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OPTIMIZATION OF CONTROLLED MOTION OF CLOSED-LOOP CHAIN MANIPULATOR ROBOTS WITH DIFFERENT DEGREE AND TYPE OF ACTUATION

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Abstract. A number of energy-optimal control problems for a new structure of closed-loop manipulator robot are considered. We present methodology and algorithm that is suitable for solving optimization problems for manipulator robots with different degree and type of actuation. This methodology is based on polynomial and Fourier series approximation of independently varying functions and conversion of the initial optimal control problem into the constrained parameter optimization problem. The methodology has been successfully used for optimization of under-, fully-, and overactuated robots having both external (powered) drives and internal (unpowered or passive) spring-damper-like drives. Comparison analysis of the simulation results of the obtained energy-optimal control processes for different manipulator robots is presented.

Keywords: manipulator, SCARA robot, closed-loop kinematic chain, semi-passive control, energy-optimal control, overactuation, redundant actuation.

1 Introduction

The development of innovative actuation subsystems and new control strategies for future robotic systems is an important research topic within the robotics community. The ability of a robotic system is highly dependent on the capabilities of its actuation and control system. The aim of the development efforts in this area is not only to improve the performance of existing robotic systems in terms of, e.g. accuracy, productivity, energy consumption and reliability but also to develop new concepts with drastically improved functionality of the robot such as the ability to perform complex tasks in unstructured environments while interacting with humans in a safe manner. Very often these new robot concepts will be very sophisticated with some kind of intelligence and they are also likely to be mobile carrying their own energy supply [25].

In terms of the mechanical subsystem for manipulator robots academia and robot manufacturers are paying special attention to closed-loop kinematic chains and degree and type of actuation [24], [27]. Advances in these areas and optimal interaction between different subsystems can lead to not only significant improvements in performance but also to new features such as compliant structure and improved dexterity. These capabilities are important for safe human interaction and the performance of complicated tasks [19].

A manipulator robot with closed-loop kinematic chain gives the manipulator higher structural stiffness, which is important for the high trajectory accuracy needed for applications such as arcwelding, cutting and sealing. The closed-loop kinematic chain also gives high performance dynamics and the possibility to bring the actuators close to the base. The latter is important for the direct-drive technology used in many robots [20], [14].

The most commonly used active (powered) drives in robotic systems are either electric, hydraulic or pneumatic. As pointed out by Waldron [26] the missing power regeneration capability of these active actuators can boost energy consumption. On the other hand some unpowered (passive) drives can be used as local energy storage devices. Examples of passive actuators are spring-damper-like drives or brakes. A manipulator robot with both active and passive actuators in the control system could be called a semi-passively controlled robotic system. The most straightforward way to realize semi-passive control is to include both active and passive actuators in the manipulator. Specialized actuators with semi-passive properties in one unit like the compliant artificial muscle actuator [12] does exists.

Existing control systems in robotics are designed with the assumption that every degree-of-freedom of the robot is governed by a separate active actuator (full actuation). This approach can lead to complicated control systems with large energy consumption [22]. By varying the degree of actuation and type of control of a closed-loop manipulator we are looking for improved performance in terms of load carrying capability, energy consumption, quickness and even a simplified control system. To design an efficient control law for closed-loop chain manipulators it seems reasonable to explore the inherent dynamics of the mechanical structure of the robotic system and the optimal interaction between different kind of actuators. The idea of utilizing the inherent dynamics of robotic systems with compliant elements as passive actuators have been investigated by several researchers [6], [22]. Variation of the number of active actuators has also been considered in the literature. For example Arai and Tachi [2] studied the position control of an underactuated two degree-of-freedom manipulator using a brake at the passive joint while Cheng *et. al.* used overactuation to overcome undesired effects caused by various kind of singularities [11].

In this paper the dynamics and control problems are studied for a new structure of manipulator robot. In comparison with the well-known SCARA-robot [23] the proposed structure is characterized by incorporation of an additional link that gives a closed-loop chain robot. In addition to the traditional active actuators in the SCARA-robot the robot under study also includes unpowered (passive) actuators. The aim of the study is to analyze how energy consumption of a manipulator robot can be reduced by utilizing semi-passive actuation (spring-damper-like drives) in combination with overactuation. We also intend to investigate the possibility to simplify the control system by reducing the number of active actuators, i.e. study the characteristics of the underactuated robot. The performance of the robots with respect to different structure of actuation are evaluated by formulating and solving a number of energy-optimal control problems. For comparison the energy-optimal control problems are also solved for the fully actuated robot without passive actuators. The kinematic, dynamic and energetic characteristics for the different robots are finally analyzed and compared. Even though some basic techniques and results related to optimal control of manipulator robots can be found in general textbooks in robotics and mechatronics [23], [19] the general optimal control problem still remains a true challenge [9]. For semi-passively actuated robotic systems with n-degrees of freedom the general energy-optimal control problem is formulated and solved for given motion in [6].

The present paper is an extension of the results obtained in [16] with the solution of several variants of an energy-optimal control problem with periodic multi-point boundary conditions for the phase coordinates and restrictions imposed on the control torques of the robot. Previously analytical studies and numerical simulations show [16] that the energy consumption of the arbitrary given motion of the considered robot in case of optimal overactuation is less than in case of full actuation. It was also shown that the robot in question with optimal spring-like passive drives has less energy consumption than the fully actuated robot executing the same arbitrary given motion. In contradiction to this earlier paper we are in particular studying the dynamic and energetic characteristics of the performed admissible motions of the robot in question that is needed to execute a typical working task (pick-and-place operation). The proposed optimization approach makes it possible to study the relationship between the type of actuation of the robot and the dynamic and energetic characteristics of its motion.

We begin by describing the modelling of controlled motion of the closed-loop chain manipulator robot under study. A suitable methodology for solution of optimal control problems for this robot is then outlined. In the main part of the paper we formulate and present the solution of a number of energy-optimal control problems for the manipulator robot with different degree and type of actuation using a constrained parameter optimization procedure. Finally, we analyze all the results obtained and make conclusions.

2 Model of the Robot and Statement of the Problem

Consider the manipulator robot depicted in Figure 1. The robot comprises four links modelled by the rigid bodies OA, AB, OD, and EC. There are one degree-of-freedom rotational joints at the points O and A, and translational joints at the point B. All joints are considered frictionless.

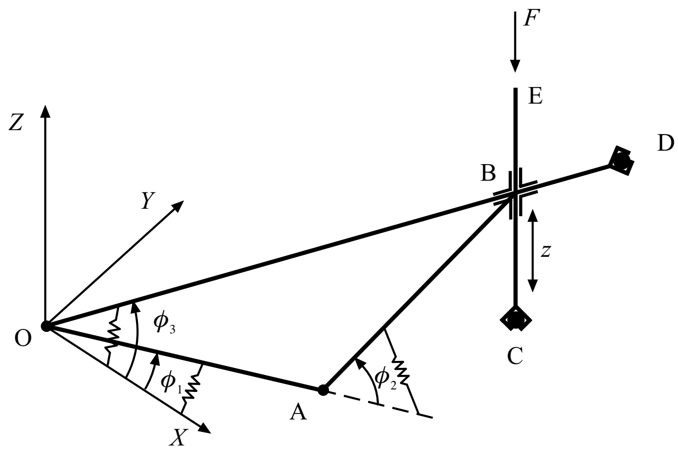


Figure 1: The Sketch of the SCARA-Like Robot

Let OXYZ be a fixed rectangular Cartesian coordinate system. It is assumed that the links OA, AB and OD move in the horizontal plane OXY under the action of the control torques $u_1(t), u_2(t)$ and $u_3(t)$ applied to the links OA, AB, and OD, respectively. Under the action of the control force $F(t)$ the link EC moves along the direction of the axes OZ. The control torques $u_i(t), i = 1, 2, 3$ and control force $F(t)$ are exerted by the powered (external) drives of the robot. The robotic system also comprises spring-damper-like actuators at joints O and A. The control torques exerted by these actuators $p_1(t)$, $p_2(t)$ and $p_3(t)$ act on the links OA, AB and OD, respectively. They will be treated as the control torques of the unpowered (internal) drives of the robot.

The following notations are employed: ϕ_1, ϕ_2, ϕ_3 and z are the angles and linear displacement that determine the position of the links OA, AB, OD and EC respectively (Figure 1); $l_i, m_i, J_i, i = 1, 2, 3,$ denote the length, the mass and the moment of inertia of the links OA, AB and OD relative to the vertical axis passing through their mass center, respectively; r_1, r_2, r_3 are the distances from the points O, A and O to the center of mass of the links OA, AB and OD, respectively; m_4, m_C, m_D denote the mass of the link EC and the mass of point-loads located at the end-effectors C and D of the robot, respectively.

Using ϕ_1, ϕ_2 and z as the generalized coordinates the equations of motion of the considered system can be derived using the Lagrange formalism [16]. Here we study the motion of the robot in the horizontal plane OXY only. The equations of the plane motion of the robot can be written as follows:

$$f_1(\phi_i, \dot{\phi}_i, \ddot{\phi}_i) = u_1 + p_1 + u_3 + p_3 \tag{1}$$

$$f_2(\phi_i, \dot{\phi}_i, \ddot{\phi}_i) = u_2 + p_2 + b\,(\phi_i)\,(u_3 + p_3) \tag{2}$$

The functions $f_1(\phi_i, \dot{\phi}_i, \ddot{\phi}_i)$, $f_2(\phi_i, \dot{\phi}_i, \ddot{\phi}_i)$ and $b(\phi_i)$ are determined by means of the Lagrange operator [16].

The inherent dynamics of the passive drives $p_i(t)$ of the robot can be modeled in different ways, e.g. by the following differential constraints:

$$p_i(t) + k_i\phi_i(t) + c_i\dot{\phi}_i(t) = 0,\ i = 1, 2, 3 \tag{3}$$

where k_i and c_i are the constant spring and the damper coefficients of the i-th passive drive.

The differential equations (1)–(3) describe the motion of semi-passively controlled SCARA-like robot in the horizontal plane OXY.

An analysis of the robot structure (Figure 1) shows that the position, the velocity and the acceleration of the robot's end-effectors (points C and D) can be uniquely determined by specification of the functions $l(t)$ and $\phi_3(t)$, (the cylindrical coordinates of the point C). Using this fact the cyclic pick-and-place operations of the robot can be specified by the following conditions:

$$f(0) = f(T) = f_0,\ f(\tau) = f_\tau,\ \dot{f}(0) = \dot{f}(\tau) = \dot{f}(T) = 0 \tag{4}$$

$$m_C = \begin{cases} m_C^0 & ,0 \le t \le \tau \\ 0 & ,\tau < t \le T \end{cases},\ m_D = \begin{cases} m_D^0 & ,0 \le t \le \tau \\ 0 & ,\tau < t \le T \end{cases} \tag{5}$$

where f is a vector-function having as its components $\phi_3(t)$ and $l(t)$; f_0, f_τ are pairs of numbers that determine the initial (and final) and intermediate states of the end-effector, τ, T, m_C^0, m_D^0 are the duration of transferring the loads, the duration of the pick-and-place operation, and the mass of the loads at the end-effectors C and D, respectively. The considered pick-and-place operation of the robot depends on the following parameters:

$$\phi_{30}, l_0, \phi_{3\tau}, l_\tau, \tau, T, m_C^0, m_D^0 \tag{6}$$

which are at disposal as input data.

As follows from the description of the considered robot the number of degrees-of-freedom is less than the number of powered drives. It means that the robot in question is an overactuated mechanical system, i.e. a system with dynamic redundancy.

The energy-optimal control problem for the considered robot can be formulated as follows.

Problem 1. The value of the structural parameters $l_i, r_i, m_i, J_i,\ i = 1, 2, 3$ of the robot and the input data (6) of the pick-and-place operation (4), (5) are given.
Determine the motion:

$$\phi_1(t),\ \phi_2(t), \quad t \in [0, T] \tag{7}$$

the control torques of the powered drives

$$u_i(t), \quad i = 1, 2, 3, \quad t \in [0, T] \tag{8}$$

and the spring and damper parameters

$$k_i,\ c_i, \quad i = 1, 2, 3 \tag{9}$$

which altogether minimize the functional

$$E_2 = \int_0^T \sum_{i=1}^{3} u_i^2(t)\mathrm{d}t \tag{10}$$

subject to the differential constraints (1)–(3), boundary conditions (4), and the following restrictions:

$$0 \le k_i \le k_i^{\max}, \quad 0 \le c_i \le c_i^{\max}, \qquad i = 1, 2, 3 \tag{11}$$

$$u_i^{\min} \le u_i(t) \le u_i^{\max}, \quad i = 1, 2, 3 \tag{12}$$

Here the parameters $k_i^{\max}$, $c_i^{\max}$, $u_i^{\min}, u_i^{\max}, i = 1, 2, 3$ are supposed to be given in advance.

In addition to the functional (10) the following functionals have been used in this paper:

$$E_1 = \int_0^T \sum_{i=1}^{3} |u_i(t)| \, \mathrm{d}t \tag{13}$$

$$E_3 = \int_0^T \sum_{i=1}^{3} \left| u_i(t)\dot{\phi}_i \right| \mathrm{d}t \tag{14}$$

In several cases [3], [4], [5] the functionals (10), (13), (14) can be used to estimate the energy consumption for the controlled motion of mechanical systems. The quality of the controlled motion can also be measured by other criteria such as time consumption, end-effector trajectory accuracy, etc. We have considered the time-optimal control of the semi-passively actuated SCARA-like robot in a separate study [17], [7].

The solution of **Problem 1** is called the energy-optimal control process of the SCARA-like robot for the given pick-and-place operation.

3 Methodology

Several methods can be used to solve **Problem 1**. Most of them are based on either the direct or the indirect approach [10], [1]. Using the direct approach the control variables are parameterized leading to a direct search in the parameter space. By applying Pontryagin's maximum principle [21] we can derive the conditions that must be satisfied by an optimal control. These conditions lead to a two-point boundary-value problem that must be solved to determine the optimal control. This is the indirect approach.

An estimate of the minimum value of the functional under consideration can be obtained by constructing an admissible set of motions of the system and respective control torques by approximation of the joint trajectories and the redundant control torques, and then solving the resulting nonlinear programming problem. Here this approach is adopted using a polynomial-Fourier series based method to convert the optimal control problem into a nonlinear programming problem [8], [18]. This method and a similar method using smoothing splines rather than Fourier series have been successfully used by researchers to study dynamics and control of bipedal locomotion systems [5]. Henceforth, **Problem 1** is converted into the following nonlinear programming problem:

$$E(\boldsymbol{x}) \to \min_{\boldsymbol{x}}, \; \boldsymbol{c}(\boldsymbol{x}) \leq 0 \tag{15}$$

Here the functions E and $\boldsymbol{c}$ are determined by means of one of the functionals (10), (13), (14) the differential constraints (1)–(3) and the restrictions (11), (12); $\boldsymbol{x}$ is a vector of variable parameters.

For the considered robotic system problem (15) is solved in two stages. In the first stage we solve the modeling task for the robot, i.e. we calculate the admissible motion $\phi_1(t), \phi_2(t)$ and the control torques $u_i(t)$, $p_i(t), i = 1, 2, 3$, that satisfy the equations (1)–(3) and the boundary conditions (4). In the second stage the respective constrained nonlinear optimization problem is considered based on one of the functionals (10), (13), (14) and restrictions (11), (12).

The modelling task for the robot includes:

- solution of inverse kinematics for the initial, intermediate and final phase states of the robot end-effectors;
- path planning by means of polynomial and Fourier series approximation of the robots generalized coordinates;
- regularization of the dynamic redundancy by polynomial and Fourier series approximation of the redundant control torque;

- and, finally, solution of the inverse dynamics problem for the remaining control torques.

The outline of the above approach is supported by the following details.

For the input parameters f_0, f_τ of the pick-and-place operation (4) the inverse kinematics problem is solved for the moments of time $t = 0,\ t = \tau$ and $t = T$. As a consequence the following parameters are calculated:

$$\phi_{n0} = \phi_n(0), \dot{\phi}_{n0} = \dot{\phi}_n(0),\ \phi_{n\tau} = \phi_n(\tau), \dot{\phi}_{n\tau} = \dot{\phi}_n(\tau),\ n = 1, 2 \tag{16}$$

The path planning problem is solved using an approximation of the generalized coordinates of the robot by a sum of a fifth order polynomial and a finite Fourier series given by the following formula:

$$q(t) = \sum_{j=0}^{5} C_{q\nu j}(t - t_0)^j + \sum_{k=1}^{N_{q\nu}} [a_{q\nu k}\cos(k\omega_\nu(t - t_0)) + b_{q\nu k}\sin(k\omega_\nu(t - t_0))] \tag{17}$$

for both intervals of time $t \in [0, \tau]$ and $t \in [\tau, T]$. Here $q = (\phi_1, \phi_2),\ \nu = 1,\ t_0 = 0$, and $\omega_\nu = 2\pi/\tau$ for $t \in [0, \tau]$, and $\nu = 2,\ t_0 = \tau$, and $\omega_\nu = 2\pi/(T - \tau)$ for $t \in [\tau, T]$; $N_{q\nu}$ are given positive integers. Taking into account the conditions (4) and (16), then from formula (17) follows that the parameters

$$C_{q\nu 4}, C_{q\nu 5}, a_{q\nu k}, b_{q\nu k},\ k = 1, 2, ..., N_{q\nu},\ \nu = 1, 2,\ q = (\phi_1, \phi_2) \tag{18}$$

can serve as optimization variables.

The dynamic redundancy of the considered robot is regularized by polynomial and Fourier series approximation of one of the control torques:

$$u_r(t) = \sum_{j=0}^{5} C_{r\nu j}(t - t_0)^j + \sum_{m=1}^{N_{r\nu}} [a_{r\nu m}\cos(m\omega_\nu(t - t_0)) + b_{r\nu m}\sin(m\omega_\nu(t - t_0))] \tag{19}$$

for both intervals of time $t \in [0, \tau]$ and $t \in [\tau, T]$. Here $r \in (1, 2, 3),\ \nu = 1,\ t_0 = 0$, and $\omega_\nu = 2\pi/\tau$ for $t \in [0, \tau]$, and $\nu = 2,\ t_0 = \tau$, and $\omega_\nu = 2\pi/(T - \tau)$ for $t \in [\tau, T]$; $N_{r\nu}$ are given positive integers. The parameters

$$C_{r\nu j}, a_{r\nu m}, b_{r\nu m},\ m = 1, 2, ..., N_{r\nu}, j = 0, 1, ..., 5,\ \nu = 1, 2 \tag{20}$$

can serve as additional optimization variables.

The inverse dynamics problem can then be solved by using the differential constraints (1)–(3). This is the final step of solving the modelling task for the SCARA-like robot. The controlled motion of the robot is then determined and the value of all functionals (10), (13), (14) can be evaluated. The proposed approach gives no unique solution of the modelling task. The solution will depend on the value of the parameters (18) and (20).

For this particular study we will also assume that the functions $q(t)$ and $u_r(t)$ are smooth, i.e twice-continuously differentiable. This requirement is enforced with the following constraints:

$$\begin{aligned} &\ddot{q}(0) = \ddot{q}(T) = \ddot{q}_0,\ \ddot{q}(\tau) = \ddot{q}_\tau,\ q = (\phi_1, \phi_2) \\ &u_r(0) = u_r(T) = u_{r0},\ u_r(\tau) = u_{r\tau}, \\ &\dot{u}_r(0) = \dot{u}_r(T) = \dot{u}_{r0},\ \dot{u}_r(\tau) = \dot{u}_{r\tau}, \\ &\ddot{u}_r(0) = \ddot{u}_r(T) = \ddot{u}_{r0},\ \ddot{u}_r(\tau) = \ddot{u}_{r\tau} \end{aligned} \tag{21}$$

and consequently the following parameters can be added to the set of optimization variables:

$$\ddot{q}_0, \ddot{q}_\tau, u_{r0}, u_{r\tau}, \dot{u}_{r0}, \dot{u}_{r\tau}, \ddot{u}_{r0}, \ddot{u}_{r\tau},\ q = (\phi_1, \phi_2) \tag{22}$$

In the second stage of the methodology for solving **Problem 1** we use the external penalty functions approach to reduce the nonlinear programming problem (15) into the following constrained nonlinear finite dimensional parameter optimization problem:

$$\tilde{E}(\boldsymbol{x}) = E(\boldsymbol{x}) + \sum_{i=1}^{3} \mu_i g_i(\boldsymbol{x}) \to \min_{\boldsymbol{x}}, \tag{23}$$

$$g_i(\boldsymbol{x}) = \int_0^T ((u_i^{\min} - u_i(t))_+ + (u_i(t) - u_i^{\max})_+ \, \mathrm{d}t,$$

$$y_+ = \begin{cases} y^2, & y \geq 0, \\ 0, & y < 0, \end{cases}, \ \boldsymbol{x}^{\min} \leq \boldsymbol{x} \leq \boldsymbol{x}^{\max}$$

where $\mu_i > 0$ are given numbers.

Here the functions E and g_i are determined by means of one of the functionals (10), (13), (14), the differential constraints (1)–(3) and the restrictions (12); $\boldsymbol{x} = (k_i, c_i, \ddot{q}_0, \ddot{q}_\tau, a_{q\nu k}, b_{q\nu k},\ u_{r0},\ \dot{u}_{r0},\ \ddot{u}_{r0}, a_{r\nu m}, b_{r\nu m} : i = 1,2,3,\ \ j = 0,1,...,5,\ \ k = 1,2,..,N_{q\nu},\ \ m = 1,2,..,N_{r\nu},\ \ q = (\phi_1, \phi_2),\ \ \nu = 1,2)$ is the vector of independent optimization variables from (18), (20), (22) taking into account the constraints (21). To solve problem (23) we have used the sequential quadratic programming method SNOPT [13] implemented in the software package TOMLAB [15].

In the next three paragraphs we present the numerical results of the solution of several variants of **Problem 1** that have been obtained based on the above methodology. For the numerical solution of all these variants it is assumed that the links OA, AB and OD of the robots in question are homogeneous bars and that their centre of mass is located at the midpoint of the links. The length and mass of the links are as follows: $l_1 = 1$ m, $l_2 = 1.5$ m, $l_3 = 3$ m, $m_1 = 5$ kg, $m_2 = m_3 = 10$ kg. For the pick-and-place operation the following input data are used: $\phi_{30} = -\frac{\pi}{4}$, $l_0 = 2$ m, $\phi_{3\tau} = \frac{\pi}{3}$, $l_\tau = 1.5$ m, $\tau = 4$ s, $m_C^0 = 5$ kg, $m_D^0 = 0$ kg. The restrictions (12) are determined by $u_i^{\min} = -100$ Nm, $u_i^{\max} = 100$ Nm, $i = 1,2,3$.

4 Energy-Optimal Control of the Fully Actuated Robots

In this paragraph we present the solution of several energy-optimal control problems for the considered robot when one of the external actuators is absent. In this case the number of actuators equals to the number of degrees of freedom of the robotic system, i.e the robotic system is fully actuated. Consider the following optimal control problems:

Problem 2. The statement of this problem is the same as **Problem 1** but with the following additional constraints:

$$u_3(t) = 0, p_i(t) = 0,\ i = 1,2,3,\ t \in [0,T] \tag{24}$$

We will denote the solution of **Problem 2** by $\mathcal{F}_2^*$ and call $\mathcal{F}_2^*$ the energy-optimal control process of the fully actuated SCARA-like robot with anthropomorphic placement of external actuators.

Problem 3. This is **Problem 1** but with the only additional constraint:

$$u_3(t) = 0,\ t \in [0,T] \tag{25}$$

The solution of **Problem 3** will be denoted by $\mathcal{F}_3^*$ and called the energy-optimal control process of the fully actuated semi-passively controlled SCARA-like robot with anthropomorphic placement of external actuators.

Problem 4. The statement of this problem follows from the statement of **Problem 1** by adding the constraints:

$$u_2(t) = 0, p_i(t) = 0,\ i = 1,2,3,\ t \in [0,T] \tag{26}$$

The solution of **Problem 4** is denoted by $\mathcal{F}_4^*$ and we will call $\mathcal{F}_4^*$ the energy-optimal control process of the fully actuated SCARA-like robot with base-placement of external actuators.

Problem 5. This is **Problem 1** but with the only additional constraint:

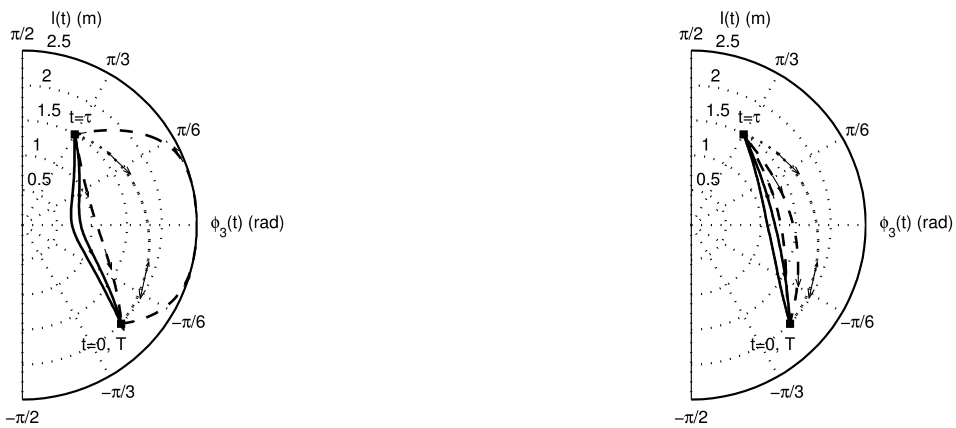
$$u_2(t) = 0,\ \ t \in [0, T] \tag{27}$$

Here the solution of this problem is denoted by $\mathcal{F}_5^*$ and it is called the energy-optimal control process of the fully actuated semi-passively controlled SCARA-like robot with base-placement of external actuators.

To perform the optimization required to solve **Problem 2-3** (**Problem 4-5**) we use as initial guess the control process of the robot determined by the formula (17) and the equations (1),(3) with zero values of the vector of variable parameters $\boldsymbol{x}$. The path of the end-effector C and the time histories of the control torques of the robot for the considered control process (initial guess) are depicted in Figure 2-3 by dotted curves. Some quantitative and qualitative results of obtained solutions of above formulated energy-optimal control problems for the fully actuated robots in question are presented in Table 4 and in Figures 2-4.

Table 1: Energy consumption and CPU-time usage (Pentium Mobile IV 1.6 GHz)

$\mathcal{F}_\#$	**1**	**2**	**3**	**4**	**5**	**6**	**7**
E_1	108	284	112	223	145	222	180
E_2^*	1100	13300	1960	8920	2680	5730	9600
E_3	71.8	243	88.9	123	76.5	136	151
k_1^*	5.31	—	8.36	—	0.0	—	0.0
k_2^*	0.0	—	0.0	—	0.0	—	0.44
k_3^*	64.0	—	61.2	—	50.8	—	57.7
c_1^*	0.0	—	0.0	—	0.0	—	0.0
c_2^*	0.0	—	1.52	—	17.0	—	34.3
c_3^*	2.86	—	—	—	0.0	—	0.0
CPU time (min)	48	11	14	12	15	27	180

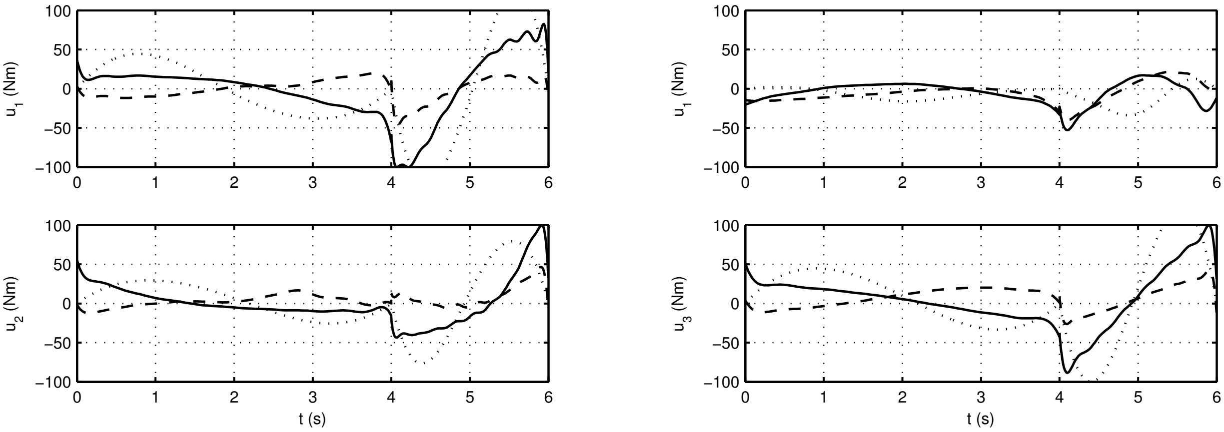


(a) Anthropomorphic placement of external actuators (**Problem 2-3**)

(b) Base-placement of external actuators (**Problem 4-5**)

Figure 2: The path of the end-effector C of the robot for the energy-optimal control processes of the fully actuated SCARA-like robot. ··· Initial guess, —— Active control, - - - Semi-passive control.

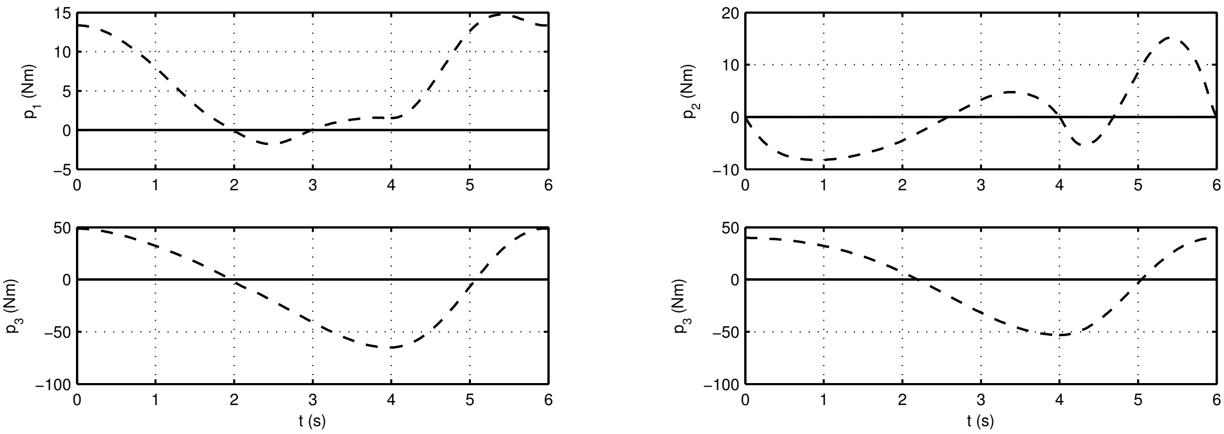
The data in the column $j, (j = 2, 3, 4, 5)$ of Table 4 correspond to the determined energy-optimal control process $\mathcal{F}_j^*$ of the fully actuated robotic system. In addition to the obtained minimal value E_2^* of the functional (10) the values of the cost functions (13) and (14) are also calculated and presented in Table 4 for the control processes $\mathcal{F}_j^*$. The control processes used as initial guess for solving **Problem 2-3** (**Problem 4-5**) are characterized by the following value of the functionals (10), (13), (14): $E_1 = 442, E_2 = 27100, E_3 = 181, (E_1 = 307, E_2 = 17100, E_3 = 215)$. Here and below all dimensions are given in SI units.



(a) Anthropomorphic placement of external actuators (**Problem 2-3**)

(b) Base-placement of external actuators (**Problem 4-5**)

Figure 3: The control torques of external actuators $u_i(t)$ for the energy-optimal control processes of the fully actuated SCARA-like robot. ··· Initial guess, —— Active control, - - - Semi-passive control.



(a) Anthropomorphic placement of external actuators (**Problem 2-3**)

(b) Base-placement of external actuators (**Problem 4-5**)

Figure 4: The control torques of internal actuators $p_i(t)$ for the energy-optimal control processes of the fully actuated semi-passively controlled SCARA-like robot.

5 Energy-Optimal Control of the Overactuated Robots

In this paragraph we present some numerical results of the solution of the energy-optimal control problems for the considered overactuted robot, namely the solution of **Problems 6,1**. The former one differ from **Problem 1** only due to the imposed additional constraints:

$$p_i(t) = 0,\ i = 1, 2, 3,\ t \in [0, T] \tag{28}$$

The solution of **Problem 1** we will denote by $\mathcal{F}_1^*$ and call the energy-optimal control process of the overactuated semi-passively controlled robot. We will denote the solution of **Problem 6** by $\mathcal{F}_6^*$ and call it the energy-optimal control process of the overactuated actively controlled SCARA-like robot.

To perform the optimization required to solve **Problems 6,1** we use as initial guess the control process of the robot determined by the formulas (17),(19) for $r = 3$ and the equations (1),(3) with zero values of the vector of variable parameters $\boldsymbol{x}$.

Some quantitative and qualitative results of obtained solutions of the energy-optimal control problems for the overactuated robots in question are presented in Table 4 and in Figures 5-7.

In Table 4 the column $j,\ j = 1, 6$ present data corresponding to the obtained energy-optimal control processes $\mathcal{F}_1^*$ and $\mathcal{F}_6^*$, respectively. The control process used as initial guess for solving **Problems 6,1** are characterized by the following value of the functionals (10), (13), (14): $E_1 = 442, E_2 = 27100, E_3 = 181$.

The path of the end-effector and the time-histories of the control torques of the overactuted robots for the determined solution of **Problem 6,1** are shown in Figures 5-7. Here the dashed curves correspond to energy-optimal control process $\mathcal{F}_1^*$ while the solid curves correspond to energy-optimal control process $\mathcal{F}_6^*$. For comparison, the kinematic and dynamic characteristics of the control process of the robot used as initial guess for the optimization procedure are also depicted in Figure 5-6 by dotted curves.

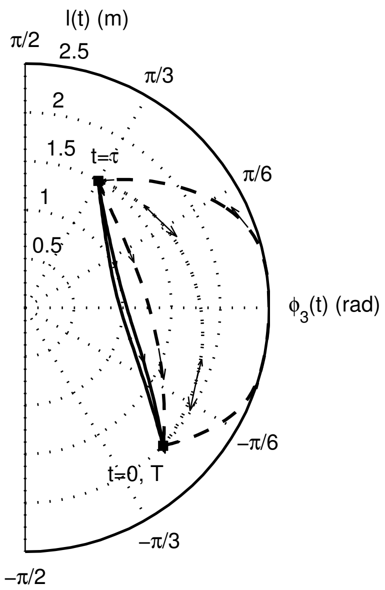


Figure 5: The path of the end-effector C of the robot for the energy-optimal control processes of the overactuated SCARA-like robots. · · · Initial guess, —— Active control (**Problem 6**), - - - Semi-passive control (**Problem 1**).

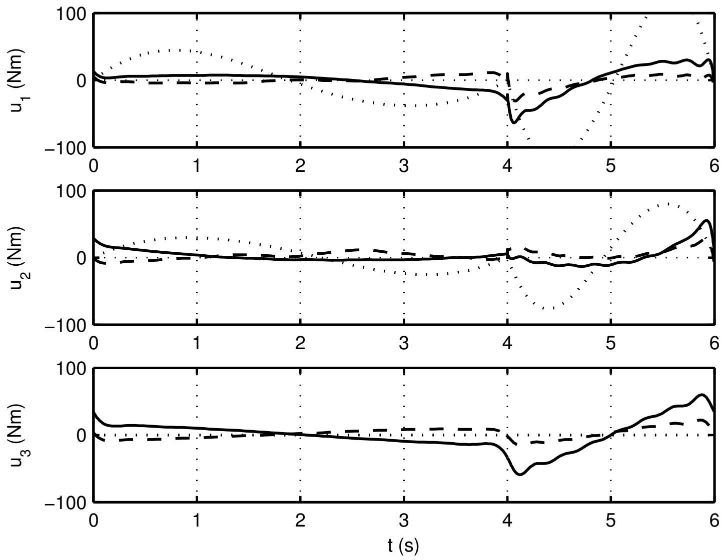


Figure 6: The control torques of external actuators $u_i(t)$ for the energy-optimal control processes of the overactuated SCARA-like robots. ⋯ Initial guess, —— Active control (**Problem 6**), **- - -** Semi-passive control (**Problem 1**).

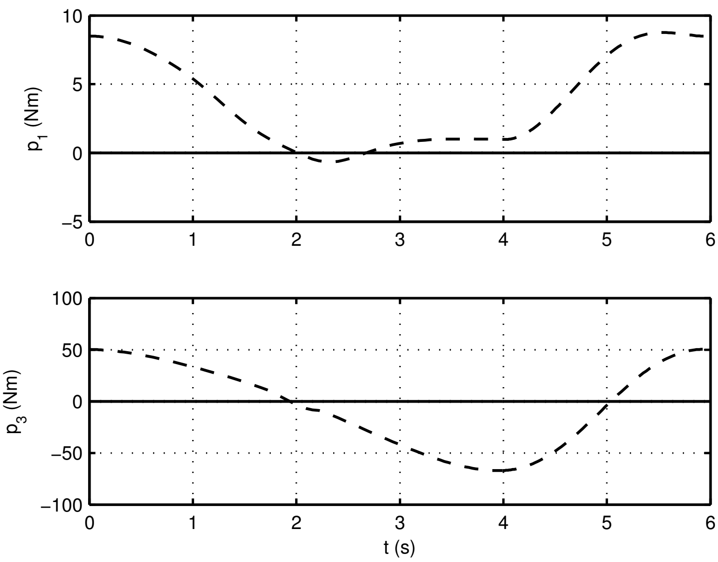


Figure 7: The control torques of internal actuators $p_i(t)$ for the energy-optimal control process of the overactuated semi-passively controlled SCARA-like robot (**Problem 1**).

6 Energy-Optimal Control of the Underactuated Robot

In contradiction to the previous paragraph were the number of external actuators of the considered robotic system exceeded the number of degrees of freedom we study here the energy-optimal control problems for the SCARA-like manipulator robot with less external actuators than the number of degrees of freedom. For the underactuated robot the restrictions (12) are determined by $u_1^{\min} = -100$ Nm, $u_1^{\max} = 100$, $u_2^{\min} = u_2^{\max} = u_3^{\min} = u_3^{\max} = 0$ Nm.

Problem 7. This is **Problem 1** with the following additional constraints:

$$u_2(t) = u_3(t) = 0, \ t \in [0, T] \tag{29}$$

The solution of **Problem 7** will be denoted $\mathcal{F}_7^*$ and called the energy-optimal control process of the underactuated semi-passively controlled SCARA-like robot.

In this paragraph we present the numerical results of the solution of **Problem 7** that have been obtained based on the above methodology.

To perform the optimization required to solve **Problem 7** we use as initial guess the control process of the robot determined by formula (17) and the equations (1),(3) with zero values of the vector of variable parameters $\boldsymbol{x}$. The path of the end-effector C and the time-histories of the control torques of the underactuated robot for the considered control process (initial guess) are depicted in Figures 8-9 by dotted curves. Some quantitative and qualitative results of obtained solutions of energy-optimal control problem for the underactuated robot in question are presented in Table 4 and in Figures 8-10.

The data of the column 7 correspond to the determined energy-optimal control process of the underactuated robotic system $\mathcal{F}_7^*$. In addition to the obtained minimal value E_2^* of the functional (10) the values of the cost functions (13) and (14) are also calculated and presented in Table 4 for the energy-optimal control process $\mathcal{F}_7^*$. The control process used as initial guess for solving **Problem 7** is characterized by the following value of the functionals (10), (13), (14): $E_1 = 442, E_2 = 27100, E_3 = 181$.

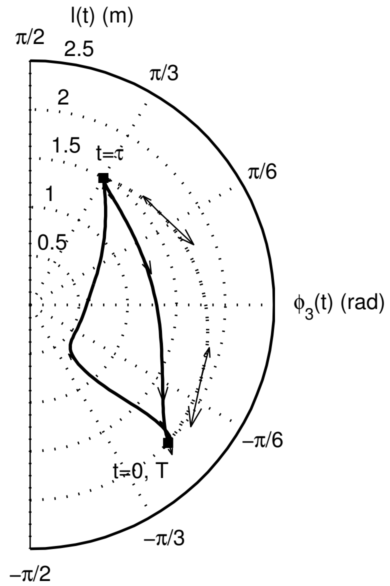


Figure 8: The path of the end-effector C of the robot for the energy-optimal control process of the underactuated semi-passively controlled SCARA-like robot (**Problem 7**).

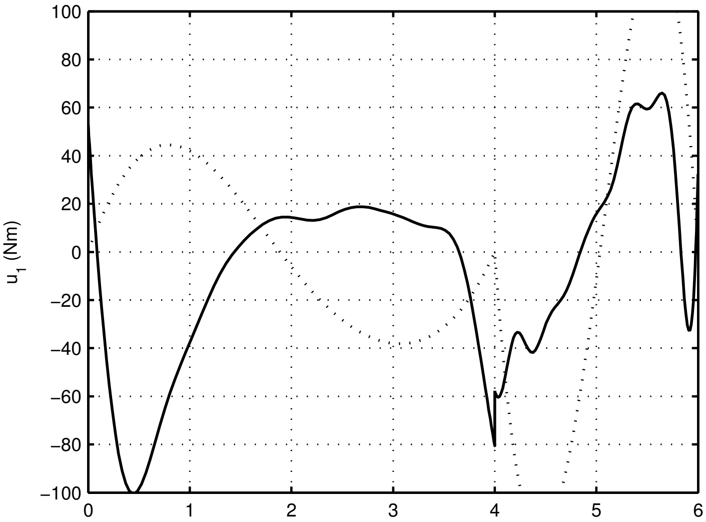


Figure 9: The control torque of external actuator $u_1(t)$ for the energy-optimal control process of the underactuated semi-passively controlled SCARA-like robot (**Problem 7**).

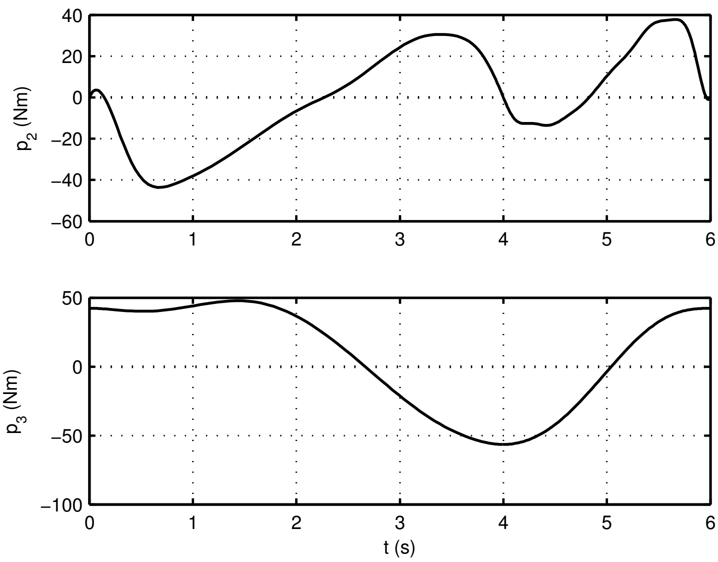


Figure 10: The control torques of internal actuators $p_i(t)$ for the energy-optimal control process of the underactuated semi-passively controlled SCARA-like robot (**Problem 7**).

7 Analysis and Discussion

Using our methodology we have successfully designed the energy-optimal control processes for the fully actuated and overactuated SCARA-like robots with and without unpowered spring-damper-like drives. Energy-optimal control problems for the underactuated robots were also considered although the proposed methodology turned out to be computationally less effective for this degree of actuation. The optimization problems obtained using our methodology for the underactuated robots are inherently ill-conditioned due to the large penalty parameters required to obtain admissible control processes with only one non-zero control torque. The ill-conditioned optimization problem is solvable for the underactuated semi-passively controlled robot (**Problem 7**) but the solution process was very demanding in terms of CPU-time usage, see Table 4. For all the optimization problems stated in this paper the CPU-time usage ranged from 11 to 180 minutes using a standard laptop computer (Table 4). The number of optimization variables varied from 60 to 100 depending on the variant of the energy-optimal control problem.

The core of the proposed methodology is a conversion of the original energy-optimal control problem to a nonlinear programming problem by a polynomial-Fourier series approximation of the independently varying functions. The convergence to a near optimal solution is confirmed empirically by appending another term in the Fourier series approximation, i.e. increase $N_{q\nu}$ and $N_{r\nu}$ in (17) and (19), to the previous solution and re-execute the constrained nonlinear optimization algorithm with the previous solution as initial guess. This procedure is continued until the value of the performance index converges, indicating that the optimal solution has been reached. The augmented energy consumption $\tilde{E}$ (formula (23)) is monotonically decreasing for increasing number of terms in the Fourier series for all studied variants of the energy-optimal control problem. The energy consumption measured by E_2 (formula (10)) versus number of terms in the Fourier- series expansion is depictured in Figure 11. The energy consumption of the control processes used as initial guess for the convergence study and the energy-optimal control problems studied in this paper are included in the graph as the energy consumption for $N_{q\nu} = N_{r\nu} = 0$, i.e. polynomial approximation only of the independently varying functions.

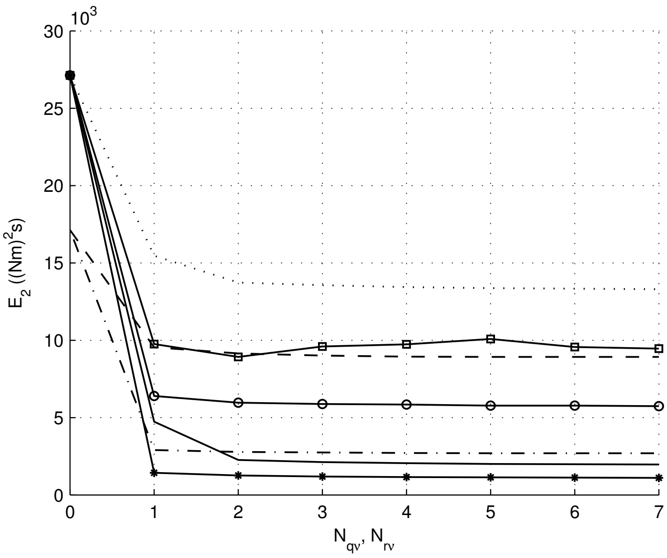


Figure 11: Convergence of optimal energy consumption for increasing number of terms in the Fourier series approximations of the independently varying functions. $\cdots$ (**Problem 2**), —— (**Problem 3**), **- - -** (**Problem 4**), $- \cdot$ (**Problem 5**), **-○-** (**Problem 6**), $-\!\ast\!-$ (**Problem 1**), –□– (**Problem 7**)

For the fully and the overactuated robots only 3-4 terms in the Fourier series expansion of independently varying functions are required for convergence.

Analysis of the numerical results in Table 4 gives that the energy consumption measured by the functional E_2 (formula (10)) is significantly less for the energy-optimal control processes with semi-passive control ($\mathcal{F}_3^*$, $\mathcal{F}_5^*$, $\mathcal{F}_1^*$) compared to the energy consumption of the corresponding energy-optimal control processes with only active control ($\mathcal{F}_2^*$, $\mathcal{F}_4^*$, $\mathcal{F}_6^*$), respectively. The utilization of semi-passive control instead of active control reduced the energy consumption by 80% for both the fully actuated robot with anthropomorphic placement of the external actuators ($\mathcal{F}_3^*$ vs. $\mathcal{F}_2^*$) and for the overactuated robot with an additional external actuator ($\mathcal{F}_1^*$ vs. $\mathcal{F}_6^*$). For the fully actuated robot with base-placement of the external actuators the reduction in energy consumption due to semi-passive control is less in percentage, approximately 70% ($\mathcal{F}_5^*$ vs. $\mathcal{F}_4^*$). For the underactuated robot semi-passive control turned out to be even more critical. Using our methodology we were not able to find an admissible control process for the underactuated robot with only active control (**Problem 7** with the additional constraints $p_i(t) = 0,\ i = 1, 2, 3,\ t \in [0, T]$). In this sense the internal actuators made the difference between a controllable and an uncontrollable robotic system.

By exploration of the dynamic redundancy for the considered robotic system the energy consumption of the energy-optimal control processes for the overactuated robots ($\mathcal{F}_6^*$, $\mathcal{F}_1^*$) are at least 66% less compared to the energy consumption of the corresponding energy-optimal control processes of the fully actuated robots with the same type of control ($\mathcal{F}_2^*$,$\mathcal{F}_4^*$ and $\mathcal{F}_3^*$,$\mathcal{F}_5^*$), respectively.

Minimizing the energy consumption measured by functional (10) with restrictions on the control torques (12) it is reasonable to assume that the path of the end-effector C for the solution of **Problem 1** and its variants will reduce the total effective inertia of the robotic system about the OZ axis. This can be confirmed by analyzing the path obtained for the energy-optimal control processes of the robots with only active control. For all energy-optimal control problems with active control studied in this paper the obtained optimal path is closer to the base than the path given by the initial guesses used in the optimization (Figure 2(a), 2(b), 5). However, due to the restrictions on the control torques and the given duration of the pick-and-place operation the value of the polar coordinate $l(t)$ of the end-effector that moves along the optimal path is limited below.

The time histories of the obtained optimal control torques for the energy-optimal control processes with active control, (Figures 3, 6) indicate that most of the energy spent to control the goal-directed motion of the robot is consumed in the return phase of the pick-and-place operation. The total effective inertia about the OZ axis is reduced in the return phase because the grip has released the load at the intermediate position but this can not compensate the quicker motion required to complete the return phase in the relatively short return-time specified by the used input data. For the energy-optimal control processes with semi-passive control ($\mathcal{F}_3^*, \mathcal{F}_5^*, \mathcal{F}_7^*$) the energy consumption are more or less evenly distributed between the two phases of the pick and place operation. This is due to the parameters of the internal drives which are mainly adjusted by the optimization procedure to decrease the energy consumption in the return phase.

For all optimal control processes under consideration with semi-passive control the optimal control torques of the unpowered actuators are dominated by a spring torque applied at the link OD. This spring-damper-like drive, with optimal parameters, is relatively stiff resulting in a relatively large torque acting on the link OD during the loading phase of the operation, see k_3^* in Table 4 and Figures 4,7,10). This dominating torque causes the fully actuated robot with anthropomorphic placement of the powered actuators and the overactuated robots to actually maximize the inertia during a significant part of the loading phase to balance the large torque applied at the link OD. For the fully actuated robot with base-placement of the powered actuators this is not possible because of the lack of a powered actuator acting on link OA which seems necessary to accomodate this kind of motion primitive for the studied pick-and-place operation.

The closed-loop kinematic chain of the SCARA-like robot gives the possibility to relocate the powered actuators to the base. This is desirable because the relocation can decrease the total effective inertia of the system. For the fully actuated actively controlled robots (**Problem 2** and **Problem 4**) the relocation of the powered drives to the base decreased the energy consumption without taking the inertia of the drives into account. However, the advantage of base-placement of the powered drives is not obvious for the robots under study with semi-passive control. The fully actuated semi-passively controlled robot with anthropomorphic placement of the drives has actually lower energy consumption with this type of actuation as discussed above. Using more advanced unpowered actuators it is possible that base-placement turns out to be a preferable choice also in combination with semi-passive control.

In the comparative study above we measured the energy consumption using a quadratic cost function (10) for all variants of the energy-optimal control problem. However, the energy consumption measured by the other cost functions (formulae (13), (14)) and the maximum torques of the optimal control processes are also less than the energy consumption and maximum torques obtained for the corresponding initial guesses used in the optimization.

By analyzing the curves in Figure 3(a), 3(b) and 6 we can study the character of the torques and estimate the maximum torque required for the obtained energy-optimal control processes. The maximum torque is decreased by a factor up to 2 due to semi-passive control for the fully and the overactuated robots. The possibility to redistribute the control torques using an additional drive is also decreasing the maximum torque required by a factor up to 2 for the overactuated robots. The resulting torques of the external actuators for all optimal control processes are smooth for both phases of the pick and place operation. They have relatively few switchings between positive and negative torque values but for several of the obtained energy-optimal control processes their extreme values are on the bounds. The control torques of the unpowered actuators are comparable in magnitude to the control torques of the powered actuators.

8 Conclusions

In this paper we have presented a suitable methodology to solve energy-optimal control problems for two degrees of freedom closed-loop robots with different degree and type of actuation. The effectiveness of the methodology is demonstrated by a comparative study of the energetic, kinematic and dynamic characteristics of the energy-optimal control processes for several robot configurations executing the same pick and place operation.

The analysis of the obtained numerical results show that the energy consumption of the robot in question can be substantially reduced by incorporating optimally designed spring-damper-like drives into the structure of the robot. This statement is valid both for the fully actuated and the overactuated robots. For some of the robot configuration the kinematics of the energy-optimal control process changed dramatically with the incorporation of passive drives into the structure. Semi-passive control can also be used to improve the controllability of the underactuated robots.

The energy consumption of the motion of the robot in question can also be significantly decreased using overactuation. In addition to lower energy consumption we also decreased the maximum torque required by adding passive elements and by performing overactuation. The character of the optimal control torques and the parameters of the spring-damper-like drives are realistic possible to use in practise. Finally we can conclude that the results presented in this paper indicate that overactuation and semi-passive control can decrease the energy consumption and improve controllability for more general robotic systems undergoing cyclic motion.

9 Acknowledgement

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References

[1] F.A. Aliev and V.B. Larin. *Optimization of Linear Control Systems: Analytical Methods and Computational Algorithms.* Gordon and Breach Science Publishers, Amsterdam, 1998.

[2] H. Arai and S. Tachi. Position control system of a two degree of freedom manipulator with a passive joint. *IEEE Trans. Ind. Elec.*, pages 15–20, 1991.

[3] M. Athans, P. L. Falb, and R. T. Lacoss. Time-, fuel- and energy-optimal control of nonlinear norminvariant systems. *IEEE Trans. Automat. Contr.*, 8:196–202, 1963.

[4] V. Berbyuk. Multibody system modelling and optimization problems of lower limb prostheses. In D. Bestle and W. Schiehlen, editors, *IUTAM Symposium on Optimization of Mechanical Systems*, pages 25–32, Amsterdam, 1996. Kluwer.

[5] V. Berbyuk, A. Boström, B. Lytwyn, and B. Peterson. Optimization of control laws of bipedal locomotion systems. In Jorge A.C. Ambrósio and Werner O. Schiehlen, editors, *Advances in Computational Multibody Dynamics*, pages 713–728, Instituto Superior Técnico, Lisbon, 1999.

[6] V. Berbyuk, A. Boström, and B. Peterson. Modelling and Design of Robotic Systems Having Spring-Damper Actuators. In *Mechatronics 2000*, Atlanta, 2000. Elsevier.

[7] V. Berbyuk and M. Lidberg. Time-optimal control of semi-passively actuated closed-loop chain robots. In *Proc. 33rd ISR (International Symposium on Robotics)*, pages 221–226, Stockholm, 2002.

[8] V. YE. Berbyuk and M. V. Demidyuk. Parametric optimization in problems of dynamics and control of motion of an elastic manipulator with distributed parameters. *J. Mech. Solids*, 21(2):78–86, 1986.

[9] J. T. Betts. *Practical methods for optimal control using nonlinear programming.* Society for Industrial and Applied Mathematics, Philadelphia, 2001.

[10] J.T. Betts. Survey of numerical methods for trajectory optimization. *J. Guidance and Control*, 21(2):193–207, 1998.

[11] H. Cheng, G. F. Liu, Y. K. Yiu, Z. H. Xiong, and Z. X. Li. Advantages and dynamics of parallell manipulators with redundant actuation. In *2001 IEEE/RSJ Int. Conf. Int. Robots Syst. (Maui, Hawaii)*, pages 171–176, 2001.

[12] F. Daerden. *Conception and realization of pleated pneumatic artificial muscles and their use as compliant actuation elements.* PhD thesis, Vrije Universiteit Brussel, 1999.

[13] P. E. Gill, W. Murray, and M. A. Saunders. SNOPT: An SQP Algorithm for Large-Scale Constrained Optimization. *SIAM J. Optimization*, 12(4):979–1006, 2002.

[14] C. M. Gosselin. Parallel computational algorithms for the kinematics and dynamics of planar and spatial parallel manipulators. *ASME J. Dynamics Syst., Measur. Contr.*, 118:22–28, 1996.

[15] K. Holmström. The TOMLAB optimization environment in Matlab. *Advanced Modeling and Optimization*, 1(1):47–69, 1999.

[16] M. Lidberg and V. Berbyuk. Modeling of controlled motion of semi-passively actuated SCARA-like robot. In *Mechatronics 2000*, Amsterdam, 2000. Elsevier.

[17] M. Lidberg and V. Berbyuk. Time-optimal control of overactuated manipulator robots with closed-loop chain. In *Multibody Dynamics in Sweden II*, Lund, 2001.

[18] M. L. Nagurka and V. Yen. Fourier-based optimal control of nonlinear dynamic systems. *ASME J. Dynamics Syst. Measur. and Contr.*, 112:17–26, 1990.

[19] Y. Nakamura. *Advanced Robotics -Redundancy and Optimization.* Addison-Wesley, New York, 1991.

[20] Y. Nakamura and M. Ghodoussi. Dynamic computation of closed-link robot mechanism with nonredundant and redundant actuators. *IEEE Trans. Robotics and Automation*, 5:294–302, 1989.

[21] L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamrelidze, and E. F Mishchenko. *Selected works. 4, The Mathematical Theory of Optimal Processes.* Gordon and Breach, New York, 1986.

[22] W. O. Schiehlen and N. Guse. Power demand of actively controlled multibody systems, (DETC2001/VIB-21343). In *Proc. DETC'01.* ASME, 2001.

[23] W. Stadler. *Analytical Robotics and Mechatronics.* McGraw-Hill, New York, 1995.

[24] T. Tran and G. Kehl. Drive dimension and path preparation for parallel manipulators. In J. Adolfsson and J. Karlsén, editors, *Mechatronics 98*, pages 293–298. Elsevier, 1998.

[25] L-W Tsai. *Robot Analysis.* Wiley, New York, 1999.

[26] K.J. Waldron. Some thoughts on the design of power systems for legged vehicles. In *Advances in multibody systems and mechatronics.* Gerhard-Mercator-Universität, 1999.

[27] Y. Zhang, W. Gruver, and F. Gao. Dynamic simplification of three degree of freedom manipulators with closed chains. *Robotics Auton. Syst.*, 28:261–269, 1999.