Time-Optimal Control of Overactuated Manipulator Robots with Closed-Loop Chain

Mathias Lidberg[*] and Viktor Berbyuk[**]

[*]Department of Applied Mechanics
[**]Department of Machine and Vehicle Systems
Chalmers University of Technology, 412 96 Göteborg

Introduction

The development of new robotic systems with closed-loop chains and different types of actuation is currently one of the main activities of many research institutions and industrial manufacturers. Potential applications of closed-loop chain manipulators arise whenever there is a need for large structural stiffness or high performance dynamics, and when it is desirable to bring the actuators as close as possible to the base of the robot [1-3]. If a closed-loop robot has more actuated joints than degrees-of-freedom the joint torques are no longer uniquely determined. Such kind of robot is called an overactuated robot. To design effective control laws for overactuated closed-loop chain manipulators it seems reasonable to explore the inherent dynamics of the mechanical structure of the system and the optimal interaction between different kind of actuators [4, 5].

In this paper the time-optimal control of a new structure of manipulator robot is under study. The proposed robotic system has the following new features in comparison with the well-known SCARA robot. In addition to powered drives it comprises several unpowered (passive) spring-damper-like drives. An additional link has also been incorporated into the structure that gives the possibility to obtain a semi-passively actuated closed-loop chain robot. Emphasis is put on a study of the interaction between the controlling stimuli of the powered drives and the torques exerted by the unpowered drives needed to provide the time-optimal motion of the robots with different degrees of actuation.

Statement of the problem

Consider the manipulator robot depicted in Fig. 1. The robot comprises four links that are modeled by the rigid bodies OA, AB, OD and EC. There are one degree-of-freedom rotational joints at the points O and A, and translational joints at the point B. All joints are considered frictionless.

Let OXYZ be a fixed rectangular Cartesian coordinate system. It is assumed that the robot's links OA, AB and OD move in the horizontal plane OXY under the action of the torques $u_1(t)$, $u_2(t)$ and $u_3(t)$ applied to the links OA, AB, and OD, respectively. Under the action of the force $F(t)$ the link EC moves along the direction of the axes OZ. The controlling stimuli $u_i(t)$, $i = 1, 2, 3$ and $F(t)$ are exerted by the powered drives of the robot. The robotic system also comprises spring-damper actuators at joints O and A. The torques exerted by these actuators p_1 , p_2 and p_3 act on the links OA, AB and OD, respectively. They will be treated as the controlling stimuli of unpowered (passive) drives of the robot. Using ϕ_1, ϕ_2 and z as the generalized coordinates the equations of motion of the considered system can be derived by using the Lagrange formalism [5]. Here we study motion of the robot in the horizontal plane OXY only. The equations of the plane motion of the robot can be written as follows:

$$f_1(\phi_i,\dot{\phi}_i,\ddot{\phi}_i) = u_1 + p_1 + u_3 + p_3, \qquad f_2(\phi_i,\dot{\phi}_i,\ddot{\phi}_i) = u_2 + p_2 + b(\phi_i)(u_3 + p_3) \qquad (1)$$

The functions f_1 and f_2 are determined by means of the Lagrange operator [5].

The inherent dynamics of the passive drives of the robot can be modeled in different ways, e.g. by the differential constraints:

$$p_j + k_j(\theta_j - \theta_{0j}) + c_j\dot{\theta}_j = 0,\ j = 1,2,3 \qquad (2)$$

where k_j are the stiffness coefficients, c_j are the damping coefficients and θ_{0j} are the no-load angles of the torsional springs.

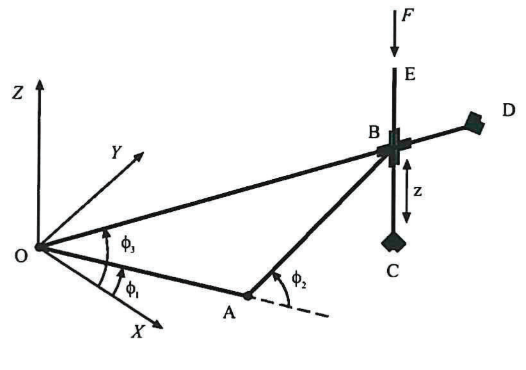

Figure 1: Manipulator Robot with Closed-Loop Chain

The differential equations that are determined by the formulas (1), (2) describe the controlled motion in the horizontal plane OXY of the semi-passively actuated SCARA-like robot.

An analysis of the robot structure (See Fig. 1) shows that the position, the velocity and the acceleration of the robot's end-effectors (points C and D) can be uniquely determined by specification of the cylindrical coordinates of the point C (functions $l(t)$, $\varphi_3(t)$ and $z(t)$). The set of cyclic pick-and-place operations of the robot can be given by the following conditions:

$$m_C = \begin{cases} m_{0C}, 0 \le t \le \tau \\ 0, \ \ \tau < t \le T \end{cases}, \qquad m_D = \begin{cases} m_{0D}, 0 \le t \le \tau \\ 0, \ \ \tau < t \le T \end{cases} \qquad (3)$$

$$\begin{aligned} f(0) &= f(T) = f_0 & f(\tau) &= f_\tau \\ \dot{f}(0) &= \dot{f}(T) = \dot{f}_0, & \dot{f}(\tau) &= \dot{f}_\tau \\ \ddot{f}(0) &= \ddot{f}(T) = \ddot{f}_0 & \ddot{f}(\tau) &= \ddot{f}_\tau \end{aligned} \qquad (4)$$

where $f(t)$ is a vector-function having as its components $\varphi_3(t)$ and $l(t)$ and $f_0, \dot{f}_0, \ddot{f}_0,\ f_\tau, \dot{f}_\tau, \ddot{f}_\tau$ are given parameters that determine the initial (final) and intermediate phase states of the end effector, τ, T, m_C, m_D are the duration of transferring of the loads, the duration of pick-and-place operation, and the loads at the end effectors C and D, respectively.

The time-optimal control problem for the considered SCARA-like robot can be formulated as follows.

Problem 1. The value of all structural parameters of the robot and the input data of the pick-and-place operation (3), (4) are given. Determine the motion $\varphi_i(t)$, the controlling stimuli of the powered drives $u_i(t)$, and the parameters of the unpowered drives of the robot k_i, c_i, i = 1,2,3, which provide the execution of the given pick-and-place operation for the minimal time T subject to the differential constraints (1), (2), the boundary conditions (3), (4), and the restrictions:

$$u_{MIN} \le u_i(t) \le u_{MAX}, i = 1,2,3, t \in [0,T] \qquad (5)$$

$$0 < \tau < T \qquad (6)$$

Here the parameters u_{MIN} and u_{MAX} are given in advance and parameter τ is determined during the solution of the *Problem 1.*

Methodology

There are various approaches to solve *Problem 1*. Firstly, one can, using Pontryagins maximum principle derive the conditions that must be satisfied by an optimal control and its associated state-costate equations. These conditions lead to a two-point boundary-value problem that must be solved to determine the optimal control. Secondly, an estimate of the minimum value of the functional under consideration can be obtained by constructing or estimating the admissible set of the dynamical system and solving the corresponding non-linear programming problem. Here the later approach is adopted using a Fourier-based method to convert the optimal control problem into an algebraic nonlinear programming problem [5]. This approach and a similar approach using smoothing splines rather than Fourier series have been used successfully by researchers to study bipedal locomotion [6,7].

Based on polynomial-Fourier approximation of the generalized coordinates and with an approximation of one of the joint torques (in case of overactuation) the controlling process of the manipulator robot can be calculated [5]. Using the external penalty functions approach *Problem 1* is converted to the respective non-linear programming problem (*Problem 2*). To solve *Problem 2* we have used a sequential quadratic programming method implemented in the software package TOMLAB [8].

Results

The quantitative and qualitative results presented in Table 1, Fig. 2 and Fig. 3 represent the obtained solutions of two time-optimal control problems for the manipulator in question. The first row of Table 1 and the thin solid curves in the figures shows the time durations (T, τ), energy consumption (E_2), path of the end-effector (Fig. 2) and respective controlling torques (Fig. 3) for the fully actuated robot. These data were used as initial guess for solution of the considered time-optimal control problems. The data in row 2 and the thick solid curves in the figures correspond to the obtained solution of *Problem 1* for the fully actuated robot. The third row of Table 1 and the dotted curves in the figures correspond to the obtained results for the same problem but for the overactuated manipulator.

To solve *Problem 1* we minimized the objective function:

$$T_1 = T + \int_0^T \sum_{i=1}^{3} \mu_i g_i dt \rightarrow \min$$

$$g_i = \int_0^T ((u_{MIN} - u_i(t))_+ + (u_i(t) - u_{MAX})_+)dt \qquad (7)$$

$$y_+ = \begin{cases} y^2, y \geq 0, \\ 0, y < 0, \end{cases}$$

where $\mu_i > 0$ are given numbers.

The objective function (7) were minimized using 34 and 56 optimization variables in case of time-optimal control of the fully actuated and the overactuated manipulator robot, respectively. Using the suggested methodology and optimization algorithm the selected objective function (7) is substantially decreased (Table 1). The duration of the task is decreased the most in case of overactuation. The decreased duration of the operation is achieved at the expense of higher energy consumption indicated in the table by the value of E_2 determined by the expression:

$$E_2 = \int_0^T \sum_{i=1}^{3} u_i^2(t)dt \qquad (8)$$

	T^*	τ^*	E_2
1	5	3	17500
2	3.57	1.79	47900
3	2.90	1.45	57000

Table 1: Time duration and energy consumption

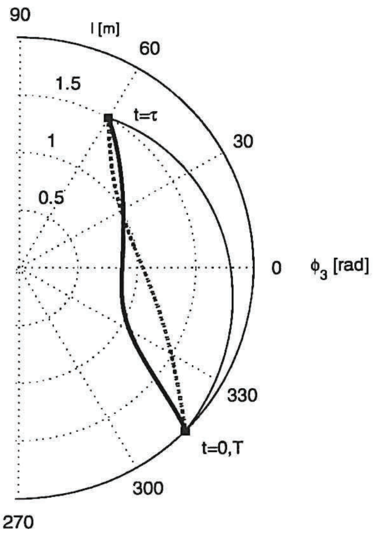

Figure 2: The path of the end-effector C

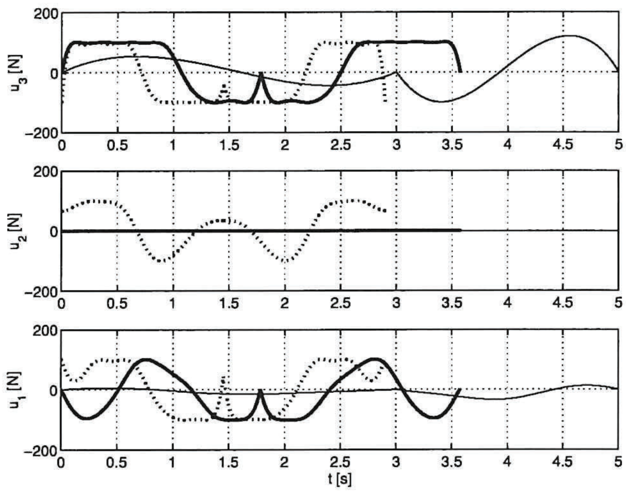

Figure 3: Controlling torques $u_i(t)$

The obtained results can help to estimate the advantage of overactuation compared to full actuation with respect to time-optimal control of the considered manipulator robot.

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