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ENERGY-OPTIMAL CONTROL OF BIPEDAL LOCOMOTION SYSTEMS

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Abstract. The mathematical statement of the problem of energy-optimal control for a bipedal locomotion system is given. The proposed statement of the problem is characterized by broad utilization of experimental data of normal human locomotion. It is done mainly by means of the mathematical formulation of the constraints imposed both on the phase coordinates and on the controlling stimuli of a system. A numerical method for the solution of optimal control problems for highly nonlinear and complex bipedal locomotion systems is proposed. The method is based on a special procedure of converting the initial optimal control problem into a standard nonlinear programming problem. This is made by an approximation of the independent variable functions using smoothing cubic splines and by the solution of inverse dynamics problem. The key features of the method are its high numerical effectiveness and the possibility to satisfy a lot of restrictions imposed on the phase coordinates of the system automatically and accurately. The proposed method is illustrated by computer simulation of the energy-optimal anthropomorphic motion of the bipedal walking robot over a horizontal surface.

Keywords: Bipedal locomotion system, energy-optimal control, smoothing cubic spline approximation, inverse dynamics, nonlinear programming problem.

1 Introduction

The problems of dynamics and control of bipedal locomotion systems (BLS) have been studied by many investigators. A major trend in the study of BLS is the creation of better mathematical models and their use in combination with more effective kinematic and dynamic analysis techniques to provide a more precise description of the system [1]. Traditionally, studies of the BLS have been concentrated on providing basic information that can be used in synthesis of artificial bipedal gait in order to design active exoskeleton [2-6], orthoses or prostheses [7-9]. This could help handicapped persons to restore their locomotor activity, and gives insight into building legged vehicles that can travel on unconventional terrain that is unsuitable for conventionally wheeled or crawler vehicles [10-12].

Many different models of BLS were proposed in the last years [1-21]. Among them the 3-D human musculoskeletal models [17, 18] look interesting but they are extremely complicated. The biomechanical model of the human body that is suitable for crashworthiness applications is described in papers [19, 20]. The simplest walking models [21] are also very important in helping to understand stability and control problems of the BLS.

Most researchers investigate the dynamical behavior and control laws of the BLS using the inverse, semi-inverse or direct dynamics approach. In recent years the interest in optimal processes of the BLS has increased remarkably [3, 6, 8, 9, 15-17, 22-30]. Probably one of the first attempts to formulate a principle of optimality for muscle-driven systems was the postulation of "a minimal principle" [22]. The authors assumed that an individual will always determine his motion so as to minimize the total "muscula effort", which is defined as the product of a constant and the square of the joint moment, and the time interval over which the minimization is to be carried out. It is, however, questionable whether muscular effort as defined above relates to any biological performance criterion at all [25].

One of the important contributions made to the optimization of BLS concerning human motion is a study presented in paper [24]. The authors employ a five-degree-of freedom model to describe the motion of the trunk, the thighs, and the shanks. Certain additional constraints are imposed on the model, and the total walking cycle is broken down into three phases: the stance phase, deploy phase and swing phase. The constrained optimization is then carried out and the optimal profiles of the joint torques as well as the optimal angular displacements of the system are obtained. Unfortunately, the paper [24] displays only the time functions of the limb angles as predicted by the model, and do not show the corresponding experimental curves. For this reason it is not possible to say how closely the model solution approximates the solution as observed from the living system.

A mathematical model that contains two control parameters for each of the five muscle groups involved simulates the dynamical behavior of the right leg of a human subject [25]. A time-optimal problem in which the right-hand end point of the state trajectory is variable is formulated and an optimization performed. The computation procedure is based on an algorithm of differential dynamic programming. It would appear from the study of the paper [25] that mathematical optimization of the motions of the complex locomotion systems is indeed possible.

The present paper is an extension of the research into optimization of bipedal locomotion that was undertaken in the articles [8, 9, 27-29]. In contrast to the papers [23-28] a plane nonlinear model of the BLS comprising nine rigid bodies interconnected by eight kinematic joints is considered. The mathematical statement of the energy-optimal control problem of the anthropomorphic motion of the BLS is given. A key feature of the proposed optimal control problem is a high level of utilization of experimental data of normal human locomotion [7]. This is done mainly by means of the mathematical formulation of the constraints and the restrictions imposed both on the phase coordinates and on the controlling stimuli of the BLS. The performance index, i.e. the objective function of the optimization, in contrast to the papers [22, 24], is the time integral over a double step of the sum of the absolute values of the mechanical power of all controlling torques acting at the joints of the BLS. The approach proposed in this paper draws our attention upon the power of nonlinear programming to determine optimal trajectories of high order, nonlinear BLS. Central to the idea of the proposed algorithm is the approximation of the minimal-necessarily amount of generalized coordinates of the BLS by the smoothing cubic splines in time. In this way, it is possible to formulate the considered optimal control problem of the BLS as an algebraic nonlinear programming problem. This problem is solved by using of technique of external penalties and minimization of the objective function in the orthogonal directions.

By means of the developed algorithm the problem of designing the energy-optimal anthropomorphic law of motion of the BLS has been solved. We display graphically the time functions of the hip, the knee and the ankle angles, and the controlling torques at the joints and the ground reaction forces as predicted by the model. For comparison we give the corresponding experimental curves of human gait. Therefore it is possible to show how closely the model solution approximates the solution as observed from the living system. The comparison of the results demonstrates the effectiveness of the proposed numerical approach for synthesizing the optimal control laws of the BLS.

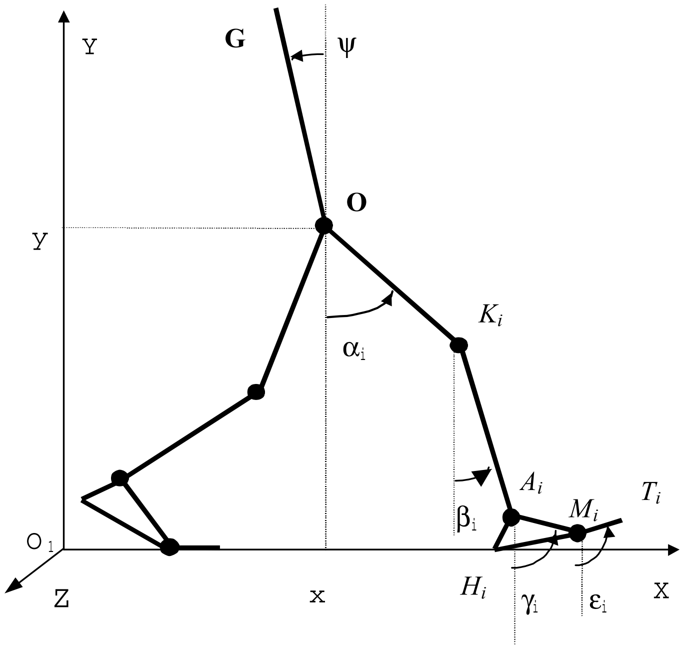

Figure 1: Model of the BLS

2 Mathematical Model

Consider a plane nine-element model of the BLS (Figure 1). This system comprises a trunk (body G) and two legs. Each leg consists of four elements. Two elements with mass and inertia model the thigh and shank. The elements $H_iA_iM_i$ and M_iT_i that model the feet of the BLS are assumed to be without inertia. The total mass m_{fi} of the foot is located at the ankle joint of the i-th leg. In addition to the weights of the trunk, thighs, shanks and feet the ground reaction forces and the control moments at the joints of the legs act on the system.

Let O_1XYZ be a fixed rectangular Cartesian coordinate system. It is assumed that the BLS moves in the O_1XY plane along the O_1X axis over a horizontal surface (the $X-Z$ plane). We will employ the following notations: $(x, y, \psi, \alpha_i, \beta_i, \gamma_i, \varepsilon_i,\ i = 1, 2)$ is the set of generalized coordinates (Figure 1); m is the mass of the trunk; r is the distance from the suspension point O of the legs to the center of mass of the trunk; J is the moment of inertia of the trunk relative to the Z axis at point O; $m_{ai}, r_{ai}, a_i, J_{ai}$ are the mass, the distance from O to the center of mass, the length and the moment of inertia of the thigh relative to the Z axis at point O, respectively; $m_{bi}, r_{bi}, b_i, J_{bi}$ are the mass, the distance from K_i to the center of mass, the length and the moment of inertia of the shank relative to the Z axis at point K_i, respectively.

The equations of motion of the system, derived using the technique of the Lagrange equations of the second kind, are as follows:

$$M\ddot{x} + \sum_{i=1}^{2} \left[K_{ai}(\dot{\alpha}_i cos\alpha_i)^{\cdot} + K_{bi}(\dot{\beta}_i cos\beta_i)^{\cdot} \right] - K_r(\dot{\psi} cos\psi)^{\cdot} = R_{1x} + R_{2x}, \qquad (1)$$

$$M(\ddot{y} + g) + \sum_{i=1}^{2} \left[K_{ai}(\dot{\alpha}_i sin\alpha_i)^{\cdot} + K_{bi}(\dot{\beta}_i sin\beta_i)^{\cdot} \right] - K_r(\dot{\psi} sin\psi)^{\cdot} = R_{1y} + R_{2y},$$

$$J\ddot{\psi} - K_r(\ddot{x} cos\psi + \ddot{y} sin\psi) - gK_r sin\psi = -q_1 - q_2,$$

$$J_i\ddot{\alpha}_i + K_{ai}\left(\ddot{x}cos\alpha_i + \ddot{y}sin\alpha_i\right) + a_iK_{bi}\left(\ddot{\beta}_icos\left(\alpha_i-\beta_i\right) + \dot{\beta}_i^2sin\left(\alpha_i-\beta_i\right)\right) +$$
$$+gK_{ai}sin\alpha_i = q_i - u_i + a_i\left(R_{ix}cos\alpha_i + R_{iy}sin\alpha_i\right),$$

$$J_{ci}\ddot{\beta}_i + K_{bi}\left(\ddot{x}cos\beta_i + \ddot{y}sin\beta_i\right) + a_iK_{bi}\left(\ddot{\alpha}_icos\left(\alpha_i-\beta_i\right) + \dot{\alpha}_i^2sin\left(\alpha_i-\beta_i\right)\right) +$$
$$+gK_{bi}sin\beta_i = u_i - p_i + b_i\left(R_{ix}cos\beta_i + R_{iy}sin\beta_i\right),$$

$$p_i - w_i + \left(y_i - y_{mi}\right)R_{ix} + \left(x_{mi} - x_i\right)R_{iy} = 0,$$

$$w_i + \left(y_{mi} - y_{Ri}\right)R_{ix} + \left(x_{Ri} - x_{mi}\right)R_{iy} = 0, \quad i = 1,2.$$

Here q_i,u_i,p_i,w_i are the control moments that act at the hip (point O), the knee (point K_i), the ankle (point A_i) and the metatarsal (point M_i) joints, respectively; R_{ix},R_{iy} are the horizontal and vertical component of the reaction forces; (x_i,y_i), (x_{mi},y_{mi}), (x_{Ri},y_{Ri}) are the Cartesian coordinates of the ankle and the metatarsal joints, and of the point of application of the vector of the reaction forces R_i of the i-th leg, respectively; g is the acceleration due to gravity; "" is a derivation with respect to time. In equations (1) we have also used: $M = m + m_{a1} + m_{b1} + m_{f1} + m_{a2} + m_{b2} + m_{f2}$, $K_r = mr$, $K_{ai} = m_{ai}r_{ai} + a_i\left(m_{bi} + m_{fi}\right)$, $K_{bi} = m_{bi}r_{bi} + b_im_{fi}$, $J_i = J_{ai} + a_i^2\left(m_{bi} + m_{fi}\right)$, $J_{ci} = J_{bi} + b_i^2m_{fi}$, $i = 1,2$.

It should be noted that in contrast to the papers [24, 27, 28] the considered model of the BLS comprises the two-link feet with both ankle and metatarsal joints. In what follows we shall show that our model of the foot makes it possible to synthesize the motion of the BLS more precisely both from kinematic and dynamic points of view.

3 Statement of the problem

The mechanical system under consideration has eleven degrees of freedom. Based on the analysis of kinematic and dynamic characteristics of human gait [7] let us set up the constraints and the restrictions needed for the mathematical statement of the optimization problem of anthropomorphic motions of the BLS.

The human motion is periodic. It leads to the following boundary conditions imposed on the phase coordinates:

$$f(0) = f(T), \quad \dot{f}(0) = \dot{f}(T), \quad \dot{x}(0) = \dot{x}(T), \quad f = (y, \psi, \alpha_1, \alpha_2, \beta_1, \beta_2, \gamma_1, \gamma_2, \varepsilon_1, \varepsilon_2), \tag{2}$$

where T is the duration of the double step. We shall also assume that all functions determined by the displacements and velocities of the points of the BLS are continuous in time. Discontinuities in the accelerations are admissible [5,6].

Human gait is characterized by a stable sequence of the phases of the leg's action during a double step. We shall assume that there are the following five phases of the i-th leg action: rotation over the heel during the period of time $t \in \left[0,\ \tau_i^h\right)$; support phase on both heel and metatarsal joint for $t \in \left[\tau_i^h,\ \tau_i^m\right)$; motion on the phalangs for $t \in \left[\tau_i^m,\ \tau_i^t\right)$; rotation over the ends of the toes for $t \in \left[\tau_i^t,\ \tau_i^s\right)$, and the swing phase of the foot over the surface during the period of time $t \in [\tau_i^s,\ T]$. For human gait the rhythm parameters $\tau_i^h, \tau_i^m, \tau_i^t, \tau_i^s$ should satisfy the following inequalities:

$$\tau_i^h \le \tau_i^m, \qquad \tau_{3-i}^t \le \tau_{3-i}^s \le \tau_i^m, \qquad i = 1,\ \ 2. \tag{3}$$

Below we shall assume that the motion of one leg of the BLS completely mimics that of the other with the time delay $\tau = T/2$. Let (x_{hi},y_{hi}), (x_{ti},y_{ti}) be the Cartesian coordinates of the points H_i

and T_i of the i-th leg of the BLS. Taking into account the above mentioned sequence of the leg action during the time of the double step the kinematically anthropomorphic cyclogram of the BLS can be described by the following constraints:

$$x_{hi}(t) \equiv x_{hi}^0, \quad y_{hi}(t) \equiv 0, \quad y_{ti}(t) > y_{mi}(t) > 0, \quad t \in \left[\tau(i-1),\, \tau_i^h\right), \tag{4}$$

$$x_{hi}(t) \equiv x_{hi}^0, \quad y_{hi} \equiv y_{mi}(t) \equiv y_{ti}(t) \equiv 0, \quad t \in \left[\tau_i^h,\, \tau_i^m\right),$$

$$x_{mi}(t) \equiv x_{mi}^0, \quad y_{hi}(t) > 0, \quad y_{mi}(t) \equiv 0, \quad y_{ti}(t) \equiv 0, \quad t \in \left[\tau_i^m,\, \tau_i^t\right),$$

$$x_{ti}(t) \equiv x_{ti}^0, \quad y_{hi}(t) > y_{mi}(t) > 0, \quad y_{ti}(t) \equiv 0, \quad t \in \left[\tau_i^t,\, \tau_i^s\right),$$

$$y_{hi}(t) \geq 0, \quad y_{mi}(t) > 0 y_{ti}(t) > 0, \quad t \in \left[\tau_i^s,\, T - \tau(i-1)\right].$$

Here x_{hi}^0, x_{mi}^0, x_{ti}^0 are abscissas of the points H_i,M_i,T_i of the i-th leg during its support phase, respectively; $x_{h2}^0 = x_{h1}^0 + L$, $x_{mi}^0 = x_{hi}^0 + l_{hi}$, $x_{ti}^0 = x_{mi}^0 + l_{mi}$, $l_{hi} = |H_i M_i|$, $l_{mi} = |M_i T_i|$; L is the length of single step of the gait.

Without any restriction of the generality the following additional conditions for $t = 0$ and $t = T$ are given by:

$$x_{h1}^0 = 0, \quad y_{t2}(0) = 0, \quad x_{m2}(0) = x_{h1}^0 - L + l_{h2}, \quad y_{m2}(0) = 0, \tag{5}$$

$$x_{h1}(T) = x_{h1}^0 + 2L, \quad y_{h1}(T) = 0, \quad x_{m2}(T) = x_{h1}^0 + L + l_{h1}, \quad y_{m2}(T) = 0.$$

We shall restrict the angular displacements of the considered BLS during the double step ($t \in [0, T]$) by the following set of constraints:

$$\theta_i^o(t) \leq \mu_i^o(t) \leq \Theta_i^o(t), \quad \theta_i^k(t) \leq \mu_i^k(t) \leq \Theta_i^k(t), \quad \theta_i^a(t) \leq \mu_i^a(t) \leq \Theta_i^a(t). \tag{6}$$

Here $\mu_i^o(t) \equiv \alpha_i(t) - \psi(t)$, $\mu_i^k(t) \equiv \alpha_i(t) - \beta_i(t)$, $\mu_i^a(t) \equiv \gamma_i(t) - \beta_i(t) + \varphi_m - \pi/2$, $i = 1,\ 2$. The functions $\theta_i^o(t), \Theta_i^o(t), \theta_i^k(t), \Theta_i^k(t), \theta_i^a(t), \Theta_i^a(t)$ are given in advance by experimental data of human gait [7], $\varphi_m = \angle A_i M_i H_i$.

Additionally we shall require that the laws of motion of the feet of the BLS should satisfy the conditions:

$$\begin{array}{ll} \varepsilon_1(t) - \gamma_1(t) - \varphi_m = 0, & t \in \left[0, \tau_1^h\right], \\ \varepsilon_1(t) = \pi/2, & t \in \left[\tau_1^h, T/2\right], \\ \varepsilon_1(t) - \gamma_1(t) - \varphi_m \geq 0, & t \in \left[T/2, \tau_1^s\right], \\ \varepsilon_2(t) - \gamma_2(t) - \varphi_m \geq 0, & t \in \left[0, \tau_2^s\right], \\ \varepsilon_2(t) - \gamma_2(t) - \varphi_m = 0, & t \in \left[\tau_2^s, \tau_2^h\right], \\ \varepsilon_2(t) = \pi/2, & t \in \left[\tau_2^h, T\right]. \end{array} \tag{7}$$

The laws of motion of the BLS should be such that the specific constraints on the forces acting from the surface on the feet are observed. We shall assume that all forces acting from the surface on the foot of the support leg satisfy the "non-suction-cup" conditions:

$$R_{iy}(t) \geq 0, \qquad i = 1,\ 2, \qquad t \in [0,\ T]. \tag{8}$$

Moreover, the following restrictions imposed on the abscissa of the point at which the resultant of the reaction forces of the support intersects the surface should be observed for the i-th leg of the BLS:

$$x_{Ri}(t) \equiv x_{hi}^0, \quad t \in \left[\tau(i-1), \tau_i^h\right], \tag{9}$$

$$x_{hi}^0 \le x_{Ri}(t) \le x_{mi}^0, \quad t \in \left[\tau_i^h, \tau_i^m\right), \quad x_{mi}^0 \le x_{Ri}(t) \le x_{ti}^0, \quad t \in [\tau_i^m, \tau_i^s).$$

Obviously, the above requirements (2)-(9) do not uniquely specify the law of motion of the BLS. To estimate the quality of the controlled motion of the BLS we shall use the objective function

$$E = \frac{1}{2L}\int_0^T \left\{ \sum_{i=1}^{2} \left[|q_i(\dot\psi - \dot\alpha_i)| + |u_i(\dot\alpha_i - \dot\beta_i)| + |p_i(\dot\beta_i - \dot\gamma_i)| + |w_i(\dot\gamma_i - \dot\varepsilon_i)| \right] \right\} dt. \tag{10}$$

In a number of cases the performance index (10) estimates the energy expenditures in bipedal locomotion [4-6, 9, 23, 27-29].

Let $Z(t) = \{x, \dot x, y, \dot y, \psi, \dot\psi, \alpha_i, \dot\alpha_i, \beta_i, \dot\beta_i, \gamma_i, \dot\gamma_i, \varepsilon_i, \dot\varepsilon_i,\ i = 1,\ 2\}$ be a vector of the phase state and $U(t) = \{q_i, u_i, p_i, w_i,\ i = 1,\ 2\}$ be a vector of the control stimuli of the BLS. The following problem can be stated.

Problem A. Assume that we are given the step length $L = L_0$ and the duration of the double step $T = T_0$. It is required to determine the control process $\{Z(t),\ U(t)\}$, $t \in [0, T]$, and the parameters τ_i^h, τ_i^m,τ_i^s which satisfy the equations (1), the boundary conditions (2), the given restrictions on the rhythm parameters and the phase coordinates (3)-(6), the given constraints on the controlling stimuli (8), (9) and which minimize the functional (10).

From a mathematical point of view problem A is a nonlinear nondifferentiable optimal control problem with restrictions imposed both on the phase coordinates and the controlling stimuli in which the left and the right-hand end points of the state trajectories are variable.

4 Methodology and algorithm

Central to the proposed approach for solving problem A is the idea that any optimal control problem can be converted into a standard nonlinear programming problem by parameterizing each of the free variable functions. Analysis of the equations of motion (1), the boundary conditions (2) and the constraints (3)-(9) shows that the following functions can be chosen as free variable functions in problem A:

$$\mu_1^k(t), \ \ \mu_2^k(t), \ \ x(t), \ \ x_g(t), \qquad t \in [0,\ T], \tag{11}$$

$$\alpha_2(t), \ \ \gamma_1(t), \qquad t \in [0,\ T/2], \quad \alpha_1(t), \ \ \gamma_2(t), \qquad t \in [T/2,\ T].$$

Here $x_g(t)$ is the abscissa of some point located on the axes OG of the trunk of the BLS. Using the constraints (2)-(9) it is possible to demonstrate that the laws of motion of the considered BLS can be determined in the final formulas if the free variable functions (11) are given. Note that because the feet are without inertia, we shall not concern ourselves with their motion during the swing phase of the leg of the BLS.

Obviously, if the law of motion of the BLS is completely specified the inverse dynamics problem can be solved using the equations (1). The only question arises for the double support phase of the BLS (phase of support on both legs simultaneously). To determine the unknown control stimuli we don't have enough equations, i.e. there is indeterminacy. It is necessary to give supplementary information for the unknown variables. This can be done in various ways [5, 6]. In what follows we shall specify the horizontal components of the support reaction $R_{ix}(t)$ and the abscissa of the point of its application $x_{Ri}(t)$ of the extremity that is preparing itself to become a shifted leg in the next single step. We shall supplement the quantities in question as follows:

$$R_{ix}(t) = R_{ix}((2-i)T/2) + \frac{R_{ix}(\tau_i^s) - R_{ix}((2-i)T/2)}{\tau_i^s - (2-i)T/2}(t - (2-i)T/2), \tag{12}$$

$$x_{Ri}(t) = x_{Ri}^0 + \frac{x_{ti}(t) - x_{Ri}^0}{\tau_i^s - (2-i)T/2}(t - (2-i)T/2), \qquad i = 1, 2.$$

To approximate the free variable functions (11) we use the solution of the following auxiliary variation problem.

Problems B. It is required to determine the function $S(t) \in C^2[a,b]$ which minimize the objective functional

$$J(S) = \int_a^b \left(\ddot{S}(t)\right)^2 dt + \sum_{j=0}^{n} \rho_j^{-1} (S(t_j) - z_j)^2 \tag{13}$$

with given boundary conditions either

$$\dot{S}(a) = \dot{S}_a, \quad \dot{S}(b) = \dot{S}_b, \tag{14}$$

or

$$\dot{S}(a) = \dot{S}(b), \qquad \ddot{S}(a) = \ddot{S}(b). \tag{15}$$

Here $\rho_j \geq 0$, z_j, t_j, $a = t_0 < t_1 < ... < t_n = b$, $j = 0, 1, ..., n$ are given numbers. If the values ρ_j and t_j are given then the solution of the problems (13)-(14) or (13), (15) exists and is determined by the smoothing cubic spline $S(t, z)$, where $z = (z_0, z_1, ..., z_n)$ is a vector of variable parameters [31].

Every free variable function (11) has been approximated by the smoothing spline

$$f = S_f\left(t, z^f\right), \qquad f = \left(\mu_1^k, \mu_2^k, x_g, x, \alpha_1, \alpha_2, \gamma_1, \gamma_2\right).$$

To approximate the functions $\alpha_i(t)$, $\gamma_i(t)$, $i = 1, 2$ we have used the solution of the problem (13), (14). The functions $\mu_i^k(t)$, $x_g(t), x(t)$, $i = 1, 2$ have been approximated by the solution of the problem (13), (15).

The variable parameters z_j^f have been represented by the following expression

$$z_j^f = \bar{z}_j^f + C_j^f, \;\; j = 0, 1, ..., n_f. \tag{16}$$

Here $\bar{z}_j^f$ are some initial values of the variable function f at the knots $t = t_j$ which are calculated using the equality constraints, i.e. equality (4), (5), (7); C_j^f are new optimization parameters.

The weighting coefficients ρ_j in the problem B have been chosen as follows:

$$\rho_0^f = \rho_{n_f}^f = 0, \qquad \rho_j^f = \rho^f, \qquad j = 1, ..., n_f - 1, \quad f = \left(\mu_1^k, \mu_2^k, x_g, x, \alpha_1, \alpha_2, \gamma_1, \gamma_2\right). \tag{17}$$

The boundary conditions in the problem B have been given taking into account the experimental data of the function f and the imposed constraints (4)-(7). Based on the free variable functions (11) and the described methodology (13)-(17) of their approximation the controlling process $\{Z(t), U(t)\}$ of the BLS can be calculated. Henceforth, the problem A is converted into the following nonlinear programming problem:

$$Q(C) \to \min_C, \qquad H(C) \leq 0. \tag{18}$$

Here functions Q and $H$are determined by means of equations (1), constraints (3)-(9), functional (10) and expressions (12)-(17); $C = \{\psi_0, C_j^f, j = 1, 2, ..., n_f - 1, \; C_0^{\gamma_1}, C_{n_{\gamma_1}}^{\gamma_1}, C_0^{\gamma_2}, C_{n_{\gamma_2}}^{\gamma_2}, C_0^{\mu_1^k}, C_0^{\mu_2^k}\}$ is a vector of variable parameters.

Using the penalty function approach [32] the problem (18) is reduced to the following problem

$$Q_1(C) = Q(C) + \sum_{l=1}^{6} \lambda_l G_l(C) \to \min_C, \tag{19}$$

$$G_1(C) = \int_0^T \sum_{i=1}^{2} \left\{ \left(\theta_i^o(t) - \mu_i^o(t)\right)_+ + \left(\mu_i^o(t) - \Theta_i^o(t)\right)_+ \right\} dt,$$

$$G_2(C) = \int_0^T \sum_{i=1}^{2} \left\{ \left(\theta_i^k(t) - \mu_i^k(t)\right)_+ + \left(\mu_i^k(t) - \Theta_i^k(t)\right)_+ \right\} dt,$$

$$G_3(C) = \int_0^T \sum_{i=1}^{2} \left\{ \left(\theta_i^a(t) - \mu_i^a(t)\right)_+ + \left(\mu_i^a(t) - \Theta_i^a(t)\right)_+ \right\} dt,$$

$$G_4(C) = \int_0^T \sum_{i=1}^{2} \left[\left(-y_{hi}(t)\right)_+ + \left(-y_{ti}(t)\right)_+ \right] dt, \quad G_5(C) = \int_0^T \sum_{i=1}^{2} \left(-R_{iy}(t)\right)_+ dt,$$

$$G_6(t) = \sum_{i=1}^{2} \left\{ \int_{\tau_{3-i}^s}^{\tau_i^m} \left[\left(x_{hi}(t) - x_{Ri}(t)\right)_+ + \left(x_{Ri}(t) - x_{mi}(t)\right)_+ \right] dt + \right.$$

$$\left. + \int_{\tau_i^m}^{iT/2} \left[\left(x_{mi}(t) - x_{Ri}(t)\right)_+ + \left(x_{Ri}(t) - x_{ti}(t)\right)_+ \right] dt \right\}, \quad x_+ = \begin{cases} x, & x \geq 0, \\ 0, & x < 0, \end{cases}$$

where $\lambda_l > 0$ are given numbers.

Henceforth, the optimal control problem (problem A) has been converted into the unconstrained optimization problem (19). To solve the problem (19) the Rozenbrock's method has been used [32].

5 Numerical results and discussion

The methodology and algorithm described above have been used to solve a number of optimal control problems for the BLS. Below numerical results are presented for the following anthropomorphic values of the linear and mass-inertia parameters of the BLS: m=46.7kg, r=0.39m, J=7.096Nm^2, m_{ai}=8.49kg, a_i=0.47m, r_{ai}=0.258m, J_{ai}=0.57Nm^2, m_{bi}=3.51kg, b_i=0.53m, r_{bi}=0.214m, J_{bi}=0.16Nm^2, m_{fi}=1.24kg, $|H_iA_i|$ =0.12m, $|A_iM_i|$ =0.17m, l_{mi}=0.1m, $\angle H_iA_iM_i$ =82°, $i = 1,\ 2$. The used parameter values of the BLS correspond to the respective parameters of a human body with a total mass M=73kg and height of 1.76m [8, 29].

Here we describe in detail the resultant energetically optimal law of motion of the BLS which has been obtained by the solution of Problem A for the step length of L=0.76m and duration of the double step T=1.14s (for so called human gait with natural cadence [7]). Figure 2 shows a cyclogram of resultant energetically optimal motion for the BLS during the time period for the double step. The obtained optimal law of motion of the BLS is characterized by the following energy and rhythm parameters: E=127J/m, τ_1^h=0.1T, τ_1^m=0.23T, τ_1^t=0.5T, τ_1^s =0.58T, τ_2^h =0.6T, τ_2^m= 0.73T, τ_2^s=0.08T.

Figures 3-5 show the ways in which the hip, the knee and the ankle angles of the leg change in time over a double step for the obtained energetically optimal law of motion for the BLS (solid curves). In these figures the domains of the values of the respective angular characteristics obtained by experiments for normal human gait [7] are also depicted (the domains are bounded by thin curves corresponding to the functions $\theta_i^o(t)$, $\Theta_i^o(t)$, $\theta_i^k(t)$, $\Theta_i^k(t)$, $\theta_i^a(t)$, $\Theta_i^a(t)$).

The analysis of these data and the cyclogram depicted in Figure 2 indicates that the kinematic characteristics of the obtained energetically optimal law of motion for the BLS are within reasonable proximity to the corresponding characteristics of human gait [7]. The way in which the specific horizontal component $R_{1x}(t)/M$ of the support reaction varies (Figure 6, solid curve) indicates that

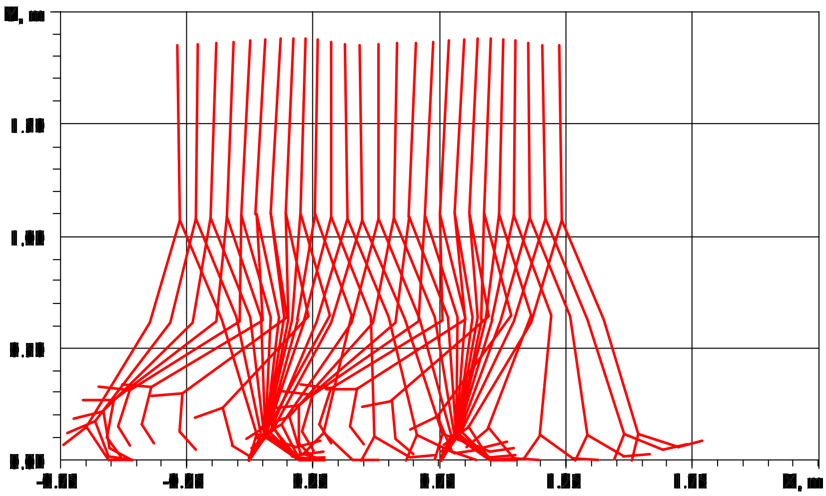

Figure 2: Cyclogram of energetically optimal motion of the BLS

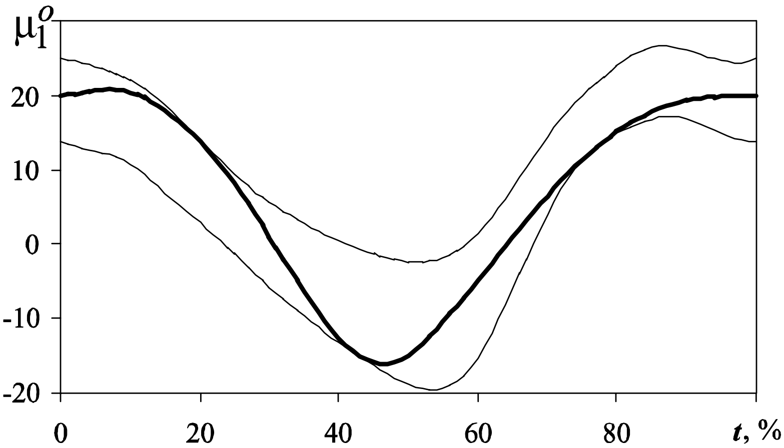

Figure 3: Hip angle $\mu_1^o(t)$, in degrees

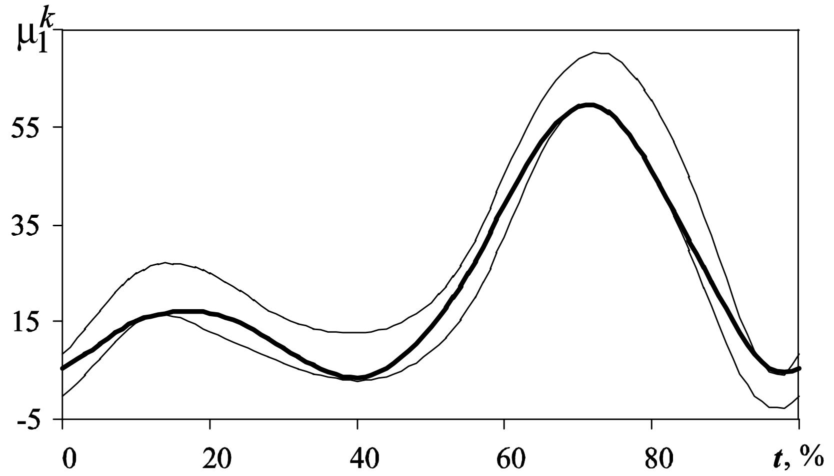

Figure 4: Knee angle $\mu_1^k(t)$, in degrees

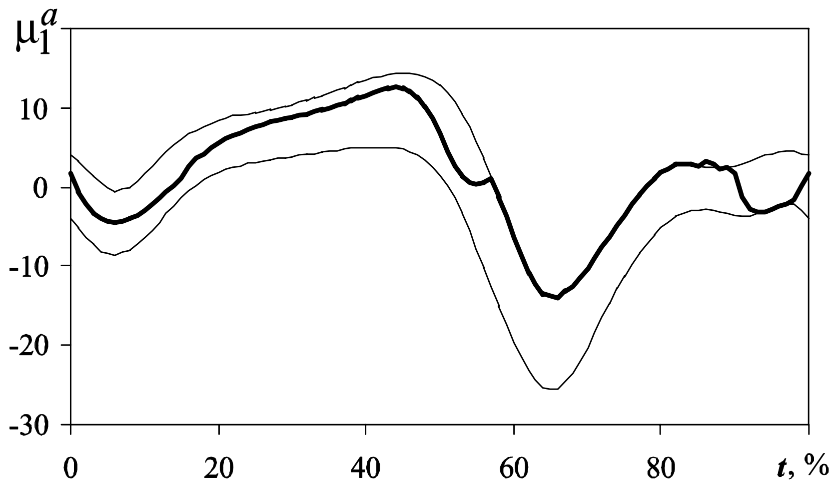

Figure 5: Ankle angle $\mu_1^a(t)$, in degrees

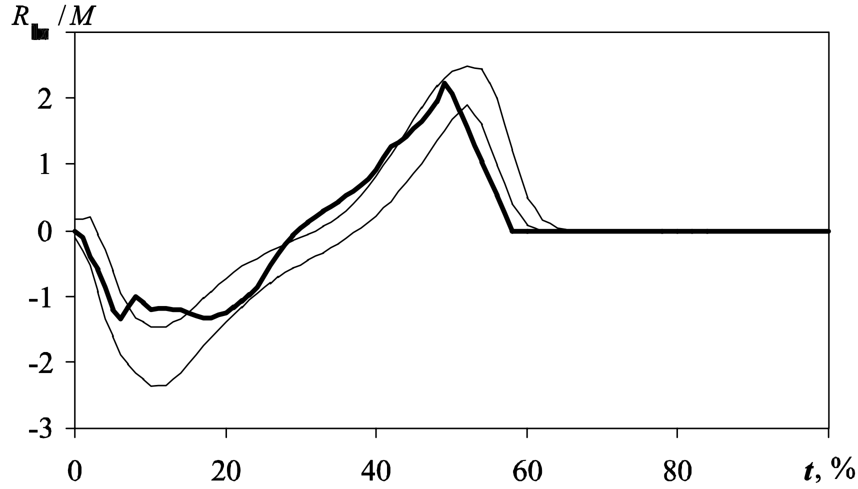

Figure 6: Force $R_{1x}(t)/M$, in N/kg

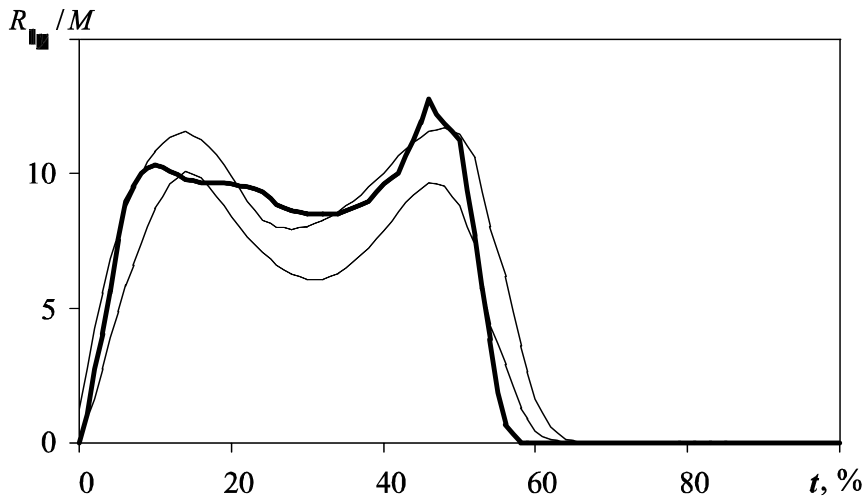

Figure 7: Force $R_{1y}(t)/M$, in N/kg

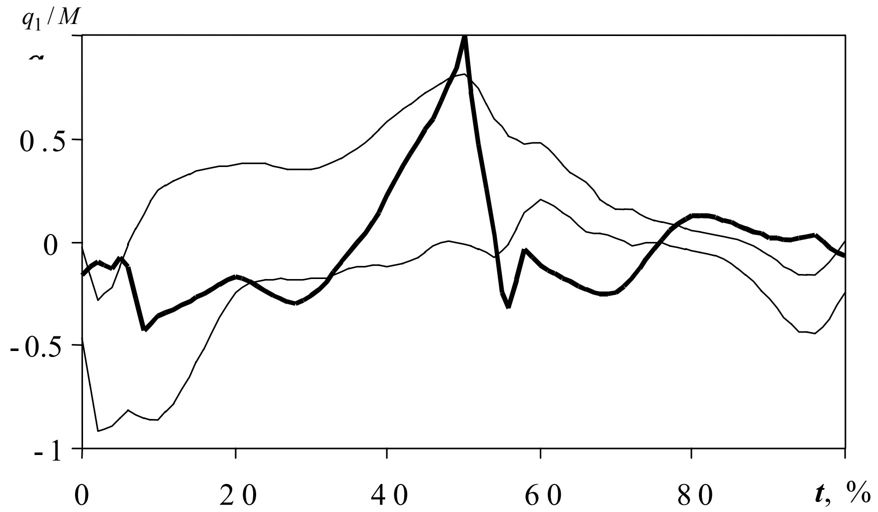

Figure 8: Hip torque $q_1(t)/M$, in Nm/kg

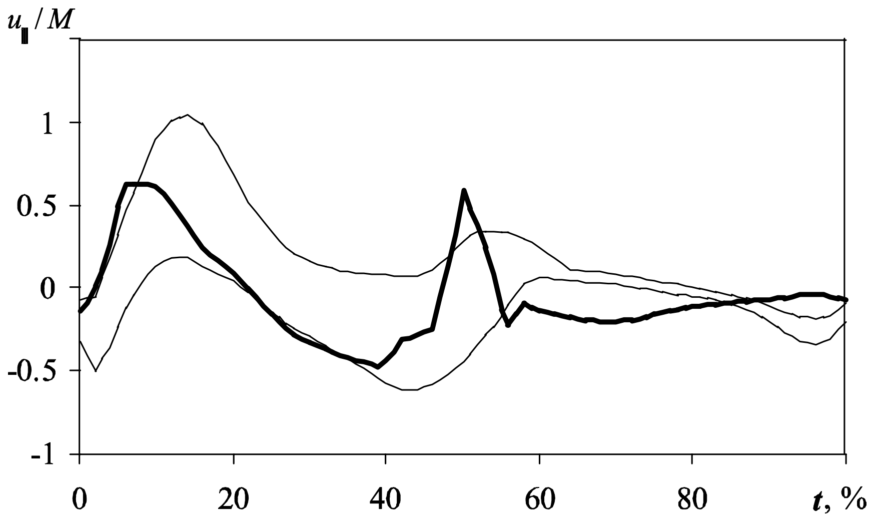

Figure 9: Knee torque $u_1(t)/M$, in Nm/kg

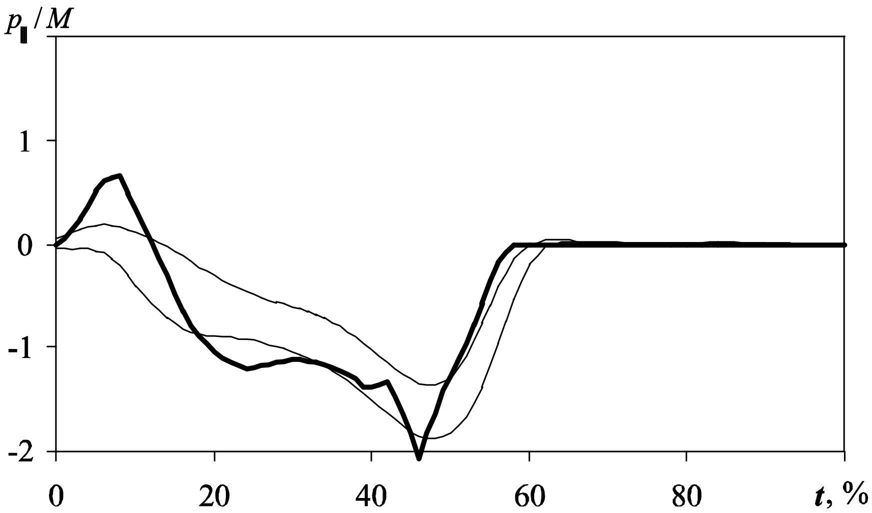

Figure 10: Ankle torque $p_1(t)/M$, in Nm/kg

in each single step the support leg successively executes two tasks: deceleration of the BLS (time interval in which $R_{ix}(t)/M<0$) and separation (time interval in which $R_{ix}(t)/M>0$). The vertical component of the support reaction $R_{1y}(t)/M$ is depicted in Figure 7 (solid curve).

Figures 8-10 show the specific control torques $q_1(t)/M$, $u_1(t)/M$, $p_1(t)/M$(solid curves) acting at the joints of the leg during the obtained energetically optimal law of motion for the BLS. For comparison purposes in Figures 6-10 the domains of the values of the respective dynamic characteristics obtained by experiments for normal human gait are shown (the domains are bounded by the thin curves). The analysis of Figures 6-10 indicates that the dynamic characteristics (forces and torques) of the obtained energetically optimal law of motion for the BLS are also within reasonable proximity to the corresponding characteristics of human gait [7].

6 Conclusions

The dynamics, control and optimization problems for the BLS are interesting and important for many applications. For instance, to design the optimal legged mobile robots for difficult terrain, to recognize the neuro-system's laws governing the goal-directed motion of human locomotor apparatus, to design the optimal prostheses and orthoses of lower limbs. All the above mentioned are examples of a broad variety of applications of multibody system dynamics [33].

In this paper the problem of optimization of the controlled motion of BLS has been investigated. From a mathematical point of view the considered object is a nonlinear multidimensional controlled system with a lot of constraints and restrictions imposed both on the phase coordinates and the controlling stimuli. The design of optimal control laws for these kinds of systems is a challenging research task that has attracted an increasing interest in recent decades.

A numerical method for the solution of optimal control problems of highly nonlinear and complex BLS has been proposed. The method is based on a special procedure of converting the initial optimal control problem into a standard nonlinear programming problem. This is made by the approximation of the independent variable functions using smoothing cubic splines and by the solution of inverse dynamics problems for the BLS. The key features of the method are its high numerical effectiveness and the possibility to satisfy a lot of restrictions imposed on the phase coordinates of the BLS automatically and accurately.

An important benefit of recasting the optimal control problem for the BLS (Problem A) as a nonlinear programming problem is that it eliminates the requirement of solving a two-point boundary-value problem that must be solved to determine an explicit expression for the optimal control. In contrast to dynamic programming, the proposed method does not require massive computer storage. It thereby offers a streamlined approach for solving different optimal control problems for the BLS. The reader who is interested in more details concerning the algorithm described in this work should consult the paper [29].

This work has demonstrated the effectiveness of the proposed approach for the optimization of anthropomorphic laws of motion for the BLS. The kinematic and dynamic characteristics of the obtained solution closely approximate the respective characteristics as observed from the living system. It is an evidence of the fruitfullness of the utilization of existing data from the behavior of biological systems for synthesizing the optimal control laws and structure of legged locomotion robots [34].

At last but not least we want to emphasize that very little is known about a criterion used by a human body for "optimizing" its motion. Nevertheless it looks reasonable that a human body minimizes energy expenditure during locomotion with natural cadence.

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References

[1] A. Morecki and K. Waldron (eds.), *Human and Machine Locomotion*, CISM Courses and Lectures No. 375, Springer-Verlag, Wien New York (1997).

[2] M. Vukobratović, *Legged Locomotion Robots and Anthropomorphic Mechanisms*, Mihailo Pupin Institute, Belgrade (1975).

[3] V. Larin, *Control of Walking Robots*, Naukova Dumka, Kiev (1980).

[4] A. Formal'sky, *Displacement of Anthropomorphic Mechanisms*, Nauka, Moscow (1982).

[5] V. Beletskii, *Two-Legged Walking: Model Problems of Dynamics and Control*, Nauka, Moscow (1984).

[6] V. Berbyuk, *Dynamics and Optimization of Robototechnical Systems*, Naukova Dumka, Kiev (1989).

[7] D. Winter, *The Biomechanics and Motor Control of Human Gait*, University of Waterloo Press, Canada (1991).

[8] V. Berbyuk, Multibody system modelling and optimization problems of lower limb prostheses, *IUTAM Symposium on Optimization of Mechanical Systems*, D. Bestle and W. Schiehlen (eds.), Kluwer Academic Publishers, 25-32 (1996).

[9] V. Berbyuk, Dynamics and optimal control of biotechnical systems "Man-Prosthesis", *IUTAM Symposium on Interaction Between Dynamics and Control in Advanced Mechanical Systems*, Ed. D. van Campen, Kluwer Academic Publishers, 35-43 (1997).

[10] A. Morecki, Modeling and simulation of human and walking robot locomotion, *Human and Machine Locomotion*, A. Morecki and K. J. Waldron (eds.), CISM Courses and Lectures No. 375, Springer-Verlag, Wien New York, 1-78 (1997).

[11] K. Waldron, Design of walking machines, *Human and Machine Locomotion*, A. Morecki and K. J. Waldron, (eds.), CISM Courses and Lectures No. 375, Springer-Verlag, Wien New York, 283-315 (1997).

[12] F. Pfeiffer, T. Rossmann and J. Steuer, Theory and practice of machine walking, *Human and Machine Locomotion*, A. Morecki and K. J. Waldron, (eds.), CISM Courses and Lectures No. 375, Springer-Verlag, Wien New York, 231-281 (1997).

[13] W. Blajer and W. Schiehlen, Walking without impacts as a motion/force control problem, *Trans. ASME, J. of Dynamic Systems, Measurements, and Control,* **114,** 660-665 (1992).

[14] V. Berbyuk, B. Peterson and N. Nishchenko, Linear viskoelastic actuator-based control system of a bipedal walking robot, *Mechatronics' 98*, J. Adolfsson and J. Karlsen, (eds.), Elsevier Science Ltd., 379-384 (1998).

[15] V. Larin, Problem of control of a hopping apparatus, *J. Franklin Inst.* **335B**(3) 579-595 (1998).

[16] V. Larin, Use of linear matrix inequalities at synthesis of system of stabilization of the hopping apparatus, *Int. Appl. Mechanics*. **37**(10), 1352-1358 (2001).

[17] H. Hatze, Neuromusculoskeletal control systems modeling - a critical survey of recent developments, *IEEE Trans. on Automatic Control,* **AC-25** (5), 375-385 (1980).

[18] M. Dietrich, K. Kedzior and C. Rzymkovski, Some theoretical and practical aspects of modeling and simulation of the human musculoskeletal system, *Human and Machine Locomotion*, A. Morecki and K. J. Waldron, (eds.), CISM Courses and Lectures No. 375, Springer-Verlag, Wien New York, 107-123 (1997).

[19] J. Ambrosio and M. Pereira, Flexible multibody dynamics with nonlinear deformations: Vehicle dynamics and crashworthiness applications, *Proceeding of the NATO ASI on Computational Methods in Mechanisms*, J. Angeles and E. Zakhariev, (eds.), Varna, Bulgaria, **1**, 1-38 (1997).

[20] M. Silva, J. Ambrosio and M. Pereira, A multibody approach to the vehicle and occupant integrated simulation, *Int. J. of Crashwothiness*, **2**(1), 73-90 (1997).

[21] M. Garcia, A. Chatterjee, A. Ruina and M. Coleman, The simplest walking model: stability, complexity and scaling, *ASME, J. of Biomechanical Engineering*, **120**, 281-288 (1998).

[22] Y. Nuber and R. Contini, A minimal principle in biomechanics, *Bull. Math. Biophys.*, **23**, 377-391 (1961).

[23] R. Beckett and K. Chang, An evaluation of the kinematics of gait by minimum energy, *J. of Biomechanics*, **1**(2), 147-159 (1968).

[24] C. Chow and D. Jacobson, Studies of human locomotion via optimal programming, *Mathem. Biosciences*, **10**(3/4), 239-306 (1971).

[25] H. Hatze, The complete optimization of a human motion, *Mathem. Biosciences,***28**, 99-135 (1976).

[26] V. Larin, Method of periodic optimization in stabilization problems of biped apparatus, Lecture notes in control and information sciences - 23, *Optimization techniques*. Berlin. ets.: Springer, Pt. 2., 593 - 599 (1980).

[27] V. Beletskii, V. Berbyuk and V. Samsonov, Parametric optimization of motions of a bipedal walking robot, *J. Mechanics of Solids*, **17**(1), 24-35 (1982).

[28] V. Berbyuk, G. Krasyuk and N. Nishchenko, Mathematical modeling of the human gait dynamics in sagittal plane, *Mathematical Methods and Physicomechanical Fields*, **40**(4), 127-138 (1997).

[29] V. Berbyuk and B. Lytwyn, The mathematical modeling of the human motions based on the optimization of the biodynamic system's controlled processes, *Mathematical Methods and Physicomechanical Fields*, **41**(3), 153-161 (1998).

[30] M. Pandy, F. Anderson, and D. Hull, A parameter optimization approach for the optimal control of large-scale musculoskeletal systems, *ASME, J. of Biomechanical Engineering,* **114**, 450-460 (1992).

[31] Y. Zavyalov, B. Kvasov and V. Miroshnichenko, *Methods of the spline functions*, Nauka, Moscow (1980).

[32] S.Bazara and C. Shetty, *Nonlinear Programming. Theory and Algorithms*, John Wiley and Sons, New York, Chichester Brisbane, Toronto (1979).

[33] W. Shiehlen, Multibody system dynamics: roots and perspectives, *Multibody System Dynamics,* **1**, 149-188 (1997).

[34] F. Hardarson, T. Benjelloun, T. Wadden and J. Wikander, Biologically inspired design and construction of a compliant robot leg for dynamic walking, *Mechatronics'98*, J.Adolfsson and J. Karlsen, (eds.), Elsevier Science Ltd., 364-372 (1998).