing empathy is an excellent way to build a stable teacher-student relationship. Early in the course, I noticed that many students were struggling to keep up with the fast pace of university studies, so I told them that I had quite literally sat in the same chairs that they are sitting in right now as a 19-year-old student, that this was also my first university level course, and that the pace at university is much higher than what they are used to from classes in school. Phrases such as "I can see that you are doing your best and that the studies are stressful, but we will manage this together. So keep up the good work." can make a huge difference in this regard. If the instructor radiates positivity, then the students will reciprocate, and the conditions in such classes are exceedingly conducive to facilitate active learning. Since most undergraduate calculus courses cover many topics, there is not always sufficient time to discuss active learning during lectures, and therefore, we used another complementary methodology to instil positivity in the teaching of the calculus course.

3.5 Promoting active learning through nudges

An effective strategy to encourage student engagement is that of nudging. Essentially, by sending motivational emails to students explaining the benefits of and theory behind active learning, students are more likely to engage in the course. Tailored nudges in undergraduate courses have been shown to improve assignment scores by approximately 3% and are most effective for students with low academic performance [17].

Due to limited time resources, I took the decision not to use tailored nudges, i.e. individual emails to each student, but instead use general nudges corresponding to mass e-mails. Each Friday afternoon during the nine weeks of the course when lectures and demonstration classes were given (the tenth week, and also he last, was exam week), I sent out a mass email to all students in the course. These general nudges contained two major elements. A summary of the topics covered in class during the past week and motivation for student engagement. The latter element discussed study techniques, in general, and the theory behind active learning, in particular. In terms of study techniques, I discussed the importance of taking charge of your own studies, which is arguably the most crucial method for successful learning [10]. Early in the course I discovered that only around 15 students answered the preclass quizzes. To increase the response rate, I explained the importance of taking these preclass quizzes in class, and then I started sending out mass emails the day before each lecture where the students were encouraged to take the preclass quizzes for tomorrow's lecture. This increased the response rate to between 50 and 70 students, so roughly a third of the students took the

preclass quizzes. We discuss this problem of low participation in the preclass quizzes in further detail in the Discussion. In addition, every weekly summary sent to the students contained motivational statements such as "Keep up the good work".

In summary, the components of the implementation of active learning in this work fall into two categories: cognitive science-related components and psychological components. The cognitive science components include interactive learning meaning preclass quizzes and in-class problem solving plus feedback using Mentimeter, demonstrations accounting for the curse of knowledge and giving context before content, and the inclusion of a popular science-style inspiration lecture in the course. Psychological components include the promotion of cycles of positivity in class and motivational nudges. Provided this arsenal of educational interventions, we will later compare the learning outcomes of the students engaged in active learning in 2024 to the learning outcomes of the students taught by means of traditional lectures the previous two years. To make sure that the comparison between learning outcomes of the students in the different cohorts was fair, the final exam was designed to ensure comparable levels of difficulty.

3.6 Ensuring comparable levels of difficulty for the final exams

Since the main metric for the learning outcomes of the students is the results on the final exam, attention was put on constructing a final exam in 2024 (AL) with comparable levels of difficulty to the previous two years. In fact, I would argue that the final exam in 2024 (AL) was slightly more difficult than in 2022 (TL) and 2023 (TL) as the final exam of the course was deliberately made easier relative to the re-exams in the courses these years. The re-exams in the course which are potentially taken after the final exam, according to this line of reasoning, should be more difficult compared to the final exam taken at the end of the course since the students taking the re-exams have had more time to study for the re-exams compared to the students only taking the final exam. Therefore, I noted, which many students agreed with, a substantial *increase* in the difficulty levels when you compared the final exams in 2022 (TL) and 2023 (TL) to the re-exams the same years.

A notable example was the inclusion of questions requiring limited calculations or reasoning on the final exams from 2022 (TL) and 2023 (TL). Such exam questions were worth disproportionally many points and required almost no calculations or reasoning but simply knowledge of certain terms. A typical example of what such an exam question from 2022 (TL) and 2023 (TL) could look like is the following.

Consider the line

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \quad t \in \mathbb{R},$$

on vector parametric form. What is the position vector and the direction vector?

For any students that know the terms "position vector" and "direction vector", the solution to this task is simply a matter of finding the correct vectors in the equation above.

In the implementation of the course in 2024 (AL), I did not include exam questions requiring no calculations on the final exam and thus the final exam included only tasks where computations and reasoning were required to find the correct answers. Thus the final exam in 2024 (AL) was much closer to a re-exam the previous years in terms of difficulty level, which is a topic I discussed with the students during the course. And many

of the students even wrote in the course survey that they did find the final exam in 2024 (AL) more difficult compared to the original exams the previous years.

Given a final exam in 2024 (AL) of sufficient level of difficulty, a special attention to the marking of the final exams in 2024 (AL) was also paid to make the comparison of learning outcomes between years fair.

3.7 Avoiding the Halo effect when marking exams

To mark the exams fairly, we made sure to be aware of the so-called *Halo Effect* described by Daniel Kahneman. Basically, there are two approaches in which a pile of written student exams can be marked. The first approach is to correct "one student at a time" which means that we correct the answer to question 1 of student 1 followed by the answer to question 2 and so on before correcting the final answer of question 8 of student 1. Then, we proceed by correcting the answers of student 2 until the exams of all students have been corrected. The second approach is to correct "one question at a time" meaning that you first correct all answers to question 1, then all answers to question 2 and so on. It might seem counterintuitive, but Daniel Kahneman explains that the second approach, which is the one we implemented in the case of the calculus course of interest, is more fair than the first approach. The reason why correcting exams for one student at a time is unfair is due to the Halo effect where the person marking the exams will be biased towards students who performed well on the initial questions of the exam and biased against the students that performed poorly on the initial questions of the exam [18].

Therefore, the final exams of the active learning students were marked using the "one question at a time" approach. Technically, I marked the solutions to five of the tasks on the exam and the remaining three questions were marked by another teaching assistant in the course. As mentioned in the previous section, the difficulty level of the final exam was slightly increased by removing questions where no calculations or reasoning were required.

Having ensured comparable levels of difficulty on the final exams in 2022 (TL), 2023 (TL) and 2024 (AL), we next compare the learning outcomes of the students between the various years.

4 Results

Here, we answer four questions. What were the prerequisite skills of the students engaged in active learning compared to the students taught by means of traditional lectures in previous years? What was the performance of the students engaged in active learning compared to the performance of the students attending traditional lectures in previous years? How satisfied were the students with the teaching using active learning strategies compared to the students attending traditional lectures? What were the opinions of the students in 2024 about the various elements of active learning that were introduced in the calculus course?

4.1 Average scores on diagnostic tests reveal similar prerequisite skills for students taught with and without elements of active learning

To measure prerequisite skills, we consider test scores on diagnostic tests before the calculus course started. The data from these diagnostic tests are the average test scores of students in various undergraduate programs at Chalmers University of Technology in

Sweden. Specifically, we are interested in the three undergraduate programs biotechnology (Bt), chemical engineering (K) and chemical engineering with engineering physics (Kf) since the students in the calculus course of interest are enrolled in these undergraduate programs.

No other data are collected, such as the number of students taking the diagnostic tests, with the exception of the year 2024 when I personally corrected the diagnostic tests. Since the number of students taking the diagnostic tests and the distribution of these students among the three undergraduate students are roughly constant over time, these numbers from 2024 give us an indication of these data from previous years. In 2024, the numbers of students taking the diagnostic tests from the three respective undergraduate programs were 66 from Bt, 67 from K and 30 from Kf. In addition, the diagnostic tests in 2022, 2023 and 2024 were different, whereas the diagnostic tests in 2014 and 2024 were identical.

The average scores on diagnostic tests in 2023 and 2024 are approximately the same, while the test scores from 2022 are slightly worse in comparison (Fig. 1). In general, the scores on the diagnostic test in 2014 were better compared to the corresponding test scores on the same test in 2024 (Fig. 1). However, the differences in test scores on these diagnostic tests are relatively small. Based on these data, we conclude that the

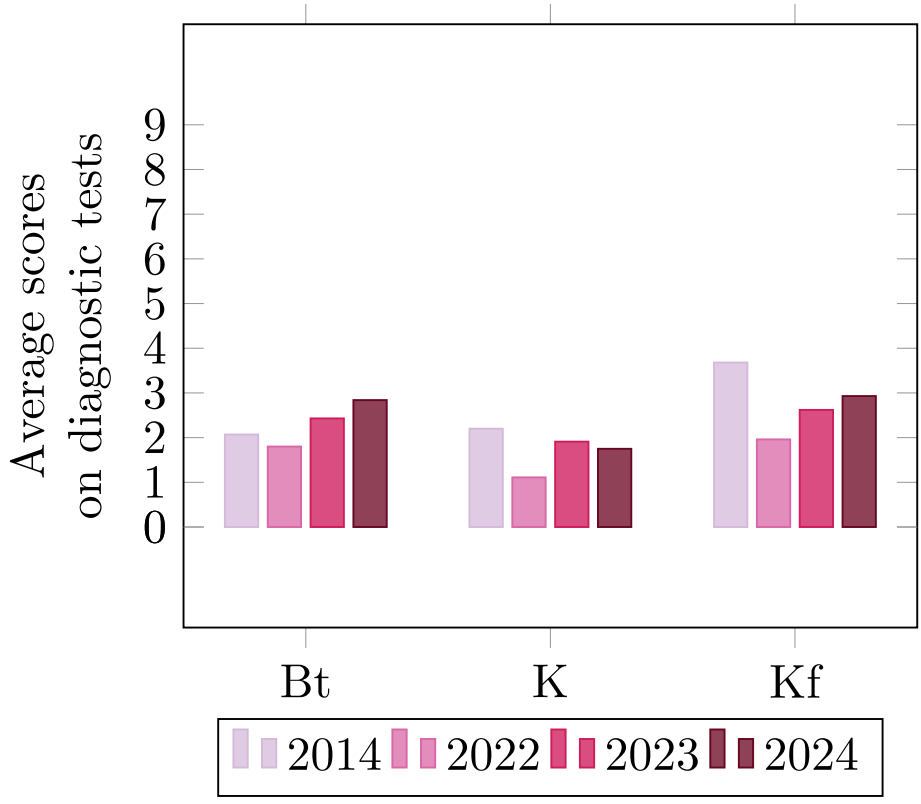

Fig. 1 Average test scores on diagnostic tests show that prerequisite skills do not vary significantly over time. The average test scores on the diagnostic tests for the undergraduate programs biotechnology (bt), chemical engineering (K) and chemical engineering with engineering physics (kf) are presented for the years 2014, 2022, 2023 and 2024. The diagnostics tests were identical in 2014 and 2024 and the diagnostic tests in 2022, 2023 and 2024 were different. The average test scores from 2014 were 2.07 for Bt, 2.20 for K and 3.68 for Kf. The average test scores from 2022 were 1.80 for Bt, 1.11 for K and 1.96 for Kf. The average test scores from 2023 were 2.43 for Bt, 1.91 for K and 2.62 for Kf. The average test scores from 2024 were 2.84 for Bt, 1.75 for K and 2.93 for Kf. The maximum number of points on the diagnostic tests is 9 points

Table 1 Final student grades in 2022(TL), 2023(TL) and 2024(AL). The number of students in the calculus course getting a specific final grade is presented. The four grades in the course are U ("fail"), 3, 4, 5 which are mainly determined by the test score on the final exam in the undergraduate calculus course

Grade	2022(TL)	2023(TL)	2024(AL)
U	66	71	59
3	53	48	52
4	26	36	47
5	12	7	26

Table 2 The final grades of students engaged in active learning (AL) are significantly better than those of students attending traditional lectures (TL). Using the contingency table in and various subtables constructed by considering two columns at a time, the statistical hypothesis that the final student grades (i.e. the rows) are independent of the year the course was given (i.e. the columns) was tested using a χ^2 test of independence. The *p*-values from these χ^2 tests of independence determine the statistical significance, and a *p*-value satisfying $p < 0.05$ means that the null hypothesis that rows and columns are independent is rejected

Years the calculus course was given	p-value	*Statistically significant?* (Yes, null hypothesis rejected if $p < 0.05$)
2022(TL), 2023(TL), 2024(AL)	0.00795	Yes
2022(TL), 2023(TL)	0.35028	No
2022(TL), 2024(AL)	0.02309	Yes
2023(TL), 2024(AL)	0.00638	Yes

prerequisite skills of the students taking the calculus course in 2024 were comparable to those of the corresponding students in previous years.

4.2 Final student grades reveal significant improvements in performance of students engaged in active learning

To compare the performance of teaching promoting active learning (AL) and traditional lectures (TL), we consider a contingency table of the final grades of the students in the undergraduate calculus course (Table 1). The number of students who took the final exam was 157 in 2022 (TL), 162 in 2023 (TL), and 184 in 2024 (AL), which, in itself, is an indicator that student engagement was higher among students engaged in active learning (AL) compared to students attending traditional lectures (TL). This is due to the fact that more students took the final exam in 2024 (AL), implying that fewer students, relative to previous years, had given up before the final exam.

From the contingency table, it is also clear that there were fewer students who failed the course, i.e. got the grade U, and more students who got the higher grades 4 and 5 in 2024 (AL), compared to the previous two years when students attended traditional lectures (TL).

Given these data, we ask ourselves whether the final student grades are dependent on or independent of the year the course was given. To this end, we conducted χ^2 independence tests on the categorical data of the final student grades (Table 2) which test the null hypothesis that the final student grades are independent of the year the course was given. The final grades of the students and the year the course was given are independent if the corresponding *p*-value satisfies $p > 0.05$. Vice versa, if $p < 0.05$, then the final student grades depend significantly on the year the course was given, and thus the

teaching methodology in terms of active learning (AL) strategies or traditional lectures (TL). These tests were based on the full contingency table containing data of final student grades in 2022 (TL), 2023 (TL), and 2024 (AL), as well as on sub-contingency tables where the final student grades from two different years are compared pairwise (i.e., 2022 (TL) compared to 2023 (TL), 2022 (TL) compared to 2024 (AL), and 2023 (TL) compared to 2024 (AL)).

The outcome of these statistical tests indicates that students engaged in active learning have significantly better final grades compared to students who attended traditional lectures (Tables 1 and 2). By converting the number of students in the contingency table of final grades (Table 1) to percentages of students receiving a certain grade a specific year, we visualise the statistically significant improvement in final grades for students engaged in active learning compared to students who attended traditional lectures (Fig. 2). The conclusion is that students engaged in active learning in 2024 (AL) perform significantly better on the final exam in the undergraduate calculus course of interest than students who attended traditional lectures (TL) in previous years.

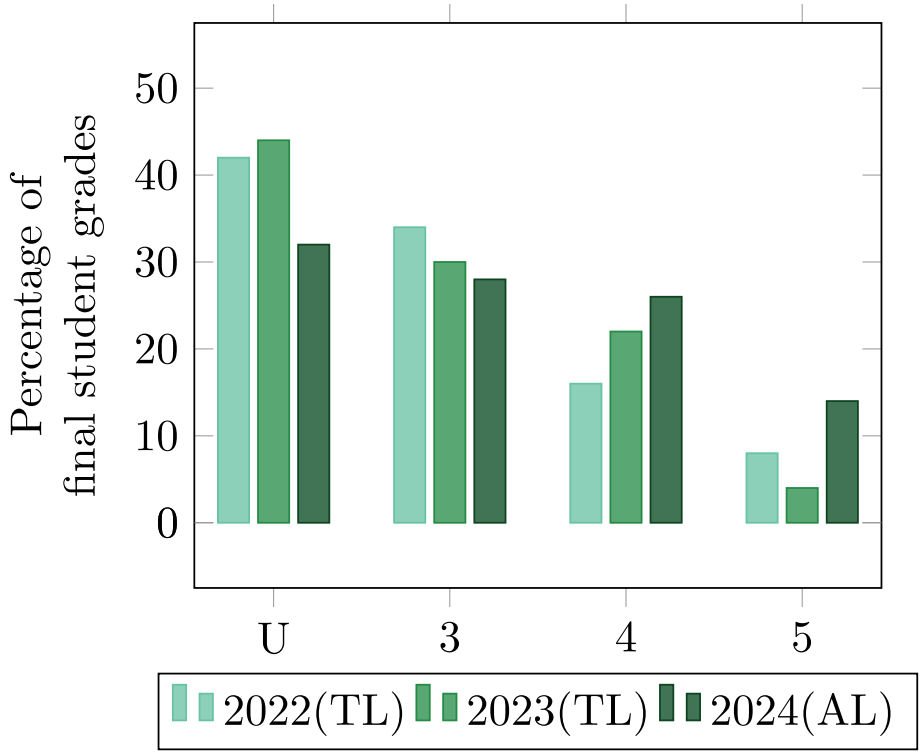

Fig. 2 The final student grades in the undergraduate calculus course of students engaged in active learning (AL) are significantly better than the final grades of students who attended traditional lectures (TL). The number of students writing the final exam in the calculus course who attended TL in 2022 and 2023 were 152 and 167, respectively, and the corresponding number of students engaged in AL in 2024 was 184. The distribution of final grades in terms of percentage of students writing the final exam in 2022 were U ("fail"): 42%, 3: 34%, 4: 16% and 5: 8%. The distribution of final grades in terms of percentage of students writing the final exam in 2023 were U ("fail"): 44%, 3: 30%, 4: 22% and 5: 4%. The distribution of final grades in terms of percentage of students writing the final exam in 2024 were U ("fail"): 32%, 3: 28%, 4: 26% and 5: 14%. The illustrated data are also presented in a contingency table (Table 1)

4.3 Grades by students in course surveys reveal significant improvements in student satisfaction for students engaged in active learning

To quantify student satisfaction, we analyse the answers in the course surveys taken by students after the undergraduate calculus course of interest. In these course surveys, students grade statements or questions about various aspects of the course on a scale from 1 to 5, where the grade 1 means "completely disagree" or "very bad" and the grade 5 means "completely agree" or "very good". Here, we have selected grades in course surveys from three categories, namely learning, teaching, and the overall impression of the students. The number of respondents to the course survey was 88 students in 2022 (TL), 101 students in 2023 (TL), and 123 students in 2024 (AL), but in some cases students have not answered certain questions, and thus there are sometimes fewer grades of certain statements in a given course survey than the number of respondents. The number of respondents to these course surveys suggests that student engagement was better in 2024 (AL) compared to the previous two years, since more students answered the course survey this year. Consequently, students engaged in active learning were more willing to express their opinions about the course compared to their counterparts attending traditional lectures.

Moreover, in 2024 when active learning was promoted, there were few grade 1:s and grade 2:s given by students in the corresponding course survey and this, in turn, renders the statistical χ^2 test of independence on all of these categorical data inaccurate[2]. To this end, we have merged grades 1, 2 and 3 into one category referred to as "grade 1, 2 or 3", and next we analyse contingency tables of grades given by students in the course surveys from 2022 (TL), 2023 (TL) and 2024 (AL), respectively.

In the learning category, students graded the statement "*the course structure (as divided into lectures, exercises, lab sessions, simulations etc.) is appropriate in order to reach the intended learning outcome of the course*" (Table 3). The average grades of this statement by students in 2022 (TL), 2023 (TL), and 2024 (AL) were 3.67, 3.78 and 4.60, respectively, demonstrating that students engaged in active learning were more satisfied with their learning in the calculus course compared to students attending traditional lectures. In fact, hypothesis testing reveals that differences in student satisfaction with learning in 2024 (AL) compared to the previous two years are statistically significant (Table 4).

Concerning teaching in the undergraduate calculus course of interest, students graded the statement "*the teaching worked well*" (Table 5). The average grades by students in 2022 (TL), 2023 (TL) and 2024 (AL) were 3.01, 3.49 and 4.60, respectively, demonstrating that students engaged in active learning were more satisfied with the teaching in the

Table 3 Grades of students in the learning category of course surveys from 2022(TL), 2023(TL) and 2024(AL). The students gave a grade from 1 ("completely disagree") to 5 ("completely agree") to the statement "the course structure (as divided into lectures, exercises, lab sessions, simulations etc.) is appropriate in order to reach the intended learning outcome of the course"

Grade	2022(TL)	2023(TL)	2024(AL)
1,2 or 3	33	34	6
4	35	42	31
5	20	24	85

[2] The rule of thumb for when one can use the χ^2 test of independence is that the smallest expected frequency of any value in the contingency table should be larger than 5. By creating the category "grade 1, 2 or 3", the resulting contingency tables satisfy this rule.

Table 4 The grades in the learning category of course surveys given by students engaged in active learning (AL) are significantly better than those given by students attending traditional lectures (TL). Using the contingency table in Table 3 and various subtables constructed by considering two columns at a time, the statistical hypothesis that the grades in course surveys (i.e. the rows) are independent of the year the course was given (i.e. the columns) was tested using a χ^2 test of independence. The *p*-values from these χ^2 tests of independence determine the statistical significance, and a *p*-value satisfying $p < 0.05$ means that the null hypothesis that rows and columns are independent is rejected

Years the calculus course was given	p-value	*Statistically significant?* (Yes, null hypothesis rejected if $p < 0.05$)
2022(TL), 2023(TL), 2024(AL)	$4.85 \cdot 10^{-15}$	Yes
2022(TL), 2023(TL)	0.883	No
2022(TL), 2024(AL)	$1.08 \cdot 10^{-12}$	Yes
2023(TL), 2024(AL)	$2.14 \cdot 10^{-12}$	Yes

Table 5 Grades of students in the teaching category of course surveys from 2022(TL), 2023(TL) and 2024(AL). The students gave a grade from 1 ("completely disagree") to 5 ("completely agree") to the statement "the teaching worked well"

Grade	2022(TL)	2023(TL)	2024(AL)
1,2 or 3	60	45	9
4	21	45	23
5	7	11	89

Table 6 The grades in the teaching category of course surveys given by students engaged in active learning (AL) are significantly better than those given by students attending traditional lectures (TL). Using the contingency table in Table 5 and various subtables constructed by considering two columns at a time, the statistical hypothesis that the grades in course surveys (i.e. the rows) are independent of the year the course was given (i.e. the columns) was tested using a χ^2 test of independence. The *p*-values from these χ^2 tests of independence determine the statistical significance, and a *p*-value satisfying $p < 0.05$ means that the null hypothesis that rows and columns are independent is rejected

Years the calculus course was given	p-value	*Statistically significant?* (Yes, null hypothesis rejected if $p < 0.05$)
2022(TL), 2023(TL), 2024(AL)	$1.86 \cdot 10^{-32}$	Yes
2022(TL), 2023(TL)	0.00426	Yes
2022(TL), 2024(AL)	$1.40 \cdot 10^{-23}$	Yes
2023(TL), 2024(AL)	$1.83 \cdot 10^{-20}$	Yes

calculus course compared to students attending traditional lectures. Hypothesis testing reveals that the differences in student satisfaction with teaching in 2024 (AL) compared to the previous two years are statistically significant (Table 6). Interestingly enough, the difference in student satisfaction with teaching between the courses given in 2022 (TL) and 2023 (TL) is also statistically significant (Table 6), and thus the average grades in combination with this statistically significant finding indicate that the students were significantly less satisfied with the teaching in the calculus course in 2022 (TL) compared to the counterpart in 2023 (TL).

Finally, the students graded the question "*what is your overall impression of the course?*" (Tables 7 and 8). The average grades by students in 2022 (TL), 2023 (TL) and

Table 7 Grades of students in the overall impression category of course surveys from 2022(TL), 2023(TL) and 2024(AL). The students gave a grade from 1 ("very bad") to 5 ("very good") to the question "what is your overall impression of the course?"

Grade	2022(TL)	2023(TL)	2024(AL)
1,2 or 3	45	47	8
4	35	46	26
5	8	8	89

Table 8 The grades in the overall impression category of course surveys given by students engaged in active learning (AL) are significantly better than those given by students attending traditional lectures (TL). Using the contingency table in Table 7 and various subtables constructed by considering two columns at a time, the statistical hypothesis that the grades in course surveys (i.e. the rows) are independent of the year the course was given (i.e. the columns) was tested using a χ^2 test of independence. The *p*-values from these χ^2 tests of independence determine the statistical significance, and a *p*-value satisfying $p < 0.05$ means that the null hypothesis that rows and columns are independent is rejected

Years the calculus course was given	p-value	*Statistically significant?* (Yes, null hypothesis rejected if $p < 0.05$)
2022(TL), 2023(TL), 2024(AL)	$7.23 \cdot 10^{-30}$	Yes
2022(TL), 2023(TL)	0.724	No
2022(TL), 2024(AL)	$1.35 \cdot 10^{-20}$	Yes
2023(TL), 2024(AL)	$2.30 \cdot 10^{-22}$	Yes

2024 (AL) were 3.31, 3.56 and 4.60, respectively, demonstrating that students engaged in active learning had a better overall impression of the calculus course compared to students attending traditional lectures previous years. Again, hypothesis testing reveals that the differences in overall impression in 2024 (AL) compared to the previous two years are statistically significant (Table 6).

In total, course surveys reveal that the three aspects of student satisfaction that correspond to learning, teaching, and overall impression are significantly better among students in 2024 (AL) compared to those of 2022 (TL) and 2023 (TL). This is visually clear from observing the average grades by students in these three categories of course surveys (Fig. 3). Next, we consider the opinions of the students in 2024 on the active learning strategies implemented.

4.4 Comments in the course survey reveal that students appreciate the implemented active learning strategies

In addition to the standard questions that make up course surveys every year, I added specific questions about active learning in the course survey of 2024. The responses to these questions shed light on the opinions of students about the active learning strategies implemented.

The students' opinions about active learning were generally positive. Regarding the format of the lectures, 14 respondents (that is, 11%) were in favour of traditional lectures based entirely on handwritten notes on blackboards, 0 respondents wanted lectures based solely on slides and 114 respondents (that is, 93%) preferred a hybrid format alternating between slides and handwritten notes. Similarly, in response to the question of which method of learning the students preferred, 11 respondents (that is 9%) preferred inactive learning in the form of traditional lectures and 114 respondents (that is 95%)

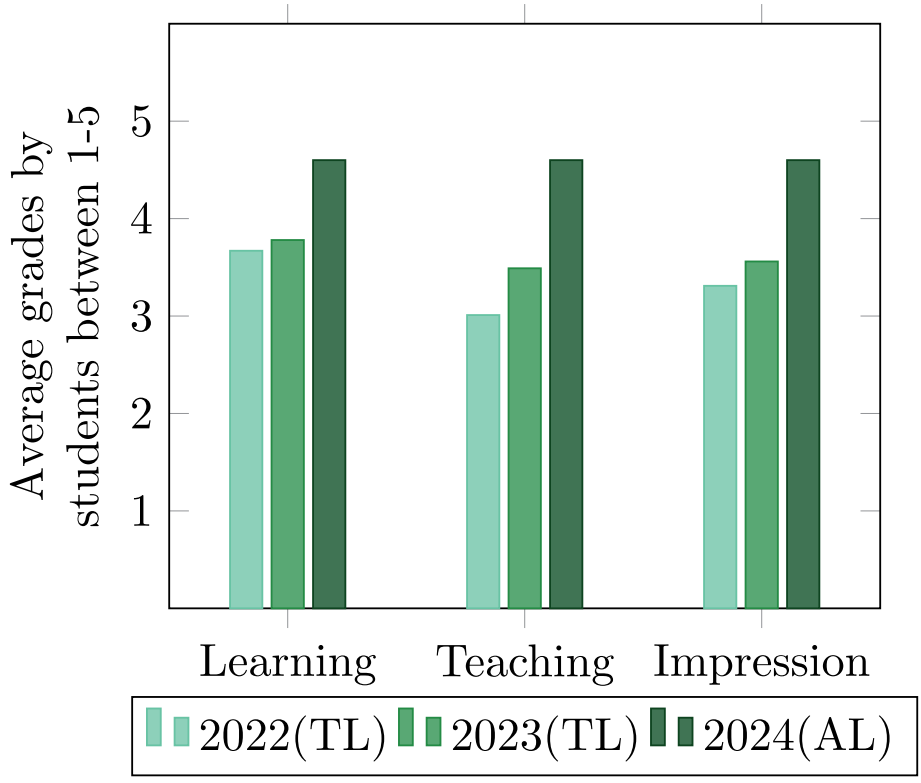

Fig. 3 *The student satisfaction is higher for students engaged in active learning compared to students attending traditional lectures*. In the course survey taken by the students after the undergraduate course, students evaluate different aspects of the course such as their learning, the teaching and their overall impression. Within these categories, statements or questions are graded from a scale from 1 to 5, where 1 means "completely disagree" or "very bad" and 5 means "completely agree" or "very good". In the learning category, the students gave the statement "the structure of the course (e.g. disposition in terms of lectures, exercises, laborations, simulations etc.) is appropriate in order to reach the learning objectives of the course" the following average grades: 3.67 in 2022 (TL), 3.78 in 2023 (TL) and 4.60 in 2024 (AL). In the teaching category, the students gave the statement "the teaching worked well" the following average grades: 3.01 in 2022 (TL), 3.49 in 2023 (TL) and 4.60 in 2024 (AL). Lastly, the question "what was your overall impression of the course?" was graded 3.31 in 2022 (TL), 3.56 in 2023 (TL) and 4.60 in 2024 (AL). The number of respondents of the course survey was 88 students in 2022 (TL), 101 students in 2023 (TL) and 123 students in 2024 (AL)

preferred active learning. Thus, the students were overwhelmingly in favour of hybrid lectures incorporating active learning strategies when the alternative is traditional lectures. The comments of these students on the implemented active learning strategies (pre-class quizzes and in-class problem solving followed by instructor feedback) were very illuminating, and two illustrative positive comments were the following.

> *I think active learning is the future. To answer quizzes and read before and then to actually be able to discuss the mathematics during the lecture does not only create a clearer understanding, but it also makes it much more fun. It also becomes easier to establish social connections since you are often sitting in different groups during lectures and thus you get to discuss with different people.*

> *I thought it was really good, I especially appreciated the short quizzes that were uploaded before each lecture (and that you were reminded several times to take them). And being able to think/reflect on questions during lectures before the solutions were presented (either together with someone sitting next to you or by yourself)*

improved your understanding of the presented solution a lot. So that was great. I would still have needed a little more time to think about each question.

There were also valuable comments with concrete suggestions on how to improve the hybrid format.

I thought the concept sounded promising but the excecution could have been better. It was stressful and hard to try to take notes while the slides were shown and at the same time try to understand what the content of the slides is. When it was time for questions, you did not always have enough time to think before the solution was demonstrated. It was not instructive that everyone could see all answers from the entire class and that many "followed" someone else rather than trying to solve the task on your own. If the slides had been uploaded before the lecture so that you could go through it and only focus on the remaining parts (such as answering questions and taking notes from the blackboard), it would have been more efficient. The notes would have been clearer and you would have been able to follow what was being said during the lectures more easily.

With more time to prepare, I would definitely be able to upload presentation slides before lectures and to think about how to implement in-class quizzes in a more effective way so as to make sure that students actually try to solve the task at hand rather than copying the answer from someone else.

In addition, the psychologically oriented learning strategies that were implemented proved crucial to promote student engagement. The cycles of positivity that permeated the lectures resulted in high levels of psychological safety for the individuals, which, in turn, promoted student engagement, as this quote from the course survey demonstrates.

You dared to ask more questions which made it easier for you to understand more of the course material than if you wouldn't have dared to ask. Also made you more focused as the lectures were more entertaining and you didn't zone out and missed material or understanding of what was being taught. Also, made it possible to "get to know" the lecturer better and therefore made you more motivated to pass the final exam because of that.

With regard to the use of general nudges in terms of weekly emails, the students were overwhelmingly optimistic. In response to the question whether the students had appreciated the weekly emails summarising the course content and giving encouragement to study, 120 students (that is, 98%) answered yes and 2 students (that is, 2%) answered no. Similarly, in response to the question whether the students appreciated the advice on study technique given in these emails as well as during lectures, 117 students (that is, 96%) answered yes and 5 students (that is, 4%) answered no.

Finally, the students were tangibly enthusiastic about the popular-science-style inspiration lecture about my previous research in mathematical biology. These two quotes demonstrate that the students gained inspiration and motivation from the inspiration lecture.

It was absolutely the best part of the whole course. To try to understand complicated concepts and to put together the pieces has its charm, but to actually get to see that the maths can be applied within other (and for me more interesting as I like biology

a lot) areas made a huge difference for my motivation especially during the studying right before the exam, but most of all for my own inspiration. It is so inspiring to see someone else's fire when they talk about something in the most engaged way possible. I am moreover really interested in taking in new information generally and, on the one hand, to be met with the same enthusiasm by the one who is giving the information, and, on the other, to see that that curiosity is not dampened despite long hours of studying and work. No matter the results in the course, Johannes's attitude to what he does and the joy he is spreading has rubbed off on my general philosophy of life.

I loved the inspiration lecture. It made me consider studying mathematics instead of chemistry

In addition to motivation, the inspiration lecture also gave students a valuable context for the course content, as these three quotes demonstrate.

The inspiration lecture was fun and a good opportunity to become motivated. It is easy to question why you take certain courses if you have not seen any application of the education you get. Therefore, I think inspiration lectures should be given more often.

I thought it was interesting and fun to get more perspective and to see how mathematics, actually, can be applied. It was fun to see the existing possibilities. I also experience that it helped my motivation to continue fighting even when it was hard. The dedication has made me so much more motivated, and the weekly emails too. It has given so much!

Think that every examiner should offer such a lecture! It gives a good perspective on why the things we learn are important and motivates you to understand!

Altogether, the implemented active learning strategies improved student performance and engagement in the calculus course of interest compared to previous years when it was taught by traditional lectures.

5 Discussion

A growing number of studies suggest that active learning improves student performance and engagement in a large lecture context. This work agrees with this sentiment, since both student performance measured by final grades (Fig. 2) and student engagement and satisfaction measured by course surveys (Fig. 3) were significantly better in 2024 when active learning strategies were introduced in the large-lecture undergraduate calculus course considered in this work compared to the previous two years when the same course was taught by traditional lectures. There is even more intangible evidence of improved learning conditions, as student attendance, a variable that was not considered in this work, was much higher in 2024 (AL) compared to previous years. Before the course, experienced colleagues told me that towards the end of the course close to exam week, student attendance will drop, and I was also told to expect a decrease in student attendance for all early classes starting at 8 a.m. in the morning. This did not happen during the calculus course in 2024, as the lecture halls were more or less full for the entirety of the course, including early lectures. Even more notable was the sound volume in the lecture halls, another variable that was not considered in this work, as the entire

room was buzzing during lectures due to student-student discussions and it was rarely quiet. As a lecturer, I was often interrupted, in the best sense possible, as students asked questions when they did not understand certain concepts or calculations, and they were also quick to point out errors that I made on the blackboard or in presentation slides. In addition, having given traditional lectures in mathematics before, I can testify to the fact that the hybrid format implemented in this course is not only more satisfactory for the students, but it is also much more enjoyable to teach this way for the instructor. Even better, it is straightforward to speculate on how to further improve the active learning strategies implemented in the calculus course.

Student performance and engagement will likely improve more by making the final course grade more cumulative. Currently, most of the course grade in the calculus and analytical geometry course in this work is based on the test score on the final exam. This final exam has a maximum score of 50 points, and students can gain two bonus points on the final exam by taking two voluntary online tests during the course. That means that merely 4% of the final grade can be determined by these smaller online tests meant to promote student participation during the course, while 96% to 100% of the final grade is determined by the test score on the final exam. All elements of the calculus course, such as attending lectures and demonstration classes, completing preclass quizzes, or taking online tests during the course, are completely voluntary. This makes the task of encouraging student participation extremely difficult, as many students will not participate in voluntary activities. In a recent and similar study of active learning in "calculus for life sciences courses", 20% of the final grade was determined by student participation during the course such as attending lectures and taking preclass quizzes and 80% of the final grade was determined by the test score on the final exam [3]. The reasoning of the authors behind this study is sound as no learning strategy can be effective unless students actually follow it. In my course, 20% of the final grade would correspond to 10 bonus points on the final exam, and a wise strategy to distribute these points will most probably lead to tangible improvements. For instance, the 10 bonus points could be distributed as follows: 4 bonus points for passing 4 online tests (implying that two more online tests are added), 3 points for taking all of the preclass quizzes and 3 points for writing weekly summaries of the course content of that particular week. These summaries would be a few paragraphs long and written individually by each student, where they summarise the course content in their own words. Such summaries would force students to engage in *reflection*, an essential component of successful learning [10]. As a concrete example of how making the course grade more cumulative would lead to improvements, at first around 10 to 15 students took the preclass quizzes before each lecture in my calculus course, and using frequent nudging, I managed to increase this number to between 50 to 70 students. Given that around 180 students took the final exam, the number of students that actually took the preclass quizzes amounts to merely a third of the whole class at best. If taking these preclass quizzes would result in extra bonus points, this would serve as an additional incentive for students to take them, which would, in all probability, increase both student engagement and ultimately performance.

Despite these suggested improvements, the general conclusion from this work and related studies is clear. Active learning in large classes is much more effective than traditional lectures. Nevertheless, active learning strategies in large undergraduate mathematics classes are generally not standard, and in my experience they are even

infrequently implemented. In fact, I have never taken a mathematics course at university where active learning strategies were implemented as the content of these courses was taught exclusively by traditional lectures, and this is true for mathematics courses at the bachelor/undergraduate, masters and PhD level. Likewise, during my 10 years as a PhD student and thereafter a postdoctoral researcher, I have never taught mathematics at university (with the exception of the calculus course described in this work) by means of active learning strategies, either as a teaching assistant or as a lecturer. In addition, I have taken two mandatory courses in university-level pedagogy at the University of Gothenburg in Sweden, and active learning constitutes a negligible part of the content of these courses. Importantly, I suspect this might partially be a European perspective, as many colleagues of mine from North America to whom I have spoken at international conferences have told me that active learning strategies in the teaching of university-level mathematics are routinely implemented in both Canada and the United States of America. In light of this observation, an arguably more interesting discussion to have is why are active learning strategies, in spite of their efficiency, so seldom implemented in undergraduate mathematics?

I argue that there are two main reasons behind the absence of active learning in large lecture mathematics classes. First, teaching at university is not valued as highly as research. In preparation for being the lecturer in the undergraduate calculus course described in this work, I asked one of my academic mentors and research collaborators for advice. They told me to "do the bare minimum, no one ever got a job from teaching". This opinion has permeated all of my experiences of teaching at Swedish universities, as most teaching assistants or lecturers teach because they have to and not because they are passionate about teaching. In my experience, there are no incentives to excel in teaching university-level mathematics, and therefore the same course material is taught by traditional lectures year after year by people who are "doing the bare minimum". Worse still, many researchers teaching mathematics at universities who want to perform well and who have even taken courses in pedagogy are simply unaware of the concept of active learning, and I used to be part of this group of researchers.

Second, many experienced mathematicians dislike active learning. The late psychologist and leading expert on the science of expertise focusing on deliberate practice by the name of K. Anders Ericsson discussed this reluctance of mathematicians to accept the efficiency of active learning strategies in his book "Peak: Secrets from the New Science of Expertise" co-written with the science author Robert Pool [7]. In relation to the seminal study of the large lecture class in undergraduate physics at the University of British Columbia [1] which served as the main inspiration for this work, Ericsson explained that mathematicians are particularly set in their ways and the only way to teach mathematics properly, according to them, is by using traditional lectures. But in 2016, Ericsson stated in his book that studies such as the one from the University of British Columbia are beginning to convince many mathematicians favouring traditional lectures of the efficiency of active learning strategies [7]. It is reassuring to know that this change of attitude has at least partially occurred in North American universities, but it has still not occurred in Swedish universities, where I have been active both as a student and a teacher. The resistance to pedagogical change among mathematicians that Ericsson described is something I can attest to first-hand during the calculus course described in this work. Previously, I described a senior lecturer with extensive teaching experience

who expressed strong reservations regarding active learning, and in my experience skepticism about active learning strategies among mathematicians is commonplace. For an outsider not steeped in academic culture, this might shed some light on why active learning is still not prevalent in undergraduate mathematics courses given at some departments.

Perhaps, this work can serve as motivation for other lecturers in undergraduate level mathematics to implement active learning strategies during their large-lecture courses. Provided the mounting evidence of the efficiency of active learning strategies, we, as instructors, owe it to students to implement such strategies as they deserve high-quality mathematics courses. Besides, more mathematically knowledgeable students from all disciplines can only benefit society as a whole, and this work is a stepping-stone towards this vision.

Supplementary Information

The online version contains supplementary material available at https://doi.org/10.1007/s44217-026-02354-4.

Supplementary Material 1

Supplementary Material 2

Supplementary Material 3

Supplementary Material 4

Supplementary Material 5

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Author contributions

J.G.B. did all of the work including writing the main manuscript, reviewing the manuscript and preparing the figures.

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Data availability

All student grades based on the test scores on the final exam and the grades of the students in the course evaluations are available in the data file "calculus_course_data_active_learning.xlsx" attached to this work. The fully anonymised course evaluations of the calculus and analytical geometry course without any student comments from 2022 (TL), 2023 (TL) and 2024 (AL) are available in the documents "MVE460_2022_redacted_no_links.pdf", "MVE460_2023_redacted_no_links.pdf" and "MVE461_2024_redacted_no_links.pdf", respectively. Based on correspondence with the data protection officer at Chalmers University of Technology, I decided not to make the course evaluation with all students' comments from 2024 (AL) available. The full course evaluation from 2024 (AL) with all students' comments is available upon request, but it is worth emphasising that all comments are written in Swedish.

Code availability

All statistical tests, that is, the χ^2 independence tests on the course data in the file "`calculus_course_data_active_learning.xlsx`", can be run using the open source Python script "`calculus_course_data_analysis_active_learning.py`" attached to this work.

Declarations

Ethical approval

The study was exempted from an ethical approval by Chalmers Institutional Ethical Advisory Board (IEAB) in accordance with the Swedish Ethics Review Act. All methods were performed in accordance with the relevant guidelines and regulations of the Chalmers Institutional Ethical Advisory Board (IEAB). The Head of the Department of Mathematical Sciences at Chalmers University of Technology issued the following signed decision (Dr.nr: MV 2025–0105) on 2025–11-11 presented in the quote below.

> **Decision (by Head of Department):**
>
> *Based on the project information provided by Johannes Borgqvist and the recommendation of the Chalmers Institutional Ethical Advisory Board (IEAB), I have determined that this study should not require an application to the Swedish Ethical Review Authority (Etikprövningsmyndigheten).*

Consent to participate

Chalmers University of Technology uses course surveys to gather student opinions as a guide for how courses can be improved. The policy regarding when students can access a course survey of Chalmers University of Technology is quoted below.

> *The survey is sent directly to your e-mail address given in Ladok on the Monday after the exam week of each study period.*

Students are informed about the purpose of course surveys beforehand which is, among other things, to use student comments to improve courses. Importantly, the course surveys are both *voluntary and anonymised*, and thus students make informed consent before answering course surveys.

Consent to publish

Not applicable (NA).

Competing interests

The authors declare no competing interests.

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